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Research article

Variance-constrained robust H_∞ state estimation for discrete time-varying uncertain neural networks with uniform quantization

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Abstract: In this paper, we consider the robust H_∞ state estimation (SE) problem for a class of discrete time-varying uncertain neural networks (DTVUNNs) with uniform quantization and time-delay under variance constraints. In order to reflect the actual situation for the dynamic system, the constant time-delay is considered. In addition, the measurement output is first quantized by a uniform quantizer and then transmitted through a communication channel. The main purpose is to design a time-varying finite-horizon state estimator such that, for both the uniform quantization and time-delay, some sufficient criteria are obtained for the estimation error (EE) system to satisfy the error variance boundedness and the H_∞ performance constraint. With the help of stochastic analysis technique, a new H_∞ SE algorithm without resorting the augmentation method is proposed for DTVUNNs with uniform quantization. Finally, a simulation example is given to illustrate the feasibility and validity of the proposed variance-constrained robust H_∞ SE method.

Keywords: discrete time-varying uncertain neural networks; uniform quantization; variance constraint; H_∞ performance requirement
Mathematics Subject Classification: 92B20

1. Introduction

In the past decades, there has been a surge of research on state estimation (SE) problems of neural networks (NNs) due to their successful applications in a variety of areas [1–3]. In general, the state information of interconnected neurons is very important for better understanding of the internal structure and dynamic behavior of NNs [4]. In many applications, notice that the neuron state of NNs is usually not fully available, hence the SE of NNs has recently attracted the increasing research interest by many scholars, see e.g. [5–7]. For example, in [8], based on Lyapunov stability theory, the finite-time resilient H_∞ SE problem has been studied for discrete-time delayed NNs. Subsequently, novel robust H_∞ SE method has been presented in [9] for uncertain discrete-time stochastic NNs with probabilistic measurement delays, and a sufficient condition has been given to ensure the robust mean square exponential stability for dynamic estimation error (EE) system. However, it is worth noting that many reported methods are only suitable for time-invariant situations [10–13], which may lead to the limitations in applications. In addition, the research on uncertainty needs to be further developed [14–16]. Therefore, it is of very great significance to study the SE of discrete time-varying uncertain neural networks (DTVUNNs).

In the research of discrete-time NNs, due to the limited network bandwidth and channel communication effect [17–19], the measurement output is usually quantized before the further transmission. The quantized control techniques can not only fully reduce channel congestion, but also improve the utilization of the transmission capacity of the network [20–22]. Therefore, many results have been proposed to analyze the dynamical behaviours of discrete-time NNs with quantization [23–25]. For instance, by constructing the Lyapunov-Krasovskii functionals (LKFs), the H_∞ SE problem has been investigated in [26] for discrete-time NNs with randomly occurring quantization, and the sufficient conditions have been given to ensure the existence of the desired estimator. Moreover, two different control strategies have been designed in [27], in which a new stochastic exponential synchronization method has been proposed for time-varying delayed NNs with and without logarithmic effect. However, it should be noted that few methods can be available for handling the SE problem for DTVUNNs with quantization impact, not to mention the online application requirements.

As it is known to all, the time delay is often encountered, which is usually one of the major sources of oscillation and instability [28–32]. So far, the discrete and distributed delays in [33] have been addressed, where the new techniques have been proposed to tackle the effects induced by Markovian jumping parameters and mode-dependent mixed time-delays. In addition, for the time-varying delay in the state equation and measurement equation, the delay-dependent conditions have been obtained in terms of linear matrix inequalities to estimate the neuron state such that the EE system is asymptotically stable [34]. As a result, a large number of work have been done regarding the dynamical behaviors of delayed NNs, and the achievement has been obtained on examining the impact from various time-delays such as constant, discrete or distributed delays [35–40]. For instance, the event-triggered SE problem has been proposed in [41] for uncertain NNs, where the main purpose is to design a non-fragile state estimator that the state EE system is globally asymptotically stable in the mean square. Recently, the novel variance-constrained H_∞ SE methods have been proposed in [42] and [43] for time-varying systems with random varying topologies and multiple missing measurements, respectively. Based on the existing results, we make the first attempt to deal with the variance-constrained H_∞ SE problem for DTVUNNs with uniform quantization.

According to the above discussions, the purpose is to design a time-varying finite-horizon state estimator, in which both the EE variance constraint and the prescribed H_∞ performance index are guaranteed. In particular, the parameter uncertainty and uniform quantization are considered in order to reflect the real situation of dynamical systems. By solving a series of recursive linear matrix inequalities (RLMIs), the sufficient conditions have been obtained for the EE system satisfying the upper bound of the covariance and the given H_∞ performance constraint. The main contributions and novelties of the paper can be listed as follows: (1) compared with the exiting results, the H_∞ SE problem is studied for DTVUNNs with uniform quantization and time-delay under the variance constraint for the first time; (2) the usual literatures regarding the SE problem of time-invariant or time-varying NNs adopt the augmented method, but we analyze the EE system with the same order of original NNs, which might reduce the computational burden and complexity; and (3) the newly developed estimation algorithm can ensure both the H_∞ performance criterion and error variance constraint, which can provide wider application domain, moreover, it has time-varying characteristics suitable for online application.

Notation The symbols of this paper are standard. $\mathbb{N}^+$ represents the set of positive integers. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the n-dimensional Euclidean space and all $n\times m$ dimensional real matrices, respectively. Matrix A^T and vector x^T represent the transpose of matrix A and vector x, respectively. I and 0 denote the identity matrix and zero matrix of proper dimensions, respectively. $\mathbb{E}\{x\}$ stands for the expectation operator. The superscript T denotes the matrix transposition and an asterisk $*$ is the term induced by symmetry in symmetric block matrices. Let $X > 0$ denote a positive definite matrix. We use $\mathrm{diag}\{S_1, S_2, \cdots, S_n\}$ to stand for a block-diagonal matrix.

2. Problem formulation and preliminaries

In this paper, we consider the following class of n-neurons DTVUNNs with time-delay and disturbances:

$$\begin{aligned} x_{k+1} &= (A_k + \Delta A_k)x_k + A_{dk}x_{k-d} + B_k f(x_k) + \omega_k, \\ y_k &= E_k x_k + \upsilon_k, \\ z_k &= G_k x_k, \\ x_k &= \varphi_k, \ (k = -d, -d+1, \cdots, 0), \end{aligned} \tag{2.1}$$

where $x_k = \begin{bmatrix} x_{1,k} & x_{2,k} & \cdots & x_{n,k} \end{bmatrix}^T \in \mathbb{R}^n$ is the neural state vector, $y_k = \begin{bmatrix} y_{1,k} & y_{2,k} & \cdots & y_{m,k} \end{bmatrix}^T \in \mathbb{R}^m$ represents the measurement output of the NN, $z_k \in \mathbb{R}^r$ stands for the controlled output, $A_k = \mathrm{diag}\{a_{1,k}, a_{2,k}, \cdots, a_{n,k}\}$ is a self-feedback diagonal matrix, A_{dk}, E_k and G_k are known matrices with compatible dimensions, $f(x_k) = \begin{bmatrix} f_1(x_{1,k}) & f_2(x_{2,k}) & \cdots & f_n(x_{n,k}) \end{bmatrix}^T$ is the nonlinear activation function. $B_k = \begin{bmatrix} b_{ij,k} \end{bmatrix}_{n\times n}$ is the connection weight matrix, d is a positive integer representing the constant delay, ω_k and υ_k are zero mean Gaussian white noises with covariances $Q_k > 0$ and $R_k > 0$, respectively. φ_k is the initial condition. ΔA_k describes the parameter uncertainty satisfying

$$\Delta A_k = H_k F_k N_k, \tag{2.2}$$

where H_k and N_k are known matrices of proper dimensions, the unknown matrix F_k satisfies the following condition

$$F_k^T F_k \le I, \ \forall k \in \mathbb{N}^+. \tag{2.3}$$

The activation function $f(\cdot)$ with $f(0) = 0$ satisfies the following sector-bounded condition

$$[f(s) - U_{1k}s]^T[f(s) - U_{2k}s] \le 0, \ \ \forall s \in \mathbb{R}^n,$$

where U_{1k} and U_{2k} are real matrices of appropriate dimensions and $U_k = U_{1k} - U_{2k}$ is a symmetric positive definite matrix (PDM).

