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Research article

Control and adaptive modified function projective synchronization of different hyperchaotic dynamical systems

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Abstract: In this work, we consider an adaptive control method, which is simpler and generalized to obtain some conditions on the parameters for hyperchaotic models determined by using a Lyapunov direct method. Further, an adaptive controller for synchronization is designed by using Lyapunov functions by which the deriving system and the response system can realize adaptive modified function projective synchronization up to scaling matrix. Numerical simulation of each system is discussed in detail with graphical results. The graphical results are presented in detail in order to validate the theoretical results. These results in this article generalize and improve the corresponding results of the recent works.

Keywords: adaptive control; modified function projective synchronization; error dynamical system; Liu hyperchaotic; Chen hyperchaotic
Mathematics Subject Classification: 37N35, 34D06, 34H10

1. Introduction

The concepts of synchronization and chaos control has gotten considerable attention in the past three decades. They have wide applicability in different areas, such as engineering, chemical reactions, biological networks and secure communication [36, 46, 47]. To date, researchers have developed so many methods and techniques for chaos control, such as linear feedback control, back-stepping design, nonlinear feedback control and adaptive control [2, 6, 8, 18, 20, 27, 32, 49–53]. Regarding nonlinear chaotic systems, the phenomenon of synchronization is a well-known subject; see, for example [3, 5, 10–12, 17, 19, 22, 25, 29–31, 34, 37–39, 54] and the references therein. Up to now, various kinds of synchronization have been presented, which including complete and anti-synchronization [13, 14] and projective synchronization. Among these kinds, the projective synchronization topic has garnered a lot of interest from researchers [26]. Regarding the projective synchronization, a lot of methods have been considered and presented for projective synchronization, e.g., function projective synchronization (FPS) [1, 7, 9, 15, 16, 23, 40], modified projective synchronization (MPS) [4, 21, 28, 33] and modified function projective synchronization (MFPS) [41, 42], which is a more general definition of MPS and FPS. A system that has at least one positive Lyapunov exponent is known as a chaotic system [48], while a system that has more than one positive Lyapunov exponent is known as a hyperchaotic system [43–45]. However, many existing MFPS studies focus on chaotic systems only. To the best of the authors' knowledge, existing literature only contains a few investigations into the use of the adaptive control method to obtain the MFPS between hyperchaotic systems. Due to its wide applicability in many areas, in the present work, we use the adaptive control method to analyze the stability of an unstable equilibrium point. Moreover, a controller is designed to gain the MFPS between a hyperchaotic Chen system and hyperchaotic Liu system.

Some existing literature on chaos control and synchronization have been presented here: In Section 1, while the remaining sections in the paper are organized as follows. The descriptions of each system and their dynamical properties (the divergence, equilibrium points and its stability) are presented in Section 2. The adaptive control technique is applied to hyperchaotic Liu and Chen systems in Section 3, which also contains some numerical results in the form of graphs. In Section 4, AMFPS among two different hyperchaotic systems is determined and some associated graphical results are presented, which confirm the importance of the given method. Finally, the conclusion is presented in Section 5, summarizing the paper.

2. Systems and dynamical properties

In this section, we present some different dynamical systems with numerical results. In what follows, each system is described by its equations, with graphical results.

2.1. Liu system

The Liu hyperchaotic system given in [24] can be shown through the following set of equations:

$$\begin{cases} \dot{p} &= a(q - p), \\ \dot{q} &= bp + kpr + es, \\ \dot{r} &= -cr - hp^2 + ms, \\ \dot{s} &= -dq, \end{cases} \tag{2.1}$$

where the state variables are given by p, q, r and s and the parameters and their values are chosen as follows: $h = 4$, $e = 1$, $a = 10$, $k = 1$, $b = 40$, $c = 2.5$, $d = 2.5$ and $m = 1$. For these values of the parameters, we give the graphical results depicted in Figures 1 and 2, which are the phase portraits.

