Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 8(11): 26614–26631.
DOI: 10.3934/math.20231361
Received: 24 March 2023
Revised: 07 September 2023
Accepted: 11 September 2023
Published: 18 September 2023

Research article

Valued-inverse Dombi neutrosophic graph and application

Mohammad Hamidi[1,*] **and Florentin Smarandache**[2]

[1] Department of Mathematics, University of Payame Noor, Tehran, Iran
[2] Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA

* **Correspondence:** Email: m.hamidi@pnu.ac.ir; Tel: +983155425252; Fax: +983155426100.

Abstract: Utilizing two ideas of neutrosophic subsets (NS) and triangular norms, we introduce a new type of graph as valued-inverse Dombi neutrosophic graphs. The valued-inverse Dombi neutrosophic graphs are a generalization of inverse neutrosophic graphs and are dual to Dombi neutrosophic graphs. We present the concepts of (complete) strong valued-inverse Dombi neutrosophic graphs and analyze the complement of (complete) strong valued-inverse Dombi neutrosophic graphs and self-valued complemented valued-inverse Dombi neutrosophic graphs. Since the valued-inverse Dombi neutrosophic graphs depend on real values, solving the non-equation and the concept of homomorphism play a prominent role in determining the complete, strong, complementarity and self-complementarity of valued-inverse Dombi neutrosophic graphs. We introduce the truth membership order, indeterminacy membership order, falsity membership order, truth membership size, indeterminacy membership size and indeterminacy membership size of any given valued-inverse Dombi neutrosophic graph, which play a major role in the application of valued inverse Dombi neutrosophic graphs in complex networks. An application of a valued-inverse Dombi neutrosophic graph is also described in this study.

Keywords: neutrosophic graph; Dombi triangular operator; inverse fuzzy graph; Dombi fuzzy graph; valued-Dombi inverse fuzzy graph
Mathematics Subject Classification: 03E72, 05C72

1. Introduction

The fuzzy graph theory, as a generalization of graph theory, is a symmetric and binary fuzzy relation on the fuzzy subset and is introduced and extended by Kaufmann and Rosenfeld [13, 18]. Later on, some researchers extended this concept, such as the complement of fuzzy graphs, the notions of regular fuzzy graphs, the notation of strong fuzzy graphs, totally regular fuzzy graphs and degree and total degree of a vertex in some fuzzy graphs, the novel concepts of M-strong fuzzy graphs and some types on domination number based on fuzzy graphs. Smarandache has presented the central concept idea of the neutrosophic set by extending the idea of fuzzy subsets [21]. A neutrosophic subset considers vague, indeterminate and inconsistent information of any real-life problem. A neutrosophic subset has three independent membership grades: truth, indeterminate and false. The bipolar fuzzy graph that is introduced by Ghorai and Pal is based on two ideas of fuzzy subset and graph structure and is used to design real-life problems [9]. The theory of single-valued neutrosophic graphs is a generalization of fuzzy graphs and has some essential applications in the real world. M. Akram has collected and has introduced a book on the scope of single-valued neutrosophic graphs that presents readers with fundamental concepts, including single-valued neutrosophic, neutrosophic graph structures, bipolar neutrosophic graphs, domination in bipolar neutrosophic graphs, bipolar neutrosophic planar graphs, interval-valued neutrosophic graphs, interval-valued neutrosophic graph structures, rough neutrosophic digraphs, neutrosophic rough digraphs, neutrosophic soft graphs, and intuitionistic neutrosophic soft graphs [2]. Recently, many researchers have more actively worked on neutrosophic graph theory, for instance, certain properties of the single-valued neutrosophic graph with application in food and agriculture organization [22], bipolar neutrosophic graph structures [6], a note on different types of product of neutrosophic graphs [15], generalized neutrosophic planar graphs and its application [16] and regularity of Pythagorean neutrosophic graphs with an illustration in MCDM [5]. The theory of single-valued neutrosophic hypergraphs as a generalization of single-valued neutrosophic graphs is a novel concept in the scope of single-valued neutrosophics and hypergraphs, which is introduced by M. Akram [1]. There is some applied research in the scope of single-valued neutrosophic hypergraphs, which are valuable in the real world, such as the implementation of single-valued neutrosophic soft hypergraphs on the human nervous system [3], and algorithms for the computation of regular single-valued neutrosophic soft hypergraphs applied to supranational Asian bodies [4]. A study on generalized graphs representations of complex neutrosophic information [19] and a novel decision-making method based on bipolar neutrosophic information [23] was conducted. The combination of triangular norms and fuzzy graphs is a novel concept for the generalization of fuzzy graphs and has applications of triangular norms in the real world. Recently, Ashraf et. al introduced the concept of Dombi fuzzy graphs and investigated some of their properties [7]. Later, Borzooei et al. presented the concept of inverse fuzzy graphs with application and analyzed the difference between fuzzy graphs and inverse fuzzy graphs [8]. Hamidi et al. applied the notation of topological spaces and fuzzy metrizable as the class of metrizable-topological spaces and introduced the concept of KM-fuzzy metric graph and KM-fuzzy metric hypergraph [11, 12]. Pal et al. introduced the combination of triangular norms and neutrosophic graphs as a generalization of the combination of triangular norms and fuzzy graphs, as some operations on Dombi neutrosophic graph, such as union, intersection composition, cartesian product, box dot product, homomorphic product, and modular product [14].

Motivation and advantage: One of the most essential applications of mathematics in the real world is to model a real problem based on the concepts of mathematics to analyze it according to the logic governing the problem. Undoubtedly, graph theory is one of the essential branches of mathematics in this modeling, which makes it easy to solve problems. Due to the fact that real problems are based on weight and real labels, weighted graphs can be essential. One of the essential areas of mathematics is fuzzy graphs, which can be effective in this field. Due to the strong connections of the elements in the real world to each other and the various properties and characteristics of objects, fuzzy graphs alone cannot analyze all the problems, and therefore, this limitation is considered in this field. Therefore, the concept of inverse single-valued neutrosophic graphs can overcome this limitation and cover different characteristics. As an example, consider a social network whose members are humans. In this network, every human being has a different temperament, and therefore, good, bad, kindness, anger, and similar (the truth value of each person represents his positive influence on others, falsity value represents his negative influence on others and indeterminacy value represents uncertainty in his influence) traits can have a significant impact on this network. Our most essential motivation in this research is based on the fact that in a mixed network that has several essential characteristics and some properties that act opposite to other properties, we model them based on inverse single-valued neutrosophic graphs. In this modeling, we deal with the reverse effect of each characteristic and examine its effect. Regarding these points, we try to combine the notations of triangular norms and neutrosophic subsets to construct an extension of graphs. Thus, we apply the notation of the Dombi triangular norm and inverse fuzzy graph to introduce the Dombi inverse neutrosophic graph. The prominent motivation for this work is a type of generalization of crisp graphs to weighted graphs, which are essential in the real world. Indeed, for any given greater than or equal to one nonnegative real number, a valued Dombi inverse neutrosophic graph is introduced. Additionally, we investigated the properties of valued Dombi inverse neutrosophic graphs and an application of the Dombi neutrosophic graph is also described in this manuscript.

2. Preliminaries

In this section, we recall some definitions and results, which we need use in what follows.

Definition 2.1. [17] *A binary operation $T : [0,1] \times [0,1] \to [0,1]$ is a t-norm, if for all $x, y, z, w \in [0,1]$:*

(*i*) $T(1, x) = x$,
(*ii*) $T(x, y) = T(y, x)$,
(*iii*) $T(T(x, y), z) = T(x, T(y, z))$,
(*iii*) *if $w \leq x$ and $y \leq z$, then $T(w, y) \leq T(x, z)$.*

For any $x, y \in [0,1]$, the t-norm $T_{Do}(x, y) = \dfrac{xy}{x + y - xy}$ is called the Dombi t-norm as an especial of Hamacher family of t-norms.