Let the quantized measurement be

$$y_k^q \triangleq \begin{bmatrix} y_{1,k}^q & y_{2,k}^q & \cdots & y_{m,k}^q \end{bmatrix}^T,$$

where $y_{j,k}^q$ is the quantized measurement of the jth sensor, the signal would be quantized by an uniform quantizer $\mathcal{L}(\cdot)$ when it is transmitted via the network. y_k^q can be rewritten as

$$y_k^q = \mathcal{L}(y_k) = \begin{bmatrix} \epsilon\mathcal{R}(\frac{y_{1,k}}{\epsilon}) & \epsilon\mathcal{R}(\frac{y_{2,k}}{\epsilon}) & \cdots & \epsilon\mathcal{R}(\frac{y_{m,k}}{\epsilon}) \end{bmatrix}^T,$$

in which ϵ denotes the quantizing level, $y_{j,k}$ denotes the jth element of the signal y_k, the function $\mathcal{R}(\cdot)$ rounds a number to the nearest integer. Let $\Delta_k = y_k^q - y_k$ be the quantization error. It is not difficult to find $\|\Delta_k\|_\infty \leq \frac{\epsilon}{2}$.

Based on the above descriptions, the following finite-horizon time-varying state estimator is constructed

$$\begin{aligned} \hat{x}_{k+1} &= A_k\hat{x}_k + A_{dk}\hat{x}_{k-d} + B_k f(\hat{x}_k) + K_k(y_k^q - E_k\hat{x}_k), \\ \hat{z}_k &= G_k\hat{x}_k, \end{aligned} \tag{2.4}$$

where $\hat{x}_k \in \mathbb{R}^n$ is the SE of x_k and K_k is the estimator gain matrix (EGM) to be designed.

Let the EE be $e_k = x_k - \hat{x}_k$ and the estimated error of controlled output be $\tilde{z}_k = z_k - \hat{z}_k$. Next, the dynamics of the EE can be acquired in the following way from (2.1) and (2.4)

$$\begin{aligned} e_{k+1} &= (A_k + \Delta A_k - K_kE_k)e_k + \Delta A_k\hat{x}_k + A_{dk}e_{k-d} + B_k\bar{f}(e_k) + \omega_k - K_k\Delta_k - K_k\upsilon_k, \\ \tilde{z}_k &= G_ke_k, \end{aligned} \tag{2.5}$$

where $\bar{f}(e_k) = f(x_k) - f(\hat{x}_k)$ and $e_{k-d} = x_{k-d} - \hat{x}_{k-d}$.

Next, the covariance matrix P_k is described as:

$$P_k = \mathbb{E}\{e_ke_k^T\}. \tag{2.6}$$

The main purpose is to design an H_∞ SE algorithm against the uniform quantization, and the EE system satisfies the following requirements.

1) For a given disturbance attenuation level $\gamma > 0$, let the matrices $\mathcal{U}_\phi > 0$ and $\mathcal{U}_\varphi > 0$ be given. The EE $\tilde{z}_k$ satisfies the following constraint:

$$\mathbb{E}\left\{\sum_{k=0}^{N-1}(\|\tilde{z}_k\|^2 - \gamma^2\|\nu_k\|^2_{\mathcal{U}_\phi})\right\} - \gamma^2\mathbb{E}\{e_0^T\mathcal{U}_\varphi e_0\} < 0, \tag{2.7}$$

where $\|\nu_k\|^2_{\mathcal{U}_\phi} = \nu_k^T\mathcal{U}_\phi\nu_k$.

2) The EE covariance satisfies the following performance criterion:

$$\mathbb{E}\{e_ke_k^T\} \leq \Phi_k, \tag{2.8}$$

where $\Phi_k(0 \leq k < N)$ is a series of admissible estimation precision demand corresponding to the actual situation.

At the end of this section, in order to facilitate the subsequent processing, the following Lemmas are introduced as in [44].

Lemma 1. *[45] Let S, W, T and P be real matrices of proper dimensions, P and W satisfy $P = P^T$ and $WW^T \leq I$. Then $P + SWT + T^T W^T S^T < 0$ holds if and only if there exists $\epsilon > 0$, such that*

$$P + \epsilon S S^T + \epsilon^{-1} T^T T < 0.$$

Lemma 2. *Setting $e_k = x_k - \hat{x}_k$, the sector-bounded condition is equivalent to*

$$\begin{bmatrix} e_k \\ f(e_k + \hat{x}_k) \\ 1 \end{bmatrix}^T \begin{bmatrix} R_{1k} & R_{2k} & -R_{2k} f(\hat{x}_k) \\ R_{2k}^T & I & -f(\hat{x}_k) \\ -f^T(\hat{x}_k) R_{2k}^T & -f^T(\hat{x}_k) & f^T(\hat{x}_k) f(\hat{x}_k) \end{bmatrix} \begin{bmatrix} e_k \\ f(e_k + \hat{x}_k) \\ 1 \end{bmatrix} \leq 0, \tag{2.9}$$

where

$$R_{1k} = \frac{U_{1k}^T U_{2k} + U_{2k}^T U_{1k}}{2}, \quad R_{2k} = -\frac{U_{1k}^T + U_{2k}^T}{2}.$$

Moreover, if the activation function $f(s)$ satisfies the sector-bounded condition, we can get:

$$f(s)^T f(s) \leq \left\{ \frac{2}{2\rho - \rho^2 - 1} \mathrm{tr}(U_{2k}^T U_{2k}) + \frac{2\rho^2}{2\rho - \rho^2 - 1} \mathrm{tr}(U_{1k}^T U_{1k}) \right\} \|s\|, \quad \rho \in (0, 1). \tag{2.10}$$

Remark 1. *The first result of Lemma 2 is derived based on [44]. In order to find the upper bound of nonlinear activation function, we deduce the second result, and the second result of Lemma 2 is mainly applied to find the upper bound of error covariance. It should be pointed out that, in almost all the existing literature, the original system is often transformed into a certain higher order one by augmenting the system state and measurement, and then the estimator is constructed for the augmented system. Different from the existing approaches, in this paper, we design the time-varying estimator directly for the original system (2.1) without resorting the augmentation of system state and measurement, and the order of the estimator can be reduced significantly which would lead to the much less computational burden. In this way, the problem that the EE system satisfies the H_∞ performance constraint and the error covariance has an upper bound can be solved.*

3. Main results

In this section, let us deal with the H_∞ performance analysis problem of the EE system (2.5) and the EE covariance constraint.

3.1. H_∞ performance analysis

Theorem 1. *Consider the DTVUNNs (2.1) with uniform quantization. Let the scalar $\gamma > 0$, the matrices $\mathcal{U}_\phi > 0$ and $\mathcal{U}_\varphi > 0$, and state EGM K_k in (2.4) be given. If $S_0 < \gamma^2 \mathcal{U}_\varphi$, there are a series of PDMs $\{S_k\}_{1 \leq k \leq N+1}$ and T_i satisfying the following RLMI:*

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & R_{2k} f(\hat{x}_k) & 0 & 0 & 0 & 0 \\ * & \Omega_{22} & f(\hat{x}_k) & 0 & 0 & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 \\ * & * & * & * & * & * & \Omega_{77} \end{bmatrix} < 0, \tag{3.1}$$

where

$$
\begin{aligned}
\Omega_{11} &= 7A_k^T S_{k+1} A_k + 8E_k^T K_k^T S_{k+1} K_k E_k + 8\Delta A_k^T S_{k+1} \Delta A_k - S_k + G_k^T G_k + T_k - R_{1k},\\
\Omega_{12} &= A_k^T S_{k+1} B_k - R_{2k},\\
\Omega_{22} &= 7B_k^T S_{k+1} B_k - I,\\
\Omega_{33} &= 8\hat{x}_k^T \Delta A_k^T S_{k+1} \Delta A_k \hat{x}_k + \frac{m\epsilon^2}{4}\lambda + 8f^T(\hat{x}_k) B_k^T S_{k+1} B_k f(\hat{x}_k) - f^T(\hat{x}_k) f(\hat{x}_k),\\
\Omega_{44} &= 8A_{dk}^T S_{k+1} A_{dk} - T_{k-d},\\
\Omega_{55} &= 8K_k^T S_{k+1} K_k - \lambda I,\\
\Omega_{66} &= K_k^T S_{k+1} K_k - \gamma^2 \mathcal{U}_\phi,\\
\Omega_{77} &= S_{k+1} - \gamma^2 \mathcal{U}_\phi,
\end{aligned}
$$

it can be shown that the H_∞ performance defined in (2.7) within finite-horizon holds for all nonzero ν_k.

