Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(8): 14460–14473.
DOI:10.3934/math.2022797
Received: 08 April 2022
Revised: 24 May 2022
Accepted: 30 May 2022
Published: 06 June 2022

Research article

Soliton solutions of conformable time-fractional perturbed Radhakrishnan-Kundu-Lakshmanan equation

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Abstract: In this paper, our main purpose is to study the soliton solutions of conformable time-fractional perturbed Radhakrishnan-Kundu-Lakshmanan equation. New soliton solutions have been obtained by the extended (G'/G)-expansion method, first integral method and complete discrimination system for the polynomial method, respectively. The solutions we obtained mainly include hyperbolic function solutions, solitary wave solutions, Jacobi elliptic function solutions, trigonometric function solutions and rational function solutions. Moreover, we draw its three-dimensional graph.

Keywords: time-fractional perturbed Radhakrishnan-Kundu-Lakshmanan equation; extended (G'/G)-expansion method; first integral method; complete discrimination system; conformable fractional derivative
Mathematics Subject Classification: 35C05, 35C07, 35R11

1. Introduction

Due to the wide application in the fields of physics, communication and engineering, the study of soliton solutions of the Radhakrishnan-Kundu-Lakshmanan (RKL) equation has attracted much attention [1–7]. Especially in nonlinear optical fibers, the RKL equation usually describes the propagation of optical pulses, which is represented by the higher-order nonlinear Schrödinger equation. In recent years, many powerful mathematical methods have been proposed to derive soliton solutions [8–21] for the RKL equation, such as the first integral method [22], the generalized exponential rational function method [23], the Laplace-Adomian decomposition method [24], the dynamical system method [25], the Painlevé analysis [26], the auxiliary equation method and extended simple equation method [27], the modified simple equation and $\exp(-\varphi(q))$ method [28].

In the paper, we consider the fractional perturbed Radhakrishnan-Kundu-Lakshmanan (FPRKL) equation [29–32]:

$$iD_t^{\alpha}\phi + a\phi_{xx} + b|\phi|^2\phi - i\delta\phi_x - i\lambda(|\phi|^2\phi)_x - i\sigma(|\phi|^2)_x\phi - i\gamma\phi_{xxx} = 0,\ 0 < \alpha \le 1, \tag{1.1}$$

where ϕ is the complex-valued wave function.

Definition 1.1. *Let* $\psi : [0,\infty) \to \mathbf{R}$. *Then, the conformable fractional derivative of* ψ *of order* α *is defined as*

$$D_t^{\alpha}\psi(t) = \lim_{\varepsilon\to 0}\frac{\psi(t+\varepsilon t^{1-\alpha}) - \psi(t)}{\varepsilon}, \tag{1.2}$$

for all $t > 0$ *and* $\alpha \in (0,1]$. *Further, some properties of conformable fractional derivative is given*

(i) $D_t^{\alpha}(t^{\delta}) = \mu t^{\delta-\alpha},\ \forall\delta \in \mathbf{R}$.

(ii) $D_t^{\alpha}(\psi(t) + \varphi(t)) = D_t^{\alpha}\psi(t) + D_t^{\alpha}\varphi(t)$.

(iii) $D_t^{\alpha}(\psi \circ \varphi)(t) = t^{1-\alpha}\varphi(t)^{\alpha-1}\varphi'(t)D_t^{\alpha}(\psi(t))|_{t=\varphi(t)}$.

This article is arranged as follows. In Section 2, we employ three different methods to solve the FPRKL equation. In Section 3, we draw three-dimensional graph of Eq (1.1). In Section 4, we give a brief conclusion.

2. Soliton solutions of Eq (1.1)

Making the complex transformation

$$\phi(x,t) = u(\xi)e^{i\tau},\ \xi = \mu(x - \nu\frac{t^{\alpha}}{\alpha}),\ \tau = -kx + \varpi\frac{t^{\alpha}}{\alpha} + \theta. \tag{2.1}$$

Substituting Eq (2.1) into Eq (1.1), separating into real and imaginary parts yields

$$\mu^2(a - 3k\gamma)u'' + (b - k\lambda)u^3 - (\varpi + ak^2 + \delta k - \gamma k^3)u = 0, \tag{2.2}$$

$$\mu^2\gamma u''' + (\nu + 2ak + \delta - 3k^2\gamma)u' + (3\lambda + 2\sigma)u^2u' = 0. \tag{2.3}$$

Integrating Eq (2.3) once, we have

$$3\mu^2\gamma u'' + 3(\nu + 2ak + \delta - 3k^2\gamma)u + (3\lambda + 2\sigma)u^3 = 0. \tag{2.4}$$

Since the function U satisfies both Eq (2.3) and Eq (2.4), the following constraint condition is obtained

$$\frac{a + 3k\gamma}{3 - \gamma} = \frac{\varpi + ak^2 + \delta k - \gamma k^3}{\nu + 2ak + \delta + 3k^2 - \gamma} = -\frac{b - k\lambda}{3\lambda + 2\sigma}. \tag{2.5}$$

So, k and c in Eq (2.5) can be obtained

$$k = -\frac{3b - \gamma + 2a\sigma + 3a\lambda}{6 - \gamma(\lambda + \sigma)},\ \nu = -\frac{\gamma(\varpi + ak^2 + \delta k - \gamma k^3)}{a + 3k - \gamma} - (2ak + \delta + 3 - \gamma k^2). \tag{2.6}$$

