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Research article

Key generic technology research and development decision-making in Stackelberg competition

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Abstract: Research and development (R&D) of key generic technology (KGT) is conducive to improving the innovation capacity of countries and regions and has a significant impact on economic development and social progress. Compared to other technologies, the factors affecting the R&D decisions of KGT are more complex and need to be explored in depth. This study constructs a Stackelberg model considering R&D effort level, R&D efficiency and product differentiation to investigate the leader's and follower's decisions on R&D of KGT under three types of R&D modes. The results reveal that firms' strategic decisions are affected by product differentiation, R&D efficiency and different R&D modes. Product differentiation and R&D efficiency have impacts on equilibrium results and government subsidies for KGT R&D are optimal for social welfare.

Keywords: key generic technology; joint R&D; government subsidies; product differentiation; Stackelberg competition
Mathematics Subject Classification: 90B30, 91A35

1. Introduction

Globally, key generic technology (KGT) is of great strategic significance and practical value for social progress and sustainable economic development. KGT has attracted increasing attention from academia, government and business in recent decades [1–4]. KGT refers to the dominant and foundational technology that can be widely applied in multiple technological fields and spread and shared among various manufacturing industries, which has a strong spillover effect. KGT is often considered as a significant innovation that facilitates plentiful incremental innovations, considerably improving social benefits and driving economic growth [5–8]. In particular, in the range of emerging technologies, the breakthrough of KGT is the key to promoting technological advancement from imitative to independent innovation, from single to integrated innovation and from product to industrial competition [9]. Therefore, numerous economies have implemented policies to encourage the research and development (R&D) of KGT. For instance, in 2021, the Chinese government updated the "Industrial Critical Generic Technology Development Guide," which provides priority and support to the R&D of 174 KGT(s), including the raw material, equipment manufacturing, electronic information, communication industries and so on. Similarly, the Unites States Congress raised the "American Technology Preeminence Act," which proposes that KGT can promote industrial development and enhance national welfare. The European Union has formulated the "Cooperation of Science and Technology" to unite the technological forces of European countries and enhance the international competitiveness of European firms.

As forward-looking technologies in a fundamental and core position, KGT in the emerging technology fields are pivotal links between basic and applied research. Previous studies demonstrate that KGT plays a crucial supporting role in the technological innovation chain and maintains the core competitiveness of strategic emerging industries [10,11]. However, in the emerging technological fields, the valuable KGT often falls into the R&D dilemma. Specifically, the R&D of KGT is generally recognized as high threshold and cost, long cycle, high risk and obvious spillover [12]. Therefore, firms have a strong incentive of free-riding and aim to enjoy the "spillover effect" bonus of KGT, which would trigger the market failure of the R&D of KGT [13]. Joint R&D is an effective form of cooperation to promote the R&D of KGT. Joint R&D refers to two or more entities working together to conduct R&D activities [14]. Joint R&D projects can be divided into three types (invention, innovation and diffusion projects) [15]. Joint R&D is an important approach to technological innovation by sharing the development costs and risks, which can effectively promote the R&D of KGT in the context of the contradiction between rapidly growing technological demand and the R&D dilemma. Furthermore, it plays a predominant role in improving and maintaining the competitiveness of firms and is the most significant element of diffusion processes [16]. Meanwhile, with the accelerating development of technology, a shorter response time is required to capture changing market demands, driving a growing need for interfacing people and disciplines and integrating critical skills, and joint R&D is the key means. Hagedoorn [17] indicated that the benefits of developing joint R&D activities for firms in technology include accelerating the development of innovation and improving market transactions. In reality, the government and firms usually take measures such as intellectual property protection mechanisms, risk and responsibility sharing, benefits and intellectual property sharing to promote joint R&D activities [18].

Furthermore, along with these characteristics of R&D, governments and their policymakers, also conscious of this reality, are advocating the development of joint R&D in the technological innovation field by providing government subsidies. Therefore, since the 1980s, several economies, such as Europe, the US and Japan, have launched various favorable science and technology policies to encourage cooperation in R&D activities among firms, research institutes and universities. Governments should use a flexible policy mix to achieve their goals [19]. Government subsidies help firms reduce R&D investment and risk, and official policies can strengthen firms' confidence in technological innovation. Because KGT has a strong spillover effect and is beneficial to society as a whole, cooperation between the government, firms, research institutes and universities is reasonable and necessary. Interactions among these different types of participants through the development of joint R&D projects allow industry to obtain mutual benefits from the results of government-funded behavior [20]. Joint R&D is a common market approach, and government subsidies are part of the government solution. How does the R&D mode (independent R&D, joint R&D and government subsidies) affect technological innovation, firm profits, consumer surplus and social welfare? What are the effects of product differentiation and R&D efficiency on firms and government behavior under different R&D models? Which R&D model is more effective? These issues are important for firms' decisions on R&D of KGT aimed at improving competitiveness. Therefore, we construct a multiple oligopoly model that includes the R&D effort level, technology spillover, government subsidies and product differentiation. Further, we analyze firms' decision-making regarding the mode selection of KGT R&D and the government's science and technology subsidy policy.

The abundant literature on the R&D of KGT is embodied in two aspects: definition/measurement and diffusion/supply, neither of which pay sufficient attention to the R&D dilemma and market failure of KGT. In the former aspect, theorists have employed diversified methods to recognize and measure KGT, such as system integration, text analysis, co-classification index, process parameter, technology-specific performance indicator and so on [21–26]. In the latter, scholars have analyzed the diffusion and supply of KGT from the perspective of resource recombination (knowledge, capital), government funds, industry-university-research consociation, patent cooperation networks, innovation competition, intellectual property rights and so on [27–31].

Moreover, some studies have analyzed the factors and outcomes of R&D behavior on firms and industrial peers. Cohen and Levinthal [32] documented that a firm's R&D contributes to realizing spillovers from other firms' R&D efforts and improving its innovative ability. Konstantinos et al. [33] and Del Carmen Haro-Domínguez et al. [34] demonstrated that absorptive capacity is directly related to the R&D process and affects firm innovation. Grunfeld [35] probed the effects of investment and spillover on a firm's R&D decisions. Caputo et al. [36] highlighted the critical issue of how the transfer of technology from R&D to manufacturing affects technology-driven firms' innovative strategies. Pourkarimi and Kam [37] quantitatively demonstrated that R&D activities, such as increasing the R&D share and number of patents granted in advanced technology, have a significant positive effect on firm performance. Yan and Yang [38] analyzed the optimal licensing schemes for a mixed-ownership firm when facing uncertain R&D outcomes and technology spillover and established that the probability of R&D success plays a critical role in the process of determining the licensing strategy for the mixed firm. Blanco et al. [39] highlighted the necessity of revising government R&D policies toward greater coordination and resources and the implementation of new instruments, considering the impact of R&D expenditure on economic growth, development and integration. Kučera and Fiľa [40] took European Union countries as samples to identify the possible impact of R&D expenditure on innovation performance and influence of the innovation performance on economic development, and established a significant interdependence between R&D expenditure, innovation performance and level of economic development.

Other studies have focused on government policies promoting firms' R&D innovation. For example, Kim and Park [41] revealed that government investment in joint R&D contributes to the improvement of the short-term business performance of firms engaging in joint R&D projects supported by the government. Tong et al. [42] empirically documented that government subsidies can realize the purpose of stimulating the innovation of high-tech enterprises and highlighted the subsidy mechanism includes resource and signal transmission effects. Furthermore, Wu and Zhao [43] examined the impact of government R&D subsidies on enterprise R&D activities from both theoretical and empirical perspectives and proposed that R&D subsidies play a critical positive role in motivating the R&D projects of firms, and the incentive effect of subsidies will be enhanced with the improvement of R&D investment and efficiency.

However, concerning the R&D dilemma and market failure of KGT, the core role of leading firms with certain advantages and government policy remains to be further examined during joint R&D of KGT, especially the choice of R&D mode. Currently, the effects of different R&D modes on a firm's technology strategy and government behavior, and the multiple characteristics of the R&D mode, such as R&D effort level, R&D efficiency, technology spillover and government subsidies remain under consideration. In other words, we do not have a satisfactory answer to the question of "how to promote the R&D of KGT in building emerging technology knowledge mansions." Moreover, considering the key features of the spillover effect and "quasi-public goods" of KGT, the R&D of KGT requires a greater range of technological resource gathering and the participation of multiple parties, including government, firms, universities and research institutions. Therefore, technological innovation cooperation is a significant enabler for KGT R&D. The main contents of the literature review and the innovation points of this study are listed in Table 1.

Table 1. Summary of the literature.

Research orientation	Research contents	References	Innovation points
Definition and measurement of KGT	Using methods such as system integration, text analysis, co-classification index, process parameter, technology-specific performance indicators, etc.	Leydesdorff, 2008; Bekar et al., 2018; Heikkilä et al., 2023; da Ponte et al., 2023; Shafique and Hagedoorn, 2022; Vu et al., 2020	
Diffusion and supply of KGT	Analyzing from the perspective of resource recombination (knowledge, capital), government funds, industry-university-research consociation, patent cooperation network, innovation competition, intellectual property rights, etc.	Appio et al., 2017; Kokshagina et al., 2017; Zheng et al., 2023; Franzò et al., 2023; Cen et al., 2022	Construct a Stackelberg model considering more realistic factors using sequential game theory.
Factors affecting KGT R&D	Impacts of other firms' behavior, absorptive capacity, investment and spillover effects on R&D decisions of KGT.	Cohen and Levinthal, 1989; Konstantinos et al., 2011; del Carmen et al., 2007; Grunfeld, 2003	Degree of product differentiation, R&D effort level, R&D efficiency, technology spillover and government subsidies are incorporated into the model.
Government policy	Impacts of government R&D investment and subsidies on KGT.	Kim and Park, 2021; Tong et al., 2021; Wu and Zhao, 2021	Analyze the boundaries of joint R&D and government subsidies.

This paper contributes to existing research as follows: First, we conduct a comparative analysis of one firm's independent R&D, two firm's joint R&D and one firm's independent R&D with government subsidies. This helps reveal the boundaries of joint R&D and government subsidies effectively, which provides the guiding value for understanding the role of joint R&D, and government and market mechanisms in a firm's technological innovation. Second, our duopoly model includes various factors that are more comprehensive and closer to reality. Parameters such as the degree of product differentiation, R&D effort level, R&D efficiency, technology spillover and government subsidies are introduced in the model, which enriches and expands existing research.

The remainder of this paper is organized as follows. Section 2 describes our model construction, which includes three models: one firm's independent R&D (Model IR), a model (Model JR) in which two firms conduct joint R&D and one (Model SR) in which one firm conducts technology R&D with government subsidies. Section 3 presents the equilibrium results for the three models and analyzes the effects of product differentiation and R&D efficiency on each equilibrium outcome. Section 4 compares and analyzes the equilibrium results to reveal the boundaries of different R&D modes. Finally, our conclusions are presented in Section 5. Related proofs are provided in the Appendix.

2. Model

Construct a Stackelberg oligopolistic market model with Firm 1 (leader) and Firm 2 (follower), which produce differentiated products. The linear inverse demand function is $p_i = a - q_i - rq_j$ [44,45], where q_i is the output of each firm, and $r \in [0,1]$ is the degree of product differentiation (the higher the value, the weaker the heterogeneity), $i, j = 1, 2$ and $i \neq j$. Firm 1 is faced with the R&D of KGT. The R&D cost is $c_1 = t\frac{b^2}{2}$ [46,47], where $b > 0$ is the R&D effort level and $t > 0$ is the R&D efficiency (the higher the value, the lower the efficiency). For simplicity, no production cost is considered in this analysis. Thus, Firm 1's profit function is $\pi_1 = (p_1 + b)q_1 - c_1$. However, because of the spillover effect of KGT, Firm 2's profit function is defined as $\pi_2 = (p_2 + \lambda b)q_2$, where $\lambda \in (0,1)$ is the degree of spillover (the higher the value, the stronger the spillover effect). The consumer surplus function is $cs = \frac{{q_1}^2 + {q_2}^2 + 2rq_1q_2}{2}$, social welfare function is $sw = \sum_{i=1}^{2} \pi_i + cs$.

Consider three types of firm R&D modes: (i) One firm conducts independent R&D and we define the case as Model IR (independent R&D), and the equilibrium results are denoted by the superscript IR , . (ii) Two firms conduct joint R&D and share the R&D cost, and we define the case as Model JR (joint R&D). If $0 < \beta < 1$ is the proportion of R&D cost undertaken by Firm 1, then the cost functions of the two firms are $c_1 = t\frac{\beta b^2}{2}$ and $c_2 = t\frac{(1-\beta)b^2}{2}$ [48]. Regardless of the spillover effect, Firm 2's profit function changes as $\pi_2 = (p_2 + b)q_2 - c_2$, and the equilibrium results are denoted by superscript JR. (iii) The government subsidizes Firm 1 and the subsidy rate is $0 < s < 1$, and we define the case as Model SR (subsidizing R&D). Then, Firm 1's cost and profit functions are changed into $c_{1s} = (1-s)t\frac{b^2}{2}$ and $\pi_1 = (p_1 + b)q_1 - c_{1s}$, and the social welfare function is $sw = \sum_{i=1}^{2} \pi_i + cs - st\frac{b^2}{2}$. The equilibrium results are denoted by the superscript SU.

