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Research article

Research on VIKOR group decision making using WOWA operator based on interval Pythagorean triangular fuzzy numbers

Jun Hu[1,2,*], Jie Wu[2] and Mengzhe Wang[1]

[1] Huaiyin Normal University, Huai'an, Jiangsu, China
[2] Jiangsu University of Science and Technology, Zhenjiang, Jiangsu, China

* **Correspondence:** Email: 1434120126@qq.com; Tel: 18361819679.

Abstract: A new decision-making method based on interval Pythagorean triangular fuzzy numbers is proposed for fuzzy information decision-making problems, taking the advantages of interval Pythagorean fuzzy numbers and triangular fuzzy numbers into account. The VIse Kriterijumski Optimizacioni Racun (VIKOR) group decision-making method is based on the Weighted Ordered Weighted Average (WOWA) operator of interval Pythagorean triangular fuzzy numbers (IVPTFWOWA). First, this article provides the definition of the IVPTFWOWA operator and proves its degeneracy, idempotence, monotonicity, and boundedness. Second, the decision steps of the VIKOR decision method using the IVPTFWOWA operator are presented. Finally, the scientificity and effectiveness of the proposed method were verified through case studies and comparative discussions. The research results indicate that the following: (1) the IVPTFWOWA operator combines interval Pythagorean fuzzy numbers and triangular fuzzy numbers, complementing the shortcomings of the two fuzzy numbers, and can characterize fuzzy information on continuous geometry, thereby reducing decision errors caused by inaccurate and fuzzy information; (2) the VIKOR decision-making method based on the IVPTFWOWA operator applies comprehensive weights, fully considering the positional weights of the scheme attributes and the weights of raters, and fully utilizing the attribute features of decision-makers and cases; and (3) compared to other methods, there is a significant gap between the decision results obtained using this method, making it easier to identify the optimal solution.

Keywords: triangular fuzzy numbers; interval fuzziness; WOWA operator; VIKOR
Mathematics Subject Classification: 47B92, 47B93, 47A13, 47N70

1. Introduction

The accurate characterization and aggregation of group decision-making information is crucial for decision results; however, accurate decision-making information is often difficult to obtain in real-world decision-making. For multi-attribute group decision-making problems in fuzzy information environments, the ability to characterize fuzzy information and aggregate decision evaluation values is important. At present, scholars internationally have conducted extensive research on this type of problem, with fruitful results. The proposal of the Weighted Average (WA) operator, the Ordered Weighted Average (OWA) operator, the Weighted Ordered Weighted Average (WOWA) operator, the Generalized Ordered Weighted Average (GWOWA) operator, and the fusion operator basically solve the problem of aggregation of evaluation values in group decision-making process. In response to the difficulty of characterizing fuzzy information, Zadeh [1] first proposed the concept of fuzzy sets in 1965; since then, scholars at home and abroad have started a wave of research on fuzzy sets. On the basis of fuzzy sets, scholars have gradually focused their attention on fuzzy evaluation and extended intuitionistic fuzzy sets [2,3], interval Pythagorean fuzzy sets [4], and triangular fuzzy sets [12–18]. The fusion research of various fuzzy numbers and aggregation operators has also been a hot topic in the field of fuzzy mathematics in recent years. Many international scholars have proposed the study of aggregation operators based on intuitionistic fuzzy, Pythagorean fuzzy, interval Pythagorean fuzzy, triangular fuzzy, and other backgrounds, such as applying WA operators [5], OWA operators [6], WOWA operators [7], GWOWA operators [8], etc. to fuzzy environments, as well as multi decision models in the context of the recurrent fuzzy information environment [9,10] and multi criteria decision-making based on hesitant fuzzy language entropy [11].

Considering that triangular fuzzy numbers have advantages that other forms of data do not have, some scholars have begun to study the combination of triangular fuzzy sets and various operators. For example, Jianming Zhang et al. [12–14] studied the ternary decision method of regret theory for triangular fuzzy numbers; Linjia Jiang et al. [15] studied the aggregation model of triangular intuitionistic fuzzy sets; Xiaoyan Su et al. [16] studied interval Python triangular fuzzy ensemble operators; Meijuan Li [17], Chunquan Li [18], and others have also conducted research on the fuzzy research and application of triangular fuzzy operators.

There are many research methods used for evaluation. For example, S. K. Sahoo et al. [19] studied multiple criteria decision-making (MCDM) methods. M. J. Ranjan et al. [20] studied probabilistic linguistic q-rung orthopair fuzzy archimedean aggregation operators for group decision-making. Yingxue Du et al. [21] studied the Pythagorean triangular fuzzy VIse Kriterijumski Optimizacioni Racun (VIKOR) decision; Gou et al. [22] studied the probabilistic, two-level language terminology set and its application to improve the design of the VIKOR method. B. F. Yildirim et al. [23] evaluated the satisfaction level of citizens in municipality services by using the picture fuzzy VIKOR method. Through the research of these scholars, it can be found that the VIKOR method can simultaneously consider maximizing group utility and minimizing individual regret, combined with the subjective preferences of decision-makers, thus possessing high ranking stability and credibility. The VIKOR method proposes a compromise solution with advantages based on the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method. Compared with these methods, the VIKOR method not only has the concepts of positive and negative ideal values of the former, but also has a compromise solution that negotiates the interests of individuals and groups, making it more suitable for solving multi criteria decision-making problems with conflicts of interest.

It can be seen that the existing research focuses more on the aggregation of fuzzy environments and operators, while there is relatively little research on the combination of interval Python fuzzy and triangular fuzzy numbers. However, interval Python triangular fuzzy numbers have advantages that other forms of data do not have. They can not only break through the boundaries of membership and non-membership, but also describe decision information on continuous sets. Therefore, considering the characteristics that triangular fuzzy numbers can describe decision information on continuous sets, this paper combines interval Pythagorean fuzzy and triangular fuzzy numbers, and reconstructs interval Pythagorean triangular fuzzy numbers. As a cushion, a new method for MCDM under interval fuzzy conditions is reconstructed by integrating interval Pythagorean triangular fuzzy numbers and WOWA operators. This method has the following advantages: first, it can minimize the decision-making errors caused by the lack of information; second, we can fully derive the attribute advantages and scoring attribute characteristics of each scorer, then fully consider the location weight of the scheme attribute and the weight of the scorers; and third, compared with other methods, the decision results obtained by using this method have a large gap, which makes it easier to identify the optimal scheme.

2. Relevant definitions

In order to solve and accurately describe the attribute evaluated problem under uncertain information environment, researchers have proposed the notion of fuzzy sets. After the introduction and in-depth study of fuzzy concepts, experts and scholars gradually proposed intuitive fuzzy sets and hesitant fuzzy sets. This paper reconstructs the weighted ordered and weighted average operator of interval triangular fuzzy numbers by fusing triangular fuzzy and interval fuzzy numbers after integrating previous research results.

2.1. The notion of interval Pythagorean fuzzy sets

Definition 1. [7] If X is a domain, then the definable interval Pythagorean fuzzy set is $\tilde{P}=\left\{\left\langle x,\tilde{P}(\mu_a^-(x),\mu_a^+(x)),\tilde{P}(v_a^-(x),v_a^-(x))\right\rangle|x\in X\right\}$. μ_a^- and μ_a^+ are the lower and higher limits of membership degree of interval Pythagorean fuzzy sets, respectively, and v_a^-, v_a^+ are the lower and higher limits of non-membership, respectively. $\tilde{P}$ conforms to $0\le(\mu_a^+)^2+(v_a^-)^2\le 1$. Then, the hesitation degree for $\tilde{P}$ is $\pi_a(x)=\left[\pi_a^-(x),\pi_a^+(x)\right]=\left[\sqrt{1-(\mu_a^+)^2-(v_a^+)^2},\sqrt{1-(\mu_a^-)^2-(v_a^-)^2}\right]$.

Definition 2. [7] Scoring function and precision function. We defined the scoring function of $\tilde{\alpha}=\left(\tilde{P}(\mu_a^-(x),\mu_a^+(x)),\tilde{P}(v_a^-(x),v_a^+(x))\right)$ as $S(\tilde{\alpha})=\frac{1}{2}\left((\mu_a^-)^2-(v_a^-)^2+(\mu_a^+)^2-(v_a^+)^2\right)$, and the accuracy function of $\tilde{\alpha}=\left(\tilde{P}(\mu_a^-,\mu_a^+),\tilde{P}(v_a^-,v_a^+)\right)$ as $H(\tilde{\alpha})=\frac{1}{2}\left((\mu_a^-)^2+(v_a^-)^2+(\mu_a^+)^2+(v_a^+)^2\right)$, $S(\tilde{\alpha})\in[-1,1]$ and $H(\tilde{\alpha})\in[0,1]$.

Definition 3. [7] Algorithm. For $\tilde{\alpha}_i=\left(\tilde{P}(\mu_{ai}^-(x),\mu_{ai}^+(x)),\tilde{P}(v_{ai}^-(x),v_{ai}^+(x))\right)(i=1,2)$, which must satisfy the following algorithm:

1) $\tilde{\alpha}_1\oplus\tilde{\alpha}_2=\left(\begin{bmatrix}\sqrt{(\mu_{a1}^-)^2+(\mu_{a2}^-)^2-(\mu_{a1}^-)^2(\mu_{a2}^-)^2},\\ \sqrt{(\mu_{a1}^+)^2+(\mu_{a2}^+)^2-(\mu_{a1}^+)^2(\mu_{a2}^+)^2}\end{bmatrix},\begin{bmatrix}v_{a1}^-v_{a2}^-,\\ v_{a1}^+v_{a2}^+\end{bmatrix}\right)$.

