AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(8): 13821–13831.
DOI: 10.3934/math.2022762
Received: 13 March 2022
Revised: 28 April 2022
Accepted: 5 May 2022
Published: 23 May 2022

Research article

Interpolative Hardy Roger's type contraction on a closed ball in ordered dislocated metric spaces and some results

Abdullah Shoaib[1,*], **Poom Kumam**[2,*] **and Kanokwan Sitthithakerngkiet**[3]

[1] Department of Mathematics and Statistics, Riphah International University, Islamabad, Pakistan
[2] Center of Excellence in Theoretical and Computational Science (TaCS-CoE) & KMUTT Fixed Point Research Laboratory, Room SCL 802 Fixed Point Laboratory, Science Laboratory Building, Departments of Mathematics, Faculty of Science, King Mongkut's University of Technology Thonburi (KMUTT), 126 Pracha-Uthit Road, Bang Mod, Thung Khru, Bangkok 10140, Thailand
[3] Intelligent and Nonlinear Dynamic Innovations Research Center, Department of Mathematics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok (KMUTNB), 1518, Wongsawang, Bangsue, Bangkok 10800, Thailand

* **Correspondence:** Email: poom.kum@kmutt.ac.th; Tel: +66024708994; Fax: +66024284025.

Abstract: The aim of this paper is to find out fixed point results with interpolative contractive conditions for pairs of generalized locally dominated mappings on closed balls in ordered dislocated metric spaces. We have explained our main result with an example. Our results generalize the result of Karapınar et al. (Symmetry 2018, 11, 8).

Keywords: dominated mappings; common fixed point; interpolative Hardy Roger's type contraction; closed ball; ordered dislocated metric spaces
Mathematics Subject Classification: 54H25, 47H10

1. Introduction and preliminaries

In the 19th century the study of fixed point theory was initiated by Poincare and in 20th century, it was developed by many mathematicians like Brouwer, Schauder, Banach, Kannan, and others. The theory of fixed point is one of the most powerful subject of functional analysis. Theorems ensuring the existence of fixed points of functions are known as fixed point theorems, see [25, 29–31, 33, 36, 39]. Fixed point theory is a beautiful mixture of topology, geometry and analysis which has a large number of applications in many fields such as game theory, mathematics engineering, economics, biology, physics, optimization theory and many others, see [8, 15, 20, 26]. In 2000, Hitzler and Seda [22] established the notation of dislocated metric space. Dislocated metric space plays very important role in electronics engineering and in logical programming [23]. For further results on dislocated metric spaces, see [1,6,37].

Arshad et al. [6] examined some functions having fixed point but there was no result to guarantee the presence of fixed point of such functions. They defined a restriction and involved a closed ball in his result to guarantee the presence of fixed points of such functions. For further results on closed ball, see [3,4,7,38].

Ran and Reurings [35] and Nieto et al. [32] gave an extension to the results in fixed point theory and obtained results in partially ordered sets, see also [11–13,40].

Many researchers have used interpolative technique to obtain generalized results by using different form of contractions [9, 10, 18]. Karapınar et al. [27] introduced a interpolative Hardy Roger's type contraction mapping and proved a fixed point result. Hardy Roger's theorem has been generalized in different ways by many researchers, see [5,19,21,28,34].

In this paper, we obtain common fixed point for a pair of dominated functions satisfying interpolative Hardy Roger's type contraction on a closed ball in ordered dislocated metric spaces. Now, we recall the following definitions and results which will be useful to understand the paper.

Definition 1.1. [6] Consider Υ be a nonempty set and $d_l : \Upsilon \times \Upsilon \to [0, +\infty)$. Then d_l is known as a d_l-metric, if the following conditions hold for $m, f, k \in \Upsilon$:

(i) if $d_l(m, f) = 0$, then $m = f$,

(ii) $d_l(m, f) = d_l(f, m)$,

(iii) $d_l(m, f) \leq d_l(m, k) + d_l(k, f) - d_l(k, k)$.

The dislocated metric space is represented by the pair (Υ, d_l). We will use DMS instead of dislocated metric space for now onward. It is evident that if $d_l(m, f) = 0$, then from (i) $m = f$. But the converse is not true in general.

Remark 1.2. [6] From (iii) of Definition 1.1, we deduce

$$d_l(m, f) + d_l(k, k) \leq d_l(m, k) + d_l(k, f),$$

for all $m, f, k \in \Upsilon$.

Example 1.3. [6] If $\Upsilon = [0, +\infty)$, then $d_l(m, g) = m + g$ define a dislocated metric d_l on Υ.

Definition 1.4. [6] Consider $\{f_n\}$ be a sequence in a DMS (Υ, d_l), we call $\{f_n\}$ be a Cauchy sequence if, $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$, so that for all $n, m \geq n_0$, we get $d_l(f_m, f_n) < \varepsilon$.