Definition 2.2. [20] *Let V be a universal set. A neutrosophic subset (NS) X in V is defined by $X = \{(x, T_X(x), I_X(x), F_X(x)) \mid x \in V, 0 \leq T_X(x) + I_X(x) + F_X(x) \leq 3\}$, which $X : V \to [0,1] \times [0,1] \times [0,1]$ is characterized by a truth-membership function T_X, an indeterminacy-membership function I_X and a falsity-membership function F_X.*

Definition 2.3. [2] *A single-valued neutrosophic graph on a nonempty X is a pair $G = (A, B)$, where A is single-valued neutrosophic set in X and B single-valued neutrosophic relation on X, such that $T_B(xy) \leq \min\{T_A(x), T_A(y)\}, I_B(xy) \leq \min\{I_A(x), I_A(y)\}, F_B(xy) \leq \max\{F_A(x), F_A(y)\}$ for all $x, y \in X$. A is called single-valued neutrosophic vertex set of G and B is called single-valued neutrosophic edge set of G, respectively.*

Definition 2.4. [10] *Let $k \in \mathbb{R}^{>0}, G^* = (V, E)$ be a simple graph, $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ be fuzzy subsets. Then $G = (\sigma, \mu, T_{Do})$ is called a k-Dombi inverse fuzzy graph, if for any $xy \in E$, get $k\mu(xy) \geq T_D(\sigma(x), \sigma(y))$. If $k = 1$, then $G = (\sigma, \mu, T_{Do})$ is called a Dombi inverse fuzzy graph.*

Definition 2.5. [14] *Let $G^* = (V, E)$ be a simple graph, $\sigma = \{(x, T_V(x), I_V(x), F_V(x)) \mid x \in V\}$ and $\mu = \{(xy, T_E(xy), I_E(xy), F_E(xy)) \mid xy \in E\}$ be two NS. Then $G = (\sigma, \mu, T_{Do})$ is called a Dombi neutrosophic graph (DNG), if for any $xy \in E$, get $T_E(xy) \leq T_{Do}(T_V(x), T_V(y)), I_E(xy) \leq T_{Do}(I_V(x), I_V(y))$ and $F_E(xy) \geq T_{Do}(F_V(x), F_V(y))$.*

3. Dombi inverse neutrosophic graph

In this section, for any $k, l, m \in \mathbb{R}^{>0}$, introduce the concept of (k, l, m)-Dombi inverse neutrosophic graph, via Dombi triangular norm and inverse neutrosophic graph.

Definition 3.1. *Let $k, l, m \in \mathbb{R}^{>0}, G^* = (V, E)$ be a simple graph, $\sigma = \{(x, T_V(x), I_X(x), F_V(x)) \mid x \in V\}$ and $\mu = \{(xy, T_E(xy), I_E(xy), F_E(xy)) \mid xy \in E\}$ be two NS. Then*

(i) $G = (\sigma, \mu, T_{Do})$ is called a (k, l, m)-Dombi inverse neutrosophic graph ((k, l, m)-DING), if for any $xy \in E$, get $kT_E(xy) \geq T_{Do}(T_V(x), T_V(y)), lI_E(xy) \geq T_{Do}(I_V(x), I_V(y))$ and $mF_E(xy) \leq T_{Do}(F_V(x), F_V(y))$,

(ii) $G = (\sigma, \mu, T_{Do})$ is called a strong (k, l, m)-Dombi inverse neutrosophic graph (strong (k, l, m)-DING), if for any $xy \in E$, $kT_E(xy) = T_{Do}(T_V(x), T_V(y)), lI_E(xy) = T_{Do}(I_V(x), I_V(y))$ and $mF_E(xy) = T_{Do}(F_V(x), F_V(y))$,

(iii) $G = (\sigma, \mu, T_{Do})$ is called a complete (k, l, m)-Dombi neutrosophic graph (complete (k, l, m)-DING), if for any $x, y \in V$, $kT_E(xy) = T_{Do}(T_V(x), T_V(y)), lI_E(xy) = T_{Do}(I_V(x), I_V(y))$ and $mF_E(xy) = T_{Do}(F_V(x), F_V(y))$.

In the above definition, if $(k, l, m) = (1, 1, 1)$, we will call $G = (\sigma, \mu, T_{Do})$ is a (complete) strong Dombi inverse neutrosophic graph.

Example 3.2. *Let $V = \{a, b, c, d\}$. Then $G = (\sigma, \mu, T_{Do})$ is a $(4, 3, 2)$-DING as shown in Figure 1.*

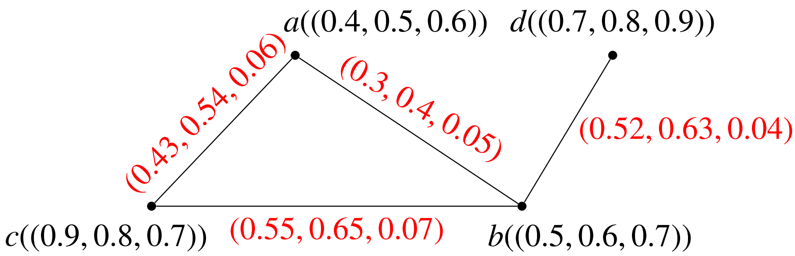

Figure 1. $(4, 3, 2)$-DING $G = (\sigma, \mu, T_{Do})$.

Theorem 3.3. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING and $k, k', l, l', m, m' \in \mathbb{R}^{>0}$. If $k' \geq k, l' \geq l, m \geq m'$, then $G = (\sigma, \mu, T_{Do})$ is a (k', l', m')-DING.*

Proof. Let $k, k,' l, l', m, m' \in \mathbb{R}^{>0}$. Since $G = (\sigma, \mu, T_{Do})$ is a (k, l, m)-DING and $k' \geq k, l' \geq l, m \geq m'$, we get that

$$k'T_E(xy) \geq kT_E(xy) \geq T_{Do}(T_V(x), T_V(y)), l'I_E(xy) \geq lI_E(xy) \geq I_{Do}(I_V(x), I_V(y))$$
$$\text{and } m'F_E(xy) \leq mF_E(xy) \leq T_{Do}(F_V(x), F_V(y)),$$

and so $G = (\sigma, \mu, T_{Do})$ is a (k', l', m')-DING. □

Proposition 3.4. *Let $G = (\sigma, \mu, T_{Do})$ be a DNG and a (k, l, m)-DING. Then*

(*i*) $k \geq 1, l \geq 1$ *and* $0 < m \leq 1$.
(*ii*) $G = (\sigma, \mu, T_{Do})$ *is strong DNG and strong* (k, l, m)*-DING if and only if* $(k, l, m) = (1, 1, 1)$.

Proof. (*i*) Let $x, y \in V$. Then

$$\frac{1}{k}T_{Do}(T_V(x), T_V(y)) \leq T_E(xy) \leq T_{Do}(T_V(x), T_V(x)), \frac{1}{l}T_{Do}(I_V(x), I_V(y)) \leq I_E(xy) \leq T_{Do}(I_V(x), I_V(x)),$$
$$\text{and } \frac{1}{m}T_{Do}(F_V(x), F_V(y)) \geq F_E(xy) \geq T_{Do}(F_V(x), F_V(x)).$$

It follows that

$$\frac{1}{k}T_{Do}(T_V(x), T_V(y)) \leq T_{Do}(T_V(x), T_V(x)), \frac{1}{l}T_{Do}(I_V(x), I_V(y)) \leq T_{Do}(I_V(x), I_V(x)),$$
$$\text{and } \frac{1}{m}T_{Do}(F_V(x), F_V(y)) \geq T_{Do}(I_V(x), I_V(x)).$$

Hence $k \geq 1, l \geq 1$ and $m \leq 1$, because of $0 < x, y \leq 1$.
(*ii*) It is clear by (*i*). □

Definition 3.5. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING and $k, k,' l, l', m, m' \in \mathbb{R}^{>0}$. Then $G^{(c,(k',l',m'))} = (\sigma^c, \mu^c, T_{Do})$ is called a (k', l', m')-complement of (k, l, m)-DING of G, if $\sigma^c = \sigma$ and for all $x, y \in V$,*

$$T^c_E(xy) = \begin{cases} k'T_E(xy) - T_{Do}(T_V(x), T_V(y)) & \text{if } xy \in E \\ T_{Do}(T_V(x), T_V(y)) & \text{if } xy \notin E, \end{cases},$$

$$I^c_E(xy) = \begin{cases} l'I_E(xy) - T_{Do}(I_V(x), I_V(y)) & \text{if } xy \in E \\ T_{Do}(I_V(x), I_V(y)) & \text{if } xy \notin E, \end{cases}$$

and $F^c_E(xy) = \begin{cases} F_E(xy) - m'T_{Do}(F_V(x), F_V(y)) & \text{if } xy \in E \\ T_{Do}(F_V(x), F_V(y)) & \text{if } xy \notin E, \end{cases}$, *where* $\mu^c = (T^c_E, I^c_E, F^c_E)$ *and* $\sigma^c = (T^c_V, I^c_V, F^c_V)$.

