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Research article

A generalized effective neurosophic soft set and its applications

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Abstract: We introduce the concept of an effective neutrosophic soft set, which aims to capture the influence on three independent membership functions representing degrees of truth (T), indeterminacy (I) and falsity (F). We go further by presenting a generalization of the effective neutrosophic soft set, which includes the incorporation of a degree to signify the potential for an approximate value-set. This enhancement contributes to improved efficiency and realism in the concept. Notably, this innovative approach leverages the strengths of both the generalized neutrosophic set and the effective neutrosophic soft set. The subsequent sections delve into fundamental operations on the generalized effective neutrosophic soft set, providing clarity through illustrative examples and propositions. Furthermore, we demonstrate the practical application of the generalized effective neutrosophic soft set in addressing decision-making problems and medical diagnoses.

Keywords: soft set; neutrosophic soft set; effective set; effective fuzzy soft set; generalized neutrosophic soft set effective neutrosophic soft set; generalized effective neutrosophic soft set
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1. Introduction

Fuzzy sets were developed by Zadeh [1] to address problems involving uncertain information. Atanassov [2] extended the concept of fuzzy sets to intuitionistic fuzzy sets, which offer a broader framework for handling uncertainty. In 1999, Molodtsov [3] introduced soft sets as another valuable tool for managing data uncertainties. The concept of soft sets has been thoroughly explored and advanced by numerous researchers [4–11], who have applied it across various domains. Maji [12] innovatively combined fuzzy sets and soft sets, giving rise to the notion of fuzzy soft sets. Furthermore, Roy and Maji's formulation of fuzzy soft sets [13] has found practical applications in addressing decision-making challenges.

The concept of generalized fuzzy soft sets was introduced by Majumdar and Samanta [14]. Their work also included the development of operations and applications in decision-making and medical diagnosis problems using this concept. Fuzzy soft sets were further advanced to fuzzy soft expert sets by Alkhazaleh and Salleh [15], effectively combining the principles of fuzzy sets and soft expert sets. They introduced operations, discussed properties, and explored applications of this concept in decision-making problems. Additionally, they introduced a mapping for this concept.

The transition from intuitionistic fuzzy sets to Neutrosophic sets (NS) was pioneered by Smarandach [16]. The concept of neutrosophic soft sets (NSS), which merges neutrosophic sets and soft sets, was introduced by Maji [17]. Sahin [18] extended the idea of neutrosophic soft sets to generalized neutrosophic soft sets (GNSS). Broumi [19] further extended this by introducing generalized neutrosophic soft sets with defined operations and applications in decision-making problems. Several developments of neutrosophic soft sets, along with their applications in decision-making, have been explored in subsequent works [20–26]. Currently, different applications on neutrosophic graph in decision making introduced in [27–29]. In the field of medical diagnosis (MD), determining diseases based on a person's symptoms is a crucial task. Neutrosophic sets find application in medical diagnosis, particularly when dealing with extensive datasets containing uncertainty, inconsistency and indeterminacy. Broumi [30] has successfully applied neutrosophic sets to address problems in medical diagnosis (MD).

Effective Fuzzy Soft Set (EFSS), introduced by Alkhazaleh in 2022 [31], was designed to extend the notion of external effectiveness within the realm of soft sets. Alkhazaleh also presented operations on Effective Fuzzy Soft Sets (ENSS) and investigated various properties of this concept. Furthermore, practical applications of EFSS were explored in decision-making problems (DM) and medical diagnosis (MD). Furthermore, the concept of Effective Fuzzy Soft Expert Sets (ENSES) [32] introduced the opinions of experts in one model.

In the same year, Effective Fuzzy Soft Set (EFSS) was extended to Effective Neutrosophic Soft Set (ENSS) [33], incorporating the notion of effectiveness on the three independent membership functions representing the degrees of truth (T), indeterminacy (I) and falsity (F). Moreover, the concept of Effective Neutrosophic Soft Expert Set (ENSES) [34] introduced with operations and sutiable examples.

In this research, we introduce the concept of an effective neutrosophic soft set, a mathematical framework designed to capture the nuances of uncertain information by utilizing three distinct membership functions representing degrees of truth (T), indeterminacy (I) and falsity (F). Going beyond conventional approaches, we propose a generalization of the effective neutrosophic soft set, which incorporates an additional degree to signify the potential for an approximate value-set. This extension enhances the framework's efficiency and realism, making it more adept at modeling complex real-world scenarios. Notably, our innovative approach seamlessly integrates the strengths of both the generalized neutrosophic set and the effective neutrosophic soft set, offering a versatile and comprehensive representation of uncertainty. To enhance understanding, we elucidate fundamental operations on the generalized effective neutrosophic soft set through illustrative examples and propositions. Furthermore, we demonstrate the practical applicability of our generalized framework in addressing diverse decision-making problems and making informed medical diagnoses. This study contributes to the advancement of mathematical models for handling uncertainty and bridges the gap between theory and practicality, offering valuable insights and solutions with real-world relevance.

While the study encompasses the broader realm of fuzzy sets and their generalizations, such as GENSS, it is pertinent to acknowledge the specific contributions and unique focus of the present work. Fuzzy sets and their extensions have played a pivotal role in Multi-Criteria Decision Making (MCDM) and related fields. However, the distinctive feature of this research lies in its innovative exploration of effective neutrosophic soft sets and their generalization. By introducing a comprehensive framework that combines elements of neutrosophic sets and soft sets, the study offers a fresh perspective on addressing uncertainty and ambiguity in decision-making processes. While the immediate application focus may not be apparent, the significance of this work becomes evident when considering its potential impact on refining MCDM methodologies and expanding the toolbox of decision-makers. Therefore, this research, while grounded in the broader context of fuzzy set theory, brings a unique and valuable perspective to the study line by opening up new avenues for addressing complex decision-making challenges.

We introduce the concept of Generalized Effective Neutrosophic Soft Set by extending the concept of Effective Neutrosophic Soft Set as defined in [34]. We also introduce the degree of possibility ($\mu(e_i)$) associated with each parameter e attached to ENSS, enhancing the realism of the concept.

We then define fundamental concepts such as soft sets, neutrosophic soft sets, effective fuzzy soft sets and effective neutrosophic soft sets. Then, we proceed to define basic operations for the new concept, including subset, equality, complement, union and intersection, with illustrative examples and accompanying propositions. Finally, we present an application of this new concept in decision-making problems and medical diagnosis.

2. Preliminary

In this section, we provide some necessary definitions for the understanding of this paper. Let U be a set of universe, E be a set of parameters and $P(U)$ denote the power set of U and $A \subset E$.

Definition 2.1. [3] A pair (F, A) is called a soft set over U, where F is a mapping

$$F : A \to P(U).$$

Definition 2.2. [12] Let U be an initial universal set, E be a set of parameters and I^U denote the power set of fuzzy set of U and $A \subset E$. A pair (F, E) is called a fuzzy soft set over U, where F is a mapping given by

$$F : A \to I^U.$$

Definition 2.3. [5] A neutrosophic set A on universe of discourse U is defined as

$$A = \{x : T_A(x), I_A(x), F_A(x); x \in U\},$$

where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$ and $T_A(x)$ is the truth-membership function, $I_A(x)$ is an indeterminacy-membership and $F_A(x)$ is a falsity-membership function.

Definition 2.4. [16] Let U be an initial universe set and E be a set of parameters. Consider $A \subseteq E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is denotes to be the set of all neutrosophic soft sets over U, where F is a mapping given by

$$F : A \to N(U).$$

Definition 2.5. [31] An effective set is a fuzzy set Λ in a universe of discourse A where Λ is a function where $\Lambda : A \longrightarrow [0,1]$. Here, A is the set of effective parameters that may change the membership values by making positive effect (or no effect) on values of memberships after applying it and defined as follows: $\Lambda = \{< a, \delta_\Lambda(a) : a \in A >\}$, where $\delta_\Lambda(a)$ is a membership degree.

Definition 2.6. [31] Let A be a set of effective parameters, and Λ be the effective set over A. Let I^U denote all fuzzy subsets of U; a pair $(F, E)_\Lambda$ is called an effective fuzzy soft set (EFSS in short) over U, where F is mapping given by

$$F : E \to I^U,$$

define as follows:

$$F(e_i)_\Lambda = \left\{ \frac{x_j}{\mu_U(x_j)_\Lambda} : x_j \in U,\ e_i \in E \right\}.$$

Where $\forall a_k \in A$

$$\mu_U(x_j)_\Lambda = \begin{cases} \mu_U(x_j) + \left[\dfrac{\left(1 - \mu_U(x_j)\right) \sum\limits_k \delta_{\Lambda_{x_j}}(a_k)}{|A|} \right], & \text{if } \mu_U(x_j) \in (0,1), \\ \mu_U(x_j), & \text{O.W.} \end{cases}$$

Definition 2.7. [31] The complement of effective set Λ over the set of effective parameters A is the effective set Λ^c where c is any fuzzy complement.

Definition 2.8. [33] Let U be an initial universal set and let E be a set of parameters. Let A be a set of effective parameters and Λ be the effective set over A. Let $N(U)$ denotes the set of all neutrosophic subsets of U, a pair $(\psi, E)_\Lambda$ is called an effective neutrosophic soft set (ENSS) over U, where ψ is a mapping given by $\psi : E \to N(U)$ defined as follows:

$$\psi(e)(x_j)_\Lambda = \left\{ \frac{x_j}{< T_U\left(x_j\right)_\Lambda, I_U\left(x_j\right)_\Lambda,\ F_U\left(x_j\right)_\Lambda >} : x_j \in U,\ e \in E \right\}, \tag{2.1}$$

where

$$T_U\left(x_j\right)_\Lambda = \begin{cases} T_U\left(x_j\right) + \left[\frac{\left(1 - T_U\left(x_j\right)\right) \sum_k \delta_{\Lambda_{x_j}}(a_k)}{|A|} \right], & \text{if } T_U\left(x_j\right) \in (0,1) \\ T_U\left(x_j\right), & \text{O.W.} \end{cases}$$

$$I_U\left(x_j\right)_\Lambda = I\left(x_j\right),$$

$$F_U\left(x_j\right)_\Lambda = \begin{cases} F_U\left(x_j\right) - \left[\frac{F_U\left(x_j\right) \sum_k \delta_{\Lambda_{x_j}}(a_k)}{|A|} \right], & \text{if } F_U\left(x_j\right) \in (0,1) \\ F_U\left(x_j\right), & \text{O.W.} \end{cases}$$

Definition 2.9. [33] Let $(\psi, E_1)_{\Lambda_1}$ and $(\phi, E_2)_{\Lambda_2}$ be two ENSSs over the common universe U. Then $(\psi, E_1)_{\Lambda_1}$ is said to be ENS subset of $(\phi, E_2)_{\Lambda_2}$ if

1) $E_1 \subset E_2$;

2) $\Lambda_1(x) \leq \Lambda_2(x)$;

3) $T_{\psi_{\Lambda_1}(e)}(x) \leq T_{\phi_{\Lambda_2}(e)}(x)$, $I_{\psi_{\Lambda_1}(e)}(x) \leq I_{\phi_{\Lambda_2}(e)}(x)$, $F_{\psi_{\Lambda_1}(e)}(x) \geq F_{\phi_{\Lambda_2}(e)}(x)$ $\forall e \in E_1, x \in U$.

We denote it by $(\psi, E_1)_{\Lambda_1} \subseteq (\phi, E_2)_{\Lambda_2}$.

Definition 2.10. [33] The $\Lambda_{complement}$ of the ENSS $(\psi, E)_\Lambda$ is the ENSS denoted by $(\psi, E)_{\Lambda^c}$, where Λ^c is fuzzy complement of Λ.

To get $\Lambda_{complement}$ of ENSS, we keep the neutrosophic soft set (ψ, E) as is and find Λ^c. Then, we apply Eq (2.1) to get a new ENSS.

Definition 2.11. [33] The $Soft_{complement}$ of the ENSS $(\psi, E)_\Lambda$is the ENSS denoted by $(\psi^c, E)_\Lambda$, where ψ^c is the neutrosophic soft complement of ψ.

To get $Soft_{complement}$ of ENSS, we keep the effective set Λ as is and compute ψ^c. Then we apply Eq (2.1) to get a new ENSS.

Definition 2.12. [33] The $Total_{complement}$ of the ENSS $(\psi, E)_\Lambda$ is the ENSS denoted by $(\psi^c, E)_{\Lambda^c}$, where ψ^c is the neutrosophic soft complement of ψ and Λ^c is any fuzzy complement of Λ.

To get $Total_{complement}$ of ENSS, we compute ψ^c and Λ^c. Then, we apply Eq (2.1) to get a new ENSS.

Definition 2.13. [33] The union of two ENSSs $(\psi, E_1)_{\Lambda_1}$ and $(\phi, E_2)_{\Lambda_2}$ over the common universe U is the ENSS $(\sigma, E)_{\Lambda_s}$, where $E = E_1 \cup E_2$ and $\forall \nu \in E$, is given as follows:

$$\sigma_{\Lambda_s}(\nu) = \begin{cases} \psi_{\Lambda_s}(\nu), & \text{if } \nu \in E_1 - E_2; \\ \phi_{\Lambda_s}(\nu), & \text{if } \nu \in E_2 - E_1; \\ (\psi \cup \sigma)_{\Lambda_s}(\nu), & \text{if } \nu \in E_1 \cap E_2. \end{cases}$$

Here, s is any s-norm and σ is a neutrosophic soft union between ψ and ϕ.

Definition 2.14. [33] The intersection of two ENSSs $(\psi, E_1)_{\Lambda_1}$ and $(\phi, E_2)_{\Lambda_2}$ over the common universe U is the ENSS $(\omega, E)_{\Lambda_t}$, where $E = E_1 \cup E_2$ and $\forall \nu \in E$, $(\omega, E)_{\Lambda_t}$ is given as follows:

$$\omega_{\Lambda_t}(\nu) = \begin{cases} \psi_{\Lambda_t}(\nu), & \text{if } \nu \in E_1 - E_2; \\ \phi_{\Lambda_t}(\nu), & \text{if } \nu \in E_2 - E_1; \\ (\psi \cap \sigma)_{\Lambda_t}(\nu), & \text{if } \nu \in E_1 \cap E_2. \end{cases}$$

Here, t is any t-norm and ω is a neutrosophic soft intersection between ψ and ϕ.

Definition 2.15. [33] Let $(\psi, E_1)_{\Lambda_1}$ and $(\phi, E_2)_{\Lambda_2}$ be two ENSSs over the common universe U. Then, "$(\psi, E_1)_{\Lambda_1}$ AND $(\phi, E_2)_{\Lambda_2}$" is ENSS denoted by $(\psi, E_1)_{\Lambda_1} \wedge (\phi, E_2)_{\Lambda_2}$ and defined by

$$(\psi, E_1)_{\Lambda_1} \wedge (\phi, E_2)_{\Lambda_2} = (\omega, E_2 \times E_1)_{\Lambda_t},$$

where $\omega_{\Lambda_t}(\alpha,\beta) = (\psi(\alpha) \cap \phi(\beta))_{\Lambda_t}, \forall (\alpha,\beta) \in E_1 \times E_2$. Here, t is any t-norm, ω_{Λ_t} is the effective neutrosophic soft intersection between ψ_{Λ_1} and ϕ_{Λ_2}.

Definition 2.16. [33] Let $(\psi, E_1)_{\Lambda_1}$ and $(\phi, E_2)_{\Lambda_2}$ be two ENSSs over the common universe U. Then, "$(\psi, E_1)_{\Lambda_1}$ OR $(\phi, E_2)_{\Lambda_2}$" is ENSS denoted by $(\psi, E_1)_{\Lambda_1} \vee (\phi, E_2)_{\Lambda_2}$ and defined by

$$(\psi, E_1)_{\Lambda_1} \vee (\phi, E_2)_{\Lambda_2} = (\sigma, E_1 \times E_2)_{\Lambda_t},$$

where $\sigma_{\Lambda_t}(\alpha,\beta) = (\psi(\alpha) \cup \phi(\beta))_{\Lambda_t} : \forall (\alpha,\beta) \in E_1 \times E_2$. Here, t is any t-norm, σ_{Λ_t} is the effective neutrosophic soft union between ψ_{Λ_1} and ϕ_{Λ_2}.

Definition 2.17. [13] Let $N(U)$ be the set of all neutrosophic sets of U. A generalized neutrosophic soft set ψ^μ over U is defined by the set of ordered pairs.
$\psi^\mu(e) = \{(\psi(e), \mu(e)) : e \in E, \psi(e) \in N(U), \mu(e) \in [0, 1]\}$,
where ψ is a mapping given by
$\psi : E \to N(U)$ and μ is a fuzzy set such that $\mu : E \to I = [0, 1]$.
Here, ψ^μ is a mapping defined by $\psi^\mu : E \to N(U) \times I$. For any parameter $e \in E$, $\psi(e)$ is referred as the neutrosophic value set of parameter e, that is $\psi(e) = \{< x, T_A(x), I_A(x), F_A(x) >: x \in U\}$, where T, I, F are membership values for truthness, indeterminacy and falsity respectively such that $T, I, F : U \to [0, 1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

In fact, ψ^μ is a parameterized family of neutrosophic sets on U, which has the degree of possibility of the approximate value set which is represented by $\mu(e)$ for each parameter e. So, we can write it as follows:

$$\psi^\mu(e) = \left\{ \frac{x_1}{\psi(e)(x_1)}, \frac{x_2}{\psi(e)(x_2)}, \ldots, \frac{x_n}{\psi(e)(x_n)}, \mu(e) \right\}.$$

3. Generalized effective neutrosophic soft set (GENSS)

In this section, we extend the concept of effective neutrosophic soft sets as introduced in [33]. In our generalization of effective neutrosophic soft sets, we attach a degree to the parameterization of fuzzy sets while defining effective neutrosophic soft sets.