Definition 1.5. [6] Consider $\{f_n\}$ be a sequence in a DMS (Υ, d_l). We call this sequence to be converges with respect to d_l, if there exists $f \in \Upsilon$ such that $d_l(f_n, f) \to 0$ as $n \to +\infty$. Where, f is known as limit of $\{f_n\}$, and we write $f_n \to f$.

Definition 1.6. [6] A DMS (Υ, d_l) is called complete, if every Cauchy sequence in Υ converges to a point in Υ.

Definition 1.7. [6] Consider Υ be a nonempty set. The triplet $(\Upsilon, \preceq, d_l)$ is said to be ordered DMS, if:

(i) if d_l to be a dislocated metric of Υ,

(ii) if $\preceq$ is a partial order on Υ.

Definition 1.8. [6] Consider a partial ordered set $(\Upsilon, \preceq)$. If $m \preceq g$ or $g \preceq m$ holds then m and g are called comparable.

Definition 1.9. [2] Consider a partially ordered set $(\Upsilon, \preceq)$. Let g be self mapping on Υ. Then we call g is dominated mapping, if $gm \preceq m$ for every m in Υ.

2. Main result

Now, we define interpolative dominated contractive condition on a closed ball in ordered dislocated metric space and prove our main result.

Theorem 2.1. Let $(\Upsilon, \preceq, d_l)$ be a complete ordered DMS, T and S are dominated mappings on Υ, $f_0 \in \Upsilon$ and $r > 0$. Assume that f and y are comparable element in $\overline{B(f_0, r)}$, such that

$$d_l(Sf, Ty) \leq \lambda (d_l(f, y))^{\beta} . (d_l(f, Sf))^{\alpha} . (d_l(y, Ty))^{\gamma} . \left[\frac{1}{2}(d_l(y, Sf) + d_l(f, Ty))\right]^{1-\alpha-\beta-\gamma}, \tag{2.1}$$

for some $\alpha, \beta, \gamma, \lambda \in [0, 1)$, with $\alpha + 2\beta + 2\gamma < 1$ and

$$d_l(f_0, Sf_0) \leq r(1 - \lambda). \tag{2.2}$$

Then there exists a non increasing sequence $\{f_n\} \subseteq \overline{B(f_0, r)}$, such that $f_n \to f^* \in \overline{B(f_0, r)}$. Also, if $f^* \preceq f_n$, then $f^* = Tf^* = Sf^*$ and $d_l(f^*, f^*) = 0$.

Proof. Consider a point f_1 on Υ such that $f_1 = Sf_0$. As $Sf_0 \preceq f_0$ so $f_1 \preceq f_0$ and let $f_2 = Tf_1$. Now $Tf_1 \preceq f_1$ gives $f_2 \preceq f_1$, continuing this method and choosing f_n in Υ such that $f_{2h+1} = S_{f_{2h}}$, $f_{2h+2} = Tf_{2_{h+1}}$, where $h = 0, 1, 2, \ldots$ clearly, $f_{2h+1} = S_{f_{2h}} \preceq f_{2h} = Tf_{2h-1} \preceq f_{2h-1}$, and this implies that the sequence $\{f_n\}$ is non increasing. By using inequality (2.2), we have $d_l(f_0, f_1) \leq r$, or $f_1 \in \overline{B(f_0, r)}$. Assume that $f_2, \ldots, f_j \in \overline{B(f_0, r)}$ for some $j \in \mathbb{N}$. Now, if $2h + 1 \leq j$, by using inequality (2.1), we obtain

$$\begin{aligned} d_l(f_{2h+1}, f_{2h+2}) &= d_l(Sf_{2h}, Tf_{2h+1}) \\ &\leq \lambda (d_l(f_{2h}, f_{2h+1}))^{\beta} . (d_l(f_{2h}, Sf_{2h}))^{\alpha} . (d_l(f_{2h+1}, Tf_{2h+1}))^{\gamma} . \\ &\quad \left[\begin{array}{c} \frac{1}{2}(d_l(f_{2h}, Tf_{2h+1}) + d_l(f_{2h+1}, Sf_{2h})) - \\ d_l(f_{2h+1}, f_{2h-1}) + d_l(f_{2h+1}, f_{2h-1}) \end{array}\right]^{1-\alpha-\beta-\gamma} . \end{aligned}$$

By Remark 1.2, we have

$$d_l(f_{2h+1}, f_{2h+2}) \leq \lambda (d_l(f_{2h}, f_{2h+1}))^{\beta} . (d_l(f_{2h}, f_{2h+1}))^{\alpha} . (d_l(f_{2h+1}, f_{2h+2}))^{\gamma} \left[\frac{1}{2}(d_l(f_{2h}, f_{2h+1}) + d_l(f_{2h+1}, f_{2h+2}))\right]^{1-\alpha-\beta-\gamma} . \tag{2.3}$$