2) $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = \left(\begin{bmatrix} \mu_{a1}^- \mu_{a2}^-, \\ \mu_{a1}^+ \mu_{a2}^+ \end{bmatrix}, \begin{bmatrix} \sqrt{(v_{a1}^-)^2 + (v_{a2}^-)^2 - (v_{a1}^-)^2 (v_{a2}^-)^2}, \\ \sqrt{(v_{a1}^+)^2 + (v_{a2}^+)^2 - (v_{a1}^+)^2 (v_{a2}^+)^2} \end{bmatrix} \right).$

3) $\lambda \tilde{\alpha} = \left(\left[\sqrt{1-(1-(\mu_a^-)^2)^\lambda}, \sqrt{1-(1-(\mu_a^+)^2)^\lambda} \right], \left[(v_a^-)^\lambda, (v_a^+)^\lambda \right] \right).$

4) $\tilde{\alpha}^\lambda = \left(\left[(\mu_a^-)^\lambda, (\mu_a^+)^\lambda \right], \left[\sqrt{1-(1-(v_a^-)^2)^\lambda}, \sqrt{1-(1-(v_a^+)^2)^\lambda} \right] \right).$

Definition 4. [7] Distance definition. For $\tilde{\alpha}_i = \left(\tilde{P}(\mu_{ai}^-, \mu_{ai}^+), \tilde{P}(v_{ai}^-, v_{ai}^+) \right)$, whose Hamming distance $d(\tilde{\alpha}_1, \tilde{\alpha}_2)$ can be defined as follows:

$$d(\tilde{\alpha}_1, \tilde{\alpha}_2) = \frac{1}{4}\left(\left|\mu_{a1}^- - \mu_{a2}^-\right| + \left|\mu_{a1}^+ - \mu_{a2}^+\right| + \left|v_{a1}^- - v_{a2}^-\right| + \left|v_{a1}^+ - v_{a2}^+\right| \right).$$

2.2. The notion of interval Pythagorean triangular fuzzy numbers

On the basis of interval triangular fuzzy numbers proposed by experts and scholars, some researchers have already started to explore interval Pythagorean triangular fuzzy numbers by integrating interval concepts and triangular fuzzy numbers.

Definition 5. [19] If $\tilde{a} = \left\langle (\underline{\tau}, \tau, \overline{\tau}); [\mu_a^-, \mu_a^+], [v_a^-, v_a^+] \right\rangle$, define $\tilde{a}$ as an interval Pythagorean triangular fuzzy number on R, so record it as IVPTFN. μ_a^- and μ_a^+ represent the lower and higher limits of the maximum membership degree of $\tilde{a}$, respectively, and v_a^- and v_a^+ represent the lower and higher limits of the minimum membership degree of $\tilde{a}$, respectively, where $0 \le [\mu_a^-, \mu_a^+] \le 1$, $0 \le [v_a^-, v_a^+] \le 1$ and $0 \le (\mu_a^+)^2 + (v_a^-)^2 \le 1$.

Definition 6. [19] For $\tilde{a} = \left\langle (\underline{\tau}, \tau, \overline{\tau}); [\mu_a^-, \mu_a^+], [v_a^-, v_a^+] \right\rangle$, $S(\tilde{a}) = \frac{\underline{\tau} + 2\tau + \overline{\tau}}{4} \bullet \frac{\left((\mu_a^-)^2 - (v_a^-)^2 + (\mu_a^+)^2 - (v_a^+)^2\right)}{2}$ is its score function, and $H(\tilde{a}) = \frac{\underline{\tau} + 2\tau + \overline{\tau}}{4} \bullet \frac{\left((\mu_a^-)^2 + (v_a^-)^2 + (\mu_a^+)^2 + (v_a^+)^2\right)}{2}$ is its exact function.

Definition 7. [19] For two interval Pythagorean triangular fuzzy numbers $\tilde{a}_i = \left\langle (\underline{\tau}_i, \tau_i, \overline{\tau}_i); [\mu_{a_i}^-, \mu_{a_i}^+], [v_{a_i}^-, v_{a_i}^+] \right\rangle$, the algorithm is as follows:

1) $\tilde{a}_1 \oplus \tilde{a}_2 = \left\langle \begin{matrix} (\underline{\tau}_1 + \underline{\tau}_2, \tau_1 + \tau_2, \overline{\tau}_1 + \overline{\tau}_2); \\ [\sqrt{(\mu_{a_1}^-)^2 + (\mu_{a_2}^-)^2 - (\mu_{a_1}^-)^2 (\mu_{a_2}^-)^2}, \sqrt{(\mu_{a_1}^+)^2 + (\mu_{a_2}^+)^2 - (\mu_{a_1}^+)^2 (\mu_{a_2}^+)^2}], [v_{a_1}^- v_{a_2}^-, v_{a_1}^+ v_{a_2}^+] \end{matrix} \right\rangle.$

2) $\tilde{a}_1 \otimes \tilde{a}_2 = \left\langle \begin{matrix} (\underline{\tau_1 \tau_2}, \tau_1 \tau_2, \overline{\tau_1 \tau_2}); \\ [\mu_{a_1}^- \mu_{a_2}^-, \mu_{a_1}^+ \mu_{a_2}^+], [\sqrt{(v_{a_1}^-)^2 + (v_{a_2}^-)^2 - (v_{a_1}^-)^2 (v_{a_2}^-)^2}, \sqrt{(v_{a_1}^+)^2 + (v_{a_2}^+)^2 - (v_{a_1}^+)^2 (v_{a_2}^+)^2}] \end{matrix} \right\rangle.$

3) $\lambda \tilde{a} = \left\langle (\lambda \underline{\tau}, \lambda \tau, \lambda \overline{\tau}); \left[\sqrt{1-(1-(\mu_a^-)^2)^\lambda}, \sqrt{1-(1-(\mu_a^+)^2)^\lambda} \right], \left[(v_a^-)^\lambda, (v_a^+)^\lambda \right] \right\rangle.$

4) $\tilde{a}^\lambda = \left\langle (\underline{\tau}^\lambda, \tau^\lambda, \overline{\tau}^\lambda); \left[(\mu_a^-)^\lambda, (\mu_a^+)^\lambda \right], \left[\sqrt{1-(1-(v_a^-)^2)^\lambda}, \sqrt{1-(1-(v_a^+)^2)^\lambda} \right] \right\rangle.$

Definition 8. [19] For two interval Pythagorean triangular fuzzy numbers $\tilde{a}_i = \left\langle (\underline{a}_i, a_i, \overline{a}_i); [u_{a_i}^-, u_{a_i}^+], [v_{a_i}^-, v_{a_i}^+] \right\rangle (i = 1, 2)$, the Hamming distance $d(\tilde{a}_1, \tilde{a}_2)$ is defined as follows:

$$d(\tilde{a}_1,\tilde{a}_2)=\frac{1}{4}\left(\begin{array}{l}\left|\frac{\underline{\tau_1}+2\tau_1+\overline{\tau_1}}{4}\mu_{a_1}^{-}-\frac{\underline{\tau_2}+2\tau_2+\overline{\tau_2}}{4}\mu_{a_2}^{-}\right|+\left|\frac{\underline{\tau_1}+2\tau_1+\overline{\tau_1}}{4}\mu_{a_1}^{+}-\frac{\underline{\tau_2}+2\tau_2+\overline{\tau_2}}{4}\mu_{a_2}^{+}\right|\\+\left|\frac{\underline{\tau_1}+2\tau_1+\overline{\tau_1}}{4}v_{a_1}^{-}-\frac{\underline{\tau_2}+2\tau_2+\overline{\tau_2}}{4}v_{a_2}^{-}\right|+\left|\frac{\underline{\tau_1}+2\tau_1+\overline{\tau_1}}{4}v_{a_1}^{+}-\frac{\underline{\tau_2}+2\tau_2+\overline{\tau_2}}{4}v_{a_2}^{+}\right|\end{array}\right).$$

2.3. The notion of WOWA operator

Many years ago, academia has put forward the concept of the weighted average operator (WA), and studied a series of evaluated value aggregation operators, including the weighted ordered weighted average operator (WOWA). With the rise of the grey decision theory and the interval fuzzy theory, there is more and more research on the integration of the aggregation operator and interval fuzzy theory.

Definition 9. [10] There is a group of data $(\alpha_1,\alpha_2,\alpha_3,...,\alpha_n)$, where $\psi=(\psi_1,\psi_2,\psi_3,...,\psi_n)^T$ and $\omega=(\omega_1,\omega_2,\omega_3,...,\omega_n)^T$ are the general weight and position weight of data, respectively, and $\psi_i\in[0,1]$, $\omega_i\in[0,1]$, $\sum_{i=1}^{n}\psi_i=1$, and $\sum_{i=1}^{n}\omega_i=1$. Suppose there is a mapping WOWA: $R^n\to R$, $WOWA(\alpha_1,\alpha_2,\alpha_3,...,\alpha_n)=\sum_{i=1}^{n}w_i\alpha_{\sigma(i)}$, where $(\alpha_{\sigma(1)},\alpha_{\sigma(2)},\alpha_{\sigma(3)},...,\alpha_{\sigma(n)})$ is simply the movement of $(\alpha_1,\alpha_2,\alpha_3,...,\alpha_n)$ in a position, and any $i<j$, $\alpha_{\sigma(i)}>\alpha_{\sigma(j)}$ is satisfied; $w=(w_1,w_2,w_3,...,w_n)^T$ is the comprehensive weight vector of WOWA and satisfies $w_i=\omega^*(\sum_{j\le i}\psi_{\sigma(j)})-\omega^*(\sum_{j<i}\psi_{\sigma(j)})$.

3. WOWA operator based on IVPTF

According to the fuzzy uncertainty characteristics of decision information, this study proposes a new operator, abbreviated as the IVPTFWOWA operator. The definition and property proof of the IVPTFWOWA operator are given below.

