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Research article

On asymptotic fixed-time controller design for uncertain nonlinear systems with pure state constraints

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Abstract: This study investigates the problem of asymptotic fixed-time tracking control (AFXTTC) for uncertain nonlinear systems (UNS) subject to pure state constraints. To study this problem, we define asymptotic fixed-time stability (AFXTS) and thus a criterion for determining AFXTS. Dynamic surface control (DSC) is combined with fuzzy logic systems (FLSs) to construct a new adaptive fuzzy asymptotic fixed-time controller. A barrier Lyapunov function (BLF) is introduced to ensure that constraints on all states are satisfied. The proposed criterion is used to analyze the AFXTS of the system, and the effectiveness and superiority of the theoretical analysis results are verified through simulations.

Keywords: asymptotic fixed-time stability; uncertain nonlinear system; pure state constraints; adaptive fuzzy control; backstepping
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1. Introduction

The modeling of the vast majority of real engineering systems involves the use of various types of linear or nonlinear dynamic differential equations [1–10]. Throughout the modeling process, it is inevitable to encounter unknown parameters or functions. If these unknowns are ignored in controller design, control performance may be degraded, or even lead to system instability. Thankfully, many methods have been devised to handle unknowns. FLSs have become a significant method used to deal with uncertainties in the dynamics of nonlinear systems. Since the introduction of fuzzy sets by Zadeh [11], FLSs have been able to approximate any real continuous function on a tight set, which has been further demonstrated in research [12]. In particular, the combination of FLSs with adaptive backstepping techniques to construct adaptive fuzzy controllers has become a considerably effective control technique, and has been applied in a variety of different types of nonlinear systems [13–17].

Despite the aforementioned solutions, it is essential to note that only the infinite-time stability of the system is taken into account. However, in many engineering applications, it is unrealistic for the tracking error to converge to the origin or desired value in an infinite time. Consequently, finite-time techniques have been developed and applied to adaptive control of various types of dynamical systems [18–22]. Li's [23] contribution is notable as he established a new finite-time stability (FTS) criterion for finite-time asymptotic tracking control by introducing a scalar function. Unfortunately, the convergence time may be unacceptably long and the target performance of the system may be difficult to achieve if the initial conditions in finite-time control are too large. Moreover, the convergence time cannot be calculated in cases where the initial conditions are difficult to obtain or unknown.

In 2012, Polyakov [24] introduced fixed-time control as an effective solution to the tracking control problem. His work has been influential, as many scholars have applied fixed-time control to this problem and achieved significant results over the past few years [25–27]. Fixed-time techniques have been extensively employed in delay systems [28] and multi-intelligent systems [29,30]. Sun [31] solves the problem of tracking control of UNS affected by actuator saturation. However, unknown factors in actual modeling projects can make tracking errors only converge to an adjustable region rather than zero. With increasing demand for accuracy, achieving practical fixed-time stability (PFXTS) while asymptotically converging to zero is of considerable significance. In recent years, some relevant results [32, 33] have emerged. In [32], an event-triggered adaptive fuzzy asymptotic tracking control scheme with prescribed performance was developed for nonlinear pure feedback systems. In [33], an asymptotic predefined-time tracking controller was designed for high-speed aircraft with input quantization and faults. Although both of them can make the tracking error converge to a small neighborhood of zero within a finite time and ultimately converge asymptotically to zero, they are both achieved by combining some control techniques with traditional asymptotic tracking control (ATC) techniques, rather than directly analyzing the AFXTS of the system. It complicates the design process of the controller. Therefore, the main motivation of this paper is to construct an AFXTS determination criterion to simplify the controller design process.

In practical systems, the presence of various constraints is inevitable and requires careful consideration during controller design, as system performance can be affected. Over the past several years, numerous fixed-time control problems with state constraints have been addressed. Current research [34–37] frequently employs BLF within the framework of backstepping control design to develop controllers for constrained nonlinear systems, which approach infinity near a certain limit. Typically, state constraints in these problems are expressed as constants [38–40] or time-varying functions [41–44]. However, the representation of constraint bounds as a time-dependent and state-variant functional form has recently gained attention as a trending research topic. Pure state constraints, where the constraints on state variables depend directly on time and state variables of systems such as industrial robots, oscillators, and spacecraft, have been studied previously [45, 46]. Limited studies have been dedicated to fixed-time control issues under pure state constraints. In this study, the AFXTTC problem is addressed under pure state constraints, building on prior research.

Based on the aforementioned observations, the key contributions of the present study are as follows:

(1) Due to the presence of unknown nonlinear functions and disturbances, it is often challenging for the tracking error to converge to zero, rendering traditional fixed-time methods inapplicable. To address this issue, a new fixed-time stability (FXTS) criterion is proposed in Lemma 1, and a useful

tool for analyzing AFXTS is constructed in Lemma 2 on its basis.

(2) We introduce a first-order filter along with the backstepping method to construct a DSC framework to avoid the complexity explosion problem. Based on this, we have developed an adaptive fuzzy asymptotic fixed-time controller using BLF and FLSs. It not only can achieve AFXTTC, but also guarantees that the entire state is confined within a specified range.

(3) Different from [32, 33], this paper directly uses the AFXTS criterion to analyze the system's AFXTS. It not only simplifies the controller design process, but also ensures that the remaining signals of the closed-loop system are not only bounded but also PFXTS.

(4) Different from constant state constraints [38–40] and time-varying state constraints [41–44], the proposed control scheme in this paper can guarantee pure state constraints, which is more in line with the needs of some practical systems. Currently, there is relatively little research on this type of state constraint, especially for the tracking control problem of UNS.

Additional sections of this paper are organized as follows. Section 2 gives problem formulation and necessary preparation. In Section 3, we provide the design process for the controller. And the stability analysis is given in Section 4. The simulations are illustrated in Section 5. Finally, the conclusion is given in Section 6.

2. System description and preliminaries

2.1. Asymptotic fixed-time stability

Consider the following nonlinear system:

$$\dot{v} = f\left(t, v\right), v\left(0\right) = v_0, \tag{2.1}$$

where $v = [v_1, v_2, \ldots, v_n]^T \in \mathbb{R}^n$ is the system state, and f: $\mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function vector.
Definition 1. [24] The origin of system (2.1) is the FXTS if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon, 0) > 0$ such that for all $\|v_0\| < \delta$ and $t \geq 0$, the solution $\|v(t, v_0)\| < \varepsilon$, and $\exists\, T_p > 0$, $\forall\, v_0 \in \mathbb{R}^n$, $\|v\| = 0$ for all $t > T_p$.
Definition 2. [31] The origin of system (2.1) is the PFXTS if $\exists\, \Delta > 0$, $\exists\, T_p > 0$, $\forall\, v_0 \in \mathbb{R}^n$, $\|v\| < \Delta$ for all $t > T_p$.
Definition 3. The origin of system (2.1) is the AFXTS if $\exists\, \Delta > 0$, $\exists\, T_p > 0$, $\forall\, v_0 \in \mathbb{R}^n$, $\|v\| < \Delta$ for all $t > T_p$, and $\|v\| \to 0$ as $t \to \infty$.
Lemma 1. The origin of system (2.1) is the FXTS if there exists a continuous, continuous differentiable, and positive definitefunction $W : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that

$$z = e^{\mu t} v, \tag{2.2}$$

$$\dot{W}(z) \leq -e^{\mu t} r_1 W^m(z) - e^{\mu t} r_2 W^n(z), \tag{2.3}$$

where $r_1, r_2 > 0$, $0 < m < 1$, $n > 1$, $\mu > 0$. Moreover, the settling time can be given by the following equation

$$T \leq T_p := \frac{1}{\mu} \ln\left(\frac{\mu}{r_2\left(n-1\right)} + \frac{\mu}{r_1\left(1-m\right)} + 1\right).$$

Proof. See the Appendix. □

Lemma 2. The origin of system (2.1) is the AFXTS if there exists a continuous, continuous differentiable, and positive definite function $W : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that

$$z = e^{\mu t} v, \tag{2.4}$$

$$\dot{W}(z) \leq -e^{\mu t} r_1 W^m(z) - e^{\mu t} r_2 W^n(z) - e^{\mu t} r_3 W(z) + e^{\mu t} b, \tag{2.5}$$

where $r_1, r_2, r_3, b > 0$, $0 < m < 1$, $n > 1$, $\mu > 0$. Moreover, the settling time can be given by the following equation

$$T \leq T_p := \frac{1}{\mu} \ln\left(\frac{\mu}{r_1(1-m)} + \frac{\mu}{r_2(n-1)} + 1\right).$$

Proof. See the Appendix. □

Remark 1. Inspired by reference [47], the notion of AFXTS was introduced in the Definition 3, providing theoretical support for the design of AFXTTC schemes and stability analysis. The advantages of the AFXTTC scheme over some existing tracking control schemes are apparent. On one hand, compared to ATC schemes based on asymptotic stability, the AFXTTC scheme not only guarantees asymptotic convergence of tracking errors to zero but also ensures fast convergence of tracking errors to a tiny neighborhood of zero point. On the other hand, compared to practical fixed-time tracking control (PFXTTC) schemes based on PFXTS, the AFXTTC scheme, while ensuring convergence of tracking errors to a tiny neighborhood of zero point within a fixed time, also achieves asymptotic convergence of tracking errors to zero. The aforementioned advantages are clearly demonstrated in Figure 5 of the Section 5.