Suppose that

$$d_l(f_{2h}, f_{2h+1}) < d_l(f_{2h+1}, f_{2h+2}).$$

This implies that

$$\frac{1}{2}(d_l(f_{2h}, f_{2h+1}) + d_l(f_{2h+1}, f_{2h+2})) < d_l(f_{2h+1}, f_{2h+2}).$$

Consequently, the inequality (2.1) yield that

$$(d_l(f_{2h+1}, f_{2h+2}))^{\alpha+\beta} \leq \lambda (d_l(f_{2h}, f_{2h+1}))^{\alpha+\beta},$$

so we conclude that

$$d_l(f_{2h}, f_{2h+1}) > d_l(f_{2h+1}, f_{2h+2}),$$

which is a contradiction, thus we have

$$d_l(f_{2h+1}, f_{2h+2}) < d_l(f_{2h}, f_{2h+1}).$$

This implies that

$$\frac{1}{2}(d_l(f_{2h}, f_{2h+1}) + d_l(f_{2h}, f_{2h+1})) \leq d_l(f_{2h}, f_{2h+1}).$$

By simple elimination, the inequality (2.1) becomes

$$(d_l(f_{2h+1}, f_{2h+2}))^{1-\gamma} \leq \lambda (d_l(f_{2h}, f_{2h+1}))^{1-\gamma}.$$

This implies that

$$d_l(f_{2h+1}, f_{2h+2}) \leq \lambda d_l(f_{2h}, f_{2h+1}).$$

Similarly, if $2h \leq j$, we deduce

$$d_l(f_{2h+1}, f_{2h}) \leq \lambda d_l(f_{2h}, f_{2h-1}).$$

By the previous inequality, we get

$$\begin{aligned} d_l(f_{2h+1}, f_{2h+2}) &\leq \lambda d_l(f_{2h}, f_{2h+1}) \leq \ldots \leq \lambda^{2h+1} d_l(f_0, f_1) \\ d_l(f_{2h+1}, f_{2h}) &\leq \lambda d_l(f_{2h}, f_{2h-1}) \leq \ldots \leq \lambda^{2h} d_l(f_0, f_1). \end{aligned} \tag{2.4}$$

Thus from inequality (2.4), we have

$$d_l\left(f_j, f_{j+1}\right) \leq \lambda^j d_l(f_0, f_1), \tag{2.5}$$

for some $j \in \mathbb{N}$. Now, using (2.5), and (2.2), we get

$$\begin{aligned} d_l\left(f_0, f_{j+1}\right) &\leq d_l(f_0, f_1) + \ldots + d_l\left(f_j, f_{j+1}\right) - \left[d_l(f_1, f_1) + \ldots d_l\left(f_j, f_j\right)\right] \\ &\leq d_l(f_0, f_1)\left[1 + \ldots + \lambda^{j-1} + \lambda^j\right] \\ &\leq (1-\lambda) r \frac{\left(1-\lambda^{j+1}\right)}{1-\lambda} < r. \end{aligned}$$

Thus $f_{j+1} \in \overline{B(f_0, r)}$. Therefore $f_h \in \overline{B(f_0, r)}$, for all $h \in \mathbb{N}$. Since $f_{h+1} \preceq f_h$ for all $h \in \mathbb{N}$, then it follow that

$$\begin{aligned} d_l(f_{h+i}, f_h) &\leq d_l(f_{h+i}, f_{h+i-1}) + \cdots + d_l(f_{h+1}, f_h) \\ &\quad - d_l(f_{h+i-1}, f_{h+i-1}) - \cdots - d_l(f_{h+1}, f_{h+1}) \\ &\leq \lambda^{h+i-1} d_l(f_0, f_1) + \ldots + \lambda^h d_l(f_0, f_1) \\ &\leq \lambda^h d_l(f_0, f_1) \frac{1-\lambda^i}{1-\lambda} \to 0, \text{ as } h \to +\infty. \end{aligned}$$

This shows that $\{f_n\}$ is a Cauchy sequence in $\left(\overline{B(f_0, r)}, d_l\right)$. Now, $\left(\overline{B(f_0, r)}, d_l\right)$ is complete because $\overline{B(f_0, r)}$ is closed. Therefore there exist a point $f^* \in \overline{B(f_0, r)}$ with

$$\lim_{n\to+\infty} d_l(f_n, f^*) = 0. \tag{2.6}$$

By assumption $f^* \preceq f_n$ as $f_n \to f^*$, we have

$$\begin{aligned} d_l(Sf^*, f^*) &\leq d_l(Sf^*, Tf_{2h+1}) + d_l(f_{2h+2}, f^*) - d_l(f_{2h+2}, f_{2h+2}) \\ &\leq \lambda d_l(f^*, f_{2h+1})^\beta . (d_l(f^*, Sf^*))^\alpha . (d_l(f_{2h+1,}Tf_{2h+1}))^\gamma . \\ &\quad \left[\frac{1}{2}d_l(f^*, Tf_{2h+1}) + d_l(f_{2h+1}, Sf^*)\right]^{1-\alpha-\beta-\gamma} + d_l(f_{2h+2}, f^*) \\ &\leq d_l(f^*, f_{2h+2}) + \lambda(d_l(f^*, f_{2h+1}))^\beta . (d_l(f^*, Sf^*))^\alpha . (d_l(f_{2h+1}, f_{2h+2}))^\gamma . \\ &\quad \left[\frac{1}{2}d_l(f^*, f_{2h+2}) + d_l(f_{2h+1}, Sf^*)\right]^{1-\alpha-\beta-\gamma} . \end{aligned}$$