Theorem 3.6. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING and $k, l, m, k', l', m' \in \mathbb{R}^{>0}$. Then*

(*i*) $G^{(c,(k',l',m'))} = (\sigma^c, \mu^c, T_{Do})$ *is a* $(\frac{k}{k'-k}, \frac{l}{l'-l}, \frac{m}{(1-mm')})$*-DING.*
(*ii*) $G^{(c,(2k,2l,\frac{1-m}{m}))} = (\sigma^c, \mu^c, T_{Do})$ *is a DING.*
(*iii*) *If* $G = (\sigma, \mu, T_{Do})$ *is strong, then* $G^{(c,(2k,2l,\frac{1-m}{m}))} = (\sigma^c, \mu^c, T_{Do})$ *is strong.*

Proof. (*i*) Let $xy \in E$. Then

$$\begin{aligned} T^c_E(xy) &= k'T_E(xy) - T_{Do}(T_V(x), T_V(y)) \geq \frac{k'}{k}T_{Do}(T_V(x), T_V(y)) - T_{Do}(T_V(x), T_V(y)) \\ &\geq (\frac{k'}{k} - 1)T_{Do}(T_V(x), T_V(y)), \\ I^c_E(xy) &= l'I_E(xy) - T_{Do}(I_V(x), I_V(y)) \geq \frac{l'}{l}T_{Do}(I_V(x), I_V(y)) - T_{Do}(I_V(x), I_V(y)) \end{aligned}$$

$$
\begin{aligned}
&\geq (\frac{l'}{l}-1)T_{Do}(I_V(x),I_V(y)) \text{ and}\\
F_E^c(xy) &= F_E(xy)-m'T_{Do}(F_V(x),F_V(y)) \leq (1-mm')F_E(xy)\\
&\leq \frac{(1-mm')}{m}T_{Do}(F_V(x),F_V(y)).
\end{aligned}
$$

It follows that

$$
\frac{k}{k'-k}T_E^c(xy) \geq T_{Do}(T_V(x),T_V(y)),\ \frac{l}{l'-l}I_E^c(xy) \geq T_{Do}(I_V(x),I_V(y)),
$$
$$
\frac{m}{(1-mm')}F_E^c(xy) \leq T_{Do}(F_V(x),F_V(y)),
$$

and so $G^{(c,(k',l',m'))} = (\sigma^c,\mu^c,T_{Do})$ is a $(\frac{k}{k'-k},\frac{l}{l'-l},\frac{m}{(1-mm')})$-DING.

(ii) Since $2k = k', 2l = l', m' = \frac{1-m}{m}$, by (i), $G^{(c,(2k,2l,\frac{1-m}{m}))} = (\sigma^c,\mu^c,T_{Do})$ is a DING.

(iii) It is clear by (i), (ii). □

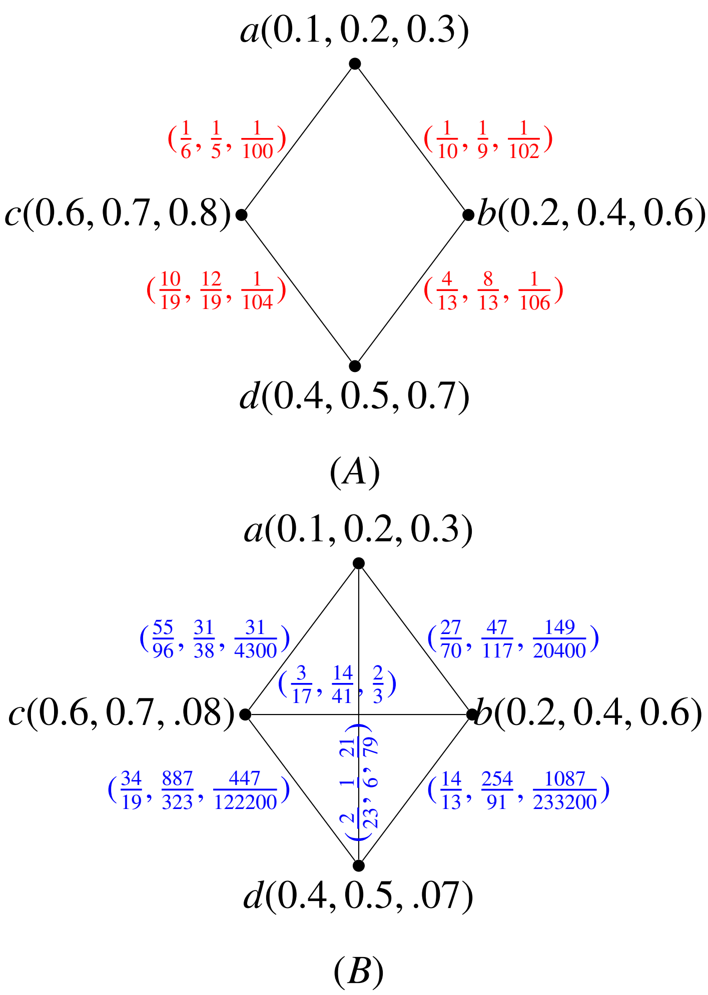

Figure 2. (A) $G = (\sigma,\mu,T_{Do})$, (B) $G^{(c,(4,5,\frac{1}{100}))} = (\sigma^c,\mu^c,T_{Do})$.

Corollary 3.7. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING and $k, l, m \in \mathbb{R}^{>0}$. Then*

(i) $G^{(c,(k,l,m))} = (\sigma^c, \mu^c, T_{Do})$ *is't a DING.*
(ii) $G^{(c,(k',l',m'))} = (\sigma^c, \mu^c, T_{Do})$ *is't a DING, which* $k' < k, l' < l, m' > \frac{1}{m}$.

Example 3.8. *Let $V = \{a, b, c, d\}$. Then $G = (\sigma, \mu, T_{Do})$ is a $(2, 3, 4)$-DING as shown in Figure 2 (A). Then $G^{(c,(4,5,\frac{1}{100}))} = (\sigma^c, \mu^c, T_{Do})$ is the $(4, 5, \frac{1}{100})$-complement of $(2, 3, 4)$-DING G in Figure 2 (B).*

Definition 3.9. *Let $k, k', k'', m, m', m'', l, l', l'' \in \mathbb{R}^{>0}$, $G = (\sigma, \mu, T_{Do})$ and $G' = (\sigma', \mu', T_{Do})$ be (k, l, m)-DING and (k', l', m')-DING on simple graph $G^* = (V, E)$ and $G'^* = (V', E')$, respectively.*

(i) *A homomorphism $h : G \to G'$ is a map $h : V \to V'$, which for all $x \in V$, $T_V(x) \geq T'_V(h(x)), I_V(x) \geq I'_V(h(x)), F_V(x) \geq F'_V(h(x))$ and for all $xy \in E$, $kT_E(xy) \geq k'T'_E(h(x)h(y))$, $lI_E(xy) \geq l'T'_E(h(x)h(y)), mF_E(xy) \geq m'F'(h(x)h(y))$.*
(ii) *A bijective mapping $h : V \to V'$ is called an isomorphism, if for any $xy \in E$ and $x \in V$, $T_V(x) = T'_V(h(x)), I_V(x) = I'_V(h(x)), F_V(x) = F'_V(h(x))$ and for all $xy \in E$, $kT_E(xy) = k'T'_E(h(x)h(y)), lI_E(xy) = l'I'_E(h(x)h(y))$ and $mF_E(xy) = m'F'(h(x)h(y))$. We will say that $G = (\sigma, \mu, T_{Do})$ and $G' = (\sigma', \mu', T_{Do})$ are isomorphic and will denote it by $G \cong G'$.*
(iii) *G is called a self (k'', l'', m'')-complemented (k, l, m)-DING, if $G \cong G^{(c,(k'',l'',m''))}$.*

Example 3.10. (i) *Let $V = \{a, b, c\}$. Then $G_1 = (\sigma_1, \mu_1, T_{Do}) \cong G_2 = (\sigma_2, \mu_2, T_{Do})$, where G_1 is a $(2, 3, 4)$-DING and G_2 is a $(4, 6, 8)$-DING as shown in Figure 3.*

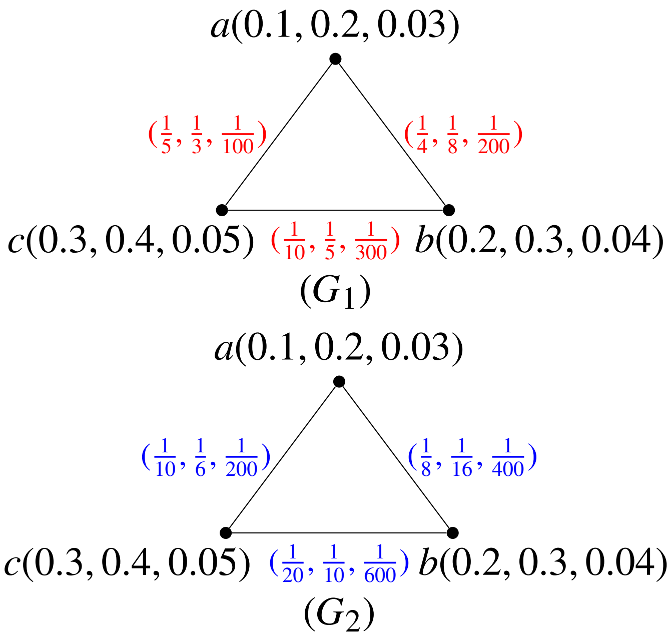

Figure 3. $(2, 3, 4)$-DING G_1 , $(4, 6, 8)$-DING G_2.

