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Research article

Maps on C^*-algebras are skew Lie triple derivations or homomorphisms at one point

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Abstract: In this paper, we show that every continuous linear map between unital C^*-algebras is skew Lie triple derivable at the identity is a $*$-derivation and that every continuous linear map between unital C^*-algebras which is a skew Lie triple homomorphism at the identity is a Jordan $*$-homomorphism.

Keywords: C^*-algebra; skew Lie triple derivation; Jordan derivation; skew Lie triple homomorphism
Mathematics Subject Classification: 46L57, 47C15

1. Introduction and basic definitions

Homomorphisms and derivations are the most intensively studied classes of operators on Banach algebras or other algebras. Let A be an algebra and M be an A-bimodule, $d : A \to M$ be a linear map. If for any $x, y \in A$,

$$d(xy) = d(x)y + xd(y), \tag{1.1}$$

then d is called a derivation. If for any $x \in A$, $d(x^2) = d(x)x + xd(x)$, then d is called a Jordan derivation. Additionally, if for any $x, y \in A$, $d([x, y]) = [d(x), y] + [x, d(y)]$, where $[x, y] = xy - yx$ is the Lie product of x and y, then d is called a Lie derivation. Clearly, a derivation is a Jordan derivation and is a Lie derivation. However, a Jordan or Lie derivation need not certainly be a derivation. The standard problem is to find conditions implying that a Jordan or a Lie derivation is actually a derivation. Along this line, there are fruitful results. See for example [1–7].

Recently, many studies are concerned with finding the standard form of linear maps satisfying the derivation type equation for special pairs of x and y. For example, [8–13] studied linear maps which satisfy the derivation type equation for any x, y with $xy = 0$ and [14–16] studied linear maps which satisfy the derivation type equation for any x, y with $xy = 1$.

A map φ from a $*$-algebra A into a bimodule M over A is called a skew Lie derivation if

$$\varphi([a,b]_*) = [\varphi(a), b]_* + [a, \varphi(b)]_* \tag{1.2}$$

for all $a, b \in A$, is called skew Lie derivable at the point $z \in A$ if (1.2) holds for all $a, b \in A$ with $ab = z$, where $[a,b]_* = ab - ba^*$. A linear map φ from A into another C^*-algebra B is called a skew Lie homomorphism at z if $\varphi([a,b]_*) = [\varphi(a), \varphi(b)]_*$ for all $a, b \in A$ with $ab = z$. If φ is a skew Lie homomorphism at all elements of A, then it is called a skew Lie homomorphism on A. For their special importance, these maps attracted many authors' attention in the past decades (see [17–20]).

In [21], Li, Zhao and Chen introduced the concept of nonlinear skew Lie triple derivations and showed every nonlinear skew Lie triple derivation between factors is an additive $*$-derivation.

Difinition 1.1. An additive map $\varphi : A \to M$ is called a nonlinear skew Lie triple derivation if

$$\varphi([[a,b]_*, c]_*) = [[\varphi(a), b]_*, c]_* + [[a, \varphi(b)]_*, c]_* + [[a,b]_*, \varphi(c)]_* \tag{1.3}$$

for all a, b, c in A, and is called skew Lie triple derivable at the point $z \in A$ if Eq (1.3) holds for all $a, b, c \in A$ with $ab = z$ and $c = a$.

For unital C^*-algebras A and B, in the present paper, we study two types of continuous maps. One of them is the type of continuous linear maps from A into B, which are skew Lie triple derivable at the identity. Another is the type of continuous linear maps from A into B, which are skew Lie triple homomorphisms at the identity (see Section 3 for more details).

2. Skew Lie triple derivations at the identity

In this section, we will study linear maps between unital C^*-algebras which are skew Lie triple derivable at the identity. Throughout this section, B will be a unital C^*-algebra, A will be a C^*-subalgebra of B and $1_A = 1_B$, A_{sa} will be the set of all self-adjoint elements in A, $\varphi : A \to B$ will be a linear map. First, let us explore the behavior of the identity 1 under φ when φ is skew Lie triple derivable at the identity.

Lemma 2.1. *If $\varphi : A \to B$ is skew Lie triple derivable at the identity, then $\varphi(1) = \varphi(1)^*$.*

Proof. Since $1 \cdot 1 = 1$ and $[[1,1]_*, 1]_* = 0$, we have

$$0 = [[\varphi(1), 1]_*, 1]_* + [[1, \varphi(1)]_*, 1]_* + [[1,1]_*, \varphi(1)]_* = \varphi(1) - \varphi(1)^* - (\varphi(1) - \varphi(1)^*)^*,$$

from which it follows that $\varphi(1) = \varphi(1)^*$.