Remark 2. In recent years, FTS/FXTS problems have received much research attention. The study of such problems usually requires the assistance of various forms of Lemma 1 [19] or Lemma 1 [31], which requires a positive definite function W satisfying $\dot{W}(x) \leq -\alpha W(x) - \beta W^m(x) + b$ or $\dot{W}(x) \leq -\alpha W^m(x) - \beta W^n(x) + b$ where $\alpha, \beta > 0$, $0 < m < 1$, $n > 1$ and b is a normal number. Inspired by [23], we establish a new FXTS characterization criterion by introducing a scalar function. Based on this, we provide Lemma 2 as a direct means of analyzing the AFXTS property of a system, which distinguishes our findings from those of [32,33]. Furthermore, for ease of reading, we have provided a glossary of abbreviations of terms in Table 1.

Table 1. Glossary of abbreviations of terms.

Abbreviation	Abbreviation of term
uncertain nonlinear systems	UNS
dynamic surface control	DSC
barrier Lyapunov function	BLF
fuzzy logic systems	FLSs
finite-time stability	FTS
fixed-time stability	FXTS
practical fixed-time stability	PFXTS
asymptotic fixed-time stability	AFXTS
asymptotic tracking control	ATC
practical fixed-time tracking control	PFXTTC
asymptotic fixed-time tracking control	AFXTTC

2.2. Problem statements

Regard the following nonlinear strict feedback systems:

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(\bar{x}_i) + d_i(t), i = 1, 2, \ldots, n-1, \\ \dot{x}_n = u + f_n(\bar{x}_n) + d_n(t), \\ y = x_1, \end{cases} \tag{2.6}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in \mathbb{R}^i$ $(i = 1, 2, \ldots, n)$ is the state vector, $u \in \mathbb{R}$ is the control input, and $y \in \mathbb{R}$ is the system output. $f_i(\bar{x}_i)$ is an unknown but smooth nonlinear function. Furthermore, $d_i(t)$ is a bounded function such that $|d_i(t)| \le d_i^*$ with $d_i^* \in \mathbb{R}_+$, which represents an unknown external disturbance.

Then, the control objective of this paper is to conceive an AFXTTC scheme for the proposed system (2.6) such that it satisfies the following conditions:

O1: The tracking error $y - y_d$ must converge to a tiny neighborhood of zero point within a fixed time and asymptotically converge to zero, where y is the output of system (2.6) and y_d is the reference signal.

O2: All signals of closed-loop systems must remain bounded within a fixed time.

O3: Full state are constrained as $|x_i| < \Psi_i(\check{x}_{i-1}, t)$ with $\check{x}_1 = y_d$ and $\check{x}_{i-1} = [y_d, x_1, \ldots, x_{i-1}]^T \in \mathbb{R}^i$ $(i = 2, \ldots, n)$, where $\Psi_i(\check{x}_{i-1}, t) \in \mathbb{R}$ is a known time-varying function of state variables and time.

To accomplish these goals, some of the required assumptions are given below:

Assumption 1. [46] For $\forall t \ge 0$, $\frac{\partial \psi_i}{\partial y_d}, \frac{\partial \psi_i}{\partial x_1}, \ldots, \frac{\partial \psi_i}{\partial x_{i-1}}$ are exist and bounded, where ψ_i is a constraint on error variable, i.e., $\psi_i = \Psi_i - \omega_i$, where the definition of ω_i will be given at the beginning of Section 3.

Assumption 2. [39] y_d and $\dot{y}_d$ are bounded, smooth and known with $|y_d| < \Psi_1(y_d, t)$. In addition, $|x_i(0)| < \Psi_i(\check{x}_{i-1}(0), 0)$.

Remark 3. Based on our survey, there is a noticeable dearth of research on tracking control problems involving pure state constraints. Compared to general state constraints, pure state constraints are more practical, rendering tracking problems with pure state constraints more significant and worthy of investigation. Different from previous studies, such as [45], the present inquiry focuses on systems containing uncertain nonlinear functions, making the systems more general while also incorporating an unknown disturbance term to enhance the control system's reliability. Moreover, previous research studies [45, 46] focused on infinite time stability, whereas our study explores AFXTS problems. All these aforementioned discrepancies underscore the heightened significance of our present study, which are reflected in Table 2.

Table 2. Comparison with state-of-the-art issues.

	Types of State Constraint			Convergence Time			Convergence Accuracy	
	Constant	Time-varying	Pure State Constraint	Infintie - time	Finite-time	Fixed-time	Bound-edness	Zero
[35,36,38,40]	√	×	×	√	×	×	√	×
[39]	√	×	×	×	√	×	√	×
[9]	√	×	×	×	×	√	√	×
[42]	×	√	×	√	×	×	√	×
[41,43]	×	√	×	√	×	×	×	√
[44]	×	√	×	×	√	×	√	×
[26]	×	√	×	×	×	√	√	×
[45,46]	×	×	√	√	×	×	√	×
[23]	×	×	×	×	√	×	×	√
[32,33]	×	×	×	×	×	√	×	√
This paper	×	×	√	×	×	√	×	√

Remark 4. Similar to [46], it is necessary to use FLSs for handling the unknown terms that arise from taking derivatives of pure state constraint functions, which requires assuming Assumption 1 to ensure that these terms are confined to a compact set for approximation with FLSs. This is also the difficulty inherent in solving pure state constraint problems, and relaxing these assumption conditions represents one of our future research directions. Assumption 2 is indispensable in solving state constraint problems since if the initial value of the system state does not satisfy the constraint conditions, achieving the control objective is impossible.

2.3. Some useful lemmas

Then, we introduce some useful lemmas:

Lemma 3. [46] Consider the continuous function $\mathcal{H}(\chi)$, which is provided for the compact set Ω. Then, for $\forall\varepsilon>0$, there exists the FLS: $\mathcal{F}(\chi) = \Theta^T\Phi(\chi)$ such that $\sup_{\chi\in\Omega}\left|\mathcal{H}(\chi) - \Theta^T\Phi(\chi)\right| \le \varepsilon$, where $\chi = [\chi_1, \ldots, \chi_n]^T$ and $\mathcal{F}$ are the input and output of the FLS, $\Theta = [\Theta_1, \ldots, \Theta_q]^T$, $q \ge 1$ is the number of fuzzy rules, $\Phi = [\Phi_1, \ldots, \Phi_q]^T$, and $\Phi_l = \frac{\prod_{i=1}^{n}\mu_{F_i^l}(\chi_i)}{\sum_{l=1}^{q}\left[\prod_{i=1}^{n}\mu_{F_i^l}(\chi_i)\right]}$, where $\mu_{F_i^l}(\chi_i)$ is commonly selected as a Gaussian function.

Lemma 4. [26] For arbitrary $\tau > 0$ and $x \in \mathbb{R}$, an inequality holds for the following: $0 < |x| - x\tanh\left(\frac{x}{\tau}\right) \le 0.2785\tau$.

Lemma 5. [26] For arbitrary variable $a_k \in \mathbb{R}$, $k = 1, \ldots, m$, and a positive real number $\gamma > 0$, an inequality holds for the following:

$$\left(\sum_{k=1}^{m}|a_k|\right)^{\gamma} \le \max\left\{m^{\gamma-1}, 1\right\}\left(\sum_{k=1}^{m}|a_k|^{\gamma}\right).$$

Lemma 6. [26] For any real variables ζ_1, ζ_2, positive constants a_1, a_2, and a_3, there exists an inequality as follows:

$$|\zeta_1|^{a_1}|\zeta_2|^{a_2} \leq \frac{a_1 a_3 |\zeta_1|^{a_1+a_2}}{a_1+a_2} + \frac{a_2 a_3^{\frac{-a_1}{a_2}} |\zeta_2|^{a_1+a_2}}{a_1+a_2}.$$

Lemma 7. [26] Consider any variable of real numbers $\alpha \geq 0, \beta > 0$ and arbitrary real numbers $\gamma > 0$, the following inequality holds: $\alpha^\gamma(\beta - \alpha) \leq \frac{1}{1+\gamma}\left(\beta^{1+\gamma} - \alpha^{1+\gamma}\right)$.

Lemma 8. [26] For any real numbers $\rho > 0$, $\varrho \leq \rho$ and any constant $m > 1$, the following inequality holds: $(\rho - \varrho)^m \geq \varrho^m - \rho^m$.

Lemma 9. [45] For arbitrary $x, y \in \mathbb{R}$, $|x| < |y|$, an inequality holds for the following: $\frac{x^2}{y^2} < \ln\frac{y^2}{y^2-x^2} < \frac{x^2}{y^2-x^2}$.

3. Control design

In this section, the controller u will be synthesized through the implementation of both DSC and adaptive fuzzy control scheme techniques, while BLF methodology will be employed to ensure the constraint is consistently satisfied.

Define the following coordinate transformation:

$$v_1 = x_1 - \omega_1, \tag{3.1}$$

$$v_i = x_i - \omega_i, \tag{3.2}$$

$$s_i = \omega_i - \alpha_{i-1}, \tag{3.3}$$

where v_1 is the tracking error, $v_i(i = 2, 3, \ldots, n)$ is dynamic surface error, α_{i-1} is the virtual controller to be designed in step $i-1$, $\omega_1 = y_d$ and ω_i is the output of the following first-order command filter

$$\iota_i\dot{\omega}_i + \omega_i = \alpha_{i-1}, \quad 2 \leq i \leq n, \tag{3.4}$$

where $\iota_i > 0$ is a design constant and s_i is first order filter output error.

