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Research article

A new fuzzy decision support system approach; analysis and applications

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Abstract: The current study proposes the idea of the N-cubic Pythagorean fuzzy set with their basic arithmetic operations to aggregate these sets. We define the score and accuracy functions for the comparison purpose. Finally, we discuss Chang's extent analysis of AHP under the environment of the N-cubic Pythagorean fuzzy set using the idea of triangular N-cubic Pythagorean fuzzy set. As an application, we discuss the reason for the downfall of international airlines using the developed approach.

Keywords: cubic Pythagorean fuzzy sets; N-cubic Pythagorean fuzzy sets; triangular N-cubic Pythagorean fuzzy set; AHP, application
Mathematics Subject Classification: 03B52, 03E72, 08A72

1. Introduction

The idea of a fuzzy set was initiated by Zadeh [1] to measure ambiguity and vagueness. Fuzzy sets use only membership values for the estimation of uncertainty. But it's difficult for experts to use crisp values for their decision purpose thus Zadeh [8–11] proposed the idea of interval-valued fuzzy sets. Atanassov [2,3] provided the further generalization of fuzzy sets by adding non-membership grades with the membership grades known as intuitionistic fuzzy sets. Yager [4–7,33–35] further extended the idea of intuitionistic fuzzy sets to Pythagorean fuzzy sets which are further extended to Q-rung orthopair fuzzy sets. The idea of interval-valued intuitionistic fuzzy sets [12–14] and interval-valued Pythagorean fuzzy sets [15–18] was also a very valuable addition. More details can be seen in [33–35] about applications of Pythagorean fuzzy sets. The idea of cubic sets (combination of interval-valued fuzzy sets and fuzzy sets) given by Jun [19–21] uses interval-valued fuzzy data as well as the crisp term of fuzzy set. Detailed about the applications of cubic sets can be seen in [36–44]. On the same lines, the idea of cubic Pythagorean fuzzy sets [22] and neutrosophic cubic set [23–25] was developed. For the applications of neutrosophic cubic sets we refer the reader [45–47]. Chang's extent analysis of the analytical hierarchy process, [27–30] used hierarchy form of data in terms of a fuzzy environment instead of crisp data. For more applications of AHP we refer the reader [50]. Jun et al. [31] extends the idea of Zadeh's fuzzy sets and introduced a new approach which is called negative-valued function, and constructed N-structures. Further, they applied N-structure theory to subtraction algebra and BCK/BCI-algebra and study their related properties. They also discussed N-ideals of subtraction algebra and used N-fuzzy sets which is the extension of fuzzy sets where they used [-1,0] instead of [0,1]. In 2013, [32] William and Saeid further gave the concept of generalized N-ideals of subtraction algebras. Rashid et al. [26] applied N-structure on cubic fuzzy set as N-cubic fuzzy set. Applications of N-structures can be seen in [48,49]. The following are some examples of applications that deal with the negative aspects of objects or their effects.

(1) Due to the lack of any approved treatment, health workers propose some possible treatment (Clinical Management Protocol 2020) to cure the unexpected virus infection, where the choice of drugs has a significant impact on the patients' recovery rate. A few researchers have experimented with the selection of drugs for COVID-19 affected patients, according to the literature. The recommended drugs for treating COVID-19 patients have a variety of functions, including effectiveness, adverse effects, and some unknown consequences. As a result, we use these sets to look for drugs that have a detrimental influence on COVID-19 or other disorders.

(2) Another example is that this set might be used to assess the drawbacks of utilizing social media, such as Facebook, which can lead to addiction and privacy issues.

It has already been studied that after crisp sets, the need for fuzzy sets was felt where true membership was discussed due to its imprecise and vague properties. Moreover, sets for false membership were defined and investigated. Many other fuzzy sets were defined in a way rather than a negative side to describe the positive behavior that is used in many real-life problems, but another thing is that these sets only illustrate positive behavior of things to show only positive features. Obviously, in exact decision-making, we observe the limitations of these things and keep them in mind.

So how do make any decision based on only the positive side?

Is it possible for anything that has the specialty of good quality features only?

These things show negative features as well as a positive behavior.

To notice this side, we define N-cubic Pythagorean fuzzy set (NCPFNs), which apply to real-life problems to describe the drawbacks that are precisely used in decision making in a better way. This paper is organizes as follows: In Section 1, we discussed brief history of different sets and motivation about our work. In Section 2, we recall some basic definitions. In Section 3, we introduce the novel concept of N-cubic Pythagorean fuzzy set (NCPFNs) with some interesting properties. In

Section 4, we develop triangular N-cubic Pythagorean fuzzy set. Further application of the proposed work is discussed in Section 5 using AHP method. Section 6, discussed comparison analysis and conclusions are providing in Section 7.

2. Preliminaries

We recall some basic definitions as:

Definition: [4] Suppose G be the non-empty universal set, then Pythagorean fuzzy set is defined as: $\{< g, \mu_G(g), \eta_G(g) > g \in G\}$, For $\mu_G(g): G \to [0,1]$ and $\eta_G(g): G \to [0,1]$ be the membership and non-membership values of g in G.

Also, $0 \leq \mu_G(g)^2 + \eta_G(g)^2\} \leq 1$ and $\pi_P(g) = \sqrt{1 - \mu_G(g)^2 - \eta_G(g)^2}$ is the degree of indeterminacy.

Definition: [20] Suppose X be the set; cubic sets are the structure given as:

$$C(X) = \{x, \mu_X(x), \eta_X(x)\}$$

where $\mu_X(x)$be the interval valued fuzzy set and $\eta_X(x)$ be the fuzzy set.

Definition: [22] Let X be a fixed non-empty set. By a cubic Pythagorean fuzzy set, we mean a structure of the form

$$C(P) = \{< x, \mu_C(x), \eta_C(x)) > / x \in X\}$$

where $\mu_C(x)$ is an interval valued Pythagorean fuzzy set in X and $\eta_C(x)$ is Pythagorean fuzzy set in X. Let $\pi_{CP}(x) = < [\pi_{CP}^-(x), \pi_{C(P)}^+(x)], \pi_{CP}(x) >$, then $\pi_{CP}(x)$ is said to be cubic Pythagorean fuzzy index of element x $\in$ X to set C(P), where

$$\pi_{CP}^-(x) = \sqrt{1 - (\mu_1^+)^2 - (\mu_2^+)^2},\ \pi_{C(P)}^+(x) = \sqrt{1 - (\mu_1^-)^2 - (\mu_2^-)^2},\ \pi_{CP}(x) = \sqrt{1 - \eta_1^2 - \eta_2^2}$$

and also, $[0,0] \leq [\mu_1(x)^-, \mu_1(x)^+]^2 + [\mu_2(x)^-, \mu_2(x)^+]^2 \leq [1,1], 0 \leq \eta_1(x)^2 + \eta_2(x)^2 \leq 1$.

3. N-Cubic Pythagorean fuzzy sets

In this section we initiated the study of N-cubic Pythagorean fuzzy sets (NCPFN). As cubic sets consist of two sets, i.e., interval valued fuzzy sets and fuzzy sets. For achieving NCPFN, we must first define N-Pythagorean fuzzy sets (NPFS) and N-interval valued Pythagorean fuzzy sets (NIVPFS). We discuss some basic operations and properties of N-cubic Pythagorean fuzzy sets (NCPFN).

(Note that the superscript 2(2) denotes the even power of membership values and non-membership value, while 2(2)+1 be the odd power of (-1) that are used in the whole manuscript.)

3.1. N-Pythagorean fuzzy sets

Definition: Let A be a fixed set. Then N-Pythagorean fuzzy set is a structure of the form

$$X = \{\mu_X(x), \eta_X(x) > x \in X\},$$

where $\mu_X(x): X \to [-1,0]$ and $\eta_X(x): X \to [-1,0]$ be the N-valued fuzzy membership and

N-valued fuzzy non-membership of x $\in$ X with the condition that

$$-1 \leq (-1)^{2(2)+1}\{\mu_X(x)^{2(2)} + \eta_X(x)^{2(2)}\} \leq 0$$

and

$$\pi_X(x) = (-1)^{2(2)+1}\sqrt{1 - \mu_X(x)^{2(2)} - \eta_X(x)^{2(2)}}$$

are the N- Pythagorean degree of indeterminacy. The spacing graph of N-Pythagorean fuzzy set can be demonstrated by Figure 1.

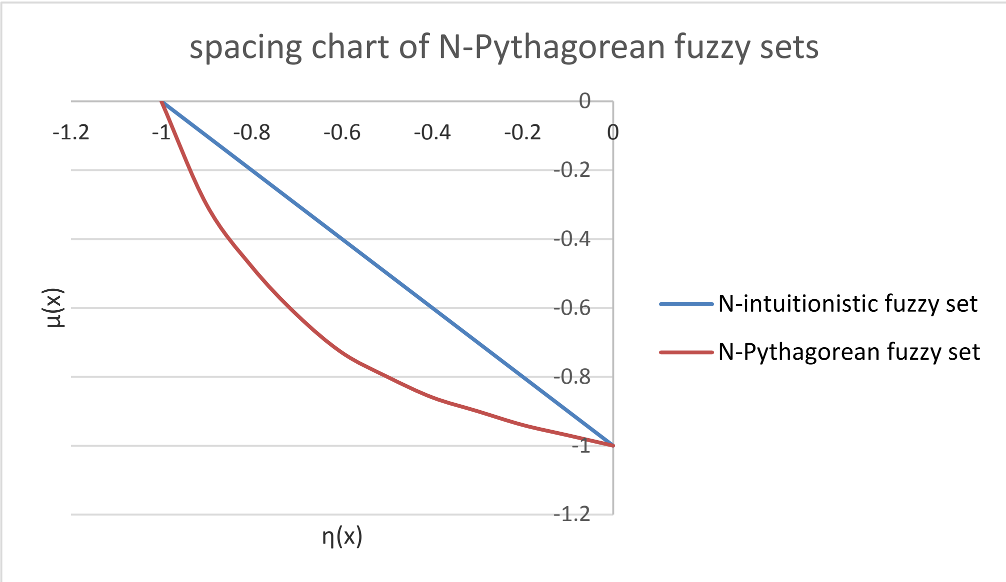


Figure 1. Spacing chart of N-Pythagorean fuzzy set.

Remark: In general, for N-q rung ortho pair fuzzy set $-1 \leq (-1)^{2q+1}\{\mu_X(x)^{2q} + \eta_X(x)^{2q}\} \leq 0$ for $q \geq 1$ and $\pi_p(x) = (-1)^{2q+1}\sqrt[q]{\mu_X(x)^{2q} + \eta_X(x)^{2q} - \mu_X(x)^{2q}\,\eta_X(x)^{2q}}$ for $q \geq 3$.

3.2. N-Interval valued Pythagorean fuzzy sets

Definition: Let X be a fixed set. Then N-interval valued Pythagorean fuzzy set is a structure of the form

$A(x) = \{x, \mu_A(x) = [\mu_A(x)^-, \mu_A(x)^+],\ \eta_A(x) = [\eta_A(x)^-, \eta_A(x)^+]\}$
where $\mu_A(x) \in D[-1,0], \eta_A(x) \in D[-1,0]$ be the N-interval valued fuzzy membership and N-interval valued fuzzy non-membership of x $\in$ X with the condition that

$-1 \leq (-1)^{2(2)+1}\{(\mu_A(x)^-)^{2(2)} + (\eta_A(x)^-)^{2(2)}\} \leq 0.$

Also,

$$\pi_A(x) = [\pi_A(x)^-, \pi_A(x)^+],$$

where

$$\pi_A(x)^- = (-1)^{2(2)+1}\sqrt{(1-(\mu_A(x)^+)^{2(2)} - (\eta_A(x)^+)^{2(2)}},$$

$$\pi_A(x)^+ = (-1)^{2(2)+1}\sqrt{(1-(\mu_A(x)^-)^{2(2)} - (\eta_A(x)^-)^{2(2)}}$$

be the N- interval valued Pythagorean degree of indeterminacy. The spacing graph of N-interval valued Pythagorean fuzzy set can be demonstrated by Figure 2.

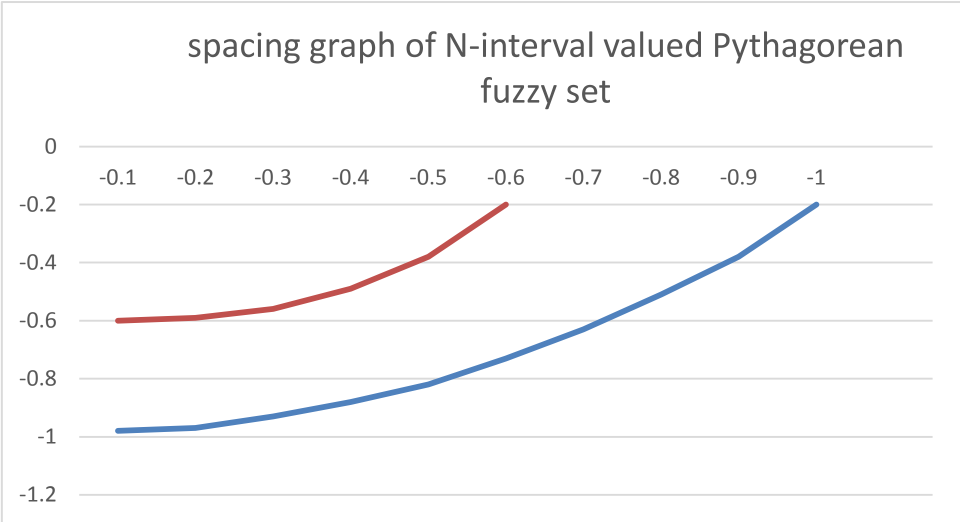


Figure 2. Spacing chart of N-interval valued Pythagorean fuzzy set.