Theorem 2.1. *Let $\varphi : A \to B$ be a continuous linear map which is skew Lie triple derivable at the identity, then it is a $*$-derivation.*

Proof. (1) First, we show φ is selfadjoint, i.e., $\varphi(x^*) = \varphi(x)^*$ for all $x \in A$. Put any $a \in A_{sa}$, e^{ita} is a unitary for each $t \in \mathbb{R}$ and $[[\mathrm{e}^{ita}, \mathrm{e}^{-ita}]_*, \mathrm{e}^{ita}]_* = \mathrm{e}^{3ita} - \mathrm{e}^{-ita}$. Thus we deduce that

$$\begin{aligned}
&\varphi(\mathrm{e}^{3ita}) - \varphi(\mathrm{e}^{-ita}) \\
=&[[\varphi(\mathrm{e}^{ita}), \mathrm{e}^{-ita}]_*, \mathrm{e}^{ita}]_* + [[\mathrm{e}^{ita}, \varphi(\mathrm{e}^{-ita})]_*, \mathrm{e}^{ita}]_* + [[\mathrm{e}^{ita}, \mathrm{e}^{-ita}]_*, \varphi(\mathrm{e}^{ita})] \\
=& - \mathrm{e}^{-ita}\varphi(\mathrm{e}^{ita})^*\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{ita})\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})\mathrm{e}^{ita} - \mathrm{e}^{ita}\varphi(\mathrm{e}^{ita})^*\mathrm{e}^{-ita} \\
&- \mathrm{e}^{-2ita}\varphi(\mathrm{e}^{ita}) + \varphi(\mathrm{e}^{ita})\mathrm{e}^{2ita} + \varphi(\mathrm{e}^{ita}) - \varphi(\mathrm{e}^{-ita}) + \mathrm{e}^{2ita}\varphi(\mathrm{e}^{-ita})^* - \mathrm{e}^{2ita}\varphi(\mathrm{e}^{ita})^*.
\end{aligned} \tag{2.1}$$

By taking derivative of Eq (2.1) at t, we obtain that

$$\begin{aligned}
&\varphi(3a\mathrm{e}^{3ita}) + \varphi(a\mathrm{e}^{-ita})\\
=&a\mathrm{e}^{-ita}\varphi(\mathrm{e}^{ita})^*\mathrm{e}^{ita} + \mathrm{e}^{-ita}\varphi(a\mathrm{e}^{ita})^*\mathrm{e}^{ita} - \mathrm{e}^{-ita}\varphi(\mathrm{e}^{ita})^*a\mathrm{e}^{ita}\\
&+ a\mathrm{e}^{ita}\varphi(\mathrm{e}^{ita})\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(a\mathrm{e}^{ita})\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{ita})a\mathrm{e}^{ita}\\
&+ a\mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})\mathrm{e}^{ita} - \mathrm{e}^{ita}\varphi(a\mathrm{e}^{-ita})\mathrm{e}^{ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})a\mathrm{e}^{ita}\\
&- a\mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})^*\mathrm{e}^{-ita} - \mathrm{e}^{ita}\varphi(a\mathrm{e}^{-ita})^*\mathrm{e}^{-ita} + \mathrm{e}^{ita}\varphi(\mathrm{e}^{-ita})^*a\mathrm{e}^{-ita}\\
&+ 2a\mathrm{e}^{-2ita}\varphi(\mathrm{e}^{ita}) - \mathrm{e}^{-2ita}\varphi(a\mathrm{e}^{ita}) + \varphi(a\mathrm{e}^{ita})\mathrm{e}^{2ita} + 2\varphi(\mathrm{e}^{ita})a\mathrm{e}^{2ita} + \varphi(a\mathrm{e}^{ita}) + \varphi(a\mathrm{e}^{-ita})\\
&+ 2a\mathrm{e}^{2ita}\varphi(\mathrm{e}^{-ita})^* + \mathrm{e}^{2ita}\varphi(a\mathrm{e}^{-ita})^* - 2a\mathrm{e}^{2ita}\varphi(\mathrm{e}^{ita})^* + \mathrm{e}^{2ita}\varphi(a\mathrm{e}^{ita})^*. \qquad (2.2)
\end{aligned}$$

Put $t = 0$ and $a = 1$ in Eq (2.2), then we get $\varphi(1) = 0$. Again put $t = 0$ in Eq (2.2), noted that $\varphi(1) = 0$, we get

$$\varphi(a) = \varphi(a)^*, \quad a \in A_{sa}.$$

For each $x \in A$, there are $a, b \in A_{sa}$ such that $x = a + ib$. Hence, $\varphi(x^*) = \varphi(a) - i\varphi(b) = \varphi(x)^*$.

