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Research article

Theoretical and numerical aspects of the Malaria transmission model with piecewise technique

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Abstract: In this paper, we apply piecewise derivatives with both singular and non-singular kernels to investigate a malaria model. The singular kernel is the Caputo derivative, while the non-singular kernel is the Atangana-Baleanu operator in Caputo's sense (ABC). The existence, uniqueness, and numerical algorithm of the proposed model are presented using piecewise derivatives with both kernels. The stability is also presented for the proposed model using Ulam-Hyers stability. The numerical simulations are performed considering different fractional orders and compared the results with the real data to evaluate the efficiency of the proposed approach.

Keywords: piecewise operator; Caputo and Atangana-Baleanu operators; Malaria mathematical model
Mathematics Subject Classification: 26A33, 34Axx, 34Lxx, 52Bxx, 65Yxx, 92-10

1. Introduction

Although there has been significant progress in the struggle with infectious diseases in the 20^{th} century, various diseases still pose a substantial threat to public health in underdeveloped countries [1, 2]. In particular, diseases like yellow fever, HIV/AIDS, Malaria, and Ebola continue to cause suffering and increase mortality rates. One reason these diseases continue is due to the shortage of access to healthcare and basic requirements like fresh and clean water and hygiene in underdeveloped countries. In addition, poverty, starvation, and other social factors of health can also worsen the impact of infectious diseases. The struggles are in progress to report these problems, though, including improving access to healthcare and executing precautionary measures like vaccinations and mosquito control. Additionally, there is constant research to improve novel treatments and vaccines for these diseases. It is important to identify that infectious diseases are a global health dispute that needs a joint effort from all countries to resolve the conflict [3, 4]. However, although improvements have been made, there is still much work to be done to confirm that everybody has access to quality healthcare and the tools essential to inhibit and treat infectious diseases. In recent years, the mathematical modeling has become a countless way for investigation of infectious diseases and control schemes improvement. Numerous scientists and scholars have been working on infectious diseases considering different mathematical models including SIQR Covid-19 model [5], Rubella disease [6], Agitation of SARS-CoV-2 disease [7], spatiotemporal HIV CD4+ T cell model [8], the monkeypox disease [9, 10] and some other fractional order models with control tactics [11, 12] to analyze the dynamics of diseases considering various factors.

Malaria is an infectious disease caused by the Plasmodium parasites. These parasites are transferred to humans through bites by Anopheles mosquitoes which are already infected. The major kinds of species of the Plasmodium are four that can infect humans in which falciparum and vivax are the most common species found in different countries. Plasmodium falciparum is generally considered to be the most dangerous species, as it can cause severe malaria and is responsible for most malaria-related deaths globally. Plasmodium vivax, while generally causing less severe illness, can lead to long-term health problems if left untreated. Malaria is serious public health issue in the world. The different governments have implemented a number of measures to control malaria's spread, which includes distribution of bed nets which are insecticide-treated and use of antimalarial medicines. Despite these efforts, malaria remains a significant cause of illness and death, particularly among vulnerable populations such as young children and pregnant women [13].

Malaria is mainly contracted through the bites of female mosquitoes that are infected with the Plasmodium parasite. If diagnosed and treated properly on time, the disease can be well managed [14]. It is a persistent illness having substantial economic, social, and health concerns, particularly in tropical regions and countries. Despite centuries of research, malaria is a major public health problem, with 109 countries classified as having endemic levels of the disease in 2008. In that year alone, an estimated one million people, mostly children under the age of five, died from malaria, and 243 million infected cases were reported [15].

Presently, the major recommendation for individuals who are sick with malaria is anticipation through the use of bed nets, since there is no known vaccine and many available anti-malarial drugs are becoming less effective due to parasite drug tolerance. The relevance of malaria has grown in recent years due to concerns about how climate change or global warming might affect its prevalence. Temperature changes can affect both the parasite and vector life cycles. For over a century, mathematical models have been used to study the patterns of human malaria transmission. However, it is important to critically review existing models and investigate their effectiveness in describing host-parasite biology, as the disease continues to be a significant threat to health and wellbeing in the face of shifting environmental and socioeconomic conditions [16].

Here, we consider [15]

$$\begin{aligned}
&\mathbb{D}_t^{\varrho}(\mathscr{S}_{h_1}(t)) = \mathcal{A}_h + b_1\mathscr{R}_{h_1} - wr_1\mathscr{S}_{h_1}\mathscr{I}_{h_1} - \zeta r_2\mathscr{S}_{h_1}\mathscr{I}_{m_1} - (d_1 + \varrho_h)\mathscr{S}_{h_1},\\
&\mathbb{D}_t^{\varrho}(\mathscr{E}_{h_1}(t)) = wr_1\mathscr{S}_{h_1}\mathscr{I}_{h_1} + \zeta r_2\mathscr{S}_{h_1}\mathscr{I}_{m_1} - (\varrho_h + b_2 + d_2)\mathscr{E}_{h_1},\\
&\mathbb{D}_t^{\varrho}(\mathscr{I}_{h_1}(t)) = b_2\mathscr{E}_{h_1}(t) - (\varrho_h + \alpha + \omega\nu)\mathscr{I}_{h_1},\\
&\mathbb{D}_t^{\varrho}(\mathscr{R}_{h_1}(t)) = \nu\omega\mathscr{I}_{h_1} + d_1\mathscr{S}_{h_1} + d_2\mathscr{E}_{h_1} - (b_1 + \varrho_h)\mathscr{R}_{h_1},\\
&\mathbb{D}_t^{\varrho}(\mathscr{S}_{m_1}(t)) = \mathcal{A}_m - qr_3\mathscr{S}_{m_1}\mathscr{I}_{h_1} - (\varrho_M + \upsilon)\mathscr{S}_{m_1},\\
&\mathbb{D}_t^{\varrho}(\mathscr{I}_{m_1}(t)) = qr_3\mathscr{S}_{m_1}\mathscr{I}_{h_1} - (\varrho_M + \upsilon)\mathscr{I}_{m_1},
\end{aligned} \tag{1.1}$$

with $N_H = \mathscr{S}_{h_1} + \mathscr{E}_{h_1} + \mathscr{I}_{h_1} + \mathscr{R}_{h_1}$ is four classes of humans, and $N_M = \mathscr{S}_{m_1} + \mathscr{I}_{m_1}$ is two classes of mosquitos, in which, $\mathscr{S}_{h_1}$ represents the susceptible human individuals, $\mathscr{E}_{h_1}$ represents exposed humans, $\mathscr{I}_{h_1}$ and $\mathscr{R}_{h_1}$ stands for the infected and recovered humans, $\mathscr{S}_{m_1}$ and $\mathscr{I}_{m_1}$ represents susceptible and infected mosquitoes, $\mathcal{A}_h$ is recruitment rate into $\mathscr{S}_{h_1}$, $\mathcal{A}_m$ recruitment rate into $\mathscr{S}_{m_1}$, b_1 rate of recovery of humans, b_2 rate of transition from $\mathscr{E}_{h_1}$ to $\mathscr{I}_{h_1}$, w is blood transfusion's average number from $\mathscr{S}_{h_1}$ to $\mathscr{I}_{h_1}$ in specific period, r_1 is rate of transfer of disease from $\mathscr{I}_{h_1}$ to $\mathscr{S}_{h_1}$, r_2 is rate of transfer of disease from $\mathscr{I}_{m_1}$ to $\mathscr{S}_{h_1}$, r_3 probability of $\mathscr{I}_{m_1}$, ζ the average rate of biting of $\mathscr{S}_{h_1}$ by infected mosquito, d_1 is recovery rate of $\mathscr{S}_{h_1}$, d_2 is recovery rate of $\mathscr{E}_{h_1}$, ς_h natural rate of mortality of humans, ς_M natural rate of mortality of mosquitoes, α is the death rate of $\mathscr{I}_{h_1}$ because of disease, ω is the human's recovery rate, ν the rate of medicines potency which are anti-malarial, the $\mathscr{S}_{m_1}$ and $\mathscr{I}_{m_1}$ classes die at ν because of the spraying use in specific time, and q at which $\mathscr{S}_{m_1}$ bites those who are infected with the disease in specific time. The compartmental model, as shown in Figure 1, represents malaria disease transmission.

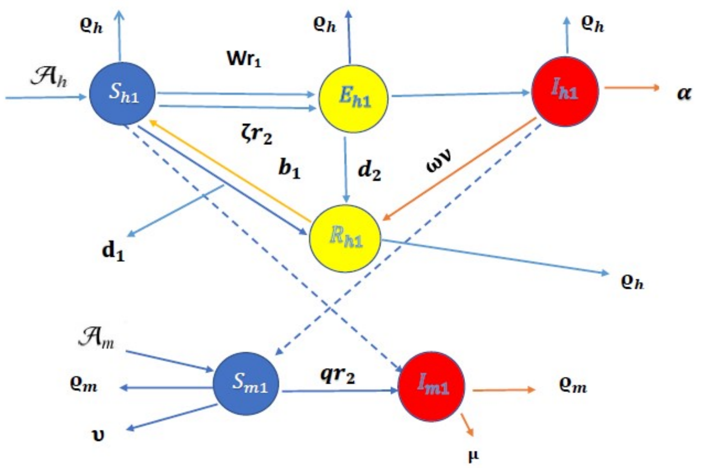

Figure 1. Compartmental flowchart for the malaria mathematical model's transmission.

Various operators, including fractal derivative, non-integer order derivative with kernel of singularity and non-singularity, fractal-fractional operator have been proposed to address crossover behavior in different fields such as infectious disease models, heat transfer, fluid dynamics, and other advection problems [17, 18].

Similarly, fractional operators have been used to analyze bifurcations and control mechanisms [19, 20] and discrete predator-prey competitive models [21]. However, despite the addition of stochastic terms to capture randomness and provide more realistic dynamics, crossover behavior remain a significant challenge [22, 23]. To address this issue, a novel approach called piece-wise differentiation and the integration was recently developed and discussed in reference [24]. The authors explore classical and global piecewise derivatives and provide examples. Further, the analysis of piecewise tumor-immune interaction [25, 26], dengue internal transmission and Leftospirosis models have been extensively studied using different fractional derivatives [27, 28].

Here, we interpret the system (1.1) for both theoretical and numerical analysis using the Caputo and ABC piece-wise derivative. The remaining sections are organized as follows: Section 2 presents model (1.1) using the piecewise derivative, considering singular as well as non-singular kernels. The basic results are covered in Section 3. In Section 4 the existence and uniqueness of the model is presented. Section 5 includes the numerical investigations of the model, while the simulations and discussions of the system are included in Section 6. Finally, Section 7 provides concluding remarks and future work.