3.3. N-Cubic Pythagorean fuzzy sets

Definition: Let X is a fixed nonempty set. By an N-cubic Pythagorean fuzzy set we mean a structure of the form

$$N_P(x) = \{< x, \mu_{NP}(x), \eta_{NP}(x) > / x \in X\}$$

Where $\mu_{NP}(x)$ is an N-interval valued Pythagorean fuzzy set in X and $\eta_{NP}(x)$ is N-Pythagorean fuzzy set in X.
Let $\pi_{NP}(x) = < [\pi_{NP}^-(x), \pi_{NP}^+(x)], \pi_{NP}(x) >$.
Then $\pi_{NP}(x)$ is said to be N-cubic Pythagorean fuzzy index of element x $\in$ X to set $N_C{}^P(x)$, where

$$\pi_{NP}^-(x) = (-1)^{2(2)+1}\sqrt{1-(\mu_1^+)^{2(2)} - (\mu_2^+)^{2(2)}},$$

$$\pi_{NP}^+(x) = (-1)^{2(2)+1}\sqrt{1-(\mu_1^-)^{2(2)} - (\mu_2^-)^{2(2)}},$$

$$\pi_{NP}(x) = (-1)^{2(2)+1}\sqrt{1-\eta_1^{2(2)} - \eta_2^{2(2)}}.$$

Also,

$$\mu_{NP}(x) = ((\mu_2(x), \mu_2(x))$$

where,

$\mu_1(x) = [\,\mu_1(x)^-, \mu_1(x)^+], \mu_2(x) = [\,\mu_2(x)^-, \mu_2(x)^+],\ \eta_{NP}(\mathrm{x}) = (\eta_1(\mathrm{x}), \eta_2(\mathrm{x}))$

and,

$[-1,-1] \leq (-1)^{2(2)+1}\{[\,\mu_1(x)^-, \mu_1(x)^+]^2 + [\,\mu_2(x)^-, \mu_2(x)^+]^2\} \leq [0,0],$

$-1 \ \leq \ (-1)^{2(2)+1}\{\eta_1^{2(2)} + \eta_2^{2(2)}\} \leq \ 0.$

We denote N-cubic Pythagorean fuzzy set as NCPFs = <N-IVPFs, N-PFs>. The spacing graph of N-cubic Pythagorean fuzzy set can be demonstrated by Figure 3.

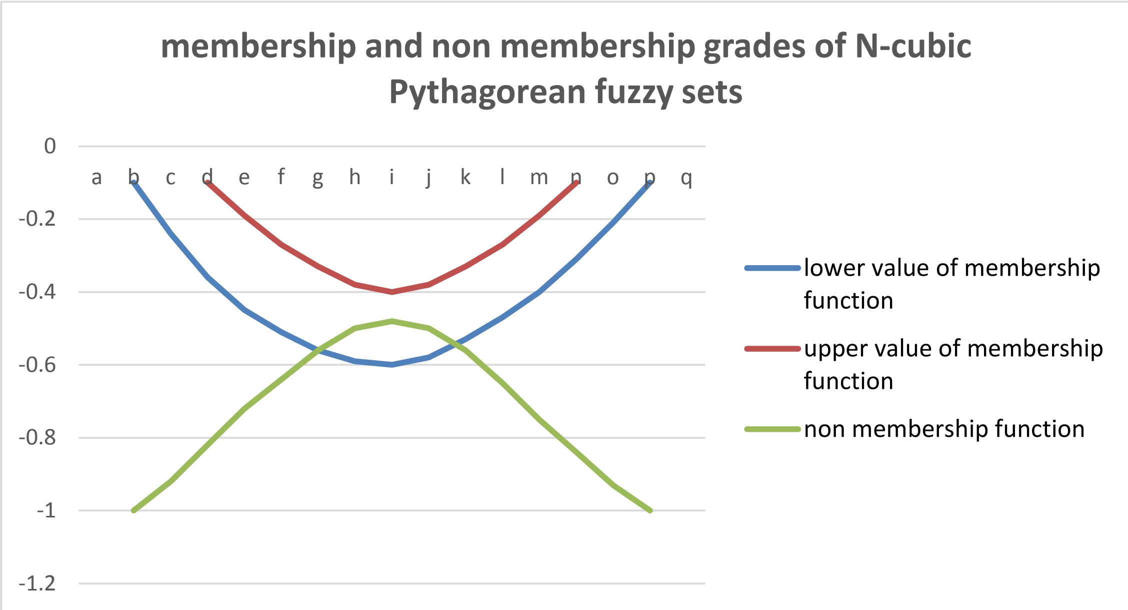


Figure 3. Spacing chart of N-cubic Pythagorean fuzzy set.

Some arithmetic operations on N-cubic Pythagorean fuzzy sets

Definition: Let A and B be two N-cubic Pythagorean fuzzy sets then, we define:

(1) $(N_A) + (N_B) = < [(-1)^{2(2)+1}\sqrt{\mu_A^-(x)^{2(2)} + \mu_B^-(x)^{2(2)} - \mu_A^-(x)^{2(2)}\mu_B^-(x)^{2(2)}},$

$(-1)^{2(2)+1}\sqrt{\mu_A^+(x)^{2(2)} + \mu_B^+(x)^{2(2)} - \mu_A^+(x)^{2(2)}\mu_B^+(x)^{2(2)}}\,],$

$(-1)^{2(2)+1}\,(\eta_A^-\eta_B^-,\ \eta_A^+\eta_B^+), ((-1)^{2(2)+1}\sqrt{\mu_{NA}^{2(2)} + \mu_{NB}^{2(2)} - \mu_{NA}^{2(2)}\mu_{NB}^{2(2)}}\,)$

(2)

$(N_A) * (N_B) = < (-1)^{2(2)+1}\,[\mu_A^-(\mathrm{x})\,\mu_B^-(\mathrm{x}), \mu_A^+(\mathrm{x})\,\mu_B^+(\mathrm{x})],$

$(-1)^{2(2)+1}\sqrt{\eta_A^-(x)^{2(2)} + \eta_B^-(x)^{2(2)} - \eta_A^-(x)^{2(2)}\,\eta_B^-(x)^{2(2)}}\,,$

$(-1)^{2(2)+1}\sqrt{\eta_A^+(x)^{2(2)} + \eta_B^+(x)^{2(2)} - \eta_A^+(x)^{2(2)}\,\eta_B^+(x)^{2(2)}}\,,$

$(-1)^{2(2)+1}\,(\mu_{NA}\mu_{NB}), (-1)^{2(2)+1}\sqrt{\eta_{NA}^{2(2)} + \eta_{NB}^{2(2)} - \eta_{NA}^{2(2)}\eta_{NB}^{2(2)}} \ >$

(3)

$$N_A^C = < [-1,-1] - \mu_{NP}, -1 - \eta_{NP} >$$

(4)

$\partial N_A =< [\ (-1)^{2(2)+1}\sqrt{1-(1-\mu_A^-(x)^{2(2)})^{\partial}},\ (-1)^{2(2)+1}\sqrt{1-(1-\mu_A^+(x)^{2(2)})^{\partial}}],\ (-1)^{2(2)+1}$

$[\ \mu_A^-(x)^{2\partial},\ \mu_A^+(x)^{2\partial}],\ [(-1)^{2(2)+1}\sqrt{1-(1-\mu_{NA}^{2(2)})^{\partial}},\ (-1)^{2(2)+1}\ \eta_{NA}^{2\partial}\ >$

(5)

$N_A{}^{\partial} =<\ (-1)^{2(2)+1}[(\ \mu_A^-\ )^{2\partial}, (\ \mu_A^+\ )^{2\partial}], [\ (-1)^{2(2)+1}\sqrt{1-(1-\eta_A^-(x)^{2(2)})^{\partial}},$

$(-1)^{2(2)+1}\sqrt{1-(1-\eta_A^+(x)^{2(2)})^{\partial}},\ (-1)^{2(2)+1}\ \mu_{NA}^{2\partial},\ (-1)^{2(2)+1}\sqrt{1-(1-\eta_{NA}^{2(2)})^{\partial}}$ > where $\partial > 0$.

3.4. N-Cubic Pythagorean fuzzy numbers (NCPFN's)

In this section we define the concept of N-cubic Pythagorean fuzzy number. We define some basic properties of proposed NCPFN's.

Definition: The set NCPFs = <N-IVPFs, N-PFs> is said to be N-cubic Pythagorean fuzzy number, if following conditions fulfilled.

(1) N-Cubic Pythagorean subset of real lines.

(2) Normal, their exist x $\in$ R such that $< \mu_{NP}(\text{x}) >= [-1, -1], < \eta_{NP}(x) >= 0$.

(3) Concave for the membership, i.e.,

$\mu_{NP}(\alpha x + (1-\alpha)y)\ \leq max\{\ p(x),\ p(y)\}\ \forall\alpha\ \in\ [-1, 0],\ x\ , y\ \in\ R.$

(4) Convex for non-membership, i.e.,

$\eta_{NP}(\alpha x + (1-\alpha)y)\ \geq min\{\ Np(x),\ Np(y)\}\ \forall\alpha\ \in\ [-1, 0],\ x\ , y\ \in\ R.$

Theorem: For any two NCPFN's N_A and N_B, and ∂, ∂_1, ∂_2 >0 then

(1) $N_A + N_B = N_B + N_A$,

(2) $N_A * N_B = N_B * N_A$,

(3) $\partial(N_A + N_B) = \partial N_A + \partial N_B, \partial > 0$,

(4) $(\partial_1 + \partial_2)N_A = \partial_1 N_A + \partial_2 N_A, \partial_1, \partial_2 > 0$,

(5) $(N_A * N_B)^{\partial} = N_A^{\partial} * N_B^{\partial}, \partial > 0$,

(6) $N_A^{\partial 1+\partial 2} = N_A^{\partial 1} * N_A^{\partial 2}, \partial_1, \partial_2 > 0$.

Proof: Easy to prove.

3.5. Score and accuracy

The accuracy and score functions play an important role in decision-making. We introduce a novel accuracy and scoring function for this set, which will be used to compare two N-cubic Pythagorean fuzzy sets.

3.5.1. Score function of N-cubic Pythagorean fuzzy sets

Definition: Let $N = <\ \grave{N}_P(x),\ N_P(x)\ >$ be the NCPFN then we define the score function of N as follows:

S(N) = 1/2 < S$\left(\grave{N}_P(x)\right)$, S($N_P(x)$) > where S$\left(\grave{N}_P(x)\right)$ = ½ {(-1)$^{2(2)+1}$ [$\mu_{\grave{N}}^{-}(x)^{2(2)}$+ $\mu_{\grave{N}}^{+}(x)^{2(2)}$ − $\eta_{\grave{N}}^{-}(x)^{2(2)}$ − $\eta_{\grave{N}}^{+}(x)^{2(2)}$] }, and S($N_P(x)$ = (-1)$^{2(2)+1}$ { $\mu_N(x)^{2(2)} - \eta_N(x)^{2(2)}$}

S(N) = 1/2 (-1)$^{2(2)+1}${1/2 [$\mu_{\grave{N}}^{-}(x)^{2(2)}$ + $\mu_{\grave{N}}^{+}(x)^{2(2)}$ − $\eta_{\grave{N}}^{-}(x)^{2(2)}$ − $\eta_{\grave{N}}^{+}(x)^{2(2)}$] + ($\mu_N(x)^{2(2)} -$ $\eta_N(x)^{2(2)}$)},

where S ($\grave{N}_P(x)$) ∈ [-1,1], S($N_P(x)$) ∈ [-1,1] and S(N) ∈ [-1,1].

Definition: Let N_A , N_B ∈ NCPFN's, then $N_A \subseteq N_B$, if and

$\mu_A^{-}(x) \leq \mu_B^{-}(x), \mu_A^{+}(x) \leq (\mu_B^{+}(x), \eta_A^{-}(x) \leq \eta_B^{-}(x)\,,\ \eta_A^{+}(x) \leq \eta_B^{+}(x),$

and

{ $\mu_A(x) \leq \mu_B(x)$, $\eta_A(x) \leq \eta_B(x)$ OR $\mu_A(x) \geq \mu_B(x)$, $\eta_A(x) \geq \eta_B(x)$} ∀ x ∈ X.

Definition: Let $N_A = <\ \grave{N}_{AP}(x),\ N_{AP}(x)\ >$ and $N_B = <\ \grave{N}_{BP}(x),\ N_{BP}(x)\ >$ be two NCPFN. Then if

(1) S (N_A) > S (N_B) ⇒ $N_A > N_B$,

(2) If we have S (N_A) = S (N_B) . Then there is no difference disagreement, indicating that such a score function cannot achieve NCPFN's rating.

Due to the score function's inadequacy, the accuracy function was added in the following sense, which has two requirements.

(1) If a(N_A) = a(N_B) ⇒ $N_A = N_B$.

(2) If a(N_A) > a(N_B) ⇒ $N_A > N_B$.

To proceed with this scoring function based on the inability to compare sets due to certain situations, we develop the following accuracy function for the implications of the provided condition:

Definition: Let $N = <\ \grave{N}_P(x),\ N_P(x)\ >$ then we define the accuracy function as follows:

a $(N) = 1/2\ <\ a(\grave{N}_P(x)),\ a(N_P(x)) >$,

where

a ($\grave{N}_P(x)$) = 1/2 {(-1)$^{2(2)+1}$ [$\mu_{\grave{N}}^{-}(x)^{2(2)}$ + $\mu_{\grave{N}}^{+}(x)^{2(2)}$ + $\eta_{\grave{N}}^{-}(x)^{2(2)}$ + $\eta_{\grave{N}}^{+}(x)^{2(2)}$] },

and

a ($N_P(x)$) = (-1)$^{2(2)+1}$ { $\mu_N(x)^{2(2)} + \eta_N(x)^{2(2)}$} then, a($N$) = 1/2 (-1)$^{2(2)+1}${1/2 [$\mu_{\grave{N}}^{-}(x)^{2(2)}$ + $\mu_{\grave{N}}^{+}(x)^{2(2)}$ + $\eta_{\grave{N}}^{-}(x)^{2(2)}$ + $\eta_{\grave{N}}^{+}(x)^{2(2)}$] + ($\mu_N(x)^{2(2)} + \eta_N(x)^{2(2)}$)},

Where a ($\grave{N}_P(x)$) ∈ [-1, 0], a ($N_P(x)$) ∈ [-1, 0] and a (N) ∈ [-1, 0].

Definition: Let S (N_A) be the score of N_A, S (N_B) be the score of N_B. Then

S (N_A) < S (N_B) if S $\left(\grave{N}_{AP}(x)\right)$ ≤ S $\left(\grave{N}_{BP}(x)\right)$, S ($N_{AP}(x)$) ≤ S ($N_{BP}(x)$), or S ($N_{AP}(x)$) ≥ S ($N_{BP}(x)$) ∀ x ∈ X.