In the above game, that under Model IR is divided into two stages. In the first stage, Firm 1 determines the optimal R&D effort level b^* to maximize its profits. In the second stage, Firm 1 and Firm 2 determine the optimal output q_i^* according to profit maximization sequentially.

When the two firms conduct the joint R&D project, Model JR has three stages. In the first stage, Firm 1 determines the optimal proportion of R&D cost β^*. In the second stage, Firm 1 determines the optimal R&D effort level b^* to maximize its profits. In the third stage, the firms choose their optimal outputs q_i^* under Stackelberg competition.

When the government subsidizes the firm's R&D activity, the game under Model SR contains three stages. In the first stage, the government determines the optimal subsidy rate s^* to maximize social welfare. In the second stage, Firm 1 determines the optimal R&D effort level b^* required to maximize its profits. In the third stage, each firm chooses the optimal output q_i^* in the Stackelberg competition. The notations used in the models are listed in Table 2.

Table 2. Model notation.

i, j	Index for firms, $i, j = 1, 2$ and $i \neq j$
q_i	Deterministic production for Firm i
p_i	Deterministic price for Firm i
r	Degree of product differentiation
c_i	The R&D cost function of Firm i
β	The proportion of R&D cost undertaken by Firm 1
b	R&D effort level
t	R&D efficiency
λ	Degree of spillover
s	Government subsidy rate
π_i	Profit function of Firm i
cs	Consumer surplus function
sw	Social welfare function
IR	One firm conducts independent R&D
JR	Two firms conduct joint R&D
SR	One firm conducts R&D with government subsidies
$*$	Optimal values of different variables

3. Model analysis

3.1. Model IR

When the leading firm conducts R&D of KGT, in the second stage, each firm pursues maximal profits by satisfying $\frac{\partial \pi_i}{\partial q_i} = 0$. The optimal quantity can then be deduced as

$$q_1 = \frac{2(a+b)-r(a+\lambda b)}{2(2-r^2)},\ q_2 = \frac{(4-r^2)(a+\lambda b)-2r(a+b)}{4(2-r^2)}. \tag{1}$$

In the first stage, Firm 1 determines the optimal R&D effort level according to $\frac{\partial \pi_1}{\partial b} = 0$. Then, we can obtain

$$b^{IR} = \frac{a(2-r)(\lambda r-2)}{(\lambda^2+4t)r^2-4(r\lambda+2t-1)}. \tag{2}$$

Normal production requires $t > t_0^{IR} = \frac{2-\lambda r}{2-r^2}$; then, equilibrium results can be obtained in the case of Model IR, as presented in Lemma 1.

Lemma 1. The equilibrium results in the case of Model IR are:

$$q_1^{IR} = \frac{2at(r-2)}{(\lambda^2+4t)r^2-4(\lambda r+2t-1)},\ q_2^{IR} = \frac{a(r^2t+(\lambda^2-\lambda+2t)r-2(2t+\lambda-1))}{(\lambda^2+4t)r^2-4(\lambda r+2t-1)},$$

$$p_1^{IR}=\frac{a(2-r)(r^2t-\lambda r+2(1-t))}{(\lambda^2+4t)r^2-4(\lambda r+2t-1)},\ p_2^{IR}=\frac{a((\lambda^2+t)r^2-(\lambda^2+3\lambda-2t)r-2(2t-\lambda-1)}{(\lambda^2+4t)r^2-4(\lambda r+2t-1)},$$

$$\pi_1^{IR}=\frac{-a^2t(2-r)^2}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))},\ \pi_2^{IR}=\frac{a^2(r^2t+(\lambda^2-\lambda+2t)r-2(2t+\lambda-1))^2}{((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2},$$

$$cs^{IR}=\frac{\begin{array}{c}a^2(4(2-r)^2t^2+4rt(r-2)\left((r^2+2r-4)t+(\lambda-1)(\lambda r-2)\right)\\+((r^2+2r-4)t+(\lambda-1)(\lambda r-2))^2)\end{array}}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2},$$

$$sw^{IR}=\frac{\begin{array}{c}a^2((7r^4-20r^3-16(r^2-4r+2))t^2-((2-\lambda r)(\lambda r^3-2(\lambda+2)r^2\\+4(5-2\lambda)r+8(\lambda-2)))t-(\lambda-1)^2(\lambda r-2)^2)\end{array}}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2}.$$

The equilibrium results are obviously affected by the values of r and t. Proposition 1 can be obtained by analyzing these effects.

Proposition 1. *Under Model IR, the effects of r and t on the equilibrium results are:*

(i) The effects of r:

If $0<\lambda<\frac{4r-r^2-2}{2-r^2}$, $2-\sqrt{2}<r<1$ and $t_0^{IR}<t<t_1$, then $\frac{\partial\pi_1^{IR}}{\partial r}>0$; if $0<\lambda<\frac{4r-r^2-2}{2-r^2}$, $2-\sqrt{2}<r<1$ and $t>t_1$, then $\frac{\partial\pi_1^{IR}}{\partial r}<0$; $\frac{\partial\pi_2^{IR}}{\partial r}<0$; $\frac{\partial cs^{IR}}{\partial r}<0$; $\frac{\partial sw^{IR}}{\partial r}>0$; when $0<r<2-\sqrt{2}$, if $t_0^{FR}<t<t_2$, then $\frac{\partial b^{IR}}{\partial r}<0$; if $t>t_2$, then $\frac{\partial b^{IR}}{\partial r}>0$.

(ii) the effects of t:

$\frac{\partial\pi_1^{IR}}{\partial t}<0$; $\frac{\partial\pi_2^{IR}}{\partial t}<0$; when $f_3(t)<0$, $\frac{\partial cs^{IR}}{\partial t}>0$; otherwise, $\frac{\partial cs^{IR}}{\partial t}<0$; when $f_4(t)<0$, then $\frac{\partial sw^{IR}}{\partial t}>0$; otherwise, $\frac{\partial SW^{IR}}{\partial t}<0$; $\frac{\partial b^{IR}}{\partial t}<0$.

Proof. See Appendix A

As demonstrated in Proposition 1, if the leading firm conducts an R&D project that competes with the no R&D firm in Stackelberg competition with a spillover effect when $t>t_0^{IR}$, the product differentiation strategy can boost the quantity of Firm 2 and consumer surplus. But the impact of r on π_1 depends on the value ranges of λ and t. Specifically, when the degree of spillover is under the critical value and R&D efficiency is relatively low, increasing product differentiation can improve the R&D-conducting firm's profit. Furthermore, the positive influences of t on π_i and b manifest such that increasing R&D efficiency enhances the two firms' profits and the R&D effort level of Firm 1. This means that the growth in R&D efficiency can motivate firms to conduct more R&D activities of technological innovation, and peer firms can enjoy the increasing spillover effect bonus. However, the relationships between R&D efficiency and consumer surplus and social welfare display an inverted U-shaped trend. This finding indicates that the influences of R&D efficiency on consumers and the whole society are complex and nonlinear. On the one hand, a high R&D efficiency level helps firms enhance the technological innovation adopted in their products and services, which benefits downstream firms and consumers. On the other hand, an R&D efficiency level that is too high will widen the technological gap between the firm and its stakeholders, leading to disadvantages such as a cooperation dilemma, which has a negative impact on consumers and society as a whole. Therefore, firms should improve their R&D efficiency within a certain range by integrating the resources and advantages of multiple actors to promote the continuous improvement of scientific and technological innovation abilities and further develop the R&D of KGT.

3.2. Model JR

When two firms conduct joint R&D of KGT, in the third stage, Firm i pursues maximal profits according to $\frac{\partial \pi_i}{\partial q_i} = 0$. Then, the quantity of each firm is

$$q_1 = \frac{(a+b)(2-r)}{2(2-r^2)},\ q_2 = \frac{(4-r^2-2r)(a+b)}{4(2-r^2)}. \tag{3}$$

In the second stage, the leading Firm 1 determines the optimal R&D effort level for the maximal profits. Substituting q_1 into $\pi_1(q_1, b)$, the following can be derived when $\frac{\partial \pi_1(b)}{\partial b} = 0$:

$$b(\beta) = \frac{a(2-r)^2}{4\beta t(2-r^2)-(2-r)^2}. \tag{4}$$

Substituting Formula (4) into $\pi_1(b)$ and Formula (3), we can obtain:

$$q_1(\beta) = \frac{2(r-2)at\beta}{4\beta t(r^2-2)+(2-r)^2},\ q_1(\beta) = \frac{2(r-2)at\beta}{4\beta t(r^2-2)+(2-r)^2}, \tag{5}$$

$$\pi_1^{JR}(\beta) = \frac{\beta t(2-r)^2 a^2}{2(4\beta t(2-r^2)-(2-r)^2)},\ \pi_2^{JR}(\beta) = \frac{ta^2(\left(\beta^2 t+\frac{1}{2\beta}-\frac{1}{2}\right)r^4+4\left(\beta^2 t-\beta+1\right)r^3-4\left(\beta^2 t-3\beta+3\right)r^2-16(\beta^2 t+\beta-1)r+8(\beta^2 t+\beta-1))}{(4\beta t(r^2-2)+(2-r)^2)^2}. \tag{6}$$

In the first stage, Firm 1 (the leader) determines the optimal proportion of R&D costs. This must be profitable for both Firms 1 and 2, which satisfies

$$\pi_1^{JR} - \pi_1^{IR} \geq 0, \tag{7}$$

$$\pi_2^{JR} - \pi_2^{IR} \geq 0. \tag{8}$$

After calculation, when $\beta_0 = \frac{(2-r)^2}{4t(2-r^2)} < \beta \leq \beta_1 = \frac{(2-r)^2}{(2-\lambda r)^2}$, $\pi_1^{JR} \geq \pi_1^{IR}$ and $\pi_2^{JR} \geq \pi_2^{IR}$ exists. Therefore, the optimal proportion of R&D cost is $\beta^{JR} = \frac{(2-r)^2}{(2-\lambda r)^2}$.

Normal production requires $0 < r < 0.914394$ and $t > t_0^{JR} = \frac{(2-\lambda r)^2}{(2-r)(2-r^2)}$, then equilibrium results can be obtained in Model JR, as presented in Lemma 2.

Lemma 2. *The equilibrium results under Model JR are:*

$$\beta^{JR} = \frac{(2-r)^2}{(2-\lambda r)^2},\ b^{JR} = -\frac{(2-\lambda r)^2 a}{(\lambda^2+4t)r^2-4(\lambda r+2t-1)},\ q_1^{CJR} = \frac{2at(2-r)}{(\lambda^2+4t)r^2-4(\lambda r+2t-1)},$$

$$q_2^{JR} = \frac{at(r^2+2r-4)}{(\lambda^2+4t)r^2-4(\lambda r+2t-1)},\ p_1^{JR} = \frac{-a(r^3 t-\left(\lambda^2+2t\right)r^2+2(2\lambda-t)r+4(t-1))}{(\lambda^2+4t)r^2-4(\lambda r+2t-1)},$$

$$p_2^{JR} = \frac{a(\left(\lambda^2+t\right)r^2-2(2\lambda-t)r-4(t-1))}{(\lambda^2+4t)r^2-4(\lambda r+2t-1)},\ \pi_1^{JR} = \frac{-a^2(2-r)^2 t}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))},$$

$$\pi_2^{JR} = \frac{-a^2 t((\lambda^4-\lambda^2-2t)r^4-4\left(2\lambda^3-\lambda^2-\lambda+2t\right)r^3+4\left(5\lambda^2-4\lambda+2t-1\right)r^2-16(\lambda-2t-1)-32t)}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2},$$

$$cs^{JR} = \frac{a^2 t^2(5r^4+4r^3-32(r^2-1))}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2},$$

$$sw^{JR} = \frac{-ta^2((\lambda^4-3t)r^4-4(2\lambda^3+7t)r^3+24(\lambda^2+2t)r^2+32(2t-\lambda)r-16(6t-1))}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2}.$$

Proposition 2. *Under Model JR, the effects of* r *and* t *on the equilibrium results are:*

(i) The effects of r*:*

$\frac{\partial \pi_1^{JR}}{\partial r} < 0$*;* $\frac{\partial \pi_2^{JR}}{\partial r} < 0$*; if* $\lambda_1 < \lambda < 1$ *and* $t_0^{JR} < t < t_5$*, then* $\frac{\partial cs^{JR}}{\partial r} < 0$*; if* $\lambda_1 < \lambda < 1$ *and* $t > t_5$*, then* $\frac{\partial cs^{JR}}{\partial r} > 0$*;* $\frac{\partial sw^{JR}}{\partial r} < 0$*; if* $\lambda < r < 0.914394$*, then* $\frac{\partial b^{JR}}{\partial r} > 0$*; if* $0 < r < \lambda$*, then* $\frac{\partial b^{JR}}{\partial r} < 0$*;* $\frac{\partial \beta^{JR}}{\partial r} < 0$.