For achieving asymptotic tracking control, we perform the following error transformation

$$z_i = e^{\mu t} v_i, \tag{3.5}$$

where $\mu > 0$.

Step 1. From (3.1), (3.3) and (3.5), one has

$$\begin{aligned}\dot{z}_1 &= \mu e^{\mu t} v_1 + e^{\mu t}(\dot{x}_1 - \dot{y}_d)\\ &= e^{\mu t}(\mu v_1 + x_2 + f_1(x_1) + d_1(t) - \dot{y}_d).\end{aligned} \tag{3.6}$$

Then choose the Lyapunov function as

$$W_1 = \frac{1}{2}\ln\frac{\psi_1^2(y_d,t)}{\psi_1^2(y_d,t) - z_1^2} + \frac{1}{2}\tilde{\theta}_1^2, \tag{3.7}$$

where $\psi_1(y_d,t)=\Psi_1(y_d,t)-y_d$, $\tilde{\theta}_1 = \theta_1^* - \hat{\theta}_1$, θ_1^* is the norm of the unknown optimal parameters of FLSs and $\hat{\theta}_1$ is the estimate of θ_1^*.

Combining (3.6), the time derivative of W_1 is

$$\dot{W}_1 = \frac{e^{\mu t} z_1}{Q_1}\left[\mu v_1 + x_2 + f_1(x_1) + d_1(t) - \dot{y}_d - \frac{v_1}{\psi_1}\left(\frac{\partial \psi_1}{\partial y_d}\dot{y}_d + \frac{\partial \psi_1}{\partial t}\right)\right] - \tilde{\theta}_1\dot{\hat{\theta}}_1, \tag{3.8}$$

where $Q_1 = \psi_1^2 - z_1^2$.

Using Young's inequality, we get

$$\frac{e^{\mu t} z_1}{Q_1} d_1(t) \leq \frac{e^{\mu t} z_1^2}{2Q_1^2} + \frac{e^{\mu t}}{2} d_1^{*2}. \tag{3.9}$$

Then, it produces

$$\dot{W}_1 \leq \frac{e^{\mu t} z_1}{Q_1}\left[\mu v_1 + x_2 + f_1(x_1) - \dot{y}_d + \frac{z_1}{2Q_1} - \frac{v_1}{\psi_1}\left(\frac{\partial \psi_1}{\partial y_d}\dot{y}_d + \frac{\partial \psi_1}{\partial t}\right)\right]$$
$$- \tilde{\theta}_1\dot{\hat{\theta}}_1 + \frac{e^{\mu t}}{2} d_1^{*2}. \tag{3.10}$$

Let

$$\mathcal{H}_1(\chi_1) = \mu v_1 + f_1(x_1) - \dot{y}_d - \frac{v_1}{\psi_1}\frac{\partial \psi_1}{\partial y_d}\dot{y}_d + \frac{z_1}{2Q_1}. \tag{3.11}$$

According to Lemma 3, we have

$$\mathcal{H}_1(\chi_1) = \Theta_1^T \Phi_1(\chi_1) + \varepsilon_1, \tag{3.12}$$

where $\chi_1 = [\bar{x}_1, y_d, \dot{y}_d, \psi_1, \frac{\partial \psi_1}{\partial y_d}, \frac{1}{e^{\mu t}}]^T$.

Clearly we have

$$\Theta_1^T \Phi_1(\chi_1) \leq \theta_1^* \phi_1, \tag{3.13}$$

with $\theta_1^* = \left\|\Theta_1^T\right\|$, $\phi_1 = \|\Phi_1\|$.

Then substituting (3.11)–(3.13) into (3.10) yields

$$\dot{W}_1 \leq \frac{e^{\mu t} z_1}{Q_1}\left(\theta_1^*\phi_1 + \varepsilon_1 + v_2 + s_2 + \alpha_1 - \frac{v_1}{\psi_1}\frac{\partial \psi_1}{\partial t}\right) - \tilde{\theta}_1\dot{\hat{\theta}}_1 + \frac{e^{\mu t}}{2} d_1^{*2}. \tag{3.14}$$

The virtual control α_1 the parameter adaptive rules for θ_1^* is chosen as

$$\alpha_1 = -k_{11}\frac{z_1^{2p_1-1}}{Q_1^{p_1-1}} - k_{12}\frac{z_1^{2p_2-1}}{Q_1^{p_2-1}} - k_{13} z_1 - \hat{\theta}_1\phi_1$$
$$- B_1 \tanh\left(\frac{z_1 B_1}{\tau_1 Q_1}\right) - \frac{z_1}{2Q_1} + \frac{z_1}{\psi_1}\frac{\partial \psi_1}{\partial t}, \tag{3.15}$$

$$\dot{\hat{\theta}}_1 = -e^{\mu t}\sigma_{11}\hat{\theta}_1 - e^{\mu t}\sigma_{12}\hat{\theta}_1^{2p_2-1} + \frac{e^{\mu t} z_1 \phi_1}{Q_1}, \tag{3.16}$$

where k_{11}, k_{12}, k_{13}, B_1, τ_1, σ_{11}, σ_{12} are positive constants, $p_1 = \frac{2m-1}{2m+1}$, $p_2 = \frac{2m+1}{2m-1}$ with $m \geq 2$ is an integer.

Next, substituting (3.15) and (3.16) into (3.14) yields

$$\begin{aligned}\dot{W}_1 \leq& -e^{\mu t}k_{11}\left(\frac{z_1^2}{Q_1}\right)^{p_1} - e^{\mu t}k_{12}\left(\frac{z_1^2}{Q_1}\right)^{p_2} - e^{\mu t}k_{13}\frac{z_1^2}{Q_1} + e^{\mu t}\sigma_{11}\tilde{\theta}_1\hat{\theta}_1 + e^{\mu t}\sigma_{12}\tilde{\theta}_1\hat{\theta}_1^{2p_2-1}\\ &+ \frac{z_1 z_2}{Q_1} + \frac{e^{\mu t}z_1 s_2}{Q_1} + \frac{e^{\mu t}}{2}d_1^{*2} + e^{\mu t}\left[\left|\frac{z_1\varepsilon_1}{Q_1}\right| - \frac{z_1 B_1}{Q_1}\tanh\left(\frac{z_1 B_1}{Q_1\tau_1}\right)\right] - \frac{e^{\mu t}z_1^2}{2Q_1^2}. \end{aligned} \tag{3.17}$$

Utilizing Young's inequality and Lemma 4 results in

$$\begin{aligned}\left|\frac{z_1\varepsilon_1}{Q_1}\right| - \frac{z_1 B_1}{Q_1}\tanh\left(\frac{z_1 B_1}{Q_1\tau_1}\right) =& \left|\frac{z_1 B_1}{Q_1}\right| - \frac{z_1 B_1}{Q_1}\tanh\left(\frac{z_1 B_1}{Q_1\tau_1}\right) + \left|\frac{z_1\varepsilon_1}{Q_1}\right| - \left|\frac{z_1 B_1}{Q_1}\right|\\ \leq& 0.2785\tau_1 + \left|\frac{z_1 C_1}{Q_1}\right|\\ \leq& 0.2785\tau_1 + \frac{z_1^2}{2Q_1^2} + \frac{1}{2}C_1^2, \end{aligned} \tag{3.18}$$

where $C_1 = \max\{0, (\varepsilon_1 - B_1)\}$ is a positive number. If B_1 is selected to satisfy $B_1 \geq \varepsilon_1$, we have $C_1 = 0$.

Putting (3.18) into (3.17) yields

$$\begin{aligned}\dot{W}_1 \leq& -e^{\mu t}k_{11}\left(\frac{z_1^2}{Q_1}\right)^{p_1} - e^{\mu t}k_{12}\left(\frac{z_1^2}{Q_1}\right)^{p_2} - e^{\mu t}k_{13}\frac{z_1^2}{Q_1} + e^{\mu t}\sigma_{11}\tilde{\theta}_1\hat{\theta}_1 + e^{\mu t}\sigma_{12}\tilde{\theta}_1\hat{\theta}_1^{2p_2-1}\\ &+ \frac{z_1 z_2}{Q_1} + \frac{e^{\mu t}z_1 s_2}{Q_1} + \frac{e^{\mu t}}{2}d_1^{*2} + e^{\mu t}\left(0.2785\tau_1 + \frac{z_1^2}{2Q_1^2} + \frac{1}{2}C_1^2\right) - \frac{e^{\mu t}z_1^2}{2Q_1^2}\\ \leq& -e^{\mu t}k_{11}\left(\frac{z_1^2}{Q_1}\right)^{p_1} - e^{\mu t}k_{12}\left(\frac{z_1^2}{Q_1}\right)^{p_2} - e^{\mu t}k_{13}\frac{z_1^2}{Q_1} + e^{\mu t}\sigma_{11}\tilde{\theta}_1\hat{\theta}_1\\ &+ e^{\mu t}\sigma_{12}\tilde{\theta}_1\hat{\theta}_1^{2p_2-1} + \frac{z_1 z_2}{Q_1} + \frac{e^{\mu t}z_1 s_2}{Q_1} + e^{\mu t}D_1, \end{aligned} \tag{3.19}$$

where $D_1 = \frac{1}{2}d_1^{*2} + 0.2785\tau_1 + \frac{1}{2}C_1^2$.