And, let a (N_A) be the accuracy of N_A, a (N_B) be the accuracy of N_B then
a (N_A) ≤ a (N_B) if, a ($\grave{N}_{AP}(x)$) ≤ a ($\grave{N}_{BP}(x)$), a ($N_{AP}(x)$) ≤ a ($N_{AP}(x)$),
or a ($N_{AP}(x)$) ≥ a ($N_{AP}(x)$) ∀ x ∈ X.

Example: Suppose N_A = < [-0.8, -0.6], [-0.5, -0.2], (-0.6, -0.2) >, N_B = < [-0.8, -0.7], [-0.6, -0.4], (-0.8, -0.4)>, be two NCPFN's. Then

$S(N_A)$ = 1/2 < S$\left(\grave{N}_{AP}(x)\right)$, S ($N_{AP}(x)$) > $S(N_A)$ = -0.28.

Also,

$S(N_B)$ = 1/2 < S$\left(\grave{N}_{BP}(x)\right)$, S ($N_{BP}(x)$) > = -0.22 $\Rightarrow$ $S(N_B) > S(N_A)$ $\Rightarrow$ $N_B > N_A$.

Now if we have,

N_A = [-0.6, -0.4], [-0.6, -0.4], (-0.4, -0.4), N_B = [-0.7, -0.6], [-0.7, -0.6], (-0.6, -0.6)$\Rightarrow$ $S(N_A)$ = 0, $S(N_B)$ = 0 $\not\Rightarrow$ (N_A) =(N_B).

Then, a (N_A) = -0.08 also, a (N_B) = -0.24 $\Rightarrow$ a (N_A) > a (N_B) $\Rightarrow$ (N_A) > (N_B).

Remark:

(1) If NIVPF membership is greater than non-membership, then score must be positive.
(2) If non-membership is greater than membership, then score must be negative.
(3) Similarly, if membership of NCPFN, s is greater than non-membership of NCPFN's, then score must be positive.
(4) Similarly, if non-membership of NCPFN, s is greater than membership, then score must be negative; similarly, if membership of NCPFN, s is greater than non-member.

4. Triangular N-cubic Pythagorean fuzzy set

In this section we initiated the concept of triangular N-cubic Pythagorean fuzzy numbers to discuss AHP method.

Definition: The set T= < (a, b, c), Ǹp(x), Np(x) > is said to be triangular NCPFN, s defined as:

$$M(\grave{N}_p(x)) = \begin{cases} \frac{x-a}{b-a}\left(\grave{N}p(x)\right), & a \leq x < b \\ \frac{x-b}{c-b}\left(\grave{N}p(x)\right), & b < x \leq c \\ 0, & x < a,\ x > c \end{cases}$$

Also,

$$M(N_p(x)) = \begin{cases} \frac{x-a}{b-a}\left(\mathrm{Np}(x)\right), & a \leq x < b \\ \frac{x-b}{c-b}\left(\mathrm{Np}(x)\right), & b < x \leq c \\ -1, & x < a,\ x > c \end{cases}$$

Where -1 ≤ a≤ b≤ c ≤ 0 and a represent lower value, b represents middle and c represent upper value of triangular NCPFN, s and M (3_p(x)) and M (N_p(x)) be the membership and non-membership of NCPFN. The graphical representation is given by Figure 4.

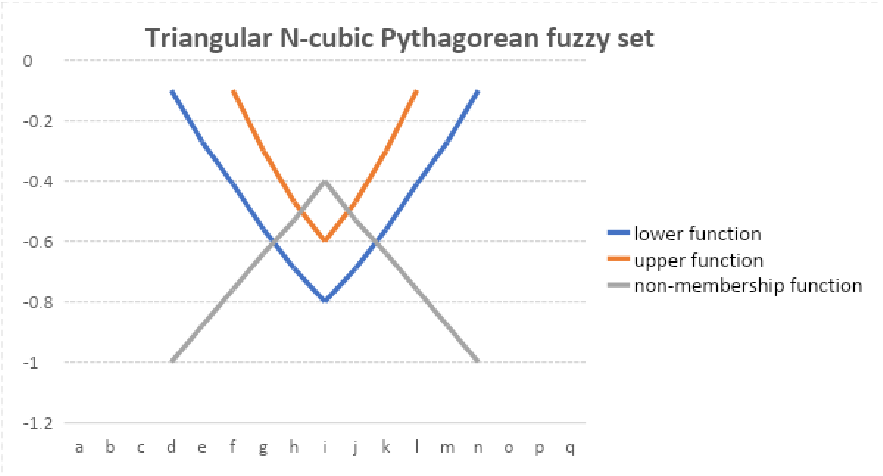


Figure 4. Triangular N-cubic Pythagorean fuzzy sets.

The region between lower value and upper value of the function demonstrates us that this is the membership region of triangular N-cubic Pythagorean fuzzy set, whereas grey shade form of triangle represents the non-membership value of the function. We formulate the three parametric functions in order to specify the triangular N-cubic Pythagorean fuzzy set.

Definition: Let $T_1 =< (a_1, a_2, a_3), \mu_{P1}(x), \eta_{P1}(x) >$, $T_2 =< (b_1, b_2, b_3), \mu_{P2}(x), \eta_{P2}(x) >$ be two triangular NCPFNs. Then,

(1) $T_1 + T_2 = <(a_1 + a_2, b_1 + b_2, c_1 + c_2)$, max[$\mu^-_{p1}(x), \mu^-_{p2}(x)$], max[$\mu^+_{p1}(x), \mu^+_{p2}(x)$],

min [η_{Np1} , η_{Np2}] for p', p'' $\in$ μ_{p1}(x), q', q'' $\in$ η_{p1}(x)

(2) $T_1T_2 = <$ (a_1a_2, b_1b_2, c_1c_2), max [$\mu^-_{p1}(x), \mu^-_{p2}(x)$], max [$\mu^+_{p1}(x), \mu^+_{p2}(x)$],

min [η_{Np1} , η_{Np2}] for p, p $\in$ μ_{p1}(x), q', q'' $\in$ η_{p1}(x)

(3) $\partial T = <(a\partial, b\partial, c\partial)$,

(4) $T^{-1} = <(\frac{1}{c}, \frac{1}{b}, \frac{1}{a})$, μ_{p1}(x), ηp_1(x)>

(5) $T_1 - T_2 = <(a_1 - a_2, b_1 - b_2, c_1 - c_2)$, max[$\mu^-_{p1}(x), \mu^-_{p2}(x)$], max[$\mu^+_{p1}(x), \mu^+_{p2}(x)$],

min [η_{Np1} , η_{Np2}] for p', p'' $\in$ μ_{p1}(x), q', q'' $\in$ η_{p1}(x) >

Definition: A function $\tilde{\omega}$ for N-interval valued Pythagorean fuzzy sets are given below,

$\tilde{\omega} = ([\mu^-_{1p}(x) + \mu^+_{1p}(x)])(1 - ß) + ß(2 - (\mu^-_{2p}(x) + \mu^+_{2p}(x)$ If ß = 0(pessimistic value),

$\tilde{\omega} = (\mu^-_{1p}(x) + \mu^+_{1p}(x))/2$, if ß $= 1$ (Optimistic value),

$\tilde{\omega} = (2 - \mu^-_{2p}(x) - \mu^+_{2p}))/2)$, and if ß = (1/2),

$\tilde{\omega} = (2 + \mu^-_{1p}(x) + \mu^+_{1p}(x) + \mu^-_{2p}(x) - \mu^+_{2p}(x))/4)$, generally used.

Also, ß is a real number between 0 and 1 (always positive) that can never be negative used as a variable based on the contribution of each once.

Definition: Let $\eta = <a, b>$ be the N-Pythagorean fuzzy sets then,

$$\tilde{v}(\eta) = 0.5\,(1 - \pi)(1 + a), \text{ for } \pi = (-1)^{2(2)+1}\sqrt{1 - \mu_X(x)^{2(2)} - \eta_X(x)^{2(2)}}$$

and,

$\eta = 1 - \tilde{v}(\eta)$ where, $0 \le \tilde{v}(\eta), \square \le 1$

to find weight, we define a function for this set: $Y = 0.5\{(2 + \mu_{1p}^{-}(x) + \mu_{1p}^{+}(x) - \mu_{2p}^{-}(x) - \mu_{2p}^{+}(x))/4) + (1 - \tilde{v}(\eta))\}$,

where, $0 \le \tilde{v}(\eta), \square \le 1$ and $\tilde{v}$ (η) already defined.

5. Applications

Investigation of downfall of IA (international airlines) using analytical hierarchy process under N-cubic Pythagorean fuzzy sets

The analytical hierarchy technique, first proposed by Thomas L. Saaty in 1970, was used to analyze difficult decisions and rank alternatives using mathematics and human reasoning in a hierarchy framework, however it failed to measure uncertainty. Following that, a fuzzy analytical hierarchy approach was established, although it is difficult to cover many challenges in decision-making. To put it another way, we're starting an NCPF analytical hierarchical process for complex decision-making. Our key goal is to determine what is causing international airlines to fall. To do so, we must first identify the essential key elements (both external and internal) that are related with airlines.

5.1. Determining the factor affecting airlines

Following are the key factors that affect airline companies given in Table 1 with brief explanation.

Table 1. Factors that affect airline companies.

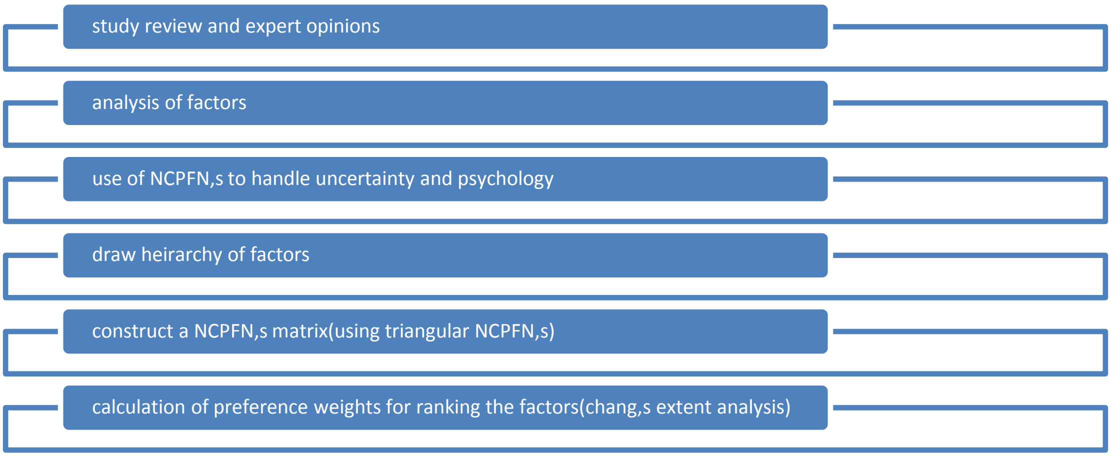
S. no	Factor category	Category code	Factors
1	Operational Factors	OF1	Load factor
	(OP)	OF2	Average number of passengers carried per departure
		OF3	Average Number of hours flown per pilot
		OF4	Number of departures per aircraft
		OF5	Number of pilots per aircraft
		OF6	The average age of the aircraft fleet
		OF7	Number of different brands of aircraft operated
		OP8	International operations

Continued on next page

S. no	Factor category	Category code	Factors
2	Economic/Government Factors (EF)	EF1	Annual inflation
		EF2	GDP Growth Rate
		EF3	Aviation Fuel price (INR per liter)
		EF4	Average Growth in Value of Passengers carried in country
3	Performance Related Factors (PF)	PF1	Available Seat Kilometer (ASK)
		PF2	Revenue per Kilometer (RPK)
		PF3	Available Seat KM per employee
		PF4	Average stage length flown in kilometer
		PF5	Fuel Efficiency (liters per KM flown)
		PF6	Breakeven load factor
		PF7	Labor cost per KM flown
4	Financial Factors (FF)	FF1	Operating revenues/operating cost
		FF2	Operating Profit/ Total Assets
		FF3	Retained earnings/total assets
		FF4	Market Value of Equity/Total Book value of debt
		FF5	Current assets/current liabilities
		FF6	Earnings before interest and taxes/ operating revenues
		FF7	Interest/total liabilities or debt service
		FF8	Operating revenues per air kilometer
		FF9	Earnings stability (the deviation around a 10-year trend line of return on assets)
		FF10	Firm size (measured by the log of the firm's total assets).
5	Market-Related Factors (MRF)	MF1	Number of airlines operating
		MF2	Company Passenger growth (%)/Industry growth (%)
		MF3	Market share
		MF4	Govt. policies regarding slot allocation
		MF5	Airport preference of airlines
6	External Factors (EX)	ExF1	Environment or weather conditions
		ExF2	Geographical location
		ExF3	Threats to national security
		ExF4	Political influence (hiring& benefits)

5.2. Investigation of N-cubic Pythagorean fuzzy analytic hierarchy process approach

These steps are followed to determine the perspective approach.



Assessment scale given by Saaty in 1970, by this we define linguistic triangular scale under NCPFN is given in Table 2.

Table 2. Linguistic Triangular scale under NCPFN.

Saaty scale	Credit	Linguistic term set under NCPFN’s
1	Equally significant	1*= <(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6,-0.3)>
3	Slightly significant	3* = <(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)>
5	Strongly significant	5*=<(4, 5, 6), [-0.8, -0.5], [-0.2, -0.1], (-0.9, -0.1)>
7	Very strongly significant	7*= <(6, 7, 8), [-0.6, -0.3], [-0.6, -0.4], (-0.7, -0.6)>
9	Absolutely significant	9*= <(9, 9, 9), [-0.9, -0.7], [-0.2, 0], (-0.8, -0.1)>
2		2*=<(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)>
4	In-between values	4*=<(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)>
6		6* = <(5, 6, 7), [-0.8, -0.5], [-0.6, -0.2], (-0.7, -0.6)>
8		8* = <(7, 8, 9), [-0.9, -0.4], [-0.3, -0.1], (-0.9, -0.4)>

Following are the steps of method (Chang’s extent analysis of AHP under NCPFN’s) are given by:

Step (a): the NCPF (Q_K) extent value of kth criterion is given by, $\sum_l^m \boldsymbol{M}_{h_k}^{\boldsymbol{l}} \, [\sum_k^n \sum_l^m \boldsymbol{M}_{h_k}^{\boldsymbol{l}}]^{-1}$, where l, m and n represent lower, middle, and upper values of fuzzy relation.