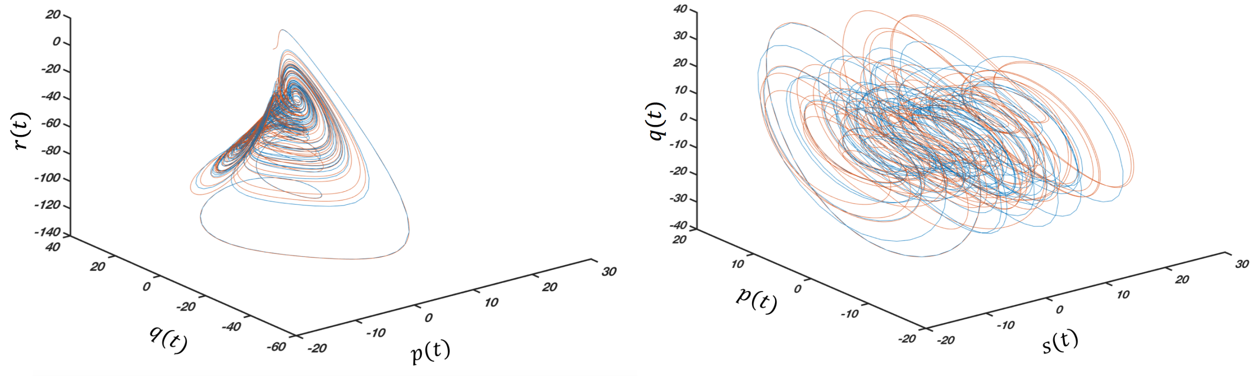


Figure 1. Liu hyperchaotic system in three dimensions.

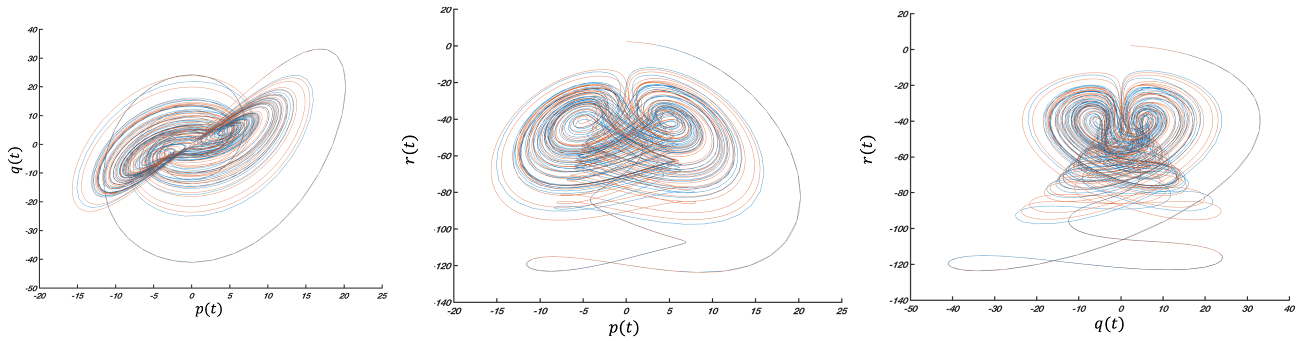


Figure 2. The Liu hyperchaotic system in two dimensions.

Dynamical properties

In this subsection, we give some important properties of the dynamical systems. We give the following definitions:

- **The divergence:** The divergence of a vector field can be obtained as follows:

$$\nabla \cdot V = \frac{\partial \dot{p}}{\partial p} + \frac{\partial \dot{q}}{\partial q} + \frac{\partial \dot{r}}{\partial r} + \frac{\partial \dot{s}}{\partial s} = -a - c = -12.5 < 0;$$

  therefore, system (2.1) is dissipative.
- **Equilibrium points and stability:** By assuming that $\dot{p} = \dot{q} = \dot{r} = \dot{s} = 0$, we can obtain $O(0,0,0,0)$ as the equilibrium point of (2.1).

At the equilibrium point O, the evaluation of the model (2.1) and determination of the Jacobian matrix lead to the characteristics, which give eigenvalues that can show the model stability at that particular equilibrium. We have

$$\lambda^4 + 12.5\lambda^3 - 372.5\lambda^2 - 968.75\lambda + 62.50 = 0.$$

The solution of the fourth-order polynomial leads to the following eigenvalues:

$$\lambda_1 = -2.5, \quad \lambda_2 = 15.51557,$$
$$\lambda_3 = -25.5785, \quad \lambda_4 = 0.06299.$$

It shows clearly that the equilibrium O is unstable.