2. Malaria model with piecewise derivative

The equation represented by model (1.1) can be expressed using a piecewise derivative having the singular and the non-singular kernels in the form:

$$
\begin{aligned}
{}_0^{CABC}D_t^{\varrho}(\mathscr{S}_{h_1}(t)) &= \mathcal{A}_h + b_1\mathscr{R}_{h_1} - wr_1\mathscr{S}_{h_1}\mathscr{I}_{h_1} - \zeta r_2\mathscr{S}_{h_1}\mathscr{I}_{m_1} - (d_1+\varrho_h)\mathscr{S}_{h_1},\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{E}_{h_1}(t)) &= wr_1\mathscr{S}_{h_1}\mathscr{I}_{h_1} + \zeta r_2\mathscr{S}_{h_1}\mathscr{I}_{m_1} - (\varrho_h + b_2 + d_2)\mathscr{E}_{h_1},\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{I}_{h_1}(t)) &= b_2\mathscr{E}_{h_1}(t) - (\varrho_h + \alpha + \omega\nu)\mathscr{I}_{h_1},\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{R}_{h_1}(t)) &= \nu\omega\mathscr{I}_{h_1} + d_1\mathscr{S}_{h_1} + d_2\mathscr{E}_{h_1} - (b_1+\varrho_h)\mathscr{R}_{h_1},\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{S}_{m_1}(t)) &= \mathcal{A}_m - qr_3\mathscr{S}_{m_1}\mathscr{I}_{h_1} - (\varrho_M + \upsilon)\mathscr{S}_{m_1},\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{I}_{m_1}(t)) &= qr_3\mathscr{S}_{m_1}\mathscr{I}_{h_1} - (\varrho_M + \upsilon)\mathscr{I}_{m_1},
\end{aligned}
\tag{2.1}
$$

for $0 < \varrho \leq 1,\ t \in [0, T]$. To elaborate further, Eq (2.1) can be expressed in the form

$$
\begin{aligned}
{}_0^{CABC}D_t^{\varrho}(\mathscr{S}_{h_1}(t)) &= \begin{cases} {}_0^{C}D_t^{\varrho}(\mathscr{S}_{h_1}(t)) = {}^{C}\mathbb{M}_1(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & 0 < t \leq t_1,\\ {}_0^{ABC}D_t^{\varrho}(\mathscr{S}_{h_1}(t)) = {}^{ABC}\mathbb{M}_1(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & t_1 < t \leq T,\end{cases}\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{E}_{h_1}(t)) &= \begin{cases} {}_0^{C}D_t^{\varrho}(\mathscr{E}_{h_1}(t)) = {}^{C}\mathbb{M}_2(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & 0 < t \leq t_1,\\ {}_0^{ABC}D_t^{\varrho}(\mathscr{E}_{h_1}(t)) = {}^{ABC}\mathbb{M}_2(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & t_1 < t \leq T,\end{cases}\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{I}_{h_1}(t)) &= \begin{cases} {}_0^{C}D_t^{\varrho}(\mathscr{I}_{h_1}(t)) = {}^{C}\mathbb{M}_3(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & 0 < t \leq t_1,\\ {}_0^{ABC}D_t^{\varrho}(\mathscr{I}_{h_1}(t)) = {}^{ABC}\mathbb{M}_3(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & t_1 < t \leq T,\end{cases}\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{R}_{h_1}(t)) &= \begin{cases} {}_0^{C}D_t^{\varrho}(\mathscr{R}_{h_1}(t)) = {}^{C}\mathbb{M}_4(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & 0 < t \leq t_1,\\ {}_0^{ABC}D_t^{\varrho}(\mathscr{R}_{h_1}(t)) = {}^{ABC}\mathbb{M}_4(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & t_1 < t \leq T,\end{cases}\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{S}_{m_1}(t)) &= \begin{cases} {}_0^{C}D_t^{\varrho}(\mathscr{S}_{m_1}(t)) = {}^{C}\mathbb{M}_5(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & 0 < t \leq t_1,\\ {}_0^{ABC}D_t^{\varrho}(\mathscr{S}_{m_1}(t)) = {}^{ABC}\mathbb{M}_5(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & t_1 < t \leq T,\end{cases}\\
{}_0^{CABC}D_t^{\varrho}(\mathscr{I}_{m_1}(t)) &= \begin{cases} {}_0^{C}D_t^{\varrho}(\mathscr{I}_{m_1}(t)) = {}^{C}\mathbb{M}_6(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & 0 < t \leq t_1,\\ {}_0^{ABC}D_t^{\varrho}(\mathscr{I}_{m_1}(t)) = {}^{ABC}\mathbb{M}_6(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t), & t_1 < t \leq T,\end{cases}
\end{aligned}
\tag{2.2}
$$

where ${}_{0}^{C}D_t^{\varrho}$ and ${}_{0}^{ABC}D_t^{\varrho}$ are Caputo's and ABC derivatives, respectively, and

$$
\begin{aligned}
&{}^{C}\mathbb{M}_1(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t) = {}^{ABC}\mathbb{M}_1(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t)\\
&= \mathcal{A}_h + b_1\mathscr{R}_{h_1} - wr_1\mathscr{S}_{h_1}\mathscr{I}_{h_1} - \zeta r_2\mathscr{S}_{h_1}\mathscr{I}_{m_1} - (d_1+\varrho_h)\mathscr{S}_{h_1},\\
&{}^{C}\mathbb{M}_2(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t) = {}^{ABC}\mathbb{M}_2(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t)\\
&= wr_1\mathscr{S}_{h_1}\mathscr{I}_{h_1} + \zeta r_2\mathscr{S}_{h_1}\mathscr{I}_{m_1} - (\varrho_h + b_2 + d_2)\mathscr{E}_{h_1},\\
&{}^{C}\mathbb{M}_3(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t) = {}^{ABC}\mathbb{M}_3(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t)\\
&= b_2\mathscr{E}_{h_1}(t) - (\varrho_h + \alpha + \omega\nu)\mathscr{I}_{h_1},\\
&{}^{C}\mathbb{M}_4(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t) = {}^{ABC}\mathbb{M}_4(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t)\\
&= \nu\omega\mathscr{I}_{h_1} + d_1\mathscr{R}_{h_1} + d_2\mathscr{E}_{h_1} - (b_1+\varrho_h)\mathscr{R}_{h_1},\\
&{}^{C}\mathbb{M}_5(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t) = {}^{ABC}\mathbb{M}_5(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t)\\
&= \mathcal{A}_m - qr_3\mathscr{S}_{m_1}\mathscr{I}_{h_1} - (\varrho_M + \upsilon)\mathscr{S}_{m_1},\\
&{}^{C}\mathbb{M}_6(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t) = {}^{ABC}\mathbb{M}_6(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t)\\
&= qr_3\mathscr{S}_{m_1}\mathscr{I}_{h_1} - (\varrho_M + \upsilon)\mathscr{I}_{m_1}.
\end{aligned}
$$

3. Definitions and preliminaries

Here, we provide some important definitions and preliminaries of the Caputo and the *ABC* fractional order derivatives and integrals.

Definition 3.1. *Suppose $U(t) \in \mathscr{H}^1(0,\tau)$, the ABC derivative of function $U(t)$ is given by [17]*

$$
{}^{ABC}{}_0D_t^{\varrho}(U(t)) = \frac{ABC(\varrho)}{1-\varrho}\int_0^t \frac{d}{d\phi}U(\phi)\mathbb{E}_{\varrho}\Big[\frac{-\varrho}{1-\varrho}\Big(t-\phi\Big)^{\varrho}\Big]d\phi. \tag{3.1}
$$

Definition 3.2. *Let $U(t)$ is differentiable function, then the Caputo and the ABC fractional piecewise derivative is given by [24] :*

$$
{}_{0}^{CABC}D_t^{\varrho}U(t) = \begin{cases} {}_{0}^{C}D_t^{\varrho}U(t), & 0 < t \leq t_1,\\ {}_{0}^{ABC}D_t^{\varrho}U(t) & t_1 < t \leq T. \end{cases}
$$

Here, ${}_{0}^{CABC}D_t^{\varrho}U(t)$ is piecewise derivative in which the Caputo operator for $0 < t \leq t_1$ and the ABC operator for $t_1 < t \leq T$.

Definition 3.3. *Let $U(t)$ be a differentiable function, then the Caputo and the ABC piece-wise integration is defined by [24]:*

$$
{}_{0}^{PCABC}I_tU(t) = \begin{cases} \dfrac{1}{\Gamma(\varrho)}\displaystyle\int_{t_1}^{t}(t-\phi)^{\varrho-1}U(\phi)d(\phi), & 0 < t \leq t_1,\\ \dfrac{1-\varrho}{ABC(\varrho)}U(t) + \dfrac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\displaystyle\int_{t_1}^{t}(t-\phi)^{\varrho-1}U(\phi)d(\phi) & t_1 < t \leq T, \end{cases}
$$

where ${}_{0}^{CABC}I_t^{\varrho}U(t)$ is piecewise integral in which Caputo operator is in $0 < t \leq t_1$ and ABC operator in $t_1 < t \leq T$.