Step(b): Degree of possibility can be analyzed by $v(Q_1 \leq Q_2)=inf_{r\leq t}[\max((\grave{\upsilon}_{R!}(r)),(\grave{\upsilon}_{R2}(t)))], sup_{r\leq t}[\min((\eta_{S1}(r)),(\eta_{S2}(t))]$. Where,

$$v_{\grave{v}}(R_1 \le R_2) = \; inf_{r\le t}[\max((\grave{v}_{R!}(r)),(\grave{v}_{R2}(t)))] = \grave{v}_{(R1\cap R2)}[d^-,d^+]$$
$$= \left\{ \begin{array}{c} \max\{P_1^-,P_2^-\},\max\{P_1^+,P_2^+\}, for\ m_1 \ge m_2 \\ 0, for\ l_2 \ge u_1 \\ \frac{(U_1-l_2)P_1^-P_2^-}{(U_1-m_1)P_2^- + (m_2-l_2)P_1^-}, \frac{(U_1-l_2)P_1^+P_2^+}{(U_1-m_1)P_2^+ + (m_2-l_2)P_1^+}, otherwise \end{array} \right\}$$

And for non-membership $v_\eta(S_1 \ge S_2) = sup_{r\le t}[\min((\eta_{S1}(r),(\eta_{S2}(t))] = \eta_{S1\cap S2}(d) =$

$$\left\{ \begin{array}{c} \min\{q_{1,}q_2\}, for\ m_1 \ge m_2 \\ -1, for\ l_2 \ge u_1 \\ \frac{(m_2-l_2q_2)(-1-q_1)+(u_1q_1-m_1)(-1-q_2)}{(u_1-m_1)(-1-q_2)+(m_2-l_2)(-1-q_2)}, otherwise \end{array} \right\}.$$

Note: Another matter we discuss here is that this structure is a combination of interval valued Pythagorean fuzzy sets and Pythagorean fuzzy sets, so how do we find the weight of these two sets in a single term? (For both the cases)

Step(c): R = Max $\{R_5 \le R_2, R_3, R_4, R_1, R_6\}$, S = min $\{S_5 \ge S_2, S_3, S_4, S_1, S_6\}$

Step (d): Analyzation of weights:

The weights of important elements can be determined for final ranking. The same scenario also determines the weights of sub criterion. Adding each sub-relative criterion's weight to the main criteria.

Step (e):

The primary criterion, sub criterion, and relative ranking are all ranked in this stage. Criteria are ranked, Ranking on the ground.

Before performing Chang's study, we create a comparison matrix of key factors based on expert judgments.

5.3. Comparison matrix between main factors (see Table 3)

Table 3. Comparison matrix of main factors.

	OP-F	EC/G-F	PR-F	F-F	MR-F	EX-F
OP-F	(1, 1, 1), [-0.8,-0.5],[-0.6,-0.3],(-0.6,-0.3)	(3, 4, 5),[-0.7,-0.3],[-0.5,-0.1],(-0.7,-0.6)	(1, 2, 3),[-0.5,-0.3],[-0.4,-0.2],(-0.6,-0.3)	(3, 4, 5),[-0.7,-0.3],[-0.5,-0.1](-0.7,-0.6)	(3, 4, 5), [-0.7,-0.3], [-0.5,-0.1], (-0.7,-0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [0.7, -0.4],[-0.6,-0.2], (-0.8,-0.4)
EC/G-F	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	$(\frac{1}{3},\frac{1}{2}, 1)$, [0.5,-0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$,[0.7,-0.3],[-0.5,-0.1],(-0.7,-0.6)	(2, 3, 4), [-0.7,-0.4],[-0.6,-0.2],(-0.8,-0.4)	(3, 4, 5), [-0.7,-0.3],[-0.5,-0.1],(-0.7,-0.6)
PR-F	$(\frac{1}{3},\frac{1}{2}, 1)$, [-0.5,-0.3],[-0.4,-0.2],(-0.6, -0.3)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(1, 1, 1), [-0.8,-0.5],[-0.6,-0.3],(-0.6,-0.3)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7,-0.4],[-0.6, -0.2],(-0.8,-0.4)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)

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	OP-F	EC/G-F	PR-F	F-F	MR-F	EX-F
F-F	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)
MR-F	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{3},\frac{1}{2}, 1)$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
EX-F	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)

5.3.1. Comparison matrix between sub-criteria (operational factor) (see Table 4)

Table 4. Comparison matrix of operational factors.

	Op-1	Op-2	Op-3	Op-4	Op-5	Op-6	Op-7	Op-8
Op-1	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(1/3, 1/2, 1), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1/5, 1/4, 1/3),[-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1/5, 1/4, 1/3), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)
Op-2	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1),[-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
Op-3	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)

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	Op-1	Op-2	Op-3	Op-4	Op-5	Op-6	Op-7	Op-8
Op-4	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)
Op-5	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)
Op-6	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$,), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)
Op-7	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)
Op-8	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)

5.3.2. Comparison matrix between sub-criteria (economic and government factor) (see Table 5)

Table 5. Comparison matrix of economic and government factors.

	EF-1	EF-2	EF-3	EF-4
EF-1	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)
EF-2	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
EF-3	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
EF-4	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)

5.3.3. Comparison matrix between sub-criteria (performance related factor) (see Table 6)

Table 6. Comparison matrix of performance related factor.

	PR-1	PR-2	PR-3	PR-4	PR-5	PR-6	PR-7
PR-1	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
PR-2	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)

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	PR-1	PR-2	PR-3	PR-4	PR-5	PR-6	PR-7
PR-3	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)
PR-4	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)
PR-5	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
PR-6	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)
PR-7	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)

5.3.4. Comparison matrix between sub-criteria (financial factor) (see Table 7)

Table 7. Comparison matrix of financial factor.

	FF-1	FF-2	FF-3	FF-4	FF-5	FF-6	FF-7	FF-8	FF-9	FF-10
FF-1	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
FF-2	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.9, -0.4], [-0.7, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)
FF-3	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)

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	FF-1	FF-2	FF-3	FF-4	FF-5	FF-6	FF-7	FF-8	FF-9	FF-10
FF-4	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{3},\frac{1}{2},\frac{1}{1}$), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
FF-5	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{3},\frac{1}{2},\frac{1}{1}$) , [-0.8, -0.5], [-0.7, -0.4], (-0.8, -0.5)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)
FF-6	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$),[-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)1	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
FF-7	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1)), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)

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	FF-1	FF-2	FF-3	FF-4	FF-5	FF-6	FF-7	FF-8	FF-9	FF-10
FF-8	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
FF-9	($\frac{1}{3},\frac{1}{2},\frac{1}{1}$), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{3},\frac{1}{2},\frac{1}{1}$), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	($\frac{1}{4},\frac{1}{3},\frac{1}{2}$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)1	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)
FF-10	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$),[-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{3},\frac{1}{2},\frac{1}{1}$)), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	($\frac{1}{5},\frac{1}{4},\frac{1}{3}$), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)

5.3.5. **C**omparison matrix between sub-criteria (market related factors) (see Table 8)

Table 8. Comparison matrix of market related factors.

	MR-1	MR-2	MR-3	MR-4	MR-5
MR-1	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)
MR-2	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
MR-3	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
MR-4	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(1, 2, 3), [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)
MR-5	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{3},\frac{1}{2},\frac{1}{1})$, [-0.5, -0.3], [-0.4, -0.2], (-0.6, -0.3)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)

5.3.6. Comparison matrix between sub-criteria (external factors) (see Table 9)

Table 9. Comparison matrix of external factors.

	E1	E2	E3	E4
E1	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
E2	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)
E3	(2, 3, 4), [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)
E4	(3, 4, 5), [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	$(\frac{1}{4},\frac{1}{3},\frac{1}{2})$, [-0.7, -0.4], [-0.6, -0.2], (-0.8, -0.4)	$(\frac{1}{5},\frac{1}{4},\frac{1}{3})$, [-0.7, -0.3], [-0.5, -0.1], (-0.7, -0.6)	(1, 1, 1), [-0.8, -0.5], [-0.6, -0.3], (-0.6, -0.3)

According to Chang's extent analysis (1992), we define some steps given below: By given Chang's method, the value of criterion h_k can be evaluated by given scenario.

$M_{h_k}^{l}$, **(**l = 1, 2, …, m**)**, **(**k=1, 2, …., n).

Step 1: The NCPF (Q_K) extent value of kth criterion is given by $\sum_l^m \boldsymbol{M}_{h_k}^{\boldsymbol{l}} [\sum_k^n \sum_l^m \boldsymbol{M}_{h_k}^{\boldsymbol{l}}]^{-1}$,

Q (1) =(0.133, 0.23, 0.42), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)}, Q (2) = (0.08, 0.16, 0.30), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6),Q (3) = (0.06, 0.13, 0.27), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6), Q (4) = (0.12, 0.22, 0.39), [-0.5, -0.3], [-0.4,-0.1], (-0.8, -0.6),Q (5) = (0.08, 0.14, 0.25), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6),Q (6) = (0.05, 0.09, 0.16), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6).

Step 2: Analyzation of probabilistic degree:

Calculations for membership and non-membership

$v_{\grave{o}}(R_1 \leq R_2)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{o}}(S_1 \geq S_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_1 \leq R_3)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{o}}(S_1 \geq S_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_1 \leq R_4)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{o}}(S_1 \geq S_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_1 \leq R_5)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{o}}(S_1 \geq S_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_1 \leq R_6)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(S_1 \geq S_2)$ = (-0.8, -0.6)
Max $\{R_1 \leq R_2, R_3, R_4, R_5, R_6\}$ = [-0.5, -0.3], [-0.4, -0.1], min $\{S_1 \geq S_2, S_3, S_4, S_5, S_6\}$ = (-0.8, -0.6)
$v_{\grave{o}}(R_2 \leq R_1)$ = [-0.36, -0.14], [-0.25, -0.06], $v_{\grave{o}}(S_2 \geq S_1)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_2 \leq R_3)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{o}}(S_2 \geq S_3)$ = (-0.20, -0.55)
$v_{\grave{o}}(R_2 \leq R_4)$ = [-0.33, -0.22], [-0.2, -0.07],$v_{\grave{o}}(S_2 \geq S_4)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_2 \leq R_5)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(S_2 \geq S_5)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_2 \leq R_6)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(S_2 \geq S_6)$ = (-0.19, -0.54)
Max $\{R_2 \leq R_1, R_3, R_4, R_5, R_6\}$ = [-0.33, -0.14], [-0.2, -0.06], min $\{S_2 \geq S_1, S_3, S_4, S_5, S_6\}$ = (-0.8, -0.6)

$v_{\grave{o}}(R_3 \leq R_1)$ = [-0.27, -0.16], [-0.25, -0.1],$v_{\grave{o}}(S_3 \geq S_1)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_3 \leq R_2)$ = [-0.16, -0.08], [-0.13, -0.03],$v_{\grave{o}}(S_3 \geq S_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_3 \leq R_4)$ = [-0.25, -0.14], [-0.22, -0.04],$v_{\grave{o}}(S_3 \geq S_4)$ = (-0.18, -0.50)
$v_{\grave{o}}(R_3 \leq R_5)$ = [-0.4, -0.2], [-0.4, -0.1],$v_{\grave{o}}(S_3 \geq S_5)$ = (-0.15, -0.44)
$v_{\grave{o}}(R_3 \leq R_6)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(S_3 \geq S_6)$ = (-0.8, -0.6)
Max $\{R_3 \leq R_1, R_2, R_4, R_5, R_6\}$ = [-0.16, -0.08], [-0.13, -0.03], min $\{S_3 \geq S_1, S_2, S_4, S_5, S_6\}$ = (-0.8, -0.6)

$v_{\grave{o}}(R_4 \leq R_1)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(S_4 \geq S_1)$ = (-0.18, -0.49)
$v_{\grave{o}}(R_4 \leq R_2)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(S_4 \geq S_2)$ = (-0.15, -0.43)
$v_{\grave{o}}(R_4 \leq R_3)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(S_4 \geq S_3)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_4 \leq R_5)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(S_4 \geq S_5)$ = (-0.21, -0.59)
$v_{\grave{o}}(R_4 \leq R_6)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{o}}(S_4 \geq S_6)$ = (-0.8, -0.6)
Max $\{R_4 \leq R_1, R_2, R_3, R_5, R_6\}$ = [-0.5, -0.3], [-0.4, -0.1], min $\{S_4 \geq S_2, S_3, S_1, S_5, S_6\}$ = (-0.8, -0.6)

$v_{\grave{o}}(R_5 \leq R_1)$ = [-0.22, -0.2], [-0.14, -0.05],$v_{\grave{o}}(S_5 \geq S_1)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_5 \leq R_2)$ = [-0.5, -0.2], [-0.3, -0.09],$v_{\grave{o}}(S_5 \geq S_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(R_5 \leq R_3)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(S_5 \geq S_3)$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(R_5 \leq R_4)$ = [-0.3, -0.16], [-0.25, -0.05],$v_{\grave{\upsilon}}(S_5 \geq S_4)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(R_5 \leq R_6)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(S_5 \geq S_6)$ = (-0.21, -0.59)
Max $\{R_5 \leq R_2, R_3, R_4, R_1, R_6\}$ = [-0.22, -0.16], [-0.14, -0.05], min $\{S_5 \geq S_2, S_3, S_4, S_1, S_6\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(R_6 \leq R_1)$ = [-0.07, -0.04], [-0.06, -0.01],$v_{\grave{\upsilon}}(S_6 \geq S_1)$ = (-0.18, -0.52)
$v_{\grave{\upsilon}}(R_6 \leq R_2)$ = [-0.2, -0.1], [-0.2, -0.08],$v_{\grave{\upsilon}}(S_6 \geq S_2)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(R_6 \leq R_3)$ = [-0.3, -0.2], [-0.4, -0.07],$v_{\grave{\upsilon}}(S_6 \geq S_3)$ = (-0.20, -0.57)
$v_{\grave{\upsilon}}(R_6 \leq R_4)$ = [-0.1, -0.06], [-0.1, -0.04],$v_{\grave{\upsilon}}(S_6 \geq S_4)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(R_6 \leq R_5)$ = [-0.3, -0.24], [-0.3, -0.08],$v_{\grave{\upsilon}}(S_6 \geq S_5)$ = (-0.8, -0.6)
Max $\{R_6 \leq R_2, R_3, R_4, R_5, R_1\}$ = [-0.07, -0.04], [-0.06, -0.01], min $\{S_6 \geq S_2, S_3, S_4, S_5, S_1\}$ = (-0.8, -0.6)