Proof. Define

$$V_k = e_k^T S_k e_k + \sum_{i=k-d}^{k-1} e_i^T T_i e_i,$$

where $S_k > 0$ and $T_i > 0$ are the matrices to be determined. According to the EE system (2.5), we get the following result

$$
\begin{aligned}
\mathbb{E}\{\Delta V_k\} &= \mathbb{E}\{V_{k+1} - V_k\}\\
&= \mathbb{E}\{e_k^T A_k^T S_{k+1} A_k e_k + e_k^T E_k^T K_k^T S_{k+1} K_k E_k e_k + e_k^T \Delta A_k^T S_{k+1} \Delta A_k e_k + \hat{x}_k^T \Delta A_k^T S_{k+1} \Delta A_k \hat{x}_k\\
&\quad + e_{k-d}^T A_{dk}^T S_{k+1} A_{dk} e_{k-d} + f^T(e_k + \hat{x}_k) B_k^T S_{k+1} B_k f(e_k + \hat{x}_k) + f^T(\hat{x}_k) B_k^T S_{k+1} B_k f(\hat{x}_k)\\
&\quad + \omega_k^T S_{k+1} \omega_k + \Delta_k^T K_k^T S_{k+1} K_k \Delta_k + \upsilon_k^T K_k^T S_{k+1} K_k \upsilon_k + 2e_k^T A_k^T S_{k+1} \Delta A_k e_k\\
&\quad + 2e_k^T A_k^T S_{k+1} \Delta A_k \hat{x}_k + 2e_k^T A_k^T S_{k+1} A_{dk} e_{k-d} + 2e_k^T A_k^T S_{k+1} B_k f(e_k + \hat{x}_k)\\
&\quad - 2e_k^T A_k^T S_{k+1} B_k f(\hat{x}_k) - 2e_k^T A_k^T S_{k+1} K_k \Delta_k - 2e_k^T A_k^T S_{k+1} K_k E_k e_k\\
&\quad + 2e_k^T \Delta A_k^T S_{k+1} \Delta A_k \hat{x}_k + 2e_k^T \Delta A_k^T S_{k+1} A_{dk} e_{k-d} + 2e_k^T \Delta A_k^T S_{k+1} B_k f(e_k + \hat{x}_k)\\
&\quad - 2e_k^T \Delta A_k^T S_{k+1} B_k f(\hat{x}_k) - 2e_k^T \Delta A_k^T S_{k+1} K_k \Delta_k - 2e_k^T \Delta A_k^T S_{k+1} K_k E_k e_k\\
&\quad + 2\hat{x}_k^T \Delta A_k^T S_{k+1} A_{dk} e_{k-d} + 2\hat{x}_k^T \Delta A_k^T S_{k+1} B_k f(e_k + \hat{x}_k) - 2\hat{x}_k^T \Delta A_k^T S_{k+1} B_k f(\hat{x}_k)\\
&\quad - 2\hat{x}_k^T \Delta A_k^T S_{k+1} K_k \Delta_k - 2\hat{x}_k^T \Delta A_k^T S_{k+1} K_k E_k e_k + 2e_{k-d}^T A_{dk}^T S_{k+1} B_k f(e_k + \hat{x}_k)\\
&\quad - 2e_{k-d}^T A_{dk}^T S_{k+1} B_k f(\hat{x}_k) - 2e_{k-d}^T A_{dk}^T S_{k+1} K_k \Delta_k - 2e_{k-d}^T A_{dk}^T S_{k+1} K_k E_k e_k\\
&\quad - 2f^T(e_k + \hat{x}_k) B_k^T S_{k+1} B_k f(\hat{x}_k) - 2f^T(e_k + \hat{x}_k) B_k^T S_{k+1} K_k \Delta_k\\
&\quad - 2f^T(e_k + \hat{x}_k) B_k^T S_{k+1} K_k E_k e_k + 2f^T(\hat{x}_k) B_k^T S_{k+1} K_k \Delta_k + 2f^T(\hat{x}_k) B_k^T S_{k+1} K_k E_k e_k\\
&\quad + 2\Delta_k^T K_k^T S_{k+1} K_k E_k e_k - e_k^T S_k e_k + e_k^T T_k e_k - e_{k-d}^T T_{k-d} e_{k-d}\}. \qquad (3.2)
\end{aligned}
$$

According to the inequality $2x^T P y \le x^T P x + y^T P y$ ($P > 0$), it is not difficult to obtain

$$
\begin{aligned}
\mathbb{E}\{\Delta V_k\} \le \mathbb{E}\{&7e_k^T A_k^T S_{k+1} A_k e_k + 8e_k^T E_k^T K_k^T S_{k+1} K_k E_k e_k + 8e_k^T \Delta A_k^T S_{k+1} \Delta A_k e_k + 8\hat{x}_k^T \Delta A_k^T S_{k+1} \Delta A_k \hat{x}_k\\
&+ 8f^T(\hat{x}_k) B_k^T S_{k+1} B_k f(\hat{x}_k) + 8e_{k-d}^T A_{dk}^T S_{k+1} A_{dk} e_{k-d} + 7f^T(e_k + \hat{x}_k) B_k^T S_{k+1} B_k f(e_k + \hat{x}_k)
\end{aligned}
$$

$$+8\Delta_k^T K_k^T S_{k+1} K_k \Delta_k + \upsilon_k^T K_k^T S_{k+1} K_k \upsilon_k + \omega_k^T S_{k+1} \omega_k + e_k^T T_k e_k - e_{k-d}^T T_{k-d} e_{k-d}$$
$$-e_k^T S_k e_k + 2e_k^T A_k^T S_{k+1} B_k f(e_k + \hat{x}_k)\}.$$

Adding the zero term $\tilde{z}_k^T \tilde{z}_k - \gamma^2 \nu_k^T \mathcal{U}_\phi \nu_k - \tilde{z}_k^T \tilde{z}_k + \gamma^2 \nu_k^T \mathcal{U}_\phi \nu_k$ to $\mathbb{E}\{\Delta V_k\}$ leads to

$$\mathbb{E}\{\Delta V_k\} = \left\{ \begin{bmatrix} \bar{e}_k^T & \nu_k^T \end{bmatrix} \tilde{\Omega} \begin{bmatrix} \bar{e}_k \\ \nu_k \end{bmatrix} - \tilde{z}_k^T \tilde{z}_k + \gamma^2 \nu_k^T \mathcal{U}_\phi \nu_k \right\},$$

where

$$\begin{aligned}
\bar{e}_k &= \begin{bmatrix} e_k^T & f^T(e_k + \hat{x}_k) & 1 & e_{k-d}^T & \Delta_k^T \end{bmatrix}^T, \\
\nu_k &= \begin{bmatrix} \upsilon_k^T & \omega_k^T \end{bmatrix}^T, \\
\tilde{\Omega} &= \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} & 0 & 0 & 0 & 0 & 0 \\ * & \tilde{\Omega}_{22} & 0 & 0 & 0 & 0 & 0 \\ * & * & \tilde{\Omega}_{33} & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 \\ * & * & * & * & \tilde{\Omega}_{55} & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 \\ * & * & * & * & * & * & \Omega_{77} \end{bmatrix}, \\
\tilde{\Omega}_{11} &= 7A_k^T S_{k+1} A_k + 8E_k^T K_k^T S_{k+1} K_k E_k + 8\Delta A_k^T S_{k+1} \Delta A_k - S_k + G_k^T G_k + T_k, \\
\tilde{\Omega}_{12} &= A_k^T S_{k+1} B_k, \\
\tilde{\Omega}_{22} &= 7B_k^T S_{k+1} B_k, \\
\tilde{\Omega}_{33} &= 8\hat{x}_k^T \Delta A_k^T S_{k+1} \Delta A_k \hat{x}_k + 8f^T(\hat{x}_k) B_k^T S_{k+1} B_k f(\hat{x}_k), \\
\tilde{\Omega}_{55} &= 8K_k^T S_{k+1} K_k,
\end{aligned} \tag{3.3}$$

moreover, Ω_{44}, Ω_{66} and Ω_{77} are already given below (3.1).

Based on Lemma 2, we can obtain

$$\begin{aligned}
\mathbb{E}\{\Delta V_k\} \leq \mathbb{E}\Bigg\{ & \begin{bmatrix} \bar{e}_k^T & \nu_k^T \end{bmatrix} \tilde{\Omega} \begin{bmatrix} \bar{e}_k \\ \nu_k \end{bmatrix} - \tilde{z}_k^T \tilde{z}_k + \gamma^2 \nu_k^T \mathcal{U}_\phi \nu_k - \Big[e_k^T R_{1k} e_k + 2e_k^T R_{2k} f(e_k + \hat{x}_k) \\
& -2e_k^T R_{2k} f(\hat{x}_k) + f^T(e_k + \hat{x}_k) f(e_k + \hat{x}_k) + f^T(\hat{x}_k) f(\hat{x}_k) - 2f^T(e_k + \hat{x}_k) f(\hat{x}_k) \\
& +\lambda(\Delta_k^T \Delta_k - \frac{m\epsilon^2}{4}) \Big] \Bigg\} \\
= \mathbb{E}\Bigg\{ & \begin{bmatrix} \bar{e}_k^T & \nu_k^T \end{bmatrix} \Omega \begin{bmatrix} \bar{e}_k \\ \nu_k \end{bmatrix} - \tilde{z}_k^T \tilde{z}_k + \gamma^2 \nu_k^T \mathcal{U}_\phi \nu_k \Bigg\},
\end{aligned} \tag{3.4}$$

where Ω is described in (3.1).