On taking limit $h \to +\infty$ and by using inequalities (2.4) and (2.6), we obtain $d_l(f^*, Sf^*) \leq 0$ which implies,

$$f^* = Sf^*.$$

Similarly from

$$d_l(f^*, Tf^*) \leq d_l(f^*, f_{2h+1}) + d_l(f_{2h+1}, Tf^*) - d_l(f_{2h+1}, f_{2h+1}),$$

we can obtain $f^* = Tf^*$. Hence S and T have a common fixed point in $\overline{B(f_0, r)}$. Now,

$$\begin{aligned} d_l(f^*, f^*) &= d_l(Sf^*, Tf^*) \\ &\leq \lambda(d_l(f^*, f^*))^\beta . (d_l(f^*, Sf^*))^\alpha . (d_l(f^*, Tf^*))^\gamma . \\ &\quad \left[\frac{1}{2}d_l(f^*, Sf^*) + d_l(f^*, Tf^*)\right]^{1-\alpha-\beta-\gamma}, \end{aligned}$$

and this implies that.

$$d_l(f^*, f^*) = 0.$$

In Theorem 2.1, the condition 2.1 is applicable only for all comparable points in a closed ball and the condition 2.2 is used to obtain a sequence in a closed ball and Example 2.10 will show the importance of this restriction. Now, in the next result the condition 2.2 is relaxed and the condition 2.1 is applied for all comparable points in the ground set.

Corollary 2.2. Let $(\Upsilon, \preceq, d_l)$ be a complete ordered DMS, T and S are dominated mappings on Υ. Assume that f and y are comparable element in Υ, such that

$$\begin{aligned} d_l(Sf, Ty) &\leq \lambda(d_l(f, y))^\beta . (d_l(f, Sf))^\alpha . (d_l(y, Ty))^\gamma . \\ &\quad \left[\frac{1}{2}(d_l(y, Sf) + d_l(f, Ty))\right]^{1-\alpha-\beta-\gamma}, \end{aligned}$$

for some $\alpha, \beta, \gamma, \lambda \in [0, 1)$, with $\alpha + 2\beta + 2\gamma < 1$. Then there exists a non increasing sequence $\{f_n\} \subseteq X$ such that $f_n \to f^* \in X$. Also, if $f^* \preceq f_n$, then $f^* = Sf^* = Tf^*$ and $d_l(f^*, f^*) = 0$.

The metric space version of Corollary 2.2 is given below.

Corollary 2.3. Let $(\Upsilon, \preceq, \rho)$ be a complete ordered metric space, T and S are dominated mappings on Υ. Assume that f and y are comparable elements in Υ, such that

$$\rho(Sf, Ty) \leq \lambda(\rho(f, y))^\beta . (\rho(f, Sf))^\alpha . (\rho(y, Ty))^\gamma .$$

$$\left[\frac{1}{2}(\rho(y,Sf)+\rho(f,Ty))\right]^{1-\alpha-\beta-\gamma},$$

for some $\alpha,\beta,\gamma,\lambda\in[0,1)$, with $\alpha+2\beta+2\gamma<1$. Then there exists a non increasing sequence $\{f_n\}\subseteq X$ such that $f_n\to f^*\in X$. Also, if $f^*\le f_n$, then $f^*=Sf^*=Tf^*$.

In Theorem 2.1, if we replace S by T, then the following result is obtained.

Corollary 2.4. Let $(\Upsilon,\le,d_l)$ be a complete ordered DMS, T is a dominated mappings on Υ, $f_0\in\Upsilon$ and $r>0$. Assume that f and y are comparable element in $\overline{B(f_0,r)}$, such that

$$\begin{aligned} d_l(Tf,Ty) \le\ & \lambda(d_l(f,y))^{\beta}.(d_l(f,Tf))^{\alpha}.(d_l(y,Ty))^{\gamma}. \\ & \left[\frac{1}{2}(d_l(y,Tf)+d_l(f,Ty))\right]^{1-\alpha-\beta-\gamma}, \end{aligned}$$

for some $\alpha,\beta,\gamma,\lambda\in[0,1)$, with $\alpha+2\beta+2\gamma<1$ and

$$d_l(f_0,Tf_0)\le(1-\lambda)r.$$

Then there exists a non increasing sequence $\{f_n\}\subseteq\overline{B(f_0,r)}$, such that $f_n\to f^*\in\overline{B(f_0,r)}$. Also, if $f^*\le f_n$, then $f^*=Tf^*$ and $d_l(f^*,f^*)=0$.