2.1. Extended (G'/G)-expansion method

Balancing u^3 and u'' in Eq (2.4), we have $N = 1$. So, the solution form of Eq (2.4) is

$$u(\xi) = a_1(\frac{G'}{G}) + a_0 + a_{-1}(\frac{G'}{G})^{-1}. \tag{2.7}$$

Here $G = G(\xi)$ satisfies the following nonlinear ordinary differential equation

$$GG'' = A(G')^2 + BGG' + CG^2, \tag{2.8}$$

where A, B, C are real parameters, Eq (2.8) satisfies the following equation

$$\frac{G'(\xi)}{G(\xi)} = \begin{cases} \frac{B}{2(1-A)} + \frac{\sqrt{\Delta_1}}{2(1-A)} \frac{C_1 \sinh \frac{\sqrt{\Delta_1}}{2}\xi + C_2 \cosh \frac{\sqrt{\Delta_1}}{2}\xi}{C_1 \cosh \frac{\sqrt{\Delta_1}}{2}\xi + C_2 \sinh \frac{\sqrt{\Delta_1}}{2}\xi}, \text{ when } \Delta_1 = \mathrm{B}^2 - 4(\mathrm{A}-1)\mathrm{C} > 0, \\ A \neq 1, \\ \frac{\beta}{2(1-A)} + \frac{\sqrt{\Delta_2}}{2(1-A)} \frac{-C_1 \sin \frac{\sqrt{\Delta_2}}{2}\xi + C_2 \cos \frac{\sqrt{\Delta_2}}{2}\xi}{C_1 \cos \frac{\sqrt{\Delta_2}}{2}\xi + C_2 \sin \frac{\sqrt{\Delta_2}}{2}\xi}, \text{when } \Delta_2 = 4(\mathrm{A}-1)\mathrm{C} - \mathrm{B}^2 > 0, \\ A \neq 1, \\ \frac{1}{1-A}(\frac{C_1}{C_1\xi + C_2} + \frac{B}{2}), \text{when } 4(\mathrm{A}-1)\mathrm{C} - \mathrm{B}^2 = 0, \mathrm{A} \neq 1. \end{cases} \tag{2.9}$$

Then we can obtain a nonlinear algebraic equations.
$(\frac{G'}{G})^3$: $6\mu^2 - \gamma(A-1)^2 a_1 - (3\lambda + 2\sigma)a_1^3 = 0.$
$(\frac{G'}{G})^2$: $9\mu^2 - \gamma B(A-1)^2 a_1 - 3(3\lambda + 2\sigma)a_1^2 a_0 = 0.$
$(\frac{G'}{G})^1$: $3\mu^2 - \gamma[2C(A-1) + B^2]a_1 - 3(\nu + 2ak + \delta + 3k^2 - \gamma)a_1 - 3(3\lambda + 2\sigma)(a_0^2 a_1 + a_1^2 a_{-1}) = 0.$
$(\frac{G'}{G})^0$: $3\mu^2 - \gamma[BCa_1 + B(A-1)a_{-1}] - 3(\nu + 2ak + \delta + 3k^2 - \gamma)a_0 - (3\lambda + 2\sigma)(6a_0^2 a_1 a_{-1} + a_0^3) = 0.$
$(\frac{G'}{G})^{-1}$: $3\mu^2 - \gamma[2C(A-1) + B^2]a_{-1} - 3(\nu + 2ak + \delta + 3k^2 - \gamma)a_{-1} - 3(3\lambda + 2\sigma)(a_0^2 a_{-1} + a_{-1}^2 a_1) = 0.$
$(\frac{G'}{G})^{-2}$: $9\mu^2 - \gamma B(A-1)a_{-1} - 3(3\lambda + 2\sigma)a_{-1}^2 a_0 = 0.$
$(\frac{G'}{G})^{-3}$: $6\mu^2 - \gamma C^2 a_{-1} - (3\lambda + 2\sigma)a_{-1}^3 = 0.$

Next, we get the following the results:

Case 1.1. $a_1 = \pm\sqrt{\frac{6\mu^2 - \gamma(A-1)^2}{3\lambda + 2\sigma}}$, $a_0 = \pm\sqrt{\frac{\mu^2 - \gamma[B^2 + 2C(A-1)] - (\nu + 2ak + \delta + 3k^2 - \gamma)}{3\lambda + 2\sigma}}$, $a_{-1} = 0$,
$\gamma = -\frac{\mu^2 - \gamma[B^2 + 2C(A-1)] + 2(\nu + 2ak + \delta + 3k^2 - \gamma)}{6\mu^2(A-1)C}$.

Case 1.2. $a_1 = 0$, $a_0 = \pm\sqrt{\frac{\mu^2 - \gamma[B^2 + 2C(A-1)] - (\nu + 2ak + \delta + 3k^2 - \gamma)}{3\lambda + 2\sigma}}$, $a_{-1} = \pm\sqrt{\frac{6\mu^2 - \gamma C^2}{3\lambda + 2\sigma}}$,
$\gamma = -\frac{\mu^2 - \gamma[B^2 + 2C(A-1)] + 2(\nu + 2ak + \delta + 3k^2 - \gamma)}{6\mu^2(A-1)C}$.