Summarizing both sides of (3.4) regarding k from 0 to $N-1$, we can easily get

$$\begin{aligned}
\sum_{k=0}^{N-1} \mathbb{E}\{\Delta V_k\} &= \mathbb{E}\{e_N^T S_N e_N - e_0^T S_0 e_0\} \\
&\leq \mathbb{E}\left\{ \sum_{k=0}^{N-1} \begin{bmatrix} \bar{e}_k^T & \nu_k^T \end{bmatrix} \Omega \begin{bmatrix} \bar{e}_k \\ \nu_k \end{bmatrix} \right\} - \mathbb{E}\left\{ \sum_{k=0}^{N-1} \left(\tilde{z}_k^T \tilde{z}_k - \gamma^2 \nu_k^T \mathcal{U}_\phi \nu_k \right) \right\}.
\end{aligned} \tag{3.5}$$

Consequently, we have the following inequality

$$\begin{aligned} J &= \mathbb{E}\left\{\sum_{k=0}^{N-1}(\|\tilde{z}_k\|^2 - \gamma^2\|\nu_k\|^2_{\mathcal{U}_\phi})\right\} - \gamma^2\mathbb{E}\{e_0\mathcal{U}_\varphi e_0\} \\ &\leq -\mathbb{E}\{e_N^T S_N e_N - e_0^T S_0 e_0\} - \gamma^2\mathbb{E}\{e_0\mathcal{U}_\varphi e_0\} + \mathbb{E}\left\{\sum_{k=0}^{N-1}\begin{bmatrix}\bar{e}_k^T & \nu_k^T\end{bmatrix}\Omega\begin{bmatrix}\bar{e}_k \\ \nu_k\end{bmatrix}\right\} \\ &= \mathbb{E}\left\{\sum_{k=0}^{N-1}\begin{bmatrix}\bar{e}_k^T & \nu_k^T\end{bmatrix}\Omega\begin{bmatrix}\bar{e}_k \\ \nu_k\end{bmatrix} + e_0^T(S_0 - \gamma^2\mathcal{U}_\varphi)e_0\right\} - \mathbb{E}\{e_N^T S_N e_N\}. \end{aligned} \tag{3.6}$$

According to $\Omega < 0$, $S_N > 0$ and the initial condition $S_0 < \gamma^2\mathcal{U}_\varphi$, one has $J < 0$. The proof is now complete. □

3.2. Analysis of covariance constraint

In this subsection, a sufficient criterion is given to ensure the boundedness of P_k.

Theorem 2. *Consider the DTVUNNs (2.1) with uniform quantization, suppose that the EGM K_k in (2.4) is given. Under the initial condition $Z_0 = P_0$, if there exists a set of PDMs $\{Z_k\}_{1\leq k\leq N+1}$ satisfying the following condition:*

$$Z_{k+1} \geq \Phi(Z_k), \tag{3.7}$$

where

$$\begin{aligned} \Phi(Z_k) &= 7A_kZ_kA_k^T + 7K_kE_kZ_kE_k^TK_k^T + 7\Delta A_kZ_k\Delta A_k^T + 7\Delta A_k\hat{x}_k\hat{x}_k^T\Delta A_k^T + 7A_{dk}Z_{k-d}A_{dk}^T \\ &\quad +7h\mathrm{tr}(Z_k)B_kB_k^T + \frac{7m\epsilon^2}{4}K_kK_k^T + K_kR_kK_k^T + Q_k, \\ h &= \frac{2}{2\rho-\rho^2-1}\mathrm{tr}(U_{2k}^TU_{2k}) + \frac{2\rho^2}{2\rho-\rho^2-1}\mathrm{tr}(U_{1k}^TU_{1k}), \end{aligned} \tag{3.8}$$

then we have $Z_k \geq P_k$ $(\forall k \in 1, 2, \cdots, N+1)$.

Proof. According to (2.6), the state covariance P_k can be calculated as follows:

$$\begin{aligned} P_{k+1} &= \mathbb{E}\{e_{k+1}e_{k+1}^T\} \\ &= \mathbb{E}\{A_ke_ke_k^TA_k^T + K_kE_ke_ke_k^TE_k^TK_k^T + \Delta A_ke_ke_k^T\Delta A_k^T + \Delta A_k\hat{x}_k\hat{x}_k^T\Delta A_k^T + A_{dk}e_{k-d}e_{k-d}^TA_{dk}^T \\ &\quad +B_k\bar{f}(e_k)\bar{f}^T(e_k)B_k^T + \omega_k\omega_k^T + K_k\Delta_k\Delta_k^TK_k^T + K_kv_kv_k^TK_k^T + A_ke_ke_k^T\Delta A_k^T + A_ke_k\hat{x}_k^T\Delta A_k^T \\ &\quad +A_ke_ke_{k-d}^TA_{dk}^T + A_ke_k\bar{f}^T(e_k)B_k^T - A_ke_k\Delta_k^TK_k^T - A_ke_ke_k^TE_k^TK_k^T + \Delta A_ke_ke_k^TA_k^T \\ &\quad +\Delta A_ke_k\hat{x}_k^T\Delta A_k^T - \Delta A_ke_ke_k^TE_k^TK_k^T + \Delta A_ke_ke_{k-d}^TA_{dk}^T + \Delta A_ke_k\bar{f}^T(e_k)B_k^T - \Delta A_ke_k\Delta_k^TK_k^T \\ &\quad +\Delta A_k\hat{x}_ke_k^TA_k^T + \Delta A_k\hat{x}_ke_k^T\Delta A_k^T + \Delta A_k\hat{x}_ke_{k-d}^TA_{dk}^T + \Delta A_k\hat{x}_k\bar{f}^T(e_k)B_k^T - \Delta A_k\hat{x}_k\Delta_k^TK_k^T \\ &\quad -\Delta A_k\hat{x}_ke_k^TE_k^TK_k^T + A_{dk}e_{k-d}e_k^TA_k^T - A_{dk}e_{k-d}e_k^TE_k^TK_k^T + A_{dk}e_{k-d}e_k^T\Delta A_k^T + A_{dk}e_{k-d}\hat{x}_k^T\Delta A_k^T \\ &\quad +A_{dk}e_{k-d}\bar{f}^T(e_k)B_k^T - A_{dk}e_{k-d}\Delta_k^TK_k^T + B_k\bar{f}(e_k)e_k^TA_k^T - B_k\bar{f}(e_k)e_k^TE_k^TK_k^T + B_k\bar{f}(e_k)e_k^T\Delta A_k^T \\ &\quad +B_k\bar{f}(e_k)\hat{x}_k^T\Delta A_k^T + B_k\bar{f}(e_k)e_{k-d}^TA_{dk}^T - B_k\bar{f}(e_k)\Delta_k^TK_k^T - K_k\Delta_ke_k^TA_k^T + K_k\Delta_ke_k^TE_k^TK_k^T \end{aligned}$$

$$
-K_k\Delta_k e_k^T \Delta A_k^T - K_k\Delta_k \hat{x}_k^T \Delta A_k^T - K_k\Delta_k e_{k-d}^T A_{dk}^T - K_k\Delta_k \bar{f}^T(e_k) B_k^T - K_k E_k e_k e_k^T A_k^T
-K_k E_k e_k e_k^T \Delta A_k^T - K_k E_k e_k \hat{x}_k^T \Delta A_k^T - K_k E_k e_k e_{k-d}^T A_{dk}^T - K_k E_k e_k \bar{f}^T(e_k) B_k^T
+K_k E_k e_k \Delta_k^T K_k^T\}.
$$

Next, it is straightforward to obtain that

$$
\begin{aligned}
P_{k+1} \le\ & \mathbb{E}\{7A_k e_k e_k^T A_k^T + 7K_k E_k e_k e_k^T E_k^T K_k^T + 7\Delta A_k e_k e_k^T \Delta A_k^T + 7\Delta A_k \hat{x}_k \hat{x}_k^T \Delta A_k^T + 7A_{dk} e_{k-d} e_{k-d}^T A_{dk}^T \\
& +7B_k \bar{f}(e_k) \bar{f}^T(e_k) B_k^T + \frac{7m\epsilon^2}{4} K_k K_k^T + K_k R_k K_k^T + Q_k\}.
\end{aligned}
$$