Lemma 3.1. *The solution of a piecewise differentiable equation*

$$
{}_{0}^{PCABC}D_t^{\varrho}G(t) = H(t, G(t)), \quad 0 < \varrho \leq 1,
$$

can be obtained in the form [24]

$$G(t)=\begin{cases}G(0)+\dfrac{1}{\Gamma(\varrho)}\displaystyle\int_0^t H(\phi,G(\phi))(t-\phi)^{\varrho-1}d\phi, \ 0<t\le t_1,\\ G(t_1)+\dfrac{1-\varrho}{ABC(\varrho)}H(t,G(t))+\dfrac{\varrho}{ABC\varrho\Gamma(\varrho)}\displaystyle\int_{t_1}^t(t-\phi)^{\varrho-1}H(\phi G(\phi))d(\phi) \ \ t_1<t\le T.\end{cases}$$

4. Qualitative analysis

In this section, we establish both the existence and uniqueness of the proposed model (2.2) using the piecewise approach. For this, we can represent model (2.2) as shown in Lemma 3.1, which can be further elaborated as follows:

$$ {}_0^{PCABC}D_t^{\varrho}\mathcal{K}(t)=\mathbb{M}(t,\mathcal{K}(t)), \ \ 0<\varrho\le 1,$$

with

$$\mathcal{K}(t)=\begin{cases}\mathcal{K}_0+\dfrac{1}{\Gamma(\varrho)}\displaystyle\int_0^t \mathbb{M}(\phi,\mathcal{K}(\phi,))(t-\phi,)^{\varrho-1}d\phi, \ 0<t\le t_1,\\ \mathcal{K}(t_1)+\dfrac{1-\varrho}{ABC(\varrho)}\mathbb{M}(t,\mathcal{K}(t))+\dfrac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\displaystyle\int_{t_1}^t(t-\phi)^{\varrho-1}\mathbb{M}(\phi,\mathcal{K}(\phi))d\phi, \ \ t_1<t\le T,\end{cases} \tag{4.1}$$

where

$$\mathcal{K}(t)=\begin{cases}\mathscr{S}_{h_1}(t)\\ \mathscr{E}_{h_1}(t)\\ \mathscr{I}_{h_1}(t)\\ \mathscr{R}_{h_1}(t)\\ \mathscr{S}_{m_1}(t)\\ \mathscr{I}_{m_1}(t)\end{cases}, \quad \mathcal{K}_0=\begin{cases}\mathscr{S}_{h_1}(0)\\ \mathscr{E}_{h_1}(0)\\ \mathscr{I}_{h_1}(0)\\ \mathscr{R}_{h_1}(0)\\ \mathscr{S}_{m_1}(0)\\ \mathscr{I}_{m_1}(0)\end{cases}, \quad \mathcal{K}(t_1)=\begin{cases}\mathscr{S}_{h_1}(t_1)\\ \mathscr{E}_{h_1}(t_1)\\ \mathscr{I}_{h_1}(t_1)\\ \mathscr{R}_{h_1}(t_1)\\ \mathscr{S}_{m_1}(t_1)\\ \mathscr{I}_{m_1}(t_1)\end{cases}, \tag{4.2}$$

$$\mathbb{M}(t,\mathcal{K}(t))=\begin{cases}\mathbb{M}_1=\begin{cases}{}^{C}\mathbb{M}_1(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t),\\ {}^{ABC}\mathbb{M}_1(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t),\end{cases}\\ \mathbb{M}_2=\begin{cases}{}^{C}\mathbb{M}_2(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t),\\ {}^{ABC}\mathbb{M}_2(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t),\end{cases}\\ \mathbb{M}_3=\begin{cases}{}^{C}\mathbb{M}_3(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t),\\ {}^{ABC}\mathbb{M}_3(\mathscr{S}_{h_1},\mathscr{E}_{h_1},\mathscr{I}_{h_1},\mathscr{R}_{h_1},\mathscr{S}_{m_1},\mathscr{I}_{m_1},t).\end{cases}\end{cases} \tag{4.3}$$

Consider $0<t\le T<\infty$ and define $E_1=C[0,T]$ (Banach space) with

$$||\mathcal{K}||=max_{t\in[0,T]}|\mathcal{K}(t)|.$$

Let the growth and Lipschitz condition be in the form:

C_1: $\exists\ \mathcal{L}_{\mathbb{M}}>0$; $\forall\ \mathbb{M},\ \bar{\mathcal{K}}\in E$, we have $|\mathbb{M}(t,\mathcal{K})-\mathbb{M}(t,\bar{\mathcal{K}})|\le\mathcal{L}_{\mathbb{M}}|\mathcal{K}-\bar{\mathcal{K}}|$,

C_2: $\exists\, C_{\mathbb{M}} > 0$ & $M_{\mathbb{M}} > 0$,

$$|\mathbb{M}(t, \mathcal{K}(t))| \leq C_{\mathbb{M}}|\mathcal{K}| + M_{\mathbb{M}}.$$

If $\mathbb{M}$ is piecewise continuous on $0 < t \leq t_1$ and $t_1 < t \leq T$ in $[0, T]$, also satisfy (C_2), then model (2.2) has at least single solution in the sub-intervals.

Proof. Consider closed sub-set in both sub-intervals on $[0, T]$ in the form of E and $\mathbb{B}$. Then using Schauder FP theorem [29], we have

$$B = \{\mathcal{K} \in E : \ \|\mathcal{K}\| \leq R_{1,2},\ R > 0\},$$

next consider operator $Q : \mathbb{B} \to \mathbb{B}$ and apply to (4.1) in the form

$$Q(\mathcal{K}) = \begin{cases} \mathcal{K}_0 + \dfrac{1}{\Gamma(\varrho)} \displaystyle\int_0^{t_1} \mathbb{M}(\phi, \mathbb{U}(\phi))(t-\phi)^{\phi-1} d\phi, \ 0 < t \leq t_1, \\ \mathcal{K}(t_1) + \dfrac{1-\rho}{ABC(\rho)} \mathbb{M}(t, \mathcal{K}(t)) + \dfrac{\rho}{ABC(\rho)\Gamma(\rho)} \displaystyle\int_{t_1}^{t} (t-\phi)^{\rho-1} \mathbb{M}(\phi, \mathcal{K}(\phi)) d(\phi), \quad t \in (t_1, T]. \end{cases} \tag{4.4}$$

For any $\mathcal{K} \in B$, one can get

$$\begin{aligned} |Q(\mathcal{K}(t))| &\leq \begin{cases} |\mathcal{K}_0| + \dfrac{1}{\Gamma(\varrho)} \displaystyle\int_0^{t_1} (t-\phi)^{\varrho-1} |\mathbb{M}(\phi, \mathcal{K}(\phi))| d\phi, \\ |\mathcal{K}_(t_1)| + \dfrac{1-\varrho}{ABC(\varrho)} |\mathbb{M}(t, \mathcal{K}(t))| + \dfrac{\varrho}{ABC(\varrho)\Gamma(\varrho)} \displaystyle\int_{t_1}^{t} (t-\phi)^{\varrho-1} |\mathbb{M}(\phi \mathcal{K}(\phi))| d(\phi), \end{cases} \\ &\leq \begin{cases} |\mathcal{K}_0| + \dfrac{1}{\Gamma(\varrho)} \displaystyle\int_0^{t_1} (t-\phi)^{\varrho-1} [C_{\mathbb{M}}|\mathcal{K}| + M_{\mathbb{M}}] d\phi, \\ |\mathcal{K}_(t_1)| + \dfrac{1-\varrho}{ABC(\varrho)} [C_{\mathbb{M}}|\mathcal{K}| + M_{\mathbb{M}}] + \dfrac{\varrho}{ABC(\varrho)\Gamma(\varrho)} \displaystyle\int_{t_1}^{t} (t-\phi)^{\varrho-1} [C_{\mathbb{M}}|\mathcal{K}| + M_{\mathbb{M}}] d(\phi), \end{cases} \\ &\leq \begin{cases} |\mathcal{K}_0| + \dfrac{\mathbf{T}^{\varrho}}{\Gamma(\varrho+1)} [C_H|\mathbb{U}| + M_{\mathbb{M}}] = R_1, \ 0 < t \leq t_1, \\ |\mathcal{K}_(t_1)| + \dfrac{1-\varrho}{ABC(\varrho)} [C_{\mathbb{M}}|\mathcal{K}| + M_{\mathbb{M}}] + \dfrac{\varrho(T-\mathbf{T})^{\varrho}}{ABC(\varrho)\Gamma\varrho + 1} [C_{\mathbb{M}}|\mathcal{K}| + M_{\mathbb{M}}] d(\phi) = R_2, t \in (t_1, T], \end{cases} \\ &\leq \begin{cases} R_1, \ 0 < t \leq t_1, \\ R_2, \ t \in (t_1, T]. \end{cases} \end{aligned} \tag{4.5}$$

From Eq (4.5), since $\mathcal{K} \in \mathbf{B}$ and $Q(\mathbf{B}) \subset \mathbf{B}$. Therefore, it shows that Q is a close as well as complete. Furthermore to prove that it is completely continuous, we take $t_i < t_j \in [0, t_1]$, so

$$\begin{aligned} |Q(\mathcal{K})(t_j) - Q(\mathcal{K})(t_i)| &= \Bigg| \frac{1}{\Gamma(\varrho)} \int_0^{t_j} (t_j - \phi)^{\varrho-1} \mathbb{M}(\phi, \mathcal{K}(\phi)) d\phi, \\ &\quad - \frac{1}{\Gamma(\varrho)} \int_0^{t_i} (t_i - \phi)^{\varrho-1} \mathbb{M}(\phi, \mathcal{K}(\phi)) d\phi \Bigg| \\ &\leq \frac{1}{\Gamma(\varrho)} \int_0^{t_i} [(t_i-\phi)^{\varrho-1} - (t_j-\phi)^{\varrho-1}] |\mathbb{M}(\phi, \mathcal{K}(\phi))| d\phi \end{aligned}$$

$$
\begin{aligned}
& +\frac{1}{\Gamma(\varrho)}\int_{t_i}^{t_j}(t_j-\phi)^{\varrho-1}|\mathbb{M}(\phi,\mathcal{K}(\phi))|d\phi \\
\leq\ & \frac{1}{\Gamma(\varrho)}\Big[\int_0^{t_i}[(t_i-\phi)^{\varrho-1}-(t_j-\phi)^{\varrho-1}]d\phi \\
& +\int_{t_i}^{t_j}(t_j-\phi)^{\varrho-1}d\phi\Big](C_{\mathbb{M}}|\mathcal{K}|+M_{\mathbb{M}}) \\
\leq\ & \frac{(C_{\mathbb{M}}\mathcal{K}+M_{\mathbb{M}})}{\Gamma(\varrho+1)}[t_j^{\varrho}-t_i^{\varrho}+2(t_j-t_i)^{\varrho}]. \qquad (4.6)
\end{aligned}
$$