(2) Now we show φ is a $*$-derivation. Taking derivative of Eq (2.2) in $t = 0$ yields that

$$\varphi(a^2) = \varphi(a)a + a\varphi(a). \qquad (2.3)$$

Put any $a, b \in A_{sa}$, then

$$\varphi((a + b)^2) = \varphi(a + b)(a + b) + (a + b)\varphi(a + b).$$

So

$$\varphi(ab + ba) = \varphi(a)b + a\varphi(b) + \varphi(b)a + b\varphi(a). \qquad (2.4)$$

For any $x \in A$, there are $a, b \in A$ such that $x = a + ib$. Hence,

$$\begin{aligned}
\varphi(x^2) =&\varphi((a^2 - b^2) + i(ab + ba))\\
=&(\varphi(a)a + a\varphi(a) - \varphi(b)b - b\varphi(b)) + i(\varphi(a)b + a\varphi(b) + \varphi(b)a + b\varphi(a))\\
=&\varphi(x)x + x\varphi(x).
\end{aligned}$$

Therefore, φ is a Jordan derivation. By [4, Theorem 6.3], φ is a $*$-derivation.

3. Skew Lie triple homomorphisms at the identity

Let A, B be unital C^*-algebras, $\varphi : A \to B$ a linear map. If

$$\varphi([[x, y]_*, x]_*) = [[\varphi(x), \varphi(y)]_*, \varphi(x)]_*$$

for all $x, y \in A$ with $xy = 1$, then φ is called a skew Lie triple homomorphism at the identity. Recall that a Jordan $*$-homomorphism between C^*-algebras is a linear map φ such that $\varphi(x^2) = \varphi(x)^2$ and $\varphi(x^*) = \varphi(x)^*$. In this section we prove that every linear continuous skew Lie triple homomorphism at the identity is a Jordan $*$-homomorphism. Throughout this section A and B will be unital C^*-algebras.

Lemma 3.1. *Let $\varphi : A \to B$ be a linear continuous skew Lie triple homomorphism at the identity. Then $\varphi(1)$ is a partial isometry.*

Proof. Since the product of 1 and itself is 1 and $[[1,1]_*,1]_*=0$, then

$$0=[[\varphi(1),\varphi(1)]_*,\varphi(1)]_*=\varphi(1)^3-\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)\varphi(1)^*\varphi(1)^*+\varphi(1)\varphi(1)\varphi(1)^*.$$

Hence,

$$\varphi(1)^3-\varphi(1)\varphi(1)^*\varphi(1)^*=\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)\varphi(1)\varphi(1)^*. \tag{3.1}$$

For any $a\in A_{sa}$, e^{ita} is a unitary for each real number t and $[[e^{ita},e^{-ita}]_*,e^{ita}]_*=e^{3ita}-e^{-ita}$. Thus we deduce that

$$\begin{aligned}\varphi(e^{3ita})-\varphi(e^{-ita})=&[[\varphi(e^{ita}),\varphi(e^{-ita})]_*,\varphi(e^{ita})]_*\\=&\varphi(e^{ita})\varphi(e^{-ita})\varphi(e^{ita})-\varphi(e^{-ita})\varphi(e^{ita})^*\varphi(e^{ita})\\&-\varphi(e^{ita})\varphi(e^{-ita})^*\varphi(e^{ita})^*+\varphi(e^{ita})\varphi(e^{ita})\varphi(e^{-ita})^*.\end{aligned}$$

Take derivative at t, then

$$\begin{aligned}3\varphi(ae^{3ita})+\varphi(ae^{-ita})=&\varphi(ae^{ita})\varphi(e^{-ita})\varphi(e^{ita})-\varphi(e^{ita})\varphi(ae^{-ita})\varphi(e^{ita})\\&+\varphi(e^{ita})\varphi(e^{-ita})\varphi(ae^{ita})+\varphi(ae^{-ita})\varphi(e^{ita})^*\varphi(e^{ita})\\&+\varphi(e^{-ita})\varphi(ae^{ita})^*\varphi(e^{ita})-\varphi(e^{-ita})\varphi(e^{ita})^*\varphi(ae^{ita})\\&-\varphi(ae^{ita})\varphi(e^{-ita})^*\varphi(e^{ita})^*-\varphi(e^{ita})\varphi(ae^{-ita})^*\varphi(e^{ita})^*\\&+\varphi(e^{ita})\varphi(e^{-ita})^*\varphi(ae^{ita})^*+\varphi(ae^{ita})\varphi(e^{ita})\varphi(e^{-ita})^*\\&+\varphi(e^{ita})\varphi(ae^{ita})\varphi(e^{-ita})^*+\varphi(e^{ita})\varphi(e^{ita})\varphi(ae^{-ita})^*.\end{aligned} \tag{3.2}$$