Step i. From (3.2), (3.3), and (3.5), one has

$$\dot{z}_i = e^{\mu t}\left(\mu v_i + x_{i+1} + f_i(\bar{x}_i) + d_i(t) - \dot{\omega}_i\right). \tag{3.20}$$

Then choose the Lyapunov function as

$$W_i = W_{i-1} + \frac{1}{2}\ln\frac{\psi_i^2(\check{x}_{i-1}, t)}{\psi_i^2(\check{x}_{i-1}, t) - z_i^2} + \frac{1}{2}\tilde{\theta}_i^2, \tag{3.21}$$

where $\psi_i(\check{x}_{i-1}, t) = \Psi_i(\check{x}_{i-1}, t) - \omega_i$, $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$, θ_i^* is the norm of the unknown optimal parameters of FLSs and $\hat{\theta}_i$ is the estimate of θ_i^*.

The time derivative of W_i is

$$\dot{W}_i = \dot{W}_{i-1} + \frac{e^{\mu t} z_i}{Q_i}\Bigg[\mu v_i + x_{i+1} + f_i(\bar{x}_i) + d_i(t) - \dot{\omega}_i - \frac{v_i}{\psi_i}\Big(\frac{\partial \psi_i}{\partial y_d}\dot{y}_d + \sum_{j=1}^{i-1}\frac{\partial \psi_i}{\partial x_j}\Big(x_{j+1} + f_j(\bar{x}_j) + d_j(t)\Big)\Big)\Bigg] - \tilde{\theta}_i \dot{\hat{\theta}}_i, \tag{3.22}$$

where $Q_i = \psi_i^2 - z_i^2$.

Utilizing Young's inequality, we have

$$\frac{e^{\mu t} z_i}{Q_i}\left(d_i(t) - \sum_{j=1}^{i-1}\frac{v_i}{\psi_i}\frac{\partial \psi_i}{\partial x_j} d_j(t)\right) \le \frac{e^{\mu t}}{2}\left(\frac{z_i^2}{Q_i^2} + \sum_{j=1}^{i-1}\left(\frac{z_i v_i}{Q_i \psi_i}\frac{\partial \psi_i}{\partial x_j}\right)^2\right) + \sum_{j=1}^{i}\frac{e^{\mu t}}{2} d_j^{*2}. \tag{3.23}$$

Then, one has

$$\dot{W}_i \le \dot{W}_{i-1} + \frac{e^{\mu t} z_i}{Q_i}\Bigg[\mu v_i + x_{i+1} + f_i(\bar{x}_i) - \dot{\omega}_i + \frac{z_i}{2Q_i} - \frac{v_i}{\psi_i}\Big(\frac{\partial \psi_i}{\partial y_d}\dot{y}_d + \sum_{j=1}^{i-1}\frac{\partial \psi_i}{\partial x_j}\Big(x_{j+1} + f_j\left(\bar{x}_j\right)\Big) + \frac{\partial \psi_i}{\partial t}\Big) + \sum_{j=1}^{i-1}\frac{z_i}{2Q_i}\left(\frac{v_i}{\psi_i}\frac{\partial \psi_i}{\partial x_j}\right)^2\Bigg] + \sum_{j=1}^{i}\frac{e^{\mu t}}{2} d_j^{*2} - \tilde{\theta}_i \dot{\hat{\theta}}_i. \tag{3.24}$$

Let

$$\mathcal{H}_i(\chi_i) = \mu v_i + f_i(\bar{x}_i) - \dot{\omega}_i - \frac{v_i}{\psi_i}\left(\frac{\partial \psi_i}{\partial y_d}\dot{y}_d + \frac{\partial \psi_i}{\partial t} + \sum_{j=1}^{i-1}\frac{\partial \psi_i}{\partial x_j}(x_{j+1} + f_j(\bar{x}_j))\right) + \frac{Q_i z_{i-1} s_i}{z_i Q_{i-1}} + \frac{Q_i v_{i-1}}{Q_{i-1}} + \sum_{j=1}^{i-1}\frac{z_i}{2Q_i}\left(\frac{v_i}{\psi_i}\frac{\partial \psi_i}{\partial x_j}\right)^2 + \frac{z_i}{2Q_i}. \tag{3.25}$$

Combining Lemma 3, one has

$$\mathcal{H}_i(\chi_i) = \Theta_i^T \Phi_i(\chi_i) + \varepsilon_i, \tag{3.26}$$

where $\chi_i = \left[\bar{x}_i, \dot{y}_d, \omega_i, \alpha_{i-1}, \psi_i, \frac{\partial \psi_i}{\partial y_d}, \frac{\partial \psi_i}{\partial x_1}, \ldots, \frac{\partial \psi_i}{\partial x_{i-1}}, \psi_{i-1}, \omega_{i-1}, \frac{1}{e^{\mu t}}\right]^T$.

Obviously, we have

$$\Theta_i^T \Phi_i(\chi_i) \le \theta_i^* \phi_i, \tag{3.27}$$

with $\theta_i^* = \left\|\Theta_i^T\right\|$, $\phi_i = \|\Phi_i\|$.

From (3.25)–(3.27), we can obtain that

$$\dot{W}_i \le \dot{W}_{i-1} + \frac{e^{\mu t} z_i}{Q_i}\Big(\theta_i^* \phi_i + \varepsilon_i + v_{i+1} + s_{i+1} + \alpha_i - \frac{v_i}{\psi_i}\frac{\partial \psi_i}{\partial t} - \frac{Q_i z_{i-1} s_i}{z_i Q_{i-1}} - \frac{Q_i v_{i-1}}{Q_{i-1}}\Big) - \tilde{\theta}_i \dot{\hat{\theta}}_i + \sum_{j=1}^{i}\frac{e^{\mu t}}{2} d_j^{*2}. \tag{3.28}$$

The virtual control α_i the parameter adaptive rules for θ_i^* is chosen as

$$\begin{aligned}\alpha_i =& -k_{i1}\frac{z_i^{2p_1-1}}{Q_i^{p_1-1}} - k_{i2}\frac{z_i^{2p_2-1}}{Q_i^{p_2-1}} - k_{i3}z_i - \hat{\theta}_i\phi_i \\ & - B_i\tanh\left(\frac{z_iB_i}{\tau_iQ_i}\right) - \frac{z_i}{2Q_i} + \frac{z_i}{\psi_i}\frac{\partial\psi_i}{\partial t},\end{aligned} \tag{3.29}$$

$$\dot{\hat{\theta}}_i = -e^{\mu t}\sigma_{i1}\hat{\theta}_i - e^{\mu t}\sigma_{i2}\hat{\theta}_i^{2p_2-1} + \frac{e^{\mu t}z_i\phi_i}{Q_i}, \tag{3.30}$$

where k_{i1}, k_{i2}, k_{i3}, B_i, τ_i, σ_{i1}, σ_{i2} are positive constants.

Then substituting (3.29) and (3.30) into (3.28) yields

$$\begin{aligned}\dot{W}_i \leq& \dot{W}_{i-1} - e^{\mu t}k_{i1}\left(\frac{z_i^2}{Q_i}\right)^{p_1} - e^{\mu t}k_{i2}\left(\frac{z_i^2}{Q_i}\right)^{p_2} - e^{\mu t}k_{i3}\frac{z_i^2}{Q_i} + e^{\mu t}\sigma_{i1}\tilde{\theta}_i\hat{\theta}_i \\ & + e^{\mu t}\sigma_{i2}\tilde{\theta}_i\hat{\theta}_i^{2p_2-1} + \frac{z_iz_{i+1}}{Q_i} + \frac{e^{\mu t}z_is_{i+1}}{Q_i} - \frac{z_{i-1}z_i}{Q_{i-1}} - \frac{e^{\mu t}z_{i-1}s_i}{Q_{i-1}} \\ & + \sum_{j=1}^{i}\frac{e^{\mu t}}{2}d_j^{*2} - \frac{e^{\mu t}z_i^2}{2Q_i^2} + e^{\mu t}\left[\left|\frac{z_i\varepsilon_i}{Q_i}\right| - \frac{z_iB_i}{Q_i}\tanh\left(\frac{z_iB_i}{Q_i\tau_i}\right)\right].\end{aligned} \tag{3.31}$$

Similar to Step 1, utilizing Young's inequality and Lemma 4 results in

$$\left|\frac{z_i\varepsilon_i}{Q_i}\right| - \frac{z_iB_i}{Q_i}\tanh\left(\frac{z_iB_i}{Q_i\tau_i}\right) \leq 0.2785\tau_i + \frac{z_i^2}{2Q_i^2} + \frac{1}{2}C_i^2, \tag{3.32}$$

where $C_i = \max\{0, (\varepsilon_i - B_i)\}$ is a positive number. If B_i is selected to satisfy $B_i \geq \varepsilon_i$, we have $C_i = 0$.

Putting (3.32) into (3.31) obtains

$$\begin{aligned}\dot{W}_i \leq& -e^{\mu t}\sum_{j=1}^{i}k_{j1}\left(\frac{z_j^2}{Q_j}\right)^{p_1} - e^{\mu t}\sum_{j=1}^{i}k_{j2}\left(\frac{z_j^2}{Q_j}\right)^{p_2} - e^{\mu t}\sum_{j=1}^{i}k_{j3}\frac{z_j^2}{Q_j} \\ & + e^{\mu t}\sum_{j=1}^{i}\sigma_{j2}\tilde{\theta}_j\hat{\theta}_j^{2p_2-1} + e^{\mu t}\sum_{j=1}^{i}\sigma_{j1}\tilde{\theta}_j\hat{\theta}_j + \frac{z_iz_{i+1}}{Q_i} + \frac{e^{\mu t}z_is_{i+1}}{Q_i} + e^{\mu t}D_i,\end{aligned} \tag{3.33}$$

where $D_i = \frac{1}{2}\sum_{j=1}^{i}(i-j+1)d_j^{*2} + 0.2785\sum_{j=1}^{i}\tau_j + \frac{1}{2}\sum_{j=1}^{i}C_j^2$.