2.2. Chen system

Here, we describe the Chen system and its analysis about the specific equilibrium point and give its numerical result. Chen et al. reported a hyperchaotic system in [35]. This hyperchaotic system is shown by the following equations:

$$\begin{cases} \dot{p} &= -qr + fp, \\ \dot{q} &= pr + gq, \\ \dot{r} &= \frac{1}{3}pq + jr + 0.2s, \\ \dot{s} &= lp + 0.5qr + 1.05s. \end{cases} \tag{2.2}$$

Here, p, q, r and s represent the state variables, and the parameters with their specific values are given by $f = 5$, $g = -10$, $j = -3.8$ and $l = 0.1$. The graphical results corresponding to these specific parameters are presented in Figures 3 and 4.

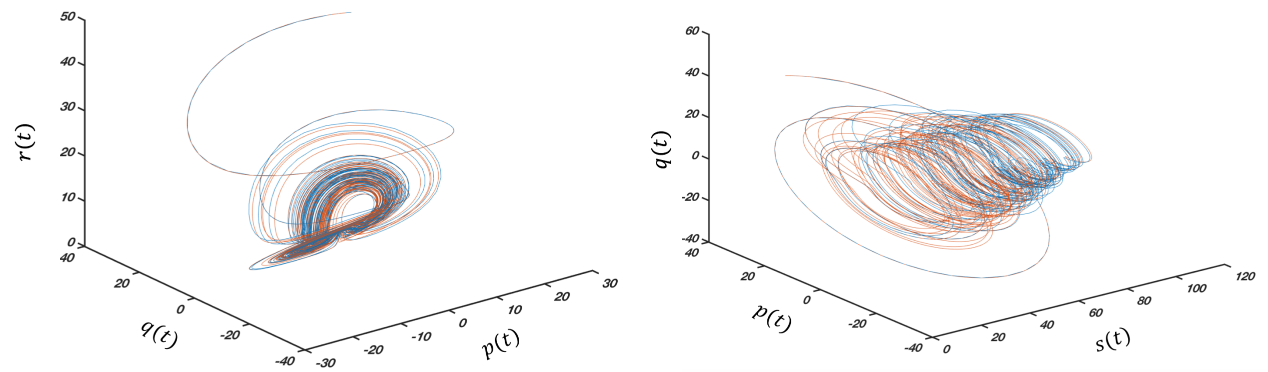


Figure 3. The graphical representation of the Chen hyperchaotic system in three dimensions.

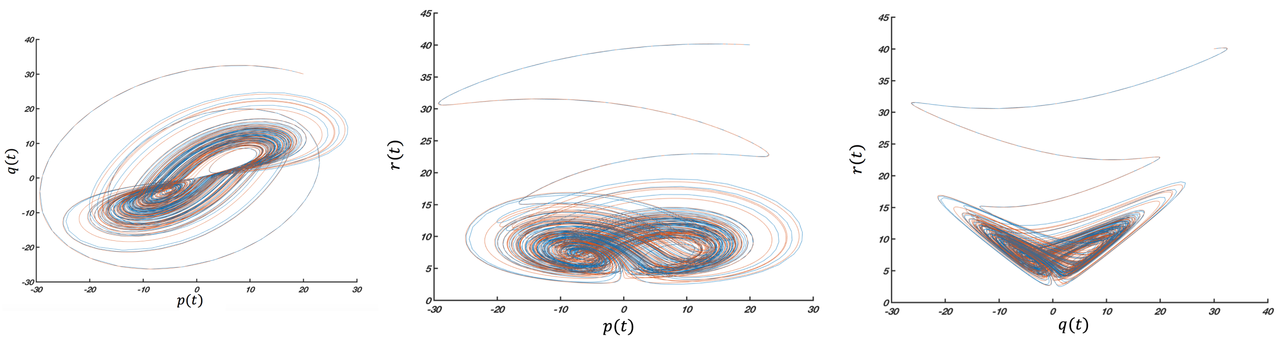


Figure 4. The graphical representation of the Chen hyperchaotic system in two dimensions.

Dynamical properties

- **The divergence:** The system (2.2) is dissipative since its divergence is negative, as shown below:

$$\nabla \cdot V = \frac{\partial \dot{p}}{\partial p} + \frac{\partial \dot{q}}{\partial q} + \frac{\partial \dot{r}}{\partial r} + \frac{\partial \dot{s}}{\partial s} = a + b + c + 1.05 = -7.75 < 0.$$

- **Equilibrium points and stability:** By assuming that $\dot{p} = \dot{q} = \dot{r} = \dot{s} = 0$, an equilibrium point can be easily obtained, say, $E(0, 0, 0, 0)$. The evaluation of system (2.2) at this equilibrium point leads to the Jacobian matrix and, further, to the following characteristics, which show the model stability that can be described by the sign of eigenvalues.

$$\lambda^4 + 7.75\lambda^3 - 40.24\lambda^2 - 157.45\lambda + 199.5 = 0.$$

Therefore, the eigenvalues are obtained as follows:

$$\lambda_1 = -3.8, \quad \lambda_2 = 1.05,$$
$$\lambda_3 = 5, \quad \lambda_4 = -10.$$

Here, it can be seen clearly that the two eigenvalues are positive, which shows that the system (2.2) at the equilibrium point E is unstable.