Put $t=0$ and $a=1$, then

$$4\varphi(1)=\varphi(1)^3+\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)\varphi(1)^*\varphi(1)^*+3\varphi(1)\varphi(1)\varphi(1)^*. \tag{3.3}$$

By taking derivative of Eq (3.2) at $t=0$, we get

$$\begin{aligned}-8\varphi(a^2)=&-\varphi(a^2)\varphi(1)^2+\varphi(1)\varphi(a^2)^*\varphi(1)+\varphi(1)\varphi(1)^*\varphi(a^2)^*-\varphi(1)\varphi(a^2)\varphi(1)^*\\&-\varphi(1)\varphi(a^2)\varphi(1)+\varphi(a^2)\varphi(1)^*\varphi(1)+\varphi(1)\varphi(a^2)^*\varphi(1)^*-\varphi(1)^2\varphi(a^2)^*\\&-\varphi(1)^2\varphi(a^2)+\varphi(1)\varphi(1)^*\varphi(a^2)+\varphi(a^2)[\varphi(1)^*]^2-\varphi(a^2)\varphi(1)\varphi(1)^*\\&+2[\varphi(a)^2\varphi(1)+\varphi(a)\varphi(a)^*\varphi(1)-\varphi(1)[\varphi(a)^*]^2-\varphi(1)\varphi(a)\varphi(a)^*]\\&-2[\varphi(a)\varphi(1)\varphi(a)+\varphi(1)\varphi(a)^*\varphi(a)+\varphi(a)\varphi(1)^*\varphi(a)^*+\varphi(a)^2\varphi(1)^*]\\&+2[\varphi(1)\varphi(a)^2-\varphi(a)\varphi(1)^*\varphi(a)+\varphi(a)\varphi(a)^*\varphi(1)^*-\varphi(a)\varphi(1)\varphi(a)^*].\end{aligned} \tag{3.4}$$

By putting $a=1$ in Eq (3.4), we get

$$8\varphi(1)=\varphi(1)^3-\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)\varphi(1)^*\varphi(1)^*+9\varphi(1)\varphi(1)\varphi(1)^*. \tag{3.5}$$

Multiplying Eq (3.3) by 2 and subtracting Eq (3.5) yield that

$$0=\varphi(1)^3+3\varphi(1)\varphi(1)^*\varphi(1)-\varphi(1)(\varphi(1)^*)^2-3\varphi(1)^2\varphi(1)^*,$$

which implies

$$\varphi(1)^3-\varphi(1)[\varphi(1)^*]^2=3\varphi(1)^2\varphi(1)^*-3\varphi(1)\varphi(1)^*\varphi(1). \tag{3.6}$$

It follows from Eqs (3.1) and (3.6) that

$$\varphi(1)^3 = \varphi(1)[\varphi(1)^*]^2, \quad \varphi(1)^2\varphi(1)^* = \varphi(1)\varphi(1)^*\varphi(1).$$

Now combine the above two equations and Eq (3.3), then we get $\varphi(1) = \varphi(1)\varphi(1)^*\varphi(1)$. Hence, $\varphi(1)$ is a partial isometry.

If furthermore $\varphi(1) = 1$, then we can show the following main theorem.

Theorem 3.2. *Let $\varphi : A \to B$ be a linear continuous skew Lie triple homomorphism at the identity. If $\varphi(1) = 1$, then φ is a Jordan $*$-homomorphism.*

Proof. (1) Since $\varphi(1) = 1$, by putting $t = 0$ in Eq (3.2), we obtain that

$$3\varphi(a) + \varphi(a) = \varphi(a) - \varphi(a) + \varphi(a) + \varphi(a) + \varphi(a)^* - \varphi(a) - \varphi(a) - \varphi(a)^* + \varphi(a)^* + \varphi(a) + \varphi(a) + \varphi(a)^*,$$

i.e., $\varphi(a) = \varphi(a)^*$. As in the proof of Theorem 2.2, we can see that $\varphi(x^*) = \varphi(x)^*$ for all $x \in A$.