Definition 10. There are interval Pythagorean triangular fuzzy numbers $\tilde{a}=\langle(\underline{\tau},\tau,\overline{\tau});[\mu_a^-,\mu_a^+],[v_a^-,v_a^+]\rangle$, where $\psi=(\psi_1,\psi_2,\psi_3,...,\psi_n)^T$ and $\omega=(\omega_1,\omega_2,\omega_3,...,\omega_n)^T$ are the general weight and position weight of IVPTF, respectively. $w_i(i=1,2,3,...,n)$ is the comprehensive weight of $\tilde{a}_i$, which satisfies $w_i\in[0,1]$, $\sum_{i=1}^{n}w_i=1$ and $w_i=\omega^*(\sum_{j\le i}\psi_{\sigma(j)})-\omega^*(\sum_{j<i}\psi_{\sigma(j)})$, and ω^* is the monotone increasing function passing through the point $(\frac{i}{n},\sum_{j\le i}\omega_j)$ and the point (0,0). If $(\tilde{a}_{\sigma(1)},\tilde{a}_{\sigma(2)},\tilde{a}_{\sigma(3)},...,\tilde{a}_{\sigma(n)})$ is the position exchange of $(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,...,\tilde{a}_n)$ and satisfies any $i<j$, then $\tilde{a}_{\sigma(i)}>\tilde{a}_{\sigma(j)}$. Then, the IVPTFWOWA operator of $\tilde{a}_i$ can be defined as follows:

$$IVPTFWOWA(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,...,\tilde{a}_n)=w_1\tilde{a}_{\sigma(1)}\oplus w_2\tilde{a}_{\sigma(2)}\oplus w_3\tilde{a}_{\sigma(3)}\oplus...\oplus w_n\tilde{a}_{\sigma(n)}$$
$$=\left\langle\begin{array}{l}(\sum_{i=1}^{n}w_i\underline{\tau_{\sigma(i)}},\sum_{i=1}^{n}w_i\tau_{\sigma(i)},\sum_{i=1}^{n}w_i\overline{\tau_{\sigma(i)}});\\\left[\sqrt{1-\prod_{i=1}^{n}(1-(\mu_{a_{\sigma(i)}}^{-})^2)^{w_i}},\sqrt{1-\prod_{i=1}^{n}(1-(\mu_{a_{\sigma(i)}}^{+})^2)^{w_i}}\right],\left[\prod_{i=1}^{n}(v_{a_{\sigma(i)}}^{-})^{w_i},\prod_{i=1}^{n}(v_{a_{\sigma(i)}}^{+})^{w_i}\right]\end{array}\right\rangle.$$

Theorem 1. Through a proof, it can be found that the new aggregation operator (IVPTFWOWA operator) formed by the fusion of interval Pythagorean triangular fuzzy numbers and WOWA operators is an evolutionary operator of IVPTFWA operators, while possessing the monotonicity, idempotence, and boundedness of aggregation operators. (The proof process can be found in Appendix I).

4. VIKOR decision steps based on IVPTFWOWA operator

VIKOR and TOPSIS decision-making methods are widely used. This paper attempts to integrate the IVPTFWOWA operator and the VIKOR method to meet the decision-making problem. The steps of the VIKOR decision-based method on the IVPTFWOWA operator are given below.

Suppose there is a multi-attribute group decision making problem with interval Pythagorean triangular fuzzy numbers. Let the decision-maker set be $K=\{K_1,K_2,K_3,...,K_n\}$, the scheme set be $M=\{M_1,M_2,M_3,...,M_n\}$, scheme attribute set be $C=\{C_1,C_2,C_3,...,C_n\}$, and the position weight be $\omega=(\omega_1,\omega_2,\omega_3,...,\omega_n)^T$. Multiple decision-makers evaluate and score each scheme according to the attribute of the scheme, and obtain an evaluated matrix of the decision-makers as follows:

$$N(k)=\begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{bmatrix},$$

where $\tilde{a}_{ij}=\left\langle(\underline{a_{ij}},a_{ij},\overline{a_{ij}});[u^-_{a_{ij}},u^+_{a_{ij}}],[v^-_{a_{ij}},v^+_{a_{ij}}]\right\rangle$ represents the evaluated value of the attribute i of the scheme j by a decision-maker.

The attribute of the scheme may either be the benefit type or the cost type. First, we need to normalize the decision matrices given by the scorers. According to the standardized fuzzy evaluation value and the weight of the scorer, we can use the IVPTFWOWA operator proposed in this article to fuse the evaluation scores given by multiple scorers into a comprehensive evaluation value matrix. Therefore, according to the new IVPTFWOWA operator step, the weight of the scorer is first determined. At this point, this article selects the VIKOR means to resolve the weight of the scorer, and defines the weight set of the decision-maker as $\lambda=\{\lambda_1,\lambda_2,\lambda_3,...,\lambda_n\}$, where $\sum_{i=1}^{n}\lambda_i=1$.The detailed decision-making process diagram is shown in Figure 1.

The decision-making steps are as follows.

Step 1: Standardize matrices. In this paper, the Pythagorean triangular fuzzy number normalization method [24] is used to standardize the decision matrix. The normalized decision-makers evaluated value matrices is marked as $D(k)$.

(1) When the scheme attribute is the benefit type, set $\tilde{a}_{ij}=\left\langle(\frac{\underline{\tau_{ij}}}{\overline{\tau_j}^+},\frac{\tau_{ij}}{\overline{\tau_j}^+},\frac{\overline{\tau_{ij}}}{\overline{\tau_j}^+});[\mu^-_{a_{ij}},\mu^+_{a_{ij}}],[v^-_{a_{ij}},v^+_{a_{ij}}]\right\rangle$, where $\overline{a_j}^+=\max_{1\le i\le m}\{\overline{a_{ij}}\}$.

(2) If the scheme attribute is the cost type, set $\tilde{a}_{ij} = \left\langle \left(\frac{\underline{\tau_j}^-}{\underline{\tau_{ij}}}, \frac{\tau_j^-}{\tau_{ij}}, \frac{\overline{\tau_j}^-}{\overline{\tau_{ij}}}\right);[\mu^-_{a_{ij}},\mu^+_{a_{ij}}],[v^-_{a_{ij}},v^+_{a_{ij}}]\right\rangle$, where $\underline{a_j}^- = \min_{1\le i\le m}\left\{\underline{a_{ij}}\right\}$.

Step 2: Resolve the weight of each scorer. First, according to the normalized decision-maker evaluated value matrices, define the positive ideal value matrices and negative ideal value matrices. The relative distance index of the interval Pythagorean triangular fuzzy evaluated value is determined according to the Hamming distance. Then, use the interval fuzzy entropy to determine the weight of each scorer.

(1) Calculate the positive ideal and bilateral negative ideal matrices of the evaluation values of each scorer.

$$L^+ = \begin{bmatrix} \tilde{a}_{11}^+ & \tilde{a}_{12}^+ & \cdots & \tilde{a}_{1n}^+ \\ \tilde{a}_{21}^+ & \tilde{a}_{22}^+ & \cdots & \tilde{a}_{2n}^+ \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{a}_{m1}^+ & \tilde{a}_{m2}^+ & \cdots & \tilde{a}_{mn}^+ \end{bmatrix}, L_e^- = \begin{bmatrix} \tilde{a}_{11}^{-e} & \tilde{a}_{12}^{-e} & \cdots & \tilde{a}_{1n}^{-e} \\ \tilde{a}_{21}^{-e} & \tilde{a}_{22}^{-e} & \cdots & \tilde{a}_{2n}^{-e} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{a}_{m1}^{-e} & \tilde{a}_{m2}^{-e} & \cdots & \tilde{a}_{mn}^{-e} \end{bmatrix}, L_f^- = \begin{bmatrix} \tilde{a}_{11}^{-f} & \tilde{a}_{12}^{-f} & \cdots & \tilde{a}_{1n}^{-f} \\ \tilde{a}_{21}^{-f} & \tilde{a}_{22}^{-f} & \cdots & \tilde{a}_{2n}^{-f} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{a}_{m1}^{-f} & \tilde{a}_{m2}^{-f} & \cdots & \tilde{a}_{mn}^{-f} \end{bmatrix},$$

where $\tilde{a}_{ij}^+ = \frac{\sum_{k=1}^{C}\tilde{a}_{ij}^k}{C}$; $\tilde{a}_{ij}^+ = \max[\tilde{a}_{ij}^k]$; $\tilde{a}_{ij}^- = \min[\tilde{a}_{ij}^k]$.

(2) Calculate the Hamming distance.

$$d(\tilde{a}_{ij}^k,\tilde{a}_{ij}^+) = \frac{1}{4}\left(\begin{aligned} &\left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}\mu_{a_{ij}}^{-(k)} - \frac{\underline{\tau_{ij}^+}+2\tau_{ij}^++\overline{\tau_{ij}^+}}{4}\mu_{a_{ij}}^{-(+)}\right| + \left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}\mu_{a_{ij}}^{+(k)} - \frac{\underline{\tau_{ij}^+}+2\tau_{ij}^++\overline{\tau_{ij}^+}}{4}\mu_{a_{ij}}^{+(+)}\right| \\ &+\left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}v_{a_{ij}}^{-(k)} - \frac{\underline{\tau_{ij}^+}+2\tau_{ij}^++\overline{\tau_{ij}^+}}{4}v_{a_{ij}}^{-(+)}\right| + \left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}v_{a_{ij}}^{+(k)} - \frac{\underline{\tau_{ij}^+}+2\tau_{ij}^++\overline{\tau_{ij}^+}}{4}v_{a_{ij}}^{+(+)}\right| \end{aligned}\right).$$

$$d(\tilde{a}_{ij}^k,\tilde{a}_{ij}^e) = \frac{1}{4}\left(\begin{aligned} &\left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}\mu_{a_{ij}}^{-(k)} - \frac{\underline{\tau_{ij}^e}+2\tau_{ij}^e+\overline{\tau_{ij}^e}}{4}\mu_{a_{ij}}^{-(e)}\right| + \left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}\mu_{a_{ij}}^{+(k)} - \frac{\underline{\tau_{ij}^e}+2\tau_{ij}^e+\overline{\tau_{ij}^e}}{4}\mu_{a_{ij}}^{+(e)}\right| \\ &+\left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}v_{a_{ij}}^{-(k)} - \frac{\underline{\tau_{ij}^e}+2\tau_{ij}^e+\overline{\tau_{ij}^e}}{4}v_{a_{ij}}^{-(e)}\right| + \left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}v_{a_{ij}}^{+(k)} - \frac{\underline{\tau_{ij}^e}+2\tau_{ij}^e+\overline{\tau_{ij}^e}}{4}v_{a_{ij}}^{+(e)}\right| \end{aligned}\right).$$

$$d(\tilde{a}_{ij}^k,\tilde{a}_{ij}^f) = \frac{1}{4}\left(\begin{aligned} &\left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}\mu_{a_{ij}}^{-(k)} - \frac{\underline{\tau_{ij}^f}+2\tau_{ij}^f+\overline{\tau_{ij}^f}}{4}\mu_{a_{ij}}^{-(f)}\right| + \left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}\mu_{a_{ij}}^{+(k)} - \frac{\underline{\tau_{ij}^f}+2\tau_{ij}^f+\overline{\tau_{ij}^f}}{4}\mu_{a_{ij}}^{+(f)}\right| \\ &+\left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}v_{a_{ij}}^{-(k)} - \frac{\underline{\tau_{ij}^f}+2\tau_{ij}^f+\overline{\tau_{ij}^f}}{4}v_{a_{ij}}^{-(f)}\right| + \left|\frac{\underline{\tau_{ij}^k}+2\tau_{ij}^k+\overline{\tau_{ij}^k}}{4}v_{a_{ij}}^{+(k)} - \frac{\underline{\tau_{ij}^f}+2\tau_{ij}^f+\overline{\tau_{ij}^f}}{4}v_{a_{ij}}^{+(f)}\right| \end{aligned}\right).$$

(3) Determine the relative distance index. According to the Hamming distance, the relative distance index of the interval Pythagorean triangular fuzzy is calculated.

$$r_{ij}^{k} = \frac{d(\tilde{a}_{ij}^{k}, \tilde{a}_{ij}^{e}) + d(\tilde{a}_{ij}^{k}, \tilde{a}_{ij}^{f})}{d(\tilde{a}_{ij}^{k}, \tilde{a}_{ij}^{+}) + d(\tilde{a}_{ij}^{k}, \tilde{a}_{ij}^{e}) + d(\tilde{a}_{ij}^{k}, \tilde{a}_{ij}^{f})} .$$

(4) Determine the scorers weight. The weights of scorers are calculated by using the obtained relative distance index and the interval Pythagorean triangular fuzzy entropy method.

$$\lambda_{kj} = E_{kj} / \sum_{k=1}^{n} E_{kj} .$$

Among them, $E_{kj} = -\frac{1}{m}\sum_{i=1}^{m} p_{ij} \ln p_{ij}$; $p_{ij}^{k} = \phi(\tilde{a}_{ij}^{k}) / \sum_{i=1}^{m} \phi(\tilde{a}_{ij}^{k})$.