Definition 3.1. Let $U = \{u_1, u_2, ..., u_n\}$ be an initial universal set, and let $E = \{e_1, e_2, ..., , e_m\}$ be a set of parameters. Let $A = \{a_1, a_2, ..., a_k\}$ be a set of effective parameters and Λ be the effective set over A. Let $N(U)$ denotes the set of all neutrosophic subsets of U and μ be a fuzzy set that is $\mu : E \to I = [0, 1]$. A pair $(\Gamma^\mu, E)_\Lambda$ is called a generalized effective neutrosophic soft set (GENSS) over U, where $\Gamma^\mu(e_i)_\Lambda = \{\{(\Gamma(e_i)_\Lambda), \mu_\Lambda(e_i)\}, e_i \in E, \Gamma(e_i)_\Lambda \in N(U) \text{ and } \mu_\Lambda(\mathrm{e_i}) \in \mathrm{I} = [0, 1]\}$, for all $e_i \in E$, $\Gamma(e_i)_\Lambda$ is referred to as the effective neutrosophic soft value set of the parameter e_i and the mapping given by $\Gamma^\mu(e_i)_\Lambda : E \to N(U) \times I$, where μ is a fuzzy set such that $\mu : E \longrightarrow I$. Then, we can write the generalized effective neutrosophic soft set (GENSS) $\Gamma^\mu(e_i)_\Lambda$ as follows:

$$\Gamma^\mu(e_i)_\Lambda = \left\{ \left(\frac{x_1}{\Gamma^\mu(e_i)_\Lambda(x_1)}, \frac{x_2}{\Gamma^\mu(e_i)_\Lambda(x_2)}, ..., \frac{x_n}{\Gamma^\mu(e_i)_\Lambda(x_n)} \right), \mu_\Lambda(e_i) \right\},$$

where,

$$\mu(e_i)_\Lambda = \begin{cases} \mu(e_i) + \left[\frac{(1-\mu(e_i)) \sum_j^n \sum_k^m \delta_{\Lambda_{x_j}}(a_k)}{|A||U|} \right], & \text{if } \mu(e_i) \in (0, 1); \\ \mu(e_i), & \text{O.W.} \end{cases}$$

Example 3.1. *Let $U = \{x_1, x_2, x_3\}$ be a set of universe. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters.let $A = \{a_1, a_2, a_3,\ a_4\}$ be a set of effective parameters. Suppose that the effective set over A for all $\{x_1, x_2, x_3\}$ s given by expert as follows:*

$$\Lambda(x_1) = \left\{\frac{a_1}{0.8},\ \frac{a_2}{1}, \frac{a_3}{0.2}, \frac{a_4}{0.4}\right\}, \Lambda(x_2,) = \left\{\frac{a_1}{0.7},\ \frac{a_2}{1}, \frac{a_3}{0.9}, \frac{a_4}{0.7}\right\},$$
$$\Lambda(x_3) = \left\{\frac{a_1}{0.5},\ \frac{a_2}{0.6}, \frac{a_3}{0}, \frac{a_4}{0.6}\right\}.$$

Let the GNSS (Γ^μ, E) is given as follows:

$$(\Gamma^\mu, E) = \left\{e_1, \left\{\left(\frac{x_1}{<0.5, 0, 0.7>}\right), \left(\frac{x_2}{<0.7, 0.3, 0.1>}\right), \left(\frac{x_3}{<0.7, 0.2, 0.5>}\right), 0.7\right\}, \right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{<0.6, 0.4, 0.9>}\right), \left(\frac{x_2}{<0.3, 0.7, 0.9>}\right), \left(\frac{x_3}{<0.2, 0, 0.6>}\right), 0.1\right\},\right.$$
$$\left.\left\{e_3, \left\{\left(\frac{x_1}{<0.7, 0.1, 0.5>}\right), \left(\frac{x_2}{<0.7, 0.2, 0.3>}\right), \left(\frac{x_3}{<0.8, 0.1, 0.1>}\right), 0.6\right\}\right\}.$$

Now, we apply Definition 3.1 to find $\Gamma^\mu(e_1)_\Lambda$ as follows:

$$\Gamma(e_1)_\Lambda(x_j) = \left\{\frac{x_1}{<0.5 + [(1-0.5)(0.8+1+0.2+0.4)/4], 0.1, 0.7 - (0.7)[(0.8+1+0.2+0.4)/4]>},\right.$$
$$\frac{x_2}{<0.7 + [(1-0.7)(0.7+1+0.9+0.7)/4], 0.3, 0.1 - [(0.1)(0.7+1+0.9+0.7)/4]>},$$
$$\left.\frac{x_3}{<0.7 + [(1-0.7)(0.5+0.6+0+0.6)/4], 0.2, 0.5 - [(0.5)(0.5+0.6+0+0.6)/4]>}\right\}$$
$$= \left\{\frac{x_1}{<0.8, 0, 0.28>}, \frac{x_2}{<0.95, 0.3, 0.02>}, \frac{x_3}{<0.83, 0.2, 029>}\right\}.$$

Then, we find $\mu(e_1)_\Lambda$ as follows:

$$\mu(e_1)_\Lambda(x_1) = 0.7 + [(1-0.7)(0.8+1+0.2+0.4+0.7+1+0.9+0.7+0.5+0.6+0+0.6)/12] = 0.89.$$

Consequently,

$$\Gamma^\mu(e_1)_\Lambda(x_j) = \left\{\left(\frac{x_1}{<0.8, 0, 0.28>}\right), \left(\frac{x_2}{<0.95, 0.3, 0.02>}\right), \left(\frac{x_3}{<0.83, 0.2, 0.29>}, 0.89\right)\right\}.$$

Similarly, we get the GENSS $(\Gamma^\mu, E)_\Lambda$ as follows:

$$(\Gamma^\mu, E)_\Lambda = \left\{e_1, \left\{\left(\frac{x_1}{<0.8, 0, 0.28>}\right), \left(\frac{x_2}{<0.95, 0.3, 0.02>}\right), \left(\frac{x_3}{<0.83, 0.2, 0.29>}\right), 0.89\right\},\right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{<0.84, 0.4, 0.36>}\right), \left(\frac{x_2}{<0.88, 0.7, 0.16>}\right), \left(\frac{x_3}{<0.54, 0, 0.35>}\right), 0.66\right\},\right.$$
$$\left.\left\{e_3, \left\{\left(\frac{x_1}{<0.88, 0.1, 0.2>}\right), \left(\frac{x_2}{<0.95, 0.2, 0.05>}\right), \left(\frac{x_3}{<0.89, 0.1, 0.06>}\right), 0.85\right\}\right\}.$$

Definition 3.2. Let $(\Gamma^{\mu}, E_1)_{\Lambda_1}$ and $(\Psi^{\eta}, E_2)_{\Lambda_2}$ be two GENSSs over the common universe U. Then $(\Gamma^{\mu}, E_1)_{\Lambda_1}$ is the GENS subset of $(\Psi^{\eta}, E_2)_{\Lambda_2}$denoted by $(\Gamma^{\mu}, E_1)_{\Lambda_1} \subseteq (\Psi^{\eta}, E_2)_{\Lambda_2}$ if the following conditions are satisfied:

1) $E_1 \subseteq E_2$;

2) $\mu(e) \leq \eta(e)$;

3) $\Lambda_1(x_i)$ is fuzzy subset of $\Lambda_2(x_i)$ for all i;

4) $\Gamma(e)_{\Lambda_1} \subseteq \Psi(e)_{\Lambda_2} \quad \forall e \in E_1 \cap E_2$.

Example 3.2. *Let* $E_1 = \{e_1, e_2, e_4\}$ *and* $E_2 = \{e_1, e_2, e_4, e_5\}$*, over the common universe* $U = \{x_1, x_2, x_3\}$*. Clearly,* $E_1 \subset E_2$*. Let*

$$\begin{aligned}
&\Lambda_1(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.7}\right\}, \Lambda_1(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\},\\
&\Lambda_1(x_3) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0}, \frac{a_3}{0.6}, \frac{a_4}{0.4}\right\}, \Lambda_2(x_1) = \left\{\frac{a_1}{0.5}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.9}\right\},\\
&\Lambda_2(x_2) = \left\{\frac{a_1}{0.6}, \frac{a_2}{0.7}, \frac{a_3}{1}, \frac{a_4}{1}\right\}, \Lambda_2(x_3) = \left\{\frac{a_1}{0.8}, \frac{a_2}{0}, \frac{a_3}{0.7}, \frac{a_4}{0.5}\right\}.
\end{aligned}$$

It's clear $\Lambda_1(x_i)$ is a fuzzy subset of $\Lambda_2(x_i), \forall i = 1, 2, 3$.

Let (Γ^{μ}, E_1) and (ψ^{η}, E_2) be defined as follows:

$$\begin{aligned}
(\Gamma^{\mu}, E_1) = &\left\{e_1, \left\{\left(\frac{x_1}{<0.5, 0.2, 0.6>}\right), \left(\frac{x_2}{<0.3, 0.1, 0.8>}\right), \left(\frac{x_3}{<0.3, 0.3, 0.8>}\right), 0.1\right\},\right.\\
&\left\{e_2, \left\{\left(\frac{x_1}{<0.2, 0.4, 0.7>}\right), \left(\frac{x_2}{<0.1, 0.3, 0.8>}\right), \left(\frac{x_3}{<0.2, 0.1, 0.6>}\right), 0.2\right\},\right.\\
&\left\{e_4, \left\{\left(\frac{x_1}{<0.4, 0.2, 0.8>}\right), \left(\frac{x_2}{<0.2, 0.1, 0.9>}\right), \left(\frac{x_3}{<0.8, 0.2, 0.5>}\right), 0.4\right\}\right\}.
\end{aligned}$$

$$\begin{aligned}
(\Psi^{\eta}, E_2) = &\left\{e_1, \left\{\left(\frac{x_1}{<0.6, 0.3, 0.5>}\right), \left(\frac{x_2}{<0.4, 0.2, 0.5>}\right), \left(\frac{x_3}{<0.5, 0.4, 0.6>}\right), 0.2\right\},\right.\\
&\left\{e_2, \left\{\left(\frac{x_1}{<0.3, 0.5, 0.6>}\right), \left(\frac{x_2}{<0.4, 0.3, 0.6>}\right), \left(\frac{x_3}{<0.5, 0.4, 0.6>}\right), 0.4\right\},\right.\\
&\left\{e_4, \left\{\left(\frac{x_1}{<0.5, 0.3, 0.7>}\right), \left(\frac{x_2}{<0.3, 0.2, 0.5>}\right), \left(\frac{x_3}{<0.9, 0.3, 0.4>}\right), 0.5\right\},\right.\\
&\left\{e_5, \left\{\left(\frac{x_1}{<0.3, 0.6, 0.5>}\right), \left(\frac{x_2}{<0.5, 0.3, 0.2>}\right), \left(\frac{x_3}{<0.8, 0.5, 0.4>}\right), 0.6\right\}\right\}.
\end{aligned}$$

By appling Definition 3.1, we get $\Gamma^{\mu}(e_1)_{\Lambda_1}$ as follows:

$$\Gamma(e_1)_{\Lambda_1}(x_j)=\left\{\frac{x_1}{<0.5+[(1-0.5)(0.3+0+1+0.7)/4],0.2,0.6-[(0.6)(0.3+0+1+0.7)/4]>},\right.$$
$$\frac{x_2}{<0.3+[(1-0.3)(0.4+0.5+1+1)/4],0.1,0.8-[(0.8)(0.4+0.5+1+1)/4]>},$$
$$\left.\frac{x_3}{<0.3+[(1-0.3)(0.7+0+0.6+0.4)/4],0.3,0.8-[(0.8)(0.7+0+0.6+0.4)/4]>}\right\}$$
$$=\left\{\frac{x_1}{<0.75,0.2,0.3>},\frac{x_2}{<0.79,0.1,0.24>},\frac{x_3}{<0.6,0.3,0.46>}\right\}.$$

Then, we find $\mu(e_1)_{\Lambda_1}$ as follows:

$$\mu(e_1)(x_1)_{\Lambda_1}=0.1+[(1-0.1)(0.3+0+1+0.7+0.4+0.5+1+1+0.7+0+0.6+0.4)/12]=0.6.$$

Consequently,

$$\Gamma^{\mu}(e_1)_{\Lambda_1}(x_j)=\left\{\left(\frac{x_1}{<0.75,0.2,0.3>}\right),\left(\frac{x_2}{<0.79,0.1,0.24>}\right),\left(\frac{x_3}{<0.6,0.3,0.46>}\right),0.6\right\}.$$

Similarly, we get the GENSS $(\Gamma^{\mu},E)_{\Lambda_1}$ and $(\Psi^{\mu},E_2)_{\Lambda_2}$ as follows:

$$(\Gamma^{\mu},E)_{\Lambda_1}=\left\{e_1,\left\{\left(\frac{x_1}{<0.75,0.2,0.3>}\right),\left(\frac{x_2}{<0.79,0.1,0.24>}\right),\left(\frac{x_3}{<0.6,0.3,0.46>}\right),0.6\right\},\right.$$
$$\left\{e_2,\left\{\left(\frac{x_1}{<0.6,0.4,0.35>}\right),\left(\frac{x_2}{<0.73,0.3,0.24>}\right),\left(\frac{x_3}{<0.54,0.1,0.35>}\right),0.64\right\},\right.$$
$$\left.\left\{e_4,\left\{\left(\frac{x_1}{<0.7,0.2,0.4>}\right),\left(\frac{x_2}{<0.76,0.1,0.27>}\right),\left(\frac{x_3}{<0.89,0.2,0.29>}\right),0.73\right\}\right\}\right..$$

$$(\Psi^{\mu},E_2)_{\Lambda_2}=\left\{e_1,\left\{\left(\frac{x_1}{<0.84,0.3,0.2>}\right),\left(\frac{x_2}{<0.9,0.2,0.08>}\right),\left(\frac{x_3}{<0.75,0.4,0.3>}\right),0.71\right\},\right.$$
$$\left\{e_2,\left\{\left(\frac{x_1}{<0.72,0.5,0.24>}\right),\left(\frac{x_2}{<0.9,0.3,0.11>}\right),\left(\frac{x_3}{<0.75,0.4,0.3>}\right),0.79\right\},\right.$$
$$\left\{e_4,\left\{\left(\frac{x_1}{<0.8,0.3,0.28>}\right),\left(\frac{x_2}{<0.88,0.2,0.28>}\right),\left(\frac{x_3}{<0.95,0.3,0.2>}\right),0.82\right\},\right.$$
$$\left.\left\{e_5,\left\{\left(\frac{x_1}{<0.72,0.6,0.2>}\right),\left(\frac{x_2}{<0.91,0.3,0.03>}\right),\left(\frac{x_3}{<0.9,0.5,0.2>}\right),0.86\right\}\right\}\right..$$

Therefore, $(\Gamma^{\mu},E_1)_{\Lambda_1}\subset(\Psi^{\eta},E_2)_{\Lambda_2}$.

Note: Let $(\Gamma^{\mu},E_1)_{\Lambda_1}$ and $(\Psi^{\eta},E_2)_{\Lambda_2}$ be two GENSS over U. Then, $(\Gamma^{\mu},E_1)_{\Lambda_1}$ is said to be equal to $(\Psi^{\eta},E_2)_{\Lambda_2}$, denoted by $(\Gamma^{\mu},E_1)_{\Lambda_1}=(\Psi^{\eta},E_2)_{\Lambda_2}$ if $(\Gamma^{\mu},E_1)_{\Lambda_1}$ is a GENS subset of $(\Psi^{\eta},E_2)_{\Lambda_2}$ and $(\Psi^{\eta},E_2)_{\Lambda_2}$ is a GENS subset of $(\Gamma^{\mu},E_1)_{\Lambda_1}$.

Definition 3.3. The $\Lambda_{complement}$ of the GENSS $(\Gamma^{\mu},E)_{\Lambda}$ is the GENSS $(\Gamma^{\mu},E)_{\Lambda^c}$ and define by:

$$(\Gamma^{\mu},E)_{\Lambda^c}=\{(\Gamma(e_i)_{\Lambda^c},\mu(e_i)_{\Lambda^c})\}.$$

To get $\Lambda_{complement}$ of GENSS, we keep the generalized neutrosophic soft set (Γ^{μ}, E) as is and find the fuzzy complement of the effective set Λ, which is Λ^c. Then, apply Definition 3.1.

Definition 3.4. The $Soft_{complement}$ of the GENSS $(\Gamma^{\mu}, E)_{\Lambda}$ is the GENSS $(\Gamma^{\mu}, E)^c_{\Lambda}$ and define by:

$$(\Gamma^{\mu}, E)^c_{\Lambda} = \{((\Gamma^{\mu})^c (e_i)_{\Lambda}, \mu (e_i)_{\Lambda})\}.$$

To get $Soft_{complement}$ of GENSS, we find the generalized neutrosophic soft complement of (Γ^{μ}, E) which is $(\Gamma^{\mu}, E)^c$ and keep the effective set Λ as is. Then, we apply Definition 3.1.

Definition 3.5. The $Total_{complement}$ of the GENSS $(\Gamma^{\mu}, E)_{\Lambda}$ is the GENSS $((\Gamma^{\mu}, E)_{\Lambda})^c$ and define by:

$$((\Gamma^{\mu}, E)_{\Lambda})^c = (\Gamma^{\mu}, E)^c_{\Lambda^c} = \{(\Gamma^c (e_i)_{\Lambda^c}, \mu^c (e_i)_{\Lambda^c})\}.$$

To get $Total_{complement}$ $((\Gamma^{\mu}, E)_{\Lambda})$, we find the generalized neutrosophic soft set complement of (Γ^{μ}, E) which is $(\Gamma^{\mu}, E)^c$ and the fuzzy complement of the effective set Λ which is Λ^c. Then, we apply Definition 3.1.

Example 3.3. *Let $U = \{x_1, x_2, x_3\}$ be a set of universe. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters. Let $A = \{a_1, a_2, a_3, a_4\}$ be a set of effective parameters. Suppose that the effective set over A for all $\{x_1, x_2, x_3\}$ is given by expert as follows:*

$$\Lambda(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.7}\right\}, \Lambda(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\},$$
$$\Lambda(x_3) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0}, \frac{a_3}{0.6}, \frac{a_4}{0.4}\right\}.$$

Let the generalized neutrosophic soft set is given as follows:

$$\begin{aligned}(\Gamma^{\mu}, E) = &\left\{e_1, \left\{\left(\frac{x_1}{<0.5, 0.2, 0.6>}\right), \left(\frac{x_2}{<0.3, 0.1, 0.8>}\right), \left(\frac{x_3}{<0.3, 0.3, 0.8>}\right), 0.1\right\},\right.\\ &\left\{e_2, \left\{\left(\frac{x_1}{<0.2, 0.4, 0.7>}\right), \left(\frac{x_2}{<0.1, 0.3, 0.8>}\right), \left(\frac{x_3}{<0.2, 0.1, 0.6>}\right), 0.2\right\},\right.\\ &\left\{e_3, \left\{\left(\frac{x_1}{<0.4, 0.2, 0.8>}\right), \left(\frac{x_2}{<0.2, 0.1, 0.9>}\right), \left(\frac{x_3}{<0.8, 0.2, 0.5>}\right), 0.4\right\}\right\}.\end{aligned}$$

Then, by using fuzzy complement of the effective set we have:

$$\Lambda^c(x_1) = \left\{\frac{a_1}{0.7}, \frac{a_2}{1}, \frac{a_3}{0}, \frac{a_4}{0.3}\right\}, \Lambda^c(x_2) = \left\{\frac{a_1}{0.6}, \frac{a_2}{0.5}, \frac{a_3}{0}, \frac{a_4}{0}\right\},$$
$$\Lambda^c(x_3) = \left\{\frac{a_1}{0.3}, \frac{a_2}{1}, \frac{a_3}{0.4}, \frac{a_4}{0.6}\right\}.$$

The complement of the generalized neutrosophic soft set is given as follows:

$$\begin{aligned}(\Gamma^{\mu}, E)^c = &\left\{e_1, \left\{\left(\frac{x_1}{<0.6, 0.2, 0.5>}\right), \left(\frac{x_2}{<0.8, 0.1, 0.3>}\right), \left(\frac{x_3}{<0.8, 0.3, 0.3>}\right), 0.9\right\},\right.\\ &\left\{e_2, \left\{\left(\frac{x_1}{<0.7, 0.4, 0.2>}\right), \left(\frac{x_2}{<0.6, 0.3, 0.1>}\right), \left(\frac{x_3}{<0.6, 0.1, 0.2>}\right), 0.8\right\},\right.\\ &\left\{e_3, \left\{\left(\frac{x_1}{<0.8, 0.2, 0.4>}\right), \left(\frac{x_2}{<0.9, 0.1, 0.2>}\right), \left(\frac{x_3}{<0.5, 0.2, 0.8>}\right), 0.6\right\}\right\}.\end{aligned}$$

Then, by applying Definitions 3.3–3.5 and Definition 3.1, we compute $\Lambda_{complement}$, $Soft_{complement}$ and $Total_{complement}$, respectively.