Family 1. When $A \neq 1$, $\Delta_1 = B^2 + 4C - 4AC > 0$, we obtain

$$\begin{aligned} \phi_1(x,t) = \{ &\pm \sqrt{\frac{\mu^2 - \gamma[B^2 + 2C(A-1)] - (\nu + 2ak + \delta + 3k^2 - \gamma)}{3\lambda + 2\sigma}} \pm \sqrt{\frac{3\mu^2 - \gamma B^2}{2(3\lambda + 2\sigma)}} \\ &\pm \sqrt{\frac{3\mu^2 - \gamma(B^2 + 4C - 4AC)}{2(3\lambda + 2\sigma)}} (\frac{C_1 \sinh \frac{\sqrt{B^2+4C-4AC}}{2}\xi + C_2 \cosh \frac{\sqrt{B^2+4C-4AC}}{2}\xi}{C_1 \cosh \frac{\sqrt{B^2+4C-4AC}}{2}\xi + C_2 \sinh \frac{\sqrt{B^2+4C-4AC}}{2}\xi})\} e^{i(-kx + \varpi \frac{t^\alpha}{\alpha} + \theta)}. \end{aligned} \tag{2.10}$$

$$\phi_2(x,t) = \{\pm\sqrt{\frac{\mu^2-\gamma[B^2+2C(A-1)]-(\nu+2ak+\delta+3k^2-\gamma)}{3\lambda+2\sigma}} \pm \sqrt{\frac{6\mu^2-\gamma C^2}{3\lambda+2\sigma}} \\ [\frac{\sqrt{B^2+4C-4AC}}{2(1-A)}(\frac{C_1\sinh\frac{\sqrt{B^2+4C-4AC}}{2}\xi + C_2\cosh\frac{\sqrt{B^2+4C-4AC}}{2}\xi}{C_1\cosh\frac{\sqrt{B^2+4C-4AC}}{2}\xi + C_2\sinh\frac{\sqrt{B^2+4C-4AC}}{2}\xi}) + \frac{B}{2(1-A)}]^{-1}\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}, \tag{2.11}$$

where C_1 and C_2 are arbitrary constants.

Especially, if $C_1 \neq 0$, and $C_2 = 0$ in Eq (2.7), we have

$$\phi_{1_1}(x,t) = \{\pm\sqrt{\frac{\mu^2-\gamma[B^2+2C(A-1)]-(\nu+2ak+\delta+3k^2-\gamma)}{3\lambda+2\sigma}} \pm \sqrt{\frac{3\mu^2-\gamma B^2}{3\lambda+2\sigma}} \\ \pm \sqrt{\frac{3\mu^2-\gamma C^2(B^2+4C-4AC)}{2(3\lambda+2\sigma)}}\tanh\frac{\sqrt{B^2+4C-4AC}}{2}\xi\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.12}$$

$$\phi_{1_2}(x,t) = \{\pm\sqrt{\frac{\mu^2-\gamma[B^2+2C(A-1)]-(\nu+2ak+\delta+3k^2-\gamma)}{3\lambda+2\sigma}} \pm \sqrt{\frac{3\mu^2-\gamma B^2}{3\lambda+2\sigma}} \\ \pm \sqrt{\frac{3\mu^2-\gamma C^2(B^2+4C-4AC)}{2(3\lambda+2\sigma)}}\coth\frac{\sqrt{B^2+4C-4AC}}{2}\xi\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.13}$$

$$\phi_{2_1}(x,t) = \{\pm\sqrt{\frac{\mu^2-\gamma[B^2+2C(A-1)]-(\nu+2ak+\delta+3k^2-\gamma)}{3\lambda+2\sigma}} \pm \sqrt{\frac{6\mu^2-\gamma C^2}{3\lambda+2\sigma}} \\ [\frac{\sqrt{B^2+4C-4AC}}{2(1-A)}\tanh\frac{\sqrt{B^2+4C-4AC}}{2}\xi + \frac{B}{2(1-A)}]^{-1}\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.14}$$

$$\phi_{2_2}(x,t) = \{\pm\sqrt{\frac{\mu^2-\gamma[B^2+2C(A-1)]-(\nu+2ak+\delta+3k^2-\gamma)}{3\lambda+2\sigma}} \pm \sqrt{\frac{6\mu^2-\gamma C^2}{3\lambda+2\sigma}} \\ [\frac{\sqrt{B^2+4C-4AC}}{2(1-A)}\coth\frac{\sqrt{B^2+4C-4AC}}{2}\xi + \frac{B}{2(1-A)}]^{-1}\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.15}$$

Family 2. When $A \neq 1$, and $\Delta_2 = B^2 + 4C - 4AC < 0$, we obtain

$$\phi_3(x,t) = \{\pm\sqrt{\frac{\mu^2-\gamma[B^2+2C(A-1)]-(\nu+2ak+\delta+3k^2-\gamma)}{3\lambda+2\sigma}} \pm \sqrt{\frac{3\mu^2-\gamma B^2}{2(3\lambda+2\sigma)}} \\ \pm \sqrt{\frac{3\mu^2-\gamma(4AC-B^2-4C)}{2(3\lambda+2\sigma)}}(\frac{-C_1\sin\frac{\sqrt{4AC-B^2-4C}}{2}\xi + C_2\cos\frac{\sqrt{4AC-B^2-4C}}{2}\xi}{C_1\cos\frac{\sqrt{4AC-B^2-4C}}{2}\xi + C_2\sin\frac{\sqrt{4AC-B^2-4C}}{2}\xi})\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.16}$$