3. Adaptive control

Here, we apply the adaptive control method to the different chaotic systems of Liu and Chen. The numerical results for the given parameters are presented. We discuss each system in detail in the following subsections.

3.1. Controlling the hyperchaotic Liu system

Consider the system (2.1) in terms of adaptive control representations:

$$\begin{cases} \dot{p} &= a(q - p) + u_1(t), \\ \dot{q} &= bp + kpr + es + u_2(t), \\ \dot{r} &= -cr - hp^2 + ms + u_3(t), \\ \dot{s} &= -dq + u_4(t). \end{cases} \tag{3.1}$$

In system (3.1), the controllers to be designed are given by u_1, u_2, u_3 and u_4, and the state variables are given by p, q, r and s. The asymptotic stability of the system (3.1) can be shown through the Lyapunov function. Define the Lyapunov function:

$$V(p, q, r, s) = \frac{1}{2}(p^2 + q^2 + r^2 + s^2);$$

then,

$$\begin{aligned} \dot{V} = &\, p(a(q - p) + u_1(t)) + q(bp + kpr + es + u_2(t)) + r(-cr - hp^2 + ms + u_3(t)) \\ &+ s(-dq + u_4(t)). \end{aligned} \tag{3.2}$$

Thus, the control functions can be formulated as follows:

$$u_1(t) = -aq, \quad u_2(t) = -bq - bp - kpr - es, \quad u_3(t) = -ms + hp^2, \quad u_4(t) = dq - ds. \tag{3.3}$$

Substituting (3.3) into (3.2), we obtain

$$\dot{V} = -(ap^2 + bq^2 + cr^2 + ds^2).$$

It can be seen that $\dot{V} \leq 0$, which shows the asymptotic stability of the model (3.1) at the given equilibrium point.

3.2. Controlling the hyperchaotic Chen system

In terms of adaptive control, the model (2.2) can be described by the below equations:

$$\begin{cases} \dot{p} &= -qr + fp + u_1(t), \\ \dot{q} &= pr + gq, \\ \dot{r} &= \frac{1}{3}pq + jr + 0.2s + u_2(t), \\ \dot{s} &= lp + 0.5qr + 1.05s + u_3(t). \end{cases} \tag{3.4}$$

In system (3.4), the state variables are given by p, q, r and s, while the controllers to be designed are u_1, u_2 and u_3. We use a Lyapunov function to show whether the model (3.4) is asymptotically stable. To do this, let us define the Lyapunov function below:

$$V(p, q, r, s) = \frac{1}{2}(p^2 + q^2 + r^2 + s^2);$$

after taking the time derivative, we get

$$\begin{aligned} \dot{V} = p(-qr + fp + u_1(t)) + q(pr + gq) + r(\frac{1}{3}pq + jr + 0.2s + u_2(t)) \\ + s(lp + 0.5qr + 1.05s + u_3(t)). \end{aligned} \tag{3.5}$$

Thus, the control functions can be formulated as follows:

$$u_1(t) = -2fp, \;\; u_3(t) = -\frac{1}{3}pq - 0.2s, \;\; u_3(t) = -2.1s - 0.5qr - lp. \tag{3.6}$$

Substituting (3.6) into (3.5), we obtain

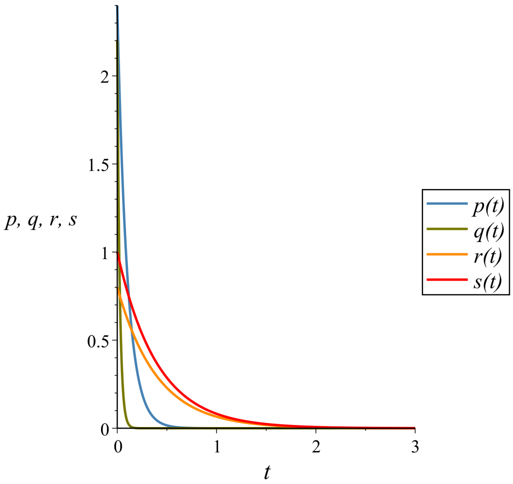
$$\dot{V} = -(fp^2 - gq^2 - jr^2 + 1.05s^2).$$

Obviously, $\dot{V} \leq 0$, which ensures the asymptotic stability of the model (3.4) at the given equilibrium point.