(ii) (i) *Consider the $(4, 5, 1)$-DING $G = (\sigma, \mu, T_{Do})$ and $(8, 9, 0.5)$-DING $G^c = (\sigma^c, \mu^c, T_{Do})$ in Figures 4 (A) and (B), respectively. Then G is a self $(8, 9, 0.5)$-complemented $(4, 5, 1)$-DING.*

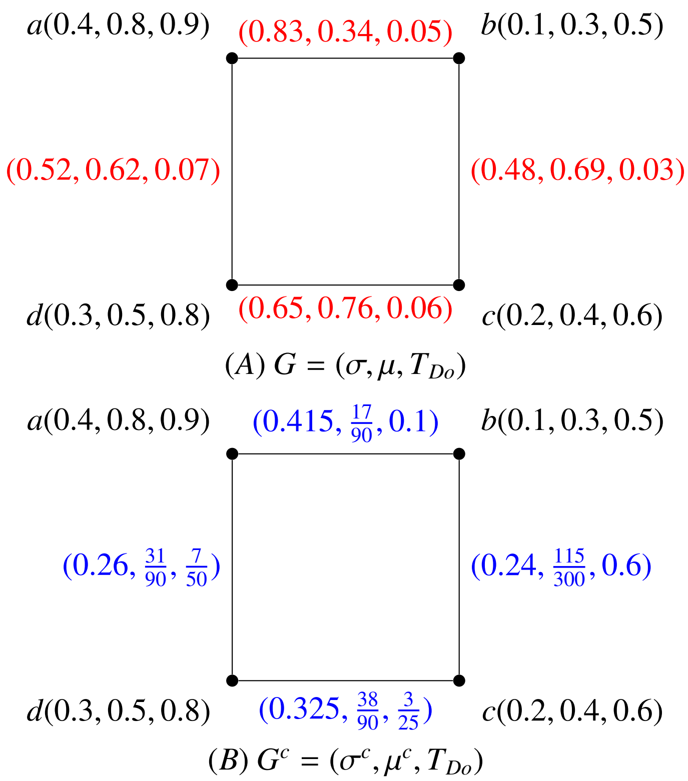

Figure 4. $(4, 5, 1)$-DING G and $(8, 9, 0.5)$-DING G^c.

Theorem 3.11. *Let $k_1, l_1, m_1, k_2, l_2, m_2, k'_1, l', m'_1, k'_2, l'_2, m'_2 \in \mathbb{R}^{>0}, G_1 = (\sigma_1, \mu_1, T_{Do}) \cong G_2 = (\sigma_2, \mu_2, T_{Do})$, where G_1 and G_2 are (k_1, l_1, m_1)-DING and (k_2, l_2, m_2)-DING, respectively. Then*

(i) If $k'_1 k_2 = k_1 k'_2, l'_1 l_2 = l_1 l'_2$ and $m_1 m'_1 = m_2 m'_2$, then $G_1^{(c,(k'_1,l'_1,m'_1))} \cong G_2^{(c,(k'_2,l'_2,m'_2))}$.
(ii) $G_1^{(c,(k_1,l_1,m_1))} \cong G_2^{(c,(k_1,l_1,m_1))}$.
(iii) $G_1^{(c,(\frac{k_1 k'_2}{k_1}, \frac{l_1 l'_2}{k_1}, \frac{m_1 m'_1}{m'_2}))} \cong G_2^{(c,(k_2,l_2,m_2))}$.

Proof. (*i*) Let $G_1 \cong G_2$, so there exists $h : V_1 \to V_2$, such that for any $x_1 \in V_1$ and for all $x_1y_1 \in E_1$, $T_{1_V}(x_1) = T_{2_V}(h(x_1)), I_{1_V}(x_1) = I_{2_V}(h(x_1)), F_{1_V}(x_1) = F_{2_V}(h(x_1))$ and $k_1 T_{1_E}(x_1y_1) = k_2 T_{2_E}(h(x_1)h(y_1)), l_1 I_{1_E}(x_1y_1) = l_2 I_{2_E}(h(x_1)h(y_1)), m_1 F_{1_E}(x_1y_1) = m_2 F_{2_E}(h(x_1)h(y_1))$. It's clear that $T^c_{1_V}(x_1) = T^c_{12_V}(h(x_1)), I^c_{1_V}(x_1) = I^c_{12_V}(h(x_1)), F^c_{1_V}(x_1) = F^c_{12_V}(h(x_1))$ for all $x_1 \in V_1$. Let $x_1y_1 \in E_1, T_{1_E}(x_1y_1) \neq 0, I_{1_E}(x_1y_1) \neq 0$ and $F_{1_E}(x_1y_1) \neq 0$. By Theorem 3.6, $G^{(c,(k',l',m'))} = (\sigma^c, \mu^c, T_{Do})$ is a $(\dfrac{k}{k'-k}, \dfrac{l}{l'-l}, \dfrac{m}{(1-mm')})$-DING. Thus,

$$\begin{aligned}
\frac{k_1}{k'_1 - k_1} T^c_{1_E}(x_1y_1) &= (\frac{k_1}{k'_1 - k_1}) k'_1 T_{1_E}(x_1y_1) - (\frac{k_1}{k'_1 - k_1}) T_{Do}(T_{1_V}(x_1), T_{1_V}(y_1)) \\
&= (\frac{k_1}{k'_1 - k_1}) \frac{k'_1 k_2}{k_1} T_{2_E}(h(x_1)h(y_1)) - (\frac{k_1}{k'_1 - k_1}) T_{Do}(T_{2_V}(h(x_1)), T_{2_V}(h(y_1))).
\end{aligned}$$

Thus,

$$\begin{aligned}
&(\frac{k_1}{k_1'-k_1})T^c_{1_E}(x_1y_1)=(\frac{k_2}{k_2'-k_2})T^c_{2_E}(h(x_1)h(y_1))\\
\Leftrightarrow\quad &(\frac{k_1}{k_1'-k_1})\frac{k_1'k_2}{k_1}T_{2_E}(h(x_1)h(y_1))-(\frac{k_1}{k_1'-k_1})T_{Do}(T_{2_V}(h(x_1)),T_{2_V}(h(y_1)))=(\frac{k_2}{k_2'-k_2})T^c_{2_E}(h(x_1)h(y_1))\\
\Leftrightarrow\quad &(\frac{k_1}{k_1'-k_1})\frac{k_1'k_2}{k_1}T_{2_E}(h(x_1)h(y_1))-(\frac{k_1}{k_1'-k_1})T_{Do}(T_{2_V}(h(x_1)),T_{2_V}(h(y_1)))\\
=\quad &((\frac{k_2}{k_2'-k_2})k_2'T_{2_E}(h(x_1)h(y_1))-(\frac{k_2}{k_2'-k_2})T_{Do}(T_{2_V}(h(x_1)),T_{2_V}(h(y_1)))\Leftrightarrow k_1'k_2=k_1k_2'.
\end{aligned}$$

Let $x_1y_1\in E$ and $T_{1_E}(x_1y_1)=0$. Thus,

$$\begin{aligned}
T^c_{1_E}(x_1y_1)&=T_{Do}(T_{1_V}(x_1),T_{1_V}(y_1))\\
&=T_{Do}(T_{2_V}(h(x_1)),T_{2_V}(h(y_1)))=T^c_{2_E}(h(x_1)h(y_1)).
\end{aligned}$$

In a similar way, $(\frac{l_1}{l_1'-l_1})I^c_{1_E}(x_1y_1)=(\frac{l_2}{l_2'-l_2})I^c_{2_E}(h(x_1)h(y_1))$ if and only if $l_1'l_2=l_1l_2'$. In addition,

$$\begin{aligned}
\frac{m_1}{1-m_1m_1'}F^c_{1_E}(x_1y_1)&=\frac{m_1}{1-m_1m_1'}F_{1_E}(x_1y_1)-(\frac{m_1}{1-m_1m_1'})m_1'T_{Do}(F_{1_V}(x_1),F_{1_V}(y_1))\\
&=(\frac{m_1}{1-m_1m_1'})\frac{m_2}{m_1}F_{2_E}(h(x_1)h(y_1))-(\frac{m_1}{1-m_1m_1'})m_1'T_{Do}(F_{2_V}(h(x_1)),F_{2_V}(h(y_1))).
\end{aligned}$$