From (2.10), we can easily get

$$
\mathbb{E}\{\bar{f}(e_k)\bar{f}^T(e_k)\} \le \mathbb{E}\{\mathrm{tr}(\bar{f}(e_k)\bar{f}^T(e_k))\}I = \mathbb{E}\{\bar{f}^T(e_k)\bar{f}(e_k)\}I \le h\mathbb{E}\{e_k^T e_k\}I,
$$

where h is defined in (3.8). Next, we have

$$
\begin{aligned}
P_{k+1} \le\ & \mathbb{E}\{7A_k e_k e_k^T A_k^T + 7K_k E_k e_k e_k^T E_k^T K_k^T + 7\Delta A_k e_k e_k^T \Delta A_k^T + 7\Delta A_k \hat{x}_k \hat{x}_k^T \Delta A_k^T + 7A_{dk} e_{k-d} e_{k-d}^T A_{dk}^T \\
& +7hB_k e_k^T e_k B_k^T + \frac{7m\epsilon^2}{4} K_k K_k^T + K_k R_k K_k^T + Q_k\}. \qquad (3.9)
\end{aligned}
$$

According to the characteristics of the trace, we can get

$$
\mathbb{E}\{e_k^T e_k\} = \mathbb{E}\{\mathrm{tr}(e_k e_k^T)\} = \mathrm{tr}(P_k). \qquad (3.10)
$$

Combining (3.9) with (3.10) results in

$$
\begin{aligned}
P_{k+1} \le\ & 7A_k P_k A_k^T + 7K_k E_k P_k E_k^T K_k^T + 7\Delta A_k P_k \Delta A_k^T + 7\Delta A_k \hat{x}_k \hat{x}_k^T \Delta A_k^T + 7A_{dk} P_{k-d} A_{dk}^T \\
& +7h\mathrm{tr}(P_k) B_k B_k^T + \frac{7m\epsilon^2}{4} K_k K_k^T + K_k R_k K_k^T + Q_k \\
=\ & \Phi(P_k).
\end{aligned}
$$

Noting $Z_0 \ge P_0$ and letting $Z_k \ge P_k$, we can obtain the following inequality

$$
\Phi(Z_k) \ge \Phi(P_k) \ge P_{k+1}. \qquad (3.11)
$$

After that, from (3.7) and (3.11), we arrive at

$$
Z_{k+1} \ge \Phi(Z_k) \ge \Phi(P_k) \ge P_{k+1}. \qquad (3.12)
$$

Therefore, the proof of this theorem is complete. □

On the basis of the above theorems, the prescribed H_∞ performance index and the covariance constraint of the EE can be ensured by solving certain matrix inequalities.

Theorem 3. *Consider the DTVUNNs (2.1) and assume that the EGM K_k is given. For give a scalar $\gamma > 0$, matrices $\mathcal{U}_\varphi > 0$ and $\mathcal{U}_\phi > 0$, under the initial conditions $S_0 \le \gamma^2\mathcal{U}_\varphi$ and $Z_0 = P_0$, if there exist* *two sets of PDMs $\{S_k\}_{1\le k\le N+1}$ and $\{Z_k\}_{1\le k\le N+1}$ satisfying the following matrix inequalities:*

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & 0 & \Theta_{14} & \Theta_{15} & 0 & 0 \\ * & \Theta_{22} & 0 & \Theta_{24} & 0 & \Theta_{26} & 0 \\ * & * & \Theta_{33} & 0 & 0 & 0 & \Theta_{37} \\ * & * & * & \Theta_{44} & 0 & 0 & \Theta_{47} \\ * & * & * & * & \Theta_{55} & 0 & 0 \\ * & * & * & * & * & \Theta_{66} & 0 \\ * & * & * & * & * & * & \Theta_{77} \end{bmatrix} < 0, \tag{3.13}$$

$$\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \Psi_{14} \\ * & \Psi_{22} & 0 & 0 \\ * & * & \Psi_{33} & 0 \\ * & * & * & \Psi_{44} \end{bmatrix} < 0, \tag{3.14}$$

where

$$\begin{aligned}
\Theta_{11} &= G_k^T G_k - S_k - R_{1k} + T_k, \\
\Theta_{22} &= \begin{bmatrix} -I & f(\hat{x}_k) \\ * & \Pi_{33} \end{bmatrix}, \\
\Theta_{33} &= \mathrm{diag}\{-T_{k-d}, -\lambda I\}, \\
\Theta_{44} &= \mathrm{diag}\{-\gamma^2 \mathcal{U}_\phi, -\gamma^2 \mathcal{U}_\phi, -S_{k+1}^{-1}\}, \\
\Theta_{55} &= \mathrm{diag}\{-S_{k+1}^{-1}, -S_{k+1}^{-1}, -S_{k+1}^{-1}\}, \\
\Theta_{66} &= \mathrm{diag}\{-S_{k+1}^{-1}, -S_{k+1}^{-1}, -S_{k+1}^{-1}\}, \\
\Theta_{77} &= \mathrm{diag}\{-S_{k+1}^{-1}, -S_{k+1}^{-1}, -S_{k+1}^{-1}, -S_{k+1}^{-1}\}, \\
\Theta_{12} &= \begin{bmatrix} -R_{2k} & R_{2k} f(\hat{x}_k) \end{bmatrix}, \\
\Theta_{14} &= \begin{bmatrix} 0 & 0 & A_k^T \end{bmatrix}, \\
\Theta_{15} &= \begin{bmatrix} \sqrt{6} A_k^T & 2\sqrt{2}\Delta A_k^T & 2\sqrt{2} E_k^T K_k^T \end{bmatrix}, \\
\Theta_{24} &= \begin{bmatrix} 0 & 0 & B_k^T \\ 0 & 0 & 0 \end{bmatrix}, \\
\Theta_{26} &= \begin{bmatrix} \sqrt{6} B_k^T & 0 & 0 \\ 0 & 2\sqrt{2}\hat{x}_k^T \Delta A_k^T & 2\sqrt{2} f^T(\hat{x}_k) B_k^T \end{bmatrix}, \\
\Theta_{37} &= \begin{bmatrix} 2\sqrt{2} A_{dk}^T & 0 & 0 & 0 \\ 0 & 2\sqrt{2} K_k^T & 0 & 0 \end{bmatrix}, \\
\Theta_{47} &= \begin{bmatrix} 0 & 0 & K_k^T & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\Pi_{33} &= -f^T(\hat{x}_k) f(\hat{x}_k) + \frac{m\epsilon^2}{4}\lambda, \\
\Psi_{11} &= -Z_{k+1} + 7h\mathrm{tr}(Z_k) B_k B_k^T + Q_k, \\
\Psi_{12} &= \begin{bmatrix} \sqrt{7} A_k Z_k & \sqrt{7}\Delta A_k Z_k \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\Psi_{13} &= \begin{bmatrix} \sqrt{7}K_kE_kZ_k & \sqrt{7}\Delta A_k\hat{x}_k \end{bmatrix}, \\
\Psi_{14} &= \begin{bmatrix} \sqrt{7}A_{dk}Z_{k-d} & \frac{\epsilon}{2}\sqrt{7m}K_k & K_kR_k \end{bmatrix}, \\
\Psi_{22} &= \text{diag}\{-Z_k, -Z_k\}, \\
\Psi_{33} &= \text{diag}\{-Z_k, -I\}, \\
\Psi_{44} &= \text{diag}\{-Z_{k-d}, -I, -R_k\},
\end{aligned}$$

then, both the EE covariance constraint and the H_∞ performance index can be satisfied simultaneously.

Proof. According to the initial conditions, both the H_∞ performance index and EE covariance constraints are analyzed, the inequality (3.13) implies (3.1) and (3.14) implies (3.8). Therefore, the H_∞ performance index and covariance constraint are guaranteed meanwhile. □

4. Design of the estimation algorithm

In this section, the sufficient condition of the design method is proposed regarding the discrete finite-horizon time-varying state estimator.