Without closed ball version of Corollary 2.4 is given below.

Corollary 2.5. Let $(\Upsilon,\le,d_l)$ be a complete ordered DMS, T are dominated mappings on Υ. Assume that f and y are comparable element in Υ, such that

$$\begin{aligned} d_l(Tf,Ty) \le\ & \lambda(d_l(f,y))^{\beta}.(d_l(f,Tf))^{\alpha}.(d_l(y,Ty))^{\gamma}. \\ & \left[\frac{1}{2}(d_l(y,Tf)+d_l(f,Ty))\right]^{1-\alpha-\beta-\gamma}, \end{aligned}$$

for some $\alpha,\beta,\gamma,\lambda\in[0,1)$, with $\alpha+2\beta+2\gamma<1$. Then there exists a non increasing sequence $\{f_n\}\subseteq X$, such that $f_n\to f^*\in X$. Also, if $f^*\le f_n$, then $f^*=Tf^*$ and $d_l(f^*,f^*)=0$.

If we put the value of α is equal to zero. Then the following result is obtained.

Corollary 2.6. Let $(\Upsilon,\le,d_l)$ be a complete ordered DMS, T and S are dominated mappings on Υ, $f_0\in\Upsilon$ and $r>0$. Assume that f and y are comparable element in $\overline{B(f_0,r)}$, such that

$$d_l(Sf,Ty)\le\lambda(d_l(f,y))^{\beta}.(d_l(y,Ty))^{\gamma}.\left[\frac{1}{2}(d_l(y,Sf)+d_l(f,Ty))\right]^{1-\beta-\gamma},$$

for some $\beta,\gamma,\lambda\in[0,1)$, with $2\beta+2\gamma<1$. Then there exists a non increasing sequence $\{f_n\}\subseteq\overline{B(f_0,r)}$, such that $f_n\to f^*\in\overline{B(f_0,r)}$. Also, if $f^*\le f_n$, then $f^*=Sf^*=Tf^*$ and $d_l(f^*,f^*)=0$.

If we put the value of β is equal to zero. Then the following result is obtained.

Corollary 2.7. Let $(\Upsilon,\le,d_l)$ be a complete ordered DMS, T and S are dominated mappings on Υ, $f_0\in\Upsilon$ and $r>0$. Assume that f and y are comparable element in $\overline{B(f_0,r)}$, such that

$$d_l(Sf,Ty)\le\lambda(d_l(f,Sf))^{\alpha}.(d_l(y,Ty))^{\gamma}.\left[\frac{1}{2}(d_l(y,Sf)+d_l(f,Ty))\right]^{1-\alpha-\gamma},$$

for some $\alpha, \gamma, \lambda \in [0, 1)$, with $\alpha + 2\gamma < 1$. Then there exists a non increasing sequence $\{f_n\} \subseteq \overline{B(f_0, r)}$, such that $f_n \to f^* \in \overline{B(f_0, r)}$. Also, if $f^* \preceq f_n$, then $f^* = Sf^* = Tf^*$ and $d_l(f^*, f^*) = 0$.

If we put the value of γ is equal to zero. Then the following result is obtained.

Corollary 2.8. Let $(\Upsilon, \preceq, d_l)$ be a complete ordered DMS, T and S are dominated mappings on Υ, $f_0 \in \Upsilon$ and $r > 0$. Assume that f and y are comparable element in $\overline{B(f_0, r)}$, such that

$$d_l(Sf, Ty) \leq \lambda (d_l(f, y))^{\beta} . (d_l(f, Sf))^{\alpha} . \left[\frac{1}{2}(d_l(y, Sf) + d_l(f, Ty))\right]^{1-\alpha-\beta},$$

for some $\alpha, \beta, \lambda \in [0, 1)$, with $\alpha + 2\beta < 1$. Then there exists a non increasing sequence $\{f_n\} \subseteq \overline{B(f_0, r)}$, such that $f_n \to f^* \in \overline{B(f_0, r)}$. Also, if $f^* \preceq f_n$, then $f^* = Sf^* = Tf^*$ and $d_l(f^*, f^*) = 0$.

If we take complete DMS (Υ, d_l) instead of complete ordered DMS $(\Upsilon, \preceq, d_l)$. Then the following result is obtained.