Thus,

$$\begin{aligned}
&(\frac{m_1}{1-m_1m_1'})F^c_{1_E}(x_1y_1)=(\frac{m_2}{1-m_2m_2'})F^c_{2_E}(h(x_1)h(y_1))\\
\Leftrightarrow\quad &(\frac{m_1}{1-m_1m_1'})\frac{m_2}{m_1}F_{2_E}(h(x_1)h(y_1))-(\frac{m_1}{1-m_1m_1'})m_1'T_{Do}(F_{2_V}(h(x_1)),F_{2_V}(h(y_1)))\\
=\quad &((\frac{m_2}{1-m_2m_2'})F_{2_E}(h(x_1)h(y_1))-(\frac{m_2}{1-m_2m_2'})m_2'T_{Do}(F_{2_V}(h(x_1)),F_{2_V}(h(y_1)))\Leftrightarrow m_1m_1'=m_2m_2'.
\end{aligned}$$

Let $x_1y_1\in E$ and $F_{1_E}(x_1y_1)=0$. Thus,

$$\begin{aligned}
F^c_{1_E}(x_1y_1)&=T_{Do}(F_{1_V}(x_1),F_{1_V}(y_1))\\
&=T_{Do}(F_{2_V}(h(x_1)),F_{2_V}(h(y_1)))=F^c_{2_E}(h(x_1)h(y_1)).
\end{aligned}$$

$(ii),(iii)$ are obtained from (i). □

Theorem 3.12. *Let $G=(\sigma,\mu,T_{Do})$ be a self (k',l',m')-complemented of strong (k,l,m)-DING on $G^*=(V,E)$. Then*

(i) $\sum_{\substack{x\neq y\\ xy\in E}}T_E(xy)=(\frac{k'^2-kk'}{k^2})\sum_{x\neq y}T_{Do}(T_V(x),T_V(y))$.

(ii) $\sum_{\substack{x\neq y\\ xy\in E}}I_E(xy)=(\frac{l'^2-ll'}{k^2})\sum_{x\neq y}T_{Do}(I_V(x),I_V(y))$.

(iii) $\sum_{\substack{x\neq y\\ xy\in E}} F_E(xy) = (\frac{m' - mm'^2}{m^2}) \sum_{x\neq y} T_{Do}(F_V(x), F_V(y))$.

(iv) $k' > k, l' > l$ *and* $mm' < 1$.

Proof. $(i), (ii), (iii)$ Let $G = (\sigma, \mu, T_{Do})$ be a self (k', l', m')-complemented (k, l, m)-DING on $G^* = (V, E)$. Since $G \cong G^{(c,(k',l',m'))}$, there exists an isomorphism $f : V \to V$ such that for any $xy \in E$,

$$k'T_E^c(f(x)f(y)) = kT_E(xy), l'I_E^c(f(x)f(y)) = lI_E(xy), m'F_E^c(f(x)f(y)) = mF_E(xy),$$

and for any $x \in V$,

$$T_V^c(f(x)) = T_V(x), I_V^c(f(x)) = I_V(x), F_V^c(f(x)) = F_V(x).$$

It follows that

$$\begin{aligned} & kT_E(xy) = k'T_E^c(f(x)f(y)) = k'(k'T_E(f(x)f(y)) - T_{Do}(T_V(f(x)), T_V(f(y)))) \\ = \quad & k'(\frac{k'}{k}T_{Do}(T_V(f(x)), T_V(f(y)) - T_{Do}(T_V(f(x)), T_V(f(y)))) = k'(\frac{k'}{k} - 1)T_{Do}(T_V(f(x)), T_V(f(y)). \end{aligned}$$

Hence $T_E(xy) = (\frac{k'^2 - kk'}{k^2})T_{Do}(T_V(f(x)), T_V(f(y))$. In a similar way, one can see that $I_E(xy) = (\frac{k'^2 - kk'}{k^2})T_{Do}(I_V(f(x)), I_V(f(y))$ and so for any $xy \in E$, we have

$$\sum_{\substack{x\neq y\\ xy\in E}} T_E(xy) = (\frac{k'^2 - kk'}{k^2}) \sum_{x\neq y} T_{Do}(T_V(x), T_V(y)) \text{ and } \sum_{\substack{x\neq y\\ xy\in E}} I_E(xy) = (\frac{l'^2 - ll'}{k^2}) \sum_{x\neq y} T_{Do}(I_V(x), I_V(y)).$$

In addition,

$$\begin{aligned} & mF_E(xy) = m'F_E^c(f(x)f(y)) = m'(F_E(f(x)f(y)) - m'T_{Do}(F_V(f(x)), F_V(f(y)))) \\ = \quad & m'(\frac{1}{m}T_{Do}(F_V(f(x)), F_V(f(y)) - m'T_{Do}(F_V(f(x)), F_V(f(y)))) = m'(\frac{1 - mm'}{m})T_{Do}(F_V(f(x)), F_V(f(y)). \end{aligned}$$

Thus, $F_E(xy) = (\frac{m' - mm'^2}{m^2})T_{Do}(F_V(f(x)), F_V(f(y))$ and so

$$\sum_{\substack{x\neq y\\ xy\in E}} F_E(xy) = (\frac{m' - mm'^2}{m^2}) \sum_{x\neq y} T_{Do}(F_V(x), F_V(y)).$$

(iv) Since $\frac{k'^2 - kk'}{k^2} > 0$, $\frac{l'^2 - ll'}{l^2} > 0$ and $\frac{m' - mm'^2}{m^2} > 0$, we get that $k' > k, l' > l$ and $mm' < 1$. □

Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING on $G^* = (V, E), k', l', m' \in \mathbb{R}^{>0}, \alpha = \frac{k'}{k'^2 - k}, \beta = \frac{l'}{l'^2 - l}, \gamma = \frac{m' - mm'^2}{m^2}, \alpha' = \frac{1}{k - 1}, \beta' = \frac{1}{l - 1}$ and $\gamma' = \frac{1 - m^2}{m}$. Thus, we have the following theorem.

Theorem 3.13. *Let* $G = (\sigma, \mu, T_{Do})$ *be a* (k, l, m)*-DING on* $G^* = (V, E), k', l', m' \in \mathbb{R}^{>0}$.

(*i*) *If* $\mu(xy) = (\alpha(T_{Do}(T_V(x), T_V(y))), \beta(T_{Do}(I_V(x), I_V(y))), \gamma(T_{Do}(F_V(x), F_V(y))))$*, then* $k' > \sqrt{k}, l' > \sqrt{l}, m' < \dfrac{1}{m}$ *and* G *is a self* (k', l', m')*-complemented* (k, l, m)*-DING.*

(*ii*) *If* $\mu(xy) = (\alpha'(T_{Do}(T_V(x), T_V(y))), \beta'(T_{Do}(I_V(x), I_V(y))), \gamma'(T_{Do}(F_V(x), F_V(y))))$*, then* $k > 1, l > 1, m < 1$ *and* G *is a self* (k, l, m)*-complemented* (k, l, m)*-DING.*

Proof. (*i*) Let $T_E(xy) = (\dfrac{k'}{k'^2 - k})(T_{Do}(T_V(x), T_V(y)))$ and $f : V \underset{x \mapsto x}{\rightarrow} V$. Then

$$\begin{aligned} k'T^c_E(xy) &= k'(k'T_E(xy) - T_{Do}(T_V(x), T_V(y))) \\ &= k'(k'(\frac{k'}{k'^2 - k})(T_{Do}(T_V(x), T_V(y))) - T_{Do}(T_V(x), T_V(y)))) \\ &= (\frac{kk'}{k'^2 - k})T_{Do}(T_V(x), T_V(y))) = kT_E(xy). \end{aligned}$$

It follows that G is a self k'-complement DING. It is similar to equation $l'I^c_E(xy) = lTI_E(xy)$ if and only if $I_E(xy) = \dfrac{l'}{l'^2 - l}T_{Do}(I_V(x), I_V(y)))$. In addition, $F_E(xy) = (\dfrac{k'}{k'^2 - k})(T_{Do}(F_V(x), F_V(y)))$, implies that

$$\begin{aligned} m'F^c_E(xy) &= m'(F_E(xy) - m'T_{Do}(F_V(x), F_V(y))) \\ &= m'((\frac{1}{m})(T_{Do}(F_V(x), F_V(y))) - m'T_{Do}(F_V(x), F_V(y)))) \\ &= (\frac{m' - mm'^2}{m^2})T_{Do}(F_V(x), F_V(y))) = mF_E(xy). \end{aligned}$$

(*ii*) Consider $(k', l', m') = (k, l, m)$, then by (*i*), G is a self (k, l, m)-complemented (k, l, m)-DING. □