(ii) The effects of t*:*

$\frac{\partial \pi_1^{JR}}{\partial t} < 0$*; if* $t_0^{JR} < t < t_6$*, then* $\frac{\partial \pi_2^{JR}}{\partial t} > 0$*; if* $t > t_6$*, then* $\frac{\partial \pi_2^{JR}}{\partial t} < 0$*;* $\frac{\partial cs^{JR}}{\partial t} < 0$*;* $\frac{\partial sw^{JR}}{\partial t} < 0$*;* $\frac{\partial b^{JR}}{\partial t} < 0$*;* $\frac{\partial \beta^{JR}}{\partial t} = 0$.

Proof. See Appendix B

As demonstrated in Proposition 2, when the two firms conduct joint R&D in Stackelberg competition with $0 < r < 0.914394$ and $t > t_0^{JR}$, a high degree of product differentiation can boost the proportion of R&D costs undertaken by Firm 1, profits of the two firms and social welfare. However, a U-shaped relationship exists between r and cs, indicating the nonlinear effects of product differentiation on consumers. At a low level of product differentiation, Firm 1 undertakes higher competitive pressure and is more likely to increase its R&D effort level when implementing R&D activities. Furthermore, the growth in R&D efficiency increases the profit and R&D effort level of Firm 1, consumer surplus and social welfare, but has no impact on the proportion of R&D cost of the leading firm. The effect of R&D efficiency on Firm 2's profit depends on the value range of t. Specifically, when $t > t_6$, improving R&D efficiency benefits the follower's profit. This suggests that the two firms conducting joint R&D should have narrow levels of technological innovation, otherwise, the joint R&D may be ineffective owing to the unsynchronous development between the collaborators. The firms should maintain coordination and cooperation in the R&D process, which helps to ensure that both firms can be more effective in the realization of technological innovation and the achievement of goals.

Compared with Proposition 1, the relationship between r and sw changes from a positive to a negative correlation, and significant changes occur in how r and t affect cs. That is, r is no longer completely negatively correlated with cs, but t has a monotonic impact on cs. The inverted U-shaped relationship between r and π_1 becomes negative, which is also reflected in the interaction between t and sw. These changes are attributable to the changes in firm's R&D modes. Furthermore, the effects of r on π_2 are the same as that in Proposition 1.

3.3. Model SR

In the third stage, each firm maximizes its profits by satisfying $\frac{\partial \pi_i}{\partial q_i} = 0$, then the equilibrium quantities can be obtained as

$$q_1 = \frac{2(a+b)-r(a+\lambda b)}{2(2-r^2)},\ q_2 = \frac{(4-r^2)(a+\lambda b)-2r(a+b)}{4(2-r^2)}. \tag{9}$$

In the second stage, the optimal R&D effort level of Firm 1 when $\frac{\partial \pi_1}{\partial b} = 0$ can be obtained as

$$b(s) = \frac{a(r-2)(\lambda r-2)}{(4t(s-1)-\lambda^2)r^2+4(\lambda r-1)+8(1-s)t}. \tag{10}$$

In the first stage, the optimal government subsidy for maximal social welfare can be determined as

$$s^{SR} = \frac{\left(\left(7r^4+6r^3-36r^2+16(3-r)\right)\lambda+2r^2(3r-2)+16(1-r)\right)t+\left(5r^2-12\right)\lambda^3 r-\left(5r^3+8r^2-12(r+2)\right)\lambda^2+4\left(2r^2-r-6\right)\lambda+4r}{\left(3r^4+14r^3-4(7r^2+8r-12)\right)\lambda+2r^2(7r-10)+16(3-2r))t}. \tag{11}$$

Normal production of firms requires $t > t_0^{SR} = \frac{(3r^4-28r^2+48)\lambda^2+4r\left(7r^2-16\right)\lambda+4(12-5r^2)}{16(r^4-4r^2+4)}$, then equilibrium results can be obtained in Model SR, as presented in Lemma 3.

Lemma 3. The equilibrium results under Model SR are:

$$b^{SR} = \frac{(3\lambda r^4+14\lambda r^3-28\lambda r^2+14r^3-32r\lambda-20r^2+48(\lambda+1)-32r)a}{(16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1)};$$

$$q_1^{SR} = \frac{2(4r^3t+\left(5\lambda^2-5\lambda-8t\right)r^2+2(\lambda-4t-1)r-12\lambda(\lambda-1)+16t)a}{(16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1)};$$

$$q_2^{SR} = \frac{2(2r^4t+\left(\lambda^2-\lambda+4t\right)r^3+6(1-\lambda-2t)r^2-2\left(\lambda^2-\lambda+4t\right)r+4(3\lambda-3+4t))a}{(16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1)};$$

$$p_1^{SR} = \frac{-(a(4r^5t+\left(5\lambda^2-8t-2\lambda\right)r^4+4(4\lambda-4t+3)r^3+2\left(-11\lambda^2+18t-3\lambda-15\right)r^2+4(4t-9\lambda-7)r+8(3\lambda^2-4t+3\lambda+6)))}{((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))};$$

$$p_2^{SR} = \frac{a(\left(4t-3\lambda^2\right)r^4-4\left(3\lambda^2+4\lambda-2t\right)r^3+4\left(7\lambda^2+2\lambda-6t+3\right)r^2+\left(7\lambda^2+9\lambda-4t\right)r-12\lambda^2+8t-6(\lambda+1))}{((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))};$$

$$\pi_1^{SR} = \frac{a^2(4r^4t+\left(5\lambda^2-5\lambda-16t\right)r^3-2\left(10\lambda^2-6\lambda-4t+1\right)r^2-4\left(3\lambda^2-2\lambda-8t-1\right)r+8(3\lambda^2-3\lambda-4t))}{-2((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))};$$

$$\pi_2^{SR} = \frac{4a^2(2r^4t+(\lambda^2-\lambda+4t)r^3-6(\lambda-2t-1)r^2+2r\lambda(1-\lambda)-8rt+12(\lambda-1)+16t)^2}{((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2};$$

$$cs^{SR} = \frac{\begin{gathered}2a^2(20t^2r^8+16t\left(2\lambda^2-2\lambda+t\right)r^7+\left(11\lambda^4-22\lambda^3+32\lambda^2t+11\lambda^2-96\lambda t-208t^2+64t\right)r^6+(136\lambda^2-68\lambda^3-192\lambda^2t+\\256\lambda t-64(t+1)-68\lambda)r^5+\left(46\lambda^3-23\lambda^4-224\lambda^2t-11\lambda^2+576\lambda t+720t^2-24\lambda-352t+12\right)r^4+(324\lambda^3+(384t-\\648)\lambda^2+64t(t-10\lambda t)324\lambda+256t)r^3+\left(136\lambda^3-68\lambda^4+512\lambda^2t-160\lambda^2-1152\lambda t-1024t^2+184\lambda+640t-92\right)r^2\\+(768\lambda^2-384\lambda^3-256\lambda^2t+512\lambda t-384\lambda-256t)r+144(\lambda^4-2\lambda\left(\lambda^2-\lambda+1\right))-384t(\lambda^2+1)+768\lambda t+512t^2)\end{gathered}}{((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2};$$

$$sw^{SR} = \frac{a^2(3r^4t+28r^3t+16(\lambda^2-2\lambda-3t+1)r^2-64rt-36\lambda^2+72\lambda+96t-36)}{2((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))}.$$

We can obtain Proposition 3 by analyzing how the values of r and t affect the equilibrium outcomes.

Proposition 3. *Under Model SR, the effects of r and t on the equilibrium results are:*

(i) The effects of r:

$\frac{\partial \pi_1^{SR}}{\partial r} < 0$; *if* $t_0^{SR} < t < t_7$, *then* $\frac{\partial \pi_2^{SR}}{\partial r} > 0$; *if* $t > t_7$, *then* $\frac{\partial \pi_2^{SR}}{\partial r} < 0$; $\frac{\partial cs^{SR}}{\partial r} > 0$; $\frac{\partial sw^{SR}}{\partial r} < 0$;

$\frac{\partial b^{SR}}{\partial r} < 0$; $\frac{\partial s^{SR}}{\partial r} > 0$.

(ii) The effects of t:

$\frac{\partial \pi_1^{SR}}{\partial t} < 0$; *if* $t_0^{SR} < t < t_7$, *then* $\frac{\partial \pi_2^{SR}}{\partial t} > 0$; *if* $t > t_7$, *then* $\frac{\partial \pi_2^{SR}}{\partial t} < 0$; $\frac{\partial cs^{SR}}{\partial t} < 0$; $\frac{\partial sw^{SR}}{\partial t} < 0$; $\frac{\partial b^{SR}}{\partial t} < 0$; $\frac{\partial s^{SR}}{\partial t} > 0$.

Proof. See Appendix C

As demonstrated in Proposition 3, when the government subsidizes the firm's R&D activity in Stackelberg competition with $t > t_0^{SR}$, r and t are negatively correlated with π_1, which reveals that the improvements of product differentiation and R&D efficiency add to the leading firm's profit. The leading firm can adopt a product differentiation strategy to promote its profit, and social welfare will also be improved. Whether the strategy benefits the follower's profit depends on the value range of the R&D efficiency. r and t have the same nonlinear impact on the follower's profits. The follower can benefit from the R&D activities of the leader only when R&D efficiency is relatively low. Furthermore, the growth of product differentiation and R&D efficiency will motivate the leading firm to enhance R&D effort and improve social welfare, but the government thus reduces subsidies for firms' R&D activity. Consumer surplus can be improved by increasing the R&D efficiency of Firm 1.

The impacts of r and t on π_2, cs and sw in Proposition 3 are different from those in Proposition 1. The complete negative correlation between r (or t) and π_2 becomes a non-monotonic interaction, the uncertain relationship between t and cs (or sw) is replaced with a negative correlation and the U-shaped trend between r and b changes into a negative correlation. These changes are the result of government subsidies. The effects of t on π_1 and b are identical to those in Propositions 1 and 2. Therefore, the roles of product differentiation and R&D efficiency are dynamic and should be flexibly grasped in different contexts.

4. Comparative analysis

To ensure firms' normal productions, we can obtain $t > t_0^{JR} = \frac{(2-\lambda r)^2}{(2-r)(2-r^2)}$ under the three types of R&D modes. The results are presented as follows.

4.1. Comparison of Firm 1's (the leader) profits

Corollary 1. $\pi_1^{SR} > \pi_1^{IR} = \pi_1^{JR}$.

Proof. See Appendix D

As shown in Corollary 1, the leading firm's profit is maximized when the government provides subsidies for R&D activity in the Stackelberg competition. If there are no government subsidies, the profit of the leading firm will decrease regardless of whether it is conducting independent or joint R&D. Note that the profit under independent R&D mode is the same as that under joint R&D mode, indicating that the value of π_1^{IR} (π_1^{JR}) is the boundary of whether Firm 1 implements joint R&D, and Firm 1 has no reason to reject joint R&D. Additionally, the definite comparative relationship among the three types of profits shows that the government subsidy mode has the absolute advantage of improving the leading firm's profit, which can help relieve the R&D cost burden and boost confidence in the R&D process.

4.2. Comparison of Firm 2's (the follower) profits

Corollary 2. $\pi_2^{SR} > \pi_2^{JR} > \pi_2^{IR}$.

Proof. See Appendix E

As presented in Corollary 2, the follower obtains maximal profits when the government provides subsidies to Firm 1's R&D activity under Stackelberg competition. If the government subsidies are canceled, the follower's profit π_2^{JR} will rank second among the three models. The follower's profits are minimized when the leading firm conducts R&D activity independently. The government's fund support has a direct, positive impact on the follower's profit. Therefore, by enjoying the technological and capital advantages of the leading firm, small and medium-sized firms should actively cooperate with industrial leaders in the R&D of KGT to solve R&D dilemmas and improve their technological innovation ability.

4.3. Comparison of consumer surplus

Corollary 3. *If* $t_0^{JR} < t < t_8$, *then* $cs^{SR} > cs^{IR} > cs^{JR}$; *if* $t > t_8$, *then* $cs^{SR} > cs^{JR} > cs^{IR}$.