Step n. Similar to Step i, z_n is given by

$$\dot{z}_n = e^{\mu t}\left(\mu v_i + u + f_n(\bar{x}_n) + d_n(t) - \dot{\omega}_n\right). \tag{3.34}$$

Define

$$W_n = W_{n-1} + \frac{1}{2}\ln\frac{\psi_n^2(\check{x}_{n-1},t)}{\psi_n^2(\check{x}_{n-1},t) - z_n^2} + \frac{1}{2}\tilde{\theta}_n^2, \tag{3.35}$$

where $\psi_n(\check{x}_{n-1},t)=\Psi_n(\check{x}_{n-1},t)-\omega_n$, $\tilde{\theta}_n = \theta_n^* - \hat{\theta}_n$, θ_n^* is the norm of the unknown optimal parameters of FLSs and $\hat{\theta}_n$ is the estimate of θ_n^*.

The time derivative of W_n is

$$\begin{aligned}\dot{W}_n =&\dot{W}_{n-1} + \frac{e^{\mu t} z_n}{Q_n}\Big[\mu v_n + u + f_n(\bar{x}_n) + d_n(t) - \dot{\omega}_n - \frac{v_n}{\psi_n}\Big(\frac{\partial \psi_n}{\partial y_d}\dot{y}_d \\ &+ \sum_{j=1}^{n-1} \frac{\partial \psi_n}{\partial x_j}\Big(x_{j+1} + f_j(\bar{x}_j) + d_j(t)\Big) + \frac{\partial \psi_n}{\partial t}\Big)\Big] - \tilde{\theta}_n \dot{\hat{\theta}}_n,\end{aligned} \tag{3.36}$$

where $Q_n = \psi_n^2 - z_n^2$.

Using Young's inequality, one has

$$\frac{e^{\mu t} z_n}{Q_n}\left(d_n(t) - \sum_{j=1}^{n-1} \frac{v_n}{\psi_n}\frac{\partial \psi_n}{\partial x_j} d_j(t)\right) \leq \frac{e^{\mu t}}{2}\left(\frac{z_n^2}{Q_n^2} + \sum_{j=1}^{n-1}\left(\frac{z_n v_n}{Q_n \psi_n}\frac{\partial \psi_n}{\partial x_j}\right)^2\right) + \sum_{j=1}^{n} \frac{e^{\mu t}}{2} d_j^{*2}. \tag{3.37}$$

Then, we have

$$\begin{aligned}\dot{W}_n \leq&\dot{W}_{n-1} + \frac{e^{\mu t} z_n}{Q_n}\Big[\mu v_n + u + f_n(\bar{x}_n) - \dot{\omega}_n + \frac{z_n}{2Q_n} - \frac{v_n}{\psi_n}\Big(\frac{\partial \psi_n}{\partial y_d}\dot{y}_d + \sum_{j=1}^{n-1} \frac{\partial \psi_n}{\partial x_j}\Big(x_{j+1} \\ &+ f_j\left(\bar{x}_j\right)\Big) + \frac{\partial \psi_n}{\partial t}\Big) + \sum_{j=1}^{n-1} \frac{z_n}{2Q_n}\left(\frac{v_n}{\psi_n}\frac{\partial \psi_n}{\partial x_j}\right)^2\Big] + \sum_{j=1}^{n} \frac{e^{\mu t}}{2} d_j^{*2} - \tilde{\theta}_n \dot{\hat{\theta}}_n.\end{aligned} \tag{3.38}$$

Let

$$\begin{aligned}\mathcal{H}_n(\chi_n) =&\mu v_n + f_n(\bar{x}_n) - \dot{\omega}_n - \frac{v_n}{\psi_n}\Big(\frac{\partial \psi_n}{\partial y_d}\dot{y}_d + \frac{\partial \psi_n}{\partial t} + \sum_{j=1}^{n-1} \frac{\partial \psi_n}{\partial x_j}\Big(x_{j+1} + f_j(\bar{x}_j)\Big)\Big) \\ &+ \frac{Q_n z_{n-1} s_n}{z_n Q_{n-1}} + \frac{Q_n v_{n-1}}{Q_{n-1}} + \sum_{j=1}^{n-1} \frac{z_n}{2Q_n}\left(\frac{v_n}{\psi_n}\frac{\partial \psi_n}{\partial x_j}\right)^2 + \frac{z_n}{2Q_n}.\end{aligned} \tag{3.39}$$

Based on Lemma 3, one has

$$\mathcal{H}_n(\chi_n) = \Theta_n^T \Phi_n(\chi_n) + \varepsilon_n, \tag{3.40}$$

where $\chi_n = \left[\bar{x}_n, \dot{y}_d, \omega_n, \alpha_{n-1}, \psi_n, \frac{\partial \psi_n}{\partial y_d}, \frac{\partial \psi_n}{\partial x_1}, \ldots, \frac{\partial \psi_n}{\partial x_{n-1}}, \psi_{n-1}, \omega_{n-1}, \frac{1}{e^{\mu t}}\right]^T$.

Clearly we have

$$\Theta_n^T \Phi_n(\chi_n) \leq \theta_n^* \phi_n, \tag{3.41}$$

with $\theta_n^* = \left\|\Theta_n^T\right\|$, $\phi_n = \|\Phi_n\|$.

By incorporating (3.39)–(3.41) into (3.38), we have

$$\begin{aligned}\dot{W}_n \leq&\dot{W}_{n-1} + \frac{e^{\mu t} z_n}{Q_n}\left(\theta_n^* \phi_n + \varepsilon_n + u - \frac{v_n}{\psi_n}\frac{\partial \psi_n}{\partial t} - \frac{Q_n z_{n-1} s_n}{z_n Q_{n-1}} - \frac{Q_n v_{n-1}}{Q_{n-1}}\right) \\ &- \tilde{\theta}_n \dot{\hat{\theta}}_n + \sum_{j=1}^{n} \frac{e^{\mu t}}{2} d_j^{*2}.\end{aligned} \tag{3.42}$$

The virtual control α_n the parameter adaptive rules for θ_n^* is chosen as

$$u = -k_{n1}\frac{z_n^{2p_1-1}}{Q_n^{p_1-1}} - k_{n2}\frac{z_n^{2p_2-1}}{Q_n^{p_2-1}} - k_{n3}z_n - \hat{\theta}_n\phi_n - B_n\tanh\left(\frac{z_nB_n}{\tau_nQ_n}\right) - \frac{z_n}{2Q_n} + \frac{z_n}{\psi_n}\frac{\partial\psi_n}{\partial t}, \tag{3.43}$$

$$\dot{\hat{\theta}}_n = -e^{\mu t}\sigma_{n1}\hat{\theta}_n - e^{\mu t}\sigma_{n2}\hat{\theta}_n^{2p_2-1} + \frac{e^{\mu t}z_n\phi_n}{Q_n}, \tag{3.44}$$

where k_{n1}, k_{n2}, k_{n3}, B_n, τ_n, σ_{n1}, σ_{n2} are positive constants.

Substituting (3.43) and (3.44) into (3.42) yields

$$\begin{aligned}\dot{W}_n \leq& \dot{W}_{n-1} - e^{\mu t}k_{n1}\left(\frac{z_n^2}{Q_n}\right)^{p_1} - e^{\mu t}k_{n2}\left(\frac{z_n^2}{Q_n}\right)^{p_2} - e^{\mu t}k_{n3}\frac{z_n^2}{Q_n} + e^{\mu t}\sigma_{n1}\tilde{\theta}_n\hat{\theta}_n \\ &+ e^{\mu t}\sigma_{n2}\tilde{\theta}_n\hat{\theta}_n^{2p_2-1} - \frac{z_{n-1}z_n}{Q_{n-1}} - \frac{e^{\mu t}z_{n-1}s_n}{Q_{n-1}} + \sum_{j=1}^{n}\frac{e^{\mu t}}{2}d_j^{*2} - \frac{e^{\mu t}z_n^2}{2Q_n^2} \\ &+ e^{\mu t}\left[\left|\frac{z_n\varepsilon_n}{Q_n}\right| - \frac{z_nB_n}{Q_n}\tanh\left(\frac{z_nB_n}{Q_n\tau_n}\right)\right].\end{aligned} \tag{3.45}$$

Similar to Step 1, according to Young's inequality and Lemma 4

$$\left|\frac{z_n\varepsilon_n}{Q_n}\right| - \frac{z_nB_n}{Q_n}\tanh\left(\frac{z_nB_n}{Q_n\tau_n}\right) \leq 0.2785\tau_n + \frac{z_n^2}{2Q_n^2} + \frac{1}{2}C_n^2, \tag{3.46}$$

where $C_n = \max\{0, (\varepsilon_n - B_n)\}$ is a positive number. If B_n is selected to satisfy $B_n \geq \varepsilon_n$, we have $C_n = 0$.