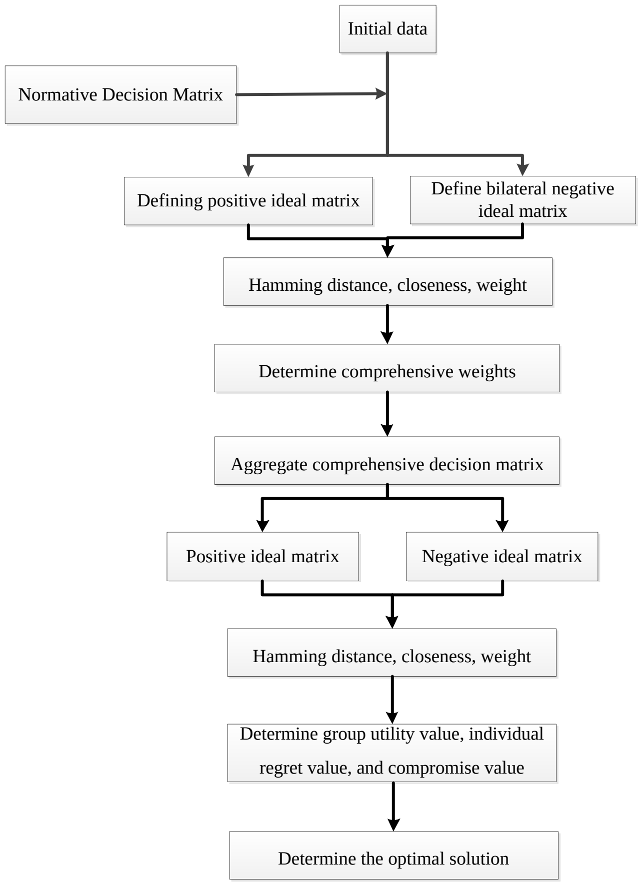

Figure 1. VIKOR group decision flow chart based on IVPTFWOWA operator.

Step 3: Calculate the comprehensive weight.

$$w_i = \omega^*(\sum_{j \le i} p_{\sigma(j)}) - \omega^*(\sum_{j < i} p_{\sigma(j)}) .$$

Step 4: Assemble the comprehensive matrices. According to the comprehensive weight obtained in the second step, the IVPTFWOWA operator is used to assemble the evaluated value matrices of multiple scorers into a single comprehensive evaluated value matrix.

$$IVPTFWOWA(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, ..., \tilde{a}_n) = w_1 \tilde{a}_{\sigma(1)} \oplus w_2 \tilde{a}_{\sigma(2)} \oplus w_3 \tilde{a}_{\sigma(3)} \oplus ... \oplus w_n \tilde{a}_{\sigma(n)}$$

$$= \left\langle \begin{array}{l} (\sum_{i=1}^{n} w_i \underline{\tau_{\sigma(i)}}, \sum_{i=1}^{n} w_i \tau_{\sigma(i)}, \sum_{i=1}^{n} w_i \overline{\tau_{\sigma(i)}}); \\ \left[\sqrt{1-\prod_{i=1}^{n}(1-(\mu^-_{a_{\sigma(i)}})^2)^{w_i}}, \sqrt{1-\prod_{i=1}^{n}(1-(\mu^+_{a_{\sigma(i)}})^2)^{w_i}}\right], \left[\prod_{i=1}^{n}(v^-_{a_{\sigma(i)}})^{w_i}, \prod_{i=1}^{n}(v^+_{a_{\sigma(i)}})^{w_i}\right] \end{array} \right\rangle .$$

Step 5: Calculate the weight of each attribute. After aggregating into a single comprehensive evaluated value matrix, the VIKOR method and the interval triangular fuzzy entropy method are used again to calculate the scheme attribute weight.

(1) According to the comprehensive evaluated value matrices, find the ideal solution of the comprehensive evaluated value matrices.

Positive ideal solution: $L^+ = (\tilde{a}_1^+, \tilde{a}_2^+, \tilde{a}_3^+, ..., \tilde{a}_n^+)$, $\tilde{a}_j^+ = \dfrac{\sum_{i=1}^{n} \max \tilde{a}_{ij}}{n}$;

Negative ideal solution: $L_e^- = (\tilde{a}_1^{-e}, \tilde{a}_2^{-e}, \tilde{a}_3^{-e}, ..., \tilde{a}_n^{-e})$, $\tilde{a}_j^+ = \max \tilde{a}_{ij}$;

Negative ideal solution: $L_f^- = (\tilde{a}_1^{-f}, \tilde{a}_2^{-f}, \tilde{a}_3^{-f}, ..., \tilde{a}_n^{-f})$, $\tilde{\alpha}_j^- = \min \tilde{\alpha}_{ij}$.

(2) Calculate the relative distance index. The relative distance index of the interval Pythagorean triangular fuzzy is calculated.

$$r_{ij}^k = \frac{d(\tilde{a}_{ij}^k, \tilde{a}_{ij}^e) + d(\tilde{a}_{ij}^k, \tilde{a}_{ij}^f)}{d(\tilde{a}_{ij}^k, \tilde{a}_{ij}^+) + d(\tilde{a}_{ij}^k, \tilde{a}_{ij}^e) + d(\tilde{a}_{ij}^k, \tilde{a}_{ij}^f)} .$$

(3) Determine the scheme attribute weight. The obtained relative distance index and interval Pythagorean triangular fuzzy entropy method are used to resolve the attribute weight of the project.

$$r_j = E_j / \sum_{j=1}^{n} E_j .$$

Among them, $E_j = -\dfrac{1}{m}\sum_{i=1}^{m} p_{ij} \ln p_{ij}$; $p_{ij} = \phi(\tilde{a}_{ij}) / \sum_{i=1}^{m} \phi(\tilde{a}_{ij})$.

Step 6: According to the attribute weight and the relative distance index calculated in Step 5, calculate the following values:

(1) The formula for calculating the group utility value is as follows:

$$S(M_i) = \sum_{j=1}^{n} r_j \frac{d(\tilde{a}_{ij}, \tilde{a}_j^+)}{d(\tilde{a}_j^+, \tilde{a}_j^-)} .$$

(2) The formula for calculating the individual regret value is as follows:

$$R(M_i) = \max_j \left\{ r_j \frac{d(\tilde{a}_{ij}, \tilde{a}_j^+)}{d(\tilde{a}_j^+, \tilde{a}_j^-)} \right\} .$$

(3) The formula for calculating the compromise value is as follows:

$$Q(M_i) = \nu \frac{S(M_i) - \max_i \{S(M_i)\}}{\min_i \{S(M_i)\} - \max_i \{S(M_i)\}} + (1-\nu) \frac{R(M_i) - \max_i \{R(M_i)\}}{\min_i \{R(M_i)\} - \max_i \{R(M_i)\}},$$

where ν is the coefficient of decision-making mechanism. When $\nu > 0.5$, it expresses that the scheme is based on the decision-making mechanism to maximize the group utility. When $\nu < 0.5$, it expresses that the scheme is based on the decision-making mechanism of minimizing individual regret. When ν=0.5, it expresses that the scheme is a decision-making mechanism that has reached a consensus through consultation.

Step 7: Determine the optimal plan. According to the VIKOR decision-making method, the smaller the value in Step 6, the better the scheme. Assume that $(M^{(1)}, M^{(2)}, M^{(3)}, ..., M^{(n)})$ is the sequence of schemes $M = \{M_1, M_2, M_3, ..., M_n\}$ from smallest to largest. If Scheme $M^{(1)}$ is the optimal scheme, the following conditions shall be met simultaneously: (a) $Q(M^{(1)}) - Q(M^{(2)}) \geq \frac{1}{n\text{-}1}$ and (b) $S(M^{(1)})$ and $R(M^{(1)})$, with at least one of them at the minimum value.

5. Example study

There are many cases where the interval hesitation fuzzy method is applied to decision models, such as medical decision-making [25,26], performance evaluation [27], etc. This article applies the VIKOR group decision-making model based on the WOWA operator of interval Pythagorean triangular fuzzy numbers to the emergency decision-making of urban network public opinion. Emergency decisions can be made for such events using the methods proposed in this article.

For example, the emergency response plan of a city's network public opinion is evaluated and analyzed. It is assumed that there are four sets of emergency plans for urban network public opinion emergencies (A1, A2, A3, A4), and through investigation and research, four attributes for evaluating this network public opinion emergency are determined, namely, rescue capability (B1), Internet users' satisfaction (B2), flexibility (B3), and time validity (B4). It is assumed that the attribute location weight is (0.3, 0.2, 0.2, 0.3). In this assessment, three decision-makers (C1, C2, C3) evaluated the urban network public opinion emergencies. In order to effectively solve the shortage and lack of basic information for decision-making, the assessment value adopts the interval Pythagorean triangular fuzzy numbers. The evaluated value matrices of the final three scorers are shown in Table 1 (standardized).

Table 1. Evaluated value matrices of three scorers.