Proof. See Appendix F

As indicated in Corollary 3, consumer surplus is always maximized when the government subsidizes Firm 1's R&D activity in Stackelberg competition, regardless of the value range of t. However, a comparison of the consumer surplus under independent and joint R&D modes is uncertain. Specifically, the consumer surplus under joint R&D mode will be greater if the value of t is above the critical value and smaller than that under Model IR. That is, the government subsidies mode has the absolute advantage of improving consumer surplus. The R&D cooperation between two firms can improve the utility of consumers only when the R&D efficiency is relatively low. Too large an R&D efficiency gap will lead to cooperation difficulties, which are not conducive to the improvement of consumer surplus.

4.4. Comparison of social welfare

Corollary 4. $sw^{SR} > sw^{JR} > sw^{IR}$.

Proof. See Appendix G

As presented in Corollary 4, the social welfare ranking is consistent with that of Firm 2's profit, revealing the strong, positive correlation between social welfare and the profit of the follower. From the perspective of social welfare, one firm conducting R&D with government subsidies is optimal, indicating that the government support promotes the maximization of social welfare. Under such support, the increase in the follower's profit will compensate for the lost consumer surplus when the leading firm conducts R&D activity, thus resulting in an overall increase in social welfare. By synthesizing Corollaries 1–4, we can determine that the function of government behavior in guiding firm R&D activities is far greater than that of firms. From the perspectives of firms, consumers and the whole of society, government behaviors always play a dominant role in motivating firm R&D and technological innovation.

5. Conclusions

This study constructs a Stackelberg model considering R&D effort level, technology spillover, R&D efficiency and product differentiation to investigate the leader's and follower's decisions on R&D of KGT under independent, joint and government subsidy R&D modes. The results reveal the condition for joint R&D and the factors that affect R&D activities. The main conclusions are as follows.

First, the firms' strategic decision on the R&D of KGT is affected by product differentiation, R&D efficiency, and different R&D modes. When the leading firm conducts independent R&D activities, increasing product differentiation can enhance the follower's profit, consumer surplus and social welfare, but may not improve the leading firm's profit, depending on the degree of spillover and R&D efficiency. In Model JR, the growth in R&D efficiency boosts the profits of the leading firm, consumer surplus and social welfare. However, improving R&D efficiency benefits follower's profits only when R&D efficiency is relatively low. This suggests that the two firms conducting joint R&D should have narrow levels of technological innovation to ensure effective cooperation. In Model SR, the leading firm can adopt the product differentiation strategy to promote its profit and social welfare will also be improved. However, whether the product differentiation strategy benefits the follower's profit depends on the value range of the R&D efficiency. Furthermore, when the government funds the firm's R&D project, the improvement in R&D efficiency helps increase consumer surplus and social welfare, which is the same as under the joint R&D mode. In other words, an improvement in R&D efficiency can produce more profits for the leading firm among the three types of R&D modes. Therefore, the leading firm should vigorously improve R&D efficiency by integrating the resources and advantages of multiple actors to promote the continuous improvement of scientific and technological innovation abilities, and further develop the R&D of KGT.

Second, product differentiation and R&D efficiency exert varied impacts on the firm's R&D behaviors and the level of government subsidies under different R&D modes. In Model IR, when R&D efficiency is high, product differentiation motivates the leading firm to increase its R&D efforts and investments. However, the R&D efforts can be enhanced with the growth of product differentiation only when the products of the two firms are highly heterogeneous. Different from independent and joint R&D modes, product differentiation can always promote the leading firm's R&D efforts but reduces government subsidies, which are unaffected by any factors. The main reason is that product differentiation has a positive effect on the leading firm's R&D efforts, profit and social welfare, which conforms to the government's goal. Thus, the government subsidies are finally decreased. We also found that an increase in R&D efficiency will motivate the leading firm's R&D efforts under the three R&D modes. This means that enhancing R&D efficiency is a significant way to help technology-driven firms solve R&D difficulty and promote R&D innovation of KGT.

Third, the comparative analyses reveal the various characteristics of independent R&D, joint R&D and government subsidies. The leading firm's profit is maximized when the government provides subsidies to the R&D activity in the Stackelberg competition. If the government provides no subsidies, the leading firm's profit will decrease regardless of whether independent or joint R&D

is conducted. Note that the profit under the independent R&D mode is the same as that under the joint R&D mode, which reveals the boundary between the two modes. In terms of the follower, its profits will be maximized under government subsidy policy but minimized when the leading firm conducts R&D activity independently. Furthermore, consumer surplus is always maximized under government subsidies. The R&D cooperation between two firms can improve the utility of consumers only when the R&D efficiency is relatively low. Note that the social welfare ranking is consistent with that of the follower's profit, displaying a positive correlation between social welfare and the follower's profit. From the perspective of social welfare, one firm conducting R&D with government subsidies is optimal, owing to the government's goal of social welfare maximization. Moreover, the comparative analyses reveal the absolute advantage of the government in improving the profits of the leader and follower, consumer surplus and social welfare, indicating the considerable functions of government behaviors in guiding firms' R&D activities. Government behaviors play a dominant role in motivating firm R&D of KGT.

Based on the above conclusions, the government should formulate effective favorable policies such as subsidy support and improve the management system for input into science and technology. Government financial investment in science and technological innovation reflects the regional and industrial levels of science and technology. Providing more funding support can promote the improvement of the innovation efficiency of technology-driven firms. Additionally, firms should start from improving R&D efficiency, and take measures from aspects of researchers, R&D funds, knowledge sharing and so on. In return for the spillover effect bonus of KGT, joint R&D should be actively conducted with the government, universities, scientific research institutions and upstream and downstream firms, giving full play to the advantages of different participants and adjusting the resource allocation structure to overcome the R&D problems of KGT. These recommendations have positive and practical value for enterprises to conduct KGT R&D and for the government to optimize technology policies in a complex market environment.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare no conflict of interest.

References

1. J. Wang, R. J. Xie, W. T. Hu, The influencing factors and promotion countermeasures of industrial generic technology diffusion based on differential game, *Discrete. Dyn. Nat. Soc.*, **2021** (2021). https://doi.org/10.1155/2021/6643922
2. J. Ma, Developing joint R&D institutes between Chinese universities and international enterprises in China's innovation system: A case at Tsinghua University. *Sustainability,* **11** (2019), 7133. https://doi.org/10.3390/su11247133
3. A. Gambardella, M. S. Giarratana, General technological capabilities, product market fragmentation, and markets for technology, *Res. Policy.*, **42** (2013), 315–325. https://doi.org/10.1016/j.respol.2012.08.002

4. E. Maine, E. Garnsey, Commercializing generic technology: The case of advanced materials ventures, *Res. Policy.*, **35** (2006), 375–393. https://doi.org/j.respol.2005.12.006
5. M. P. Feldman, J. W. Yoon, An empirical test for general purpose technology: An examination of the cohen-boyer rDNA technology, *Ind. Corp. Change.*, **21** (2012), 249–275. https://doi.org/10.1093/icc/dtr040
6. M. Coccia, The source and nature of general purpose technologies for supporting next K-waves: Global leadership and the case study of the U.S. Navy's mobile user objective system, *Technol. Forecast. Soc.*, **116** (2017), 331–339. https://doi.org/10.1016/j.techfore.2016.05.019
7. A. Schaefer, D. Schiess, R. Wehrli, Long-term growth driven by a sequence of general purpose technologies, *Econ. Model.*, **37** (2014), 23–31. https://doi.org/10.1016/j.econmod.2013.10.014
8. L. Ardito, A. M. Petruzzelli, V. Albino, Investigating the antecedents of general purpose technologies: A patent perspective in the green energy field, *J. Eng. Technol. Manage.*, **39** (2016), 81–100. https://doi.org/10.1016/j.jengtecman.2016.02.002
9. Y. Liu, J. Du, J. Yang, An incentive mechanism for general purpose technologies R&D based on the concept of super-conflict equilibrium: Empirical evidence from nano industrial technology in China, *Technol. Forecast. Soc.*, **147** (2019), 185–197. https://doi.org/10.1016/j.techfore.2019.07.014
10. S. Petralia, Mapping general purpose technologies with patent data, *Res. Policy.*, **49** (2020), 104013. https://doi.org/10.1016/j.respol.2020.104013
11. L. Wang, H. Shu, G. P. Zhu, A study on the relationship between different technological absorptive capacities and R&D investment in joint R&D, In: *2013 International Conference on Management Science and Engineering 20th Annual Conference Proceedings*, Harbin, China, 2013, 1974–1980. https://doi.org/10.1109/ICMSE.2013.6586535
12. D. Rotolo, D. Hicks, B. R. Martin, What is an emerging technology? *Res. Policy.*, **44** (2015), 1827–1843. https://doi.org/10.1016/j.respol.2015.06.006
13. Y. L. Zheng, B. Yang, L. Wang, Research on the evolution of industrial generic technology R&D behavior and multiple failures, *Sci. Res. Manag.*, **40** (2019), 164–174. https://doi.org/10.13580/j.cnki.fstc.2019.05.004
14. R. Du, S. Z. Ai, Cross-organizational knowledge acquisition through flexible hiring and joint R&D: Insights from a survey in China, *Expert. Syst. Appl.*, **35** (2008), 434–441. https://doi.org/10.1016/j.eswa.2007.07.009
15. N. Arranz, J. C. F. de Arroyabe, Joint R&D projects: Experiences in the context of European technology policy, *Technol. Forecast. Soc.*, **73** (2006), 860–885. https://doi.org/10.1016/j.techfore.2005.11.003
16. D. Zhao, M. Wang, X. Wang, Analysis and control strategy for relationship risk in manufacturer-supplier joint R&D, In: *2008 4th International Conference on Wireless Communications, Networking and Mobile Computing*, Dalian, China, 2008, 1–4. https://doi.org/10.1109/WiCom.2008.2469
17. J. Hagedoorn, Understanding the rationale of strategic technology partnering: Interorganizational modes of cooperation and sectoral differences, *Strateg. Manag. J.*, **14** (1993), 371–385. https://doi.org/10.1002/smj.4250140505
18. Y. C. Zhou, B. Zhang, J. Zou, J. Bi, K. Wang, Joint R&D in low-carbon technology development in China: A case study of the wind-turbine manufacturing industry, *Energ. Policy*, **46** (2012), 100–108. https://doi.org/10.1016/j.enpol.2012.03.037

19. J. Chen, C. Sun, Y. Wang, J. Liu, P. Zhou, Carbon emission reduction policy with privatization in an oligopoly model, *Environ. Sci. Pollut. Res.*, **30** (2023), 45209–45230. https://doi.org/10.1007/s11356-022-24256-2
20. N. Arranz, J. C. F. de Arroyabe, Joint R&D projects as complex systems: A framework of analysis, *Ieee. T. Eng. Manage*., **55** (2008), 552–565. https://doi.org/doi: 10.1109/TEM.2008.927816
21. L. Leydesdorff, On the normalization and visualization of author co-citation data: Salton's cosine versus the Jaccard index, *J. Am. Soc. Inf. Sci. Tec*., **59** (2008), 77–85. https://doi.org/10.1002/asi.20732
22. C. Bekar, K. Carlaw, R. Lipsey, General purpose technologies in theory, application and controversy: A review, *J. Evol. Econ*., **28** (2018), 1005–1033. https://doi.org/10.1007/s00191-017-0546-0
23. J. Heikkilä, J. Rissanen, T. Ali-Vehmas, Coopetition, standardization and general purpose technologies: A framework and an application, *Telecommun. Policy*., **47** (2023), N.PAG. https://doi.org/10.1016/j.telpol.2022.102488
24. A. da Ponte, G. Leon, I. Alvarez, Technological sovereignty of the EU in advanced 5G mobile communications: An empirical approach, *Telecommun. Policy*., **47** (2023), N.PAG. https://doi.org/10.1016/j.telpol.2022.102459
25. M. Shafique, J. Hagedoorn, Look at U: Technological scope of the acquirer, technological complementarity with the target, and post-acquisition R&D output, *Technovation*., **115** (2022), N.PAG. https://doi.org/10.1016/j.technovation.2022.102533
26. K. Vu, P. Hanafizadeh, E. Bohlin, ICT as a driver of economic growth—a survey of the literature and directions for future research, *Telecommun. Policy*., **44** (2020), Article 101922. https://doi.org/10.1016/j.telpol.2020.101922
27. F. P. Appio, A. Martini, G. Fantoni, The light and shade of knowledge recombination: Insights from a general-purpose technology, *Technol. Forecast. Soc*., **125** (2017), 154–165. https://doi.org/10.1016/j.techfore.2017.07.018
28. O. Kokshagina, T. Gillier, P. Cogez, P. L. Masson, B. Weil, Using innovation contests to promote the development of generic technologies, *Technol. Forecast. Soc*., **114** (2017), 152–164. https://doi.org/10.1016/j.techfore.2016.07.035
29. Y. Zheng, C. Bai, L. Wang, C. Han, M. Yang, A. Pappu, Evolutionary game analysis on the diffusion of general purpose technologies with government multiple supports, *Econ. Innov. New Tech.*, 2023, 1–19. https://doi.org/10.1080/10438599.2023.2196418
30. S. Franzò, N. Doppio, A. Natalicchio, F. Frattini, L. Mion, Designing innovation contests to support external knowledge search in small and medium-sized enterprises, *Technovation.*, **121** (2023). https://doi.org/10.1016/j.technovation.2022.102684
31. J. Cen, L. Zhou, Z. Zhang, Who are building technical knowledge mansions? Impact of patent cooperation networks on the generic technology R&D performance of emerging enterprises, *Technol. Anal. Strateg*., **34** (2022), 1384–1401. https://doi.org/10.1080/09537325.2021.1962521
32. W. M. Cohen, D. A. Levinthal, Innovation and learning: The two faces of R&D, *Econ. J.*, **99** (1989), 569–596. https://doi.org/10.2307/2233763
33. K. Konstantinos, P. Alexandros, P. Margarita, I. George, Absorptive capacity, innovation, and financial performance, *J. Bus. Res*., **64** (2011), 1335–1343. https://doi.org/10.1016/j.jbusres.2010.12.005
34. H. del Carmen, A. Daniel, L. Francisco Javier, R. M. Antonia, The impact of absorptive capacity on technological acquisitions engineering consulting companies, *Technovation*., **27** (2007), 417–425. https://doi.org/10.1016/j.technovation.2007.04.003