Theorem 3.14. *Let* $G = (\sigma, \mu, T_{Do})$ *be a* $(1, 1, \frac{1}{4})$*-DING on* $G^* = (V, E)$*. Then* $G = (\sigma, \mu, T_{Do})$ *is a self* $(\frac{1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}, 2 + \sqrt{3})$*-complemented DING.*

Proof. Let $G = (\sigma, \mu, T_{Do})$ be a DING on $G^* = (V, E)$. If $G = (\sigma, \mu, T_D)$ is a self (k, l, m)-complemented Dombi inverse fuzzy graph, then

$$\begin{aligned} &kT^c_E(xy) = k(kT_E(xy) - T_E(xy)) = k^2T_E(xy) - kT_E(xy) = T_E(xy), \\ &kI^c_E(xy) = k(kI_E(xy) - I_E(xy)) = k^2I_E(xy) - kI_E(xy) = I_E(xy), \\ &kF^c_E(xy) = k(F_E(xy) - kT_{Do}(F_V(x), F_V(y))) = kF_E(xy) - k^2T_{Do}(F_V(x), F_V(y)) = \frac{1}{4}F_E(xy), \end{aligned}$$

because of, $T_E(xy) = T_{Do}(T_V(x), T_V(y)), I_E(xy) = T_{Do}(I_V(x), I_V(y))$ and $\dfrac{1}{4}F_E(xy) = T_{Do}(F_V(x), F_V(y))$. It follows that, $k = l = \frac{1+\sqrt{5}}{2}$ and $m = 2 + \sqrt{3}$. □

Corollary 3.15. *Let* $G = (\sigma, \mu, T_{Do})$ *be a DING on* $G^* = (V, E)$*. Then* $G = (\sigma, \mu, T_{Do})$ *is't a self complement DING.*

Proof. Let $G = (\sigma, \mu, T_{Do})$ be a self complemented DING on $G^* = (V, E)$ and $T_E(xy) \neq 0, I_E(xy) \neq 0$ and $F_E(xy) \neq 0$ for any $x, y \in V$. By Theorem 3.14, $k = l = \frac{1+\sqrt{5}}{2}$. In addition, $kF^c_E(xy) = F_E(xy)$, implies that $m = \dfrac{1 + \sqrt{3}i}{2} \notin \mathbb{R}^{>0}$, which is a contradiction. □

Theorem 3.16. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING. Then $G^{(c,(c,(c,(k',l',m'))))} \cong G$ if and only if $\mu(xy) = (\delta T_{Do}(T_V(x), T_V(y)), \theta T_{Do}(I_V(x), I_V(y)), \vartheta T_{Do}(F_V(x), F_V(y))$, where $\delta = \dfrac{k'^2 + k'}{k'^3 - k}, \theta = \dfrac{l'^2 + l'}{l'^3 - l}$ and $\vartheta = \dfrac{2m'^2}{m' - m}$.*

Proof. Let $xy \in E$ and $x \in V$. Then $(T_V^c)^c(x) = T_V(x), (I_V^c)^c(x) = I_V(x), (F_V^c)^c(x) = F_V(x)$. In addition, $k'((T_E^c)^c(xy)) = kT_E(xy)$ if and only if

$$\begin{aligned} & k'\Big(k'(T_E^c(xy)) - T_{Do}(T_V(x), T_V(y))\Big) = kT_E(xy) \\ \Longleftrightarrow\quad & k'\Big(k'(k'T_E(xy) - T_{Do}(T_V(x), T_V(y)))) - T_{Do}(T_V(x), T_V(y))\Big) = kT_E(xy) \\ \Longleftrightarrow\quad & T_E(xy) = (\frac{k'^2 + k'}{k'^3 - k})(T_{Do}(T_V(x), T_V(y))). \end{aligned}$$

In a similar way, $l'((I_E^c)^c(xy)) = lI_E(xy)$ if and only if $I_E(xy) = (\dfrac{l'^2 + l'}{l'^3 - l})(T_{Do}(I_V(x), I_V(y)))$. Moreover, $m'((F_E^c)^c(xy)) = mF_E(xy)$ if and only if

$$\begin{aligned} & m'\Big(F_E^c(xy)) - m'T_{Do}(F_V(x), F_V(y))\Big) = mF_E(xy) \\ \Longleftrightarrow\quad & m'\Big((F_E(xy) - m'T_{Do}(F_V(x), F_V(y)))) - m'T_{Do}(F_V(x), F_V(y))\Big) = mF_E(xy) \\ \Longleftrightarrow\quad & F_E(xy) = (\frac{2m'^2}{m' - m})(T_{Do}(F_V(x), F_V(y))). \end{aligned}$$

It follows that $G^{(c,(c,(c,(k',l',m'))))} \cong G$ if and only if $\mu(xy) = (\delta T_{Do}(T_V(x), T_V(y)), \theta T_{Do}(I_V(x), I_V(y)),$ $\vartheta T_{Do}(F_V(x), F_V(y))$, where $\delta = \dfrac{k'^2 + k'}{k'^3 - k}, \theta = \dfrac{l'^2 + l'}{l'^3 - l}$ and $\vartheta = \dfrac{2m'^2}{m' - m}$. □

Let $n \in \mathbb{N}$. Then from now on, will denote $G^{(c,(c,(k',l',m')))}$ by $G^{(c^2,(k',l',m'))}$, $G^{(c,c,(c,(k',l',m')))}$ by $G^{(c^3,(k',l',m'))}$ and $G^{(\underbrace{c,c,\dots,c}_{(n-1)\text{-times}},(c,(k',l',m')))}$ by $G^{(c^n,(k',l',m'))}$. Additionally consider $\delta_n = \dfrac{k'(k'^n - 1)}{((k')^{n+1} - k)(k' - 1)}), \theta_n = \dfrac{l'(l'^n - 1)}{((l')^{n+1} - l)(l' - 1)})$ and $\vartheta_n = \dfrac{nm'^2}{m' - m}$.

Corollary 3.17. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING and $k \in \mathbb{R}^{>0}$. Then*

(*i*) $G^{(c^n,(k',l',m'))} \cong G \Leftrightarrow \mu(xy) = (\delta_n(T_{Do}(T_V(x), T_V(y)), \theta_n(T_{Do}(I_V(x), I_V(y)), \vartheta_n(T_{Do}(F_V(x), F_V(y)))$.

(*ii*) $G^{(c^n,(k,l,2m))} \cong G \Leftrightarrow \mu(xy) = (a(T_{Do}(T_V(x), T_V(y))), b(T_{Do}(I_V(x), I_V(y))), (c(T_{Do}(F_V(x), F_V(y))))$, *where* $a = \dfrac{1}{k-1}, b = \dfrac{1}{l-1}$ *and* $c = 4nm$.

(*ii*) $G^{(c^n,(k,l,m))} \ncong G$

Theorem 3.18. *Let $G = (\sigma, \mu, T_{Do})$ be a complete (k, l, m)-DING and $k, l, m \in \mathbb{R}^{>0}$. Then*

(*i*) $G^{(c,(k',l',m'))} = (\sigma^c, \mu^c, T_{Do})$ *is a complete* $(\dfrac{k}{k' - k}, \dfrac{l}{l' - l}, \dfrac{1 - mm'}{m})$-*DING.*

(*ii*) $G^{(c^n,(k',l',m'))}$ *is a complete* $(\dfrac{k}{k'^n - k(\sum_{0 \le i \le n-1} k'^i)}, \dfrac{j}{j'^n - j(\sum_{0 \le i \le n-1} j'^i)}, \dfrac{1 - nmm'}{m})$-*DING.*

Proof. (i) Let $xy \in E$. Then

$$\begin{aligned} T_E^c(xy) &= k'T_E(xy) - T_{Do}(T_V(x), T_V(y)) = \frac{k'}{k}T_{Do}(T_V(x), T_V(y)) - T_{Do}(T_V(x), T_V(y)) \\ &= (\frac{k'-k}{k})T_{Do}(T_V(x), T_V(y)). \end{aligned}$$

In a similar way, $I_E^c(xy) = (\frac{l'-l}{l})T_{Do}(I_V(x), I_V(y))$. Moreover,

$$\begin{aligned} F_E^c(xy) &= F_E(xy) - m'T_{Do}(F_V(x), F_V(y)) = \frac{1}{m}T_{Do}(F_V(x), F_V(y)) - m'T_{Do}(F_V(x), F_V(y)) \\ &= (\frac{1-mm'}{m})T_{Do}(T_V(x), T_V(y)). \end{aligned}$$

(ii) It is obtained from (i) and by induction. □

Corollary 3.19. *Let $n \in \mathbb{N}, G = (\sigma, \mu, T_{Do})$ be a complete (k, l, m)-DING and $k, l, m \in \mathbb{R}^{>0}$. Then $G^{(c^n,(k,l,m))}$ can't be a complete (k, l, m)-DING.*

Definition 3.20. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING on a simple graph $G^* = (V, E)$. Then $O_T(G) = \sum_{x\in V} T_V(x)$ (as truth membership order of G), $O_I(G) = \sum_{x\in V} I_V(x)$ (as indeterminacy membership order of G), $O_F(G) = \sum_{x\in V} F_V(x)$ (as falsity membership order of G), $S_T(G) = k \sum_{\substack{xy\in E\\ x\neq y}} T_E(xy)$(as truth membership size of G), $S_I(G) = l \sum_{\substack{xy\in E\\ x\neq y}} I_E(xy)$(as indeterminacy membership size of G) and $S_F(G) = m \sum_{\substack{xy\in E\\ x\neq y}} F_E(xy)$(as indeterminacy membership size of G).*

Theorem 3.21. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING and $G' = (\sigma', \mu', T_{Do})$ be a (k', l', m') on $G^* = (V, E)$ and $G'^* = (V', E')$, respectively. Then*

(i) $O_T(G) = O_T(G')$, $O_I(G) = O_T(G')$ and $O_F(G) = O_F(G')$.
(ii) $S_T(G) = S_T(G')$, $S_I(G) = S_I(G')$ and $S_F(G) = S_F(G')$.