Corollary 2.9. Let (Υ, d_l) be a complete DMS, T and S are self mappings on Υ, $f_0 \in \Upsilon$ and $r > 0$. Assume that f and y are element in $\overline{B(f_0, r)}$, such that

$$\begin{aligned} d_l(Sf, Ty) \leq\ & \lambda (d_l(f, y))^{\beta} . (d_l(f, Sf))^{\alpha} . (d_l(y, Ty))^{\gamma} . \\ & \left[\frac{1}{2}(d_l(y, Sf) + d_l(f, Ty))\right]^{1-\alpha-\beta-\gamma}, \end{aligned}$$

for some $\alpha, \beta, \gamma, \lambda \in [0, 1)$, with $\alpha + 2\beta + 2\gamma < 1$ and

$$d_l(f_0, Sf_0) \leq r(1 - \lambda).$$

Then there exists a sequence $\{f_n\} \subseteq \overline{B(f_0, r)}$, such that $f_n \to f^* \in \overline{B(f_0, r)}$, $f^* = Tf^* = Sf^*$ and $d_l(f^*, f^*) = 0$.

Example 2.10. Let $\Upsilon = [0, +\infty) \cap \mathbb{Q}$ be endowed with the order $f \preceq y$ if $d_l(f, f) \leq d_l(y, y)$, and define $d_l : \Upsilon \times \Upsilon \to \Upsilon$ as $d_l(f, y) = f + y$. Then (Υ, d_l) is an ordered completed dislocated metric space. Let $T, S : \Upsilon \to \Upsilon$ be defined by,

$$Sf = \left\{ \begin{array}{c} \frac{f}{7} \text{ if } f \in [0, 1] \cap \Upsilon \\ f - \frac{1}{3} \text{ if } f \in (1, +\infty) \cap \Upsilon \end{array} \right\}$$

$$Tf = \left\{ \begin{array}{c} \frac{2f}{7} \text{ if } f \in [0, 1] \cap \Upsilon \\ f - \frac{1}{4} \text{ if } f \in (1, +\infty) \cap \Upsilon \end{array} \right\}.$$

Clearly T and S are dominated mappings. For $f_0 = 1$, $r = 2$, $\alpha = \frac{1}{7}$, and $\beta = \frac{1}{9}$, $\gamma = \frac{1}{10}$, $\lambda = \frac{3}{7}$, $\overline{B(f_0, r)} = [0, 1] \cap \Upsilon$, and $(1 - \lambda) r = \frac{8}{7} = d_l(f_0, Sf_0)$. Now if $f = 1$, $y = 2$ then

$$\begin{aligned} d_l(Sf, Ty) =\ & \frac{f}{7} + y - \frac{1}{4} \geq \frac{3}{7}(f + y)^{\frac{1}{9}} . \left(f + \frac{f}{7}\right)^{\frac{1}{7}} . \left(2y - \frac{1}{4}\right)^{\frac{1}{10}} . \\ & \left[\frac{1}{2}\left(y + \frac{f}{7} + f + y - \frac{1}{4}\right)\right]^{1-\frac{1}{9}-\frac{1}{7}-\frac{1}{10}}, \end{aligned}$$

and so,

$$d_l(Sf,Ty) \geq \lambda (d_l(f,y))^{\beta} . (d_l(f,Sf))^{\alpha} . (d_l(y,Ty))^{\gamma} . \left[\frac{1}{2}(d_l(y,Sf)+d_l(f,Ty))\right]^{1-\alpha-\beta-\gamma}.$$

Thus, the contractive condition does not hold on Υ. Now if $f, y \in \overline{B(f_0,r)}$, then

$$\begin{aligned} d_l(Sf,Ty) &= \frac{f}{7}+\frac{2y}{7}=\frac{1}{7}(f+2y) \\ &\leq \frac{3}{7}.(f+y)^{\frac{1}{9}}.\left(f+\frac{f}{7}\right)^{\frac{1}{7}}.\left(f+\frac{2y}{7}\right)^{\frac{1}{10}}. \\ &\quad \left[\frac{1}{2}\left(y+\frac{f}{7}+f+\frac{2y}{7}\right)\right]^{1-\frac{1}{9}-\frac{1}{7}-\frac{1}{10}} \\ &= \lambda (d_l(f,y))^{\beta} . (d_l(f,Sf))^{\alpha} . (d_l(y,Ty))^{\gamma} . \\ &\quad \left[\frac{1}{2}(d_l(y,Sf)+d_l(f,Ty))\right]^{1-\alpha-\beta-\gamma}. \end{aligned}$$

Therefore all the condition of theorem are satisfied. Moreover, 0 is the common fixed point of T and S.

3. Conclusions

Arshad et al. [6] analyzed that there are mappings which are contractive only on the subsets of its domain. They deduced the fixed point results satisfying contraction on closed ball to ensure the existence of fixed point of such mappings. On the other hand, Karapınar et al. [27] recently gave the concept interpolative contraction and established some result. We extend their findings, and in this paper, fixed point results with interpolative contractive conditions for a pair of generalized locally dominated mappings on closed balls in ordered dislocated metric spaces have been established.