35. L. A. Grunfeld, Meet me halfway but don't rush: Absorptive capacity and strategic R&D investment revisited, *Int. J. Ind. Organ.*, **21** (2003), 1091–1109. https://doi.org/10.1016/S0167-7187(03)00076-6
36. F. Caputo, P. Soto-Acosta, C. Chiacchierini, A. Mazzoleni, P. Passaro, Smashing local boundaries for sustain companies' innovativeness: The role of international R&D teams, *J. Bus. Res.*, **128** (2021), 641–649. https://doi.org/10.1016/j.jbusres.2020.04.062
37. P. Pourkarimi, E. Kam, The impact of R&D and advertising on firm performance in high-tech industries—evidence from the U.S. information and communications technology industry, *J. Quant. Econ.*, **20** (2022), 723–753. https://doi.org/10.1007/S40953-022-00302-1
38. Q. Yan, L. Yang, Optimal licensing schemes for a mixed ownership firm when facing uncertain R&D outcomes and technology spillover, *Int. Rev. Econ. Financ.*, **58** (2018), 550–572. https://doi.org/10.1016/j.iref.2018.06.003
39. F. A. Blanco, F. J. Delgado, M. J. Presno, R&D expenditure in the EU: Convergence or divergence? *Econ. Res.-Ekon. Istraživanja.*, **33** (2020), 1685–1710. https://doi.org/10.1080/1331677X.2020.1756371
40. J. Kučera, M. Fiľa, R&D expenditure, innovation performance and economic development of the EU countries, *Entrep. Sustain.*, **9** (2022), 227–241. https://doi.org/10.9770/JESI.2022.9.3(14)
41. S. Kim, K. C. Park, Government funded R&D collaboration and it's impact on SME's business performance, *J. Informetr.*, **15** (2021), 101197. https://doi.org/10.1016/j.joi.2021.101197
42. X. Tong, Z. Wang, X. Li, The influence of government subsidy on enterprise innovation: Based on Chinese high-tech enterprises, *Econ. Res.-Ekon. Istraživanja.*, **35** (2021), 1481–1499. https://doi.org/10.1080/1331677X.2021.1972818
43. W. S. Wu, K. Zhao, Government R&D subsidies and enterprise R&D activities: Theory and evidence, *Econ. Res.-Ekon. Istraživanja.*, **35** (2021), 391–408. https://doi.org/10.1080/1331677X.2021.1893204
44. A. M. Spence, Product differentiation and welfare, *Am. Econ. Rev.*, **66** (1976), 407–414. https://doi.org/10.1016/0309-586X(76)90008-X
45. J. Chen, X. Xie, J. Liu, Capacity sharing with different oligopolistic competition and government regulation in a supply chain, *Manag. Decis. Econ.*, **41** (2020), 79–92. https://doi.org/10.1002/mde.3094
46. Q. Dong, J. C. Bárcena-Ruiz, Corporate social responsibility and partial privatization of state holding corporations, *J. Econ.*, **132** (2021), 223–250. https://doi.org/10.1007/s00712-020-00725-5
47. X. Wang, X. Zhang, L. Zhu, Imperfect market, emissions trading scheme, and technology adoption: A case study of an energy-intensive sector, *Energ. Econ.*, **81** (2019), 142–158. https://doi.org/10.1016/j.eneco.2019.03.014.
48. J. Chen, C. Sun, J. Shi, J. Liu, Technology R&D and sharing in carbon emission reduction in a duopoly, *Technol. Forecast. Soc.*, **183** (2022), 121936. https://doi.org/10.1016/j.techfore.2022.121936

Supplementary

Appendix A

(i) Effects of r:

$\frac{\partial \pi_1^{IR}}{\partial r} = \frac{2a^2 t(2-r)((\lambda^2-\lambda+4t)r-2(\lambda+2t-1))}{((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2}$. Let $f_1(t) \stackrel{\text{def}}{=} (\lambda^2-\lambda+4t)r-2(\lambda+2t-1)=0$, and there is $t_1 = \frac{(2-\lambda r)(1-\lambda)}{4(1-r)}$. Under the condition of $2-\sqrt{2} < r < 1$, if $0 < \lambda < \frac{4r-r^2-2}{2-r^2}$ and $t_0^{IR} < t < t_1$, then $\frac{\partial \pi_1^{IR}}{\partial r} > 0$; if $0 < \lambda < \frac{4r-r^2-2}{2-r^2}$ and $t > t_1$, then $\frac{\partial \pi_1^{IR}}{\partial r} < 0$.

$\frac{\partial \pi_2^{IR}}{\partial r} = \frac{a^2(2(\lambda^4+6\lambda^2 t-\lambda^3+t^2)r^2-8(\lambda^3+(2t-1)\lambda^2+2(2\lambda t+2t^2-t))r+8(2t+1)\lambda^2+8(2t-1)\lambda+16t(t-1))(r^2 t+(\lambda^2-\lambda+2t)r-2(2t+\lambda-1))}{(4(\lambda r+2t-1)-(\lambda^2+4t)r^2)^3}$. When $t > t_0^{IR}$, there is $2(\lambda^4 + 6\lambda^2 t - \lambda^3 + t^2)r^2 - 8(\lambda^3 + (2t-1)\lambda^2 + 2(2\lambda t + 2t^2 - t))r + 8(2t+1)\lambda^2 + 8(2t-1)\lambda + 16t(t-1) > 0$, $r^2 t + (\lambda^2 - \lambda + 2t)r - 2(2t + \lambda - 1) < 0$, and $4(\lambda r + 2t - 1) - (\lambda^2 + 4t)r^2 > 0$. Therefore, if $t > t_0^{IR}$, then $\frac{\partial \pi_2^{IR}}{\partial r} < 0$.

$\frac{\partial cs^{IR}}{\partial r} = \frac{a^2((40t^3+2\lambda(7\lambda-8)t^2+\lambda^3(\lambda-1)t)r^4-(8t^2(6t-4\lambda^2+7\lambda-3)-4\lambda^2(3\lambda^2-7\lambda+4)t-\lambda^4(\lambda-1)^2)r^3-(144t^3+24(5\lambda^2-3\lambda-5)t^2+12(\lambda^4+2\lambda^3-8\lambda^2+5\lambda)t+6\lambda^3(\lambda-1)^2)r^2+(64t^2(6t+2\lambda^2+\lambda-6)+8(\lambda^4+4\lambda^3-7\lambda^2-6\lambda+8)t+12\lambda^2(\lambda-1)^2)r-256t^3-32(\lambda^2+4\lambda-9)t^2-16(\lambda^3+\lambda^2-7\lambda+5)t-8\lambda(\lambda-1)^2)}{(4(\lambda r+2t-1)-(\lambda^2+4t)r^2)^3}$. When $t > t_0^{IR}$, there is $(40t^3 + 2\lambda(7\lambda-8)t^2 + \lambda^3(\lambda-1)t)r^4 - (8t^2(6t - 4\lambda^2 + 7\lambda - 3) - 4\lambda^2(3\lambda^2 - 7\lambda + 4)t - \lambda^4(\lambda-1)^2)r^3 - (144t^3 + 24(5\lambda^2 - 3\lambda - 5)t^2 + 12(\lambda^4 + 2\lambda^3 - 8\lambda^2 + 5\lambda)t + 6\lambda^3(\lambda-1)^2)r^2 + (64t^2(6t + 2\lambda^2 + \lambda - 6) + 8(\lambda^4 + 4\lambda^3 - 7\lambda^2 - 6\lambda + 8)t + 12\lambda^2(\lambda-1)^2)r - 256t^3 - 32(\lambda^2 + 4\lambda - 9)t^2 - 16(\lambda^3 + \lambda^2 - 7\lambda + 5)t - 8\lambda(\lambda-1)^2 < 0$, and $4(\lambda r + 2t - 1) - (\lambda^2 + 4t)r^2 > 0$. Therefore, if $t > t_0^{IR}$, then $\frac{\partial cs^{IR}}{\partial r} < 0$.

$\frac{\partial sw^{IR}}{\partial r} = \frac{a^2(\lambda^6 r^3-2r^2(r+3)\lambda^5+(r^2(rt+12t+1)+12(1-t)r+4(2t+3))r\lambda^4-(r^4 t+28r^3 t+6(4t+1)r^2-8(4t-3)r+8(2t+1))\lambda^3+(2r^3 t(7rt+8(2t+1))-24t(5t-4)r^2+4(32t^2-14t+3)r-16(2t^2+t-1))\lambda^2-(8r^3 t^2(2r-7)-18t(4t-5)r^2-16t(4t-3)r+8(16t^2-14t+1))\lambda+8t(5r^4 t^2-3t(2t-1)r^3-3t(6t+5)r^2+8(6t^2-6t+1)r-2(16t(t-1)+5)))}{((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}$. When $t > t_0^{IR}$, there is $(\lambda^2 + 4t)r^2 - 4(\lambda r + 2t - 1) < 0$, and $\lambda^6 r^3 - 2r^2(r+3)\lambda^5 + \left(r^2(rt + 12t + 1) + 12(1-t)r + 4(2t+3)\right)r\lambda^4 - \left(r^4 t + 28r^3 t + 6(4t+1)r^2 - 8(4t-3)r + 8(2t+1)\right)\lambda^3 + \left(2r^3 t\left(7rt + 8(2t+1)\right) - 24t(5t-4)r^2 + 4(32t^2 - 14t + 3)r - 16(2t^2 + t - 1)\right)\lambda^2 - \left(8r^3 t^2(2r-7) - 18t(4t-5)r^2 - 16t(4t-3)r + 8(16t^2 - 14t + 1)\right)\lambda + 8t\left(5r^4 t^2 - 3t(2t-1)r^3 - 3t(6t+5)r^2 + 8(6t^2 - 6t + 1)r - 2(16t(t-1) + 5)\right) < 0$. Therefore, if $t > t_0^{IR}$, then $\frac{\partial sw^{IR}}{\partial r} > 0$.

$\frac{\partial b^{IR}}{\partial r} = \frac{a((2\lambda^2(\lambda-1)+8t(\lambda+1))r^2-8(\lambda^2-(1-2t)\lambda+4t)r+8(2t+1)\lambda+8(2t-1))}{-((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2}$. Let $f_2(t) \stackrel{\text{def}}{=} \left(2\lambda^2(\lambda-1) + 8t(\lambda+1)\right)r^2 - 8(\lambda^2 - (1-2t)\lambda + 4t)r + 8(2t+1)\lambda + 8(2t-1) = 0$, there is $t_2 = \frac{(2-\lambda r)^2(1-\lambda)}{4(r^2+2)(\lambda+1)-8r(\lambda+2)}$. When $0 < r < 2-\sqrt{2}$, $t_2 > 0$. Therefore, when $0 < r < 2-\sqrt{2}$, if $t > t_2$, then $\frac{\partial b^{IR}}{\partial r} > 0$; if $t_0^{IR} < t < t_2$, then $\frac{\partial b^{IR}}{\partial r} < 0$.

(ii) Effects of t:

$\frac{\partial \pi_1^{IR}}{\partial t} = \frac{-a^2(2-r)^2(\lambda r-2)^2}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2} < 0$.

$\frac{\partial \pi_2^{IR}}{\partial t} = \frac{2a^2(r-2)(\lambda r-2)(r^2\lambda+(\lambda^2-\lambda+2t)r-2(2t+\lambda-1))(r^2\lambda-4\lambda+2r)}{((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}$. When $t > t_0^{IR}$, there is $r^2\lambda + (\lambda^2-\lambda+2t)r - 2(2t+\lambda-1) < 0$, and $r^2\lambda - 4\lambda + 2r < 0$. Therefore, $\frac{\partial \pi_2^{IR}}{\partial t} < 0$.