Further (4.6), we get $t_i \to t_j$, so

$$|Q(\mathcal{K})(t_j)-Q(\mathcal{K})(t_i)| \to 0, \text{ as } t_i \to t_j.$$

Hence, Q is equi-continuous in interval $[0, t_1]$. Now consider second sub-interval $t_i, t_j \in [t_1, T]$ as

$$
\begin{aligned}
|Q(\mathcal{K})(t_j)-Q(\mathcal{K})(t_i)| =\ & \Big|\frac{1-\varrho}{ABC(\varrho)}\mathbb{M}(t,\mathcal{K}(t))+\frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\int_{t_1}^{t_j}(t_j-\phi)^{\varrho-1}\mathbb{M}(\phi,\mathcal{K}(\phi))d\phi \\
& -\frac{1-\varrho}{ABC(\varrho)}\mathbb{M}(t,\mathcal{K}(t))+\frac{(\varrho)}{ABC(\varrho)\Gamma(\varrho)}\int_{t_1}^{t_i}(t_i-\phi)^{\varrho-1}\mathbb{M}(\phi,\mathcal{K}(\phi))d\phi\Big| \\
\leq\ & \frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\int_{t_1}^{t_i}[(t_i-\phi)^{\varrho-1}-(t_j-\phi)^{\varrho-1}]|\mathbb{M}(\phi,\mathcal{K}(\phi))|d\phi \\
& +\frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\int_{t_i}^{t_j}(t_j-\phi)^{\varrho-1}|\mathbb{M}(\phi,\mathcal{K}(\phi))|d\phi \\
\leq\ & \frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\Big[\int_{t_1}^{t_i}[(t_i-\phi)^{\varrho-1}-(t_j-\phi)^{\varrho-1}]d\phi \\
& +\int_{t_i}^{t_j}(t_j-\phi)^{\varrho-1}d\phi\Big](C_{\mathbb{M}}|\mathcal{K}|+M_{\mathbb{M}}) \\
\leq\ & \frac{\varrho(C_{\mathbb{M}}\mathcal{K}+M_{\mathbb{M}})}{ABC(\varrho)\Gamma(\varrho+1)}[t_j^{\varrho}-t_i^{\varrho}+2(t_j-t_i)^{\varrho}]. \qquad (4.7)
\end{aligned}
$$

Further as in (4.7), we take $t_i \to t_j$, then

$$|Q(\mathcal{K})(t_j)-Q(\mathcal{K})(t_i)| \to 0, \text{ as } t_i \to t_j.$$

So Q shows that it is equi-continuous in $[t_1, T]$. Hence, Q is uniform continuous, completely continuous, and is bounded, due to Arzela-Ascoli result. Hence, the suggested model exhibits at least single solution in each interval. □

Theorem 4.1. *With (C_1), the model's solution is unique if Q is contraction.*

Proof. Since the operator is piecewise continuous, then consider $\mathcal{K}$ and $\bar{\mathcal{K}} \in B$ on $[0, t_1]$

$$
\begin{aligned}
\|Q(\mathcal{K})-Q(\bar{\mathcal{K}})\| =\ & \max_{t\in[0,t_1]}\Big|\frac{1}{\Gamma(\varrho)}\int_0^t(t-\phi)^{\varrho-1}\mathbb{M}(\phi,\mathcal{K}(\phi))d\phi-\frac{1}{\Gamma(\varrho)}\int_0^t(t-\phi)^{\varrho-1}\mathbb{M}(\phi,\bar{\mathcal{K}}(\phi))d\phi\Big| \\
\leq\ & \frac{\mathbf{T}^{\varrho}}{\Gamma(\varrho+1)}L_{\mathbb{M}}\|\mathcal{K}-\bar{\mathcal{K}}\|. \qquad (4.8)
\end{aligned}
$$

From (4.8), we have

$$\|Q(\mathcal{K}) - Q(\bar{\mathcal{K}})\| \leq \frac{\mathbf{T}^{\varrho}}{\Gamma(\varrho+1)} L_{\mathbb{M}} \|\mathcal{K} - \bar{\mathcal{K}}\|. \tag{4.9}$$

So Q is contraction. Hence, using Banach's contraction, the suggested system has unique solution [30]. Further on $t \in [t_1, T]$, we have

$$\|Q(\mathcal{K}) - Q(\bar{\mathcal{K}})\| \leq \frac{1-\varrho}{ABC(\varrho)} L_{\mathbb{M}} \|\mathcal{K} - \bar{\mathcal{K}}\| + \frac{\varrho(\mathbf{T} - T^{\varrho})}{ABC(\varrho)\Gamma(\varrho+1)} L_{\mathbb{M}} \|\mathcal{K} - \bar{\mathcal{K}}\|. \tag{4.10}$$

or

$$\|Q(\mathcal{K}) - Q(\bar{\mathcal{K}})\| \leq L_{\mathbb{M}} \left[\frac{1-\varrho}{ABC(\varrho)} + \frac{\varrho(T - \mathbf{T})^{\varrho}}{ABC(\varrho)\Gamma(\varrho+1)}\right] \|\mathcal{K} - \bar{\mathcal{K}}\|. \tag{4.11}$$

Hence, Q is a contraction. Hence, from Eqs (4.9) and (4.11), the piecewise differentiable function has a unique solution. □

5. Stability analysis

Here, we established the Hyers-Ulam stability for the proposed model.

Definition 5.1. *The piecewise malaria transmission disease model* (2.1) *is Ulam-Hyers stable if* $\forall$ $\alpha > 0$*,*

$$\left|{}^{PCABC}\mathbf{D}_t^{\varrho}\mathcal{K}(t) - \mathbb{M}(t, \mathcal{K}(t))\right| < \alpha, \forall\ t \in \mathcal{T}, \tag{5.1}$$

and $\exists\ \ \mathbb{H} > 0$ *and a unique solution* $\overline{\mathcal{K}} \in Z$ *,*

$$\left\|\mathcal{K} - \overline{\mathcal{K}}\right\|_Z \leq \mathbb{H}\,\alpha, \forall\ t \in \mathcal{T}. \tag{5.2}$$

Additionally, for the inequality shown above, for a non-decreasing function $\Phi : [0, \infty) \to R^+$

$$\left\|\mathcal{K} - \overline{\mathcal{K}}\right\|_Z \leq \mathbb{H}\Phi(\alpha), \forall\ t \in \mathcal{T}. \tag{5.3}$$

If $\ \Phi(0) = 0$ *as fated, then the resultant solution is typically U-H stable.*

Definition 5.2. *Our suggested model (2.1) is Hyers-Ulam-Rassias stable if* $\Psi : [0, \infty) \to R^+$*,* $\forall\ \alpha > 0$*, and the inequality*

$$\left|{}^{PCABC}\mathbf{D}_t^{\varrho}\mathcal{K}(t) - \mathbb{M}(t, \mathcal{K}(t))\right| < \alpha\Psi(t), \forall\ t \in \mathcal{T}, \tag{5.4}$$

and $\exists\ \ \mathbb{H}_{\Psi} > 0$ *and a unique solution* $\overline{\mathcal{K}} \in Z$*, so that*

$$\left\|\mathcal{K} - \overline{\mathcal{K}}\right\|_Z \leq \mathbb{H}_{\Psi}\alpha\Psi(t),\ t \in \mathcal{T}. \tag{5.5}$$

If there exist $\Psi : [0, \infty) \to R^+$ *for the inequality*

$$\left|{}^{PCABC}\mathbf{D}_t^{\varrho}\mathcal{K}(t) - \mathbb{M}(t, \mathcal{K}(t))\right| < \Psi(t),\ t \in \mathcal{T}. \tag{5.6}$$

If there exist a unique solution $\overline{\mathcal{K}} \in Z$ *with constant* $\mathbb{H}_{\Psi} > 0$*, so*

$$\left\|\mathcal{K} - \overline{\mathcal{K}}\right\|_Z \leq \mathbb{H}_{\Psi}\Psi(t),\ t \in \mathcal{T}. \tag{5.7}$$

Then, the result is typically H-U-R stable.

Remark 5.1. *Suppose a function $\phi \in C(\mathcal{T})$ is not dependent on $\mathcal{K} \in \mathcal{Z}$, and $\phi(0) = 0$, then*

$$\begin{aligned} |\phi(t)| &\leq \alpha,\ t \in \mathcal{T}, \\ {}^{PCABC}\mathcal{D}_t^{\varrho}\mathcal{K}(t) &= \mathbb{M}\left(t, \mathcal{K}(t)\right) + \phi(t),\ t \in \mathcal{T}. \end{aligned}$$

Lemma 5.1. *Suppose the function*

$$ {}_0^{PCABC}\mathrm{D}_t^{\varrho}\mathcal{K}(t) = \mathbb{M}(t, \mathcal{K}(t)),\ \ 0 < \varrho \leq 1. \tag{5.8}$$

The solution of Eq (5.8) can be obtained in the form

$$\mathcal{K}(t) = \begin{cases} \mathcal{K}_0 + \dfrac{1}{\Gamma(\varrho)}\displaystyle\int_0^t \mathbb{M}(\phi, \mathcal{K}(\phi))(t-\phi)^{\varrho-1}d\phi,\ 0 < t \leq t_1, \\ \mathcal{K}(t_1) + \dfrac{1-\varrho}{ABC(\varrho)}\mathbb{M}(t, \mathcal{K}(t)) + \dfrac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\displaystyle\int_{t_1}^t (t-\phi)^{\varrho-1}\mathbb{M}(\phi, \mathcal{K}(\phi))d(\phi),\ \ t_1 < t \leq T, \end{cases} \tag{5.9}$$

$$\left\|\mathbb{M}(\mathcal{K}) - \mathbb{M}(\overline{\mathcal{K}})\right\| \leq \begin{cases} \dfrac{\mathcal{T}_1^{\varrho}}{\Gamma(\varrho+1)}\alpha, t \in \mathcal{T}_1, \\ \left[\dfrac{(1-\varrho)\Gamma(\varrho) + (\mathcal{T}_2^{\varrho})}{ABC(\varrho)\Gamma(\varrho)}\right]\alpha = \Theta\alpha,\ t \in \mathcal{T}_2. \end{cases} \tag{5.10}$$

Theorem 5.1. *Considering Lemma 5.1, if the condition $\frac{L_f \mathcal{T}^{\varrho}}{\Gamma(\varrho)} < 1$ satisfy, then the solution of model (2.1) is Hyers-Ulam as well as generalized Hyers-Ulam stable.*

Proof. Suppose $\mathcal{K} \in Z$ is the solution of model (2.1) and $\overline{\mathcal{K}} \in Z$ is a unique solution of model (2.1). Thus we have

Case 1: For $t \in \mathcal{T}$, we have

$$\begin{aligned} \left\|\mathcal{K} - \overline{\mathcal{K}}\right\| &= \sup_{t\in\mathcal{T}}\left|\mathcal{K} - \left(\mathcal{K}_\circ + \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t_1-s)^{\varrho-1}\mathbb{M}\left(s, \overline{\mathcal{K}}(s)\right)ds\right)\right| \\ &\leq \sup_{t\in\mathcal{T}}\left|\mathcal{K} - \left(\mathcal{K}_\circ + \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t_1-s)^{\varrho-1}\mathbb{M}\left(s, \overline{\mathcal{K}}(s)\right)ds\right)\right| \\ &\quad + \sup_{t\in\mathcal{T}}\left| + \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t_1-s)^{\varrho-1}\mathbb{M}\left(s, \mathcal{K}(s)\right)ds \right. \\ &\quad \left. - \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t_1-s)^{\varrho-1}\mathbb{M}\left(s, \overline{\mathcal{K}}(s)\right)ds\right| \\ &\leq \frac{\mathcal{T}_\infty{}^{\varrho}}{\Gamma(\varrho+1)}\alpha + \frac{L_f\mathcal{T}_\infty}{\Gamma(\varrho+1)}\left\|\mathcal{K} - \overline{\mathcal{K}}\right\|. \end{aligned}$$