$$\phi_4(x,t) = \{\pm\sqrt{\frac{\mu^2-\gamma[B^2+2C(A-1)]-(\nu+2ak+\delta+3k^2-\gamma)}{3\lambda+2\sigma}} \pm \sqrt{\frac{6\mu^2-\gamma C^2}{3\lambda+2\sigma}} \\ [\frac{\sqrt{B^2+4C-4AC}}{2(1-A)}(\frac{-C_1\sin\frac{\sqrt{4AC-B^2-4C}}{2}\xi + C_2\cos\frac{\sqrt{4AC-B^2-4C}}{2}\xi}{C_1\cos\frac{\sqrt{4AC-B^2-4C}}{2}\xi + C_2\sin\frac{\sqrt{4AC-B^2-4C}}{2}\xi}) + \frac{B}{2(1-A)}]^{-1}\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.17}$$

Family 3. When $A \neq 1$, and $B^2 + 4C - 4AC = 0$, we obtain the rational function solution of Eq (1.1) as

$$\phi_5(t,x,y) = \{\pm\sqrt{\frac{\mu^2 - \gamma[B^2 + 2C(A-1)] - (\nu + 2ak + \delta + 3k^2 - \gamma)}{3\lambda + 2\sigma}} \pm \sqrt{\frac{6\mu^2 - \gamma}{3\lambda + 2\sigma}} \left(\frac{C_1}{C_1\xi + C_2} + \frac{B}{2}\right)\} e^{i(-kx + \varpi\frac{t^\alpha}{\alpha} + \theta)}. \tag{2.18}$$

$$\phi_6(t,x,y) = \{\pm\sqrt{\frac{\mu^2 - \gamma[B^2 + 2C(A-1)] - (\nu + 2ak + \delta + 3k^2 - \gamma)}{3\lambda + 2\sigma}} \pm \sqrt{\frac{6\mu^2 - \gamma C^2(1-A)^2}{3\lambda + 2\sigma}} \left(\frac{C_1}{C_1\xi + C_2} + \frac{B}{2}\right)^{-1}\} e^{i(-kx + \varpi\frac{t^\alpha}{\alpha} + \theta)}. \tag{2.19}$$

2.2. *First integral method*

Now, Eq (2.4) is equivalent to the following of two dimensional system

$$\begin{cases} X_\xi(\xi) = Y(\xi), \\ Y_\xi(\xi) = \frac{\nu + 3ak + \delta + 3k^2 - \gamma}{\mu^2 - \gamma} X(\xi) + \frac{3\lambda + 2\sigma}{3\mu^2 - \gamma} X(\xi)^3, \end{cases} \tag{2.20}$$

where $Q(X, Y) = \sum_{i=0}^{m} a_i(X)Y^i$,

$$P(X(\xi), Y(\xi)) = \sum_{i=0}^{m} a_i(X)Y^i = 0, \tag{2.21}$$

Then, based on the division theorem, there exists a polynomial $g(X) + h(X)Y$ in $C[X, Y]$ such that

$$\frac{dQ}{d\xi} = \frac{\partial Q}{\partial X}\frac{dX}{d\xi} + \frac{\partial Q}{\partial Y}\frac{dY}{d\xi} = [g(X) + h(X)Y(\xi)]\sum_{i=0}^{m} a_i(X)Y^i(\xi). \tag{2.22}$$

Case 2.1. If $m = 1$.

Substituting Eq (2.21) into Eq (2.22) and calculating the coefficients of $Y^i(\xi) = (i = 0, 1, 2)$ both sides of Eq (2.22), we have

$$g(X)a_0(X) = a_1(X)[-\frac{(\nu + 3ak + \delta - 3k^2\gamma)}{\mu^2\gamma}X(\xi) - \frac{(3\lambda + 2\sigma)}{3\mu^2\gamma}X(\xi)^3]. \tag{2.23}$$

$$\frac{da_0(X)}{dX} = g(X)a_1(X) + h(X)a_0(X). \tag{2.24}$$

$$\frac{da_1(X)}{dX} = h(X)a_1(X). \tag{2.25}$$

Since $a_i(X)(i = 0, 1)$ are polynomials, then we obtain when $deg(g(X)) = 1$

$$g(X) = A_1(X) + B_0. \tag{2.26}$$

$$a_0(X) = \frac{1}{2}A_1(X)^2 + B_0(X) + A_0. \tag{2.27}$$

Substituting$a_0(X), a_1(X), g(X)$ into Eq (2.23) and setting all the coefficients of powers X to be zero, then, we obtain

$$A_1 = \pm\sqrt{-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma}}, B_0 = 0, A_0 = \pm\frac{\nu+3ak+\delta-3k^2\gamma}{\sqrt{-\frac{2}{3}(3\lambda+2\sigma)\mu^2\gamma}}. \tag{2.28}$$

Using the conditions Eq (2.28) in Eq (2.21), we obtain

$$X^{'}(\xi) = \pm\frac{\nu+3ak+\delta-3k^2\gamma}{\sqrt{-\frac{2}{3}(3\lambda+2\sigma)\mu^2\gamma}} \mp \sqrt{-\frac{(3\lambda+2\sigma)}{6\mu^2\gamma}}X^2(\xi). \tag{2.29}$$