$\frac{\partial cs^{IR}}{\partial t} = \frac{a^2(3r^5-6r^4-4r^3+8r^2)\lambda^4+\left(32r(r^2-1)-3r^5-8r^2(r^2-1)\right)\lambda^3-(2-r)((r^3((5r+2)tr+2(7-10t))+20r^2+16(rt-r-1))\lambda^2-(8r^3t(r+2)+4(1-2t)(5r^2+2r-4))\lambda-4(tr^3-6r^2t-2r+8t))}{((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}$. Let $f_3(t) \overset{\text{def}}{=} (3r^5-6r^4-4r^3+8r^2)\lambda^4 + \left(32r(r^2-1)-3r^5-8r^2(r^2-1)\right)\lambda^3 - (2-r)((r^3((5r+2)tr+2(7-10t)) + 20r^2 + 16(rt-r-1))\lambda^2 - (8r^3t(r+2) + 4(1-2t)(5r^2+2r-4))\lambda - 4(tr^3-6r^2t-2r+8t)$. When $t > t_0^{IR}$, $f_3(t)$ can be positive or negative. Therefore, if $f_3(t) > 0$, then $\frac{\partial cs^{IR}}{\partial t} < 0$; if $f_3(t) < 0$, then $\frac{\partial cs^{IR}}{\partial t} > 0$.

$\frac{\partial sw^{IR}}{\partial t} = \frac{a^2(r-2)(\lambda r-2)(\lambda^3 r(r^3+8)-4(2r^3+r^2+2r+4)\lambda^2+((2r^3(5r-6)-16(r-1)(r-2))t+4(7r^2+4))\lambda-12r(r-1)(r+3)t+16(1-2r))}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}$. Let $f_4(t) \overset{\text{def}}{=} \lambda^3 r(r^3+8) - 4(2r^3+r^2+2r+4)\lambda^2 + ((2r^3(5r-6)-16(r-1)(r-2))t + 4(7r^2+4))\lambda - 12r(r-1)(r+3)t + 16(1-2r)$. When $t > t_0^{IR}$, $f_4(t)$ can be positive or negative. Therefore, if $f_4(t) > 0$, then $\frac{\partial sw^{IR}}{\partial t} < 0$; if $f_4(t) < 0$, then $\frac{\partial sw^{IR}}{\partial t} > 0$.

$\frac{\partial b^{IR}}{\partial t} = \frac{a(r-2)(\lambda r-2)(4r^2-8)}{((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2} < 0$.

Appendix B

(i) Effects of r:

$\frac{\partial \pi_1^{JR}}{\partial r} = \frac{2a^2t(2-r)((\lambda^2+4t-\lambda)r-4t-2\lambda+2)}{((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2}$. When $0 < r < 0.914394$ and $t > t_0^{JR}$, there is $\left((\lambda^2+4t-\lambda)r-4t-2\lambda+2\right) < 0$. Therefore, $\frac{\partial \pi_1^{JR}}{\partial r} < 0$.

$\frac{\partial \pi_2^{JR}}{\partial r} = \frac{2a^2t((-\lambda^4+(1+8t)\lambda^3-2\lambda^2t+8t^2)r^4-((8t-2)\lambda^3-4\lambda^2+6(6t+1)\lambda-40t)\lambda r^3+12((4t-1)\lambda^3-4\lambda^2t+(1+2t)\lambda-4t(2t+3))r^2+8((3(1-2t)\lambda^2+6(2t-1)\lambda+12t^2+4t-1)r-2(2t+1)\lambda-8t^2+2))}{-((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}$. When $0 < r < 0.914394$ and $t > t_0^{JR}$, there is $(-\lambda^4 + (1+8t)\lambda^3 - 2\lambda^2 t + 8t^2)r^4 - ((8t-2)\lambda^3 - 4\lambda^2 + 6(6t+1)\lambda - 40t)\lambda r^3 + 12((4t-1)\lambda^3 - 4\lambda^2 t + (1+2t)\lambda - 4t(2t+3))r^2 + 8((3(1-2t)\lambda^2 + 6(2t-1)\lambda + 12t^2 + 4t - 1)r - 2(2t+1)\lambda - 8t^2 + 2)) < 0$. Therefore, $\frac{\partial \pi_2^{JR}}{\partial r} < 0$.

$\frac{\partial cs^{JR}}{\partial r}=\frac{8a^2t^2((1/4\lambda^2+t+5/2\lambda)r^4+(\lambda-4\lambda^2-6t-5)r^3+(6t-3)r^2+8(\lambda^2+2)r-16\lambda)}{-((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}$. Let $f_5(t)\overset{\text{def}}{=}((1/4\lambda^2+t+5/2\lambda)r^4+(\lambda-4\lambda^2-6t-5)r^3+(6t-3)r^2+8(\lambda^2+2)r-16\lambda)=0$, then $t_5=\frac{(2-\lambda r)((\lambda+10)r^3+16(1-\lambda)r^2+32(\lambda-r))}{4r^2(r^2-6r+6)}$. Let $g_1(\lambda)\overset{\text{def}}{=}(\lambda+10)r^3+16(1-\lambda)r^2+32(\lambda-r)=0$, there is $\lambda_1=\frac{2r(16-5r^2-3r)}{r^3-16r^2+32}\in(0,1)$. If $\lambda_1<\lambda<1$ and $t_0^{JR}<t<t_5$, then $\frac{\partial cs^{JR}}{\partial r}<0$; if $\lambda_1<\lambda<1$ and $t>t_5$, then $\frac{\partial cs^{JR}}{\partial r}>0$.

$\frac{\partial sw^{JR}}{\partial r}=\frac{2a^2t^2((8\lambda^3+7\lambda^2+28t+6\lambda)r^4+4((\lambda-2\lambda^4-18(\lambda^2+t)-3)r^3+4(12\lambda^2(\lambda-1)-6t+24\lambda-3)r^2+32(6t+2\lambda+1)r+4(32t-32\lambda+1))}{-((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}$. When $t>t_0^{JR}$, there is $(8\lambda^3+7\lambda^2+28t+6\lambda)r^4+4((\lambda-2\lambda^4-18(\lambda^2+t)-3)r^3+4(12\lambda^2(\lambda-1)-6t+24\lambda-3)r^2+32(6t+2\lambda+1)r+4(32t-32\lambda+1)<0$. Therefore, we can obtain $\frac{\partial sw^{JR}}{\partial r}<0$.

$\frac{\partial b^{JR}}{\partial r}=\frac{16at(2-\lambda r)(r-\lambda)}{((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2}$. If $\lambda<r<0.914394$, then $\frac{\partial b^{JR}}{\partial r}>0$; if $0<r<\lambda$, then $\frac{\partial b^{JR}}{\partial r}<0$. $\frac{\partial\beta^{JR}}{\partial r}=\frac{4(2-r)(1-\lambda)}{(\lambda r-2)^3}<0$.

(ii) Effects of t:

$\frac{\partial\pi_1^{JR}}{\partial t}=\frac{-a^2(\lambda r-2)^2(r-2)^2}{2((4t+1)r^2+4(r-2t+1))^2}<0$.

$\frac{\partial\pi_2^{JR}}{\partial t}=\frac{a^2(\lambda r-2)^2((\lambda^4-(4t+1)\lambda^2)r^4+4(\lambda^2-2\lambda^3+(4t+1)\lambda-8t)r^3+4\left((2t+5)\lambda^2-4\lambda+2t-1\right)r^2+16(6t+1-(2t+1)\lambda)r-64t)}{-2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}$. Let $f_6(t)\overset{\text{def}}{=}(\lambda^4-(4t+1)\lambda^2)r^4+4(\lambda^2-2\lambda^3+(4t+1)\lambda-8t)r^3+4\left((2t+5)\lambda^2-4\lambda+2t-1\right)r^2+16(6t+1-(2t+1)\lambda)r-64t=0$, then $t_6=\frac{(\lambda r-2)^2(r\lambda+r-4)(\lambda-1)r}{4(r^4\lambda^2+4(2-\lambda)r^3-2(\lambda^2+1)r^2+8(\lambda-3)r)}$. If $t_0^{JR}<t<t_6$, then $\frac{\partial\pi_2^{JR}}{\partial t}>0$; if $t>t_6$, then $\frac{\partial\pi_2^{JR}}{\partial t}<0$.

$\frac{\partial cs^{JR}}{\partial t}=\frac{a^2t(\lambda r-2)^2(5r^4+4r^3+32(1-r^2))}{((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}<0$.

$\frac{\partial sw^{JR}}{\partial t}=\frac{a^2(\lambda r-2)^2((8(2\lambda-7)r^3-2(2\lambda^2+3)r^4+8(\lambda^2+10)r^2+32((4-\lambda)r-5))t+\lambda^4r^4-8\lambda^3r^3+24\lambda^2r^2-32r\lambda+16)}{-2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^3}$.

When $t>t_0^{JR}$, there is $\left(8(2\lambda-7)r^3-2(2\lambda^2+3)r^4+8(\lambda^2+10)r^2+32\left((4-\lambda)r-5\right)\right)t+\lambda^4r^4-8\lambda^3r^3+24\lambda^2r^2-32r\lambda+16<0$. Therefore, $\frac{\partial sw^{JR}}{\partial t}<0$.

$\frac{\partial b^{JR}}{\partial t}=\frac{4a(r^2-2)(\lambda r-2)^2}{\left((\lambda^2+4t)r^2-4(\lambda r+2t-1)\right)^2}<0$. $\frac{\partial\beta^{JR}}{\partial t}=0$.

Appendix C

(i) Effects of r:

$\frac{\partial \pi_1^{SR}}{\partial r} =$

$$\frac{-a^2((15\lambda^3(\lambda-1)-1283\lambda^2 t-32\lambda t+256t^2)r^6+(-60\lambda^4+72\lambda^3+592\lambda^2 t-12\lambda^2-384\lambda t-768t^2+244t)r^5+(32\lambda^4-348\lambda^3+96\lambda^2 t+472\lambda^2+928\lambda t-512t^2-156\lambda-512t)r^4+(288\lambda^4-320\lambda^3-2688\lambda^2 t-192\lambda^2++1280\lambda t+3072t^2+224\lambda-768t)r^3+(1872\lambda^3-384\lambda^4+1600\lambda^2 t-1840\lambda^2-2624\lambda t-1024t^2+432\lambda+1920t-80)r^2+(192(\lambda^3-\lambda^2-\lambda+1)+2816\lambda^2-384\lambda^4-1536\lambda t-3072t^2+256t)r+576\lambda^4-1920\lambda^3-2304\lambda^2 t+1920\lambda^2+2560\lambda t+2048t^2-192(2\lambda+1)-1280t)}{2((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2}.$$

When $t > t_0^{SR}$, $(15\lambda^3(\lambda-1)-1283\lambda^2 t-32\lambda t+256t^2)r^6+(-60\lambda^4+72\lambda^3+592\lambda^2 t-12\lambda^2-384\lambda t-768t^2+244t)r^5+(32\lambda^4-348\lambda^3+96\lambda^2 t+472\lambda^2+928\lambda t-512t^2-156\lambda-512t)r^4+(288\lambda^4-320\lambda^3-2688\lambda^2 t-192\lambda^2++1280\lambda t+3072t^2+224\lambda-768t)r^3+(1872\lambda^3-384\lambda^4+1600\lambda^2 t-1840\lambda^2-2624\lambda t-1024t^2+432\lambda+1920t-80)r^2+(192(\lambda^3-\lambda^2-\lambda+1)+2816\lambda^2-384\lambda^4-1536\lambda t-3072t^2+256t)r+576\lambda^4-1920\lambda^3-2304\lambda^2 t+1920\lambda^2+2560\lambda t+2048t^2-192(2\lambda+1)-1280t) > 0$ exists. Therefore, $\frac{\partial \pi_1^{SR}}{\partial r} < 0$.