On further simplification, we obtain

$$\left\|\mathcal{K} - \overline{\mathcal{K}}\right\| \leq \left(\frac{\frac{\mathcal{T}_\infty}{\Gamma(\varrho+1)}}{1 - \frac{L_f\mathcal{T}_\infty}{\Gamma(\varrho+1)}}\right)\alpha. \tag{5.11}$$

Case 2:

$$\begin{aligned}\left\|\mathcal{K}-\overline{\mathcal{K}}\right\| &\le \sup_{t\in\mathcal{T}}\left|\mathcal{K}-\left[\mathcal{K}(t_1)+\frac{1-\varrho}{ABC(\varrho)}\left[\mathbb{M}\left(t,\mathcal{K}(t)\right)\right]\right.\right.\\ &\quad+\left.\left.\frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\left[\int_{t_1}^{t}(t-s)^{\varrho-1}\mathbb{M}\left(s,\overline{\mathcal{K}}(s)\right)d(s)\right]\right]\right|\\ &\quad+\sup_{t\in\mathcal{T}}\frac{1-\varrho}{ABC(\varrho)}\left|\mathbb{M}\left(t,\mathcal{K}(t)\right)-\mathbb{M}\left(t,\overline{\mathcal{K}}(t)\right)\right|\\ &\quad+\sup_{t\in\mathcal{T}}\frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\int_{t_1}^{t}(t-s)^{\varrho-1}\left|\mathbb{M}\left(s,\mathcal{K}(s)\right)-\mathbb{M}\left(s,\overline{\mathcal{K}}(s)\right)\right|ds.\end{aligned}$$

By further simplification and using $\Theta=\left[\frac{(1-\varrho)\Gamma(\varrho)+\mathcal{T}_2^{\varrho}}{ABC(\varrho)\Gamma(\varrho)}\right]$, we obtain

$$\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \le \Theta\alpha+\Theta L_f\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z, \tag{5.12}$$

and

$$\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \le \left(\frac{\Theta}{1-\frac{\Theta}{L_f}}\right)\alpha\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z,$$

by using

$$\mathbb{H}=\max\left\{\left(\frac{\frac{\mathcal{T}_1}{\Gamma(\varrho+1)}}{1-\frac{L_f\mathcal{T}_1}{\Gamma(\varrho+1)}}\right),\frac{\Theta}{1-\frac{\Theta L_f}{1-M_f}}\right\}.$$

Finally, from Eqs (5.11) and (5.12), we have

$$\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \le \mathbb{H}\alpha,\ \ t\in\mathcal{T}.$$

Hence, the solution of model (2.1) is Hyers-Ulam stable. Also, if we replace α by $\Phi(\alpha)$ then from (5.13), we obtain

$$\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \le \mathbb{H}\Phi(\alpha),\ \ t\in\mathcal{T}.$$

Now, $\Phi(0)=0$ shows that solution of our proposed model (2.1) is generalized Hyers-Ulam stable. □

We include the following remark to conclude the Rassias stability results and also the generalized form.

Remark 5.2. *Suppose a function $\phi\in C(\mathcal{T})$ does not depend upon $\mathcal{K}\in\mathcal{Z}$, and $\phi(0)=0$, then*

$$\begin{aligned}|\phi(t)| &\le \Psi(t)\alpha,\ t\in\mathcal{T},\\ {}^{PCABC}\mathcal{D}_t^{\varrho}\mathcal{K}(t) &= \mathbb{M}\left(t,\mathcal{K}(t)\right)+\phi(t),\ t\in\mathcal{T},\\ \int_0^t\Psi(s)ds &\le C_{\Psi}\Psi(t),\ t\in\mathcal{T}.\end{aligned}$$

Lemma 5.2. *Solution to the model*

$$\begin{aligned} {}^{PCABC}\mathcal{D}_t^{\varrho}\mathcal{K}(t) &= \mathbb{M}\left(t, \mathcal{K}(t)\right) + \phi(t), \\ \mathcal{K}(0) = \mathcal{K}_{\circ}, \end{aligned}$$

holds the relation

$$\left\|\mathbb{M}(\mathcal{K}) - \mathbb{M}(\overline{\mathcal{K}})\right\| \leq \begin{cases} \dfrac{\mathcal{T}_1^{\varrho}}{\Gamma(\varrho+1)} C_{\Psi}\Psi(t)\alpha, \ t \in \mathcal{T}_1, \\ \left[\dfrac{(1-\varrho)\Gamma(\varrho) + (\mathcal{T}_2^{\varrho})}{ABC(\varrho)\Gamma(\varrho)}\right] C_{\Psi}\Psi(t)\alpha = \Theta C_{\Psi}\Psi(t)\alpha, \ t \in \mathcal{T}_2. \end{cases} \tag{5.13}$$

Where $\mathbb{H}_{f,\Psi,\Theta} = \Theta\mathbb{H}_{f,\Psi}$. *With the help of Remark 5.2, one can get Eq* (5.13).

Theorem 5.2. *The solution of model* (5.13) *is H-U-R stable if the following conditions hold*
H_1 : *For each* $\mathcal{K}, v \in \mathcal{Z}$ *and a constant* $C_{\Phi} > 0$, *we get*

$$|\Phi(\mathcal{K}) - \Phi(v)| \leq C_{\Phi} |\mathcal{K} - v| ;$$

H_2 : *For each* $\mathcal{K}, v, \overline{\mathcal{K}}, \overline{v} \in \mathcal{Z}$ *and constant* $L_f > 0,\ 0 < M_f < 1$, *we get*

$$\begin{aligned} \left|\mathbb{M}(t, \mathcal{K}, v) - \mathbb{M}(t, \overline{\mathcal{K}}, \overline{v})\right| &\leq L_f \left|\mathcal{K} - \overline{\mathcal{K}}\right| + M_f |v - \overline{v}| \\ M_f &< 1. \end{aligned}$$

Proof. We prove these results in two cases.
Case 1: for $t \in \mathcal{T}$, we have

$$\begin{aligned} \left\|\mathcal{K} - \overline{\mathcal{K}}\right\| &= \sup_{t\in\mathcal{T}} \left|\mathcal{K} - \left(\mathcal{K}_{\circ} + \frac{1}{\Gamma(\varrho)} \int_0^{t_1} (t_1 - s)^{\varrho-1} \mathbb{M}\left(s, \overline{\mathcal{K}}(s)\right) ds\right)\right| \\ &\leq \sup_{t\in\mathcal{T}} \left|\mathcal{K} - \left(\mathcal{K}_{\circ} + \frac{1}{\Gamma(\varrho)} \int_0^{t_1} (t_1 - s)^{\varrho-1} \mathbb{M}\left(s, \overline{\mathcal{K}}(s)\right) ds\right)\right| \\ &\quad + \sup_{t\in\mathcal{T}} \left| + \frac{1}{\Gamma(\varrho)} \int_0^{t_1} (t_1 - s)^{\varrho-1} \mathbb{M}\left(s, \mathcal{K}(s)\right) ds \right. \\ &\quad \left. - \frac{1}{\Gamma(\varrho)} \int_0^{t_1} (t_1 - s)^{\varrho-1} \mathbb{M}\left(s, \overline{\mathcal{K}}(s)\right) ds \right| \\ &\leq \frac{\mathcal{T}_1^{\varrho}}{\Gamma(\varrho+1)} C_{\Phi}\Phi(t)\alpha + \frac{L_f \mathcal{T}_{\infty}}{\Gamma(\varrho+1)} \left\|\mathcal{K} - \overline{\mathcal{K}}\right\|. \end{aligned}$$

On further simplification

$$\left\|\mathcal{K} - \overline{\mathcal{K}}\right\| \leq \left(\frac{C_{\Phi}\Phi(t)\frac{\mathcal{T}_1}{\Gamma(\varrho+1)}}{1 - \frac{L_f \mathcal{T}_1}{\Gamma(\varrho+1)}}\right)\alpha. \tag{5.14}$$

Case 2:

$$\left\|\mathcal{K} - \overline{\mathcal{K}}\right\| \leq \sup_{t\in\mathcal{T}} \left|\mathcal{K} - \left[\mathcal{K}(t_1) + \frac{1-\varrho}{ABC(\varrho)} \left[\mathbb{M}\left(t, \mathcal{K}(t)\right)\right]\right.\right.$$

$$
\begin{aligned}
&+ \quad \frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\left[\int_{t_1}^{t}(t-s)^{\varrho-1}\mathbb{M}\left(s,\overline{\mathcal{K}}(s)\right)d(s)\right]\Bigg]\Bigg| \\
&+ \quad \sup_{t\in\mathcal{T}}\frac{1-\varrho}{ABC(\varrho)}\left|\mathbb{M}\left(t,\mathcal{K}(t)\right)-\mathbb{M}\left(t,\overline{\mathcal{K}}(t)\right)\right| \\
&+ \quad \sup_{t\in\mathcal{T}}\frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\int_{t_1}^{t}(t-s)^{\varrho-1}\left|\mathbb{M}\left(s,\mathcal{K}(s)\right)-\mathbb{M}\left(s,\overline{\mathcal{K}}(s)\right)\right|ds.
\end{aligned}
$$

By further simplification and using $\Theta = \left[\frac{(1-\varrho)\Gamma(\varrho)+\mathcal{T}_2^{\varrho}}{ABC(\varrho)\Gamma(\varrho)}\right]$, we obtain

$$
\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \leq \Theta C_{\Phi}\Phi(t)\alpha + \Theta L_f\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z, \tag{5.15}
$$

and

$$
\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \leq \left(\frac{\Theta C_{\Phi}\Phi(t)}{1-\frac{\Theta}{L_f}}\right)\alpha\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z,
$$

by using

$$
\mathbb{H}_{\Theta,C_{\Phi}} = \max\left\{\left(\frac{\frac{\mathcal{T}_1}{\Gamma(\varrho+1)}}{1-\frac{L_f\mathcal{T}_1}{\Gamma(\varrho+1)}}\right), \frac{C_{\Phi}\Phi(t)\Theta}{1-\frac{\Theta L_f}{1-M_f}}\right\}.
$$