		B1	B2	B3	B4
C1	A1	<(0.6,0.7,0.9); [0.7,0.9],[0.2,0.4]>	<(0.5,0.7,0.8); [0.7,0.8],[0.3,0.4]>	<(0.6,0.7,0.8); [0.6,0.7],[0.1,0.2]>	<(0.5,0.7,0.8); [0.6,0.8],[0.1,0.3]>
	A2	<(0.7,0.8,0.9); [0.7,0.8],[0.3,0.4]>	<(0.6,0.8,0.9); [0.7,0.9],[0.2,0.3]>	<(0.7,0.8,0.9); [0.7,0.9],[0.1,0.2]>	<(0.6,0.8,0.9); [0.8,0.9],[0.1,0.3]>
	A3	<(0.5,0.6,0.7); [0.6,0.8],[0.2,0.3]>	<(0.5,0.7,0.9); [0.7,0.9],[0.3,0.4]>	<(0.5,0.8,0.9); [0.7,0.8],[0.2,0.3]>	<(0.5,0.6,0.8); [0.6,0.8],[0.2,0.3]>
	A4	<(0.6,0.7,0.9); [0.7,0.9],[0.2,0.4]>	<(0.5,0.7,0.9); [0.7,0.8],[0.2,0.4]>	<(0.5,0.6,0.9); [0.7,0.8],[0.1,0.3]>	<(0.6,0.7,0.9); [0.7,0.8],[0.1,0.3]>

Continued on next page

		B1	B2	B3	B4
C2	A1	<(0.6,0.7,0.8); [0.8,0.9],[0.2,0.3]>	<(0.5,0.6,0.7); [0.6,0.7],[0.1,0.2]>	<(0.6,0.8,0.9); [0.7,0.8],[0.2,0.3]>	<(0.6,0.7,0.9); [0.6,0.8],[0.2,0.4]>
	A2	<(0.5,0.7,0.9); [0.7,0.8],[0.1,0.2]>	<(0.7,0.8,0.9); [0.6,0.7],[0.2,0.3]>	<(0.6,0.7,0.8); [0.7,0.8],[0.2,0.4]>	<(0.5,0.7,0.8); [0.7,0.9],[0.2,0.3]>
	A3	<(0.7,0.8,0.9); [0.7,0.8],[0.1,0.3]>	<(0.6,0.8,0.9); [0.6,0.7],[0.1,0.2]>	<(0.6,0.7,0.8); [0.6,0.7],[0.2,0.3]>	<(0.6,0.7,0.9); [0.6,0.7],[0.2,0.3]>
	A4	<(0.6,0.7,0.9); [0.7,0.8],[0.2,0.4]>	<(0.5,0.7,0.9); [0.6,0.8],[0.2,0.3]>	<(0.6,0.8,0.9); [0.7,0.8],[0.2,0.4]>	<(0.5,0.6,0.9); [0.7,0.8],[0.2,0.3]>
C3	A1	<(0.6,0.7,0.8); [0.6,0.7],[0.2,0.3]>	<(0.7,0.8,0.9); [0.6,0.7],[0.2,0.3]>	<(0.5,0.7,0.8); [0.7,0.8],[0.2,0.4]>	<(0.5,0.6,0.8); [0.7,0.8],[0.2,0.4]>
	A2	<(0.5,0.7,0.8); [0.6,0.8],[0.2,0.3]>	<(0.6,0.7,0.9); [0.7,0.8],[0.2,0.4]>	<(0.7,0.8,0.9); [0.6,0.8],[0.2,0.3]>	<(0.6,0.8,0.9); [0.6,0.8],[0.2,0.3]>
	A3	<(0.7,0.8,0.9); [0.7,0.8],[0.2,0.4]>	<(0.6,0.7,0.8); [0.6,0.8],[0.2,0.3]>	<(0.6,0.7,0.9); [0.6,0.7],[0.1,0.3]>	<(0.6,0.8,0.9); [0.7,0.8],[0.1,0.3]>
	A4	<(0.6,0.7,0.8); [0.7,0.8],[0.2,0.3]>	<(0.6,0.8,0.9); [0.6,0.7],[0.2,0.4]>	<(0.6,0.7,0.9); [0.6,0.7],[0.2,0.3]>	<(0.7,0.8,0.9); [0.6,0.7],[0.2,0.3]>

5.1. Decision-making process

According to the VIKOR decision steps based on IVPTFWOWA operator, we first need to judge the attribute type of the decision-making scheme for standardization. The attributes of the example given in this paper are rescue ability, Internet users' satisfaction, flexibility and time validity, which are all benefit indicators and have been standardized. Therefore, the weight can be directly calculated. The detailed decision-making steps and processes of this example are given below (four decimal places are reserved for all calculation results).

Step 1: Determine the attribute weight of the scores w_j^k. Since the numerical examples given in this paper have been normalized, the normalization process will not be repeated here. The weights of each expert for each attribute are directly calculated.

(1) The score of each evaluated interval Pythagorean triangular fuzzy value is obtained by the interval Pythagorean triangular fuzzy numbers scoring function (Definition 6), and the ideal matrices are resolved. (See Appendix Table II for specific results)

(2) The Hamming distance is determined according to Hamming distance calculation model (Definition 8) proposed in this paper. (See Appendix Table II for specific results)

(3) Calculate the relative distance index of the Pythagorean triangular fuzzy evaluated value of each interval according to Hamming distance. (See Appendix Table II for specific results)

(4) Calculate the attribute weight of the decision-makers. Here, the relative distance index and interval Pythagorean triangular fuzzy entropy method are used to resolve the attribute weight of each scorer. The results are shown in Table 2.

Step 2: Determine the comprehensive weight. In this paper, the IVPTFWOWA aggregation operator is studied. The determination of the comprehensive weight is based on the comprehensive calculation of the location weight and the decision-maker attribute weight. Therefore, in this example, the comprehensive weight should be obtained based on the decision maker's attribute weight result obtained in the first step and the position weight given in advance.

(1) According to the position weight vector (0.3, 0.2, 0.2, 0.3) of the scheme attribute given in advance in this paper and Definition 12, the following can be obtained:

$$\omega^* = \begin{cases} 1.2x & 0 \le x \le 1/4 \\ 0.8x + 0.1 & 1/4 \le x \le 3/4 \\ 1.2x - 0.2 & 3/4 \le x \le 1 \end{cases}.$$

(2) According to the relationship between the attribute weight and the position weight function of each decision, the comprehensive weight can be resolved (Table 3).

Step 3: Aggregation of comprehensive decision matrices. After determining the comprehensive weight, the IVPTFWOWA aggregation operator (Definition 10) can aggregate the evaluated interval Pythagorean triangular fuzzy value matrices into a comprehensive evaluated value matrix. The interval Pythagorean triangular fuzzy evaluated value matrices after aggregation are shown in Table 4 below.

Table 2. Attribute weights of each scores.

		B1	B2	B3	B4
	C1	0.3319	0.3320	0.3315	0.3329
w_j^k	C2	0.3356	0.3339	0.3320	0.3347
	C3	0.3325	0.3341	0.3364	0.3324

Table 3. Comprehensive weight matrices.

		B1	B2	B3	B4
	C1	0.3685	0.3673	0.3619	0.3678
w_j^k	C2	0.2655	0.2671	0.2656	0.2663
	C3	0.3660	0.3656	0.3652	0.3659

Table 4. Comprehensive evaluated value matrices.

	B1	B2	B3	B4
A1	<(0.6000,0.7000,0.8266); [0.7159,0.8528],[0.2000,0.3238]>	<(0.5534,0.6901,0.7901); [0.6413,0.7425],[0.1802, 0.2875]>	<(0.5734,0.7369,0.8369); [0.6678,0.7689],[0.1553, 0.2792]>	<(0.5366,0.6734,0.8366); [0.6305,0.8000],[0.1550, 0.3598]>
A2	<(0.5531,0.7266,0.8634); [0.6677,0.8000],[0.1725,0.2789]>	<(0.6366,0.7733,0.9000); [0.6677,0.8226],[0.2000, 0.3240]>	<(0.6635,0.7635,0.8635); [0.6770,0.8460],[0.1548, 0.2869]>	<(0.5734,0.7734,0.8734); [0.7159,0.8717],[0.1550, 0.3000]>
A3	<(0.6268,0.7268,0.8268); [0.6677,0.8000],[0.1549,0.3238]>	<(0.5633,0.7267,0.8634); [0.6413,0.8292],[0.1929, 0.2992]>	<(0.5631,0.7369,0.8635); [0.6414,0.7427],[0.1664, 0.3000]>	<(0.5734,0.7107,0.8734); [0.6413,0.7688],[0.1550, 0.3000]>
A4	<(0.6000,0.7000,0.8734); [0.7000,0.8459],[0.2000,0.3706]>	<(0.5366,0.7366,0.9000); [0.6413,0.7688],[0.2000, 0.3704]>	<(0.5734,0.7104,0.9000); [0.6678,0.7689],[0.1664, 0.3336]>	<(0.6100,0.7100,0.9000); [0.6677,0.7688],[0.1550, 0.3000]>

Step 4: Determine the scheme attribute weight. After the above calculation, the interval Pythagorean triangular fuzzy evaluated value matrices of multiple scorers have been aggregated into a single comprehensive evaluated value matrix.

(1) According to the comprehensive evaluated interval Pythagorean triangular fuzzy value

matrix and the interval Pythagorean triangular fuzzy numbers scoring function (Definition 6), the score of each evaluated interval Pythagorean triangular fuzzy value is calcualted. (See Appendix Table III for specific results)

(2) According to Definition 8, the Hamming distance in the comprehensive interval Pythagorean triangular fuzzy evaluation value matrix is calculated. (See Appendix Table III for specific results)

(3) Calculate the relative distance index of the evaluated Pythagorean triangular fuzzy value of each evaluated interval in the comprehensive value matrices according to Hamming distance. (See Appendix Table III for specific results)

(4) Determine the scheme attribute weight. The obtained relative distance index and the interval Pythagorean triangular fuzzy entropy method are used to determine the attribute weight of the scheme. The calculation results are shown in Table 5.

Table 5. Scheme attribute weights.

	B1	B2	B3	B4
w_j	0.2489	0.2511	0.2492	0.2509

Step 5: According to the VIKOR decision-making method, three values of each scheme are calculated in turn (here the decision-making mechanism coefficient is taken as 0.5). The calculation results are shown in Table 6.