Acknowledgments

This research was funded by National Science, Research and Innovation Fund (NSRF), and King Mongkut's University of Technology North Bangkok with Contract no. KMUTNB-FF-65-24.

Conflict of interest

The authors declare that they do not have any competing interests.

References

1. C. T. Aage, J. N. Salunke, The results on fixed points in dislocated and dislocated quasi-metric space, *Appl. Math. Sci*, **2** (2008), 2941–2948.

2. M. Abbas, S. Z. Németh, Finding solutions of implicit complementarity problems by isotonicity of the metric projection, *Nonlinear Anal.*, **75** (2012), 2349–2361. https://doi.org/10.1016/j.na.2011.10.033
3. A. E. Al-Mazrooei, A. Shoaib, J. Ahmad, Cyclic b-multiplicative (A, B)-Hardy-Rogers-type local contraction and related results in b-multiplicative and b-metric spaces, *J. Math.*, **2020** (2020), 1–9. https://doi.org/10.1155/2020/2460702
4. A. E. Al-Mazrooei, A. Shoaib, J. Ahmad, Unique fixed-point results for β-admissible mapping under $(\beta - \psi)$-contraction in complete dislocated G_d-metric space, *Mathematics*, **8** (2020), 1–13. https://doi.org/10.3390/math8091584
5. M. U. Ali, H. Aydi, M. Alansari, New generalizations of set valued interpolative Hardy-Rogers type contractions in b-metric spaces, *J. Funct. Spaces*, **2021** (2021), 1–8. https://doi.org/10.1155/2021/6641342
6. M. Arshad, A. Shoaib, P. Vetro, Common fixed points of a pair of Hardy Rogers type mappings on a closed ball in ordered dislocated metric spaces, *J. Funct. Space.*, **2013** (2013), 1–9. https://doi.org/10.1155/2013/638181
7. M. Arshad, A. Shoaib, I. Beg, Fixed point of a pair of contractive dominated mappings on a closed ball in an ordered dislocated metric space, *Fixed Point Theory A.*, **2013** (2013), 1–15. https://doi.org/10.1186/1687-1812-2013-115
8. J. P. Aubin, *Mathematical methods of games and economic theory,* Elsevier, North-Holland, Amsterdam, 1979.
9. H. Aydi, E. Karapınar, A. F. Roldán López de Hierro, ω-interpolative Ćirić-Reich-Rus-type contractions, *Mathematics*, **7** (2019). https://doi.org/10.3390/math7010057
10. H. Aydi, C. M. Chen, E. Karapınar, Interpolative Ćirić-Reich-Rus type contractions via the Branciari distance, *Mathematics*, **7** (2019). https://doi.org/10.3390/math7010084
11. A. Arif, M. Nazam, Hussain, M. Abbas, The ordered implicit relations and related fixed point problems in the cone b-metric spaces, *AIMS Math.*, **7** (2022), 5199–5219. https://doi.org/10.3934/math.2022290
12. I. Beg, A. R. Butt, Common fixed point for generalized set valued contractions satisfying an implicit relation in partially ordered metric spaces, *Math. Commun.*, **15** (2010), 65–76.
13. I. Beg, A. R. Butt, Fixed point for set-valued mappings satisfying an implicit relation in partially ordered metric spaces, *Nonlinear Anal.*, **71** (2009), 3699–3704. https://doi.org/10.1016/j.na.2009.02.027
14. T. G. Bhaskar, V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, *Nonlinear Anal.*, **65** (2006), 1379–1393. https://doi.org/10.1016/j.na.2005.10.017
15. S. Bohnenblust, S. Karlin, *Contributions to the theory of games,* Princeton University Press, Princeton, 1950.
16. L. B. Ćirić, Common fixed point theorems for set-valued mappings, *Demonstr. Math.*, **39** (2006), 419–428.
17. L. B. Ćirić, J. S. Ume, Some common fixed point theorems for weakly compatible mappings, *J. Math. Anal. Appl.*, **314** (2006), 488–499.