Sub-criteria (operational factor):
Step 1: The NCPF (O_K) extent value of kth sub-criterion (operational factor) is given by:

$$\sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}} [\sum_{k}^{n}\sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}}]^{-1}$$

O (1) = (0.06, 0.12, 0.23), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
O (2) = (0.078, 0.143, 0.26), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
O (3) = (0.073, 0.12, 0.23), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
O (4) = (0.076, 0.149, 0.278), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
O (5) = (0.052, 0.091, 0.16), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
O (6) = (0.08, 0.153, 0.274), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
O (7) = (0.077, 0.143, 0.25), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
O (8) = (0.034, 0.063, 0.132), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)

Step 2:
$v_{\grave{\upsilon}}(PO_1 \leq PO_2) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(OP_1 \geq OP_2)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PO_1 \leq PO_3) =$ [-0.2, -0.05], [-0.11, -0.007],$v_{\grave{\upsilon}}(OP_1 \geq OP_3)$ = (-0.16, -0.47)
$v_{\grave{\upsilon}}(PO_1 \leq PO_4) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(OP_1 \geq OP_4)$ =(-0.8, -0.6)
$v_{\grave{\upsilon}}(PO_1 \leq PO_5) =$ [-0.2, -0.05], [-0.11, -0.007], $v_{\grave{\upsilon}}(OP_1 \geq OP_5)$ = (-0.16, -0.48)
$v_{\grave{\upsilon}}(PO_1 \leq PO_6) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(OP_1 \geq OP_6)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(PO_1 \leq PO_7) =$ [-0.2, -0.05], [-0.11, -0.007], $v_{\grave{\upsilon}}(OP_1 \geq OP_7)$ = (-0.16, -0.48)
$v_{\grave{\upsilon}}(PO_1 \leq PO_8) =$ [-0.2, -0.05], [-0.11, -0.007], $v_{\grave{\upsilon}}(OP_1 \geq OP_8)$ = (-0.16, -0.47)
Max $\{PO_1 \leq PO_2, PO_3, PO_4, PO_5, PO_6, PO_7, PO_8\}$ = [-0.2, -0.05], [-0.11, -0.007], min $\{OP_1 \geq OP_2, OP_3, OP_4, OP_5, OP_6, OP_7, OP_8\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(PO_2 \leq PO_1) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(OP_2 \geq OP_1)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PO_2 \leq PO_3) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(OP_2 \geq OP_3)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PO_2 \leq PO_4) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(OP_2 \geq OP_4)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PO_2 \leq PO_5) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(OP_2 \geq OP_5)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PO_2 \leq PO_6) =$ [-0.23, -0.06], [-0.12, -0.008], $v_{\grave{\upsilon}}(OP_2 \geq OP_6)$ = (-0.17, -0.51)
$v_{\grave{\upsilon}}(PO_2 \leq PO_7) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(OP_2 \geq OP_7)$ = (-0.8, -0.6)

$v_{\grave{\imath}}(PO_2 \leq PO_8) =$ [-0.22, -0.06], [-0.12, -0.008], $v_{\grave{\imath}}(OP_2 \geq OP_8)$ = (-0.17, -0.51)
Max $\{PO_2 \leq PO_1, PO_3, PO_4, PO_5, PO_6, PO_7, PO_8\}$ = [-0.22, -0.06], [-0.12, -0.008], min $\{OP_2 \geq OP_1, OP_3, OP_4, OP_5, OP_6, OP_7, OP_8\}$ = (-0.8, -0.6)

$v_{\grave{\imath}}(PO_3 \leq PO_1) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\imath}}(OP_3 \geq OP_1)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_3 \leq PO_2) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\imath}}(OP_3 \geq OP_2)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_3 \leq PO_4) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\imath}}(OP_3 \geq OP_4)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_3 \leq PO_5) =$ [-0.20, -0.05], [-0.11, -0.007],$v_{\grave{\imath}}(OP_3 \geq OP_5)$ = (-0.16, -0.47)
$v_{\grave{\imath}}(PO_3 \leq PO_6) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_3 \geq OP_6)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_3 \leq PO_7) =$ [-0.21, -0.05], [-0.11, -0.007],$v_{\grave{\imath}}(OP_3 \geq OP_7)$ = (-0.16, -0.48)
$v_{\grave{\imath}}(PO_3 \leq PO_8) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_3 \geq OP_8)$ = (-0.8, -0.6)
Max $\{PO_3 \leq PO_2, PO_1, PO_4, PO_5, PO_6, PO_7, PO_8\}$ = [-0.20, -0.05], [-0.11, -0.007], min $\{OP_3 \geq OP_2, OP_1, OP_4, OP_5, OP_6, OP_7, OP_8\}$ = (-0.8, -0.6)

$v_{\grave{\imath}}(PO_4 \leq PO_1) =$ [-0.20, -0.05], [-0.11, -0.008],$v_{\grave{\imath}}(OP_4 \geq OP_1)$ = (-0.16, -0.48)
$v_{\grave{\imath}}(PO_4 \leq PO_2) =$ [-0.20, -0.05], [-0.11, -0.007],$v_{\grave{\imath}}(OP_4 \geq OP_2)$ = (-0.16, -0.47)
$v_{\grave{\imath}}(PO_4 \leq PO_3) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_4 \geq OP_3)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_4 \leq PO_5) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_4 \geq OP_5)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_4 \leq PO_6) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_4 \geq OP_6)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_4 \leq PO_7) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_4 \geq OP_7)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_4 \leq PO_8) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_4 \geq OP_8)$ = (-0.8, -0.6)
Max $\{PO_4 \leq PO_2, PO_3, PO_1, PO_5, PO_6, PO_7, PO_8\}$ = [-0.20, -0.05], [-0.11, -0.007], min $\{OP_4 \geq OP_2, OP_3, OP_1, OP_5, OP_6, OP_7, OP_8\}$ = (-0.8, -0.6)

$v_{\grave{\imath}}(PO_5 \leq PO_1) =$ [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\imath}}(OP_5 \geq OP_1)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_5 \leq PO_2) =$ [-0.22, -0.06], [-0.12, -0.008],$v_{\grave{\imath}}(OP_5 \geq OP_2)$ = (-0.17, -0.50)
$v_{\grave{\imath}}(PO_5 \leq PO_3) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_5 \geq OP_3)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_5 \leq PO_4) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_5 \geq OP_4)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_5 \leq PO_6) =$ [-0.21, -0.05], [-0.11, -0.007], $v_{\grave{\imath}}(OP_5 \geq OP_6)$ = (-0.17, -0.52)
$v_{\grave{\imath}}(PO_5 \leq PO_7) =$ [-0.18, -0.05], [-0.10, -0.007],$v_{\grave{\imath}}(OP_5 \geq OP_7)$ = (-0.18, -0.53)
$v_{\grave{\imath}}(PO_5 \leq PO_8) =$ [-0.18, -0.04], [-0.09, -0.005], $v_{\grave{\imath}}(OP_5 \geq OP_8)$ = (-0.17, -0.51)
Max $\{PO_5 \leq PO_2, PO_3, PO_4, PO_1, PO_6, PO_7, PO_8\}$ = [-0.18, -0.04], [-0.09, -0.005], min $\{OP_5 \geq OP_2, OP_3, OP_4, OP_1, OP_6, OP_7, OP_8\}$ = (-0.8, -0.6)

$v_{\grave{\imath}}(PO_6 \leq PO_1) =$ [-0.18, -0.05], [-0.11, -0.008], $v_{\grave{\imath}}(OP_6 \geq OP_1)$ = (-0.18, -0.54)
$v_{\grave{\imath}}(PO_6 \leq PO_2) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_6 \geq OP_2)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_6 \leq PO_3) =$ [-0.18, -0.05], [-0.11, -0.008],$v_{\grave{\imath}}(OP_6 \geq OP_3)$ = (-0.18, -0.54)
$v_{\grave{\imath}}(PO_6 \leq PO_4) =$ [-0.18, -0.05], [-0.11, -0.007],$v_{\grave{\imath}}(OP_6 \geq OP_4)$ = (-0.18, -0.53)
$v_{\grave{\imath}}(PO_6 \leq PO_5) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_6 \geq OP_5)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_6 \leq PO_7) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_6 \geq OP_7)$ = (-0.8, -0.6)
$v_{\grave{\imath}}(PO_6 \leq PO_8) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\imath}}(OP_6 \geq OP_8)$ = (-0.8, -0.6)
Max $\{PO_6 \leq PO_2, PO_3, PO_4, PO_5, PO_1, PO_7, PO_8\}$ = [-0.18, -0.05], [-0.11, -0.007], min $\{OP_6 \geq OP_2, OP_3, OP_4, OP_5, OP_1, OP_7, OP_8\}$ = (-0.8, -0.6)

$v_{\grave{o}}(PO_7 \le PO_1) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_7 \ge OP_1)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_7 \le PO_2) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_7 \ge OP_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_7 \le PO_3) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_7 \ge OP_3)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_7 \le PO_4) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_7 \ge OP_4)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_7 \le PO_5) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_7 \ge OP_5)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_7 \le PO_6) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_7 \ge OP_6)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_7 \le PO_8) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_7 \ge OP_8)$ = (-0.8, -0.6)
Max $\{PO_7 \le PO_2, PO_3, PO_4, PO_5, PO_6, PO_1, PO_8\}$ = [-0.5, -0.3], [-0.4, -0.1], min $\{ OP_7 \ge OP_2, OP_3, OP_4, OP_5, OP_6, OP_1, OP_8\}$ = (-0.8, -0.6)

$v_{\grave{o}}(PO_8 \le PO_1) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_8 \ge OP_1)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_8 \le PO_2) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_8 \ge OP_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_8 \le PO_3) =$ [-0.23, -0.06], [-0.13, -0.008],$v_{\grave{o}}(OP_8 \ge OP_3)$ = (-0.18, -0.54)
$v_{\grave{o}}(PO_8 \le PO_4) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_8 \ge OP_4)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_8 \le PO_5) =$ [-0.23, -0.06], [-0.13, -0.008],$v_{\grave{o}}(OP_8 \ge OP_5)$ = (-0.18, -0.54)
$v_{\grave{o}}(PO_8 \le PO_6) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_8 \ge OP_6)$ = (-0.8, -0.6)
$v_{\grave{o}}(PO_8 \le PO_7) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(OP_8 \ge OP_7)$ = (-0.8, -0.6)
Max $\{PO_8 \le PO_2, PO_3, PO_4, PO_5, PO_6, PO_7, PO_1\}$ = [-0.23, -0.06], [-0.13, -0.008], min $\{OP_8 \ge OP_2, OP_3, OP_4, OP_5, OP_6, OP_7, OP_1\}$ = (-0.8, -0.6)

Sub-criteria (economic/government factor)
Step1: The NCPF (E_K) extent value of kth sub-criterion (economic/government factor) is given by,

$$\sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}} \, [\sum_{k}^{n}\sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}}]^{-1}$$

E (1) = (0.21, 0.35, 0.34), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
E (2) = (0.18, 0.30, 0.29), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
E (3) = (0.48, 0.068, 0.061), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
E (4) = (0.15, 0.27, 0.26), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)

Step 2:
$v_{\grave{o}}(EC_1 \le EC_2) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_1 \ge GO_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(EC_1 \le EC_3) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_1 \ge GO_3)$ = (-0.8, -0.6)
$v_{\grave{o}}(EC_1 \le EC_4) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_1 \ge GO_4)$ = (-0.8, -0.6)
Max $\{EC_1 \le EC_2, EC_3, EC_4\}$ = [-0.5, -0.3], [-0.4, -0.1], min $\{GO_1 \ge GO_2, GO_3, GO_4\}$ = (-0.8, -0.6)

$v_{\grave{o}}(EC_2 \le EC_1) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_2 \ge GO_1)$ = (-0.8, -0.6)
$v_{\grave{o}}(EC_2 \le EC_3) =$ [-0.9, -0.1], [-0.4, -0.006],$v_{\grave{o}}(GO_2 \ge GO_3)$ = (-0.7, -0.5)
$v_{\grave{o}}(EC_2 \le EC_4) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_2 \ge GO_4)$ = (-0.8, -0.6)
Max $\{EC_2 \le EC_1, EC_3, EC_4\}$ = [-0.5, -0.1], [-0.4, -0.006], min $\{GO_2 \ge GO_1, GO_3, GO_4\}$ = (-0.8, -0.6)

$v_{\grave{o}}(EC_3 \le EC_1)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_3 \ge GO_1)$ = (-0.8, -0.6)
$v_{\grave{o}}(EC_3 \le EC_2)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_3 \ge GO_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(EC_3 \le EC_4)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_3 \ge GO_4)$ = (-1, -0.6)

Max $\{EC_3 \leq EC_2, EC_1, EC_4\}$ = [-0.5, -0.3], [-0.4, -0.1], min $\{GO_3 \geq GO_2, GO_1, GO_4\}$ = (-1, -0.6)

$v_{\grave{o}}(EC_4 \leq EC_1)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_4 \geq GO_1)$ = (-1, -0.6)
$v_{\grave{o}}(EC_4 \leq EC_2)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_4 \geq GO_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(EC_4 \leq EC_3)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(GO_4 \geq GO_3)$ = (-1, -0.6)
Max $\{EC_4 \leq EC_2, EC_3, EC_1\}$ = [-0.5, -0.3], [-0.4, -0.1], min $\{GO_4 \geq GO_2, GO_3, GO_1\}$ = (-1, -0.6)

Sub-criteria (performance related factor)
Step 1: The NCPF (P_K) extent value of kth sub-criterion (performance related factor) is given by

$$\sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}} [\sum_{k}^{n} \sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}}]^{-1}$$