To compute $\Lambda_{complement}(\Gamma^{\mu}(e_1)_{\Lambda})$, we need to compute $\Gamma(e_1)_{\Lambda^c}$ and $\mu_{\Lambda^c}(e_1)$ as follows:

$$\begin{aligned}\Gamma(e_1)_{\Lambda^c} =& \left\{ \frac{x_1}{< 0.5 + [(1-0.5)(0.7+1+0+0.3)/4], 0.2, 0.6 - [(0.6)(0.7+1+0+0.3)/4] >}, \right. \\ & \frac{x_2}{< 0.3 + [(1-0.3)(0.6+0.5+0+0)/4], 0.1, 0.8 - [(0.8)(0.6+0.5+0+0)/4] >}, \\ & \left. \frac{x_3}{< 0.3 + [(1-0.3)(0.3+0.1+0.4+0.6)/4], 0.3, 0.8 - [(0.8)(0.3+0.1+0.4+0.6)/4] >} \right\} \\ =& \left\{ \frac{x_1}{< 0.75, 0.2, 0.3 >}, \frac{x_2}{< 0.49, 0.1, 0.58 >}, \frac{x_3}{< 0.70, 0.3, 0.34 >} \right\}.\end{aligned}$$

$$\mu(e_1)_{\Lambda^c} = 0.1 + [(1-0.1)(0.7+1+0.3+0.6+0.5+0.3+1+0.4+0.6)/12] = 0.51.$$

Consequently,

$$\Lambda_{complement}(\Gamma^{\mu}(e_1)_{\Lambda}) = \Gamma^{\mu}(e_1)_{\Lambda^c} = \left\{ \left(\frac{x_1}{< 0.75, 0.2, 0.3 >} \right), \left(\frac{x_2}{< 0.49, 0.1, 0.58 >} \right), \left(\frac{x_3}{< 0.70, 0.3, 0.34 >}, 0.51 \right) \right\}.$$

Similarly, we get $\Lambda_{complement}((\Gamma^{\mu}, E)_{\Lambda})$ as follows:

$$\begin{aligned}\Lambda_{complement}((\Gamma^{\mu}, E)_{\Lambda}) = (\Gamma^{\mu}, E)_{\Lambda^c} =& \left\{ e_1, \left\{ \left(\frac{x_1}{< 0.75, 0.2, 0.3 >} \right), \left(\frac{x_2}{< 0.49, 0.1, 0.58 >} \right), \left(\frac{x_3}{< 0.70, 0.3, 0.34 >} \right), 0.51 \right\}, \right. \\ & \left\{ e_2, \left\{ \left(\frac{x_1}{< 0.6, 0.4, 0.35 >} \right), \left(\frac{x_2}{< 0.35, 0.3, 0.44 >} \right), \left(\frac{x_3}{< 0.66, 0.1, 0.26 >} \right), 0.56 \right\}, \right. \\ & \left\{ e_3, \left\{ \left(\frac{x_1}{< 0.7, 0.2, 0.4 >} \right), \left(\frac{x_2}{< 0.42, 0.1, 0.65 >} \right), \left(\frac{x_3}{< 0.34, 0.2, 0.21 >} \right), 0.67 \right\} \right\}.\end{aligned}$$

To compute $Soft_{complement}(\Gamma^{\mu}(e_1)_{\Lambda})$, we need to compute $\Gamma^c(e_1)_{\Lambda}$ and $\mu_{\Lambda}(e_1)$ as follows:

$$\begin{aligned}\Gamma^c(e_1)_{\Lambda} =& \left\{ \frac{x_1}{< 0.6 + [(1-0.6)(0.3+0+1+0.7)/4], 0.2, 0.5 - [(0.5)(0.3+0+1+0.7)/4] >}, \right. \\ & \frac{x_2}{< 0.8 + [(1-0.8)(0.4+0.5+1+1)/4], 0.1, 0.3 - [(0.3)(0.4+0.5+1+1)/4] >}, \\ & \left. \frac{x_3}{< 0.8 + [(1-0.8)(0.7+0+0.6+0.4)/4], 0.3, 0.3 - [(0.3)(0.7+0+0.6+0.4)/4] >} \right\} \\ =& \left\{ \frac{x_1}{< 0.8, 0.2, 0.25 >}, \frac{x_2}{< 0.94, 0.1, 0.09 >}, \frac{x_3}{< 0.89, 0.3, 0.17 >} \right\}.\end{aligned}$$

$$\mu(e_1)_{\Lambda} = 0.9 + [(1-0.9)(0.3+0+1+0.7+0.4+0.5+1+1+0.7+0+0.6+0.4)/12] = 0.6.$$

Consequently,

$$Soft_{complement}(\Gamma^{\mu}(e_1)_{\Lambda}) = (\Gamma^{\mu})^c(e_1)_{\Lambda} = \left\{ \left(\frac{x_1}{< 0.8, 0.2, 0.25 >} \right), \left(\frac{x_2}{< 0.94, 0.1, 0.09 >} \right), \left(\frac{x_3}{< 0.89, 0.3, 0.17 >}, 0.6 \right) \right\}.$$

Similarly, we get $Soft_{complement}\left((\Gamma^{\mu}, E)_{\Lambda}\right)$ as follows:

$$Soft_{complement}\left((\Gamma^{\mu}, E)_{\Lambda}\right) = (\Gamma^{\mu}, E)^{c}_{\Lambda} = \left\{e_1, \left\{\left(\frac{x_1}{< 0.8, 0.2, 0.25 >}\right), \left(\frac{x_2}{< 0.94, 0.1, 0.09 >}\right), \left(\frac{x_3}{< 0.89, 0.3, 0.17 >}\right), 0.6\right\},\right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{< 0.85, 0.4, 0.1 >}\right), \left(\frac{x_2}{< 0.88, 0.3, 0.03 >}\right), \left(\frac{x_3}{< 0.77, 0.1, 0.12 >}\right), 0.64\right\},\right.$$
$$\left.\left\{e_3, \left\{\left(\frac{x_1}{< 0.9, 0.2, 0.2 >}\right), \left(\frac{x_2}{< 0.97, 0.1, 0.06 >}\right), \left(\frac{x_3}{< 0.71, 0.2, 0.46 >}\right), 0.73\right\}\right\}.$$

To compute $Total_{complement}(\Gamma^{\mu}(e_1)_{\Lambda})$, we need to compute $\Gamma^{c}(e_1)_{\Lambda^c}$ and $\mu_{\Lambda^c}(e_1)$ as follows:

$$\Gamma^{c}(e_1)_{\Lambda^c} = \left\{\frac{x_1}{< 0.6 + [(1-0.6)(0.7+1+0+0.3)/4], 0.2, 0.5 - [(0.5)(0.7+1+0+0.3)/4] >},\right.$$
$$\frac{x_2}{< 0.8 + [(1-0.8)(0.6+0.5+0+0)/4], 0.1, 0.3 - [(0.3)(0.6+0.5+0+0)/4] >},$$
$$\left.\frac{x_3}{< 0.8 + [(1-0.8)(0.3+0.1+0.4+0.6)/4], 0.3, 0.3 - [(0.3)(0.3+0.1+0.4+0.6)/4] >}\right\}$$
$$= \left\{\frac{x_1}{< 0.8, 0.2, 0.25 >}, \frac{x_2}{< 0.94, 0.1, 0.09 >}, \frac{x_3}{< 0.89, 0.3, 0.17 >}\right\}.$$

$$\mu(e_1)_{\Lambda^c} = 0.9 + [(1-0.9)(0.7+1+0.3+0.6+0.5+0.3+1+0.4+0.6)/12] = 0.95.$$

Consequently,

$$Total_{complement}\left(\Gamma^{\mu}(e_1)_{\Lambda}\right) = (\Gamma^{\mu})^{c}(e_1)_{\Lambda^c} = \left\{\left(\frac{x_1}{< 0.8, 0.2, 0.25 >}\right), \left(\frac{x_2}{< 0.94, 0.1, 0.09 >}\right), \left(\frac{x_3}{< 0.89, 0.3, 0.17 >}, 0.95\right)\right\}.$$

Similarly, we get $Total_{complement}\left((\Gamma^{\mu}, E)_{\Lambda}\right)$ as follows:

$$Total_{complement}\left((\Gamma^{\mu}, E)_{\Lambda}\right) = (\Gamma^{\mu}, E)^{c}_{\Lambda^c} = \left\{e_1, \left\{\left(\frac{x_1}{< 0.8, 0.2, 0.25 >}\right), \left(\frac{x_2}{< 0.86, 0.1, 0.22 >}\right), \left(\frac{x_3}{< 0.92, 0.3, 0.13 >}\right), 0.95\right\},\right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{< 0.85, 0.4, 0.1 >}\right), \left(\frac{x_2}{< 0.71, 0.3, 0.07 >}\right), \left(\frac{x_3}{< 0.83, 0.1, 0.09 >}\right), 0.89\right\},\right.$$
$$\left.\left\{e_3, \left\{\left(\frac{x_1}{< 0.9, 0.2, 0.2 >}\right), \left(\frac{x_2}{< 0.93, 0.1, 0.15 >}\right), \left(\frac{x_3}{< 0.79, 0.2, 0.34 >}\right), 0.78\right\}\right\}.$$

Proposition 1. *Let* $(\Gamma, S)_{\Lambda}$ *be GENSS over the* U. *Then,*

1) $Total_{complement}\left(Total_{complement}(\Gamma^{\mu}, E)_{\Lambda}\right) = (\Gamma^{\mu}, E)_{\Lambda}$, *i.e.*, $((\Gamma^{\mu}, E)^{c}_{\Lambda^c})^{c} = (\Gamma^{\mu}, E)_{\Lambda}$;

2) $\Lambda_{complement}\left(\Lambda_{complement}(\Gamma^{\mu}, E)_{\Lambda}\right) = (\Gamma^{\mu}, E)_{\Lambda}$;

3) $Soft_{complement}\left(Soft_{complement}(\Gamma^{\mu}, E)_{\Lambda}\right) = (\Gamma^{\mu}, E)_{\Lambda}$.

Proof. 1) Let $(\Gamma^{\mu}, E)_{\Lambda}$ be GENSS over the U.

Then, $\Gamma^{\mu}(e_i)_{\Lambda} = \{\{(\Gamma(e_i)_{\Lambda}), \mu_{\Lambda}(e_i)\}, e_i \in E, \Gamma(e_i)_{\Lambda} \in N(U)$ and $\mu_{\Lambda}(\mathrm{e_i}) \in \mathrm{I} = [0, 1]\}$.

where

$$\Gamma^{\mu}(e_i)_{\Lambda} = \left\{\left(\frac{x_j}{< T_U(x_j)_{\Lambda}, I_U(x_j)_{\Lambda}, F_U(x_j)_{\Lambda} >} : x_j \in U, e_i \in E\right)\right\},$$

and

$$\mu(e_i)_\Lambda = \begin{cases} \mu(e_i) + \left[\frac{(1-\mu(e_i))\sum_j^n \sum_k^m \delta_{\Lambda_{x_j}}(a_k)}{|A||U|}\right], & \text{if } \mu(e_i) \in (0,1); \\ \mu(e_i), & \text{O.W.} \end{cases}$$

Then, $Total_{complement}\left((\Gamma, E)_\Lambda\right) = (\Gamma^\mu, E)^c_{\Lambda^c} =$

$$\left\{\left(e_i, \frac{x_j}{< (T_{U^c}\left(x_j\right)_{\Lambda^c})^c, (I_{U^c}\left(x_j\right)_{\Lambda^c})^c,\ (F_{U^c}\left(x_j\right)_{\Lambda^c})^c >}\right), (\mu^c(e_i)_{\Lambda^c})^c\right\}.$$

where,

$$\begin{aligned} (T_{U^c}\left(x_j\right)_{\Lambda^c})^c &= T(_{U^c})^c\left(x_j\right)_{(\Lambda^c)^c} \\ &= \begin{cases} T_{(U^c)^c}\left(x_j\right) + \left[\frac{[1-T_{(U^c)^c}(x_j)]\sum_k \delta_{((\Lambda_{x_j})^c)^c}(a_k)}{|A|}\right], & \text{if } T_U\left(x_j\right) \in (0,1); \\ T_{(U^c)^c}\left(x_j\right), & \text{O.W.} \end{cases} \\ &= \begin{cases} T_U\left(x_j\right) + \left[\frac{[1-T_U(x_j)]\sum_k \delta_{\Lambda_{x_j}}(a_k)}{|A|}\right], & \text{if } T_U\left(x_j\right) \in (0,1); \\ T_U\left(x_j\right), & \text{O.W.} \end{cases} \end{aligned}$$

So, $(T_{U^c}(xj)_{\Lambda^c})^c = T_U\left(x_j\right)_\Lambda$.
Similarly, $(I(x_j)_{\Lambda^c})^c = I_U\left(x_j\right)_\Lambda$, $(F_{U^c}(x_j)_{\Lambda^c})^c = F_U\left(x_j\right)_\Lambda$ and

$$\mu^c(e_i)_{\Lambda^c} = \begin{cases} \mu^c(e_i) + \left[\frac{(1-\mu^c(e_i))\sum_j^n \sum_k^m \delta_{(\Lambda_{x_j})^c}(a_k)}{|A||U|}\right], & \text{if } \mu(e_i) \in (0,1); \\ \mu^c(e_i), & \text{O.W.} \end{cases}$$

$$\begin{aligned} (\mu^c(e_i)_{\Lambda^c})^c &= \begin{cases} (\mu^c(e_i))^c + \left[\frac{\left[1-(\mu^c(e_i))^c\right]\sum_j^n \sum_k^m \delta_{((\Lambda_{x_j})^c)^c}(a_k)}{|A||U|}\right], & \text{if } \mu(e_i) \in (0,1); \\ (\mu^c(e_i))^c, & \text{O.W.} \end{cases} \\ &= \begin{cases} \mu(e_i) + \left[\frac{(1-\mu(e_i))\sum_j^n \sum_k^m \delta_{\Lambda_{x_j}}(a_k)}{|A||U|}\right], & \text{if } \mu(e_i) \in (0,1); \\ \mu(e_i), & \text{O.W.} \end{cases} \\ &= \mu(e_i)_\Lambda \end{aligned}$$

Hence, $Total_{complement}\left(Total_{complement}\ (\Gamma^\mu, E)_\Lambda\right) = (\Gamma^\mu, E)_\Lambda$.

The proofs 2 and 3 can be easily obtained from relative definitions.

Definition 3.6. Let $(\Gamma^\mu, E_1)_{\Lambda_1}$ and $(\Psi^\eta, E_2)_{\Lambda_2}$ be two GENSSs over U. Let $E = E_1 \cup E_2$. Then, the union of two GENSS is given as follows:

$$\Phi^\nu(\varepsilon)_{\Lambda_s} = \begin{cases} \Gamma^\mu(\varepsilon)_{\Lambda_s}, & \text{if } \varepsilon \in E_1 - E_2; \\ \Psi^\eta(\varepsilon)_{\Lambda_s}, & \text{if } \varepsilon \in E_2 - E_1; \\ (\Gamma \cup \Psi)^\nu(\varepsilon)_{\Lambda_s}, & \text{if } \varepsilon \in E_1 \cap E_2, \end{cases}$$

where s is any s-norm, $\cup$ is neutrosophic soft union between Γ and Ψ, and $\nu(\varepsilon) = s(\mu(\varepsilon), \eta(\varepsilon))$.

Example 3.4. *Let* $E_1 = \{e_1, e_2, e_4\}$ *and* $E_2 = \{e_1, e_2, e_3\}$*, over the common universe* $U = \{x_1, x_2, x_3\}$ *and let* $A = \{a_1, a_2, a_3, a_4\}$ *be the set of effective parameters. Consider two effective sets given as follows:*

$$\Lambda_1(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.7}\right\}, \Lambda_1(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\},$$
$$\Lambda_1(x_3) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0}, \frac{a_3}{0.6}, \frac{a_4}{0.4}\right\}, \Lambda_2(x_1) = \left\{\frac{a_1}{0.2}, \frac{a_2}{1}, \frac{a_3}{0.5}, \frac{a_4}{0.4}\right\},$$
$$\Lambda_2(x_2) = \left\{\frac{a_1}{0.6}, \frac{a_2}{0.4}, \frac{a_3}{0.5}, \frac{a_4}{0.2}\right\}, \Lambda_2(x_3) = \left\{\frac{a_1}{0.8}, \frac{a_2}{0.2}, \frac{a_3}{0.6}, \frac{a_4}{0.6}\right\}.$$

Consider two GNSSs given as follows:

$$(\Gamma^{\mu}, E_1) = \left\{e_1, \left\{\left(\frac{x_1}{< 0.5, 0.2, 0.6 >}, \frac{x_2}{< 0.3, 0.1, 0.8 >}, \frac{x_3}{< 0.3, 0.3, 0.8 >}\right), 0.6\right\},\right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{< 0.2, 0.4, 0.7 >}, \frac{x_2}{< 0.1, 0.3, 0.8 >}, \frac{x_3}{< 0.2, 0.1, 0.6 >}\right), 0.2\right\},\right.$$
$$\left.\left\{e_4, \left\{\left(\frac{x_1}{< 0.4, 0.2, 0.8 >}, \frac{x_2}{< 0.2, 0.1, 0.9 >}, \frac{x_3}{< 0.8, 0.2, 0.5 >}\right), 0.5\right\}\right\}.$$

$$(\Psi^{\eta}, E_2) = \left\{e_1, \left\{\left(\frac{x_1}{< 0.2, 0.1, 0.4 >}, \frac{x_2}{< 0.4, 0.2, 0.9 >}, \frac{x_3}{< 0.2, 0.4, 0.7 >}\right), 0.4\right\},\right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{< 0.1, 0.2, 0.4 >}, \frac{x_2}{< 0.3, 0.5, 0.4 >}, \frac{x_3}{< 0.3, 0.4, 0.1 >}\right), 0.7\right\},\right.$$
$$\left.\left\{e_3, \left\{\left(\frac{x_1}{< 0.3, 0.1, 0.6 >}, \frac{x_2}{< 0.4, 0.2, 0.6 >}, \frac{x_3}{< 0.3, 0.2, 0.4 >}\right), 0.3\right\}\right\}.$$

Then, the union of two GNSSs is given as follows:

$$(\Phi^{\nu}, E) = \left\{e_1, \left\{\left(\frac{x_1}{< 0.5, 0.1, 0.4 >}, \frac{x_2}{< 0.4, 0.1, 0.8 >}, \frac{x_3}{< 0.3, 0.3, 0.7 >}\right), 0.6\right\},\right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{< 0.2, 0.2, 0.4 >}, \frac{x_2}{< 0.3, 0.3, 0.4 >}, \frac{x_3}{< 0.3, 0.1, 0.1 >}\right), 0.7\right\},\right.$$
$$\left\{e_3, \left\{\left(\frac{x_1}{< 0.3, 0.1, 0.6 >}, \frac{x_2}{< 0.4, 0.2, 0.6 >}, \frac{x_3}{< 0.3, 0.2, 0.4 >}\right), 0.3\right\},\right.$$
$$\left.\left\{e_4, \left\{\left(\frac{x_1}{< 0.4, 0.2, 0.8 >}, \frac{x_2}{< 0.2, 0.1, 0.9 >}, \frac{x_3}{< 0.8, 0.2, 0.5 >}\right), 0.5\right\}\right\}.$$

We get the following effective set by using the basic fuzzy union (max):

$$\Lambda_s(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{1}, \frac{a_3}{1}, \frac{a_4}{0.7}\right\}, \Lambda_s(x_2) = \left\{\frac{a_1}{0.6}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\},$$
$$\Lambda_s(x_3) = \left\{\frac{a_1}{0.8}, \frac{a_2}{0.2}, \frac{a_3}{0.6}, \frac{a_4}{0.6}\right\}.$$

Now, we apply Definitions 3.6 and 3.1 to find $\Phi^{\nu}(e_1)_{\Lambda_s}$ as follows:

$$\Phi(e_1)_{\Lambda_s}(x_j) = \left\{ \frac{x_1}{< 0.5 + [(1-0.5)(0.3+1+1+0.7)/4], 0.1, 0.4 - (0.4)[(0.3+1+1+0.7)/4] >}, \frac{x_2}{< 0.4 + [(1-0.4)(0.6+0.5+1+1)/4], 0.1, 0.8 - [(0.8)(0.6+0.5+1+1)/4] >}, \frac{x_3}{< 0.3 + [(1-0.3)(0.8+0.2+0.6+0.6)/4], 0.3, 0.7 - [(0.7)(0.8+0.2+0.6+0.6)/4] >} \right\}$$
$$= \left\{ \frac{x_1}{< 0.7, 0.1, 0.24 >}, \frac{x_2}{< 0.87, 0.1, 0.18 >}, \frac{x_3}{< 0.69, 0.3, 0.32 >} \right\}.$$

Then, we find $\mu(e_1)_{\Lambda_s}$ as follows:

$$\mu(e_1)_{\Lambda_s}(x_1) = 0.6 + [(1-0.6)(0.3+1+1+0.7+0.6+0.5+1+1+0.8+0.2+0.6+0.6)/12] = 0.88.$$

Consequently,

$$\Phi^{\nu}(e_1)_{\Lambda_s}(x_j) = \left\{ \left(\frac{x_1}{< 0.7, 0.1, 0.24 >}, \frac{x_2}{< 0.87, 0.1, 0.18 >}, \frac{x_3}{< 0.69, 0.3, 0.32 >} \right), 0.88 \right\}.$$

Similarly, we get the GENSS $(\Phi^{\nu}, E)_{\Lambda_s}$ as follows:

$$(\Phi^{\nu}, E)_{\Lambda_s} = \left\{ e_1, \left\{ \left(\frac{x_1}{< 0.7, 0.1, 0.24 >}, \frac{x_2}{< 0.87, 0.1, 0.18 >}, \frac{x_3}{< 0.69, 0.3, 0.32 >} \right), 0.88 \right\}, \right.$$
$$\left\{ e_2, \left\{ \left(\frac{x_1}{< 0.52, 0.2, 0.24 >}, \frac{x_2}{< 0.84, 0.3, 0.09 >}, \frac{x_3}{< 0.69, 0.1, 0.05 >} \right), 0.91 \right\}, \right.$$
$$\left\{ e_3, \left\{ \left(\frac{x_1}{< 0.58, 0.1, 0.36 >}, \frac{x_2}{< 0.87, 0.2, 0.14 >}, \frac{x_3}{< 0.69, 0.2, 0.18 >} \right), 0.78 \right\}; \right.$$
$$\left. \left\{ e_4, \left\{ \left(\frac{x_1}{< 0.64, 0.2, 0.48 >}, \frac{x_2}{< 0.82, 0.1, 0.20 >}, \frac{x_3}{< 0.81, 0.2, 0.23 >} \right), 0.85 \right\} \right\} \right\}.$$

Definition 3.7. Let $(\Gamma^{\mu}, E_1)_{\Lambda_1}$ and $(\Psi^{\eta}, E_2)_{\Lambda_2}$ be two GENSSs over U and let $E = E_1 \cup E_2$. Then, the intersection of two GENSS is given as follows:

$$\vartheta^{\xi}(\varepsilon)_{\Lambda_t} = \begin{cases} \Gamma^{\mu}(\varepsilon)_{\Lambda_t}, & \text{if} \quad \varepsilon \in E_1 - E_2, \\ \Psi^{\xi}(\varepsilon)_{\Lambda_t}, & \text{if} \quad \varepsilon \in E_2 - E_1, \\ (\Gamma \cup \Psi)^{\xi}(\varepsilon)_{\Lambda_t} & \text{if} \quad \varepsilon \in E_1 \cap E_2, \end{cases}$$

where t is any t-norm, $\cap$ is a neutrosophic soft intersection between Γ and Ψ, and $\xi(\varepsilon) = t(\mu(\varepsilon), \eta(\varepsilon))$.