18. P. Debnath, M. de La Sen, Fixed-points of interpolative Ćirić-Reich-Rus-type contractions in b-metric spaces, *Symmetry*, **12** (2020), 12. https://doi.org/10.3390/sym12010012
19. P. Debnath, M. de La Sen, Set-valued interpolative Hardy-Rogers and set-valued Reich-Rus-Ćirić-type contractions in b-metric spaces, *Mathematics*, **7** (2019). https://doi.org/10.3390/math7090849
20. P. Debnath, N. Konwar, S. Radenović, *Metric fixed point theory: Applications in science, engineering and behavioural sciences*, Springer Nature, Singapore, 2021. https://doi.org/10.1007/978-981-16-4896-0
21. P. Debnath, Z. D. Mitrović, S. Radenović, Interpolative Hardy-Rogers and Reich-Rus-Ćirić type contractions in b-metric and rectangular b-metric spaces, *Mat. Vestn.*, **72** (2020), 368–374.
22. P. Hitzler, A. K. Seda, Dislocated topologies, *J. Electr. Eng.*, **51** (2000), 3–7.
23. P. Hitzler, *Generalized metrics and topology in logic programming semantics,* PhD thesis, School of Mathematics, Applied Mathematics and Statistics, National University Ireland, University College Cork, 2001.
24. N. Hussain, J. R. Roshan, V. Paravench, M. Abbas, Common fixed point results for weak contractive mappings in ordered *b*-dislocated metric space with applications, *J. Inequal. Appl.,* **2013** (2013), 1–21. https://doi.org/10.1186/1029-242X-2013-486
25. N. Hussain, J. R. Roshan, V. Paravench, A. Latif, A unification of *G*-metric, partial metric, and *b*-metric spaces, *Abstr. Appl. Anal.,* **2014** (2014), 1–144. https://doi.org/10.1155/2014/180698
26. A. Jeribi, B. Krichen, B. Mefteh, Existence solutions of a two-dimensional boundary value problem for a system of nonlinear equations arising in growing cell populations, *J. Biol. Dynam.*, **7** (2013), 218–232.
27. E. Karapınar, O. Alqahtani, H. Aydi, On interpolative Hardy-Rogers type contractions, *Symmetry,* **11** (2019).
28. V. N. Mishra, L. M. Sánchez Ruiz, P. Gautam, S. Verma, Interpolative Reich-Rus-Ćirić and Hardy-Rogers contraction on quasi-partial *b*-metric space and related fixed point results, *Mathematics*, **8** (2020). https://doi.org/10.3390/sym11010008
29. Z. Mustafa, V. Parvaneh, J. R. Roshan, Z. Kadelburg, b_2-Metric spaces and some fixed point theorems, *Fixed Point Theory A.*, **2014** (2014), 1–23. https://doi.org/10.1186/1687-1812-2014-144
30. Z. Mustafa, V. Parvaneh, M. Abbas, J. R. Roshan, Some coincidence point results for generalized (ψ, φ)-weakly contractive mappings in ordered *G*-metric spaces, *Fixed Point Theory A.*, **2013** (2013). https://doi.org/10.1186/1687-1812-2013-326
31. Z. Mustafa, J. R. Roshan, V. Parvaneh, Z. Kadelburg, Fixed point theorems for weakly T-Chatterjea and weakly T-Kannan contractions in b-metric spaces, *J. Inequal Appl.*, **2013** (2013). https://doi.org/10.1186/1029-242X-2014-46
32. J. J. Nieto, R. Rodríguez-López, Contractive mapping theorems in partially ordered sets and applications to ordinary differential equations, *Order,* **22** (2005), 223–239. https://doi.org/10.1007/s11083-005-9018-5
33. M. Nazam, A. Mukheimer, H. Aydi, M. Arshad, R. Riaz, Fixed point results for dualistic contractions with an application, *Discrete Dyn. Nat. Soc.*, **2020** (2020). https://doi.org/10.1155/2020/6428671

34. M. Nazam, H. Aydi, A. Hussain, Existence theorems for (Ψ, Φ)-orthogonal interpolative contractions and an application to fractional differential equations, *Optimization*, 2022, 1–31. https://doi.org/10.1080/02331934.2022.2043858

35. A. C. M. Ran, M. C. B. Reurings, A fixed point theorem in partially ordered sets and some applications to matrix equations, *Proc. Amer. Math. Soc.*, **132** (2004), 1435–1443.

36. J. R. Roshan, N. Shobkolaei, S. Sedghi, V. Parvaneh, S. Radenović, Common fixed point theorems for three maps in discontinuous G_b- metric spaces, *Acta Math. Sci.*, **34** (2014), 1643–1654.

37. A. Shoaib, I. S. Khan, Z. Hassan, Generalized contraction involving an open ball and common fixed point of multivalued mappings in ordered dislocated quasi metric spaces, *Filomat*, **34** (2020), 323–338. https://doi.org/10.2298/FIL2002323S

38. A. Shoaib, M. Arshad, T. Rasham, Some fixed point results in ordered complete dislocated quasi G_d metric space, *J. Comput. Anal. Appl.*, **29** (2021),1036–1046.

39. A. Shoaib, Q. Mahmood, A. Shahzad, M. S. M. Noorani, S. Radenović, Fixed point results for rational contraction in function weighted dislocated quasi-metric spaces with an application, *Adv. Differ. Equ.*, **2021** (2021), 1–15. https://doi.org/10.1186/s13662-021-03458-x

40. J. Tiammee, S. Suantai, Fixed point theorems for monotone multi-valued mappings in partially ordered metric spaces, *Fixed Point Theory A.*, **2014** (2014), 1–13. https://doi.org/10.1186/1687-1812-2014-110

AIMS Press

© 2022 the Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0)