Now from Eqs (5.14) and (5.15), we have

$$
\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \leq \mathbb{H}_{\Theta,C_{\Phi}}\alpha, \quad t\in\mathcal{T}.
$$

So the solution of model (2.1) is H-U-R stable. □

Remark 5.3. *Suppose a function $\phi \in C(\mathcal{T})$ does not depend upon $\mathcal{K} \in \mathcal{Z}$, and $\phi(0) = 0$, then*

$$
|\phi(t)| \leq \Psi(t), t \in \mathcal{T}.
$$

Theorem 5.3. *In light of H_1, H_2, Remark 5.3 and Lemma 5.2, the solution of model 2.1 is generalized H-U-R stable, if $M_f < 1$.*

Where

H_1 : *For each $\mathcal{K}, v \in \mathcal{Z}$ and constant $C_{\Phi} > 0$, we get*

$$
|\Phi(\mathcal{K})-\Phi(v)| \leq C_{\Phi}|\mathcal{K}-v|,
$$

and

H_2 : *For each $\mathcal{K}, v, \overline{\mathcal{K}}, \overline{v} \in \mathcal{Z}$ and constant $L_f > 0, 0 < M_f < 1$, we get*

$$
\left|\mathbb{M}(t,\mathcal{K},v)-\mathbb{M}(t,\overline{\mathcal{K}},\overline{v})\right| \leq L_f\left|\mathcal{K}-\overline{\mathcal{K}}\right| + M_f|v-\overline{v}|.
$$

Proof. Here, we will discuss two cases:

Case 1: For $t \in \mathcal{T}$, we have

$$\begin{aligned}\left\|\mathcal{K}-\overline{\mathcal{K}}\right\| &= \sup_{t\in\mathcal{T}}\left|\mathcal{K}-\left(\mathcal{K}_\circ+\frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t_1-s)^{\varrho-1}\mathbb{M}\left(s,\overline{\mathcal{K}}(s)\right)ds\right)\right| \\ &\leq \sup_{t\in\mathcal{T}}\left|\mathcal{K}-\left(\mathcal{K}_\circ+\frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t_1-s)^{\varrho-1}\mathbb{M}\left(s,\overline{\mathcal{K}}(s)\right)ds\right)\right| \\ &\quad+\sup_{t\in\mathcal{T}}\left|+\frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t_1-s)^{\varrho-1}\mathbb{M}\left(s,\mathcal{K}(s)\right)ds\right. \\ &\quad\left.-\frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t_1-s)^{\varrho-1}\mathbb{M}\left(s,\overline{\mathcal{K}}(s)\right)ds\right| \\ &\leq \frac{\mathcal{T}_1^{\varrho}}{\Gamma(\varrho+1)}C_\Phi\Phi(t)\alpha+\frac{L_f\mathcal{T}_\infty}{\Gamma(\varrho+1)}\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|.\end{aligned}$$

On further simplification

$$\left\|\mathcal{K}-\overline{\mathcal{K}}\right\| \leq \left(\frac{C_\Phi\Phi(t)\frac{\mathcal{T}_1}{\Gamma(\varrho+1)}}{1-\frac{L_f\mathcal{T}_1}{\Gamma(\varrho+1)}}\right)\alpha. \tag{5.16}$$

Case 2:

$$\begin{aligned}\left\|\mathcal{K}-\overline{\mathcal{K}}\right\| &\leq \sup_{t\in\mathcal{T}}\left|\mathcal{K}-\left[\mathcal{K}(t_1)+\frac{1-\varrho}{ABC(\varrho)}\left[\mathbb{M}\left(t,\mathcal{K}(t)\right)\right]\right.\right. \\ &\quad\left.\left.+\frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\left[\int_{t_1}^{t}(t-s)^{\varrho-1}\mathbb{M}\left(s,\overline{\mathcal{K}}(s)\right)d(s)\right]\right]\right| \\ &\quad+\sup_{t\in\mathcal{T}}\frac{1-\varrho}{ABC(\varrho)}\left|\mathbb{M}\left(t,\mathcal{K}(t)\right)-\mathbb{M}\left(t,\overline{\mathcal{K}}(t)\right)\right| \\ &\quad+\sup_{t\in\mathcal{T}}\frac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\int_{t_1}^{t}(t-s)^{\varrho-1}\left|\mathbb{M}\left(s,\mathcal{K}(s)\right)-\mathbb{M}\left(s,\overline{\mathcal{K}}(s)\right)\right|ds.\end{aligned}$$

By further simplification and using $\Theta=\left[\frac{(1-\varrho)\Gamma(\varrho)+\mathcal{T}_2^{\varrho}}{ABC(\varrho)\Gamma(\varrho)}\right]$, we have

$$\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \leq \Theta C_\Phi\Phi(t)\alpha+\Theta L_f\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z, \tag{5.17}$$

we have

$$\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \leq \left(\frac{\Theta C_\Phi\Phi(t)}{1-\Theta L_f}\right)\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z,$$

using

$$\mathbb{H}_{\Theta,C_\Phi}=\max\left\{\left(\frac{\frac{\mathcal{T}_1}{\Gamma(\varrho+1)}}{1-\frac{L_f\mathcal{T}_1}{\Gamma(\varrho+1)}}\right),\frac{C_\Phi\Phi(t)\Theta}{1-\Theta L_f}\right\}.$$

Now from Eqs (5.16) and (5.17), we have

$$\left\|\mathcal{K}-\overline{\mathcal{K}}\right\|_Z \leq \mathbb{H}_{\Theta,C_\Phi},\quad t\in\mathcal{T}.$$

Hence the solution of the model (2.1) is generalized H-U-R stable. □

6. A numerical algorithm for the piecewise Malaria model

Here, we formulate a numerical method for solving the given problem with a piecewise derivative, presented in Eq (2.1). This approach focuses on constructing a numerical scheme applicable for two sub-intervals within the range of $[0,T]$ using the Caputo and ABC operators. To achieve this, we use the insights provided by the piecewise operator numerical scheme presented in [24]. By employing the piecewise integration technique to Eq (2.2) in the Caputo and ABC form, the following procedure occurs:

$$
\begin{aligned}
\mathscr{S}_{\mathsf{h}_1}(t) &= \begin{cases} \mathscr{S}_{\mathsf{h}_1}(0) + \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t-\tau)^{\varrho-1}\,{}^{C}\mathbb{M}_1(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; 0<t\le t_1,\\ \mathscr{S}_{\mathsf{h}_1}(t_1) + \frac{1-\varrho}{AB(\varrho)}\mathbb{M}_1(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})\\ +\frac{\varrho}{AB(\varrho)\Gamma(\varrho)}\int_{t_1}^{t}(t-\tau)^{\varrho-1}\mathbb{M}_1(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; t_1<t\le T, \end{cases}\\
\mathscr{E}_{\mathsf{h}_1}(t) &= \begin{cases} \mathscr{E}_{\mathsf{h}_1}(0) + \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t-\tau)^{\varrho-1}\,{}^{c}\mathbb{M}_2(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; 0<t\le t_1,\\ \mathscr{E}_{\mathsf{h}_1}(t_1) + \frac{1-\varrho}{AB(\varrho)}\mathbb{M}_2(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})\\ +\frac{\varrho}{AB(\varrho)\Gamma(\varrho)}\int_{t_1}^{t}(t-\tau)^{\varrho-1}\mathbb{M}_2(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; t_1<t\le T, \end{cases}\\
\mathscr{I}_{\mathsf{h}_1}(t) &= \begin{cases} \mathscr{I}_{\mathsf{h}_1}(0) + \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t-\tau)^{\varrho-1}\,{}^{c}\mathbb{M}_3(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; 0<t\le t_1,\\ \mathscr{I}_{\mathsf{h}_1}(t_1) + \frac{1-\varrho}{AB(\varrho)}\mathbb{M}_3(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})\\ +\frac{\varrho}{AB(\varrho)\Gamma(\varrho)}\int_{t_1}^{t}(t-\tau)^{\varrho-1}\mathbb{M}_3(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; t_1<t\le T, \end{cases}\\
\mathscr{R}_{\mathsf{h}_1}(t) &= \begin{cases} \mathscr{R}_{\mathsf{h}_1}(0) + \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t-\tau)^{\varrho-1}\,{}^{c}\mathbb{M}_4(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; 0<t\le t_1,\\ \mathscr{R}_{\mathsf{h}_1}(t_1) + \frac{1-\varrho}{AB(\varrho)}\mathbb{M}_4(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})\\ +\frac{\varrho}{AB(\varrho)\Gamma(\varrho)}\int_{t_1}^{t}(t-\tau)^{\varrho-1}\mathbb{M}_4(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; t_1<t\le T, \end{cases}\\
\mathscr{S}_{\mathsf{m}_1}(t) &= \begin{cases} \mathscr{S}_{\mathsf{m}_1}(0) + \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t-\tau)^{\varrho-1}\,{}^{c}\mathbb{M}_5(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; 0<t\le t_1,\\ \mathscr{S}_{\mathsf{m}_1}(t_1) + \frac{1-\varrho}{AB(\varrho)}\mathbb{M}_5(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})\\ +\frac{\varrho}{AB(\varrho)\Gamma(\varrho)}\int_{t_1}^{t}(t-\tau)^{\varrho-1}\mathbb{M}_5(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; t_1<t\le T, \end{cases}\\
\mathscr{I}_{\mathsf{m}_1}(t) &= \begin{cases} \mathscr{I}_{\mathsf{m}_1}(0) + \frac{1}{\Gamma(\varrho)}\int_0^{t_1}(t-\tau)^{\varrho-1}\,{}^{c}\mathbb{M}_6(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; 0<t\le t_1,\\ \mathscr{I}_{\mathsf{m}_1}(t_1) + \frac{1-\varrho}{AB(\varrho)}\mathbb{M}_6(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})\\ +\frac{\varrho}{AB(\varrho)\Gamma(\varrho)}\int_{t_1}^{t}(t-\tau)^{\varrho-1}\mathbb{M}_6(t,\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1})d\tau \;\; t_1<t\le T. \end{cases}
\end{aligned} \tag{6.1}
$$