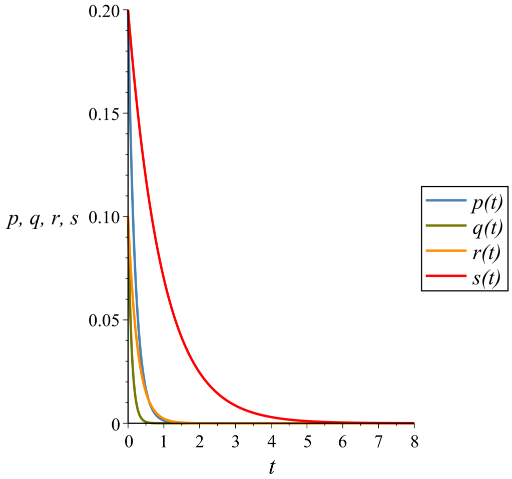
3.3. Simulation results

Here, we discuss the simulation results for the controlling hyperchaotic Liu and Chen systems. The numerical results were obtained by using Maple software version 16. The numerical results have been obtained in the form of graphics, which show the effectiveness of the method proposed. The initial conditions associated with hyperchaotic Liu system (2.1) and the hyperchaotic Chen system (2.2) are as follows: $q_1(0) = 2.2$, $p_1(0) = 2.4$, $r_1(0) = 0.8$, $s_1(0) = 0$, $q_2(0) = 0.1$ $r_2(0) = 0.1$, $p_2(0) = 0.2$ and $s_2(0) = 0.2$. Figure 5, with subgraphs (a) and (b), is presented to show the convergence of the trajectories of the controlled systems at the equilibrium points O and E. The convergence of the trajectory at O for the uncontrolled system (2.1) is shown in Figure 5(a), while, in Figure 5(b), we give the convergence of the trajectory at E for the uncontrolled system (2.2).

In the simulation, the error dynamics approximately tended to zero. The presented method of adaptive control shows that they are valid for application in hyperchaotic systems.



(a)



(b)

Figure 5. Hyperchaotic (a) Liu and (b) Chen systems controlled to a fixed point.

4. AMFPS of Liu and Chen chaotic dynamical systems

Here, we present Liu and Chen chaotic dynamical systems with adaptive MFPS. In what follows, we explain each system in detail.

The drive system for AMFPS of a Liu hyperchaotic system can be described by the following equations:

$$\begin{cases} \dot{p}_1 &= a(q_1 - p_1), \\ \dot{q}_1 &= bp_1 + kp_1r_1 + es_1, \\ \dot{r}_1 &= -cr_1 - hp_1^2 + ms_1, \\ \dot{s}_1 &= -dq_1, \end{cases} \tag{4.1}$$

and the Chen hyperchaotic model, as a response model, is described by the following equations:

$$\begin{cases} \dot{p}_2 &= -q_2r_2 + fp_2 + u_1, \\ \dot{q}_2 &= p_2r_2 + gq_2 + u_2, \\ \dot{r}_2 &= \frac{1}{3}p_2q_2 + jr_2 + 0.2s_2 + u_3, \\ \dot{s}_2 &= lp_2 + 0.5q_2r_2 + 1.05s_2 + u_4, \end{cases} \tag{4.2}$$

where the nonlinear controllers are given by u_i, for $i = 1, 2, 3, 4$, and, in the sense of MFPS, the synchronization of the two chaotic systems is given by

$$\begin{cases} \lim\limits_{t\to+\infty} \|p_2 - (\alpha_{11}p_1 + \alpha_{12})p_1\| = 0, \\ \lim\limits_{t\to+\infty} \|q_2 - (\alpha_{21}q_1 + \alpha_{22})q_1\| = 0, \\ \lim\limits_{t\to+\infty} \|r_2 - (\alpha_{31}r_1 + \alpha_{32})r_1\| = 0, \\ \lim\limits_{t\to+\infty} \|s_2 - (\alpha_{41}s_1 + \alpha_{42})s_1\| = 0. \end{cases} \tag{4.3}$$