Table 6. Calculation results.

	A1	A2	A3	A4
Group Utility Value	0.7324	0.1766	0.6982	0.5848
Individual Regret Value	0.2511	0.1766	0.2492	0.2258
Compromise Value	1	0	0.9567	0.6977

Step 6: Decide the best plan. According to the VIKOR decision-making method, the smaller the group utility value, individual regret value and compromise value, the better the scheme is. According to the group unity value, the priority of schemes is A2>A4>A3>A1, indicating that scheme A2 is the best. According to the individual regret value, the priority of the scheme is A2>A4>A3>A1, indicating that scheme A2 is the best. Arranged according to the compromise value, the priority of the scheme is A2>A4>A3>A1, indicating that scheme A2 is the best. To sum up, the scheme A2 for each group has the lowest median value and meet the judgment conditions of the VIKOR compromise scheme (a. $Q(A^{(1)})-Q(A^{(2)})$=0.6977$\geq\frac{1}{3}$; b. $S(A^{(1)})$ and $R(A^{(1)})$ are the minimum values). Therefore, it can be determined that scheme A2 is a compromise scheme.

5.2. Decision results and comparison

According to the calculation and analysis in the previous section of this paper, it can be basically determined that scheme A2 is a compromise scheme. The group utility value, individual regret value and compromise value are 0.1766, 0.1766 and 0, respectively, and are all the minimum values of each scheme.

To illustrate the accuracy of the results and the effectiveness and scientificity of the methods, we will adjust the coefficient of the decision-making mechanism and the comparative analysis.

(1) Adjustment of coefficient of decision-making mechanism. The above research is based on the calculated results when we set the decision-making mechanism coefficient to 0.5. In order to verify this, if the adjustment coefficient changes our calculation results, we set the coefficient to different values, and the results are as follows.

We know that different mechanism coefficients have a certain impact on the compromise value (obtained from A3 and A4 columns); however, the compromise value of the A2 scheme in Table 7 is always 0, and the A2 scheme is always a compromise scheme. The reason for this phenomenon is that in the previous analysis, the group utility value and the maximum and minimum individual regret values are the same, that is, the maximum group utility value and individual regret value are both A1 schemes. The minimum value of group utility value and individual regret value are A2 schemes.

Table 7. Table of compromise values and compromise schemes under different decision-making mechanism coefficients.

Decision mechanism coefficient	Compromise value				Compromise proposal
	A1	A2	A3	A4	
0.1	1	0	0.9714	0.6683	A2
0.2	1	0	0.9677	0.6757	A2
0.3	1	0	0.9640	0.6830	A2
0.4	1	0	0.9604	0.6903	A2
0.5	1	0	0.9567	0.6977	A2
0.6	1	0	0.9530	0.7050	A2
0.7	1	0	0.9494	0.7124	A2
0.8	1	0	0.9457	0.7197	A2
0.9	1	0	0.9421	0.7270	A2

(2) A comparative study of decision-making methods. Although the method has obtained a compromise scheme, it cannot explain the accuracy and effectiveness of this method at present. Therefore, this paper selects the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) decision-making method [28] and our team's previous research achievement [8] to analyze the calculation example of this paper again for comparison with the results of this paper.

When literature methods are used, the scores of each scheme can be calculated as (0.1653, 0.1849, 0.1713, 0.1667), and it can be found that A2>A3>A4>A1. The results show that the best scheme is also the A2 scheme.

When our team's previous research is used, it can be calculated that the relative closeness of each scheme is (0.4691, 0.4883, 0.4704, 0.4811), and the ranking of each scheme is A2>A4>A3>A1. The results show that the best scheme is the A2 scheme, and the ranking of its complete scheme is completely consistent with that of the method proposed in this paper.

Through the comparative study of the above methods, the VIKOR decision-making method based on the IVPTFWOWA operator, which is a new method developed in this paper, has several obvious advantages:

(1) The decision results obtained by using the VIKOR decision method of the IVPTFWOWA operator are consistent with those of the decision methods proposed by other scholars, which fully demonstrates the accuracy, effectiveness and scientificity of the new methods and operators proposed in this paper.

(2) No matter if the interval Pythagorean fuzzy geometric weighted Bonferroni average operator is used, or if the interval Pythagorean fuzzy WA operator is used, the difference between the good and bad results of the scheme is small, and it is difficult for the final decision-maker to choose the best scheme. However, the gap between the decision results obtained by the VIKOR decision method of the IVPTFWOWA operator is obviously large, which is more convenient for decision-makers to quickly and accurately determine the optimal scheme.

6. Conclusions

In this paper, we studied the IVPTFWOWA operator, defined the Hamming distance formula of the IVPTFWOWA operator, and proved the basic properties of the IVPTFWOWA operator, such as idempotence, monotony and boundedness. At the same time, this study reconstructed a new decision theory based on the IVPTFWOWA operator, and gave the decision-making steps of the compromise method. Finally, the method was validated by randomly set numerical examples, and the results were compared using other methods. To sum up, the method has the three advantages: first, the new operator studied is based on the background of information fuzziness, which can minimize the decision-making error caused by lack of information; second, the method in this paper can be used to give different weights to decision-makers. Instead, the weights of each attribute of each score can be resolved by their own evaluated values, making full use of their own attribute advantages; third, this paper applies the comprehensive weight, fully considers the location weight of the scheme attribute and the weight of the decision-maker, and maximizes the use of the attribute characteristics of the decision-maker and the case; finally, compared with other methods, the decision results obtained by using this method have a large gap, which makes it easier to identify the optimal scheme.

This paper studies and proposes the IVPTFWOWA operator. This operator not only extends the use of fuzzy numbers from discrete sets to continuous sets, but also uses the evaluated information of decision makers more accurately and effectively. However, there are still some limitations in the research process of this paper. For example, the weight of each attribute cannot be indirectly calculated using this method. In this paper, the effect of the introduction of artificial variables on the model is not considered. In the follow-up research, we will continue to study the concept fusion of triangular fuzzy numbers and other fuzzy sets, construct integration operators, and find more stable decision information aggregation operators. Next, we will consider whether the introduction of artificial variables will affect the stability and applicability of the model.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

There are no conflicts of interest between the authors of this article.

Appendix

Appendix I: Property Proof Process of IVPTFWOWA Operator

Theorem 1. (Integration Invariance) If $\tilde{a}_i = \left\langle (\underline{\tau_i}, \tau_i, \overline{\tau_i}); [\mu^-_{a_i}, \mu^+_{a_i}], [v^-_{a_i}, v^+_{a_i}] \right\rangle (i = 1, 2, 3, ..., n)$ is an IVPTF, the number after IVPTFWOWA operator integration should still be IVPTF.

It is proved that $\tilde{a}_i = \left\langle (\underline{\tau_i}, \tau_i, \overline{\tau_i}); [\mu^-_{a_i}, \mu^+_{a_i}], [v^-_{a_i}, v^+_{a_i}] \right\rangle$, $(\tilde{a}_{\sigma(1)}, \tilde{a}_{\sigma(2)}, \tilde{a}_{\sigma(3)}, ..., \tilde{a}_{\sigma(n)})$ is the position exchange of $(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, ..., \tilde{a}_n)$, and w_i is the position weight of $\tilde{a}_i$. It can be seen from definition 9 that:

$$w_i \tilde{a}_{\sigma(i)} = \left\langle (w_i \underline{\tau_{\sigma(i)}}, w_i \tau_{\sigma(i)}, w_i \overline{\tau_{\sigma(i)}}); \left[1-(1-(\mu^-_{a_{\sigma(i)}})^2)^{w_i}, 1-(1-(\mu^+_{a_{\sigma(i)}})^2)^{w_i}\right], \left[(v^-_{a_{\sigma(i)}})^{w_i}, (v^+_{a_{\sigma(i)}})^{w_i}\right] \right\rangle$$

$$\bigoplus_{i=1}^{n} w_i \tilde{a}_{\sigma(i)} = \bigoplus_{i=1}^{n} \left\langle (w_i \underline{\tau_{\sigma(i)}}, w_i \tau_{\sigma(i)}, w_i \overline{\tau_{\sigma(i)}}); \left[1-(1-(\mu^-_{a_{\sigma(i)}})^2)^{w_i}, 1-(1-(\mu^+_{a_{\sigma(i)}})^2)^{w_i}\right], \left[(v^-_{a_{\sigma(i)}})^{w_i}, (v^+_{a_{\sigma(i)}})^{w_i}\right] \right\rangle$$

$$= \left\langle \begin{array}{l} (\sum_{i=1}^{n} w_i \underline{\tau_{\sigma(i)}}, \sum_{i=1}^{n} w_i \tau_{\sigma(i)}, \sum_{i=1}^{n} w_i \overline{\tau_{\sigma(i)}}); \\ \left[\sqrt{1-\prod_{i=1}^{n}(1-(\mu^-_{a_{\sigma(i)}})^2)^{w_i}}, \sqrt{1-\prod_{i=1}^{n}(1-(\mu^+_{a_{\sigma(i)}})^2)^{w_i}}\right], \left[\prod_{i=1}^{n}(v^-_{a_{\sigma(i)}})^{w_i}, \prod_{i=1}^{n}(v^+_{a_{\sigma(i)}})^{w_i}\right] \end{array} \right\rangle$$

$$= IVPTFWOWA(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, ..., \tilde{a}_n)$$

Theorem 2. (Degeneracy) It is known that $\tilde{a}_i = \left\langle (\underline{\tau_i}, \tau_i, \overline{\tau_i}); [\mu^-_{a_i}, \mu^+_{a_i}], [v^-_{a_i}, v^+_{a_i}] \right\rangle$ is an IVPTF. When the weight is $\omega_i = \frac{1}{n}$, the IVPTFWOWA operator will degenerate to the IVPTFWA operator.