Proof. It is clear by definition. □

Example 3.22. *Let $V = \{a, b, c\}$. Then $G_1 = (\sigma_1, \mu_1, T_{Do}) \cong G_2 = (\sigma_2, \mu_2, T_{Do})$ are $(5, 5, 0.2)$-DING and G_2 and $(4, 6, 0.4)$-DING, respectively as shown in Figure 5. Computations show that $O_T(G_1) = O_T(G_2)$, $O_I(G_1) = O_T(G_2), O_F(G_1) = O_F(G_2), S_T(G_1) = S_T(G_2)$, $S_I(G_1) = S_I(G_2)$ and $S_F(G_1) = S_F(G_2)$, while $G_1 \ncong G_2$.*

Example 3.22, shows that the converse of the Theorem 3.24, is not necessarily true.

Definition 3.23. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING on a simple graph $G^* = (V, E)$ and $x \in V$. Then $d_T(x, G) = \sum_{xy\in E} T_E(xy)$ (as truth membership value of x), $d_I(x, G) = \sum_{xy\in E} I_E(xy)$ (as indeterminacy membership value of x), $d_F(x, G) = \sum_{xy\in E} F_E(xy)$ (as falsity membership value of x), $d_T(xy, G) = T_V(x) + T_V(y)$ (as truth membership value of xy), $d_I(xy, G) = I_V(x) + I_V(y)$ (as indeterminacy membership value of xy) and $d_F(xy, G) = F_V(x) + F_V(y)$ (as falsity membership value of xy).*

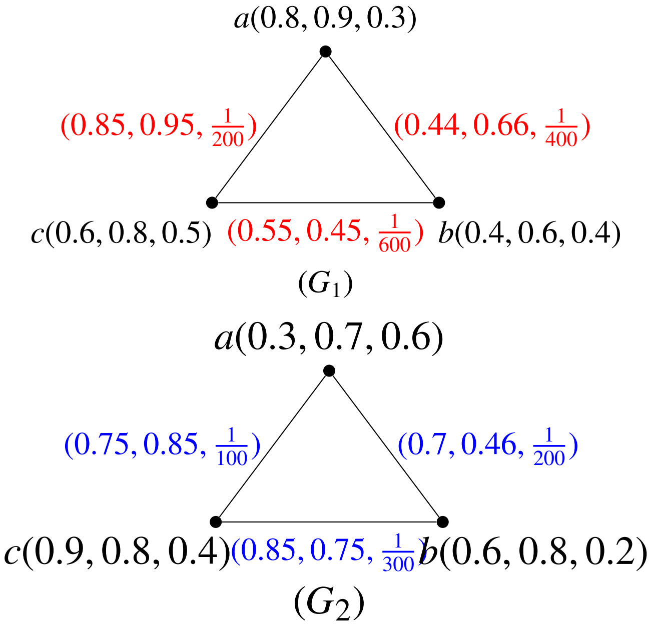

Figure 5. $(5, 5, 0.2)$-DING G_1 , $(4, 5, 0.4)$-DING G_2.

Theorem 3.24. *Let $G = (\sigma, \mu, T_{Do})$ be a (k, l, m)-DING and $G' = (\sigma', \mu', T_{Do})$ be a (k', l', m') on $G^* = (V, E)$ and $G'^* = (V', E')$, respectively. Then*

(i) $d_T(x, G) = \dfrac{k'}{k} d_T(x, G')$, $d_T(x, G) = \dfrac{l'}{l} d_T(x, G')$ *and* $d_T(x, G) = \dfrac{m'}{m} d_T(x, G')$.

(ii) $S_T(G) = 2kd_T(x, G)$, $S_I(G) = 2ld_I(x, G)$ *and* $S_F(G) = 2md_F(x, G)$.

Proof. It is clear by definition. □

4. An application of valued inverse Dombi neutrosophic graph

In this section, we apply the concept of valued inverse Dombi neutrosophic graph in the real world. In mathematics, a network is called a graph, objects are called vertices (or nodes), and the connections are called edges. For example, when we represent the social network of a collection of people as a graph, the vertices are the people under consideration, while the edges list all the friendships between them. **Social network:** Establishing social relations between people can have an impactive on a person's life. In today's developing world, social relations are one of the basic principles for the durability and sustainability of individual life. According to the experts in sociology, social relations can be defined as communication between two people or several groups, that is, social relations are the communication and interdependence of people towards each other. These relationships can be positive and constructive, like friendship, or negative and destructive, like enmity. In the unique sense of social communication, it refers to communication that leads to the transfer of meaning or message among a group of people. Therefore, social communication requires several elements such as intelligence (as a truth membership degree), incuriosity (as an indeterministic membership degree) and deception (as a falsity membership degree). Now, we consider a complex network, which we investigated as the following algorithm:

step1: In Table 1, we introduce a group of people with some characteristics of intelligence, incuriosity, and deception in social personality that are supposed to work in a social network(for instance, Benjamin$(0.6, 0.7, 0.1)$ means that, based on psychological and scientific research, Benjamin uses 60 percent of his talent, 70 percent is curious and 10 percent is deceitful).

step2: In Figure 6 (the vertex set represents the people and the edge set represents the communication between them), we there are essential factors that can create communication between people in the group such as mutual respect (as a truth membership degree), responsibility (as an indeterministic membership degree) and insincere (as a falsity membership degree). In this step, we compute the relation between of vertices with impact $(1, 1, 10)$. For instance, $T_E(BO) = \frac{0.6 \times 0.7}{0.6 + 0.7 - 0.42} = \frac{21}{44}, I_E(BO) = \frac{0.7 \times 0.8}{0.7 + 0.8 - 0.56} = \frac{28}{47}, F_E(BO) = \frac{1}{10}\frac{0.1 \times 0.2}{0.1 + 0.2 - 0.02} = \frac{1}{140}$.

Table 1. Social personality.

	Intelligence	Incuriosity	Deception
Benjamin	0.6	0.7	0.1
Oliver	0.7	0.8	0.2
James	0.8	0.9	0.3
William	0.4	0.5	0.4
Lucas	0.5	0.6	0.5

step 3: By definition 3.23, we want to identify the best one in terms of the most mutual respect, the most responsibility and the least insincere and to identify the best co-worker in terms of the most intelligence and mutual respect, incuriosity, and responsibility, and the least deception and Insincere. Thus, computations show these aims in Tables 2 and 3. For instance, $d_T(W, G) = \frac{4}{11} + \frac{6}{19} \simeq 0.7, d_T(B, G) = \frac{21}{44} + \frac{6}{19} \simeq 0.8$ and so $d_T(BW) = 0.7 + 0.8 = 1.5$.

Table 2. Best candidate in term of mutual respect, responsibility, and insincere.

$d_T(x, G)$	$d_I(x, G)$	$d_F(x, G)$
(O, 1.5)	(O, 1.8)	(B, 0.02)
(J, 1)	(J, 1.2)	(L, 0.02)
(B, 0.8)	(B, 1)	(J, 0.03)
(W, 0.7)	(W, 0.9)	(W, 0.03)
(L, 0.4)	(L, 0.5)	(O, 0.04)

Table 3. Best candidate in term of co-worker intelligence and mutual respect, co-worker incuriosity and responsibility and co-worker deception and insincere.