Theorem 4. *Let the attenuation level $\gamma > 0$, matrices $\mathcal{U}_\phi > 0$, $\mathcal{U}_\varphi > 0$ and a set of pre-defined variance upper bound matrices $\{\Phi_k\}_{0\le k\le N}$ be given. Under the initial conditions*

$$\begin{cases} S_0 - \gamma^2\mathcal{U}_\varphi \le 0, \\ \mathbb{E}\{e_0e_0^T\} = Z_0 \le \Phi_0, \end{cases} \tag{4.1}$$

if there exist the positive scalars $\{\epsilon_{1,k}, \epsilon_{2,k}, \epsilon_{3,k}, \epsilon_{4,k}\}_{1\le k\le N+1}$, PDMs $\{S_k\}_{1\le k\le N+1}$, $\{Z_k\}_{1\le k\le N+1}$ and matrices $\{K_k\}_{0\le k\le N}$ with appropriate dimensions satisfying the following RLMIs:

$$\begin{bmatrix}
\Upsilon_{11} & \Theta_{12} & 0 & \Theta_{14} & \Upsilon_{15} & 0 & 0 & 0 \\
* & \Theta_{22} & 0 & \Theta_{24} & 0 & \Upsilon_{26} & 0 & \Upsilon_{28} \\
* & * & \Theta_{33} & 0 & 0 & 0 & \Theta_{37} & 0 \\
* & * & * & \Upsilon_{44} & 0 & 0 & \Theta_{47} & 0 \\
* & * & * & * & \Upsilon_{55} & 0 & 0 & \Upsilon_{58} \\
* & * & * & * & * & \Upsilon_{66} & 0 & \Upsilon_{68} \\
* & * & * & * & * & * & \Upsilon_{77} & 0 \\
* & * & * & * & * & * & * & \Upsilon_{88}
\end{bmatrix} < 0, \tag{4.2}$$

$$\begin{bmatrix}
\Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Psi_{14} & 0 & 0 \\
* & \Psi_{22} & 0 & 0 & \mathcal{W}_{k,1}^T & 0 \\
* & * & \Psi_{33} & 0 & 0 & \mathcal{W}_{k,2}^T \\
* & * & * & \Psi_{44} & 0 & 0 \\
* & * & * & * & -\epsilon_{3,k}I & 0 \\
* & * & * & * & * & -\epsilon_{4,k}I
\end{bmatrix} < 0, \tag{4.3}$$

$$Z_{k+1} - \Phi_{k+1} \le 0, \tag{4.4}$$

with the following updating rule:

$$\bar{S}_{k+1} = S_{k+1}^{-1},$$

where

$$\begin{aligned}
\Upsilon_{11} &= G_k^T G_k - S_k - R_{1k} + T_k + \epsilon_{1,k} N_k^T N_k, \\
\Upsilon_{15} &= \begin{bmatrix} \sqrt{6}A_k^T & 0 & 2\sqrt{2}E_k^T K_k^T \end{bmatrix}, \\
\Upsilon_{26} &= \begin{bmatrix} \sqrt{6}B_k^T & 0 & 0 \\ 0 & 0 & 2\sqrt{2}f^T(\hat{x}_k)B_k^T \end{bmatrix}, \\
\Upsilon_{28} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \hat{x}_k^T N_k^T \end{bmatrix}, \\
\Upsilon_{58} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 2\sqrt{2}H_k & 0 & 0 \end{bmatrix}, \\
\Upsilon_{68} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\sqrt{2}H_k & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\Upsilon_{44} &= \mathrm{diag}\{-\gamma^2\mathcal{U}_\phi, -\gamma^2\mathcal{U}_\phi, -\bar{S}_{k+1}\}, \\
\Upsilon_{55} &= \mathrm{diag}\{-\bar{S}_{k+1}, -\bar{S}_{k+1}, -\bar{S}_{k+1}\}, \\
\Upsilon_{66} &= \mathrm{diag}\{-\bar{S}_{k+1}, -\bar{S}_{k+1}, -\bar{S}_{k+1}\}, \\
\Upsilon_{77} &= \mathrm{diag}\{-\bar{S}_{k+1}, -\bar{S}_{k+1}, -\bar{S}_{k+1}, -\bar{S}_{k+1}\}, \\
\Upsilon_{88} &= \mathrm{diag}\{-\epsilon_{1,k}I, -\epsilon_{2,k}I, -\epsilon_{2,k}I\}, \\
\Lambda_{11} &= -Z_{k+1} + 7h\mathrm{tr}(Z_k)B_k B_k^T + Q_k + \epsilon_{3,k}H_k H_k^T + +\epsilon_{4,k}H_k H_k^T, \\
\Lambda_{12} &= \begin{bmatrix} \sqrt{7}A_k Z_k & 0 \end{bmatrix}, \\
\Lambda_{13} &= \begin{bmatrix} \sqrt{7}K_k E_k Z_k & 0 \end{bmatrix}, \\
\mathcal{H}_{k,1} &= \begin{bmatrix} 0 & 0 & 2\sqrt{2}H_k^T \end{bmatrix}, \\
\mathcal{H}_{k,2} &= \begin{bmatrix} 0 & 2\sqrt{2}H_k^T & 0 \end{bmatrix}, \\
\mathcal{N}_{k,2}^T &= \begin{bmatrix} 0 & N_k\hat{x}_k \end{bmatrix}, \\
\mathcal{W}_{k,1} &= \begin{bmatrix} 0 & \sqrt{7}N_k Z_k \end{bmatrix}, \\
\mathcal{W}_{k,2} &= \begin{bmatrix} 0 & \sqrt{7}N_k\hat{x}_k \end{bmatrix},
\end{aligned}$$

and other items are represented in Theorems 1–3, it is concluded that both the H_∞ performance index and EE covariance constraints are ensured.

Proof. For convenient to tackle the parameter uncertainty, we rewrite (3.13) as below

$$\begin{bmatrix}
\Theta_{11} & \Theta_{12} & 0 & \Theta_{14} & \Theta_{15}^0 & 0 & 0 \\
* & \Theta_{22} & 0 & \Theta_{24} & 0 & \Theta_{26}^0 & 0 \\
* & * & \Theta_{33} & 0 & 0 & 0 & \Theta_{37} \\
* & * & * & \Theta_{44} & 0 & 0 & \Theta_{47} \\
* & * & * & * & \Theta_{55} & 0 & 0 \\
* & * & * & * & * & \Theta_{66} & 0 \\
* & * & * & * & * & * & \Theta_{77}
\end{bmatrix} + \bar{N}_{k,1}F_k\bar{H}_{k,1} + (\bar{N}_{k,1}F_k\bar{H}_{k,1})^T + \bar{N}_{k,2}F_k\bar{H}_{k,2}$$

$$+(\bar{N}_{k,2}F_k\bar{H}_{k,2})^T < 0,$$

where

$$\begin{aligned}
\Theta_{15}^0 &= \begin{bmatrix} \sqrt{6}A_k^T & 0 & 2\sqrt{2}E_k^T K_k^T \end{bmatrix}, \\
\Theta_{26}^0 &= \begin{bmatrix} \sqrt{6}B_k^T & 0 & 0 \\ 0 & 0 & 2\sqrt{2}f^T(\hat{x}_k)B_k^T \end{bmatrix}, \\
\bar{N}_{k,1}^T &= \begin{bmatrix} N_k^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ \bar{H}_{k,1} = \begin{bmatrix} 0 & 0 & 0 & 0 & \mathcal{H}_{k,1} & 0 & 0 \end{bmatrix}, \\
\bar{N}_{k,2}^T &= \begin{bmatrix} 0 & \mathcal{N}_{k,2}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ \bar{H}_{k,2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \mathcal{H}_{k,2} & 0 \end{bmatrix}.
\end{aligned}$$

It follows from Lemma 1 that

$$\begin{bmatrix}
\Theta_{11} & \Theta_{12} & 0 & \Theta_{14} & \Theta_{15}^0 & 0 & 0 \\
* & \Theta_{22} & 0 & \Theta_{24} & 0 & \Theta_{26}^0 & 0 \\
* & * & \Theta_{33} & 0 & 0 & 0 & \Theta_{37} \\
* & * & * & \Theta_{44} & 0 & 0 & \Theta_{47} \\
* & * & * & * & \Theta_{55} & 0 & 0 \\
* & * & * & * & * & \Theta_{66} & 0 \\
* & * & * & * & * & * & \Theta_{77}
\end{bmatrix} + \epsilon_{1,k}\bar{N}_{k,1}\bar{N}_{k,1}^T + \epsilon_{1,k}^{-1}\bar{H}_{k,1}^T\bar{H}_{k,1} + \epsilon_{2,k}\bar{N}_{k,2}\bar{N}_{k,2}^T$$
$$+\epsilon_{2,k}^{-1}\bar{H}_{k,2}^T\bar{H}_{k,2} < 0.$$