The error dynamics between (4.1) and (4.2) are given by

$$\begin{cases} \dot{e}_1 &= -q_2r_2 + fp_2 - 2\alpha_{11}ap_1q_1 + 2\alpha_{11}ap_1^2 - \alpha_{12}aq_1 + \alpha_{12}ap_1 + u_1, \\ \dot{e}_2 &= p_2r_2 + gq_2 - 2\alpha_{21}bp_1q_1 - 2\alpha_{21}kp_1q_1r_1 - 2\alpha_{21}eq_1s_1 - \alpha_{22}bp_1 - \alpha_{22}kp_1r_1 - \alpha_{22}es_1 + u_2, \\ \dot{e}_3 &= \frac{1}{3}p_2q_2 + jr_2 + 0.2s_2 + 2\alpha_{31}cr_1^2 + 2\alpha_{31}hr_1p_1^2 - 2\alpha_{31}mr_1s_1 + \alpha_{32}cr_1 \\ &+\alpha_{32}hp_1^2 - \alpha_{32}ms_1 + u_3, \\ \dot{e}_4 &= lp_2 + 0.5q_2r_2 + 1.05s_2 + 2\alpha_{41}dq_1s_1 + \alpha_{42}dq_1 + u_4. \end{cases} \tag{4.4}$$

By defining the state errors $e_1(t) = p_2 - (\alpha_{11}p_1 + \alpha_{12})p_1, e_2(t) = q_2 - (\alpha_{21}q_1 + \alpha_{22})q_1$, $e_3(t) = r_2 - (\alpha_{31}r_1 + \alpha_{32})r_1$ and $e_4(t) = s_2 - (\alpha_{41}s_1 + \alpha_{42})s_1$.

The main purpose is to determine the controls u_i $(i = 1, 2, 3, 4)$ that could stabilize the error variables of the model (4.4). So, the following control law is presented:

$$\begin{cases} u_1 &= q_2r_2 - 2fp_2 + 2a\alpha_{11}p_1q_1 - 2a\alpha_{11}p_1^2 + a\alpha_{12}q_1 - \alpha_{12}ap_1 + \alpha_{11}fp_1^2 + \alpha_{12}fp_1, \\ u_2 &= -p_2r_2 + 2\alpha_{21}bp_1q_1 + 2\alpha_{21}kp_1q_1r_1 + 2\alpha_{21}eq_1s_1 + \alpha_{22}bp_1 + \alpha_{22}kp_1r_1 \\ &+\alpha_{22}es_1 + g\alpha_{21}q_1^2 + g\alpha_{22}q_1, \\ u_3 &= -\frac{1}{3}p_2q_2 - 0.2s_2 - 2\alpha_{31}cr_1^2 - 2\alpha_{31}hr_1p_1^2 + 2\alpha_{31}mr_1s_1 - \alpha_{32}hp_1^2 \\ &+\alpha_{32}ms_1 - \alpha_{32}cr_1 + \alpha_{31}jr_1^2 + \alpha_{32}jr_1, \\ u_4 &= -lp_2 - 0.5q_2r_2 - 2.1s_2 + 1.05\alpha_{41}s_1^2 + 1.05\alpha_{42}s_1 - 2d\alpha_{41}q_1s_1 - \alpha_{42}dq_1. \end{cases} \tag{4.5}$$

So, we have the below result:

Theorm 1. *For nonzero scalars α_1, α_2, α_3 and α_4, the AMFPS among the two models (4.1) and (4.2) will be induced by the control input (4.5).*

Proof of Theorem 1. Consider the function

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2). \tag{4.6}$$

Then

$$\begin{aligned} \dot{V} &= \dot{e}_1e_1 + \dot{e}_2e_2 + \dot{e}_3e_3 + \dot{e}_4e_4 \\ &= e_1(- q_2r_2 + fp_2 - 2\alpha_{11}ap_1q_1 + 2\alpha_{11}ap_1^2 - \alpha_{12}aq_1 + \alpha_{12}ap_1 + u_1) \\ &+ e_2(p_2r_2 + gq_2 - 2\alpha_{21}bp_1q_1 - 2\alpha_{21}kp_1q_1r_1 - 2\alpha_{21}eq_1s_1 - \alpha_{22}bp_1 - \alpha_{22}kp_1r_1 - \alpha_{22}es_1 + u_2) \\ &+ e_3(\frac{1}{3}p_2q_2 + jr_2 + 0.2s_2 + 2\alpha_{31}cr_1^2 + 2\alpha_{31}hr_1p_1^2 - 2\alpha_{31}mr_1s_1 + \alpha_{32}cr_1 + \alpha_{32}hp_1^2 - \alpha_{32}ms_1 + u_3) \\ &+ e_4(lp_2 + 0.5q_2r_2 + 1.05s_2 + 2\alpha_{41}dq_1s_1 + \alpha_{42}dq_1 + u_4); \end{aligned} \tag{4.7}$$