$\frac{\partial \pi_2^{SR}}{\partial r} =$

$$\frac{8a^2(2r^4 t+(\lambda^2-\lambda+4t)r^3-6(\lambda+2t-1)r^2+2(\lambda-\lambda^2-4t)r+12(\lambda-1)+16t)((3\lambda^3(\lambda-1)-4\lambda^2 t-40\lambda t-64t^2)r^6+(40\lambda^2 t+36\lambda^2(1-\lambda)+192\lambda t+128t^2-112t)r^5+(10\lambda^3(\lambda-1)+72\lambda^2 t-148\lambda^2+16\lambda t+128t^2+148\lambda+80t)r^4+(160\lambda^3(\lambda-1)-192\lambda^2 t-160\lambda^2-704\lambda t-512t^2+384t)r^3+(88\lambda^3(1-\lambda)-288\lambda^2 t+520\lambda^2+256(2\lambda+t)-520\lambda-416t)r^2+(96\lambda^2(1-\lambda)+256\lambda^2+768\lambda t+512t^2+96(\lambda-1)-256t)r+96\lambda^3(\lambda-1)+256\lambda^2 t-672\lambda^2-896\lambda t-512t^2+672\lambda+384t)}{-((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^3}.$$

When $t > t_0^{SR}$, $(3\lambda^3(\lambda-1)-4\lambda^2 t-40\lambda t-64t^2)r^6+(40\lambda^2 t+36\lambda^2(1-\lambda)+192\lambda t+128t^2-112t)r^5+(10\lambda^3(\lambda-1)+72\lambda^2 t-148\lambda^2+16\lambda t+128t^2+148\lambda+80t)r^4+(160\lambda^3(\lambda-1)-192\lambda^2 t-160\lambda^2-704\lambda t-512t^2+384t)r^3+(88\lambda^3(1-\lambda)-288\lambda^2 t+520\lambda^2+256(2\lambda+t)-520\lambda-416t)r^2+(96\lambda^2(1-\lambda)+256\lambda^2+768\lambda t+512t^2+96(\lambda-1)-256t)r+96\lambda^3(\lambda-1)+256\lambda^2 t-672\lambda^2-896\lambda t-512t^2+672\lambda+384t < 0$ exists. Let $f_7(t) \stackrel{\text{def}}{=} 2r^4 t+(\lambda^2-\lambda+4t)r^3-6(\lambda+2t-1)r^2+2(\lambda-\lambda^2-4t)r+12(\lambda-1)+16t=0$, then $t_7 = \frac{(r+6)\lambda-\lambda^2 r-6}{2(r^2+2r-4)}$. If $t_0^{SR} < t < t_7$, then $\frac{\partial \pi_2^{SR}}{\partial r} > 0$. If $t > t_7$, then $\frac{\partial \pi_2^{SR}}{\partial r} < 0$.

$\frac{\partial cs^{SR}}{\partial r} =$

$$\frac{4a^2((48t\lambda^3(\lambda-1)-(232\lambda^2+304\lambda)t^2-128t^3)r^{10}+(33\lambda^6-66\lambda^5-80\lambda^4 t+33\lambda^4-384\lambda^3 t-16\lambda^2 t^2+464\lambda^2 t+1312\lambda t^2+768t^3-224t^2)r^9+(480\lambda^4-306\lambda^5+612\lambda^4+1440\lambda^3 t+1920\lambda^2 t^2-306\lambda^3-2592\lambda^2 t+768\lambda t^2+672\lambda t+2016t^2)r^8+(170\lambda^6-340\lambda^5-416\lambda^4 t-490\lambda^4+3136\lambda^3 t-224\lambda^2 t^2+1320\lambda^3-1568\lambda^2 t-10624\lambda t^2-4608t^3-660\lambda^2-2048\lambda t-832t^2+896t)r^7+(2242\lambda^5-5184\lambda^4 t-4484\lambda^4-8800\lambda^3 t-5152\lambda^2 t^2+1898\lambda^3+19136\lambda^2 t+3776\lambda t^2+3072t^3+688\lambda^2-4512\lambda t-11520t^2-344\lambda-640t)r^6+(4392\lambda^5-2196\lambda^6+\lambda^4(5376t+2472)-\lambda^3(9600t-336)+2304\lambda^2 t^2-576\lambda^2 t+33408\lambda t^2+9216t^3+4668\lambda^2+9600\lambda t+11520t^2-4800t)r^5+(14976\lambda^4 t-5688\lambda^5+11376\lambda^4+18944\lambda^3 t+2688\lambda^2 t^2-5152\lambda^3-49024\lambda^2 t-19456\lambda^2-8192t^3-1072\lambda^2+9984\lambda t+19328t^2+536\lambda+5120t)r^4+(5840\lambda^6-11680\lambda^5-13952\lambda^4 t-3952\lambda^4+13568\lambda^3 t-5632\lambda^2 t^2+19584\lambda^3+7808\lambda^2 t-47104\lambda t^2-6144t^3-9104\lambda^2-14848\lambda t-28160t^2-1376\lambda+7424t+688)r^3+(4896\lambda^5-16896\lambda^4 t-9792\lambda^4-13440\lambda^3 t+7680\lambda^2 t^2+5184\lambda^3+48384\lambda^2 t+30720\lambda t^2+6144t^3-576\lambda^2-7296\lambda t-4608t^2+288\lambda-10752t)r^2+(9600\lambda^5-4800\lambda^6+11008\lambda^4 t+1344\lambda^4-7680\lambda^3 t4096\lambda^2 t^2-12288\lambda^3-8192\lambda^2 t+24576\lambda t^2+4800\lambda^2+7680\lambda t+20480t^2+2688\lambda-2816t-134)r+6144\lambda^4 t-8192\lambda^2 t^2-12288\lambda t^2-16384\lambda^2-8192t^2+6144t)}{((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^3}.$$

When $t > t_0^{SR}$, the numerator of $\frac{\partial cs^{SR}}{\partial r}$ is positive. Therefore, $\frac{\partial cs^{SR}}{\partial r} > 0$.

$\frac{\partial sw^{SR}}{\partial r} =$

$$\frac{\begin{array}{c}2a^2((21\lambda t(\lambda-1)-112t^2)r^6+(24\lambda^2-48\lambda^3+\lambda^2(24-158t)+256\lambda t+288t^2-98t)r^5+(112\lambda(\lambda^2+1)+\lambda^2(52t-224)-192\lambda t+320t^2+\\140t)r^4+(216\lambda^3-108\lambda^4+\lambda^2(720t-108)-1152\lambda t-1344t^2+432t)r^3-(500\lambda(\lambda^2+1)+\lambda^2(560t-1000)-1248\lambda t-320t^2+688t)r^2\\+(120\lambda^3(\lambda-2)+\lambda^2(96-832t)+1280\lambda t+1536t^2+48\lambda-448t-24)r+576\lambda^3+\lambda^2(768t-1152)-1536\lambda t-1024t^2+576\lambda+768t)\end{array}}{2((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2}.$$

When $t > t_0^{SR}$, the numerator of $\frac{\partial sw^{SR}}{\partial r}$ is negative. Therefore, $\frac{\partial sw^{SR}}{\partial r} < 0$.

$$\frac{\partial b^{SR}}{\partial r} =$$

$$\frac{\begin{array}{c}2a((21\lambda(\lambda-1)-112t(\lambda+1))r^6+(60\lambda(1-\lambda)+256\lambda t+320t)r^5+((\lambda-1)(52\lambda^2-140)+320t(\lambda+1))r^4+(288\lambda(\lambda-1)-1152\lambda t\\-1536t)r^3+((1-\lambda)(560\lambda^2++688)320t(\lambda+1))r^2+(384\lambda(1-\lambda)+1280\lambda t+1792t)r+768(\lambda^2-1)(\lambda-1)-1024t(\lambda+1)\end{array}}{((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2}.$$

When $t > t_0^{SR}$, there is $(21\lambda(\lambda-1)-112t(\lambda+1))r^6 + (60\lambda(1-\lambda) + 256\lambda t + 320t)r^5 + ((\lambda-1)(52\lambda^2-140) + 320t(\lambda+1))r^4 + (288\lambda(\lambda-1) - 1152\lambda t - 1536t)r^3 + ((1-\lambda)(560\lambda^2 + +688)320t(\lambda+1))r^2 + (384\lambda(1-\lambda) + 1280\lambda t + 1792t)r + 768(\lambda^2-1)(\lambda-1) - 1024t(\lambda+1) < 0$. Therefore, $\frac{\partial b^{SR}}{\partial r} < 0$.

$$\frac{\partial s^{SR}}{\partial r} =$$

$$\frac{\begin{array}{c}-((15\lambda^3(\lambda-1)-80\lambda t(\lambda+1))r^6+(48\lambda^2(1-\lambda)+256\lambda t+176\lambda^2 t)r^5+(32\lambda^4-44\lambda^3+\lambda^2(192t-136)+256\lambda t+148\lambda+64t)r^4\\+(288\lambda^3-16\lambda^4-\lambda^2(832t+384)-1280\lambda t-64t+112)r^3+(656\lambda^3-384\lambda^4+\lambda^2(448t+592)+448\lambda t-784\lambda-80)r^2+\\(\lambda^2(768\lambda^2 t+384)-576\lambda^3+1024\lambda t+192\lambda-256t)r+576\lambda^4-768\lambda^2(\lambda+t)-384\lambda^2-512\lambda t+768\lambda+256t-192)\end{array}}{t(3r^4+14r^3-4(7r^2+8r-12))\lambda+2r^2(7r-10)+16(3-2r))^2}.$$

When $t > t_0^{SR}$, there is $(15\lambda^3(\lambda-1) - 80\lambda t(\lambda+1))r^6 + (48\lambda^2(1-\lambda) + 256\lambda t + 176\lambda^2 t)r^5 + (32\lambda^4 - 44\lambda^3 + \lambda^2(192t - 136) + 256\lambda t + 148\lambda + 64t)r^4 + (288\lambda^3 - 16\lambda^4 - \lambda^2(832t + 384) - 1280\lambda t - 64t + 112)r^3 + (656\lambda^3 - 384\lambda^4 + \lambda^2(448t + 592) + 448\lambda t - 784\lambda - 80)r^2 + (\lambda^2(768\lambda^2 t + 384) - 576\lambda^3 + 1024\lambda t + 192\lambda - 256t)r + 576\lambda^4 - 768\lambda^2(\lambda+t) - 384\lambda^2 - 512\lambda t + 768\lambda + 256t - 192) < 0$. Therefore, $\frac{\partial s^{SR}}{\partial r} > 0$.

(ii) Effects of t:

$$\frac{\partial \pi_1^{SR}}{\partial t} = \frac{\begin{array}{c}2a^2(3\lambda^2 r^8+8\lambda(\lambda+1)r^7-(62\lambda^2+64\lambda+28)r^6+8(\lambda^2+13\lambda+12)r^5+(224\lambda^2+192\lambda+40)r^4\\-(144\lambda^2+560\lambda+416)r^3+(224(1-\lambda^2)+64\lambda)r^2+(192\lambda^2+640\lambda+448)r-384(\lambda+1))\end{array}}{((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2}.$$ When $t > t_0^{SR}$, there exists $3\lambda^2 r^8 + 8\lambda(\lambda+1)r^7 - (62\lambda^2 + 64\lambda + 28)r^6 + 8(\lambda^2 + 13\lambda + 12)r^5 + (224\lambda^2 + 192\lambda + 40)r^4 - (144\lambda^2 + 560\lambda + 416)r^3 + (224(1-\lambda^2) + 64\lambda)r^2 + (192\lambda^2 + 640\lambda + 448)r - 384(\lambda+1) < 0$. Therefore, $\frac{\partial \pi_1^{SR}}{\partial t} < 0$.

$$\frac{\partial \pi_2^{SR}}{\partial t} = \frac{\begin{array}{c}16a^2(2r^4 t+(\lambda^2-\lambda+4t)r^3-6(\lambda+2t-1)r^2+2(\lambda-\lambda^2-4t)r+12(\lambda-1)+16t)(3\lambda^2 r^8+\\2\lambda(7\lambda+10)+(8\lambda-46\lambda^2+28)r^6-(116\lambda^2+184\lambda+40)r^5+24(10\lambda^2+2\lambda-5)r^4+(304\lambda^2\\+512\lambda+176)r^3+64(2-5\lambda-8\lambda^2)r^2-(256\lambda^2+448\lambda+192)r+384\lambda(\lambda+1))\end{array}}{-((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^3}.$$ When $t > t_0^{SR}$,

$3\lambda^2r^8 + 2\lambda(7\lambda + 10) + (8\lambda - 46\lambda^2 + 28)r^6 - (116\lambda^2 + 184\lambda + 40)r^5 + 24(10\lambda^2 + 2\lambda - 5)r^4 + (304\lambda^2 + 512\lambda + 176)r^3 + 64(2 - 5\lambda - 8\lambda^2)r^2 - (256\lambda^2 + 448\lambda + 192)r + 384\lambda(\lambda + 1) > 0$ exists. The function $2r^4t + (\lambda^2 - \lambda + 4t)r^3 - 6(\lambda + 2t - 1)r^2 + 2(\lambda - \lambda^2 - 4t)r + 12(\lambda - 1) + 16t$ has been defined as $f_7(t)$ presented in the above appendix as part of $\frac{\partial \pi_2^{SR}}{\partial r}$. Therefore, if $t_0^{SR} < t < t_7$, then $\frac{\partial \pi_2^{SR}}{\partial t} > 0$. If $t > t_7$, then $\frac{\partial \pi_2^{SR}}{\partial t} < 0$.