Here,

$$
{}^{C}\mathbb{M}_i(t) = {}^{C}\mathbb{M}_i(\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1},t), \quad {}^{ABC}\mathbb{M}_i(t) = {}^{ABC}_{i}\mathbb{M}(\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1},t)
$$

are left hand sides of Eq (6.1) for $i = 1, 2, 3$, also given in Eq (2.2). We present a scheme for model (6.1) which will be used for other compartments as well. At $t = t_{n+1}$

$$
\mathscr{S}_{\mathsf{h}_1}(t_{n+1})) = \begin{cases} \mathscr{S}_{\mathsf{h}_1}(0) + \dfrac{1}{\Gamma(\varrho)}\displaystyle\int_0^{t_1}(t-\phi)^{\varrho-1}\;{}^{C}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1},\phi)d\phi,\\ \mathscr{S}_{\mathsf{h}_1}(t_1) + \dfrac{1-\varrho}{ABC(\varrho)}\;{}^{ABC}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1},\mathscr{E}_{\mathsf{h}_1},\mathscr{I}_{\mathsf{h}_1},\mathscr{R}_{\mathsf{h}_1},\mathscr{S}_{\mathsf{m}_1},\mathscr{I}_{\mathsf{m}_1},t_n)\\ +\dfrac{\varrho}{ABC(\varrho)\Gamma(\varrho)}\displaystyle\int_{t_1}^{t_{n+1}}(t-\phi)^{\varrho-1}\;{}^{ABC}\mathbb{M}_1(\phi)d\phi, \quad t_1<t\le T. \end{cases} \tag{6.2}
$$

Using newton interpolation approximation, Eq (6.2) follows as [24]

$$\mathscr{S}_{\mathsf{h}_1}(t_{n+1})) = \begin{cases} \mathscr{S}_{\mathsf{h}_1}(0) + \left\{ \begin{array}{l} \frac{(\Delta t)^{\varrho-1}}{\Gamma(\varrho+1)} \sum_{\mathrm{K}=2}^{i} \Big[{}^{C}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-2}, t_{\mathrm{K}-2}) \Big] \mathbf{A}_1 \\ + \frac{(\Delta t)^{\varrho-1}}{\Gamma(\varrho+2)} \sum_{\mathrm{K}=2}^{i} \Big[{}^{C}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-1}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-1}, t_{\mathrm{K}-1}) \\ -{}^{C}\,\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-2}, t_{\mathrm{K}-2}) \Big] \mathbf{A}_2 \\ + \frac{\varrho(\Delta t)^{\varrho-1}}{2\Gamma(\varrho+3)} \sum_{\mathrm{K}=2}^{i} \Big[{}^{C}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}}, t_{\mathrm{K}}) \\ -2\,{}^{C}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-1}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-1}, t_{\mathrm{K}-1}) \\ + {}^{C}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-2}, t_{\mathrm{K}-2}) \Big] \Delta \end{array} \right\}, \\ \mathscr{S}_{\mathsf{h}_1}(t_1) + \left\{ \begin{array}{l} \frac{1-\varrho}{ABC(\varrho)} {}^{ABC}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{n}, \mathscr{E}_{\mathsf{h}_1}^{n}, \mathscr{I}_{\mathsf{h}_1}^{n}, \mathscr{R}_{\mathsf{h}_1}^{n} \mathscr{S}_{\mathsf{m}_1}^{n}, \mathscr{I}_{\mathsf{m}_1}^{n}, t_n) \\ + \frac{\varrho}{ABC(\varrho)} \frac{(\delta t)^{\varrho-1}}{\Gamma(\varrho+1)} \sum_{\mathrm{K}=i+3}^{n} \Big[{}^{ABC}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-2}, \\ \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-2}, t_{\mathrm{K}-2}) \Big] \mathbf{A}_1 + \frac{\varrho}{ABC(\varrho)} \frac{(\upsilon t)^{\varrho-1}}{\Gamma(\varrho+2)} \sum_{\mathrm{K}=i+3}^{n} \Big[{}^{ABC}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-1}, \\ \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-1} \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-1}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-1}, t_{\mathrm{K}-1}) \\ + {}^{ABC}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-2}, t_{\mathrm{K}-2}) \Big] \mathbf{A}_2 \\ + \frac{\varrho}{ABC(\varrho)} \frac{\varrho(\upsilon t)^{\varrho-1}}{\Gamma(\varrho+3)} \sum_{\mathrm{K}=i+3}^{n} \Big[{}^{ABC}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}}, t_{\mathrm{K}}) \\ -2\,{}^{ABC}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}-1}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-1}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-1}, t_{\mathrm{K}-1}) \\ + {}^{ABC}\mathbb{M}_1(\mathscr{S}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{E}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{R}_{\mathsf{h}_1}^{\mathrm{K}-2}, \mathscr{S}_{\mathsf{m}_1}^{\mathrm{K}-2}, \mathscr{I}_{\mathsf{m}_1}^{\mathrm{K}-2}, t_{\mathrm{K}-2}) \Big] \Delta. \end{array} \right\}. \end{cases}$$

For other five compartment, one can derive the Newton interpolation as above. Here

$$\begin{aligned} \mathbf{A}_1 &= (n+1-\mathrm{K})^{\varrho}\Big(2(n-\mathrm{K})^2 + (3\varrho+10)(-\mathrm{K}+n) + 2\varrho^2 + 9\varrho + 12\Big) \\ &\quad -(-\mathrm{K}+n)\Big(2(n-\mathrm{K})^2 + (5\varrho+10)(n-\mathrm{K}) + 6\varrho^2 + 18\varrho + 12\Big), \\ \mathbf{A}_2 &= (1-\mathrm{K}+n)^{\varrho}\Big(3-\mathrm{K}+2\varrho+n\Big) - (-\mathrm{K}+\mathrm{n})\Big(n+3\varrho-\mathrm{K}+3\Big), \\ \Delta &= (1+n-\mathrm{K})^{\varrho} - (n-\mathrm{K})^{\varrho}. \end{aligned}$$

7. Numerical simulations and discussion

The objective of this section is to present simulations of a mathematical model (2.1) that combines the Caputo and ABC piecewise operators. This model is employed to investigate the dynamics of a population consisting of susceptible, exposed, infected, and recovered humans, as well as susceptible and infected mosquitoes. The parameter values employed to generate the figures in this section are as follows: $\mathcal{A}_h = 0.027$, $\mathcal{A}_m = 0.13$, $b_1 = 1/730$, $b_2 = 0.001$, $w = 0.038$, $r_1 = 0.020$, $r_2 = 0.010$, $r_3 = 0.072$, $\zeta = 0.13$, $d_1 = 0.001$, $d_2 = 0.001$, $\varrho_h = 0.0004$, $\varrho_M = 0.04$, $\alpha = 0.05$, $p = 0.611$, $\nu = 0.01$, $\upsilon = 0.01$, $q = 0.022$. The initial population values are set as $\mathscr{S}_{h_1} = 1100$, $\mathscr{E}_{h_1} = 150$, $\mathscr{I}_{h_1} = 20$, $\mathscr{R}_{h_1} = 200$, $\mathscr{S}_{m_1} = 1000$, $\mathscr{I}_{m_1} = 100$ considered from reference [15]. The time interval is divided into two sub-intervals: for the left panels of Figs. 2-7, the interval is $[0, t_1] = [0, 23]$ and $[t_1, T] = [23, 60]$. Similarly, for the right panels, the interval is divided as $[0, t_1] = [0, 13]$ and $[t_1, T] = [13, 400]$.

In Figure 2, the behavior of the susceptible human population is depicted. These sub figures illustrate the decline in the number of susceptible individuals over time, with a more rapid decrease observed during the second interval. Figure 3 display the evolution of the exposed individuals. These figures show how the population of exposed individuals changes over time.

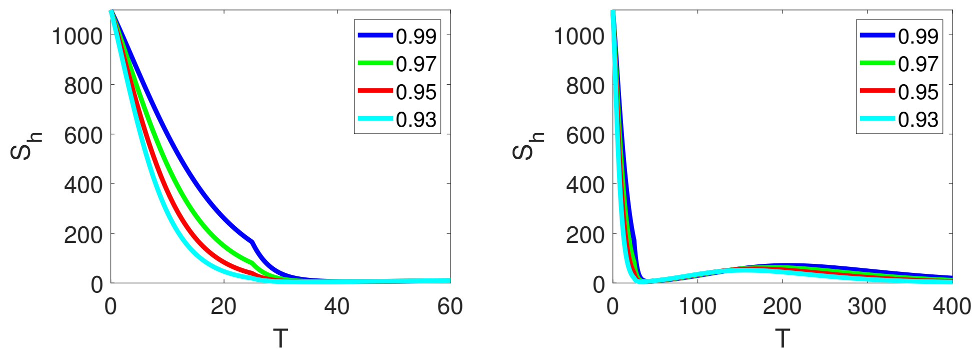

Figure 2. The behavior of susceptible human population $\mathscr{S}_{h_1}(t)$ in the C-ABC piece-wise model (2.1).

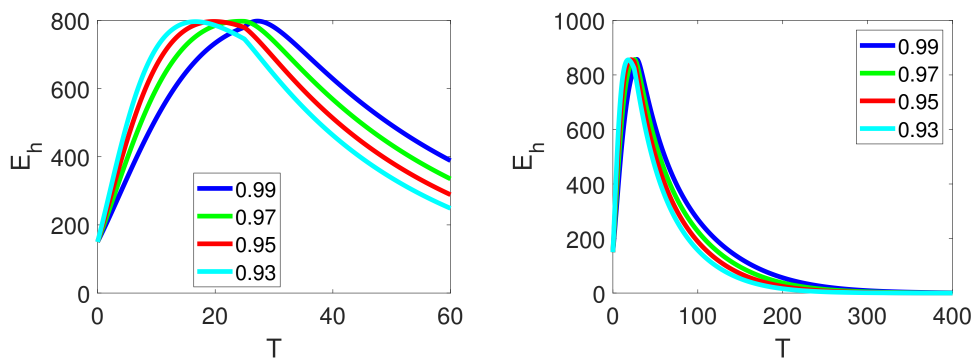

Figure 3. The behavior of the exposed human population $\mathscr{E}_{h_1}(t)$ in the C-ABC piece-wise model (2.1).

The behavior of the infected population of humans is demonstrated in Figure 4. These sub figures provide insights into the dynamics of the infected individuals and how their population varies over time.