Combining Eq (2.29) with Eq (2.20), we obtain the exact solution for FPRKL equation which can be written as

Type 1. If $\frac{c+3ak+\delta-3k^2\gamma}{\gamma} < 0, (3\lambda+2\sigma)\gamma < 0$, we get

$$\phi_7(x,t) = \pm\sqrt{\frac{\nu+3ak+\delta-3k^2\gamma}{3\lambda+2\sigma}}\tanh(\sqrt{-\frac{\nu+3ak+\delta-3k^2\gamma}{2\mu^2\gamma}}\xi - \frac{\varepsilon\ln\xi_0}{2})e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}, \text{if } \xi_0 > 0. \tag{2.30}$$

$$\phi_8(x,t) = \pm\sqrt{\frac{\nu+3ak+\delta-3k^2\gamma}{3\lambda+2\sigma}}\coth(\sqrt{-\frac{\nu+3ak+\delta-3k^2\gamma}{2\mu^2\gamma}}\xi - \frac{\varepsilon\ln\xi_0}{2})e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}, \text{if } \xi_0 < 0. \tag{2.31}$$

$$\phi_9(x,t) = \pm\sqrt{\frac{\nu+3ak+\delta-3k^2\gamma}{3\lambda+2\sigma}}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}, \text{if } \xi_0 = 0. \tag{2.32}$$

Type 2. If $\frac{c+3ak+\delta-3k^2\gamma}{\gamma} > 0, (3\lambda+2\sigma)\gamma > 0$, we get

$$\phi_{10}(x,t) = \pm\sqrt{\frac{\nu+3ak+\delta-3k^2\gamma}{3\lambda+2\sigma}}\tan(\sqrt{-\frac{\nu+3ak+\delta-3k^2\gamma}{2\mu^2\gamma}}\xi + \xi_0)e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.33}$$

$$\phi_{11}(x,t) = \pm\sqrt{\frac{\nu+3ak+\delta-3k^2\gamma}{3\lambda+2\sigma}}\cot(\sqrt{-\frac{\nu+3ak+\delta-3k^2\gamma}{2\mu^2\gamma}}\xi + \xi_0)e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.34}$$

Type 3. If $\frac{\nu+3ak+\delta-3k^2\gamma}{\gamma} = 0, (3\lambda+2\sigma)\gamma < 0$, we get

$$\phi_{12}(x,t) = \pm\frac{1}{\sqrt{-\frac{3\lambda+2\sigma}{6\mu^2\gamma}}\xi + \xi_0}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.35}$$

Case 2.2. If $m = 2$.

Comparing the coefficients of $Y^i(\xi) = (0,1,2,3)$ on both sides of Eq (2.22), we obtain

$$g(X)a_0(X) = a_1(X)[-\frac{(\nu+3ak+\delta-3k^2\gamma)}{\mu^2\gamma}X(\xi) - \frac{(3\lambda+2\sigma)}{3\mu^2\gamma}X(\xi)^3]. \tag{2.36}$$

$$\frac{da_0(X)}{dX} + 2a_2(X)[\frac{\nu + 3ak + \delta - 3k^2\gamma}{\mu^2\gamma}X(\xi) + \frac{(3\lambda + 2\sigma)}{3\mu^2\gamma}X(\xi)^3] = g(X)a_1(X) + h(X)a_0(X). \tag{2.37}$$

$$\frac{da_1(X)}{dX} = h(X)a_1(X) + g(X)a_2(X). \tag{2.38}$$

$$\frac{da_2(X)}{dX} = h(X)a_2(X). \tag{2.39}$$

Balancing the degrees of $g(X)$ and $a_1(X)$, we get $deg(g(X)) = 1$, $deg(a_1(X)) = 2$, then

$$g(X) = A_1(X) + B_0. \tag{2.40}$$

$$a_1(X) = \frac{1}{2}A_1(X)^2 + B_0(X) + A_0, A_1 \neq 0, \tag{2.41}$$

where A_1, A_0, B_0 are all constants to be determined.

Now, Eq (2.37) becomes

$$\begin{aligned} a_0(X) = [&\frac{3\lambda + 2\sigma}{6\mu^2\gamma} + \frac{1}{8}A_1^2]X(\xi)^4 + \frac{1}{2}A_1B_0X(\xi)^3 + [\frac{\nu + 3ak + \delta - 3k^2\gamma}{\mu^2\gamma} \\ &+ \frac{1}{2}A_1A_0 + \frac{1}{2}B_0^2]X(\xi)^2 + A_0B_0X(\xi) + d, \end{aligned} \tag{2.42}$$

where d is the constant of integration.

Substituting $a_0(X), g(X), a_1(X)$ into Eq (2.36) and setting all the coefficients of powers X to be zero, we obtain

$$A_1 = \pm 2\sqrt{-\frac{2(3\lambda + 2\sigma)}{3\mu^2\gamma}},\ B_0 = 0, A_0 = \pm\frac{\sqrt{6}(\nu + 3ak + \delta - 3k^2\gamma)}{\sqrt{-(3\lambda + 2\sigma)\mu^2\gamma}}, d = -\frac{(\nu + 3ak + \delta - 3k^2\gamma)^2}{2(3\lambda + 2\sigma)\mu^2\gamma}. \tag{2.43}$$

From Eq (2.43) into Eq (2.21), we obtain

$$X'(\xi) = \pm\frac{\sqrt{6}(\nu + 3ak + \delta - 3k^2\gamma)}{2\sqrt{-(3\lambda + 2\sigma)\mu^2\gamma}} \pm \sqrt{-\frac{2(3\lambda + 2\sigma)}{3\mu^2\gamma}}X^2(\xi). \tag{2.44}$$

This shows that the two cases $m = 1$ and $m = 2$ give the same solutions.