substituting the control input (4.5) gives

$$\begin{aligned} \frac{dV}{dt} &= e_1(- fp_2 + \alpha_{11}fp_1^2 + \alpha_{12}fp_1) + e_2(gq_2 + g\alpha_{21}q_1^2 + g\alpha_{22}q_1) \\ &+ e_3(jr_2 + \alpha_{31}jr_1^2 + \alpha_{32}jr_1) + e_4(- 1.05s_2 + 1.05\alpha_{41}s_1^2 + 1.05\alpha_{42}s_1), \\ \Rightarrow \frac{dV}{dt} &= -fe_1^2 + ge_2^2 + je_3^2 - 1.05e_4^2. \end{aligned} \tag{4.8}$$

Then, we have

$$\frac{dV}{dt} = -e^T Pe, \tag{4.9}$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}, \quad P = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & -g & 0 & 0 \\ 0 & 0 & -j & 0 \\ 0 & 0 & 0 & 1.05 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 3.8 & 0 \\ 0 & 0 & 0 & 1.05 \end{bmatrix};$$

since $\dot{V}$ is negative definite, it shows that the controller (4.5) induces AMFPS in the driving system via the response system. □

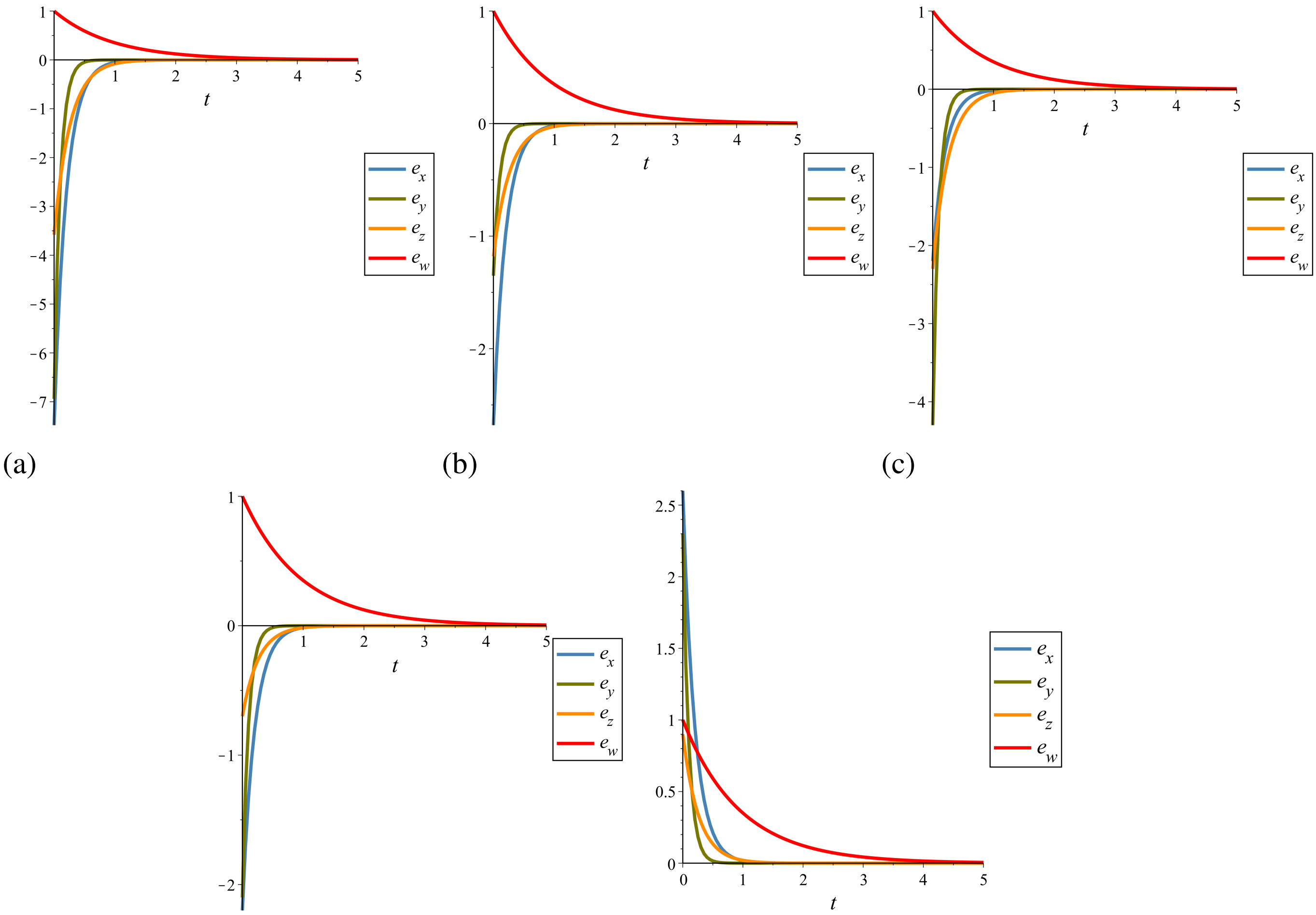
4.1. Simulation results

In the present section, we obtain the numerical results of the systems described above. To obtain the simulation results, we used Maple 16. The graphical results were obtained that verify the proposed synchronization method. The initial conditions of the variables in the hyperchaotic Liu system (2.1) and the hyperchaotic Chen system (2.2) were considered as follows: $p_1(0) = 2.4$, $q_1(0) = 2.2$, $r_1(0) = 0.8$, $s_1(0) = 0$, $p_2(0) = 0.2$, $q_2(0) = 0.1$, $r_2(0) = 0.1$ and $s_2(0) = 0.2$. The simulation results for the synchronization are shown in Figure 6. First, we chose the scaling functions as follows:

$$\varphi_1 = 0.5p_1 + 2,\ \varphi_2 = q_1 + 1,\ \varphi_3 = 2r_1 + 3,\ \varphi_4 = 2s_1 + 1;$$

then the AMFPS between (4.1) and (4.2) was achieved as shown in Figure 6(a). Figure 6(b) shows the generalized FPS, when the scaling functions are as follows: $\varphi_1 = 0.5p_1$, $\varphi_2 = 0.3q_1$, $\varphi_3 = 2r_1$, $\varphi_4 = s_1$. Furthermore, by choosing the simplified scaling functions as $\varphi_1 = 1$, $\varphi_2 = 2$, $\varphi_4 = 4$ and $\varphi_3 = 3$, we obtained MPS as shown in Figure 6(c). Moreover, by simplifying the scaling functions to $\varphi_i = 1$, for $i = 1, 2, 3, 4$, we obtained complete synchronization, as shown in Figure 6(d). Finally, Figure 6(e) shows the anti-synchronization as a result of choosing $\varphi_i = -1$ $(i = 1, 2, 3, 4)$.