$$\frac{\partial cs^{SR}}{\partial t} = \frac{\begin{gathered}16a^2(15\lambda^2r^{12}t+\big(76\lambda t(\lambda+1)-12\lambda^3(\lambda+1)\big)r^{11}+(56\lambda^4-12\lambda^3-232\lambda^2t-44\lambda^2-80\lambda t+28t)r^{10}+(48\lambda^3-184\lambda^4+\lambda^2(104\\-800)-1008\lambda t+32\lambda-208t)r^9+(160\lambda^3-464\lambda^4+(1532t+496)\lambda^2+1216\lambda t-80\lambda+64t-112)r^8+(1008\lambda^4+16\lambda^3+(3248t\\-992)\lambda^2+4784\lambda t-192\lambda+1536t+160)r^7+\big(1440\lambda^4-688\lambda^3-(5232t+1968)\lambda^2-6208\lambda t+512\lambda-1488t+704\big)r^6-(2592\lambda^4\\+256\lambda^3+(6336t-3488)\lambda^2+10496\lambda t-384\lambda+4160t+1024)r^5+(1216\lambda^3-1984\lambda^4+\lambda^2(9536t+3328)+14336\lambda t-1088\lambda\\+5056t-1472)r^4+(3200\lambda^4+256\lambda^3+\lambda^2(5888t-5376)+10752\lambda t-256\lambda+4864t+2176)r^3+(1024\lambda^4-768\lambda^3-\lambda^2(8704t+\\2048)-15360\lambda t+768\lambda-6656t+1024)r^2-(1536\lambda^4+\lambda^2(2048t-3072)-4096\lambda t-2048t-1536)r+3072)t+6144\lambda t+3072t)\end{gathered}}{-((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^3}.$$

When $t > t_0^{SR}$, the numerator of $\frac{\partial cs^{SR}}{\partial t}$ is positive. Therefore, $\frac{\partial cs^{SR}}{\partial t} < 0$.

$$\frac{\partial sw^{SR}}{\partial t} = \frac{\begin{gathered}-a^2(9\lambda^2r^8+84\lambda(\lambda+1)r^7+\big(28\lambda^2+72\lambda+196\big)r^6-\big(976\lambda^2+1536\lambda+560\big)r^5+(176\lambda^2-384\lambda-496)r^4+\\(3136\lambda^2+5760\lambda+2624)r^3-\big(1664\lambda^2+2560\lambda+896\big)r^2-\big(3072\lambda^2+6144\lambda+3072\big)r+2304\big(\lambda^2+1\big)+4608\lambda)\end{gathered}}{2((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2}.$$

When $t > t_0^{SR}$, there is $(9\lambda^2r^8 + 84\lambda(\lambda + 1)r^7 + (28\lambda^2 + 72\lambda + 196)r^6 - (976\lambda^2 + 1536\lambda + 560)r^5 + (176\lambda^2 - 384\lambda - 496)r^4 + (3136\lambda^2 + 5760\lambda + 2624)r^3 - (1664\lambda^2 + 2560\lambda + 896)r^2 - (3072\lambda^2 + 6144\lambda + 3072)r + 2304(\lambda^2 + 1) + 4608\lambda) > 0$. Therefore, $\frac{\partial sw^{SR}}{\partial t} < 0$.

$\frac{\partial b^{SR}}{\partial t} = \frac{-16a(r^4-4r^2+4)(3\lambda r^4+14(\lambda+1)r^3-4(7\lambda+5)r^2-32(\lambda+1)r+48(\lambda+1))}{((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2}$. When $t > t_0^{SR}$, there is $3\lambda r^4 + 14(\lambda + 1)r^3 - 4(7\lambda + 5)r^2 - 32(\lambda + 1)r + 48(\lambda + 1) > 0$. Therefore, $\frac{\partial b^{SR}}{\partial t} < 0$.

$\frac{\partial s^{SR}}{\partial t} = \frac{(1-\lambda)(\lambda r-2)(5\lambda r^2-12\lambda+2r)}{t^2\big(3r^4+14r^3-4(7r^2+8r-12)\big)\lambda+2r^2(7r-10)+16(3-2r))}$. When $t > t_0^{SR}$, there is $\big(3r^4 + 14r^3 - 4(7r^2 + 8r - 12)\big)\lambda + 2r^2(7r - 10) + 16(3 - 2r) > 0$ and $5\lambda r^2 - 12\lambda + 2r < 0$. Therefore, $\frac{\partial s^{SR}}{\partial t} > 0$.

Appendix D

$$\pi_1^{IR} - \pi_1^{JR} = 0.$$

$\pi_1^{IR} - \pi_1^{SR} = \frac{a^2(r-2)(\lambda r-2)(7t\lambda r^4+\big(5\lambda^2(\lambda-1)+6t(\lambda+1)\big)r^3+\big(8\lambda(1-\lambda)-4t(9\lambda+1)\big)r^2+\big(12\lambda^2(1-\lambda)-16t(\lambda+1)+4(1-\lambda)\big)r+24\lambda(\lambda-1)+16t(3\lambda+1))}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))}$. When $t > t_0^{JR}$, we can get $7t\lambda r^4 + \big(5\lambda^2(\lambda - 1) + 6t(\lambda + 1)\big)r^3 + \big(8\lambda(1 - \lambda) - 4t(9\lambda + 1)\big)r^2 + \big(12\lambda^2(1 - \lambda) - 16t(\lambda + 1) + 4(1 - \lambda)\big)r + 24\lambda(\lambda - 1) + 16t(3\lambda + 1) > 0, (\lambda^2 + 4t)r^2 - 4(\lambda r + 2t - 1) < 0$ and $(16t - 3\lambda^2)r^4 - 28\lambda r^3 + 4(7\lambda - 16t + 5)r^2 + 64(r\lambda + t) - 48(\lambda^2 + 1) > 0$. Therefore, $\pi_1^{IR} < \pi_1^{SR}$.

In conclusion, $\pi_1^{SR} > \pi_1^{IR} = \pi_1^{JR}$.

Appendix E

$\pi_2^{IR} - \pi_2^{JR} = \frac{a^2(1-\lambda)(2-\lambda r)(t\lambda(\lambda+1)r^3+2t(1-3\lambda)r^2+2(\lambda^2-\lambda+8t)r-4(4t-\lambda-1))}{\left((\lambda^2+4t)r^2-4(\lambda r+2t-1)\right)^2}$. When $t > t_0^{JR}$, the numerator of $(\pi_2^{IR} - \pi_2^{JR})$ is negative. Therefore, $\pi_2^{IR} < \pi_2^{JR}$.

$$\pi_2^{SR} - \pi_2^{JR} = \frac{\begin{gathered}8a^2(2r^4t+(\lambda^2-\lambda+4t)r^3-6(\lambda-2t-1)r^2+2r\lambda(1-\lambda)-8rt+12(\lambda-1)+16t)^2((4t+1)r^2+4(r-2t+1))^2 \\ -\left(2\left((a^2+6a+1)t-8t^2\right)-a^2\right)tr^4+4\left(2\left((a^2+a-2)t+a^2+2a\right)+1\right)tr^3+(64t^3-8\left(a^2+10a+6\right)t^2+ \\ 4(10a(a+1)+1)t+2(2a+1)^2)r^2+\left(32\left(1-a^2-a\right)t^2-16(3a+2)t+8(2a+1)^2\right)r+8(2t-1)(2(\left(a^2+4a+2\right)t \\ -2t^2)-(2a+1)^2)(\left(16t-3\lambda^2\right)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48\left(\lambda^2+1\right))^2\end{gathered}}{2((4t+1)r^2+4(r-2t+1))^2((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2}.$$

When $t > t_0^{JR}$, the numerator of $(\pi_2^{SR} - \pi_2^{JR})$ is positive. Therefore, $\pi_2^{SR} > \pi_2^{JR}$.

In conclusion, $\pi_2^{SR} > \pi_2^{JR} > \pi_2^{IR}$.

Appendix F

$cs^{IR} - cs^{JR} = \frac{a^2(1-\lambda)(2-\lambda r)(6r^2t+\left(\lambda^2-\lambda-4t\right)r-2(\lambda+4t-1))}{2\left((\lambda^2+4t)r^2-4(\lambda r+2t-1)\right)^2}$. Let $f_8(t) \overset{\text{def}}{=} 6r^2t + (\lambda^2 - \lambda - 4t)r - 2(\lambda + 4t - 1) = 0$, there is $t_8 = \frac{-(1-\lambda)(2-\lambda r)}{2(3r^2-2r-4)}$. If $t_0^{JR} < t < t_8$, then $cs^{IR} > cs^{JR}$; if $t > t_8$, then $cs^{IR} < cs^{JR}$.

$cs^{IR} - cs^{SR} =$

$$\frac{\begin{gathered}a^2(4(2-r)^2t^2+4rt(r-2)\left((r^2+2r-4)t+(\lambda-1)(\lambda r-2)\right)+\left((r^2+2r-4)t+(\lambda-1)(\lambda r-2)\right)^2((16t-3\lambda^2)r^4-28\lambda r^3+ \\ 4(7\lambda-16t+5)r^2+64(r\lambda+t)-48\left(\lambda^2+1\right))^2-2((20t^2r^8+16t\left(2\lambda^2-2\lambda+t\right)r^7+(11\lambda^4-22\lambda^3+32\lambda^2t+11\lambda^2-96\lambda t \\ -208t^2+64t)r^6+(136\lambda^2-68\lambda^3-192\lambda^2t+256\lambda t-64(t+1)-68\lambda)r^5+(46\lambda^3-23\lambda^4-224\lambda^2t-11\lambda^2+576\lambda t+720t^2 \\ -24\lambda-352t+12)r^4+(324\lambda^3+(384t-648)\lambda^2+64t(t-10\lambda t)324\lambda+256t)r^3+(136\lambda^3-68\lambda^4+512\lambda^2t-160\lambda^2-1152\lambda t \\ -1024t^2+184\lambda+640t-92)r^2+(768\lambda^2-384\lambda^3-256\lambda^2t+512\lambda t-384\lambda-256t)r+144(\lambda^4-2\lambda\left(\lambda^2-\lambda+1\right))-384t(\lambda^2 \\ +1)+768\lambda t+512t^2))(\left(\lambda^2+4t\right)r^2-4(\lambda r+2t-1))^2)\end{gathered}}{2\left((\lambda^2+4t)r^2-4(\lambda r+2t-1)\right)^2((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))^2}.$$

When $t > t_0^{JR}$, the numerator of $(cs^{IR} - cs^{SR})$ is negative. Therefore, $cs^{IR} < cs^{SR}$.

Considering $cs^{JR} - cs^{SR}$, we can find the numerator of $(cs^{JR} - cs^{SR})$ is negative. Therefore, $cs^{JR} < cs^{SR}$.

In conclusion, if $t_0^{JR} < t < t_8$, then $cs^{SR} > cs^{IR} > cs^{JR}$; if $t > t_8$, then $cs^{SR} > cs^{JR} > cs^{IR}$.

Appendix G

$$sw^{IR} - sw^{JR} = \frac{\begin{gathered}a^2((\lambda^4t+\lambda^2t+4t^2)r^4-(8\lambda^3t+2\lambda^2t+6\lambda t+48t^2)r^3+(2\lambda^3-\lambda^4+16\lambda^2t-\lambda^2+24\lambda t+ \\ 32t^2+8t)r^2+4\left(\lambda^3+2\lambda^2t-2\lambda^2-8\lambda t+32t^2+\lambda-10t\right)r-4(\lambda^2+4\lambda t+32t^2-2\lambda-12t+1))\end{gathered}}{2((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2}.$$

When $t > t_0^{JR}$, the numerator of $(sw^{IR} - sw^{JR})$ is negative. Therefore, $sw^{IR} < sw^{JR}$.

$sw^{JR} - sw^{SR} =$

$$\frac{-a^2(t((\lambda^4-3t)r^4-4(2\lambda^3+7t)r^3+24(\lambda^2+2t)r^2+32(2t-\lambda)r-16(6t-1))((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+16(4(r\lambda+t)-3(\lambda^2+1)))+a^2(3r^4t+28r^3t+16(\lambda^2-2\lambda-3t+1)r^2-8(8rt-9\lambda-12t)-36(\lambda^2+1))\left((\lambda^2+4t)r^2-4(\lambda r+2t-1)\right)^2)}{4((\lambda^2+4t)r^2-4(\lambda r+2t-1))^2((16t-3\lambda^2)r^4-28\lambda r^3+4(7\lambda-16t+5)r^2+64(r\lambda+t)-48(\lambda^2+1))}.$$

When $t > t_0^{JR}$, the numerator of $(sw^{JR} - sw^{SR})$ is negative and $(16t - 3\lambda^2)r^4 - 28\lambda r^3 + 4(7\lambda - 16t + 5)r^2 + 64(r\lambda + t) - 48(\lambda^2 + 1) > 0$. Therefore, $sw^{JR} < sw^{SR}$.

In conclusion, $sw^{SR} > sw^{JR} > sw^{IR}$.