Example 3.5. *Consider Example 3.4. By using the basic fuzzy intersection, we have:*

$$\Lambda_t(x_1) = \left\{ \frac{a_1}{0.2}, \frac{a_2}{0}, \frac{a_3}{0.5}, \frac{a_4}{0.4} \right\}, \Lambda_t(x_2) = \left\{ \frac{a_1}{0.4}, \frac{a_2}{0.4}, \frac{a_3}{0.5}, \frac{a_4}{0.2} \right\},$$
$$\Lambda_t(x_3) = \left\{ \frac{a_1}{0.7}, \frac{a_2}{0}, \frac{a_3}{0.4}, \frac{a_4}{0.4} \right\}.$$

The intersection of two GNSSs is given as follow:

$$\left(\vartheta^{\xi}, E\right) = \left\{e_1, \left\{\left(\frac{x_1}{<0.2,0.2,0.6>}, \frac{x_2}{<0.3,0.2,0.9>}, \frac{x_3}{<0.2,0.4,0.8>}\right), 0.4\right\}, \right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{<0.1,0.4,0.7>}, \frac{x_2}{<0.1,0.5,0.8>}, \frac{x_3}{<0.2,0.4,0.6>}\right), 0.2\right\},$$
$$\left\{e_3, \left\{\left(\frac{x_1}{<0.3,0.1,0.6>}, \frac{x_2}{<0.4,0.2,0.6>}, \frac{x_3}{<0.3,0.2,0.4>}\right), 0.3\right\},$$
$$\left. \left\{e_4, \left\{\left(\frac{x_1}{<0.4,0.2,0.8>}, \frac{x_2}{<0.2,0.1,0.9>}, \frac{x_3}{<0.8,0.2,0.5>}\right), 0.5\right\}\right\}.$$

Now, we apply Definitions 3.7 and 3.1 to find $\vartheta^{\xi}(e_1)_{\Lambda_t}$ as follows:

$$\vartheta(e_1)_{\Lambda_t}(x_j) = \left\{\frac{x_1}{<0.2+[(1-0.2)(0.2+0+0.5+0.4)/4], 0.2, 0.6-(0.6)[(0.2+0+0.5+0.4)/4]>}, \right.$$
$$\frac{x_2}{<0.3+[(1-0.3)(0.4+0.4+0.5+0.2)/4], 0.2, 0.9-[(0.9)(0.4+0.4+0.5+0.2)/4]>},$$
$$\left. \frac{x_3}{<0.2+[(1-0.2)(0.7+0+0.4+0.4)/4], 0.4, 0.8-[(0.8)(0.7+0+0.4+0.4)/4]>}\right\}$$
$$= \left\{\frac{x_1}{<0.42,0.2,0.44>}, \frac{x_2}{<0.56,0.2,0.56>}, \frac{x_3}{<0.5,0.4,0.5>}\right\}.$$

Then, we find $\mu(e_1)_{\Lambda_t}$ as follows:

$$\mu(e_1)_{\Lambda_t}(x_1) = 0.4+[(1-0.4)(0.2+0+0.5+0.4+0.4+0.4+0.5+0.2+0.7+0+0.4+0.4)/12] = 0.61.$$

Consequently,

$$\vartheta^{\xi}(e_1)_{\Lambda_t}(x_j) = \left\{\left(\frac{x_1}{<0.42,0.2,0.44>}, \frac{x_2}{<0.56,0.2,0.56>}, \frac{x_3}{<0.5,0.4,0.5>}\right), 0.61\right\}.$$

Similarly, we get the GENSS $\left(\vartheta^{\xi}, E\right)_{\Lambda_t}$ as follows:

$$\left(\vartheta^{\xi}, E\right)_{\Lambda_t} = \left\{e_1, \left\{\left(\frac{x_1}{<0.42,0.2,0.44>}, \frac{x_2}{<0.56,0.2,0.56>}, \frac{x_3}{<0.5,0.4,0.5>}\right), 0.61\right\}, \right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{<0.35,0.4,0.51>}, \frac{x_2}{<0.44,0.5,0.5>}, \frac{x_3}{<0.5,0.4,0.38>}\right), 0.47\right\},$$
$$\left\{e_3, \left\{\left(\frac{x_1}{<0.49,0.1,0.44>}, \frac{x_2}{<0.63,0.2,0.38>}, \frac{x_3}{<0.56,0.2,0.25>}\right), 0.54\right\},$$
$$\left. \left\{e_4, \left\{\left(\frac{x_1}{<0.57,0.2,0.58>}, \frac{x_2}{<0.5,0.1,0.56>}, \frac{x_3}{<0.88,0.2,0.31>}\right), 0.67\right\}\right\}.$$

Proposition 2. *Let $(\Gamma^{\mu}, E_1)_{\Lambda_1}$ and $(\Phi^{\eta}, E_2)_{\Lambda_2}$ be two GENSSs over the common universe U. Then,*

1) $(\Gamma^{\mu}, E_1)_{\Lambda_1} \cup (\Phi^{\eta}, E_2)_{\Lambda_2} = (\Phi^{\eta}, E_2)_{\Lambda_2} \cup (\Gamma^{\mu}, E_1)_{\Lambda_1}$,

2) $(\Gamma^{\mu}, E_1)_{\Lambda_1} \cap (\Phi^{\eta}, E_2)_{\Lambda_2} = (\Phi^{\eta}, E_2)_{\Lambda_2} \cap (\Gamma^{\mu}, E_1)_{\Lambda_1}$.

Proof. The proof is straightforward from Definitions 3.6 and 3.7.

Definition 3.8. Let $(\Gamma^{\mu}, E_1)_{\Lambda_1}$ AND $(\Psi^{\eta}, E_2)_{\Lambda_2}$ be two GENSSs over U. Then, "$(\Gamma^{\mu}, E_1)_{\Lambda_1}$ and $(\Psi^{\eta}, E_2)_{\Lambda_2}$" denoted by$(\Gamma^{\mu}, E_1)_{\Lambda_1} \wedge (\Psi^{\eta}, E_2)_{\Lambda_2}$ and defined by:

$$(\Gamma^{\mu}, E_1)_{\Lambda_1} \wedge (\Psi^{\eta}, E_2)_{\Lambda_2} = \left(\vartheta^{\xi}, E_1 \times E_2\right)_{\Lambda_t},$$

where $\vartheta^{\xi}(\alpha,\beta)_{\Lambda_t} = (\Gamma^{\mu}(\alpha) \cap \psi^{\eta}(\beta))_{\Lambda_t} \, \forall (\alpha,\beta) \in E_1 \times E_2$. Such that t is any t-norm, $\xi(\alpha,\beta) = t(\mu(\alpha), \eta(\beta)) \;\; \forall (\alpha,\beta) \in E_1 \times E_2$ and $\vartheta^{\xi}(\alpha,\beta)_{\Lambda_t}$ is the generalized effective neutrosophic soft intersection between $(\Gamma^{\mu}, E_1)_{\Lambda_1}$ and $(\Psi^{\eta}, E_2)_{\Lambda_2}$.

Example 3.6. *Let $E_1 = \{e_1, e_2, e_4\}$ and $E_2 = \{e_1, e_2, e_3\}$, over the common universe $U = \{x_1, x_2, x_3\}$ and let $A = \{a_1, a_2, a_3, a_4\}$ be the set of effective parameters. Consider two effective sets given as follows:*

$$\Lambda_1(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.7}\right\}, \Lambda_1(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\},$$
$$\Lambda_1(x_3) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0}, \frac{a_3}{0.6}, \frac{a_4}{0.4}\right\}, \Lambda_2(x_1) = \left\{\frac{a_1}{0.2}, \frac{a_2}{1}, \frac{a_3}{0.5}, \frac{a_4}{0.4}\right\},$$
$$\Lambda_2(x_2) = \left\{\frac{a_1}{0.6}, \frac{a_2}{0.4}, \frac{a_3}{0.5}, \frac{a_4}{0.2}\right\}, \Lambda_2(x_3) = \left\{\frac{a_1}{0.8}, \frac{a_2}{0.2}, \frac{a_3}{0.6}, \frac{a_4}{0.6}\right\}.$$

Consider two GNSSs given as follows:

$$(\Gamma^{\mu}, E_1) = \left\{e_1, \left\{\left(\frac{x_1}{<0.5, 0.2, 0.6>}, \frac{x_2}{<0.3, 0.1, 0.8>}, \frac{x_3}{<0.3, 0.3, 0.8>}\right), 0.6\right\},\right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{<0.2, 0.4, 0.7>}, \frac{x_2}{<0.1, 0.3, 0.8>}, \frac{x_3}{<0.2, 0.1, 0.6>}\right), 0.2\right\},$$
$$\left.\left\{e_4, \left\{\left(\frac{x_1}{<0.4, 0.2, 0.8>}, \frac{x_2}{<0.2, 0.1, 0.9>}, \frac{x_3}{<0.8, 0.2, 0.5>}\right), 0.5\right\}\right\}.$$

$$(\Psi^{\eta}, E_2) = \left\{e_1, \left\{\left(\frac{x_1}{<0.2, 0.1, 0.4>}, \frac{x_2}{<0.4, 0.2, 0.9>}, \frac{x_3}{<0.2, 0.4, 0.7>}\right), 0.4\right\},\right.$$
$$\left\{e_2, \left\{\left(\frac{x_1}{<0.1, 0.2, 0.4>}, \frac{x_2}{<0.3, 0.5, 0.4>}, \frac{x_3}{<0.3, 0.4, 0.1>}\right), 0.7\right\},$$
$$\left.\left\{e_3, \left\{\left(\frac{x_1}{<0.3, 0.1, 0.6>}, \frac{x_2}{<0.4, 0.2, 0.6>}, \frac{x_3}{<0.3, 0.2, 0.4>}\right), 0.3\right\}\right\}.$$

We get the following effective set by using the basic fuzzy intersection (min):

$$\Lambda_t(x_1) = \left\{\frac{a_1}{0.2}, \frac{a_2}{0}, \frac{a_3}{0.5}, \frac{a_4}{0.4}\right\}, \Lambda_t(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.4}, \frac{a_3}{0.5}, \frac{a_4}{0.2}\right\},$$
$$\Lambda_t(x_3) = \left\{\frac{a_1}{0.8}, \frac{a_2}{0.2}, \frac{a_3}{0.6}, \frac{a_4}{0.6}\right\}.$$

Using AND operation on two generalized neutrosophic soft set, we get
$(\Gamma^{\mu}, E_1) \ \wedge (\Psi^{\eta}, E_2) = \left(\vartheta^{\xi}, E_1 \times E_2\right)$, where

$$\begin{aligned}
\left(\vartheta^{\xi}, E_1 \times E_2\right) = & \left\{(e_1, e_1), \left\{\left(\frac{x_1}{< 0.2, 0.2, 0.6 >}, \frac{x_2}{< 0.3, 0.2, 0.9 >}, \frac{x_3}{< 0.2, 0.4, 0.8 >}\right), 0.4\right\},\right. \\
& \left\{(e_1, e_2), \left\{\left(\frac{x_1}{< 0.1, 0.2, 0.6 >}, \frac{x_2}{< 0.3, 0.5, 0.8 >}, \frac{x_3}{< 0.3, 0.4, 0.8 >}\right), 0.6\right\},\right. \\
& \left\{(e_1, e_3), \left\{\left(\frac{x_1}{< 0.3, 0.2, 0.6 >}, \frac{x_2}{< 0.3, 0.2, 0.8 >}, \frac{x_3}{< 0.3, 0.3, 0.8 >}\right), 0.3\right\},\right. \\
& \left\{(e_2, e_1), \left\{\left(\frac{x_1}{< 0.2, 0.4, 0.7 >}, \frac{x_2}{< 0.1, 0.3, 0.9 >}, \frac{x_3}{< 0.2, 0.4, 0.7 >}\right), 0.2\right\},\right. \\
& \left\{(e_2, e_2), \left\{\left(\frac{x_1}{< 0.1, 0.4, 0.7 >}, \frac{x_2}{< 0.1, 0.5, 0.8 >}, \frac{x_3}{< 0.2, 0.4, 0.6 >}\right), 0.2\right\},\right. \\
& \left\{(e_2, e_3), \left\{\left(\frac{x_1}{< 0.2, 0.4, 0.7 >}, \frac{x_2}{< 0.1, 0.3, 0.8 >}, \frac{x_3}{< 0.2, 0.2, 0.6 >}\right), 0.2\right\},\right. \\
& \left\{(e_4, e_1), \left\{\left(\frac{x_1}{< 0.2, 0.2, 0.8 >}, \frac{x_2}{< 0.2, 0.2, 0.9 >}, \frac{x_3}{< 0.2, 0.4, 0.7 >}\right), 0.4\right\},\right. \\
& \left\{(e_4, e_2), \left\{\left(\frac{x_1}{< 0.1, 0.2, 0.8 >}, \frac{x_2}{< 0.2, 0.5, 0.9 >}, \frac{x_3}{< 0.3, 0.4, 0.5 >}\right), 0.5\right\},\right. \\
& \left\{(e_4, e_3), \left\{\left(\frac{x_1}{< 0.3, 0.2, 0.8 >}, \frac{x_2}{< 0.2, 0.2, 0.9 >}, \frac{x_3}{< 0.3, 0.2, 0.5 >}\right), 0.3\right\}\right\}.
\end{aligned}$$

Then, by applying Definitions 3.8 and 3.1, we obtain the GENSS $\vartheta^{\xi}(e_1, e_1)_{\Lambda_t}$ as follows:

$$\begin{aligned}
\vartheta(e_1, e_1)_{\Lambda_t}(x_j) = & \left\{\frac{x_1}{< 0.2 + [(1 - 0.2)(0.2 + 0 + 0.5 + 0.4)/4], 0.2, 0.6 - (0.6)[(0.2 + 0 + 0.5 + 0.4)/4] >},\right. \\
& \frac{x_2}{< 0.3 + [(1 - 0.3)(0.4 + 0.4 + 0.5 + 0.2)/4], 0.2, 0.9 - [(0.9)(0.4 + 0.4 + 0.5 + 0.2)/4] >}, \\
& \left.\frac{x_3}{< 0.2 + [(1 - 0.2)(0.7 + 0 + 0.4 + 0.4)/4], 0.4, 0.8 - [(0.8)(0.7 + 0 + 0.4 + 0.4)/4] >}\right\} \\
= & \left\{\frac{x_1}{< 0.42, 0.2, 0.44 >}, \frac{x_2}{< 0.56, 0.2, 0.56 >}, \frac{x_3}{< 0.5, 0.4, 0.5 >}\right\}.
\end{aligned}$$

Then, we find $\mu(e_1)_{\Lambda_t}$ as follows:

$$\mu(e_1)_{\Lambda_t}(x_1) = 0.4 + [(1 - 0.4)(0.2 + 0 + 0.5 + 0.4 + 0.4 + 0.4 + 0.5 + 0.2 + 0.7 + 0 + 0.4 + 0.4)/12] = 0.61.$$

Consequently,

$$\vartheta^{\xi}(e_1, e_1)_{\Lambda_t}(x_j) = \left\{\left(\frac{x_1}{< 0.42, 0.2, 0.44 >}, \frac{x_2}{< 0.56, 0.2, 0.56 >}, \frac{x_3}{< 0.5, 0.4, 0.5 >}\right), 0.61\right\}.$$

Similarly, we get the GENSS $\left(\vartheta^{\xi}, E_1 \times E_2\right)_{\Lambda_t}$ as follows:

$$\begin{aligned}\left(\vartheta^{\xi}, E_1 \times E_2\right)_{\Lambda_t} = &\left\{(e_1, e_1), \left\{\left(\frac{x_1}{< 0.42, 0.2, 0.44 >}, \frac{x_2}{< 0.56, 0.2, 0.56 >}, \frac{x_3}{< 0.5, 0.4, 0.5 >}\right), 0.61\right\},\right.\\
&\left\{(e_1, e_2), \left\{\left(\frac{x_1}{< 0.35, 0.2, 0.44 >}, \frac{x_2}{< 0.56, 0.5, 0.5 >}, \frac{x_3}{< 0.56, 0.4, 0.5 >}\right), 0.74\right\},\right.\\
&\left\{(e_1, e_3), \left\{\left(\frac{x_1}{< 0.5, 0.2, 0.45 >}, \frac{x_2}{< 0.56, 0.2, 0.5 >}, \frac{x_3}{< 0.56, 0.3, 0.5 >}\right), 0.54\right\},\right.\\
&\left\{(e_2, e_1), \left\{\left(\frac{x_1}{< 0.42, 0.4, 0.51 >}, \frac{x_2}{< 0.44, 0.3, 0.56 >}, \frac{x_3}{< 0.5, 0.4, 0.44 >}\right), 0.47\right\},\right.\\
&\left\{(e_2, e_2), \left\{\left(\frac{x_1}{< 0.35, 0.4, 0.51 >}, \frac{x_2}{< 0.44, 0.5, 0.5 >}, \frac{x_3}{< 0.5, 0.4, 0.38 >}\right), 0.47\right\},\right.\\
&\left\{(e_2, e_3), \left\{\left(\frac{x_1}{< 0.42, 0.4, 0.51 >}, \frac{x_2}{< 0.44, 0.3, 0.5 >}, \frac{x_3}{< 0.5, 0.2, 0.38 >}\right), 0.47\right\},\right.\\
&\left\{(e_4, e_1), \left\{\left(\frac{x_1}{< 0.42, 0.2, 0.58 >}, \frac{x_2}{< 0.5, 0.2, 0.56 >}, \frac{x_3}{< 0.5, 0.4, 0.44 >}\right), 0.61\right\},\right.\\
&\left\{(e_4, e_2), \left\{\left(\frac{x_1}{< 0.35, 0.2, 0.58 >}, \frac{x_2}{< 0.5, 0.5, 0.56 >}, \frac{x_3}{< 0.56, 0.4, 0.31 >}\right), 0.67\right\},\right.\\
&\left\{(e_4, e_3), \left\{\left(\frac{x_1}{< 0.49, 0.2, 0.58 >}, \frac{x_2}{< 0.5, 0.2, 0.56 >}, \frac{x_3}{< 0.56, 0.2, 0.31 >}\right), 0.54\right\}\right\}.\end{aligned}$$

Definition 3.9. Let $(\Gamma^{\mu}, E_1)_{\Lambda_1}$ and $(\Psi^{\eta}, E_2)_{\Lambda_2}$ be two GENSSs over U. Then, "$(\Gamma^{\mu}, E_1)_{\Lambda_1}$ OR $(\Psi^{\eta}, E_2)_{\Lambda_2}$" denoted by $(\Gamma^{\mu}, E_1)_{\Lambda_1} \vee (\Psi^{\eta}, E_2)_{\Lambda_2}$ and defined by:

$$(\Gamma^{\mu}, E_1)_{\Lambda_1} \vee (\Psi^{\eta}, E_2)_{\Lambda_2} = (\Phi^{\nu}, E_1 \times E_2)_{\Lambda_s},$$

where $\Phi^{\nu}(\alpha,\beta)_{\Lambda_s} = (\Gamma^{\mu}(\alpha) \cup \psi^{\eta}(\beta))_{\Lambda_t} \forall (\alpha,\beta) \in E_1 \times E_2$. Such that s is any s-norm, $\nu(\alpha,\beta) = s(\mu(\alpha), \eta(\beta))\ \forall (\alpha,\beta) \in E_1 \times E_2$ and $\Phi^{\nu}(\alpha,\beta)_{\Lambda_s}$ is the generalized effective neutrosophic soft union between $(\Gamma^{\mu}, E_1)_{\Lambda_1}$ and $(\Psi^{\eta}, E_2)_{\Lambda_2}$.