P (1) = (0.089, 0.168, 0.30), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
P (2) = (0.080, 0.15, 0.293), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
P (3) = (0.093, 0.164, 0.296), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
P (4) = (0.06, 0.124, 0.23), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
P (5) = (0.098, 0.181, 0.32), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
P (6) = (0.04, 0.07, 0.13), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
P (7) = (0.07, 0.133, 0.24), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)

Step 2:
$v_{\grave{o}}(PE_1 \leq PE_2)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_1 \geq RE_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(PE_1 \leq PE_3)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_1 \geq RE_3)$ = (-0.8, -0.6)
$v_{\grave{o}}(PE_1 \leq PE_4)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_1 \geq RE_4)$ = (-0.8, -0.6)
$v_{\grave{o}}(PE_1 \leq PE_5)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_1 \geq RE_5)$ = (-0.8, -0.6)
$v_{\grave{o}}(PE_1 \leq PE_6)$ = [-0.23, -0.06], [-0.13, -0.009],$v_{\grave{o}}(RE_1 \geq RE_6)$ = (-0.19, -0.55)
$v_{\grave{o}}(PE_1 \leq PE_7)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_1 \geq RE_7)$ = (-0.8, -0.6)
Max $\{PE_1 \leq PE_2, PE_3, PE_4, PE_5, PE_6, PE_7\}$ = [-0.23, -0.06], [-0.13, -0.009], min $\{RE_1 \geq RE_2, RE_3, RE_4, RE_5, RE_6, RE_7\}$ = (-0.8, -0.6)

$v_{\grave{o}}(PE_2 \leq PE_1)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_2 \geq RE_1)$ = (-0.8, -0.6)
$v_{\grave{o}}(PE_2 \leq PE_3)$ = [-0.21, -0.06], [-0.12, -0.009],$v_{\grave{o}}(RE_2 \geq RE_3)$ = (-0.17, -0.48)
$v_{\grave{o}}(PE_2 \leq PE_4)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_2 \geq RE_4)$ = (-0.8, -0.6)
$v_{\grave{o}}(PE_2 \leq PE_5)$ = [-0.21, -0.05], [-0.11, -0.008],$v_{\grave{o}}(RE_2 \geq RE_5)$ = (-0.16, -0.48)
$v_{\grave{o}}(PE_2 \leq PE_6)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_2 \geq RE_6)$ = (-0.8, -0.6)
$v_{\grave{o}}(PE_2 \leq PE_7)$ = [-0.21, -0.06], [-0.12, -0.009],$v_{\grave{o}}(RE_2 \geq RE_7)$ = (-0.17, -0.49)
Max $\{PE_2 \leq PE_1, PE_3, PE_4, PE_5, PE_6, PE_7\}$ = [-0.21, -0.05], [-0.11, -0.009], min $\{RE_2 \geq RE_1, RE_3, RE_4, RE_5, RE_6, RE_7\}$ = (-0.8, -0.6)

$v_{\grave{o}}(PE_3 \leq PE_1)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_3 \geq RE_1)$ = (-0.8, -0.6)
$v_{\grave{o}}(PE_3 \leq PE_2)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_3 \geq RE_2)$ = (-0.8, -0.6)
$v_{\grave{o}}(PE_3 \leq PE_4)$ = [-0.23, -0.06], [-0.13, -0.009],$v_{\grave{o}}(RE_3 \geq RE_4)$ = (-0.18, -0.53)
$v_{\grave{o}}(PE_3 \leq PE_5)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{o}}(RE_3 \geq RE_5)$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(PE_3 \leq PE_6)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(RE_3 \geq RE_6)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_3 \leq PE_7)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(RE_3 \geq RE_7)$ = (-0.8, -0.6)
Max $\{PE_3 \leq PE_2, PE_1, PE_4, PE_5, PE_6, PE_7\}$ = [-0.23, -0.06], [-0.13, -0.009], min $\{RE_3 \geq RE_2, RE_1, RE_4, RE_5, RE_6, RE_7\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(PE_4 \leq PE_1)$ = [-0.22, -0.06], [-0.12, -0.009],$v_{\grave{\upsilon}}(RE_4 \geq RE_1)$ = (-0.18, -0.54)
$v_{\grave{\upsilon}}(PE_4 \leq PE_2)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(RE_4 \geq RE_2)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_4 \leq PE_3)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(RE_4 \geq RE_3)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_4 \leq PE_5)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(RE_4 \geq RE_5)$ = (-0.17, -0.51)
$v_{\grave{\upsilon}}(PE_4 \leq PE_6)$ = [-0.20, -0.05], [-0.11, -0.009],$v_{\grave{\upsilon}}(RE_4 \geq RE_6)$ = (-0.17, -0.50)
$v_{\grave{\upsilon}}(PE_4 \leq PE_7)$ = [-0.21, -0.05], [-0.11, -0.008],$v_{\grave{\upsilon}}(RE_4 \geq RE_7)$ = (-0.17, -0.51)
Max $\{PE_4 \leq PE_2, PE_3, PE_1, PE_5, PE_6, PE_7\}$ = [-0.21, -0.05], [-0.11, -0.008], min $\{RE_4 \geq RE_2, RE_3, RE_1, RE_5, RE_6, RE_7\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(PE_5 \leq PE_1)$ = [-0.19, -0.05], [-0.11, -0.008],$v_{\grave{\upsilon}}(RE_5 \geq RE_1)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_5 \leq PE_2)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(RE_5 \geq RE_2)$ = (-0.18, -0.52)
$v_{\grave{\upsilon}}(PE_5 \leq PE_3)$ = [-0.19, -0.05], [-0.11, -0.009],$v_{\grave{\upsilon}}(RE_5 \geq RE_3)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_5 \leq PE_4)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(RE_5 \geq RE_4)$ = (-0.16, -0.49)
$v_{\grave{\upsilon}}(PE_5 \leq PE_6)$ = [-0.22, -0.05], [-0.12, -0.007],$v_{\grave{\upsilon}}(RE_5 \geq RE_6)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_5 \leq PE_7)$ = [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(RE_5 \geq RE_7)$ = (-0.8, -0.6)
Max $\{PE_5 \leq PE_2, PE_3, PE_4, PE_1, PE_6, PE_7\}$ = [-0.19, -0.05], [-0.11, -0.007], min $\{RE_5 \geq RE_2, RE_3, RE_4, RE_1, RE_6, RE_7\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(PE_6 \leq PE_1)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(RE_6 \geq RE_1)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_6 \leq PE_2)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(RE_6 \geq RE_2)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_6 \leq PE_3)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(RE_6 \geq RE_3)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_6 \leq PE_4)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(RE_6 \geq RE_4)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_6 \leq PE_5)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(RE_6 \geq RE_5)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_6 \leq PE_7)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(RE_6 \geq RE_7)$ = (-0.17, -0.51)
Max $\{PE_6 \leq PE_2, PE_3, PE_4, PE_5, PE_1, PE_7\}$ = [-0.5, -0.3], [-0.4, -0.1], min $\{RE_6 \geq RE_2, RE_3, RE_4, RE_5, RE_1, RE_7\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(PE_7 \leq PE_1)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(RE_7 \geq RE_1)$ = (-0.17, -0.50)
$v_{\grave{\upsilon}}(PE_7 \leq PE_2)$ = [-0.12, -0.04], [-0.07, -0.008], $v_{\grave{\upsilon}}(RE_7 \geq RE_2)$ = (-0.17, -0.51)
$v_{\grave{\upsilon}}(PE_7 \leq PE_3)$ = [-0.13, -0.04], [-0.08, -0.007],$v_{\grave{\upsilon}}(RE_7 \geq RE_3)$ = (-0.16, -0.48)
$v_{\grave{\upsilon}}(PE_7 \leq PE_4)$ = [-0.11, -0.03], [-0.06, -0.007],$v_{\grave{\upsilon}}(RE_7 \geq RE_4)$ = (-0.18, -0.52)
$v_{\grave{\upsilon}}(PE_7 \leq PE_5)$ = [-0.17, -0.05], [-0.10, -0.007],$v_{\grave{\upsilon}}(RE_7 \geq RE_5)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(PE_7 \leq PE_6)$ = [-0.10, -0.03], [-0.06, -0.008],$v_{\grave{\upsilon}}(RE_7 \geq RE_6)$ = (-0.16, -0.48)
Max $\{PE_7 \leq PE_2, PE_3, PE_4, PE_5, PE_6, PE_1\}$ = [-0.10, -0.03], [-0.06, -0.007], min $\{RE_7 \geq RE_2, RE_3, RE_4, RE_5, RE_6, RE_1\}$ = (-0.8, -0.6)

Sub-criteria (financial factor)

Step 1: The NCPF (F_K) extent value of kth sub-criterion (financial factor) is given by,

$$\sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}} [\sum_{k}^{n} \sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}}]^{-1}$$

F (1) = (0.067, 0.122, 0.225), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
F (2) = (0.072, 0.128, 0.234),[-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
F (3) = (0.051, 0.090, 0.166), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
F (4) = (0.064, 0.112, 0.204), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
F (5) = (0.043, 0.076, 0.141), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
F (6) = (0.055, 0.095, 0.169), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
F (7) = (0.039, 0.065, 0.115), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
F (8) = (0.083, 0.153, 0.279), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
F (9) = (0.034, 0.065, 0.128), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
F (10) = (0.051, 0.090, 0.165), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)

Step 2:
$v_{\grave{\upsilon}}(IF_1 \leq IF_2)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_1 \geq FI_2)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_1 \leq IF_3)$ = [-0.21, -0.05], [-0.11, -0.007], $v_{\grave{\upsilon}}(FI_1 \geq FI_3)$ = (-0.16, -0.49)
$v_{\grave{\upsilon}}(IF_1 \leq IF_4)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_1 \geq FI_4)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_1 \leq IF_5)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_1 \geq FI_5)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_1 \leq IF_6)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_1 \geq FI_6)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_1 \leq IF_7)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_1 \geq FI_7)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_1 \leq IF_8)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_1 \geq FI_8)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_1 \leq IF_9)$ = [-0.20, -0.05], [-0.11, -0.008], $v_{\grave{\upsilon}}(FI_1 \geq FI_9)$ = (-0.17, -0.50)
$v_{\grave{\upsilon}}(IF_1 \leq IF_{10})$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_1 \geq FI_{10})$ = (-0.8, -0.6)
Max $\{IF_1 \leq IF_2, IF_3, IF_4, IF_5, IF_6, IF_7, IF_8, IF_9, IF_{10}\}$ = [-0.20, -0.05], [-0.11, -0.007], min $\{FI_1 \geq FI_2, FI_3, FI_4, FI_5, FI_6, FI_7, FI_8, FI_9, FI_{10}\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(IF_2 \leq IF_1)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_2 \geq FI_1)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_2 \leq IF_3)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_2 \geq FI_3)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_2 \leq IF_4)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_2 \geq FI_4)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_2 \leq IF_5)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_2 \geq FI_5)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_2 \leq IF_6)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_2 \geq FI_6)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_2 \leq IF_7)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_2 \geq FI_7)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_2 \leq IF_8)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_2 \geq FI_8)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_2 \leq IF_9)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_2 \geq FI_9)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_2 \leq IF_{10})$ = [-0.21, -0.05], [-0.11, -0.008], $v_{\grave{\upsilon}}(FI_2 \geq FI_{10})$ = (-0.17.-0.51)
Max $\{IF_2 \leq IF_1, IF_3, IF_4, IF_5, IF_6, IF_7, IF_8, IF_9, IF_{10}\}$ = [-0.21, -0.05], [-0.11, -0.008], min $\{FI_2 \geq FI_1, FI_3, FI_4, FI_5, FI_6, FI_7, FI_8, FI_9, FI_{10}\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(IF_3 \leq IF_1)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_3 \geq FI_1)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_3 \leq IF_2)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_3 \geq FI_2)$ = (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_3 \leq IF_4)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\grave{\upsilon}}(FI_3 \geq FI_4)$ = (-0.16, -0.48)
$v_{\grave{\upsilon}}(IF_3 \leq IF_5)$ = [-0.19, -0.05], [-0.10, -0.006], $v_{\grave{\upsilon}}(FI_3 \geq FI_5)$ = (-0.16, -0.49)

$v_{\dot{\upsilon}}(IF_3 \leq IF_6)$ = [-0.18, -0.05], [-0.10, -0.006], $v_{\dot{\upsilon}}(FI_3 \geq FI_6)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_3 \leq IF_7)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_3 \geq FI_7)$ = (-0.15, -0.47)
$v_{\dot{\upsilon}}(IF_3 \leq IF_8)$ = [-0.19, -0.05], [-0.10, -0.006], $v_{\dot{\upsilon}}(FI_3 \geq FI_8)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_3 \leq IF_9)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_3 \geq FI_9)$ = (-0.15, -0.46)
$v_{\dot{\upsilon}}(IF_3 \leq IF_{10})$ = [-0.20, -0.05], [-0.10, -0.005], $v_{\dot{\upsilon}}(FI_3 \geq FI_{10})$ = (-0.8, -0.6)
Max $\{IF_3 \leq IF_2, IF_1, IF_4, IF_5, IF_6, IF_7, IF_8, IF_9, IF_{10}\}$ = [-0.18, -0.05], [-0.10, -0.005], min $\{FI_3 \geq FI_2, FI_1, FI_4, FI_5, FI_6, FI_7, FI_8, FI_9, FI_{10}\}$ = (-0.8, -0.6)

$v_{\dot{\upsilon}}(IF_4 \leq IF_1)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_4 \geq FI_1)$ = (-0.17, -0.50)
$v_{\dot{\upsilon}}(IF_4 \leq IF_2)$ = [-0.17, -0.05], [-0.09, -0.008], $v_{\dot{\upsilon}}(FI_4 \geq FI_2)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_4 \leq IF_3)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_4 \geq FI_3)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_4 \leq IF_5)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_4 \geq FI_5)$ = (-0.16, -0.49)
$v_{\dot{\upsilon}}(IF_4 \leq IF_6)$ = [-0.21, -0.05], [-0.11, -0.006], $v_{\dot{\upsilon}}(FI_4 \geq FI_6)$ = (-0.16, -0.49)
$v_{\dot{\upsilon}}(IF_4 \leq IF_7)$ = [-0.20, -0.05], [-0.11, -0.007], $v_{\dot{\upsilon}}(FI_4 \geq FI_7)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_4 \leq IF_8)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_4 \geq FI_8)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_4 \leq IF_9)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_4 \geq FI_9)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_4 \leq IF_{10})$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_4 \geq FI_{10})$ = (-0.8, -0.6)
Max $\{IF_4 \leq IF_2, IF_3, IF_1, IF_5, IF_6, IF_7, IF_8, IF_9, IF_{10}\}$ = [-0.17, -0.05], [-0.09, -0.006], min $\{FI_4 \geq FI_2, FI_3, FI_1, FI_5, FI_6, FI_7, FI_8, FI_9, FI_{10}\}$ = (-0.8, -0.6)