Substituting (3.46) into (3.45) yields

$$\begin{aligned}\dot{W}_n \leq& -e^{\mu t}\sum_{j=1}^{n}k_{j1}\left(\frac{z_j^2}{Q_j}\right)^{p_1} - e^{\mu t}\sum_{j=1}^{n}k_{j2}\left(\frac{z_j^2}{Q_j}\right)^{p_2} - e^{\mu t}\sum_{j=1}^{n}k_{j3}\frac{z_j^2}{Q_j} \\ &+ e^{\mu t}\sum_{j=1}^{n}\sigma_{j1}\tilde{\theta}_j\hat{\theta}_j + e^{\mu t}\sum_{j=1}^{n}\sigma_{j2}\tilde{\theta}_j\hat{\theta}_j^{2p_2-1} + e^{\mu t}D_n,\end{aligned} \tag{3.47}$$

where $D_n = \frac{1}{2}\sum_{j=1}^{n}(n-j+1)\,d_j^{*2} + 0.2785\sum_{j=1}^{n}\tau_j + \frac{1}{2}\sum_{j=1}^{n}C_j^2$.

To date, we have formulated the virtual controller (3.15), (3.29), the controller (3.43), and the adaptive rule (3.16), (3.30), and (3.44). The adaptive control scheme flow chart, as depicted in Figure 1, outlines the proposed methodology. In the subsequent section, we shall conduct an analysis of system stability to substantiate the theoretical capability of our control scheme to attain the desired control objective.

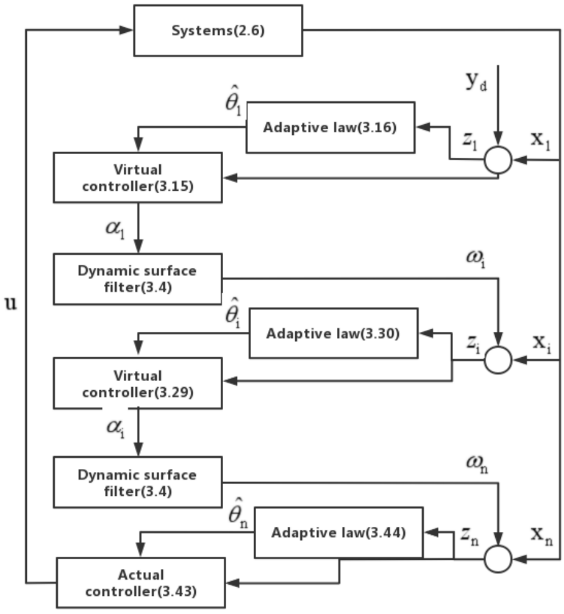

Figure 1. Flow chart.

4. Asymptotic fixed-time stability analysis

Then, the main results of this study will be summarized.

Theorem 1. For the nonlinear strict feedback system (2.6), if it is achievable that Assumptions 1 and 2, by devising an adaptive fuzzy controller (3.43), virtual controllers (3.15), (3.29), and adaptive laws (3.16), (3.30), and (3.44), it is ensured that all signals of the closed-loop system are PFXTS, and the tracking error $y - y_d$, is capable of converging to a small neighborhood of zero within a fixed time, and ultimately asymptotically converging to zero. At the same time, all of the state variables never violate their constraints.

Proof. Let

$$W = W_n = \sum_{i=1}^{n} \frac{1}{2} \ln \frac{\psi_i^2(\check{x}_{i-1}, t)}{\psi_i^2(\check{x}_{i-1}, t) - z_i^2} + \sum_{i=1}^{n} \frac{1}{2} \tilde{\theta}_i^2.$$

By combining Lemma 5, we can obtain that

$$\begin{aligned} \dot{W} \leq & - e^{\mu t} k_1 \left(\sum_{i=1}^{n} \frac{z_i^2}{Q_i} \right)^{p_1} - e^{\mu t} k_2 n^{1-p_2} \left(\sum_{i=1}^{n} \frac{z_i^2}{Q_i} \right)^{p_2} - e^{\mu t} k_3 \sum_{i=1}^{n} \frac{z_i^2}{Q_i} \\ & + e^{\mu t} \sum_{i=1}^{n} \sigma_{i1} \tilde{\theta}_i \hat{\theta}_i + e^{\mu t} \sum_{i=1}^{n} \sigma_{i2} \tilde{\theta}_i \hat{\theta}_i^{2p_2 - 1} + e^{\mu t} D_n, \end{aligned} \tag{4.1}$$

where $k_1 = \min\{k_{11}, \ldots, k_{n1}\}$, $k_2 = \min\{k_{12}, \ldots, k_{n2}\}$, $k_3 = \min\{k_{13}, \ldots, k_{n3}\}$.

Base on $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$, we have

$$\sigma_{i1} \tilde{\theta}_i \hat{\theta}_i \leq -\frac{\sigma_{i1}}{4} \tilde{\theta}_i^2 - \frac{\sigma_{i1}}{4} \tilde{\theta}_i^2 + \frac{\sigma_{i1}}{2} \theta_i^{*2}. \tag{4.2}$$

According to Lemma 6, let $\zeta_1 = \frac{\sigma_{i1}}{4}\tilde{\theta}_i^2$, $\zeta_2 = 1$, $a_1 = p_1$, $a_2 = 1 - p_1$, $a_3 = p_1^{-1}$, we get

$$(\frac{\sigma_{i1}}{4}\tilde{\theta}_i^2)^{p_1} \leq (1-p_1)p_1^{\frac{p_1}{1-p_1}} + \frac{\sigma_{i1}}{4}\tilde{\theta}_i^2. \tag{4.3}$$

From (4.2) and (4.3), we can get

$$\sum_{i=1}^{n}\sigma_{i1}\tilde{\theta}_i\hat{\theta}_i \leq -\sum_{i=1}^{n}\left(\frac{\sigma_{i1}}{4}\tilde{\theta}_i^2\right)^{p_1} - \sum_{i=1}^{n}\frac{\sigma_{i1}}{4}\tilde{\theta}_i^2 + \sum_{i=1}^{n}\frac{\sigma_{i1}}{2}\theta_i^{*2} + n(1-p_1)p_1^{\frac{p_1}{1-p_1}}. \tag{4.4}$$

Then, utilizing Lemma 7, we have

$$\begin{aligned}\tilde{\theta}_i\hat{\theta}_i^{2p_2-1} &= \left(\theta_i^* - \hat{\theta}_i\right)\hat{\theta}_i^{2p_2-1} \leq \frac{1}{2p_2}\left(\theta_i^{*2p_2} - \hat{\theta}_i^{2p_2}\right) \\ &= \frac{1}{2p_2}\theta_i^{*2p_2} - \frac{1}{2p_2}\left(\theta_i^* - \tilde{\theta}_i\right)^{2p_2}.\end{aligned} \tag{4.5}$$

And applying Lemma 8 to (4.5), one has

$$\begin{aligned}\tilde{\theta}_i\hat{\theta}_i^{2p_2-1} &\leq \frac{1}{2p_2}\theta_i^{*2p_2} - \frac{1}{2p_2}\left(\tilde{\theta}_i^{2p_2} - \theta_i^{*2p_2}\right) \\ &= \frac{1}{p_2}\theta_i^{*2p_2} - \frac{1}{2p_2}\tilde{\theta}_i^{2p_2}.\end{aligned} \tag{4.6}$$

Putting (4.4) and (4.6) in (4.1), we have

$$\begin{aligned}\dot{W} \leq &- e^{\mu t}k_1\left(\sum_{i=1}^{n}\frac{z_i^2}{Q_i}\right)^{p_1} - e^{\mu t}k_2 n^{1-p_2}\left(\sum_{i=1}^{n}\frac{z_i^2}{Q_i}\right)^{p_2} - e^{\mu t}k_3\sum_{i=1}^{n}\frac{z_i^2}{Q_i} \\ &- e^{\mu t}\sigma_1\sum_{i=1}^{n}\left(\frac{1}{2}\tilde{\theta}_i^2\right)^{p_1} - e^{\mu t}\sigma_2\sum_{i=1}^{n}\left(\frac{1}{2}\tilde{\theta}_i^2\right)^{p_2} - e^{\mu t}\sigma_3\sum_{i=1}^{n}\frac{1}{2}\tilde{\theta}_i^2 + e^{\mu t}D,\end{aligned} \tag{4.7}$$

where $\sigma_1 = \min\{\left(\frac{\sigma_{11}}{2}\right)^{p_1}, \ldots, \left(\frac{\sigma_{n1}}{2}\right)^{p_1}\}$, $\sigma_2 = \min\{\frac{\sigma_{12}}{2^{1-p_2}p_2}, \ldots, \frac{\sigma_{n2}}{2^{1-p_2}p_2}\}$, $\sigma_3 = \min\{\frac{\sigma_{11}}{2}, \ldots, \frac{\sigma_{n1}}{2}\}$, $D = D_n + \sum_{i=1}^{n}\frac{\sigma_{i1}}{2}\theta_i^{*2} + n(1-p_1)p_1^{\frac{p_1}{1-p_1}} + \sum_{i=1}^{n}\frac{\sigma_{i2}}{p_2}\theta_i^{*2p_2}$.