Similarly, (3.14) can be rewritten as

$$\begin{bmatrix}
\Psi_{11} & \Psi_{12}^0 & \Psi_{13}^0 & \Psi_{14} \\
* & \Psi_{22} & 0 & 0 \\
* & * & \Psi_{33} & 0 \\
* & * & * & \Psi_{44}
\end{bmatrix} + \tilde{N}_{k,1}F_k\tilde{H}_{k,1} + (\tilde{N}_{k,1}F_k\tilde{H}_{k,1})^T + \tilde{N}_{k,1}F_k\tilde{H}_{k,2} + (\tilde{N}_{k,1}F_k\tilde{H}_{k,2})^T < 0,$$

where

$$\begin{aligned}
\Psi_{12}^0 &= \begin{bmatrix} \sqrt{7}A_{dk}Z_k & 0 \end{bmatrix}, \\
\Psi_{13}^0 &= \begin{bmatrix} \sqrt{7}K_kE_kZ_k & 0 \end{bmatrix}, \\
\tilde{N}_{k,1}^T &= \begin{bmatrix} H_k^T & 0 & 0 & 0 \end{bmatrix}, \ \tilde{H}_{k,1} = \begin{bmatrix} 0 & \mathcal{W}_{k,1} & 0 & 0 \end{bmatrix}, \\
\tilde{H}_{k,2} &= \begin{bmatrix} 0 & 0 & \mathcal{W}_{k,2} & 0 \end{bmatrix}.
\end{aligned}$$

Then, it follows from Lemma 1 that

$$\begin{bmatrix}
\Psi_{11} & \Psi_{12}^0 & \Psi_{13}^0 & \Psi_{14} \\
* & \Psi_{22} & 0 & 0 \\
* & * & \Psi_{33} & 0 \\
* & * & * & \Psi_{44}
\end{bmatrix} + \epsilon_{3,k}\tilde{N}_{k,1}\tilde{N}_{k,1}^T + \epsilon_{3,k}^{-1}\tilde{H}_{k,1}^T\tilde{H}_{k,1} + \epsilon_{4,k}\tilde{N}_{k,2}\tilde{N}_{k,2}^T + \epsilon_{4,k}^{-1}\tilde{H}_{k,2}^T\tilde{H}_{k,2} < 0.$$

So we conclude that (4.2) implies (3.13). Similarly, inequality (4.3) implies (3.14). Therefore, the EE system (2.5) satisfies the EE covariance constraint and H_∞ performance index. □

Remark 2. *As a matter of fact, there has been increasing research interest on handling the quantization impacts because the digital computers have been widely used in control systems. Accordingly, many methods have been proposed to tackle the quantization effects, such as logarithmic quantization effects and uniform quantization effects. In this paper, we make one the first attempt to deal with the SE problem of DTVUNNs with the uniform quantization, where two combined performance indices have been introduced to meet the actual requirements. For instance, different from the SE approach in [46], the variance-constrained H_∞ SE method proposed in this paper has the advantage to reveal the influences from time-delay and uniform quantization on the performance of the estimation algorithm. In particular, the important features of the newly developed method are the time-varying characteristic and without resorting the state augmentation method.*

Remark 3. *So far, we have developed a new finite-horizon state estimation algorithm, in which both the EE variance constraint and the prescribed H_∞ performance index are guaranteed. Accordingly, in Theorem 4, the EGM of the variance-constrained H_∞ estimator subject to uniform quantization is obtained by solving RLMIs (4.2)–(4.4), and this recursive process is particularly useful for real-time implementation due to its time-varying characteristic and low order feature. One the one hand, for the techniques used, we propose the RLMIs for the purpose of computational convenience. In addition, sufficient conditions are established to guarantee the prescribed H_∞ performance requirement and error variance constraints. On the other hand, we point out that our main results can be extended to handle related problem with the communication resource constraints, and the results will appear in the near future.*

The H_∞ state estimator design algorithm of DTVUNNs can be summarized as follows.
Step 1. Give the H_∞ performance index γ, the initial states of x_k and its estimate $\hat{x}_k$, the PDMs $\mathcal{U}_\phi$ and $\mathcal{U}_\varphi$, select the matrices $\{Z_0, S_0\}$ satisfying the initial conditions (4.1).
Step 2. By solving the RLMIs (4.2)–(4.4) to obtain the matrices $\{Z_{k+1}, S_{k+1}\}$, and the EGM $\{K_k\}$ at the sampling instant k.
Step 3. Setting $k = k + 1$, if $k < N$, then return to Step 2, else go to Step 4.
Step 4. Stop.

5. An illustrative example

In this section, the validity of the theoretical results is verified by numerical simulation.
For DTVUNNs (2.1), we consider the following system parameters:

$$A_k = \begin{bmatrix} -0.068\sin(3.13k) & 0 \\ 0 & 0.020 \end{bmatrix}, \quad B_k = \begin{bmatrix} 0.13\sin(1.25k) & 0.22 \\ 0.31 & 0.11 \end{bmatrix}, \quad E_k = \begin{bmatrix} -0.2\sin(1.3k) & -0.28 \end{bmatrix},$$

$$G_k = \begin{bmatrix} 0.42 & -0.16 \end{bmatrix}, \quad A_{dk} = \begin{bmatrix} 0.1\sin(0.3k) & -0.17 \\ 0.35 & -0.23 \end{bmatrix}, \quad H_k = \begin{bmatrix} -0.24 & -0.31 \\ -0.31 & 0.12 \end{bmatrix},$$

$$N_k = \begin{bmatrix} -0.23 & 0.15 \end{bmatrix}, \quad U_{1k} = \begin{bmatrix} 0.02 & 0.34 \\ 0.34 & 0.51 \end{bmatrix}, \quad U_{2k} = \begin{bmatrix} 0.21 & 0.05 \\ 0.05 & 0.43 \end{bmatrix}, \quad \rho = 0.13, \quad d = 3.$$

Moreover, the activation function is taken as follows:

$$f(x_k) = \begin{bmatrix} 0.1\tanh(x_{1,k}) + 0.01x_{1,k} \\ 0.02x_{2,k} + 0.1\tanh(0.03x_{2,k}) \end{bmatrix},$$

where $x_k = \begin{bmatrix} x_{1,k} & x_{2,k} \end{bmatrix}^T$ is the neuron state vector of the NNs. Let the disturbance attenuation level be $\gamma = 0.32$ and $N = 90$, the connection weight matrices be $\mathcal{U}_\phi = 0.1$, upper bound matrices be $\{\Phi_k\}_{0\le k\le N} = \text{diag}\{0.35, 0.35\}$, and covariances be $Q_k = \text{diag}\{0.04, 0.06\}$ and $R_k = 0.01$. After that the matrix inequalities (4.2)–(4.4) can be resolved.

Set the initial state $x_{k,0} = \begin{bmatrix} -0.56 & -0.15 \end{bmatrix}^T$, $\hat{x}_{k,0} = \begin{bmatrix} -2.35 & -0.11 \end{bmatrix}^T$ and $x_{k-d} = 0$, $(k = 1, 2, 3)$. According to the above proposed SE method, the simulation results are obtained in Figures 1–4. Figure 1 depicts the output $z_{1,k}$ and its estimated value $\hat{z}_{1,k}$. Figure 2 represents the output $z_{2,k}$ and its estimation $\hat{z}_{2,k}$. Figure 3 shows the trajectory of output EE $\tilde{z}_k$. Figure 4 shows the trajectory of the actual covariance and the upper bound of the covariance. The simulation results show that the EE is relatively small and verify the feasibility as well as practicability of the presented SE method.