$v_{\dot{\upsilon}}(IF_5 \leq IF_1)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_5 \geq FI_1)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_5 \leq IF_2)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_5 \geq FI_2)$ = (-0.17, -0.51)
$v_{\dot{\upsilon}}(IF_5 \leq IF_3)$ = [-0.19, -0.05], [-0.11, -0.008], $v_{\dot{\upsilon}}(FI_5 \geq FI_3)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_5 \leq IF_4)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_5 \geq FI_4)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_5 \leq IF_6)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_5 \geq FI_6)$ = (-0.16, -0.48)
$v_{\dot{\upsilon}}(IF_5 \leq IF_7)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_5 \geq FI_7)$ = (-0.16, -0.48)
$v_{\dot{\upsilon}}(IF_5 \leq IF_8)$ = [-0.16, -0.04], [-0.09, -0.006], $v_{\dot{\upsilon}}(FI_5 \geq FI_8)$ = (-0.15, -0.45)
$v_{\dot{\upsilon}}(IF_5 \leq IF_9)$ = [-0.16, -0.04], [-0.09, -0.006], $v_{\dot{\upsilon}}(FI_5 \geq FI_9)$ = (-0.15, -0.47)
$v_{\dot{\upsilon}}(IF_5 \leq IF_{10})$ = [-0.19, -0.04], [-0.10, -0.005], $v_{\dot{\upsilon}}(FI_5 \geq FI_{10})$ = (-0.8, -0.6)
Max $\{IF_5 \leq IF_2, IF_3, IF_4, IF_1, IF_6, IF_7, IF_8, IF_9, IF_{10}\}$ = [-0.16, -0.04], [-0.09, -0.005], min $\{FI_5 \geq FI_2, FI_3, FI_4, FI_1, FI_6, FI_7, FI_8, FI_9, FI_{10}\}$ = (-0.8, -0.6)

$v_{\dot{\upsilon}}(IF_6 \leq IF_1)$ = [-0.17, -0.04], [-0.09, -0.005], $v_{\dot{\upsilon}}(FI_6 \geq FI_1)$ = (-0.15, -0.46)
$v_{\dot{\upsilon}}(IF_6 \leq IF_2)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_6 \geq FI_2)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_6 \leq IF_3)$ = [-0.18, -0.04], [-0.10, -0.005], $v_{\dot{\upsilon}}(FI_6 \geq FI_3)$ = (-0.17, -0.50)
$v_{\dot{\upsilon}}(IF_6 \leq IF_4)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_6 \geq FI_4)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_6 \leq IF_5)$ = [-0.14, -0.04], [-0.08, -0.007], $v_{\dot{\upsilon}}(FI_6 \geq FI_5)$ = (-0.4, -0.3)
$v_{\dot{\upsilon}}(IF_6 \leq IF_7)$ = [-0.5, -0.3], [-0.4, -0.1], $v_{\dot{\upsilon}}(FI_6 \geq FI_7)$ = (-0.17, -0.51)
$v_{\dot{\upsilon}}(IF_6 \leq IF_8)$ = [-0.19, -0.04], [-0.10, -0.005], $v_{\dot{\upsilon}}(FI_6 \geq FI_8)$ = (-0.17, -0.51)
$v_{\dot{\upsilon}}(IF_6 \leq IF_9)$ = [-0.19, -0.05], [-0.11, -0.006], $v_{\dot{\upsilon}}(FI_6 \geq FI_9)$ = (-0.8, -0.6)
$v_{\dot{\upsilon}}(IF_6 \leq IF_{10})$ = [-0.19, -0.05], [-0.10, -0.006], $v_{\dot{\upsilon}}(FI_6 \geq FI_{10})$ = (-0.16, -0.50)
Max $\{IF_6 \leq IF_2, IF_3, IF_4, IF_5, IF_1, IF_7, IF_8, IF_9, IF_{10}\}$ = [-0.14, -0.04], [-0.08, -0.005], min $\{FI_6 \geq FI_2, FI_3, FI_4, FI_5, FI_1, FI_7, FI_8, FI_9, FI_{10}\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(IF_7 \leq IF_1)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_7 \geq FI_1)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_7 \leq IF_2)$ =[-0.20, -0.05], [-0.11, -0.006],$v_{\grave{\upsilon}}(FI_7 \geq FI_2)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_7 \leq IF_3)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_7 \geq FI_3)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_7 \leq IF_4)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_7 \geq FI_4)$= (-0.18, -0.53)
$v_{\grave{\upsilon}}(IF_7 \leq IF_5)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_7 \geq FI_5)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_7 \leq IF_6)$ =[-0.18, -0.05], [-0.10, -0.008],$v_{\grave{\upsilon}}(FI_7 \geq FI_6)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_7 \leq IF_8)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_7 \geq FI_8)$= (-0.17, -0.51)
$v_{\grave{\upsilon}}(IF_7 \leq IF_9)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_7 \geq FI_9)$= (-0.17, -0.51)
$v_{\grave{\upsilon}}(IF_7 \leq IF_{10})$ =[-0.14, -0.04], [-0.08, -0.006],$v_{\grave{\upsilon}}(FI_7 \geq FI_{10})$= (-0.15, -0.49)
Max $\{IF_7 \leq IF_2, IF_3, IF_4, IF_5, IF_6, IF_1, IF_8, IF_9, IF_{10}\}$ = [-0.14, -0.04], [-0.10, -0.006], min $\{FI_7 \geq FI_2, FI_3, FI_4, FI_5, FI_6, FI_1, FI_8, FI_9, FI_{10}\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(IF_8 \leq IF_1)$ =[-0.13, -0.04], [-0.07, -0.006],$v_{\grave{\upsilon}}(FI_8 \geq FI_1)$= (-0.16, -0.50)
$v_{\grave{\upsilon}}(IF_8 \leq IF_2)$ =[-0.17, -0.04], [-0.09, -0.005],$v_{\grave{\upsilon}}(FI_8 \geq FI_2)$= (-0.15, -0.48)
$v_{\grave{\upsilon}}(IF_8 \leq IF_3)$ =[-0.15, -0.04], [-0.08, -0.005],$v_{\grave{\upsilon}}(FI_8 \geq FI_3)$= (-0.16, -0.49)
$v_{\grave{\upsilon}}(IF_8 \leq IF_4)$ =[-0.19, -0.04], [-0.10, -0.004],$v_{\grave{\upsilon}}(FI_8 \geq FI_4)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_8 \leq IF_5)$ =[-0.17, -0.04], [-0.09, -0.005],$v_{\grave{\upsilon}}(FI_8 \geq FI_5)$= (-0.18, -0.53)
$v_{\grave{\upsilon}}(IF_8 \leq IF_6)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_8 \geq FI_6)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_8 \leq IF_7)$ =[-0.11, -0.03], [-0.06, -0.007],$v_{\grave{\upsilon}}(FI_8 \geq FI_7)$= (-0.6, -0.4)
$v_{\grave{\upsilon}}(IF_8 \leq IF_9)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_8 \geq FI_9)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_8 \leq IF_{10})$ =[-0.17, -0.04], [-0.09, -0.005],$v_{\grave{\upsilon}}(FI_8 \geq FI_{10})$= (-0.8, -0.6)
Max $\{IF_8 \leq IF_2, IF_3, IF_4, IF_5, IF_6, IF_7, IF_1, IF_9, IF_{10}\}$ = [-0.11, -0.03], [-0.06, -0.004], min $\{FI_8 \geq FI_2, FI_3, FI_4, FI_5, FI_6, FI_7, FI_1, FI_9, FI_{10}\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(IF_9 \leq IF_1)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_9 \geq FI_1)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_9 \leq IF_2)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_9 \geq FI_2)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_9 \leq IF_3)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_9 \geq FI_3)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_9 \leq IF_4)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_9 \geq FI_4)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_9 \leq IF_5)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_9 \geq FI_5)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_9 \leq IF_6)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_9 \geq FI_6)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_9 \leq IF_7)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_9 \geq FI_7)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_9 \leq IF_8)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_9 \geq FI_8)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_9 \leq IF_{10})$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_9 \geq FI_{10})$= (-0.15, -0.44)
Max $\{IF_9 \leq IF_2, IF_3, IF_4, IF_5, IF_6, IF_7, IF_8, IF_1, IF_{10}\}$ = [-0.5, -0.3], [-0.4, -0.1], min $\{FI_9 \geq FI_2, FI_3, FI_4, FI_5, FI_6, FI_7, FI_8, FI_1, FI_{10}\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(IF_{10} \leq IF_1)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_{10} \geq FI_1)$= (-0.15, -0.45)
$v_{\grave{\upsilon}}(IF_{10} \leq IF_2)$ =[-0.14, -0.04], [-0.08, -0.006],$v_{\grave{\upsilon}}(FI_{10} \geq FI_2)$= (-0.14, -0.42)
$v_{\grave{\upsilon}}(IF_{10} \leq IF_3)$ =[-0.13, -0.04], [-0.07, -0.06],$v_{\grave{\upsilon}}(FI_{10} \geq FI_3)$= (-0.14, -0.43)
$v_{\grave{\upsilon}}(IF_{10} \leq IF_4)$ =[-0.17, -0.04], [-0.09, -0.005],$v_{\grave{\upsilon}}(FI_{10} \geq FI_4)$= (-0.13, -0.41)
$v_{\grave{\upsilon}}(IF_{10} \leq IF_5)$ =[-0.15, -0.04], [-0.08, -0.005],$v_{\grave{\upsilon}}(FI_{10} \geq FI_5)$= (-0.14, -0.42)
$v_{\grave{\upsilon}}(IF_{10} \leq IF_6)$ =[-0.18, -0.04], [-0.09.-0.004],$v_{\grave{\upsilon}}(FI_{10} \geq FI_6)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(IF_{10} \leq IF_7)$ =[-0.16, -0.04], [-0.09, -0.005],$v_{\grave{\upsilon}}(FI_{10} \geq FI_7)$= (-0.16, -0.46)
$v_{\grave{\upsilon}}(IF_{10} \leq IF_8)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(FI_{10} \geq FI_8)$= (-0.8, -0.6)

$v_{\grave{\upsilon}}(IF_{10} \leq IF_9)$ =[-0.12, -0.04], [-0.07, -0.007],$v_{\grave{\upsilon}}(FI_{10} \geq FI_9)$= (-0.6, -0.4)
Max $\{IF_{10} \leq IF_2, IF_3, IF_4, IF_5, IF_6, IF_7, IF_8, IF_9, IF_1\}$ = [-0.12, -0.04], [-0.07, -0.004], min $\{FI_{10} \geq FI_2, FI_3, FI_4, FI_5, FI_6, FI_7, FI_8, FI_9, FI_1\}$ = (-0.8, -0.6)

Sub-criteria (market related factor)
Step 1: The NCPF (M_K) extent value of kth sub-criterion (market related factor) is given by,

$$\sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}} [\sum_{k}^{n} \sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}}]^{-1}$$

M (1) = (0.129, 0.229, 0.397), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
M (2) = (0.102, 0.175, 0.300), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
M (3) = (0.176, 0.316, 0.56), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
M (4) = (0.041, 0.078, 0.143), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
M (5) = (0.12, 0.20, 0.347), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)

Step 2:
$v_{\grave{\upsilon}}(RM_1 \leq RM_2)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_1 \geq MR_2)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(RM_1 \leq RM_3)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_1 \geq MR_3)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(RM_1 \leq RM_4)$ =[-0.23, -0.07], [-0.14, -0.01],$v_{\grave{\upsilon}}(MR_1 \geq MR_4)$= (-0.24, -0.66)
$v_{\grave{\upsilon}}(RM_1 \leq RM_5)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_1 \geq MR_5)$= (-0.8, -0.6)
Max $\{RM_1 \leq RM_2, RM_3, RM_4, RM_5\}$ = [-0.23, -0.07], [-0.14, -0.01], min $\{MR_1 \geq MR_2, MR_3, MR_4, RM_5\}$ = (-0.8, -0.66)

$v_{\grave{\upsilon}}(RM_2 \leq RM_1)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_2 \geq MR_1)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(RM_2 \leq RM_3)$ =[-0.22, -0.06], [-0.12, -0.01],$v_{\grave{\upsilon}}(MR_2 \geq MR_3)$= (-0.21, -0.61)
$v_{\grave{\upsilon}}(RM_2 \leq RM_4)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_2 \geq MR_4)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(RM_2 \leq RM_5)$ =[-0.19, -0.06], [-0.11, -0.01],$v_{\grave{\upsilon}}(MR_2 \geq MR_5)$= (-0.25, -0.67)
Max $\{RM_2 \leq RM_1, RM_3, RM_4, RM_5\}$ = [-0.19, -0.06], [-0.11, -0.01], min $\{MR_2 \geq MR_1, MR_3, MR_4, RM_5\}$ = (-0.8, -0.67)

$v_{\grave{\upsilon}}(RM_3 \leq RM_1)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_3 \geq MR_1)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(RM_3 \leq RM_2)$ =[-0.22, -0.06], [-0.12, -0.009],$v_{\grave{\upsilon}}(MR_3 \geq MR_2)$= (-0.20, -0.59)
$v_{\grave{\upsilon}}(RM_3 \leq RM_4)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_3 \geq MR_4)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(RM_3 \leq RM_5)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_3 \geq MR_5)$= (-0.8, -0.6)
Max $\{RM_3 \leq RM_2, RM_1, RM_4, RM_5\}$ = [-0.22, -0.06], [-0.12, -0.009], min $\{MR_3 \geq MR_2, MR_1, MR_4, RM_5\}$ = (-0.8, -0.6)