As can be seen, the error dynamics have approximately tended to zero. The MFPS applied to the hyperchaotic systems is valid and gives effective results.



(d) (e)

Figure 6. The errors between the Liu and Chen hyperchaotic systems for (a) AMFPS, (b) GFPS, (c) MPS, (d) complete synchronization and (e) anti-synchronization.

5. Conclusions

We successfully presented and applied the adaptive control technique to the hyperchaotic Liu and Chen systems. The asymptotic stability of each system on the path to the specific equilibrium point is discussed. Further, we assigned specific values to the parameters and obtained the graphical results in detail. The Lyapunov function constructed for each system and its asymptotic stability are discussed. Further, MFPS was used to synchronize the two different hyperchaotic systems through the use of a Lyapunov function. Under the conditions of the controller, MFPS of the hyperchaotic Liu system and hyperchaotic Chen system was successfully achieved. Based on the simulation results, we discussed the scaling function simplified to scaling factor. All of the simulation results are demonstrated the corresponding figures to show that the system errors approached zero. The work of this paper provides a theoretical reference for the control and synchronization of hyperchaotic systems. In future work, we may apply the idea to applications in the engineering field, such as information processing and secure communication.

Use of AI tools declaration

The authors declare that they have not used artificial intelligence tools in the creation of this article.

Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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