By solving the RLMIs and based on the Matlab toolbox, the EGM K_k can be listed in Tablle 1 as follows:

Table 1. Estimator Gain Matrix.

k	K_k
1	$K_1 = \begin{bmatrix} 1.0151 & 0.9112 \end{bmatrix}^T$
2	$K_2 = \begin{bmatrix} 1.0668 & 1.0230 \end{bmatrix}^T$
3	$K_3 = \begin{bmatrix} 0.9848 & 1.1010 \end{bmatrix}^T$
$\vdots$	$\vdots$

Remark 4. *The robust H_∞ SE problem has not been thoroughly handled for DTVUNNs with uniform quantization, especially for the SE problem under different performance requirements of time-varying uncertain NNs. Hence, we make great effort to discuss the robust H_∞ SE problem for time-varying uncertain NNs with uniform quantization under variance constraint. Moreover, we propose a new estimation method with strong advantages in time-varying characteristic and low order feature, which might reduce the computational burden. For the implementation issue, we firstly give the H_∞ performance index γ, the initial states of x_k and its estimate $\hat{x}_k$, the PDMs $\mathcal{U}_\phi$ and $\mathcal{U}_\varphi$, and select the matrices $\{Z_0, S_0\}$ satisfying the initial conditions (4.1). Secondly, by solving the RLMIs (4.2)–(4.4), we obtain the matrices $\{Z_{k+1}, S_{k+1}\}$, and the EGM $\{K_k\}$ at the sampling instant k. Thirdly, setting $k = k + 1$, if $k < N$, then go back, otherwise the estimation problem is solvable.*

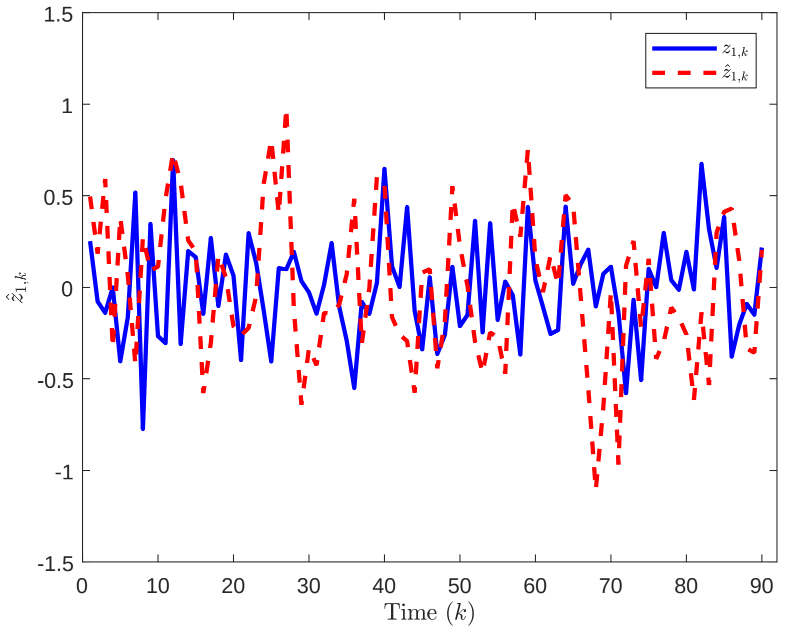

Figure 1. The controlled output $z_{1,k}$ and its estimation.

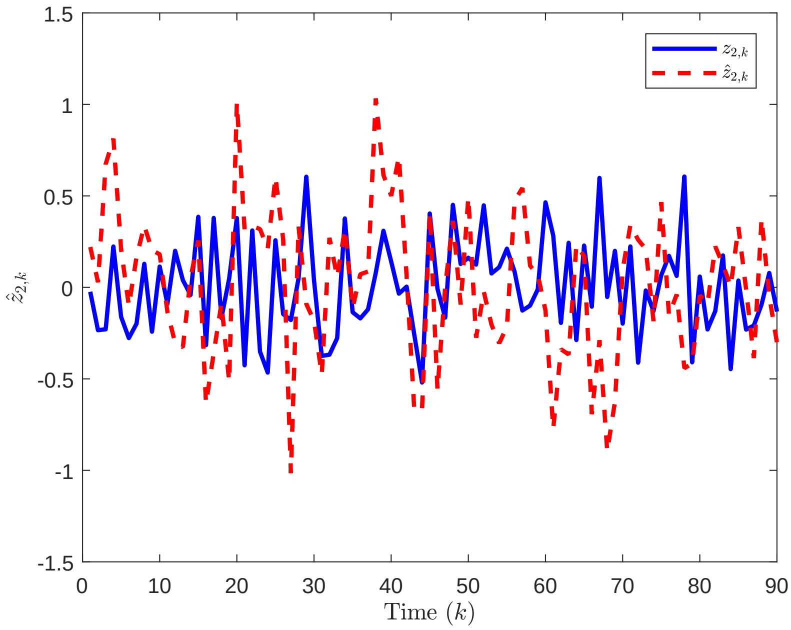

Figure 2. The controlled output $z_{2,k}$ and its estimation.

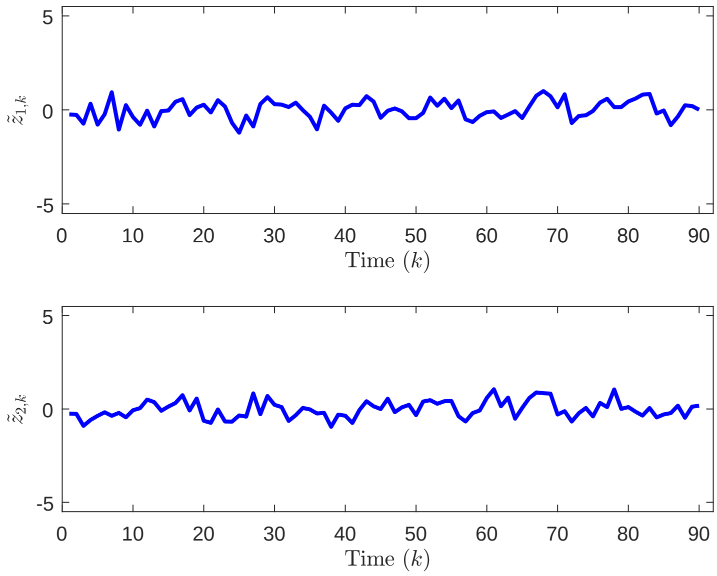

Figure 3. The output EEs.

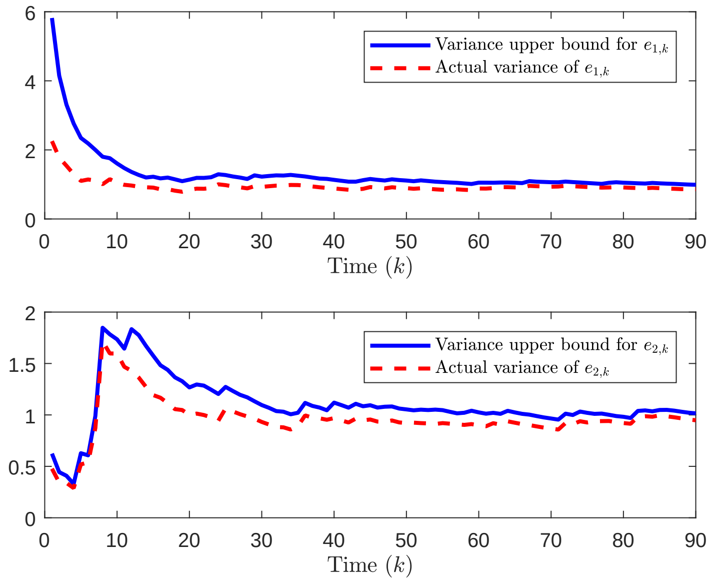

Figure 4. The upper bound of error variance and actual error variance.

Remark 5. *In this paper, we have addressed the variance-constrained robust H_∞ SE problem for DTVUNNs. In terms of stochastic analysis technique and matrix theory, a novel variance-constrained robust SE algorithm has been proposed, which can guarantee the error variance boundedness and the H_∞ performance requirement. In order to improve transmission efficiency, uniform quantization is introduced in the process of signal transmission through the network. For more details, the input signal* *is equally divided by the uniform quantization, i.e., their quantization intervals are same. The signals under the uniform quantization are limited to the interval* $[-M, M]$*, where* M *is determined by the type of the sensor. Compared with other existing literatures, a new* H_∞ *SE algorithm without resorting the augmentation method has been proposed for DTVUNNs with uniform quantization, which can greatly reduce the amount of computation. From the engineering viewpoint, the proposed SE method under variance constraint has time-varying characteristics. It is not only suitable for dealing with the estimation problem of time-varying NNs, but also suitable for online application. For the techniques used, we propose the RLMIs for the computational convenience.*

6. Conclusions

This paper has addressed the robust H_∞ SE problem for DTVUNNs with uniform quantization and time-delay under variance constraints. A time-varying finite-horizon state estimator has been constructed such that, in the presence of uniform quantization and time-delay, some sufficient criteria have been obtained for the EE system to satisfy the error variance boundedness and the prescribed H_∞ performance requirement. A novel time-varying H_∞ SE algorithm has been proposed by using the Matlab toolbox and matrix theory, where the gain matrix of the SE has been obtained by solving RLMIs. Finally, a simulation example has been given to illustrate the feasibility of the SE method proposed in this paper. This paper has investigated the variance-constrained robust H_∞ SE problem for DTVUNNs with uniform quantization, but there are still many topics worth studying. It can be also extended to deal with the in-domain coupling and the Levy-type noise as in [47, 48], and the related methods can be expected in the subsequent study. In addition, the variance-constrained state estimation problem based on event-triggering mechanism is discussed in the future [49–53].

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Conflict of interest

The authors declare that they have no conflict of interest.

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