$d_T(xy, G)$	$d_I(xy, G)$	$d_F(xy, G)$
(OJ, 2.5)	(OJ, 3)	(WB, 0.05)
(BO, 2.3)	(BO, 2.8)	(BO, 0.06)
(LO, 1.9)	(LO, 2.3)	(JW, 0.06)
(WJ, 1.7)	(JW, 2.1)	(LO, 0.06)
(BW, 1.5)	(BW, 1.9)	(JO, 0.07)

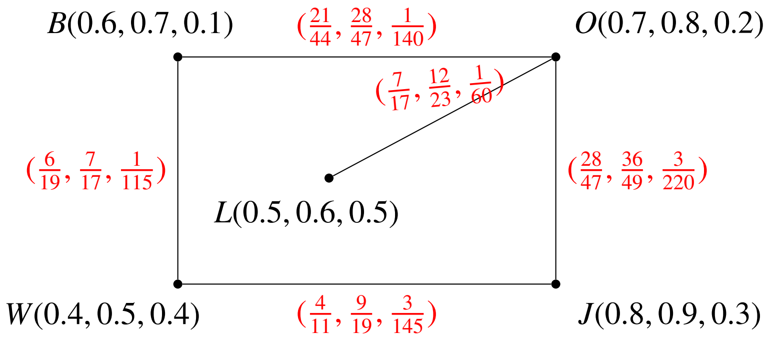

Figure 6. DING $G = (\sigma, \mu, T_{Do})$.

step 4: From Table 2, we find that the best candidate In term of mutual respect and responsibility is Oliver, and the best candidate In term of insincere is Benjamin. Additionally for any $x, y, d_T(JO) \geq d_T(xy), d_I(JO) \geq d_I(xy)$, while $d_F(JO) \leq d_T(xy)$.

5. Conclusions

The current paper has introduced a novel concept of valued-inverse Dombi neutrosophic graph and (complete) strong valued-inverse Dombi neutrosophic graph as a generalization of the inverse neutrosophic graph. This study investigates some conditions such that neutrosophic subsets be valued-inverse Dombi neutrosophic graphs and Dombi neutrosophic graphs.The complement of valued-inverse Dombi neutrosophic graphs is introduced and concerning the notation of isomorphism theorems, we can prove that the complement of a valued-inverse Dombi neutrosophic graph is a valued-inverse Dombi neutrosophic graph. The complement of valued-inverse Dombi neutrosophic graphs is extended by induction and the isomorphic complement of valued-inverse Dombi neutrosophic graphs is analyzed. Finally, we apply these concepts to the real world and design a real problem in the social network. Also

(i) we show that for any given (k, l, m)-DING, with what changes of parameters is a valued-inverse Dombi neutrosophic graph.

(ii) We show that for any given (k, l, m)-DING, conditions in parameters, is a strong valued-inverse Dombi neutrosophic graph and strong (k, l, m)-DING.

(iii) The solving of non-equations plays a prominent role in finding the ordered triple for a value of complement of valued-inverse Dombi neutrosophic graphs.

(iv) We find the conditions on ordered triples and how a complement of strong valued-inverse Dombi neutrosophic graph is strong.

(v) It is introduced as the notation of homomorphism (isomorphism), and it proves how the complement of two isomorphic valued-inverse Dombi neutrosophic graphs are isomorphic.

(vi) It introduces the concept of self-valued-inverse Dombi of strong (k, l, m)-DING and investigates the relation between the components of its neutrosophic vertices and neutrosophic edges.

(vii) It presents the truth membership order, indeterminacy membership order, falsity membership order, truth membership size, indeterminacy membership size, and indeterminacy membership size of any given (k, l, m)-DING.

($viii$) The truth membership order, indeterminacy membership order, falsity membership order, truth membership size, indeterminacy membership size, and indeterminacy membership size of any given (k, l, m)-DING play the prominent role in the application of valued-inverse Dombi neutrosophic graphs in complex networks.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

Conflict of interest

The authors declare that they have no conflict of interest.

References

1. M. Akram, S. Shahzadi, A. B. Saeid, Single valued neutrosophic Hypergraphs, *TWMS J. Appl. Eng. Math.*, **8** (2018), 122–135.
2. M. Akram, *Single-Valued Neutrosophic Graphs*, Singapore: Springer, 2018. https://doi.org/10.1007/978-981-13-3522-8
3. M. Akram, H. Saba Nawaz, Implementation of single-valued neutrosophic soft hypergraphs on human nervous system, *Artif. Intell. Rev.*, **56** (2022), 1387–1425.
4. M. Akram, H. Saba Nawaz, Algorithms for the computation of regular single-valued neutrosophic soft hypergraphs applied to supranational asian bodies, *J. Appl. Math. Comput.* , **68** (2022), 4479–4506.
5. D. Ajay, P. Chellamani, G. Rajchakit, N. Boonsatit, P. Hammachukiattikul, Regularity of Pythagorean neutrosophic graphs with an illustration in MCDM, *AIMS Mathematics*, **7** (2022), 9424–9442. https://doi.org/10.3934/math.2022523
6. M. Akram, M. H Sarwar, W. A. Dudek, Bipolar Neutrosophic Graph Structures, In: *Graphs for the Analysis of Bipolar Fuzzy Information*, Singapore: Springer, 2020, 393–446. https://doi.org/10.1007/978-981-15-8756-6_10
7. S. Ashraf, S. Naz, E. E. Kerre, Dombi Fuzzy Graphs, *Fuzzy Inf. Eng.*, **10** (2018), 58–79. https://doi.org/10.1080/16168658.2018.1509520
8. R. A. Borzooei, R. Almallah, Y. B. Jun, H. Ghaznavi, Inverse Fuzzy Graphs with Applications, *New Math. Nat. Comput.*, **16** (2020), 397–418. https://doi.org/10.1142/S1793005720500246
9. G. Ghorai, M. Pal, Certain types of product bipolar fuzzy graphs, *Int. J. Appl. Comput. Math.*, **3** (2017), 605–619.
10. S. Jahanpanah, M. Hamidi, Valued-Inverse Dombi Fuzzy Graph, 2023, Submitted.
11. M. Hamidi, S. Jahanpanah, A. Radfar, Extended graphs based on KM-fuzzy metric spaces, *Iran. J. Fuzzy Syst.*, **17** (2020), 81–95.
12. M. Hamidi, S. Jahanpanah, A. Radfar, On KM-Fuzzy Metric Hypergraphs, *Fuzzy Inf. Eng.*, **12** (2020), 300–321. https://doi.org/10.1080/16168658.2020.1867419

13. A. Kaufmann, *Introduction to the theory of fuzzy subsets*, New York: Academic Press, 1975.
14. T. S. Lakhwani, K. Mohanta, A. Dey, S. P. Mondal, A. Pal, Some operations on Dombi neutrosophic graph, *J. Ambient Intell. Human. Comput.*, **13** (2022), 425–443. https://doi.org/10.1007/s12652-021-02909-3
15. K. Mohanta, A. Dey, A. Pal, A note on different types of product of neutrosophic graphs, *Complex. Intell. Syst.*, **7** (2021), 857–871. https://doi.org/10.1007/s40747-020-00238-0
16. R. Mahapatra, S. Samanta, M. Pal, Generalized neutrosophic planar graphs and its application, *J. Appl. Math. Comput.*, **65** (2021), 693–712. https://doi.org/10.1007/s12190-020-01411-x
17. H. T. Nguyen, C. L. Walker, E. A. Walker, *A First Course in Fuzzy Logic*, 4 Eds., Boca Raton: CRC Press, 2018.
18. A. Rosenfeld, Fuzzy graphs, In: *Fuzzy sets and their applications to cognitive and decision processes*, New York: Academic Press, 1975, 77–95. https://doi.org/10.1016/B978-0-12-775260-0.50008-6
19. S. Siddique, U. Ahmad, M. Akram, A study on generalized graphs representations of complex neutrosophic information, *J. Appl. Math. Comput.*, **65** (2021), 481–514. https://doi.org/10.1007/s12190-020-01400-0
20. S. Sahin, A. Kargın, M. Yucel, Hausdorff Measures on Generalized Set Valued Neutrosophic Quadruple Numbers and Decision Making Applications for Adequacy of Online Education, *Neutrosophic Sets Syst.*, **40** (2021), 86–116.
21. F. Smarandache, Neutrosophic set a generalization of the intuitionistic fuzzy set, *Int. J. Pure. Appl. Math.*, **24** (2005), 287–297.
22. S. Zeng, M. Shoaib, S. Ali, F. Smarandache, H. Rashmanlou, F. Mofidnakhaei, Certain Properties of Single-Valued Neutrosophic Graph With Application in Food and Agriculture Organization, *Int. J. Comput. Intell.*, **14** (2021), 1516–1540. https://doi.org/10.2991/ijcis.d.210413.001
23. J. Zhan, M. Akram, M. Sitara, Novel decision-making method based on bipolar neutrosophic information, *Soft Comput.*, **23** (2019), 9955–9977.

AIMS Press