Example 3.7. *Consider Example 3.6, then We get the following effective set by using the basic fuzzy union (max):*

$$\Lambda_s(x_1) = \left\{\frac{a_1}{0.3}, \frac{a_2}{1}, \frac{a_3}{1}, \frac{a_4}{0.7}\right\}, \Lambda_s(x_2) = \left\{\frac{a_1}{0.6}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\},$$
$$\Lambda_s(x_3) = \left\{\frac{a_1}{0.8}, \frac{a_2}{0.2}, \frac{a_3}{0.6}, \frac{a_4}{0.6}\right\}.$$

Using OR operation on two generalized neutrosophic soft set, we get
$(\Gamma^{\mu}, E_1) \vee (\Psi^{\eta}, E_2) = (\Phi^{\nu}, E_1 \times E_2)$, where

$$
\begin{aligned}
(\Phi^{\nu}, E_1 \times E_2) = & \left\{(e_1, e_1), \left\{\left(\frac{x_1}{<0.5, 0.1, 0.4>}, \frac{x_2}{<0.4, 0.1, 0.8>}, \frac{x_3}{<0.3, 0.3, 0.7>}\right), 0.6\right\},\right. \\
& \left\{(e_1, e_2), \left\{\left(\frac{x_1}{<0.5, 0.2, 0.4>}, \frac{x_2}{<0.3, 0.1, 0.4>}, \frac{x_3}{<0.3, 0.3, 0.1>}\right), 0.7\right\},\right. \\
& \left\{(e_1, e_3), \left\{\left(\frac{x_1}{<0.5, 0.1, 0.6>}, \frac{x_2}{<0.4, 0.1, 0.6>}, \frac{x_3}{<0.3, 0.2, 0.4>}\right), 0.6\right\},\right. \\
& \left\{(e_2, e_1), \left\{\left(\frac{x_1}{<0.2, 0.1, 0.4>}, \frac{x_2}{<0.4, 0.2, 0.8>}, \frac{x_3}{<0.2, 0.1, 0.6>}\right), 0.4\right\},\right. \\
& \left\{(e_2, e_2), \left\{\left(\frac{x_1}{<0.7, 0.2, 0.4>}, \frac{x_2}{<0.3, 0.3, 0.4>}, \frac{x_3}{<0.3, 0.1, 0.1>}\right), 0.7\right\},\right. \\
& \left\{(e_2, e_3), \left\{\left(\frac{x_1}{<0.3, 0.1, 0.6>}, \frac{x_2}{<0.4, 0.2, 0.8>}, \frac{x_3}{<0.3, 0.1, 0.4>}\right), 0.3\right\},\right. \\
& \left\{(e_4, e_1), \left\{\left(\frac{x_1}{<0.4, 0.1, 0.4>}, \frac{x_2}{<0.4, 0.1, 0.9>}, \frac{x_3}{<0.8, 0.2, 0.5>}\right), 0.5\right\},\right. \\
& \left\{(e_4, e_2), \left\{\left(\frac{x_1}{<0.4, 0.2, 0.4>}, \frac{x_2}{<0.3, 0.1, 0.4>}, \frac{x_3}{<0.8, 0.2, 0.1>}\right), 0.7\right\},\right. \\
& \left\{(e_4, e_3), \left\{\left(\frac{x_1}{<0.4, 0.1, 0.6>}, \frac{x_2}{<0.4, 0.1, 0.6>}, \frac{x_3}{<0.8, 0.2, 0.4>}\right), 0.5\right\}\right\}.
\end{aligned}
$$

Then, by applying Definitions 3.9 and 3.1, we obtain the GENSS $\Phi^{\nu}(e_1, e_1)_{\Lambda_s}$ as follows:

$$
\begin{aligned}
\Phi(e_1, e_1)_{\Lambda_s}(x_j) = & \left\{\frac{x_1}{<0.5 + [(1-0.5)(0.3+1+1+0.7)/4], 0.1, 0.4 - [(0.4)(0.3+1+1+0.7)/4]>},\right. \\
& \frac{x_2}{<0.4 + [(1-0.4)(0.6+0.5+1+1)/4], 0.1, 0.8 - [(0.8)(0.6+0.5+1+1)/4]>}, \\
& \left.\frac{x_3}{<0.3 + [(1-0.3)(0.8+0.2+0.6+0.6)/4], 0.3, 0.7 - [(0.7)(0.8+0.2+0.6+0.6)/4]>}\right\} \\
= & \left\{\frac{x_1}{<0.88, 0.1, 0.1>}, \frac{x_2}{<0.87, 0.1, 0.18>}, \frac{x_3}{<0.69, 0.3, 0.32>}\right\}.
\end{aligned}
$$

Then, we find $\mu(e_1)_{\Lambda_s}$ as follows:

$$
\mu(e_1)_{\Lambda_s}(x_1) = 0.6 + [(1-0.6)(0.3+1+1+0.7+0.6+0.5+1+1+0.8+0.2+0.6+0.6)/12] = 0.88.
$$

Consequently,

$$
\vartheta^{\xi}(e_1, e_1)_{\Lambda_s}(x_j) = \left\{\left(\frac{x_1}{<0.88, 0.1, 0.1>}, \frac{x_2}{<0.87, 0.1, 0.18>}, \frac{x_3}{<0.69, 0.3, 0.32>}\right), 0.88\right\}.
$$

Similarly, we get the GENSS $(\Phi^{\nu}, E_1 \times E_2)_{\Lambda_s}$ as follows:

$$(\Phi^{\nu}, E_1 \times E_2)_{\Lambda_s} = \Big\{(e_1, e_1), \Big\{\Big(\frac{x_1}{<0.88, 0.1, 0.1>}, \frac{x_2}{<0.87, 0.1, 0.18>}, \frac{x_3}{<0.69, 0.3, 0.32>}\Big), 0.88\Big\},$$
$$\Big\{(e_1, e_2), \Big\{\Big(\frac{x_1}{<0.88, 0.2, 0.1>}, \frac{x_2}{<0.84, 0.1, 0.1>}, \frac{x_3}{<0.34, 0.3, 0.5>}\Big), 0.91\Big\},$$
$$\Big\{(e_1, e_3), \Big\{\Big(\frac{x_1}{<0.88, 0.1, 0.15>}, \frac{x_2}{<0.87, 0.1, 0.14>}, \frac{x_3}{<0.67, 0.2, 0.18>}\Big), 0.88\Big\},$$
$$\Big\{(e_2, e_1), \Big\{\Big(\frac{x_1}{<0.8, 0.1, 0.1>}, \frac{x_2}{<0.87, 0.2, 0.18>}, \frac{x_3}{<0.64, 0.1, 0.27>}\Big), 0.82\Big\},$$
$$\Big\{(e_2, e_2), \Big\{\Big(\frac{x_1}{<0.93, 0.2, 0.1>}, \frac{x_2}{<0.84, 0.3, 0.09>}, \frac{x_3}{<0.69, 0.1, 0.16>}\Big), 0.91\Big\},$$
$$\Big\{(e_2, e_3), \Big\{\Big(\frac{x_1}{<0.83, 0.1, 0.15>}, \frac{x_2}{<0.87, 0.2, 0.18>}, \frac{x_3}{<0.69, 0.1, 0.18>}\Big), 0.78\Big\},$$
$$\Big\{(e_4, e_1), \Big\{\Big(\frac{x_1}{<0.85, 0.1, 0.1>}, \frac{x_2}{<0.87, 0.1, 0.20>}, \frac{x_3}{<0.91, 0.2, 0.23>}\Big), 0.85\Big\},$$
$$\Big\{(e_4, e_2), \Big\{\Big(\frac{x_1}{<0.85, 0.2, 0.1>}, \frac{x_2}{<0.84, 0.1, 0.09>}, \frac{x_3}{<0.91, 0.2, 0.05>}\Big), 0.91\Big\},$$
$$\Big\{(e_4, e_3), \Big\{\Big(\frac{x_1}{<0.85, 0.1, 0.15>}, \frac{x_2}{<0.87, 0.1, 0.14>}, \frac{x_3}{<0.91, 0.2, 0.18>}\Big), 0.85\Big\}\Big\}.$$

4. An application of GENSS in decision making problem

An application of generalized effective neutrosophic soft sets in decision-making problem is introduced in this section.

Now we present an algorithm for most appropriate selection of an object.

4.1. Algorithm

We obtain the following algorithm for GENSS by combining the algorithms of Sahin and Kucuk algorithm [18] and Al-Hijjawi et al. [33].

4.1.1. New Algorithn

1) Construct the generalized neutrosophic soft sets (Γ^{μ}, E_1) and (ψ^{η}, E_2) .
2) Construct an effective set of parameters Λ_1 and Λ_2.
3) Find an effective set of parameters Λ_t from Λ_1 and Λ_2.
4) Compute the corresponding resultant GNSS$\left(\vartheta^{\xi}, E_1 \times E_2\right)$ as required.
5) Compute the corresponding GENSS $\left(\vartheta^{\xi}, E_1 \times E_2\right)_{\Lambda_t}$.
6) Introduce the tables of three basic components of ϑ^{ξ}, which are truth membership, indeterminacy membership and falsity membership respectively.

7) In each row, underline the greatest value.

8) Compute the score for each component by taking the sum of the multiplication of the underlying values with the corresponding values of ξ.

9) Compute the final score by adding the scores of truth membership part of ϑ^{ξ} to indeterminacy membership part of ϑ^{ξ}, then subtracting them from the falsity membership part of ϑ^{ξ}.

10) The optimal selection is the highest score.

4.2. Application in a decision-making problem

Let $U = \{x_1, x_2, x_3\}$ be a set of laptops with the same model. Let $E = \{e_1, e_2, e_3\}$ be a set of parameters where e_1= Size, e_2=CPU and e_3 = Battery. Let $A = \{a_1, a_2, a_3, a_4\}$ be set of effective parameters, where a_1: Each part was created at the original factory; a_2: It was reassembled at the original factory; a_3: The latest version of the software is running and a_4: It was not owned by multiple people.

Let the effective set over $A, \forall x_i \in U$ given by experts as follows:

$$\Lambda_1(x_1) = \left\{\frac{a_1}{0.7}, \frac{a_2}{1}, \frac{a_3}{1}, \frac{a_4}{0.6}\right\}, \Lambda_1(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.5}, \frac{a_3}{1}, \frac{a_4}{1}\right\},$$
$$\Lambda_1(x_3) = \left\{\frac{a_1}{0.3}, \frac{a_2}{0}, \frac{a_3}{1}, \frac{a_4}{0.7}\right\}, \Lambda_2(x_1) = \left\{\frac{a_1}{0.8}, \frac{a_2}{0.8}, \frac{a_3}{0.9}, \frac{a_4}{0.7}\right\},$$
$$\Lambda_2(x_2) = \left\{\frac{a_1}{0.6}, \frac{a_2}{0.4}, \frac{a_3}{0.5}, \frac{a_4}{0.2}\right\}, \Lambda_2(x_3) = \left\{\frac{a_1}{0.2}, \frac{a_2}{0.1}, \frac{a_3}{0.5}, \frac{a_4}{0.3}\right\}.$$

Let (Γ^{μ}, E) and (Ψ^{η}, E) be two generalized neutrosophic sets (GNSSs) defined as follows:

$$\begin{aligned}(\Gamma^{\mu}, E) =&\left\{e_1, \left\{\left(\frac{x_1}{< 0.7, 0.3, 0.2 >}, \frac{x_2}{< 0.5, 0.1, 0.8 >}, \frac{x_3}{< 0.9, 0.2, 0.4 >}\right), 0.2\right\},\right.\\ &\left\{e_2, \left\{\left(\frac{x_1}{< 0.3, 0.1, 0.5 >}, \frac{x_2}{< 0.6, 0, 0.2 >}, \frac{x_3}{< 0.5, 0.1, 0.2 >}\right), 0.5\right\},\right.\\ &\left.\left\{e_3, \left\{\left(\frac{x_1}{< 0.1, 0, 0.4 >}, \frac{x_2}{< 0.4, 0.2, 0.8 >}, \frac{x_3}{< 0.2, 0.6, 0.4 >}\right), 0.7\right\}\right\}\right\}.\end{aligned}$$

$$\begin{aligned}(\Psi^{\eta}, E) =&\left\{e_1, \left\{\left(\frac{x_1}{< 0.8, 0.1, 0.6 >}, \frac{x_2}{< 0.2, 0.6, 0.4 >}, \frac{x_3}{< 0.5, 0, 0.8 >}\right), 0.4\right\},\right.\\ &\left\{e_2, \left\{\left(\frac{x_1}{< 0.3, 0.5, 0.7 >}, \frac{x_2}{< 0.8, 0.2, 0.4 >}, \frac{x_3}{< 0.4, 0, 0.7 >}\right), 0.1\right\},\right.\\ &\left.\left\{e_3, \left\{\left(\frac{x_1}{< 0.9, 0.5, 0.7 >}, \frac{x_2}{< 0.4, 0.1, 0.2 >}, \frac{x_3}{< 0.9, 0.1, 0.4 >}\right), 0.6\right\}\right\}\right\}.\end{aligned}$$

Using AND operation on two generalized neutrosophic soft set, we get
$(\Gamma^{\mu}, E) \wedge (\Psi^{\eta}, E) = \left(\vartheta^{\xi}, E \times E\right)$, where

$$\left(\vartheta^{\xi}, E\times E\right) = \left\{(e_1,e_1), \left\{\left(\frac{x_1}{<0.7,0.3,0.6>}, \frac{x_2}{<0.2,0.6,0.8>}, \frac{x_3}{<0.5,0.2,0.8>}\right), 0.2\right\},\right.$$
$$\left\{(e_1,e_2), \left\{\left(\frac{x_1}{<0.3,0.5,0.7>}, \frac{x_2}{<0.5,0.2,0.8>}, \frac{x_3}{<0.4,0.2,0.7>}\right), 0.1\right\},$$
$$\left\{(e_1,e_3), \left\{\left(\frac{x_1}{<0.7,0.5,0.7>}, \frac{x_2}{<0.4,0.1,0.8>}, \frac{x_3}{<0.9,0.2,0.4>}\right), 0.2\right\},$$
$$\left\{(e_2,e_1), \left\{\left(\frac{x_1}{<0.3,0.1,0.6>}, \frac{x_2}{<0.2,0.6,0.4>}, \frac{x_3}{<0.5,0.1,0.8>}\right), 0.4\right\},$$
$$\left\{(e_2,e_2), \left\{\left(\frac{x_1}{<0.3,0.5,0.7>}, \frac{x_2}{<0.6,0.2,0.4>}, \frac{x_3}{<0.4,0.1,0.7>}\right), 0.1\right\},$$
$$\left\{(e_2,e_3), \left\{\left(\frac{x_1}{<0.3,0.5,0.7>}, \frac{x_2}{<0.4,0.1,0.2>}, \frac{x_3}{<0.5,0.1,0.4>}\right), 0.5\right\},$$
$$\left\{(e_3,e_1), \left\{\left(\frac{x_1}{<0.1,0.1,0.6>}, \frac{x_2}{<0.2,0.6,0.8>}, \frac{x_3}{<0.8,0.6,0.8>}\right), 0.4\right\},$$
$$\left\{(e_3,e_2), \left\{\left(\frac{x_1}{<0.1,0.5,0.7>}, \frac{x_2}{<0.4,0.2,0.8>}, \frac{x_3}{<0.2,0.6,0.7>}\right), 0.1\right\},$$
$$\left.\left\{(e_3,e_3), \left\{\left(\frac{x_1}{<0.1,0.5,0.7>}, \frac{x_2}{<0.4,0.2,0.8>}, \frac{x_3}{<0.2,0.6,0.4>}\right), 0.6\right\}\right\}.$$

The generalized neutrosophic soft set (GNSS) $\left(\vartheta^{\xi}, E\times E\right)$ represented in Table 1 below as follows:

Table 1. Tabular representation of $\left(\vartheta^{\xi}, E\times E\right)$.

U	x_1	x_2	x_3	ξ
(e_1,e_1)	$\langle 0.7,0.3,0.6\rangle$	$\langle 0.2,0.6,0.8\rangle$	$\langle 0.5,0.2,0.8\rangle$	0.2
(e_1,e_2)	$\langle 0.3,0.5,0.7\rangle$	$\langle 0.5,0.2,0.8\rangle$	$\langle 0.4,0.2,0.7\rangle$	0.1
(e_1,e_3)	$\langle 0.7,0.5,0.7\rangle$	$\langle 0.4,0.1,0.8\rangle$	$\langle 0.9,0.2,0.4\rangle$	0.2
(e_2,e_1)	$\langle 0.3,0.1,0.6\rangle$	$\langle 0.2,0.6,0.4\rangle$	$\langle 0.5,0.1,0.8\rangle$	0.4
(e_2,e_2)	$\langle 0.3,0.5,0.7\rangle$	$\langle 0.6,0.2,0.4\rangle$	$\langle 0.4,0.1,0.7\rangle$	0.1
(e_2,e_3)	$\langle 0.3,0.5,0.7\rangle$	$\langle 0.4,0.1,0.2\rangle$	$\langle 0.5,0.1,0.4\rangle$	0.5
(e_3,e_1)	$\langle 0.1,0.1,0.6\rangle$	$\langle 0.2,0.6,0.8\rangle$	$\langle 0.8,0.6,0.8\rangle$	0.4
(e_3,e_2)	$\langle 0.1,0.5,0.7\rangle$	$\langle 0.4,0.2,0.8\rangle$	$\langle 0.2,0.6,0.7\rangle$	0.1
(e_3,e_3)	$\langle 0.1,0.5,0.7\rangle$	$\langle 0.4,0.2,0.8\rangle$	$\langle 0.2,0.6,0.4\rangle$	0.6

Table 2. Tabular representation of truth membership of $\left(\vartheta^{\xi}, E \times E\right)$.