2.3. *Complete discrimination system for the polynomial method*

Multiplying u' on both sides of Eq (2.4), and again integrating it on ξ, we can get

$$(u')^2 = a_4u^4 + a_2u^2 + a_0, \tag{2.45}$$

where $a_4 = -\frac{3\lambda+2\sigma}{6\mu^2\gamma}$, $a_2 = -\frac{\nu+2ak+\delta-3k^2\gamma}{\mu^2\gamma}$, and a_0 is the constant.

Making the transformation $\psi = \pm\sqrt{-(\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{-\frac{1}{3}}}u$, $\xi_1 = -(\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi$, Eq (2.45) becomes

$$(\psi')^2 = \psi(\psi^2 + p_1\psi + p_0), \tag{2.46}$$

where $p_1 = -\frac{4(\nu+2ak+\delta-3k^2\gamma)}{\mu^2\gamma}(-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{-\frac{2}{3}}$, $p_0 = -4a_0(\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{-\frac{1}{3}}$.

Integrating Eq (2.46), we have

$$\pm(\xi_1-\xi_0)=\int\frac{du}{\sqrt{\psi(\psi^2+p_1\psi+p_0)}},\tag{2.47}$$

where ξ_0 is the integration constant.

Suppose that $\Delta = p_1^2 - 4p_0$ and $G(\psi) = \psi^2 + p_1\psi + p_0$, there are four cases for the solutions of Eq (2.4).

Case 3.1. $\Delta = 0$.

When $\psi > 0$, we have

$$\pm(\xi_1-\xi_0)=\int\frac{d\psi}{\sqrt{\psi}(\psi+\frac{p_1}{2})}.\tag{2.48}$$

If $p_1 < 0$, the corresponding solutions are

$$\phi_{16}(x,t)=\pm[\frac{3(\nu+2ak+\delta-3k^2\gamma)}{3\lambda+2\sigma}]^{\frac{1}{2}}\tanh\{[-\frac{18(\nu+2ak+\delta-3k^2\gamma)}{\mu^2\gamma(3\lambda+2\sigma)^2}]^{\frac{1}{6}}[(-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi - \xi_0]\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}.\tag{2.49}$$

$$\phi_{17}(x,t)=\pm[\frac{3(\nu+2ak+\delta-3k^2\gamma)}{3\lambda+2\sigma}]^{\frac{1}{2}}\coth\{[-\frac{18(\nu+2ak+\delta-3k^2\gamma)}{\mu^2\gamma(3\lambda+2\sigma)^2}]^{\frac{1}{6}}[(-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi - \xi_0]\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}.\tag{2.50}$$

If $p_1 > 0$, the corresponding solutions are

$$\phi_{18}(x,t)=\pm[\frac{3(\nu+2ak+\delta-3k^2\gamma)}{3\lambda+2\sigma}]^{\frac{1}{2}}\tan\{[-\frac{18(\nu+2ak+\delta-3k^2\gamma)}{\mu^2\gamma(3\lambda+2\sigma)^2}]^{\frac{1}{6}}[(-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi - \xi_0]\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}.\tag{2.51}$$

If $p_1 = 0$, we get the corresponding solutions

$$\phi_{19}(x,t)=\frac{1}{[-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma}]^{\frac{1}{2}}\xi-[-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma}]^{\frac{1}{6}}\xi_0}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}.\tag{2.52}$$

Case 3.2. $\Delta > 0$ and $p_0 = 0$. As for $\psi > -p_1$, we have

$$\pm(\xi_1-\xi_0)=\int\frac{d\psi}{\psi\sqrt{\psi+p_1}}.\tag{2.53}$$

If $p_1 > 0$, the solutions are given as follows

$$\phi_{20}(x,t)=\pm[\frac{3(\nu+2ak+\delta-3k^2\gamma)}{3\lambda+2\sigma}]^{\frac{1}{2}}\{\tanh^2[(-\frac{18(\nu+2ak+\delta-3k^2\gamma)}{\mu^2\gamma(3\lambda+2\sigma)^2})^{\frac{1}{6}}((-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi - \xi_0)]-1\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}.\tag{2.54}$$

$$\phi_{21}(x,t) = \pm[\frac{3(\nu+2ak+\delta-3k^2\gamma)}{3\lambda+2\sigma}]^{\frac{1}{2}}\{\coth^2[(-\frac{18(\nu+2ak+\delta-3k^2\gamma)}{\mu^2\gamma(3\lambda+2\sigma)^2})^{\frac{1}{6}}((-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi - \xi_0)] - 1\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.55}$$

If $p_1 < 0$, the solutions are given as follows

$$\phi_{22}(x,t) = \pm[\frac{3(\nu+2ak+\delta-3k^2\gamma)}{3\lambda+2\sigma}]^{\frac{1}{2}}\{\tan^2[(-\frac{18(\nu+2ak+\delta-3k^2\gamma)}{\mu^2\gamma(3\lambda+2\sigma)^2})^{\frac{1}{6}}((-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi - \xi_0)] - 1\}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.56}$$

Case 3.3. $\Delta > 0$, $p_0 \neq 0$.