According to Lemma 5, we get

$$\begin{aligned}\dot{W} \leq &- e^{\mu t}r_1\left(\sum_{i=1}^{n}\frac{z_i^2}{Q_i} + \frac{\tilde{\theta}_i^2}{2}\right)^{p_1} - e^{\mu t}r_2\left(\sum_{i=1}^{n}\frac{z_i^2}{Q_i} + \frac{\tilde{\theta}_i^2}{2}\right)^{p_2} \\ &- e^{\mu t}r_3\left(\sum_{i=1}^{n}\frac{z_i^2}{Q_i} + \frac{1}{2}\tilde{\theta}_i^2\right) + e^{\mu t}D,\end{aligned} \tag{4.8}$$

where $r_1 = \min\{k_1, \sigma_1\}$, $r_2 = \min\{k_2 n^{1-p_2}, \sigma_2 n^{1-p_2}\}$, $r_3 = \min\{k_3, \sigma_3\}$.

Using Lemma 9, we have

$$\dot{W} \leq -r_1 e^{\mu t}W^{p_1} - r_2 e^{\mu t}W^{p_2} - r_3 e^{\mu t}W + e^{\mu t}D. \tag{4.9}$$

By Lemma 2, W is PFXTS, so $\ln \frac{\psi_i^2}{\psi_i^2 - z_i^2}$, $\tilde{\theta}_i$ are PFXTS. Furthermore, it can be inferred that $|x_i| < \Psi_i(\check{x}_{i-1}, t)$. Meanwhilewe, we can know $\hat{\theta}_i$ is PFXTS due to the PFXTS of $\tilde{\theta}_i$. And the tracking error $x_1 - y_d$ is AFXTS with the fixed time

$$T \le T_p := \frac{1}{\mu} \ln\left(\frac{\mu}{r_1(1-p_1)} + \frac{\mu}{r_2(p_2-1)} + 1\right).$$

□

Remark 5. In the present study, the fuzzy adaptive controller constructed by constructing a Lyapunov function analysis on z can make z bounded. Since $z = e^{\mu t} v$, that is, $v = e^{-\mu t} z$, v clearly converges to zero when $t \to \infty$, which explains why the introduction of the scalar function $e^{\mu t}$ in Lemma 2 enables the follow-up error to narrow to zero. Such findings are also reflected in Figure 4(a) in the simulation.

Remark 6. Prior researches [18–22, 25–31] , have addressed the issue of finite/fixed time tracking, but only an adjustable region could be reached. However, this studys' finding indicate the eventual convergence of the tracking error to zero, which is more aligned with our increasing demand for precision.

Remark 7. Previously conducted researches [48–50] have demonstrated that the tracking error in asymptotic tracking problems can eventually converge to zero. However, it fails to provide assurance for the convergence of the error to a bound within a finite period, rendering it unsuitable for certain practical applications. As such, the finite/fixed time theory presents a crucial approach to addressing this issue.

5. Simulation results

We present in this section the simulation of a single-linked robot arm consisting of rigid links. Its dynamic equations are

$$J\ddot{\varrho} = -E\dot{\varrho} - \mathrm{MgL}\sin(\varrho) + u, \tag{5.1}$$

where $\ddot{\varrho}$, $\dot{\varrho}$, and ϱ represent the link angular acceleration, velocity, and displacement, respectively. Meanwhile, ϱ is the system output, u is the control input, M and L are the mass and length of the link, g is the acceleration of gravity, E is the constant of the damping and J is the moment of inertia.

Define $x_1 = \varrho$, $x_2 = \dot{\varrho}$, and select a disturbance as $d_1(t) = -0.5\sin(t)$, $d_2(t) = 0.2\cos(0.5t)$, we can establish (63) as

$$\begin{aligned} \dot{x}_1 &= x_2 - 0.5\sin(t), \\ \dot{x}_2 &= -\frac{E}{J}x_2 - \frac{MgL}{J}\sin(x_1) + u + 0.2\cos(0.5t), \end{aligned} \tag{5.2}$$

where $M = 1kg$, $L = 1m$, $g = 10m/s^2$, $E = 2N \cdot m \cdot s$, $J = 1kg \cdot m^2$. The states are constrained by $|x_1| < \Psi_1(y_d, t) = e^{-0.1y_d} + e^{-t} + 0.3$, $|x_2| < \Psi_2(y_d, x_1, t) = 0.2\sin(0.5t) + e^{-0.5x_1^2} + 0.2\cos(0.5y_d) + 0.5$. The reference singal is defined as $y_d = 0.3(\cos(-0.4t) + \sin(0.5t))$ and the system output $y = x_1$ is anticipated to be consistent with the reference singal y_d will be depicted by Figure 2.

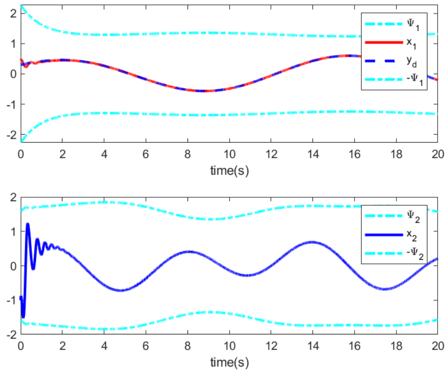

Figure 2. The trajectories of x_1 and x_2.

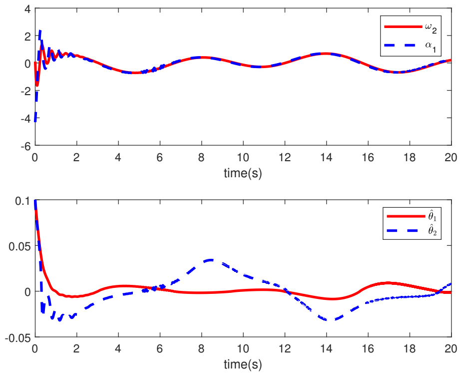

Figure 3. The trajectories of ω_2, α_1, $\hat{\theta}_1$ and $\hat{\theta}_2$.

Then, choose the fuzzy membership functions as

$$\begin{aligned}
\mu_{F_1^\iota} =& e^{\frac{-(x_1+0.25\iota)^2}{8}} \cdot e^{\frac{-(y_d+0.25\iota)^2}{8}} \cdot e^{\frac{-(dy_d+0.25\iota)^2}{8}} \cdot e^{\frac{-(\psi_1+0.25\iota)^2}{8}} \\
& \cdot e^{\frac{-(d\psi_1/dy_d+0.25\iota)^2}{8}} \cdot e^{\frac{-\left(\delta^{-1}+0.25\iota\right)^2}{8}}, \\
\mu_{F_2^\iota} =& e^{\frac{-(x_1+0.25\iota)^2}{8}} \cdot e^{\frac{-(x_2+0.25\iota)^2}{8}} \cdot e^{\frac{-(dy_d+0.25\iota)^2}{8}} \cdot e^{\frac{-(\psi_2+0.25\iota)^2}{8}} \cdot e^{\frac{-(d\psi_2/dy_d+0.25\iota)^2}{8}} \\
& \cdot e^{\frac{-(d\psi_2/dx_1+0.25\iota)^2}{8}} \cdot e^{\frac{-(\alpha_1+0.25\iota)^2}{8}} \cdot e^{\frac{-(\omega_2+0.25\iota)^2}{8}} \cdot e^{\frac{-\left(\delta^{-1}+0.25\iota\right)^2}{8}} \\
& \cdot e^{\frac{-(\psi_1+0.25\iota)^2}{8}} \cdot e^{\frac{-(y_d+0.25\iota)^2}{8}},
\end{aligned}$$

where $\iota = 1, \ldots, 7$.

$$\Phi_{1j}(\chi_1) = \mu_{F_1^j} / \sum_{\iota=1}^{7} \mu_{F_1^\iota},$$

$$\Phi_{2j}(\chi_2) = \mu_{F_2^j} / \sum_{\iota=1}^{7} \mu_{F_2^\iota},$$

where $j = 1, \ldots, 7$.

The virtual controller,adaptive fuzzy controller, and adaptive laws of this paper are described as:

$$\begin{aligned}
\alpha_1 =& -k_{11}\frac{z_1^{2p_1-1}}{Q_1^{p_1-1}} - k_{12}\frac{z_1^{2p_2-1}}{Q_1^{p_2-1}} - k_{13}z_1 - \hat{\theta}_1\phi_1(\chi_1) \\
& - B_1 \tanh\left(\frac{z_1 B_1}{\tau_1 Q_1}\right) - \frac{z_1}{2Q_1} + \frac{z_1}{\psi_1}\frac{\partial \psi_1}{\partial t}, \\
u =& -k_{21}\frac{z_2^{2p_1-1}}{Q_2^{p_1-1}} - k_{22}\frac{z_2^{2p_2-1}}{Q_2^{p_2-1}} - k_{23}z_2 - \hat{\theta}_2\phi_2(\chi_2) \\
& - B_2 \tanh\left(\frac{z_2 B_2}{\tau_2 Q_2}\right) - \frac{z_2}{2Q_2} + \frac{z_2}{\psi_2}\frac{\partial \psi_2}{\partial t},
\end{aligned}$$

$$\dot{\hat{\theta}}_i = \frac{e^{\mu t} z_i \phi_i(\chi_i)}{Q_i} - e^{\mu t}\sigma_{i1}\hat{\theta}_i - e^{\mu t}\sigma_{i2}\hat{\theta}_i^{2p_2-1}, i = 1, 2,$$

where $z_1 = e^{\mu t}(x_1 - y_d), z_2 = e^{\mu t}(x_2 - \omega_2), Q_1 = \psi_1^2 - z_1^2, Q_2 = \psi_2^2 - z_2^2, \chi_1 = [x_1, y_d, \dot{y}_d, \psi_1, \frac{\partial \psi_1}{\partial y_d}, \frac{1}{e^{\mu t}}]^T, \chi_2 = [\bar{x}_2, \dot{y}_d, \psi_2, \frac{\partial \psi_2}{\partial y_d}, \frac{\partial \psi_2}{\partial x_1}, \omega_2, \alpha_1, \frac{1}{e^{\mu t}}, \psi_1, y_d]^T$.