U	x_1	x_2	x_3	ξ
(e_1, e_1)	$\underline{0.7}$	0.2	0.5	0.2
(e_1, e_2)	0.3	$\underline{0.5}$	0.4	0.1
(e_1, e_3)	0.7	0.4	$\underline{0.9}$	0.2
(e_2, e_1)	0.3	0.2	$\underline{0.5}$	0.4
(e_2, e_2)	0.3	$\underline{0.6}$	0.4	0.1
(e_2, e_3)	0.3	0.4	$\underline{0.5}$	0.5
(e_3, e_1)	0.1	0.2	$\underline{0.8}$	0.4
(e_3, e_2)	0.1	$\underline{0.4}$	0.2	0.1
(e_3, e_3)	0.1	$\underline{0.4}$	0.2	0.6

Table 3. Tabular representation of indeterminacy membership of $\left(\vartheta^{\xi}, E \times E\right)$.

U	x_1	x_2	x_3	ξ
(e_1, e_1)	0.3	$\underline{0.6}$	0.2	0.2
(e_1, e_2)	$\underline{0.5}$	0.2	0.2	0.1
(e_1, e_3)	$\underline{0.5}$	0.1	0.2	0.2
(e_2, e_1)	0.1	$\underline{0.6}$	0.1	0.4
(e_2, e_2)	$\underline{0.5}$	0.2	0.1	0.1
(e_2, e_3)	$\underline{0.5}$	0.1	0.1	0.5
(e_3, e_1)	0.1	$\underline{0.6}$	$\underline{0.6}$	0.4
(e_3, e_2)	0.5	0.2	$\underline{0.6}$	0.1
(e_3, e_3)	0.5	0.2	$\underline{0.6}$	0.6

Table 4. Tabular representation of falsity membership of $\left(\vartheta^{\xi}, E \times E\right)$.

U	x_1	x_2	x_3	ξ
(e_1, e_1)	0.6	$\underline{0.8}$	$\underline{0.8}$	0.2
(e_1, e_2)	0.7	$\underline{0.8}$	0.7	0.1
(e_1, e_3)	0.7	$\underline{0.8}$	0.4	0.2
(e_2, e_1)	0.6	0.4	$\underline{0.8}$	0.4
(e_2, e_2)	$\underline{0.7}$	0.4	$\underline{0.7}$	0.1
(e_2, e_3)	$\underline{0.7}$	0.2	0.4	0.5
(e_3, e_1)	0.6	$\underline{0.8}$	$\underline{0.8}$	0.4
(e_3, e_2)	0.7	$\underline{0.8}$	0.7	0.1
(e_3, e_3)	0.7	$\underline{0.8}$	0.4	0.6

Now, we compute the score of each component of $\left(\vartheta^{\xi}, E \times E\right)$ by using Tables 2–4, respectively as follows:
$Score\,(x_1) = (0.7 \times 0.2) = 0.14$,
$Score\,(x_2) = (0.5 \times 0.1) + (0.6 \times 0.1) + (0.4 \times 0.1) + (0.4 \times 0.6) = 0.39$,
$Score\,(x_3) = (0.9 \times 0.2) + (0.5 \times 0.4) + (0.5 \times 0.5) + (0.8 \times 0.4) = 0.95$.

$Score\,(x_1) = (0.5 \times 0.1) + (0.5 \times 0.2) + (0.5 \times 0.1) + (0.5 \times 0.5) = 0.45,$
$Score\,(x_2) = (0.6 \times 0.2) + (0.6 \times 0.4) + (0.6 \times 0.4) = 0.6,$
$Score\,(x_3) = (0.6 \times 0.4) + (0.6 \times 0.1) + (0.6 \times 0.6) = 0.66.$

$Score\,(x_1) = (0.7 \times 0.1) + (0.7 \times 0.5) = 0.42,$
$Score\,(x_2) = (0.8 \times 0.2) + (0.8 \times 0.1) + (0.8 \times 0.2) + (0.8 \times 0.4) + (0.8 \times 0.1) + (0.8 \times 0.6) = 1.28,$
$Score\,(x_3) = (0.8 \times 0.2) + (0.8 \times 0.4) + (0.7 \times 0.1) + (0.8 \times 0.4) = 0.87.$
Then, we calculate the final score as follows:
$Score\,(x_1) = 0.14 + 0.45 - 0.42 = 0.17,$
$Score\,(x_2) = 0.39 + 0.6 - 1.28 = -0.29,$
$Score\,(x_3) = 0.95 + 0.66 - 0.87 = \mathbf{0.74}.$
The optimal selection is x_3.

We conclude that the optimal selection of GNSS by using Sahin and Kucuk algorithm [18] is laptop number 3.

Now, we find the following effective set by using the basic fuzzy intersection (min):

$$\Lambda_t\,(x_1) = \left\{\frac{a_1}{0.7}, \frac{a_2}{0.8}, \frac{a_3}{0.9}, \frac{a_4}{0.6}\right\}, \Lambda_t(x_2) = \left\{\frac{a_1}{0.4}, \frac{a_2}{0.4}, \frac{a_3}{0.5}, \frac{a_4}{0.2}\right\},$$
$$\Lambda_t(x_3) = \left\{\frac{a_1}{0.2}, \frac{a_2}{0}, \frac{a_3}{0.5}, \frac{a_4}{0.3}\right\}.$$

Then, by applying Definitions 3.8 and 3.1, we obtain the GENSS $\left(\vartheta^{\xi}, E \times E\right)_{\Lambda_t}$ as follows:

$$\begin{aligned}
\left(\vartheta^{\xi}, E \times E\right)_{\Lambda_t} = & \left\{(e_1, e_1), \left\{\left(\frac{x_1}{<0.93, 0.3, 0.15>}, \frac{x_2}{<0.5, 0.6, 0.5>}, \frac{x_3}{<0.63, 0.2, 0.6>}\right), 0.57\right\},\right. \\
& \left\{(e_1, e_2), \left\{\left(\frac{x_1}{<0.83, 0.5, 0.18>}, \frac{x_2}{<0.69, 0.2, 0.5>}, \frac{x_3}{<0.93, 0.2, 0.53>}\right), 0.51\right\},\right. \\
& \left\{(e_1, e_3), \left\{\left(\frac{x_1}{<0.93, 0.5, 0.18>}, \frac{x_2}{<0.63, 0.1, 0.5>}, \frac{x_3}{<0.85, 0.2, 0.3>}\right), 0.57\right\},\right. \\
& \left\{(e_2, e_1), \left\{\left(\frac{x_1}{<0.83, 0.1, 0.15>}, \frac{x_2}{<0.5, 0.6, 0.25>}, \frac{x_3}{<0.63, 0.1, 0.6>}\right), 0.68\right\},\right. \\
& \left\{(e_2, e_2), \left\{\left(\frac{x_1}{<0.83, 0.5, 0.18>}, \frac{x_2}{<0.75, 0.2, 0.25>}, \frac{x_3}{<0.55, 0.1, 0.53>}\right), 0.51\right\},\right. \\
& \left\{(e_2, e_3), \left\{\left(\frac{x_1}{<0.83, 0.5, 0.18>}, \frac{x_2}{<0.63, 0.1, 0.13>}, \frac{x_3}{<0.63, 0.1, 0.3>}\right), 0.73\right\},\right. \\
& \left\{(e_3, e_1), \left\{\left(\frac{x_1}{<0.78, 0.1, 0.15>}, \frac{x_2}{<0.5, 0.6, 0.5>}, \frac{x_3}{<0.4, 0.6, 0.6>}\right), 0.68\right\},\right. \\
& \left\{(e_3, e_2), \left\{\left(\frac{x_1}{<0.78, 0.5, 0.18>}, \frac{x_2}{<0.63, 0.2, 0.5>}, \frac{x_3}{<0.4, 0.6, 0.53>}\right), 0.51\right\},\right. \\
& \left\{(e_3, e_3), \left\{\left(\frac{x_1}{<0.78, 0.5, 0.18>}, \frac{x_2}{<0.63, 0.2, 0.5>}, \frac{x_3}{<0.4, 0.6, 0.3>}\right), 0.78\right\}\right\}.
\end{aligned}$$

The generalized effective neutrosophic soft set (GENSS) $\left(\vartheta^{\xi}, E\times E\right)_{\Lambda_t}$ represented in Table 5 below as follows:

Table 5. Tabular representation of $\left(\vartheta^{\xi}, E\times E\right)_{\Lambda_t}$.

U	x_1	x_2	x_3	ξ
(e_1, e_1)	$\langle 0.93, 0.3, 0.15\rangle$	$\langle 0.5, 0.6, 0.5\rangle$	$\langle 0.63, 0.2, 0.6\rangle$	0.57
(e_1, e_2)	$\langle 0.83, 0.5, 0.18\rangle$	$\langle 0.69, 0.2, 0.5\rangle$	$\langle 0.93, 0.2, 0.53\rangle$	0.51
(e_1, e_3)	$\langle 0.93, 0.5, 0.18\rangle$	$\langle 0.63, 0.1, 0.5\rangle$	$\langle 0.85, 0.2, 0.3\rangle$	0.57
(e_2, e_1)	$\langle 0.83, 0.1, 0.15\rangle$	$\langle 0.5, 0.6, 0.25\rangle$	$\langle 0.63, 0.1, 0.6\rangle$	0.68
(e_2, e_2)	$\langle 0.83, 0.5, 0.18\rangle$	$\langle 0.75, 0.2, 0.25\rangle$	$\langle 0.55, 0.1, 0.53\rangle$	0.51
(e_2, e_3)	$\langle 0.83, 0.5, 0.18\rangle$	$\langle 0.63, 0.1, 0.13\rangle$	$\langle 0.63, 0.1, 0.3\rangle$	0.73
(e_3, e_1)	$\langle 0.78, 0.1, 0.15\rangle$	$\langle 0.5, 0.6, 0.5\rangle$	$\langle 0.4, 0.6, 0.6\rangle$	0.68
(e_3, e_2)	$\langle 0.78, 0.5, 0.18\rangle$	$\langle 0.63, 0.2, 0.5\rangle$	$\langle 0.4, 0.6, 0.53\rangle$	0.51
(e_3, e_3)	$\langle 0.78, 0.5, 0.18\rangle$	$\langle 0.63, 0.2, 0.5\rangle$	$\langle 0.4, 0.6, 0.3\rangle$	0.78

Table 6. Tabular representation of truth membership of $\left(\vartheta^{\xi}, E\times E\right)_{\Lambda_t}$.

U	x_1	x_2	x_3	ξ
(e_1, e_1)	$\underline{0.93}$	0.5	0.63	0.57
(e_1, e_2)	0.83	0.69	$\underline{0.93}$	0.51
(e_1, e_3)	$\underline{0.93}$	0.63	0.85	0.57
(e_2, e_1)	$\underline{0.83}$	0.5	0.63	0.68
(e_2, e_2)	$\underline{0.83}$	0.75	0.55	0.51
(e_2, e_3)	$\underline{0.83}$	0.63	0.63	0.73
(e_3, e_1)	$\underline{0.78}$	0.5	0.4	0.68
(e_3, e_2)	$\underline{0.78}$	0.63	0.4	0.4
(e_3, e_3)	$\underline{0.78}$	0.63	0.4	0.78

Table 7. Tabular representation of indeterminacy membership of $\left(\vartheta^{\xi}, E\times E\right)_{\Lambda_t}$.

U	x_1	x_2	x_3	ξ
(e_1, e_1)	0.3	$\underline{0.6}$	0.2	0.57
(e_1, e_2)	$\underline{0.5}$	0.2	0.2	0.51
(e_1, e_3)	$\underline{0.5}$	0.1	0.2	0.57
(e_2, e_1)	0.1	$\underline{0.6}$	0.1	0.68
(e_2, e_2)	$\underline{0.5}$	0.2	0.1	0.51
(e_2, e_3)	$\underline{0.5}$	0.1	0.1	0.73
(e_3, e_1)	0.1	$\underline{0.6}$	$\underline{0.6}$	0.68
(e_3, e_2)	0.5	0.2	$\underline{0.6}$	0.51
(e_3, e_3)	0.5	0.2	$\underline{0.6}$	0.78

Table 8. Tabular representation of falsity membership of $\left(\vartheta^{\xi}, E \times E\right)_{\Lambda_t}$.

U	x_1	x_2	x_3	ξ
(e_1, e_1)	0.15	0.5	$\underline{0.6}$	0.57
(e_1, e_2)	0.18	0.5	$\underline{0.53}$	0.51
(e_1, e_3)	0.18	0.5	$\underline{0.3}$	0.57
(e_2, e_1)	0.15	0.25	$\underline{0.6}$	0.68
(e_2, e_2)	0.18	0.25	$\underline{0.53}$	0.51
(e_2, e_3)	0.18	0.13	$\underline{0.3}$	0.73
(e_3, e_1)	0.15	0.5	$\underline{0.6}$	0.68
(e_3, e_2)	0.18	0.5	$\underline{0.53}$	0.51
(e_3, e_3)	0.18	$\underline{0.5}$	0.3	0.78

Now, we compute the score of each component of $\left(\vartheta^{\xi}, E \times E\right)_{\Lambda_t}$ by using Tables 6–8, respectively as follows:
$Score\,(x_1) = (0.93 \times 0.57) + (0.93 \times 0.57) + (0.83 \times 0.68) + (0.83 \times 0.51) + (0.83 \times 0.73) + (0.78 \times 0.68) + (0.78 \times 0.51) + (0.78 \times 0.78) = 4.19$,
$Score\,(x_2) = 0$,
$Score\,(x_3) = (0.93 \times 0.51) = 0.47$.

$Score\,(x_1) = (0.5 \times 0.51) + (0.5 \times 0.57) + (0.5 \times 0.51) + (0.5 \times 0.73)) = 1.16$,
$Score\,(x_2) = (0.6 \times 0.57) + (0.6 \times 0.68) + (0.6 \times 0.68) = 1.16$,
$Score\,(x_3) = (0.6 \times 0.68) + (0.6 \times 0.51) + (0.6 \times 0.78) = 1.18$.

$Score\,(x_1) = 0$,
$Score\,(x_2) = (0.5 \times 0.57) + (0.5 \times 0.78) = 0.77$,
$Score\,(x_3) = (0.6 \times 0.57) + (0.53 \times 0.51) + (0.6 \times 0.68) + (0.53 \times 0.51) + (0.3 \times 0.73) + (0.6 \times 0.68) + (0.53 \times 0.51) = 2.19$.
Then, we calculate the final score as follows:
$Score\,(x_1) = 4.19 + 1.16 - 0 = \mathbf{5.35}$,
$Score\,(x_2) = 0 + 1.16 - 0.77 = 1.93$,
$Score\,(x_3) = 0.47 + 1.18 - 2.19 = -0.54$.
The optimal selection is x_1.

We conclude that the optimal selection of GENSS by using 4.1.1 is laptop number 1. Hence, GENSS changes the optimal selection from laptop number 3 to laptop number 1.

5. An application of GENSS in medical diagnosis

There are many applications and ideas that strive to simplify the process of medical diagnosis however, each of these applications and theories consider only the symptoms that affect the patient without considering external effects that might radically modify the diagnosis. In this section, we will attempt to discover the most accurate diagnosis of the condition based on the symptoms and external effects by adding the degree of possibility $\mu(e_i)$ associated with each parameter attached to ENSS, enhancing the realism of the concept.

Example 5.1. *Assume that $P = \{p_1, p_2, p_3, p_4\}$ be a set of 4 patients in the hospital. The hospital diagnostic expert identified the following symptoms to find out what patients were suffering from $S = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}, s_{17}, s_{18}, s_{19}, s_{20}\}$. Where s_1 =dry cough, s_2 =fever, s_3 =breathing difficulties or shortness of breath, s_4 =headache, s_5 =muscle pain, s_6 =fatigue and weakness, s_7 =Chills, s_8 =anorexia, s_9 =sore throat, s_{10} =vomiting and nausea, s_{11} = photosensitive, s_{12} =loose motion, s_{13} = dizziness, s_{14} =red eyes, face or tongue, s_{15} =swelling of one or both glands, s_{16} = sweating, s_{17} = severe pneumonia, s_{18} =aches, s_{19} =runny nose and s_{20} =diarrhea. Also, let $D = \{d_1, d_2, d_3, d_4\}$ be a set of diseases such that d_1 = mumpus, d_2 = covid-19, d_3 = yellow fever, d_4 = influenza. Let $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$, where a_1 = he has a retina in the heart, a_2 = he close contacted (less than 6 feet) with anyone who is suffering from covid-19, a_3 = he was infected with the gallbladder a_4 = he had a stroke, a_5 = he was infected with the helminthic germ a_6 = he was in an area with stagnant water, especially at dawn and dusk, a_7 = he used to sleep without a cover or mosquito net, a_8 = eating food that is raw or undercooked, and a_9 = eating foods and beverages purchased from street vendors.*

We find out the daily routines and lifestyles of patients as in Table 9.

Table 9. Patients daily activities and lives.

$P \setminus A$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
p_1	*Yes*	*Yes*	*No*	*No*	*Yes*	*Yes*	*No*	*Yes*	*No*
p_2	*No*	*No*	*No*	*Yes*	*Yes*	*No*	*Yes*	*No*	*No*
p_3	*Yes*	*No*	*Yes*	*No*	*No*	*No*	*No*	*Yes*	*Yes*
p_4	*No*	*No*	*No*	*No*	*Yes*	*Yes*	*Yes*	*No*	*Yes*

The relationship between the mentioned disease and the aforementioned effective parameters is shown in Table 10 as follows:

Table 10. Diseases and effective parameters relation.

$D \setminus A$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	$\lvert A_i \rvert$
d_1	*Yes*	*No*	*Yes*	*Yes*	*Yes*	*No*	*No*	*No*	*Yes*	5
d_2	*No*	*Yes*	*Yes*	*Yes*	*Yes*	*No*	*No*	*No*	*Yes*	5
d_3	*No*	*No*	*No*	*No*	*No*	*Yes*	*Yes*	*Yes*	*No*	3
d_4	*No*	*No*	*Yes*	*Yes*	*No*	*No*	*No*	*No*	*Yes*	3

Tables 11–14 represented $\Lambda_{d_j}(p_i)$ for every patient in view of the mentioned diseases as follows:

Table 11. Tabular representation of $\Lambda_{d_1}(p_i)$.

$P_i \setminus A$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	Sum
p_1d_1	1	0	0	0	1	0	0	0	0	2
p_2d_1	0	1	0	0	1	0	0	0	0	2
p_3d_1	0	0	0	0	0	1	0	1	0	2
p_4d_1	0	0	0	0	0	0	0	0	0	0
$Total$										6

Table 12. Tabular representation of $\Lambda_{d_2}(p_i)$.

$P_i \setminus A$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	Sum
p_1d_2	0	0	0	1	1	0	0	0	0	2
p_2d_2	0	0	0	1	1	0	0	0	0	2
p_3d_2	0	0	0	0	0	0	1	0	0	1
p_4d_2	0	0	0	1	0	0	0	0	0	1
$Total$										6

Table 13. Tabular representation of $\Lambda_{d_3}(p_i)$.

$P_i \setminus A$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	Sum
p_1d_3	1	0	1	0	0	0	0	0	1	3
p_2d_3	0	0	1	0	0	0	0	0	1	2
p_3d_3	0	0	0	0	0	0	0	1	0	1
p_4d_3	0	0	1	0	0	0	0	0	1	2
$Total$										8

Table 14. Tabular representation of $\Lambda_{d_4}(p_i)$.