Suppose that $\lambda_1 < \lambda_2 < \lambda_3$, λ_1, λ_2 and λ_3 are two roots of $G(\psi) = 0$. Here we make the transformation $\psi = \lambda_1 + (\lambda_2 - \lambda_1)\sin^2\varphi$, it is clear that

$$\pm(\xi_1 - \xi_0) = \frac{2}{\sqrt{\lambda_3 - \lambda_1}}\int \frac{d\psi}{\sqrt{1 - m_1^2\sin^2\varphi}}, \tag{2.57}$$

where $m_1^2 = \frac{\lambda_2-\lambda_1}{\lambda_3-\lambda_1}$. We get the corresponding solutions

$$\phi_{23}(x,t) = \pm[-\frac{3\mu^2\gamma}{2(3\lambda+2\sigma)}]^{\frac{1}{6}}\{\lambda_1 + (\lambda_2-\lambda_1)\mathrm{sn}^2[\frac{\sqrt{\lambda_3-\lambda_1}}{2}((-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi - \xi_0), m_1]\}^{\frac{1}{2}} e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.58}$$

If $\psi > \lambda_3$, we take the following transformation $\psi = \frac{-\lambda_2\sin^2\varphi+\lambda_3}{\cos^2\varphi}$, the corresponding solutions are

$$\phi_{24}(x,t) = \pm[-\frac{3\mu^2\gamma}{2(3\lambda+2\sigma)}]^{\frac{1}{6}}\{\frac{-\lambda_2\mathrm{sn}^2(\sqrt{\lambda_3-\lambda_1}((-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi-\xi_0)/2, m_1) - \gamma}{\mathrm{cn}^2(\sqrt{\lambda_3-\lambda_1}((-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi-\xi_0)/2, m_1)}\}^{\frac{1}{2}}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.59}$$

Case 3.4. $\Delta < 0$, taking the transformation $\psi = \sqrt{p_0}\tan^2\frac{\varphi}{2}$ it is clear that

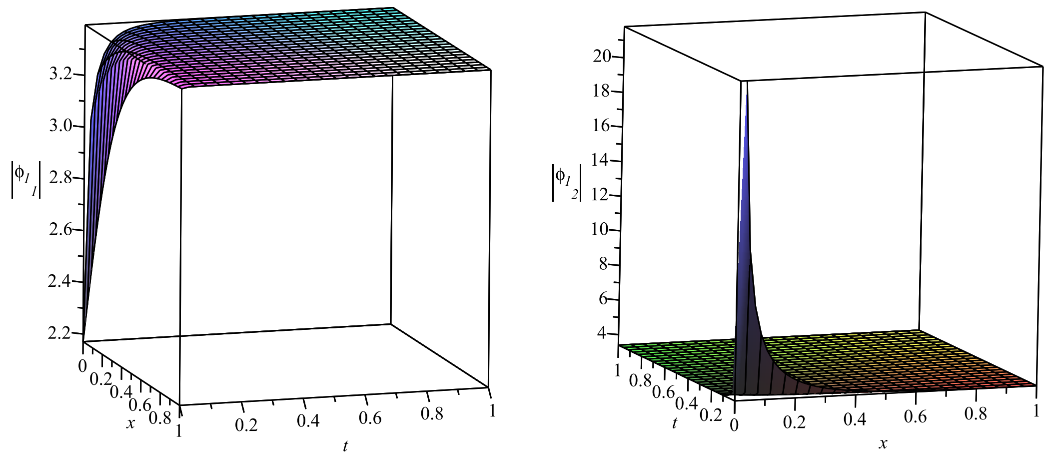
$$\pm 2(\xi_1 - \xi_0) = p_0^{-\frac{1}{4}}\int \frac{d\psi}{\sqrt{1 - m_2^2\sin^2\varphi}}, \tag{2.60}$$

where $m_2^2 = \frac{1}{2}(1 - \frac{p_1}{2\sqrt{p_0}})$. we get the corresponding solutions

$$\phi_{25}(x,t) = \pm(-\frac{6\mu^2\gamma a_0}{3\lambda+2\sigma})^{\frac{1}{4}}\{\frac{2}{1+\mathrm{cn}[(-\frac{192a_0^3\mu^2\gamma}{3\lambda+2\sigma})^{\frac{1}{12}}((-\frac{2(3\lambda+2\sigma)}{3\mu^2\gamma})^{\frac{1}{3}}\xi-\xi_0), m_2]} - 1\}^{\frac{1}{2}}e^{i(-kx+\varpi\frac{t^\alpha}{\alpha}+\theta)}. \tag{2.61}$$