The controller parameters are engineered in the simulation as: $\mu = 0.5$, $k_{11} = 0.2$, $k_{12} = 50$, $k_{13} = 1$, $k_{21} = 0.2$, $k_{22} = 50$, $k_{23} = 1$, $\sigma_{11} = \sigma_{12} = \sigma_{21} = \sigma_{22} = 2$, $\iota_2 = 0.1$, $\tau_1 = \tau_2 = 1$, $B_1 = B_2 = 2$, $p_1 = \frac{9}{11}$, $p_2 = \frac{11}{9}$. And the initial values are chosen as $x(0) = [0.5\ -1]^T, \hat{\theta}(0) = [0.1\ 0.1]^T, \omega_2 = 0.1$.

The simulation results are shown in Figures 2–5. Figure 2 shows the trajectory of the system with all states under the action of the controller u. It is easy to obtain that all states are guaranteed to be purely state constraints and that the output x_1 can track to y_d. Figure 3 illustrates the trajectories of ω_2, α_1, $\hat{\theta}_1$ and $\hat{\theta}_2$ which the closed loop system in which all signals are bounded. Figure 4(a) displays the trajectory of the tracking error v_1, which rapidly converges within the fixed time $T_p = 5.5035$ and asymptotically converges to zero. Figure 4(b) showcases the trajectory of the control input u, where its bounds are demonstrated. In Figure 5, we compare our results with the ATC [50] and PFXTTC [31]. Compared to ATC [50], the proposed controller in this article can enable rapid convergence of tracking error to a smaller value. Compared to PFXTTC [31], the proposed controller in this article can ensure asymptotic convergence of tracking error to zero. All of these demonstrate the superiority of the controller constructed in this article.

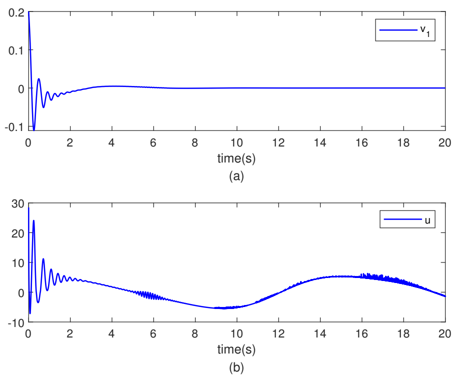

Figure 4. The trajectories of v_1 and u.

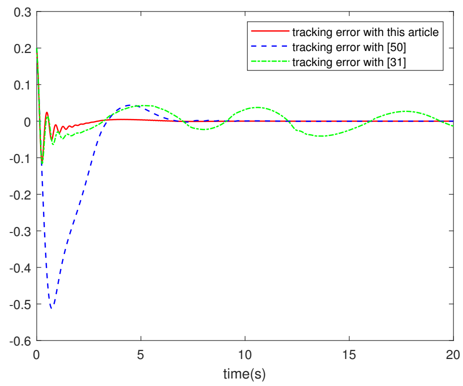

Figure 5. Simulation results with different control scheme.

6. Conclusions

In summary, this paper investigates the challenging problem of AFXTTC for a class of uncertain nonlinear systems with pure state constraints. Specifically, an improved FXTS determination theorem is proposed and an AFXTS determinacy theorem framework is established. A novel adaptive fuzzy asymptotic fixed-time controller is introduced by combining DSC, FLSs, and BLF. Our research results demonstrate that the tracking error can converge to zero within a fixed time domain independent of the initial values, and then asymptotically converge to zero while satisfying a set of specific constraints that are not only time-dependent but also state-dependent. Simulation results not only demonstrate the effectiveness of the proposed approach but also confirm its superiority by comparing the results with those obtained by the ATC and PFXTTC schemes. By the way, in recent years, constrained logical dynamic systems have been extensively studied [37, 51]. Future work will focus on generalizing the findings of this study to constrained logical dynamic systems.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare no conflict of interest in this paper.

Appendix

1. *Proof of the Lemma 1*

Since there exists a Lyapunov function W: $\mathbb{R} \to \mathbb{R}_{\geq 0}$ such that inequality (2.3) holds, the origin of system (2.1) is asymptotically stable [2]. Here we will complete our proof in two steps:

Step 1. In case of $W > 1$, we can let $v(t, v(0))$ be a solution of (2.2) and let $y_1(t) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be a function that satisfies

$$\dot{y}_1 = -e^{\mu t} r_2 y_1^n,$$

and $W(z(0)) \leq y_1(0)$. Hence, if $0 \leq t < \frac{1}{\mu} \ln\left[\frac{\mu}{r_2(n-1)}\left(1 - y_1^{1-n}(0)\right) + 1\right]$, $y_1 = \left[\frac{r_2(n-1)}{\mu}(e^{\mu t} - 1) + y_1^{1-n}(0)\right]^{\frac{1}{1-n}}$, and if $t \geq \frac{1}{\mu} \ln\left[\frac{\mu}{r_2(n-1)}\left(1 - y_1^{1-n}(0)\right) + 1\right]$, $y_1 \leq 1$. By the comparison lemma [2], we have $W(v(t, v_0)) \leq y_1(t)$. Let

$$t_1 = \frac{1}{\mu} \ln\left[\frac{\mu}{r_2(n-1)} + 1\right] \geq \frac{1}{\mu} \ln\left[\frac{\mu}{r_2(n-1)}\left(1 - y_1^{1-n}(0)\right) + 1\right],$$

thus, $W(v(t, v_0)) \leq 1$ for $t \geq t_1$.

Step 2. When $t \geq t_1$, $W \leq 1$, we can let $y_2(t) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ be a function that satisfies

$$\dot{y}_2 = -e^{\mu t} r_1 y_2^m,$$

and $W(z(t_1)) \leq y_2(t_1) = 1$. Hence, if $t_1 \leq t < \frac{1}{\mu} \ln\left[\frac{\mu}{r_1(1-m)} + \frac{\mu}{r_2(n-1)} + 1\right]$, $y_2 = \left[\frac{r_1(1-m)}{\mu}(e^{\mu t_1} - e^{\mu t}) + y_2^{1-m}(t_1)\right]^{\frac{1}{1-n}}$, and if $t \geq \frac{1}{\mu} \ln\left[\frac{\mu}{r_1(1-m)} + \frac{\mu}{r_2(n-1)} + 1\right]$, $y_2 = 0$. By the comparison lemma [2], we have $W(v(t,v_0)) \leq y_2(t)$. Let

$$T_p = \frac{1}{\mu}\ln\left[\frac{\mu}{r_1(1-m)} + \frac{\mu}{r_2(n-1)} + 1\right],$$

thus, $W(v(t,v_0)) = 0$ for $t \geq T_p$. This shows that the trajectories of (1) can reach the origin in fixed time T_p. Hence, by Definition 1, the origin of system (2.1) is FXTS. □

2. *Proof of the Lemma 2*

Our proof is divided into two steps:

Step 1. Let $B = \{x \in R \mid W(x) \leq (b/r_3)\}$. Due to the fact that W is continuous and positive definite, the set B is nonempty and closed. So, we can consider the following two cases.

Case I. If $z \in B$, due to $\dot{W}(z) \leq 0$, once the trajectory of z reaches the boundary of B, it does not exceed the set of B.

Case II. If $z \notin B$, obviously, $z(0) \notin B$, because if $z(0) \in B$, from *Case I*, $\forall\, t \in \mathbb{R}_{\geq 0}$, we get $z(t) \in B$, which contradicts the previous text. Then, there exists a minimum moment t_2 such that the inequality $W(z(t_2)) \leq (b/r_3)$ holds, i.e., $\forall\, t \in [0, t_2)$, $W(z(t_2)) > (b/r_3)$, which implies that

$$\dot{W}(z) \leq -e^{\mu t} r_1 W^m(z) - e^{\mu t} r_2 W^n(z).$$

We have $z(t) \in B$ for $t \geq t_2$ by *Case I* and $T_p \geq t_2$ by Lemma 1. Thus, we have $z(t) \in B$ for $t \geq T_p$.

From *Case I* and *Case II*, we have $z(t) \in B$ for any $t \geq T_p$. Let $\Delta_1 \in (0, +\infty)$ be a sufficiently large constant, and $B_1 = \{x \in B \mid \|x\| \leq \Delta_1\}$. It is clear that the set $B_1 \subseteq B$ is non-empty, bounded, and closed, this means that there must be a bounded constant $\Delta > 0$ such that $\|z\| \leq \Delta$ is true for all $z \in B_1$. Thus, the origin of system (2.2) transformed by inequality (2.5) is PFXTS according to Definition 2. There must be a bounded constant $\Delta > 0$ such that $\|z\| \leq \Delta$ for all $t \geq T_p$.

Step 2. From (2.4) and Definition 3, $\|v\| \leq \Delta/e^{\mu t}) \leq \Delta$ for all $t \geq T_p$. As $t \to \infty$, due to $e^{\mu t} \to \infty$ but $z = e^{\mu t} v$ is bounded, so $\|v\| \to 0$. Thus, the origin of system (2.1) is AFXTS. □

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