(2) Since $\varphi(1) = 1$ and $\varphi(a) = \varphi(a)^*$, it follows from Eq (3.4) that

$$\varphi(a^2) = \varphi(a)^2, \quad a \in A_{sa}.$$

Replacing a by $a + b$ for $a, b \in A_{sa}$, we get

$$\varphi(ab + ba) = \varphi(a)\varphi(b) + \varphi(b)\varphi(a), \quad a, b \in A_{sa}.$$

Now for each $x \in A$, there are $a, b \in A_{sa}$ such that $x = a + ib$. So

$$\varphi(x^2) = \varphi(a^2 - b^2 + i(ab + ba)) = \varphi(a)^2 - \varphi(b)^2 + i(\varphi(a)\varphi(b) + \varphi(b)\varphi(a)) = \varphi(x)^2,$$

and so φ is a Jordan $*$-homomorphism.

For any partial isometry e in a C^*-algebra A, e^*e and ee^* are projections. A can be decomposed as a direct sum of the form

$$A = ee^*Ae^*e \oplus (1 - ee^*)Ae^*e \oplus ee^*A(1 - e^*e) \oplus (1 - ee^*)A(1 - e^*e).$$

From Lemma 3.1, it follows that $\varphi(1)$ is a partial isometry. Let $\varphi(1)\varphi(1)^* = p$ and $\varphi(1)^*\varphi(1) = q$, then we can decompose B as

$$B = pBq \oplus p^\perp Bq \oplus pBq^\perp \oplus p^\perp Bq^\perp,$$

where $p^\perp = 1 - p$ and $q^\perp = 1 - q$. Let $B_0(\varphi(1)) = pBq, B_2(\varphi(1)) = p^\perp Bq^\perp$, we can show the following corollary.

Corollary 3.1. *Let $\varphi : A \to B$ be a linear continuous skew Lie triple homomorphism at the identity. If $\varphi(1)^* = \varphi(1)$, then $\varphi(a) = \varphi(a)^*$ for all $a \in A_{sa}$ and $\varphi(A) \subset B_0(\varphi(1))$.*

Proof. Since $\varphi(1)^* = \varphi(1)$, $p = q = \varphi(1)^2$. By putting $t = 0$ in Eq (3.2), we obtain that

$$\begin{aligned} 4\varphi(a) =& \varphi(a)\varphi(1)\varphi(1) - \varphi(1)\varphi(a)\varphi(1) + \varphi(1)\varphi(1)\varphi(a) + \varphi(a)\varphi(1)^*\varphi(1) \\ &+ \varphi(1)\varphi(a)^*\varphi(1) - \varphi(1)\varphi(1)^*\varphi(a) - \varphi(a)\varphi(1)^*\varphi(1)^* - \varphi(1)\varphi(a)^*\varphi(1)^* \\ &+ \varphi(1)\varphi(1)^*\varphi(a)^* + \varphi(a)\varphi(1)\varphi(1)^* + \varphi(1)\varphi(a)\varphi(1)^* + \varphi(1)\varphi(1)\varphi(a)^* \\ =& 2\varphi(a)p + 2p\varphi(a)^*. \end{aligned} \tag{3.7}$$

Hence, $\varphi(a) = \varphi(a)^*$ for all $a \in A_{sa}$. It follows from Eq (3.7) that

$$\varphi(a) = \frac{1}{2}\varphi(a)p + \frac{1}{2}p\varphi(a).$$

So $p^\perp\varphi(a)p^\perp = p\varphi(a)p^\perp = p^\perp\varphi(a)p = 0$ and so $\varphi(a) = p\varphi(a)p \in B_0(\varphi(1))$. Therefore, $\varphi(A) \subset B_0(\varphi(1))$.

Corollary 3.2. *Let $\varphi : A \to B$ be a linear continuous skew Lie triple homomorphism at the identity. Then φ is a Jordan $*$-homomorphism if and only if $\varphi(1)$ is a projection.*

Proof. If φ is a Jordan $*$-homomorphism, then $\varphi(1) = \varphi(1^2) = \varphi(1)^2$ and $\varphi(1)^* = \varphi(1)$. So $\varphi(1)$ is a projection.

Conversely, if $\varphi(1)$ is a projection, then $\varphi(1) = p$ is the identity of the subalgebra $B_0(\varphi(1))$. By Corollary 3.1, we can regard φ as a map from A into $B_0(\varphi(1))$. Hence, by Theorem 3.2, φ is a Jordan $*$-homomorphism.

4. Conclusions

It is not hard to see that the continuity of the linear map φ is very important in this paper. The automatical continuity of some maps on operator algebra is an important problem (see for example [22]). Let φ be a linear map which is skew Lie triple derivable at the identity or is a skew Lie triple homomorphism at the identity. It is natural to ask whether φ is automatically continuous.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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