Proof: when the position weight is $\omega_i = \frac{1}{n}$, the comprehensive weight is $w_i = \frac{1}{n}$.

$$
\begin{aligned}
&IVPTFWOWA(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,...,\tilde{a}_n) = w_1\tilde{a}_{\sigma(1)} \oplus w_2\tilde{a}_{\sigma(2)} \oplus w_3\tilde{a}_{\sigma(3)} \oplus ... \oplus w_n\tilde{a}_{\sigma(n)} \\
&= \left\langle \begin{array}{l} (\sum_{i=1}^{n} w_i \underline{\tau_{\sigma(i)}}, \sum_{i=1}^{n} w_i \tau_{\sigma(i)}, \sum_{i=1}^{n} w_i \overline{\tau_{\sigma(i)}}); \\ \left[\sqrt{1-\prod_{i=1}^{n}(1-(\mu^-_{a_{\sigma(i)}})^2)^{w_i}}, \sqrt{1-\prod_{i=1}^{n}(1-(\mu^+_{a_{\sigma(i)}})^2)^{w_i}}\right], \left[\prod_{i=1}^{n}(v^-_{a_{\sigma(i)}})^{w_i}, \prod_{i=1}^{n}(v^+_{a_{\sigma(i)}})^{w_i}\right] \end{array} \right\rangle \\
&= \left\langle \begin{array}{l} (\sum_{i=1}^{n} \frac{1}{n} \underline{\tau_{\sigma(i)}}, \sum_{i=1}^{n} \frac{1}{n} \tau_{\sigma(i)}, \sum_{i=1}^{n} \frac{1}{n} \overline{\tau_{\sigma(i)}}); \\ \left[\sqrt{1-\prod_{i=1}^{n}(1-(\mu^-_{a_{\sigma(i)}})^2)^{\frac{1}{n}}}, \sqrt{1-\prod_{i=1}^{n}(1-(\mu^+_{a_{\sigma(i)}})^2)^{\frac{1}{n}}}\right], \left[\prod_{i=1}^{n}(v^-_{a_{\sigma(i)}})^{\frac{1}{n}}, \prod_{i=1}^{n}(v^+_{a_{\sigma(i)}})^{\frac{1}{n}}\right] \end{array} \right\rangle \\
&= \left\langle \begin{array}{l} (\frac{1}{n}\sum_{i=1}^{n} \underline{\tau_i}, \frac{1}{n}\sum_{i=1}^{n} \tau_i, \frac{1}{n}\sum_{i=1}^{n} \overline{\tau_i}); \\ \left[\sqrt{1-(\prod_{i=1}^{n}(1-(\mu^-_{a_i})^2))^{\frac{1}{n}}}, \sqrt{1-(\prod_{i=1}^{n}(1-(\mu^+_{a_i})^2))^{\frac{1}{n}}}\right], \left[(\prod_{i=1}^{n}(v^-_{a_i}))^{\frac{1}{n}}, (\prod_{i=1}^{n}(v^+_{a_i}))^{\frac{1}{n}}\right] \end{array} \right\rangle \\
&= w_1\tilde{a}_1 \oplus w_2\tilde{a}_2 \oplus w_3\tilde{a}_3 \oplus ... \oplus w_n\tilde{a}_n \\
&= IVPTFWA(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,...,\tilde{a}_n)
\end{aligned}
$$

.

Theorem 3. (Idempotence) If the interval Pythagorean triangular fuzzy numbers $\tilde{a}_i = \langle (\underline{\tau_i}, \tau_i, \overline{\tau_i}); [\mu^-_{a_i}, \mu^+_{a_i}], [v^-_{a_i}, v^+_{a_i}] \rangle = \tilde{a}$ is assumed, then $IVPTFWOWA(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,...,\tilde{a}_n) = \tilde{a}$.

Proof: When $\tilde{a}_i = \langle (\underline{\tau_i}, \tau_i, \overline{\tau_i}); [\mu^-_{a_i}, \mu^+_{a_i}], [v^-_{a_i}, v^+_{a_i}] \rangle = \tilde{a}$,

$$
\begin{aligned}
&IVPTFWOWA(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,...,\tilde{a}_n) = IVPTFWOWA(\tilde{a},\tilde{a},\tilde{a},...,\tilde{a}) \\
&= w_1\tilde{a} \oplus w_2\tilde{a} \oplus w_3\tilde{a} \oplus ... \oplus w_i\tilde{a} \\
&= \oplus_{i=1}^{n} \left\langle (w_i\underline{\tau}, w_i\tau, w_i\overline{\tau}); \left[1-(1-(\mu^-_a)^2)^{w_i}, 1-(1-(\mu^+_a)^2)^{w_i}\right], \left[(v^-_a)^{w_i}, (v^+_a)^{w_i}\right] \right\rangle \\
&= \left\langle \begin{array}{l} (\sum_{i=1}^{n} w_i \underline{\tau}, \sum_{i=1}^{n} w_i \tau, \sum_{i=1}^{n} w_i \overline{\tau}); \\ \left[\sqrt{1-\prod_{i=1}^{n}(1-(\mu^-_a)^2)^{w_i}}, \sqrt{1-\prod_{i=1}^{n}(1-(\mu^+_a)^2)^{w_i}}\right], \left[\prod_{i=1}^{n}(v^-_a)^{w_i}, \prod_{i=1}^{n}(v^+_a)^{w_i}\right] \end{array} \right\rangle. \\
&= \left\langle \begin{array}{l} (\underline{\tau}, \tau, \overline{\tau}); \\ \left[\sqrt{1-(1-(\mu^-_a)^2)}, \sqrt{1-(1-(\mu^+_a)^2)}\right], \left[v^-_a, v^+_a\right] \end{array} \right\rangle \\
&= \tilde{\alpha}
\end{aligned}
$$

Theorem 4. (Monotonicity) For $\forall i$, if $\tilde{a}_{Ai} < \tilde{a}_{Bi}$ is satisfied, then $IVPTFWOWA(\tilde{a}_{A1},\tilde{a}_{A2},\tilde{a}_{A3},...,\tilde{a}_{An}) < IVPTFWOWA(\tilde{a}_{B1},\tilde{a}_{B2},\tilde{a}_{B3},...,\tilde{a}_{Bn})$ holds.

Proof:

$$IVPTFWOWA(\tilde{a}_{A1},\tilde{a}_{A2},\tilde{a}_{A3},...,\tilde{a}_{An})$$
$$=\left\langle \begin{array}{l} (\sum_{i=1}^{n} w_{Ai}\underline{\tau_{\sigma(Ai)}},\sum_{i=1}^{n} w_{Ai}\tau_{\sigma(Ai)},\sum_{i=1}^{n} w_{Ai}\overline{\tau_{\sigma(Ai)}}); \\ \left[\sqrt{1-\prod_{i=1}^{n}(1-(\mu^{-}_{a_{\sigma(Ai)}})^2)^{w_{Ai}}},\sqrt{1-\prod_{i=1}^{n}(1-(\mu^{+}_{a_{\sigma(Ai)}})^2)^{w_{Ai}}}\right],\left[\prod_{i=1}^{n}(v^{-}_{a_{\sigma(Ai)}})^{w_{Ai}},\prod_{i=1}^{n}(v^{+}_{a_{\sigma(Ai)}})^{w_{Ai}}\right] \end{array} \right\rangle;$$

Similarly:

$$IVPTFWOWA(\tilde{a}_{B1},\tilde{a}_{B2},\tilde{a}_{B3},...,\tilde{a}_{Bn})$$
$$=\left\langle \begin{array}{l} (\sum_{i=1}^{n} w_{Bi}\underline{\tau_{\sigma(Bi)}},\sum_{i=1}^{n} w_{Bi}\tau_{\sigma(Bi)},\sum_{i=1}^{n} w_{Bi}\overline{\tau_{\sigma(Bi)}}); \\ \left[\sqrt{1-\prod_{i=1}^{n}(1-(\mu^{-}_{a_{\sigma(Bi)}})^2)^{w_{Bi}}},\sqrt{1-\prod_{i=1}^{n}(1-(\mu^{+}_{a_{\sigma(Bi)}})^2)^{w_{Bi}}}\right],\left[\prod_{i=1}^{n}(v^{-}_{a_{\sigma(Bi)}})^{w_{Bi}},\prod_{i=1}^{n}(v^{+}_{a_{\sigma(Bi)}})^{w_{Bi}}\right] \end{array} \right\rangle.$$

Because there is $\forall i$, $\tilde{a}_{Ai} < \tilde{a}_{Bi}$, then there must be $\tilde{a}_{\sigma(Ai)} < \tilde{a}_{\sigma(Bi)}$. so $\oplus_{i=1}^{n} w_{Ai}\,\tilde{a}_{\sigma(Ai)} < \oplus_{i=1}^{n} w_{Bi}\,\tilde{a}_{\sigma(Bi)}$.

To sum up, $IVPTFWOWA(\tilde{a}_{A1},\tilde{a}_{A2},\tilde{a}_{A3},...,\tilde{a}_{An}) < IVPTFWOWA(\tilde{a}_{B1},\tilde{a}_{B2},\tilde{a}_{B3},...,\tilde{a}_{Bn})$ can be obtained.

Theorem 5. (Boundedness) If $\tilde{a}$ is an interval Pythagorean triangular fuzzy numbers and there are $\tilde{a}_A = \min \tilde{a}_i$, $\tilde{a}_B = \max \tilde{a}_i$, then there must be $\tilde{a}_A < IVPTFWOWA(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,...,\tilde{a}_n) < \tilde{a}_B$.

proof: according to the idempotence property:

$\tilde{a}_A = IVPTFWOWA(\tilde{a}_A,\tilde{a}_A,\tilde{a}_A,...,\tilde{a}_A)$, $\tilde{a}_B = IVPTFWOWA(\tilde{a}_B,\tilde{a}_B,\tilde{a}_B,...,\tilde{a}_B)$.

In addition, there is $\tilde{a}_A = \min \tilde{a}_i$, $\tilde{a}_B = \max \tilde{a}_i$, that is, $\tilde{a}_A < \tilde{a}_i < \tilde{a}_B$.

It can be seen from the monotonicity:

$IVPTFWOWA(\tilde{a}_A,\tilde{a}_A,\tilde{a}_A,...,\tilde{a}_A) < IVPTFWOWA(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,...,\tilde{a}_n) < IVPTFWOWA(\tilde{a}_B,\tilde{a}_B,\tilde{a}_B,...,\tilde{a}_B)$.

That is, $\tilde{a}_A < IVPTFWOWA(\tilde{a}_1,\tilde{a}_2,\tilde{a}_3,...,\tilde{a}_n) < \tilde{a}_B$.