$v_{\grave{\upsilon}}(RM_4 \leq RM_1)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_4 \geq MR_1)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(RM_4 \leq RM_2)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_4 \geq MR_2)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(RM_4 \leq RM_3)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{\upsilon}}(MR_4 \geq MR_3)$= (-0.8, -0.6)
$v_{\grave{\upsilon}}(RM_4 \leq RM_5)$ =[-0.07, -0.03], [-0.05, -0.01],$v_{\grave{\upsilon}}(MR_4 \geq MR_5)$= (-0.20, -0.56)
Max $\{RM_4 \leq RM_2, RM_3, RM_1, RM_5\}$ = [-0.07, -0.03], [-0.05, -0.01], min $\{MR_4 \geq MR_2, MR_3, MR_1, RM_5\}$ = (-0.8, -0.6)

$v_{\grave{v}}(RM_5 \le RM_1)$ =[-0.11, -0.03], [-0.06, -0.007],$v_{\grave{v}}(MR_5 \ge MR_1)$= (-0.18, -0.52)
$v_{\grave{v}}(RM_5 \le RM_2)$ =[-0.05, -0.03], [-0.06, -0.007],$v_{\grave{v}}(MR_5 \ge MR_2)$= (-1, -0.52)
$v_{\grave{v}}(RM_5 \le RM_3)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{v}}(MR_5 \ge MR_3)$= (-0.8, -0.6)
$v_{\grave{v}}(RM_5 \le RM_4)$ =[-0.08, -0.03], [-0.05, -0.008],$v_{\grave{v}}(MR_5 \ge MR_4)$= (-0.19, -0.54)
Max $\{RM_5 \le RM_2, RM_3, RM_4, RM_1\}$ = [-0.05, -0.03], [-0.05, -0.007], min $\{MR_5 \ge MR_2, MR_3, MR_4, RM_1\}$ = (-1, -0.6)

Sub-criteria (external factor)

Step1: The NCPF (E_K) extent value of kth sub-criterion (external factor) is given by,

$$\sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}} \, [\sum_{k}^{n} \sum_{l}^{m} \boldsymbol{M}_{h_k}^{\boldsymbol{l}}]^{-1}$$

E (1) = (0.103, 0.17, 0.28), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
E (2)=(0.186, 0.312, 0.515), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
E(3)=(0.185, 0.309, 0.507), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)
E(4)=(0.132, 0.207, 0.33), [-0.5, -0.3], [-0.4, -0.1], (-0.8, -0.6)

Step 2:

$v_{\grave{v}}(XE_1 \le XE_2) =$ [-0.5, -0.3], [-0.4, -0.1],$v_{\grave{v}}(EX_1 \ge EX_2)$= (-0.8, -0.6)
$v_{\grave{v}}(XE_1 \le XE_3)$ =[-0.6, -0.16], [-0.10, -0.01], $v_{\grave{v}}(EX_1 \ge EX_3)$= (-0.2, -0.7)
$v_{\grave{v}}(XE_1 \le XE_4)$ =[-0.6, -0.16], [-0.10, -0.01], $v_{\grave{v}}(XE_1 \ge EX_4)$= (-0.2, -0.7)
Max $\{XE_1 \le XE_2, XE_3, XE_4\}$= [-0.5, -0.16], [-0.10, -0.01], min $\{EX_1 \ge EX_2, EX_3, EX_4\}$ = (-0.8, -0.7)

$v_{\grave{v}}(XE_2 \le XE_1)$ =[-0.20, -0.05], [-0.11, -0.008], $v_{\grave{v}}(EX_2 \ge EX_1)$ = (-0.2, -0.6)
$v_{\grave{v}}(XE_2 \le XE_3)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{v}}(EX_2 \ge EX_3)$= (-0.8, -0.6)
$v_{\grave{v}}(XE_2 \le XE_4)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{v}}(EX_2 \ge EX_4)$= (-0.8, -0.6)
Max $\{XE_2 \le XE_1, XE_3, XE_4\}$= [-0.20, -0.05], [-0.11, -0.008], min $\{EX_2 \ge EX_1, EX_3, EX_4\}$ = (-0.8, -0.6)

$v_{\grave{v}}(XE_3 \le XE_1)$ =[-0.5, -0.3], [-0.4, -0.1], $v_{\grave{v}}(EX_3 \ge EX_1)$= (-0.8, -0.6)
$v_{\grave{v}}(XE_3 \le XE_2)$ =[-0.5, -0.3], [-0.4, -0.1], $v_{\grave{v}}(EX_3 \ge EX_2)$= (-0.8, -0.6)
$v_{\grave{v}}(XE_3 \le XE_4)$ =[-0.5, -0.3], [-0.4, -0.1], $v_{\grave{v}}(EX_3 \ge EX_4)$= (-0.8, -0.6)
Max $\{XE_3 \le XE_1, XE_2, XE_4\}$= [-0.5, -0.3], [-0.4, -0.1], min $\{EX_3 \ge EX_2, EX_1, EX_4\}$ = (-0.8, -0.6)

$v_{\grave{v}}(XE_4 \le XE_1)$ =[-0.2, -0.08], [-0.15, -0.01],$v_{\grave{v}}(EX_4 \ge EX_1)$ = (-0.2, -0.7)
$v_{\grave{v}}(XE_4 \le XE_2)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{v}}(EX_4 \ge EX_2)$ = (-0.8, -0.6)
$v_{\grave{v}}(XE_4 \le XE_3)$ =[-0.5, -0.3], [-0.4, -0.1],$v_{\grave{v}}(EX_4 \ge EX_3)$ = (-0.8, -0.6)
Max $\{XE_4 \le XE_2, XE_3, XE_1\}$= [-0.2, -0.008], [-0.15, -0.01], min $\{EX_4 \ge EX_2, EX_3, EX_1\}$ = (-0.8, -0.7)

By the intersection of both membership functions and both non membership function of N-cubic Pythagorean fuzzy numbers is the abscissae of the common portion of upper and lower triangles are generated respectively.

Similarly, these steps (b, c) are followed by remaining other sub-criteria including (operational factors, economic/government factors, performance related factors, financial factors, market related factors, external factors).

Step 3: The weight of main factors that play role in downfall of PIA in hierarchy form is given by Table 10:

Table 10. Weight of main factors.

Main factors	ω̃	□	¥	Weight	Rank
Operational factors	0.4250	0.8321	0.6285	0.1629	5
Economic/government factors	0.4475	0.8321	0.639	0.1658	4
Performance related factors	0.4800	0.8321	0.656	0.1702	2
Financial factors	0.4252	0.8321	0.6286	0.1631	6
Market related factors	0.4525	0.8321	0.642	0.1665	3
External factors	0.4900	0.8321	0.661	0.1715	1

The weight of sub-criteria (operational factors) is as under (see Table 11):

Table 11. Weight of sub-criteria of operational factors.

Sub-criteria (operational factors)	ω̃	□	¥	Weight	Rank
Load factor	0.4228	0.8321	0.627	0.1222	7
Average number of passengers carried per departure	0.4128	0.8321	0.622	0.1213	8
Average number of hours flown per pilot	0.4352	0.8321	0.633	0.1234	6
Number of departures per aircraft	0.4668	0.8321	0.649	0.1265	3
Number of pilots per aircraft	0.4688	0.8321	0.650	0.1267	2
The average age of the aircraft fleet	0.4718	0.8321	0.651	0.1269	1
Number of different brands of aircraft operated	0.4612	0.8321	0.646	0.1259	5
International operations	0.4620	0.8321	0.647	0.1261	4

The weight of sub-criteria (economic/government) factor is (see Table 12):

Table 12. Weight of sub-criteria of economic/government factor.

Sub-criteria (economic/government factors)	ω̃	□	¥	Weight	Rank
Annual inflation	0.4250	0.8321	0.628	0.2340	4
GDP Growth Rate	0.4515	0.8321	0.641	0.2389	3
Aviation Fuel price (INR per liter)	0.42	1	0.702	0.2616	2
Average Growth in Value of Passengers carried in country	0.4250	1	0.712	0.2653	1

The weight of sub-criteria (performance related) factor as under (see Table 13):

Table 13. Weight of performance related factor.

Sub-criteria (performance related factors)	ω̃	□	¥	Weight	Rank
Available seat kilometer (ASK)	0.4595	0.8321	0.645	0.1428	6
Revenue per kilometer (RPK)	0.4620	0.8321	0.647	0.1430	4
Available seat KM per employee	0.4608	0.8321	0.646	0.1429	5
Average stage length flown in kilometer	0.4640	0.8321	0.648	0.1431	3
Fuel efficiency (liters per KM flown)	0.4680	0.8321	0.650	0.1437	2
Break even load factor	0.4250	0.8321	0.628	0.1388	7
Labor cost per KM flown	0.4842	0.8321	0.658	0.1455	1

The weight of sub-criteria (financial factor) is below (see Table 14):

Table 14. Weight of financial factor.

Sub-criteria (financial factors)	ω̃	□	¥	Weight	Rank
Operating revenues/operating cost	0.4656	0.8321	0.648	0.0988	7
Operating Profit/ Total Assets	0.4635	0.8321	0.647	0.0986	8
Retained earnings/total assets	0.4678	0.8321	0.649	0.0989	6
Market Value of Equity/Total Book value of debt	0.4690	0.8321	0.650	0.0991	5
Current assets/current liabilities	0.4783	0.8321	0.655	0.0999	3
Earnings before interest and taxes/ operating revenues	0.4762	0.8321	0.654	0.0998	4
Interest/total liabilities or debt service	0.4815	0.8321	0.656	0.1001	2
Operating revenues per air kilometer	0.5900	0.8321	0.711	0.1084	1
Earnings stability (the deviation around a 10-year trend line of return on assets)	0.4250	0.8321	0.628	0.0957	9
Firm size (measured by the log of the firm's total assets).	0.4195	0.8321	0.625	0.0945	10

The weight of sub-criteria (market related) is as under (see Table 15):

Table 15. Weight of market related factor.

Sub-criteria (market related)	ω̃	□	¥	weight	rank
Number of airlines operating	0.4625	0.8367	0.649	0.1933	4
Company passenger growth (%)/Industry growth (%)	0.4675	0.8376	0.652	0.1942	3
Market share	0.4622	0.8321	0.647	0.1927	5
Govt. policies regarding slot allocation	0.4900	0.8321	0.661	0.1969	2
Airport preference of airlines	0.4942	1	0.747	0.2225	1

The weight of sub-criteria (external factors) as follows (see Table 16):

Table 16. Weight of external factors.

Sub-criteria (external factors)	ω̃	□	¥	weight	rank
Environment or weather conditions	0.3625	1	0.681	0.2597	1
Geographical location	0.4670	0.8321	0.649	0.2475	3
Threats to national security	0.4250	0.8321	0.628	0.2395	4
Political influence (hiring& benefits)	0.4880	0.8408	0.664	0.2532	2

Step 4: Final ranking

The final ranking of main criteria's and sub-criteria's according to weights are (see Table 17):

Table 17. Final ranking.

Factor's list	Main criteria ranking	Sub-criteria list	Sub-criteria ranking	Weights	Global ranking
Operational factors	5	OP(1)	7	0.1222	27
		OP(2)	8	0.1213	28
		OP(3)	6	0.1234	26
		OP(4)	3	0.1265	23
		OP(5)	2	0.1267	22
		OP(6)	1	0.1269	21
		OP(7)	5	0.1259	25
		OP(8)	4	0.1261	24

Continued on next page

Factor's list	Main criteria ranking	Sub-criteria list	Sub-criteria ranking	Weights	Global ranking
Economic/Govt factor	4	E/G(1)	4	0.2340	8
		E/G(2)	3	0.2389	7
		E/G(3)	2	0.2616	2
		E/G(4)	1	0.2653	1
Performance related factor	2	PR(1)	6	0.1428	19
		PR(2)	4	0.1430	17
		PR(3)	5	0.1429	18
		PR(4)	3	0.1431	16
		PR(5)	2	0.1437	15
		PR(6)	7	0.1388	20
		PR(7)	1	0.1455	14
Financial factor	6	FF(1)	7	0.0988	35
		FF(2)	8	0.0986	36
		FF(3)	6	0.0989	34
		FF(4)	5	0.0991	33
		FF(5)	3	0.0999	31
		FF(6)	4	0.0998	32
		FF(7)	2	0.1001	30
		FF(8)	1	0.1084	29
		FF(9)	9	0.0957	37
		FF(10)	10	0.0945	38
Market related factors	3	MR (1)	4	0.1933	12
		MR (2)	3	0.1942	11
		MR (3)	5	0.1927	13
		MR (4)	2	0.1969	10
		MR (5)	1	0.2225	9
External factors	1	EF (1)	1	0.2597	3
		EF (2)	3	0.2475	5
		EF (3)	4	0.2395	6
		EF (4)	2	0.2532	4

The final ranking of the classification of several factors, it is observed that ranking of external factors is 1, which means external factors plays negative role in financial position to create the downfall of companies in airline sector. If we discuss further, then we may conclude that for other factors including Performance related factor contribute to negative sense after the external factors. Managers of the companies will be able to overcome the position of downfall after gaining information that which category is strong in their negative performance as well as weak.

6. Comparison analysis

Following Table 18 shows that comparative analysis of fuzzy AHP and AHP under N-cubic Pythagorean fuzzy sets of the downfall of international airline.

Table 18. Comparative analysis.

Methods	Main Ranking	Ranking Results
Fuzzy AHP	EF = 1, MR = 3, FF = 6, PR = 2, E/G = 4, OP = 5	EF > PR > MR > E/G > OP >FF
AHP under N- cubic pythagorean fuzzy sets	EF = 1, MR = 3, FF = 6, PR = 2, E/G = 4, OP = 5	EF > PR > MR > E/G > OP > FF

7. Conclusions

Experts must make decisions about how to improve the performance of airline companies for them to grow. There is a need to examine the inner and outside factors of airline's firms. For this purpose, and according to world research, the results will be more accurate. It is critical to observe both negative and positive factors when making decisions. To discuss the negative factors we initiated the study of N-cubic Pythagorean fuzzy set. This method reveals the behavior of variables in non-positive ways, which could be effective in overcoming the severity of this industry. This method will be used for ranking reasons in other real-world applications in future.

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Conflict of interest

The authors declare that there is no conflict of interest regarding the publication of this article.

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