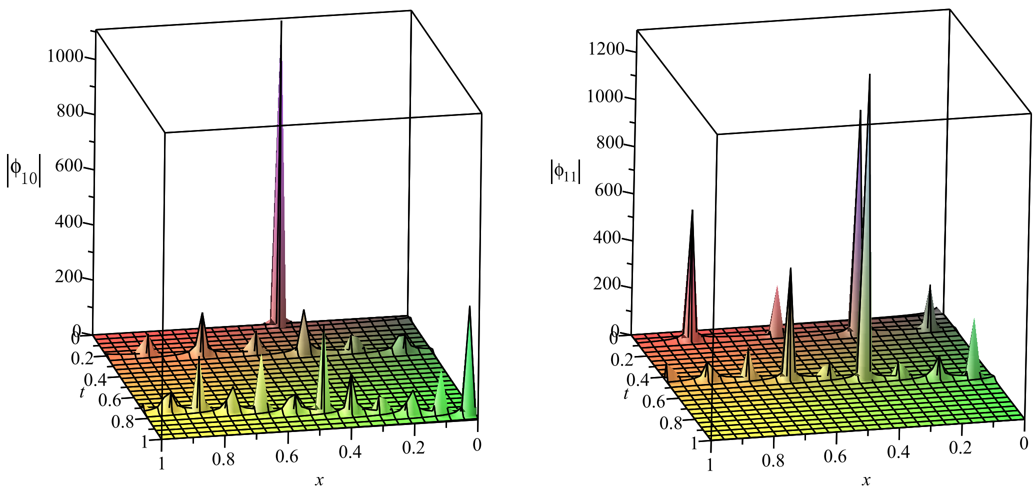
3. Physical explanations

In this section, the numerical simulations of some remarkable solutions for the FPRKL equation are presented. By the (G'/G)-expansion method, we obtained the solution $\phi_{1_1}(x,t)$ and $\phi_{1_2}(x,t)$ shown in Figure 1. The graphical solutions $\phi_{10}(x,t)$ and $\phi_{11}(x,t)$ are shown in Figure 2. Moreover, the graphical solutions $\phi_{16}(x,t)$ and $\phi_{17}(x,t)$ are shown in Figure 3.



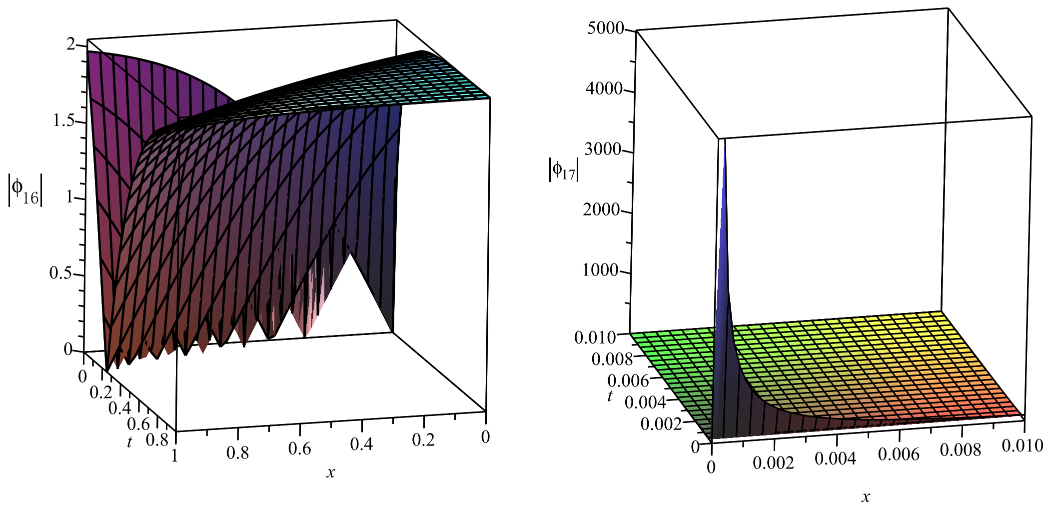
(a) (b)

Figure 1. The hyperbolic function solutions of Eq (1.1), when $\varpi = 2$, $\lambda = 1$, $\delta = 1$, $\sigma = 1$, $k = 1, \mu = 3, a = 1, \gamma = 1, A = 1, B = 2, C = 1, \alpha = \frac{1}{2}$, (a)the hyperbolic function solutions $\phi_{1_1}(x, t)$, (b) the hyperbolic function solutions $\phi_{1_2}(x, t)$.



(a) (b)

Figure 2. The trigonometric function solutions of Eq (1.1), when $\lambda = -1$, $\delta = 1$, $\sigma = 1$, $\mu = 1, a = 1, \gamma = -1, b = -\frac{22}{3}$, (a)the trigonometric function solutions $\phi_{10}(x, t)$, (b)the trigonometric function solutions $\phi_{11}(x, t)$.



(a) (b)

Figure 3. The hyperbolic function solutions of Eq (1.1), when $\lambda = 1$, $\gamma = -1$, $\delta = 1$, $\sigma = 1, a = 1, b = -\frac{14}{3}, \mu = 1, \xi_0 = 0$, (a)the hyperbolic function solutions $\phi_{16}(x, t)$, (b) the hyperbolic function solutions $\phi_{17}(x, t)$.

4. Conclusions

In this article, we have investigated the exact solutions to the FPRKL equation by three different methods. Many exact solutions have been obtained. In the paper, we get all the traveling wave solutions, which have not been seen in other literature. These solutions might be further useful and effective to study more about the various forms of solitary waves in physics. We have noticed that the proposed complete discrimination system for the polynomial method gives much more new and general exact solutions than the other two suggested methods. In future work, we will consider the bifurcation, phase diagrams and exact solutions of the FPRKL equation.

Acknowledgments

This work was supported by Science Research Fund of Education Department of Sichuan Province of China under grant No.18ZB0537 and Scientific Research Funds of Sichuan Vocational and Technical College under grant No.2022YZB009.

Conflict of interest

The author declare no conflict of interest.

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