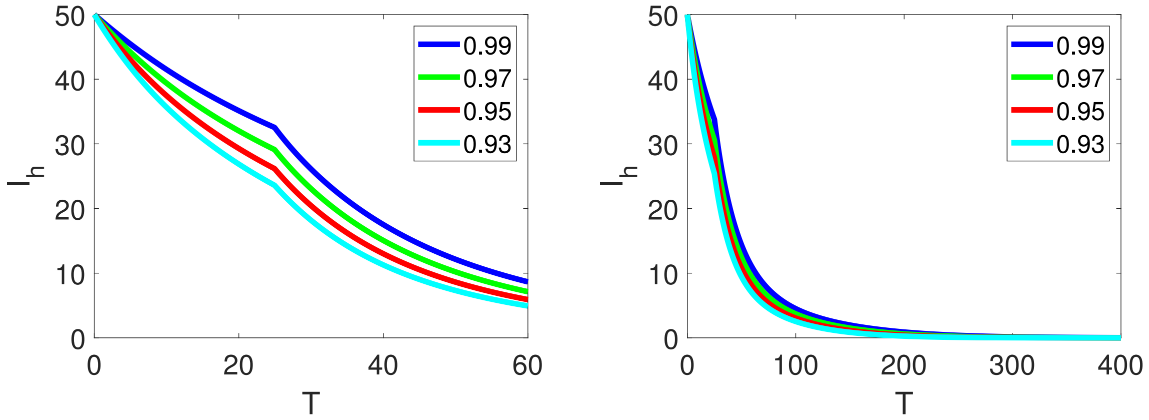

Figure 4. The behavior of the infected human population $\mathscr{I}_{h_1}(t)$ in the C-ABC piece-wise model (2.1).

Similarly, Figure 5 shows the populous of recovered humans. These graphs show the initial rise in the number of recovered individuals, which gradually enhances after $t = 23$ and stabilizes at $t = 60$. Notably, individuals with small fractional orders of recovery get stability more quickly compared to those with higher values.

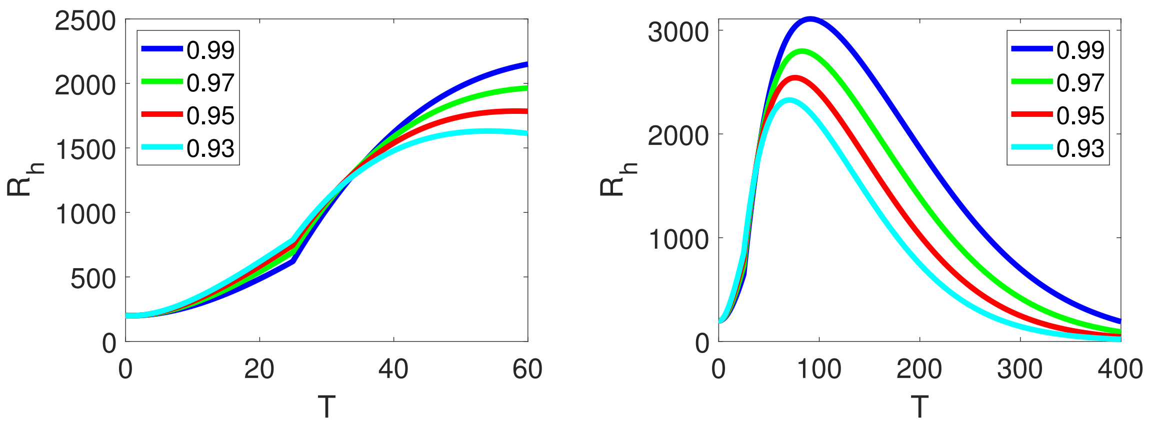

Figure 5. The behavior of the recovered human population $\mathscr{R}_{h_1}(t)$ in the C-ABC piece-wise model (2.1).

Furthermore, Figsures 6 and 7 illustrate the behavior of susceptible and infected mosquitoes, respectively. These figures provide insights into the dynamics of mosquito populations in relation to the transmission of the virus. The simulations reveal crucial insights into the behavior of the various populations involved in the present mathematical model. The results show the decrease in the susceptible population over time, the rapid decrease in the infections when considering the ABC operator, and the patterns seen in the recovered population with different fractional orders. These findings add to our understanding of the spread and control of infections of a viral disease, particularly in relation to human and mosquito populations.

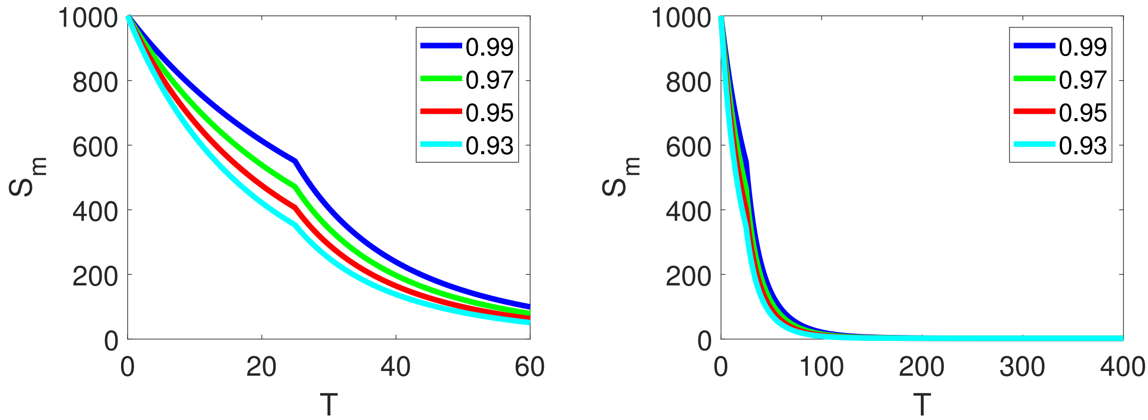

Figure 6. The behavior of the susceptible mosquitoes population $\mathscr{S}_{m_1}(t)$ in the C-ABC piecewise model (2.1).

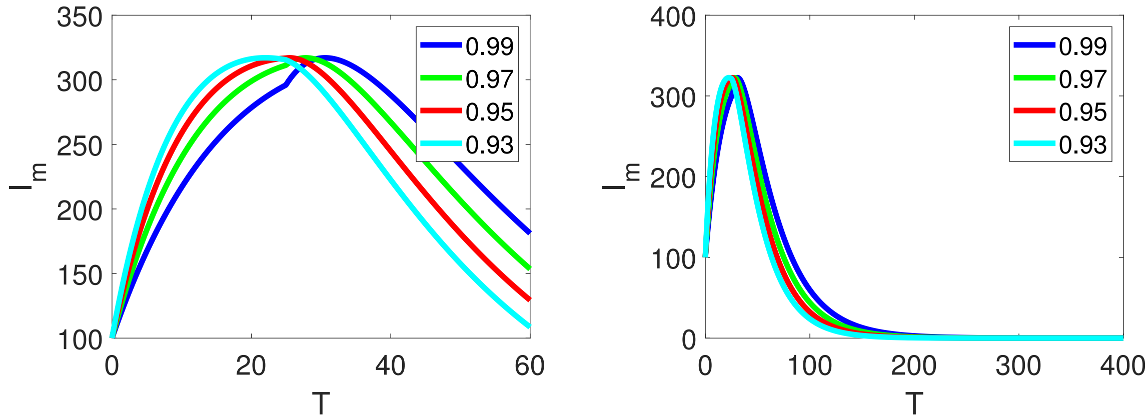

Figure 7. The behavior of the infected mosquitoes population $\mathscr{I}_{m_1}(t)$ in the C-ABC piecewise model (2.1).

The comparison between the outcomes of the proposed model, whether real-world or simulated, is illustrated in Figure 8. The left panel of the figure presents the recorded instances of infection cases within the Western Pacific region, while the corresponding data from the Mediterranean region is depicted on the right panel. These data points have been taken from [31]. It is worth noting that the scaling factor for infections is such that a value of 1 corresponds to 1000 cases. Furthermore, the temporal units are defined in such a way that 1 time unit equates to a span of 4 months.

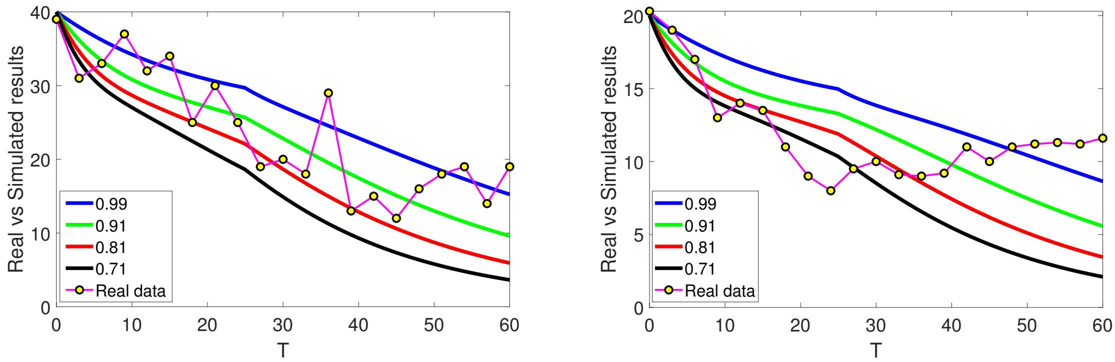

Figure 8. The comparison of the real vs simulated results of infected population of humans in C-ABC piece-wise model (2.1).

Upon close examination of the left panel, it becomes evident that a multitude of data points align precisely with a fractional order of 0.81 within the proposed model. A similar observation can be made in the right panel, where the data points conform to distinct fractional orders. This intriguing finding underscores the significance of employing piece-wise operators when analyzing biological models. The utilization of such operators allows for a more accurate representation of the intricate dynamics present within these systems, as evidenced by the varying fractional orders that align with the real data points. This emphasizes the model's capability to capture the nuanced behavior of the biological phenomena under investigation.

8. Conclusions

We have studied a malaria model using a novel approach that incorporates both singular and non-singular kernels through piecewise derivatives. By employing the Caputo derivative for the singular kernel and the Atangana-Baleanu operator for the non-singular kernel, we have thoroughly investigated the existence and uniqueness of solutions characterized by piecewise derivatives. To approximate the solutions, we used a piecewise Newton polynomial approach. This numerical scheme is specially designed to handle piecewise derivatives for singular and non-singular kernels. Further, the numerical simulations are performed considering various fractional orders. These simulations offer valuable insights into the dynamics of the malaria model, effectively taking into account crossover behaviors and the efficacy of the proposed approach. Furthermore, the results are thoroughly compared with real-world data, signifying their high effectiveness in analyzing disease dynamics.

For possible future work, one possibility could involve exploring the applications of this approach to different disease models, allowing for a broader understanding of disease dynamics and the development of adaptable strategies for various health challenges. Additionally, investigating the sensitivity of our results to variations in model parameters and exploring the impact of interventions or control measures could further enhance the practical implications of our research.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Availability of Data and Material

The data regarding this manuscript is available within the manuscript.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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