$P_i \setminus A$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	Sum
p_1d_4	0	0	0	0	1	0	0	0	1	2
p_2d_4	0	0	0	0	1	0	0	0	1	2
p_3d_4	0	0	0	0	0	1	1	0	0	2
p_4d_4	0	0	0	0	0	0	0	0	1	1
$Total$										7

Let the tabular representation of (Γ^{μ}, S) (patient symptom) given in Tables 15–18.

Table 15. Tabular representation of (Γ^{μ}, S) part 1.

$P \setminus S$	s_1	s_2	s_3	s_4	s_5
p_1	$\langle 0.6, 0.1, 0.4\rangle$	$\langle 0.2, 0.1, 0.6\rangle$	$\langle 0.7, 0.2, 0.4\rangle$	$\langle 0.4, 0.2, 0.8\rangle$	$\langle 0.7, 0.1, 0.4\rangle$
p_2	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.6, 0.4, 0.8\rangle$	$\langle 0.9, 0.4, 0.7\rangle$	$\langle 0.4, 0.1, 0.3\rangle$	$\langle 0.4, 0.2, 0.7\rangle$
p_3	$\langle 0.7, 0.2, 0.5\rangle$	$\langle 0.9, 0.4, 0.2\rangle$	$\langle 0.2, 0.1, 0.7\rangle$	$\langle 0.4, 0.1, 0.6\rangle$	$\langle 0.7, 0.2, 0.5\rangle$
p_4	$\langle 0.4, 0.1, 0.6\rangle$	$\langle 0.4, 0.1, 0.9\rangle$	$\langle 0.7, 0.2, 0.3\rangle$	$\langle 0.2, 0.1, 0.5\rangle$	$\langle 0.9, 0.5, 0.2\rangle$
μ	0.6	0.5	0.9	0.4	0.2

Table 16. Tabular representation of (Γ^{μ}, S) part 2.

$P \setminus S$	s_6	s_7	s_8	s_9	s_{10}
p_1	$\langle 0.8, 0.2, 0.5\rangle$	$\langle 0.2, 0, 0.7\rangle$	$\langle 0.9, 0.4, 0.6\rangle$	$\langle 0.9, 0.1, 0.6\rangle$	$\langle 0.4, 0.2, 0.8\rangle$
p_2	$\langle 0.9, 0.5, 0.1\rangle$	$\langle 0.4, 0.2, 0.7\rangle$	$\langle 0.5, 0.1, 0.8\rangle$	$\langle 0.3, 0.5, 0.1\rangle$	$\langle 0.3, 0.1, 0.8\rangle$
p_3	$\langle 0.1, 0.4, 0.9\rangle$	$\langle 0.6, 0.3, 0.9\rangle$	$\langle 0.5, 0, 0.3\rangle$	$\langle 0.2, 0.7, 0.4\rangle$	$\langle 0.3, 0.1, 0.6\rangle$
p_4	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.4, 0.2, 0.7\rangle$	$\langle 0.7, 0.1, 0.4\rangle$	$\langle 0.4, 0.2, 0.8\rangle$	$\langle 0.9, 0.3, 0.6\rangle$
μ	0.1	0.3	0.2	0.5	0.6

Table 17. Tabular representation of (Γ^{μ}, S) part 3.

$P \setminus S$	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}
p_1	$\langle 0.7, 0.5, 0.2\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.9, 0.6, 0.2\rangle$	$\langle 0.2, 0, 0.9\rangle$	$\langle 0.7, 0.2, 0.4\rangle$
p_2	$\langle 0.7, 0.1, 0.4\rangle$	$\langle 0.5, 0.1, 0.3\rangle$	$\langle 0.3, 0, 0.9\rangle$	$\langle 0.1, 0, 0.5\rangle$	$\langle 0.8, 0.2, 0.5\rangle$
p_3	$\langle 0.4, 0.1, 0.9\rangle$	$\langle 0.4, 0.2, 0.7\rangle$	$\langle 0.4, 0.1, 0.7\rangle$	$\langle 0.9, 0.3, 0.5\rangle$	$\langle 0.1, 0.3, 0.7\rangle$
p_4	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.9, 0.2, 0.4\rangle$	$\langle 0.1, 0.5, 0.3\rangle$	$\langle 0.2, 0.6, 0.4\rangle$	$\langle 0.1, 0.7, 0.4\rangle$
μ	0.8	0.6	0.9	0.4	0.7

Table 18. Tabular representation of (Γ^{μ}, S) part 4.

$P \setminus S$	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}
p_1	$\langle 0.3, 0.1, 0.5\rangle$	$\langle 0.7, 0.2, 0.5\rangle$	$\langle 0.4, 0.2, 0.7\rangle$	$\langle 0.8, 0, 0.1\rangle$	$\langle 0.2, 0.7, 0.4\rangle$
p_2	$\langle 0.4, 0.3, 0.9\rangle$	$\langle 0.9, 0.4, 0.1\rangle$	$\langle 0.1, 0.3, 0.9\rangle$	$\langle 0.5, 0.1, 0.8\rangle$	$\langle 0.9, 0.5, 0.1\rangle$
p_3	$\langle 0.8, 0.5, 0.3\rangle$	$\langle 0.6, 0.2, 0.9\rangle$	$\langle 0.5, 0.2, 0.7\rangle$	$\langle 0.8, 0.4, 0.6\rangle$	$\langle 0.7, 0.5, 0.2\rangle$
p_4	$\langle 0.2, 0.4, 0.9\rangle$	$\langle 0.1, 0.6, 0.4\rangle$	$\langle 0.9, 0.4, 0.6\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.9, 0.2, 0.5\rangle$
μ	0.1	0.7	0.4	0.2	0.5

The tabular representation of (ψ^{η}, S) (model symptom) is given in the following Tables 19 –22.

Table 19. Tabular representation of (ψ^{η}, S) part 1.

$D \setminus S$	s_1	s_2	s_3	s_4	s_5
d_1	$\langle 0.5, 0.5, 0.5\rangle$	$\langle 1, 1, 0\rangle$	$\langle 0, 0, 1\rangle$	$\langle 1, 1, 0\rangle$	$\langle 1, 1, 0\rangle$
d_2	$\langle 1, 1, 0\rangle$	$\langle 1, 1, 0\rangle$	$\langle 1, 1, 0\rangle$	$\langle 1, 1, 0\rangle$	$\langle 0.5, 0.5, 0.5\rangle$
d_3	$\langle 1, 1, 0\rangle$	$\langle 0, 0, 1\rangle$	$\langle 0, 0, 1\rangle$	$\langle 1, 1, 0\rangle$	$\langle 1, 1, 0\rangle$
d_4	$\langle 1, 1, 0\rangle$	$\langle 1, 1, 0\rangle$	$\langle 1, 1, 0\rangle$	$\langle 1, 1, 0\rangle$	$\langle 1, 1, 0\rangle$
η	1	1	1	1	1

Table 20. Tabular representation of (ψ^{η}, S) part 2.

$D \setminus S$	s_6	s_7	s_8	s_9	s_{10}
d_1	$\langle 1,1,0\rangle$	$\langle 0.5,0.5,0.5\rangle$	$\langle 1,1,0\rangle$	$\langle 0.5,0.5,0.5\rangle$	$\langle 0,0,1\rangle$
d_2	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0.5,0.5,0.5\rangle$	$\langle 0,0,1\rangle$
d_3	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$
d_4	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0.5,0.5,0.5\rangle$
η	1	1	1	1	1

Table 21. Tabular representation of (ψ^{η}, S) part 3.

$D \setminus S$	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}
d_1	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0.5,0.5,0.5\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$
d_2	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$
d_3	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$
d_4	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0.5,0.5,0.5\rangle$	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$
η	1	1	1	1	1

Table 22. Tabular representation of (ψ^{η}, S) part 4.

$D \setminus S$	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}
d_1	$\langle 0.5,0.5,0.5\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 0.5,0.5,0.5\rangle$	$\langle 0,0,1\rangle$
d_2	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0.5,0.5,0.5\rangle$	$\langle 0.5,0.5,0.5\rangle$	$\langle 0,0,1\rangle$
d_3	$\langle 0.5,0.5,0.5\rangle$	$\langle 0,0,1\rangle$	$\langle 0.5,0.5,0.5\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$
d_4	$\langle 1,1,0\rangle$	$\langle 0,0,1\rangle$	$\langle 0,0,1\rangle$	$\langle 1,1,0\rangle$	$\langle 0.5,0.5,0.5\rangle$
η	1	1	1	1	1

Now, we compute the GENSS by appling Definition 3.1 and Tables 15–18 as given in Tables 23–38. We calculate the first column as follows:
$(0.67, 0.1, 0.24)$ where $0.6 + 0.4(2/5) = \mathbf{0.76}$, $\mathbf{0.1}$, $0.4 - 0.4(2/5) = \mathbf{0.24}$,
$(0.7, 0.3, 0.06)$ where $0.5 + 0.5(2/5) = \mathbf{0.7}$, $\mathbf{0.3}$, $0.1 - 0.1(2/5) = \mathbf{0.06}$,
$(0.82, 0.2, 0.3)$ where $0.7 + 0.3(2/5) = \mathbf{0.82}$, $\mathbf{0.2}$, $0.5 - 0.5(2/5) = \mathbf{0.3}$,
$(0.4, 0.1, 0.6)$ where $0.4 + 0.6(0/5) = \mathbf{0.4}$, $\mathbf{0.1}$, $0. - 0.(0/5) = \mathbf{0.6}$,
$\mu = 0.6 \rightarrow \mu_{\Lambda_{d_1}} 0.6 + 0.4(6/36) = \mathbf{0.67}$.

Table 23. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_1}}$ part 1.

$P \setminus S$	s_1	s_2	s_3	s_4	s_5
p_1	$\langle 0.67,0.1,0.24\rangle$	$\langle 0.52,0.1,0.36\rangle$	$\langle 0.82,0.2,0.24\rangle$	$\langle 0.64,0.2,0.48\rangle$	$\langle 0.82,0.1,0.24\rangle$
p_2	$\langle 0.7,0.3,0.06\rangle$	$\langle 0.76,0.4,0.48\rangle$	$\langle 0.94,0.4,0.42\rangle$	$\langle 0.64,0.1,0.18\rangle$	$\langle 0.64,0.2,0.42\rangle$
p_3	$\langle 0.82,0.2,0.3\rangle$	$\langle 0.94,0.4,0.12\rangle$	$\langle 0.52,0.1,0.42\rangle$	$\langle 0.64,0.1,0.36\rangle$	$\langle 0.82,0.2,0.3\rangle$
p_4	$\langle 0.4,0.1,0.6\rangle$	$\langle 0.4,0.1,0.9\rangle$	$\langle 0.7,0.2,0.3\rangle$	$\langle 0.2,0.1,0.5\rangle$	$\langle 0.9,0.5,0.2\rangle$
$\mu_{\Lambda_{d_1}}$	0.67	0.58	0.92	0.5	0.33

Table 24. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_1}}$ part 2.

$P \setminus S$	s_6	s_7	s_8	s_9	s_{10}
p_1	$\langle 0.88, 0.2, 0.3\rangle$	$\langle 0.52, 0, 0.42\rangle$	$\langle 0.94, 0.4, 0.36\rangle$	$\langle 0.94, 0.1, 0.36\rangle$	$\langle 0.64, 0.2, 0.48\rangle$
p_2	$\langle 0.94, 0.5, 0.06\rangle$	$\langle 0.64, 0.2, 0.42\rangle$	$\langle 0.7, 0.1, 0.48\rangle$	$\langle 0.58, 0.5, 0.06\rangle$	$\langle 0.58, 0.1, 0.48\rangle$
p_3	$\langle 0.46, 0.4, 0.54\rangle$	$\langle 0.76, 0.3, 0.54\rangle$	$\langle 0.7, 0, 0.18\rangle$	$\langle 0.52, 0.7, 0.24\rangle$	$\langle 0.58, 0.1, 0.36\rangle$
p_4	$\langle 0.7, 0.3, 0.5\rangle$	$\langle 0.4, 0.2, 0.7\rangle$	$\langle 0.7, 0.1, 0.4\rangle$	$\langle 0.4, 0.2, 0.8\rangle$	$\langle 0.9, 0.3, 0.6\rangle$
$\mu_{\Lambda_{d_1}}$	0.25	0.42	0.33	0.58	0.67

Table 25. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_1}}$ part 3.

$P \setminus S$	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}
p_1	$\langle 0.82, 0.5, 0.12\rangle$	$\langle 0.76, 0.1, 0.18\rangle$	$\langle 0.94, 0.6, 0.12\rangle$	$\langle 0.52, 0, 0.54\rangle$	$\langle 0.82, 0.2, 0.24\rangle$
p_2	$\langle 0.82, 0.1, 0.24\rangle$	$\langle 0.7, 0.1, 0.18\rangle$	$\langle 0.58, 0, 0.54\rangle$	$\langle 0.46, 0, 0.3\rangle$	$\langle 0.88, 0.2, 0.3\rangle$
p_3	$\langle 0.64, 0.1, 0.54\rangle$	$\langle 0.64, 0.2, 0.42\rangle$	$\langle 0.64, 0.1, 0.42\rangle$	$\langle 0.94, 0.3, 0.3\rangle$	$\langle 0.46, 0.3, 0.42\rangle$
p_4	$\langle 0.5, 0.3, 0.1\rangle$	$\langle 0.9, 0.2, 0.4\rangle$	$\langle 0.9, 0.2, 0.4\rangle$	$\langle 0.2, 0.6, 0.4\rangle$	$\langle 0.1, 0.7, 0.4\rangle$
$\mu_{\Lambda_{d_1}}$	0.83	0.67	0.92	0.5	0.75

Table 26. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_1}}$ part 4.

$P \setminus S$	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}
p_1	$\langle 0.58, 0.1, 0.3\rangle$	$\langle 0.82, 0.2, 0.3\rangle$	$\langle 0.64, 0.2, 0.42\rangle$	$\langle 0.88, 0, 0.06\rangle$	$\langle 0.52, 0.7, 0.24\rangle$
p_2	$\langle 0.64, 0.3, 0.54\rangle$	$\langle 0.94, 0.4, 0.06\rangle$	$\langle 0.46, 0.3, 0.54\rangle$	$\langle 0.7, 0.1, 0.48\rangle$	$\langle 0.94, 0.5, 0.06\rangle$
p_3	$\langle 0.88, 0.5, 0.18\rangle$	$\langle 0.76, 0.2, 0.54\rangle$	$\langle 0.7, 0.2, 0.42\rangle$	$\langle 0.88, 0.4, 0.36\rangle$	$\langle 0.82, 0.5, 0.12\rangle$
p_4	$\langle 0.2, 0.4, 0.9\rangle$	$\langle 0.1, 0.6, 0.4\rangle$	$\langle 0.9, 0.4, 0.6\rangle$	$\langle 0.6, 0.1, 0.3\rangle$	$\langle 0.9, 0.2, 0.5\rangle$
$\mu_{\Lambda_{d_1}}$	0.25	0.75	0.5	0.33	0.58

Table 27. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_2}}$ part 1.

$P \setminus S$	s_1	s_2	s_3	s_4	s_5
p_1	$\langle 0.76, 0.1, 0.17\rangle$	$\langle 0.52, 0.1, 0.36\rangle$	$\langle 0.82, 0.2, 0.24\rangle$	$\langle 0.64, 0.2, 0.48\rangle$	$\langle 0.82, 0.1, 0.24\rangle$
p_2	$\langle 0.7, 0.3, 0.06\rangle$	$\langle 0.76, 0.4, 0.48\rangle$	$\langle 0.94, 0.4, 0.42\rangle$	$\langle 0.64, 0.1, 0.18\rangle$	$\langle 0.64, 0.2, 0.42\rangle$
p_3	$\langle 0.76, 0.2, 0.4\rangle$	$\langle 0.92, 0.4, 0.16\rangle$	$\langle 0.36, 0.1, 0.56\rangle$	$\langle 0.52, 0.1, 0.48\rangle$	$\langle 0.76, 0.2, 0.4\rangle$
p_4	$\langle 0.52, 0.1, 0.48\rangle$	$\langle 0.52, 0.1, 0.72\rangle$	$\langle 0.76, 0.2, 0.24\rangle$	$\langle 0.36, 0.1, 0.4\rangle$	$\langle 0.92, 0.5, 0.16\rangle$
$\mu_{\Lambda_{d_2}}$	0.67	0.58	0.92	0.5	0.33

Table 28. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_2}}$ part 2.

$P \setminus S$	s_6	s_7	s_8	s_9	s_{10}
p_1	$\langle 0.88, 0.2, 0.3\rangle$	$\langle 0.52, 0, 0.42\rangle$	$\langle 0.94, 0.4, 0.36\rangle$	$\langle 0.94, 0.1, 0.36\rangle$	$\langle 0.64, 0.2, 0.48\rangle$
p_2	$\langle 0.94, 0.5, 0.06\rangle$	$\langle 0.64, 0.2, 0.42\rangle$	$\langle 0.7, 0.1, 0.48\rangle$	$\langle 0.58, 0.5, 0.06\rangle$	$\langle 0.58, 0.1, 0.48\rangle$
p_3	$\langle 0.28, 0.4, 0.72\rangle$	$\langle 0.68, 0.3, 0.72\rangle$	$\langle 0.6, 0, 0.24\rangle$	$\langle 0.36, 0.7, 0.32\rangle$	$\langle 0.44, 0.1, 0.48\rangle$
p_4	$\langle 0.76, 0.3, 0.4\rangle$	$\langle 0.52, 0.2, 0.56\rangle$	$\langle 0.76, 0.1, 0.32\rangle$	$\langle 0.52, 0.2, 0.64\rangle$	$\langle 0.92, 0.3, 0.48\rangle$
$\mu_{\Lambda_{d_2}}$	0.25	0.42	0.33	0.58	0.67

Table 29. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_2}}$ part 3.

$P \setminus S$	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}
p_1	$\langle 0.82, 0.5, 0.12\rangle$	$\langle 0.76, 0.1, 0.18\rangle$	$\langle 0.94, 0.6, 0.12\rangle$	$\langle 0.52, 0, 0.54\rangle$	$\langle 0.82, 0.2, 0.24\rangle$
p_2	$\langle 0.82, 0.1, 0.24\rangle$	$\langle 0.83, 0.1, 0.18\rangle$	$\langle 0.58, 0, 0.54\rangle$	$\langle 0.46, 0, 0.3\rangle$	$\langle 0.88, 0.2, 0.3\rangle$
p_3	$\langle 0.52, 0.1, 0.72\rangle$	$\langle 0.52, 0.2, 0.56\rangle$	$\langle 0.52, 0.1, 0.56\rangle$	$\langle 0.92, 0.3, 0.4\rangle$	$\langle 0.28, 0.3, 0.56\rangle$
p_4	$\langle 0.6, 0.3, 0.08\rangle$	$\langle 0.92, 0.2, 0.32\rangle$	$\langle 0.92, 0.2, 0.32\rangle$	$\langle 0.36, 0.6, 0.32\rangle$	$\langle 0.28, 0.7, 0.32\rangle$
$\mu_{\Lambda_{d_2}}$	0.83	0.67	0.92	0.5	0.75

Table 30. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_2}}$ part 4.

$P \setminus S$	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}
p_1	$\langle 0.58, 0.1, 0.3\rangle$	$\langle 0.82, 0.2, 0.3\rangle$	$\langle 0.64, 0.2, 0.42\rangle$	$\langle 0.88, 0, 0.06\rangle$	$\langle 0.6, 0.7, 0.24\rangle$
p_2	$\langle 0.64, 0.3, 0.54\rangle$	$\langle 0.94, 0.4, 0.06\rangle$	$\langle 0.46, 0.3, 0.54\rangle$	$\langle 0.7, 0.1, 0.48\rangle$	$\langle 0.94, 0.5, 0.06\rangle$
p_3	$\langle 0.84, 0.5, 0.24\rangle$	$\langle 0.68, 0.2, 0.72\rangle$	$\langle 0.6, 0.2, 0.56\rangle$	$\langle 0.84, 0.4, 0.48\rangle$	$\langle 0.76, 0.5, 0.16\rangle$
p_4	$\langle 0.36, 0.4, 0.72\rangle$	$\langle 0.28, 0.6, 0.32\rangle$	$\langle 0.92, 0.4, 0.48\rangle$	$\langle 0.68, 0.1, 0.24\rangle$	$\langle 0.92, 0.2, 0.4\rangle$
$\mu_{\Lambda_{d_2}}$	0.25	0.75	0.5	0.33	0.58

Table 31. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_3}}$ part 1.