Appendix II: Positive Ideal Matrices, Bilateral Negative Ideal Matrices, Hamming Distance and Relative Distance Index of the Original Evaluated Value Matrices

		B1	B2	B3	B4
L^{+}	A1	<(0.6000,0.7000,0.8333); [0.7143,0.8579],[0.2000,0.3302]>	<(0.5667,0.7000,0.8000) ;[0.6377,0.7389],[0.1817,0.2884]>	<(0.5667,0.7333,0.8333); [0.6707,0.7718],[0.1587,0.2884]>	<(0.5333,0.6667,0.8333); [0.6377,0.8000],[0.1587,0.3634]>
	A2	<(0.5667,0.7333,0.8667) ;[0.6707,0.8000],[0.1817,0.2884]>	<(0.6333,0.7667,0.9000); [0.6707,0.8205],[0.2000,0.3302]>	<(0.6667,0.7667,0.8667); [0.6707,0.8421],[0.1587,0.2884]>	<(0.5667,0.7667,0.8667); [0.7143,0.8746],[0.1587,0.3000]>
	A3	<(0.6333,0.7333,0.8333); [0.6707,0.8000],[0.1587,0.3302]>	<(0.5667,0.7333,0.8667); [0.6377,0.8205],[0.1817,0.2884]>	<(0.5667,0.7333,0.8667); [0.6377,0.7389],[0.1587,0.3000]>	<(0.5667,0.7000,0.8667); [0.6377,07718], [0.1587,0.3000]>
	A4	<(0.6000,0.7000,0.8667); [0.7000,0.8421],[0.2000,0.3634]>	<(0.5333,0.7333,0.9000); [0.6377,0.7718],[0.2000,0.3634]>	<(0.5667,0.7000,0.9000); [0.6707,0.7718],[0.1587,0.3302]>	<(0.6000,0.7000,0.9000); [0.6707,0.7718], [0.1587,0.3000]>
L_{e}^{-}	A1	<(0.6,0.7,0.8);[0.8,0.9],[0.2,0.3]>	<(0.5,0.7,0.8);[0.7,0.8],[0.3,0.4]>	<(0.6,0.8,0.9);[0.7,0.8],[0.2,0.3]>	<(0.5,0.7,0.8);[0.6,0.8],[0.1,0.3]>
	A2	<(0.5,0.7,0.9);[0.7,0.8],[0.1,0.2]>	<(0.6,0.8,0.9);[0.7,0.9],[0.2,0.3]>	<(0.7,0.8,0.9);[0.7,0.9],[0.1,0.2]>	<(0.6,0.8,0.9);[0.8,0.9],[0.1,0.3]>
	A3	<(0.7,0.8,0.9);[0.7,0.8],[0.1,0.3]>	<(0.5,0.7,0.9);[0.7,0.9],[0.3,0.4]>	<(0.5,0.8,0.9);[0. 7,0.8],[0.2,0.3]>	<(0.6,0.8,0.9);[0.7,0.8],[0.1,0.3]>
	A4	<(0.6,0.7,0.9);[0.7,0.9],[0.2,0.4]>	<(0.5,0.7,0.9);[0.7,0.8],[0.2,0.4]>	<(0.6,0.8,0.9);[0.7,0.8],[0.2,0.4]>	<(0.6,0.7,0.9);[0.7,0.8],[0.1,0.3]>
L_{f}^{-}	A1	<(0.6,0.7,0.8);[0.6,0.7],[0.2,0.3]>	<(0.5,0.6,0.7);[0.6,0.7],[0.1,0.2]>	<(0.6,0.7,0.8);[0.6,0.7],[0.1,0.2]>	<(0.6,0.7,0.9);[0.6,0.8],[0.2,0.4]>
	A2	<(0.5,0.7,0.8);[0.6,0.8],[0.2,0.3]>	<(0.7,0.8,0.9);[0.6,0.7],[0.2,0.3]>	<(0. 6,0.7,0.8);[0.7,0.8],[0.2,0.4]>	<(0.6,0.8,0.9);[0.6,0.8],[0.2,0.3]>
	A3	<(0.5,0.6,0.7);[0.6,0.8],[0.2,0.3]>	<(0.6,0.7,0.8);[0.6,0.8],[0.2,0.3]>	<(0.6,0.7,0.8);[0.6,0.7],[0.2,0.3]>	<(0.6,0.7,0.9);[0.6,0.7],[0.2,0.3]>
	A4	<(0.6,0.7,0.9);[0.7,0.8],[0.2,0.4]>	<(0.6,0.8,0.9);[0.6,0.7],[0.2,0.4]>	<(0.6,0.7,0.9);[0.6,0.7],[0.2,0.3]>	<(0.7,0.8,0.9);[0.6,0.7],[0.2,0.3]>

Hamming Distance		$d(\tilde{a}_{ij}^{k},\tilde{a}_{ij}^{+})$				$d(\tilde{a}_{ij}^{k},\tilde{a}_{ij}^{e})$				$d(\tilde{a}_{ij}^{k},\tilde{a}_{ij}^{f})$			
		A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4
C1	B1	0.0265	0.0882	0.0761	0.0215	0.0400	0.1250	0.1150	0.0000	0.0838	0.1194	0.0000	0.0181
	B2	0.0519	0.0298	0.0530	0.0124	0.0000	0.0000	0.0000	0.0000	0.1313	0.0531	0.0700	0.0219
	B3	0.0586	0.0558	0.0424	0.0373	0.01075	0.0000	0.0000	0.0981	0.0000	0.1025	0.0600	0.0338
	B4	0.0270	0.0473	0.0402	0.0211	0.0000	0.0000	0.0950	0.0000	0.0588	0.0775	0.0294	0.0394

Continued on next page

Hamming Distance		$d(\tilde{a}_{ij}^{k}, \tilde{a}_{ij}^{+})$				$d(\tilde{a}_{ij}^{k}, \tilde{a}_{ij}^{e})$				$d(\tilde{a}_{ij}^{k}, \tilde{a}_{ij}^{f})$			
		A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4
C2	B1	0.0255	0.0386	0.0400	0.0151	0.0000	0.0000	0.0000	0.0181	0.0700	0.0581	0.1150	0.0000
	B2	0.0793	0.0308	0.0409	0.0253	0.1313	0.0531	0.0925	0.0350	0.0000	0.0000	0.0450	0.0444
	B3	0.0489	0.0467	0.0301	0.0608	0.0000	0.1025	0.0600	0.0000	0.1075	0.0000	0.0000	0.0806
	B4	0.0318	0.0339	0.0234	0.0271	0.0588	0.0813	0.0756	0.0481	0.0000	0.0175	0.0000	0.0350
		A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4
C3	B1	0.0573	0.0328	0.0607	0.0272	0.0700	0.0581	0.0400	0.0488	0.0000	0.0000	0.1350	0.0306
	B2	0.0407	0.0252	0.0216	0.0191	0.0250	0.0550	0.0700	0.0219	0.1200	0.0281	0.0000	0.0000
	B3	0.0265	0.0242	0.0245	0.0354	0.0519	0.0800	0.0669	0.0806	0.0744	0.0375	0.0269	0.0000
	B4	0.0174	0.0352	0.0548	0.0185	0.0444	0.0775	0.0000	0.0394	0.0356	0.0000	0.0756	0.0000
Relative Distance Index		r_{ij}^{1}				r_{ij}^{2}				r_{ij}^{3}			
		A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4
B1		0.8239	0.7348	0.6019	0.4572	0.7331	0.6007	0.7419	0.5450	0.5499	0.6393	0.7424	0.7445
B2		0.7165	0.6410	0.5692	0.6381	0.6233	0.6332	0.7709	0.7582	0.3379	0.3140	0.2705	0.3305
B3		0.6473	0.6477	0.5862	0.7795	0.6872	0.6868	0.6659	0.5700	0.8267	0.8290	0.7927	0.6947
B4		0.6853	0.6208	0.7556	0.6515	0.6490	0.7443	0.7640	0.7544	0.8214	0.6879	0.5800	0.6799

Appendix III: Positive Ideal Solution, Bilateral Negative Ideal Solution, Hamming Distance and Relative Distance Index of Comprehensive Evaluated Value Matrices

	B1	B2	B3	B4
L^+	<(0.5950,0.7133,0.8476); [0.6887,0.8266],[0.1808,0.3226]>	<(0.5725,0.7317,0.8634) ;[0.6481,0.7944],[0.1931,0.3187]>	<(0.5934,0.7369,0.8660); [0.6638,0.7861],[0.1606,0.2992]>	<(0.5733,0.7167,0.8708); [0.6660,0.8083],[0.1550,0.3140]>
L_e^-	<(0.6000,0.7000,0.8734); [0.7000,0.8459],[0.2000,0.3706]>	<(0.6366,0.7733,0.9000); [0.6677,0.8226],[0.2000,0.3240]>	<(0.6635,0.7635,0.8635); [0.6770,0.8460],[0.1548,0.2869]>	<(0.5734,0.7734,0.8734); [0.7159,0.8717],[0.1550,0.3000]>
L_f^-	<(0.5531,0.7266,0.8634); [0.6677,0.8000],[0.1725,0.2789]>	<(0.5534,0.6901,0.7901); [0.6413,0.7425],[0.1802,0.2875]>	<(0.5631,0.7369,0.8635); [0.6414,0.7427],[0.1664,0.3000]>	<(0.5366,0.6734,0.8366); [0.6305,0.8000],[0.1550,0.3598]>

Hamming Distance	$d(\tilde{a}_{ij},\tilde{a}_{ij}^{+})$				$d(\tilde{a}_{ij},\tilde{a}_{ij}^{e})$				$d(\tilde{a}_{ij},\tilde{a}_{ij}^{f})$			
	A1	A2	A3	A4	A1	A2	A3	A4	A1	A2	A3	A4
B1	0.0089	0.0178	0.0103	0.0181	0.0120	0.0359	0.0264	0.0000	0.0255	0.0000	0.0151	0.0359
B2	0.0389	0.0340	0.0115	0.01590	0.0730	0.0000	0.0349	0.0379	0.0000	0.0730	0.0381	0.0450
B3	0.0128	0.0251	0.0161	0.0121	0.0377	0.0000	0.0408	0.0369	0.0143	0.0408	0.0000	0.0150
B4	0.0282	0.0333	0.0153	0.0091	0.0615	0.0000	0.0479	0.0358	0.0000	0.0615	0.0183	0.0281

Relative Distance Index	r_{ij}			
	A1	A2	A3	A4
B1	0.8071	0.6681	0.8008	0.6652
B2	0.6520	0.6820	0.8639	0.8387
B3	0.8024	0.6190	0.7171	0.8114
B4	0.6857	0.6486	0.8126	0.8754

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