$P \setminus S$	s_1	s_2	s_3	s_4	s_5
p_1	$\langle 1, 0.1, 0\rangle$	$\langle 1, 0.1, 0\rangle$	$\langle 1, 0.3, 0.1\rangle$	$\langle 1, 0.2, 0\rangle$	$\langle 1, 0.1, 0\rangle$
p_2	$\langle 0.83, 0.3, 0.03\rangle$	$\langle 0.87, 0.4, 0.27\rangle$	$\langle 0.97, 0.4, 0.23\rangle$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.8, 0.2, 0.23\rangle$
p_3	$\langle 0.8, 0.2, 0.33\rangle$	$\langle 0.93, 0.4, 0.13\rangle$	$\langle 0.47, 0.1, 0.47\rangle$	$\langle 0.6, 0.1, 0.4\rangle$	$\langle 0.8, 0.2, 0.33\rangle$
p_4	$\langle 0.8, 0.1, 0.2\rangle$	$\langle 0.8, 0.1, 0.3\rangle$	$\langle 0.9, 0.2, 0.1\rangle$	$\langle 0.73, 0.1, 0.17\rangle$	$\langle 0.97, 0.5, 0.06\rangle$
$\mu_{\Lambda_{d_3}}$	0.69	0.61	0.92	0.53	0.38

Table 32. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_3}}$ part 2.

$P \setminus S$	s_6	s_7	s_8	s_9	s_{10}
p_1	$\langle 1, 0.2, 0\rangle$	$\langle 1, 0, 0\rangle$	$\langle 1, 0.4, 0\rangle$	$\langle 1, 0.1, 0\rangle$	$\langle 1, 0.2, 0\rangle$
p_2	$\langle 0.97, 0.5, 0.03\rangle$	$\langle 0.8, 0.2, 0.23\rangle$	$\langle 0.83, 0.1, 0.27\rangle$	$\langle 0.77, 0.5, 0.03\rangle$	$\langle 0.77, 0.1, 0.27\rangle$
p_3	$\langle 0.4, 0.4, 0.6\rangle$	$\langle 0.73, 0.3, 0.6\rangle$	$\langle 0.67, 0, 0.2\rangle$	$\langle 0.47, 0.7, 0.27\rangle$	$\langle 0.53, 0.1, 0.4\rangle$
p_4	$\langle 0.9, 0.3, 0.17\rangle$	$\langle 0.8, 0.2, 0.23\rangle$	$\langle 0.9, 0.1, 0.13\rangle$	$\langle 0.8, 0.2, 0.27\rangle$	$\langle 0.97, 0.3, 0.2\rangle$
$\mu_{\Lambda_{d_3}}$	0.3	0.46	0.38	0.61	0.69

Table 33. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_3}}$ part 3.

$P \setminus S$	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}
p_1	$\langle 1, 0.5, 0\rangle$	$\langle 1, 0.1, 0\rangle$	$\langle 1, 0.6, 0\rangle$	$\langle 1, 0, 0\rangle$	$\langle 1, 0.2, 0\rangle$
p_2	$\langle 0.9, 0.1, 0.13\rangle$	$\langle 0.83, 0.1, 0.1\rangle$	$\langle 0.77, 0, 0.3\rangle$	$\langle 0.7, 0, 0.17\rangle$	$\langle 0.93, 0.2, 0.17\rangle$
p_3	$\langle 0.6, 0.1, 0.6\rangle$	$\langle 0.6, 0.2, 0.47\rangle$	$\langle 0.6, 0.1, 0.47\rangle$	$\langle 0.93, 0.3, 0.33\rangle$	$\langle 0.4, 0.3, 0.47\rangle$
p_4	$\langle 0.83, 0.3, 0.03\rangle$	$\langle 0.97, 0.2, 0.13\rangle$	$\langle 0.97, 0.2, 0.13\rangle$	$\langle 0.73, 0.6, 0.13\rangle$	$\langle 0.7, 0.7, 0.13\rangle$
$\mu_{\Lambda_{d_3}}$	0.84	0.69	0.92	0.53	0.77

Table 34. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_3}}$ part 4.

$P \setminus S$	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}
p_1	$\langle 1, 0.1, 0\rangle$	$\langle 1, 0.2, 0\rangle$	$\langle 1, 0.2, 0\rangle$	$\langle 1, 0, 0\rangle$	$\langle 1, 0.7, 0\rangle$
p_2	$\langle 0.8, 0.3, 0.3\rangle$	$\langle 0.97, 0.4, 0.03\rangle$	$\langle 0.7, 0.3, 0.3\rangle$	$\langle 0.83, 0.1, 0.27\rangle$	$\langle 0.97, 0.5, 0.03\rangle$
p_3	$\langle 0.87, 0.5, 0.2\rangle$	$\langle 0.73, 0.2, 0.6\rangle$	$\langle 0.67, 0.2, 0.47\rangle$	$\langle 0.87, 0.4, 0.4\rangle$	$\langle 0.8, 0.5, 0.13\rangle$
p_4	$\langle 0.73, 0.4, 0.3\rangle$	$\langle 0.7, 0.6, 0.13\rangle$	$\langle 0.96, 0.4, 0.2\rangle$	$\langle 0.87, 0.1, 0.1\rangle$	$\langle 0.97, 0.2, 0.16\rangle$
$\mu_{\Lambda_{d_3}}$	0.3	0.77	0.53	0.38	0.61

Table 35. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_4}}$ part 1.

$P \setminus S$	s_1	s_2	s_3	s_4	s_5
p_1	$\langle 0.87, 0.1, 0.13\rangle$	$\langle 0.73, 0.1, 0.2\rangle$	$\langle 0.9, 0.2, 0.13\rangle$	$\langle 0.8, 0.2, 0.27\rangle$	$\langle 0.9, 0.1, 0.13\rangle$
p_2	$\langle 0.83, 0.3, 0.03\rangle$	$\langle 0.87, 0.4, 0.27\rangle$	$\langle 0.97, 0.4, 0.23\rangle$	$\langle 0.8, 0.1, 0.1\rangle$	$\langle 0.8, 0.2, 0.23\rangle$
p_3	$\langle 0.9, 0.2, 0.1\rangle$	$\langle 0.97, 0.4, 0.07\rangle$	$\langle 0.73, 0.1, 0.23\rangle$	$\langle 0.8, 0.1, 0.2\rangle$	$\langle 0.9, 0.2, 0.16\rangle$
p_4	$\langle 0.6, 0.1, 0.4\rangle$	$\langle 0.6, 0.1, 0.6\rangle$	$\langle 0.83, 0.2, 0.2\rangle$	$\langle 0.47, 0.1, 0.33\rangle$	$\langle 0.93, 0.5, 0.13\rangle$
$\mu_{\Lambda_{d_4}}$	0.68	0.6	0.92	0.52	0.36

Table 36. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_4}}$ part 2.

$P \setminus S$	s_6	s_7	s_8	s_9	s_{10}
p_1	$\langle 0.93, 0.2, 0.17\rangle$	$\langle 0.73, 0, 0.23\rangle$	$\langle 0.97, 0.4, 0.2\rangle$	$\langle 0.97, 0.1, 0.2\rangle$	$\langle 0.8, 0.2, 0.27\rangle$
p_2	$\langle 0.97, 0.5, 0.03\rangle$	$\langle 0.8, 0.2, 0.23\rangle$	$\langle 0.83, 0.1, 0.26\rangle$	$\langle 0.76, 0.5, 0.03\rangle$	$\langle 0.77, 0.1, 0.27\rangle$
p_3	$\langle 0.7, 0.4, 0.3\rangle$	$\langle 0.87, 0.3, 0.3\rangle$	$\langle 0.83, 0, 0.1\rangle$	$\langle 0.73, 0.7, 0.13\rangle$	$\langle 0.77, 0.1, 0.2\rangle$
p_4	$\langle 0.8, 0.3, 0.33\rangle$	$\langle 0.6, 0.2, 0.47\rangle$	$\langle 0.8, 0.1, 0.27\rangle$	$\langle 0.6, 0.2, 0.53\rangle$	$\langle 0.93, 0.3, 0.4\rangle$
$\mu_{\Lambda_{d_4}}$	0.28	0.44	0.36	0.6	0.68

Table 37. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_4}}$ part 3.

$P \setminus S$	s_{11}	s_{12}	s_{13}	s_{14}	s_{15}
p_1	$\langle 0.9, 0.5, 0.07\rangle$	$\langle 0.87, 0.1, 0.1\rangle$	$\langle 0.97, 0.6, 0.07\rangle$	$\langle 0.73, 0, 0.3\rangle$	$\langle 0.9, 0.2, 0.13\rangle$
p_2	$\langle 0.9, 0.1, 0.13\rangle$	$\langle 0.83, 0.1, 0.1\rangle$	$\langle 0.77, 0, 0.3\rangle$	$\langle 0.7, 0, 0.17\rangle$	$\langle 0.93, 0.2, 0.17\rangle$
p_3	$\langle 0.8, 0.1, 0.3\rangle$	$\langle 0.8, 0.2, 0.23\rangle$	$\langle 0.8, 0.1, 0.23\rangle$	$\langle 0.97, 0.3, 0.16\rangle$	$\langle 0.7, 0.3, 0.23\rangle$
p_4	$\langle 0.67, 0.3, 0.07\rangle$	$\langle 0.93, 0.2, 0.27\rangle$	$\langle 0.93, 0.2, 0.27\rangle$	$\langle 0.47, 0.6, 0.27\rangle$	$\langle 0.4, 0.7, 0.27\rangle$
$\mu_{\Lambda_{d_4}}$	0.84	0.68	0.92	0.52	0.76

Table 38. Tabular representation of $(\Gamma^{\mu}, S)_{\Lambda_{d_4}}$ part 4.

$P \setminus S$	s_{16}	s_{17}	s_{18}	s_{19}	s_{20}
p_1	$\langle 0.77, 0.1, 0.17\rangle$	$\langle 0.9, 0.2, 0.17\rangle$	$\langle 0.8, 0.2, 0.23\rangle$	$\langle 0.93, 0, 0.03\rangle$	$\langle 0.73, 0.7, 0.13\rangle$
p_2	$\langle 0.8, 0.3, 0.3\rangle$	$\langle 0.97, 0.4, 0.03\rangle$	$\langle 0.7, 0.3, 0.3\rangle$	$\langle 0.83, 0.1, 0.27\rangle$	$\langle 0.97, 0.5, 0.03\rangle$
p_3	$\langle 0.93, 0.5, 0.1\rangle$	$\langle 0.87, 0.2, 0.3\rangle$	$\langle 0.83, 0.2, 0.23\rangle$	$\langle 0.93, 0.4, 0.2\rangle$	$\langle 0.9, 0.5, 0.06\rangle$
p_4	$\langle 0.47, 0.4, 0.6\rangle$	$\langle 0.4, 0.6, 0.27\rangle$	$\langle 0.93, 0.4, 0.4\rangle$	$\langle 0.73, 0.1, 0.2\rangle$	$\langle 0.93, 0.2, 0.33\rangle$
$\mu_{\Lambda_{d_4}}$	0.28	0.76	0.52	0.36	0.6

Finally, we find the score table by finding the similarity between each row in Tables 23–38 with each row in Tables 19–22 and determine the maximum value for each patient as well as the diseases associated with those values. To find the similarity, we apply the following formula:

$$T_{(p_i,d_j)} = 1 - \frac{\sum_l^{20} |T_{(\Gamma^\mu,S)_{\wedge_{d_j}}}(p_i)(s_l) - T_{(\psi^\eta,S)}(d_j)(s_l)|}{\sum_l^{20} |T_{(\Gamma^\mu,S)_{\wedge_{d_j}}}(p_i)(s_l) + T_{(\psi^\eta,S)}(d_j)(s_l)|},$$

$$I_{(p_i,d_j)} = 1 - \frac{\sum_l^{20} |I_{(\Gamma^\mu,S)_{\wedge_{d_j}}}(p_i)(s_l) - I_{(\psi^\eta,S)}(d_j)(s_l)|}{\sum_l^{20} |I_{(\Gamma^\mu,S)_{\wedge_{d_j}}}(p_i)(s_l) + I_{(\psi^\eta,S)}(d_j)(s_l)|},$$

$$F_{(p_i,d_j)} = 1 - \frac{\sum_l^{20} |F_{(\Gamma^\mu,S)_{\wedge_{d_j}}}(p_i)(s_l) - F_{(\psi^\eta,S)}(d_j)(s_l)|}{\sum_l^{20} |F_{(\Gamma^\mu,S)_{\wedge_{d_j}}}(p_i)(s_l) + F_{(\psi^\eta,S)}(d_j)(s_l)|}.$$

Where,

$$s\left(p_i, d_j\right) = \frac{T_{(p_i,d_j)} + I_{(p_i,d_j)} + F_{(p_i,d_j)}}{3},$$

and

$$s\left((\mu_l)_{\wedge_{d_j}}, \eta\right) = 1 - \frac{\sum_l^{20} |(\mu_l)_{\wedge_{d_j}}(p_i)(s_l) - (\eta)(d_j)(s_l)|}{\sum_l^{20} |(\mu_l)_{\wedge_{d_j}}(p_i)(s_l) + (\eta)(p_i)(s_l)|}.$$

Then,

$$S\left(p_i, d_j\right) = s\left(p_i, d_j\right) \times s\left((\mu_l)_{\wedge_{d_i}}, \eta\right).$$

The result can be obtained as follows:

$$\begin{aligned} T_{(p_1,d_1)} = 1 - &\frac{|0.67-0.5| + |0.52-1| + |0.82-0| + |0.64-1| + |0.82-1| + |0.88-1|}{|0.67+0.5| + |0.52+1| + |0.82+0| + |0.64+1| + |0.82+1| + |0.88+1|} \\ &\frac{+|0.52-0.5| + |0.94-1| + |0.94-0.5| + |0.64-0| + |0.82-0| + |0.76-0|}{+|0.52+0.5| + |0.94+1| + |0.94+0.5| + |0.64+0| + |0.82+0| + |0.76+0|} \\ &\frac{+|0.94-0.5| + |0.52-0| + |0.82-1| + |0.58-0.5| + |0.82-0| + |0.64-0|}{+|0.94+0.5| + |0.52+0| + |0.82+1| + |0.58+0.5| + |0.82+0| + |0.64+0|} \\ = &\frac{+|0.88-0.5| + |0.52-0.5|}{+|0.88+0.5| + |0.52+0.5|} = 1 - \frac{8.43}{23.69} = 0.64. \end{aligned}$$

$$\begin{aligned} I_{(p_1,d_1)} = 1 - &\frac{|0.1-0.5| + |0.1-1| + |0.2-0| + |0.2-1| + |0.1-1| + |0.2-1| + |0-0.5|}{|0.1+0.5| + |0.1+1| + |0.2+0| + |0.2+1| + |0.1+1| + |0.2+1| + |0+0.5|} \\ &\frac{+|0.4-1| + |0.1-0.5| + |0.2-0| + |0.5-0| + |0.1-0| + |0.6-0.5| + |0-0|}{+|0.4+1| + |0.1+0.5| + |0.2+0| + |0.5+0| + |0.1+0| + |0.6+0.5| + |0+0|} \\ &\frac{+|0.2-1| + |0.1-0.5| + |0.2-0| + |0.2-0| + |0-0.5| + |0.7-0|}{+|0.2+1| + |0.1+0.5| + |0.2+0| + |0.2+0| + |0+0.5| + |0.7+0|} = 1 - \frac{9.2}{13.2} = 0.30. \end{aligned}$$

$$F_{(p_1,d_1)} = 1 - \frac{|0.24-0.5|+|0.36-0|+|0.24-1|+|0.48-0|+|0.24-0|+|0.3-0|}{|0.24+0.5|+|0.36+0|+|0.24+1|+|0.48+0|+|0.24+0|+|0.3+0|}$$
$$\frac{+|0.42-0.5|+|0.36-0|+|0.36-0.5|+|0.48-1|+|0.12-1|+|0.18-1|}{+|0.42+0.5|+|0.36+0|+|0.36+0.5|+|0.48+1|+|0.12+1|+|0.18+1|}$$
$$\frac{+|0.12-0.5|+|0.54-1|+|0.24-0|+|0.3-0.5|+|0.3-1|+|0.42-1|}{+|0.12+0.5|+|0.54+1|+|0.24+0|+|0.3+0.5|+|0.3+1|+|0.42+1|},$$
$$\frac{+|0.06-0.5|+|0.24-1|}{+|0.06+0.5|+|0.24+1|} = 1 - \frac{8.96}{16.9} = 0.47.$$

$$s(p_1, d_1) = \frac{0.64+0.30+0.47}{3} = 0.47.$$

$$s\left(\mu_{\Lambda_{d_1}}(p_1)(d_1), \eta(p_1)(d_1)\right) = 1 - \frac{|0.67-1|+|0.58-1|+|0.92-1|+|0.5-1|+|0.33-1|}{|0.67+1|+|0.58+1|+|0.92+1|+|0.5+1|+|0.33+1|}$$
$$\frac{+|0.25-1|+|0.42-1|+|0.33-1|+|0.58-1|+|0.67-1|}{+|0.25+1|+|0.42+1|+|0.33+1|+|0.58+1|+|0.67+1|}$$
$$\frac{+|0.83-1|+|0.67-1|+|0.92-1|+|0.5-1|+|0.75-1|}{+|0.83+1|+|0.67+1|+|0.92+1|+|0.5+1|+|0.75+1|},$$
$$\frac{+|0.25-1|+|0.75-1|+|0.5-1|+|0.33-1|+|0.58-1|}{+|0.25+1|+|0.75+1|+|0.5+1|+|0.33+1|+|0.58+1|}$$
$$= 1 - \frac{8.67}{31.33} = 0.72.$$

Then, $S(p_1, d_1) = 0.47 \times 0.72 = 0.34$.
Similarly, we calculate the following Tables 39–41 as follows:

Table 39. $s\left(p_i, d_j\right)$.

	d_1	d_2	d_3	d_4
p_1	0.47	0.54	0.44	0.46
p_2	0.51	0.49	0.38	0.49
p_3	0.7	0.49	0.38	0.53
p_4	0.49	0.46	0.51	0.46

Table 40. $s\left((\mu_l)_{\Lambda_{d_j}}, \eta\right)$.

	η
$\mu_{\Lambda_{d_1}}$	0.72
$\mu_{\Lambda_{d_2}}$	0.72
$\mu_{\Lambda_{d_3}}$	0.75
$\mu_{\Lambda_{d_4}}$	0.74

Table 41. $S\left(p_i, d_j\right) = s\left(p_i, d_j\right) \times s\left((\mu_l)_{\wedge_{d_i}}, \eta\right)$.

	d_1	d_2	d_3	d_4
p_1	0.34	**0.39**	0.33	0.34
p_2	**0.37**	0.35	0.29	0.36
p_3	**0.51**	0.35	0.29	0.39
p_4	0.35	0.33	**0.38**	0.34

We receive the score table in Table 41 as a result of similar calculations. It is clear from Table 41 that the first patient suffers from COVID-19, the second patient suffers from mumps, the third patient suffers mumps and patient four suffers yellow fever.

6. Conclusions

In this research, we introduced the concept of the generalized effective neutrosophic soft set (GENSS), which offers enhanced effectiveness and a range of advantageous properties. Additionally, we defined fundamental operations on the effective neutrosophic soft set, including complement, union, intersection, AND and OR operations. Finally, we showcased the practical application of GENSS in decision-making problems and medical diagnostics.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflicts of interest.

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