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Research article

COVID-19 propagation and the usefulness of awareness-based control measures: A mathematical model with delay

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Abstract: The current emergence of coronavirus (SARS-CoV-2 or COVID-19) has put the world in threat. Social distancing, quarantine and governmental measures such as lockdowns, social isolation, and public hygiene are helpful in fighting the pandemic, while awareness campaigns through social media (radio, TV, etc.) are essential for their implementation. On this basis, we propose and analyse a mathematical model for the dynamics of COVID-19 transmission influenced by awareness campaigns through social media. A time delay factor due to the reporting of the infected cases has been included in the model for making it more realistic. Existence of equilibria and their stability, and occurrence of Hopf bifurcation have been studied using qualitative theory. We have derived the basic reproduction number (R_0) which is dependent on the rate of awareness. We have successfully shown that public awareness has a significant role in controlling the pandemic. We have also seen that the time delay destabilizes the system when it crosses a critical value. In sum, this study shows that public awareness in the form of social distancing, lockdowns, testing, etc. can reduce the pandemic with a tolerable time delay.

Keywords: mathematical model; basic reproduction number; stability analysis; time delay; Hopf bifurcation; sensitivity analysis
Mathematics Subject Classification: 34C23, 93A30

1. Introduction

In the last month of 2019, a ruthless contagious disease popularly known as COVID-19 or SARS-CoV-2 (Severe Acute Respiratory Syndrome Coronavirus) unfolded in the Wuhan city of China's Hubei province and has transformed into a severe global crisis by spreading in more than 200 countries of the Earth. The pandemic has taken almost 0.3 million people's lives and infected more than 5.4 million people [1]. The World Health Organization (WHO) has declared it a public health emergency worldwide [2]. This disease mainly transmits via respiratory droplets through close contact is indirectly transmitted through fomites in the environment around the infected person [3–5].

Fever, fatigue, dry cough, and myalgia are the common symptoms of COVID-19, while some patients suffer from headaches, abdominal pain, diarrhoea, nausea, and vomiting. From clinical observation, within 1–2 days after patient symptoms, the patient becomes morbid after 4–6 days, and the infection may clear within 18 days, depending on the immune system [6]. It is reported that the average death rate of COVID-19 is about 3.4% [7].

One of the WHO's strategies to control overwhelming infection spread is breaking the infection cycle, e.g., minimizing human-to-human transmission by reducing secondary infections among close contacts and health care workers. Thus, attention is paid on social distancing, lockdown, personal hygiene measures, etc., In this serious phase, public awareness of the disease and its transmission routes are the foundation of controlling the outbreak [8].

Mathematical modelling is one of the finest methods to precisely analyse the dynamics of an infectious disease. There are a few mathematical models of COVID-19 developed and published that have analysed various aspects of disease dynamics and possible containment [9, 10]. Kucharski et al. [11] have analysed the dynamics of COVID-19 through modelling by considering all the positive cases of Wuhan, China. In their research, Ndairou et al. [12] have proposed a mathematical model to discuss the spread of COVID-19 transmission in Wuhan, China. Hellewell et al. [13] have investigated the effective control measures of COVID-19 outbreak using isolation as a control measure. A quarantine model of coronavirus contagion and data analysis have been reported by Volpert [14]. Recently, Fanelli and Piazza have predicted the nature of the disease in the three most affected countries (China, Italy and Iran) till March 2020, establishing a mathematical model using ordinary differential equations [15]. A stochastic regression model was formulated by Ribeiro et al. [16] to forecast the phenomena of almost ten affected states of Brazil.

Scientists are working day and night to discover proper treatments for the virus. Therefore, social distancing is the only way to reduce the disease transmission rate and to break the chain. So, in this situation, the impact of media coverage will play an important role to control the disease transmission rate. A lot of works have been done mathematically and numerically covering the prediction of COVID-19 outbreak, but mathematically none of the work has been focused on the media impacts on this disease. So, our main aim in this work is to focus on the impact of media coverage on the spread and control of infectious diseases in a given region. Public awareness affects the society in two ways: Firstly, it can cause a panic of the society by covering the news; secondly, it can decrease the chance of interaction among the notified people.

During the early stage of the epidemic, both public and media individuals were unaware of the COVID-19 infections. As the knowledge of the disease is being publicized, people respond to it and ultimately change their habits and daily life behaviour to reduce their vulnerability [17,18]. Mass media such as TV familiarize the people with the syndromes and the possible defensive means: namely, social distancing, habits of healthier sanitation, use of preventive ointments, wearing protecting masks, and self-quarantine. Individuals, who are aware of the risk of the infection, are minimizing the chance of being infected, which is deeply influencing the pattern of the pandemic [10, 19].

Since the earliest time required in understanding the hazard of the COVID-19 transmission and treatment, the reaction of the people is not immediate. It causes a time delay in the management of the disease. Also, time lag may be raised in the reporting of infected cases by the hospitals. This time delay should be taken into attention for an opposite modelling of the disease. This type of model includes delay differential equations; see, for example, [19,20] and the references therein. It is seen that time delay extremely influences the stability of steady states and solutions through periodic oscillations when the time delay parameter crosses a certain threshold value [21].

In this article, a mathematical model has been proposed in order to predict the dynamics of COVID-19 transmission in a region. In the proposed model, a time lag due to the reporting of infected cases has been included. Stability of the equilibria of the proposed model is determined using the basic reproduction number. A threshold value of the delay parameter has been determined for which the system can show limit cycle oscillation.

The article is organised as follows: In section 2, the formulation of the mathematical model is provided, and some basic properties of the model system are analysed. The dynamics of the system with and without delay have been provided in section 3. Sensitivity of the model parameters has been analysed in section 4. Numerical simulations, on the basis of the outcomes of section 3 and 4, have been included in section 5. In section 6, the discussion on the main outcomes is made to conclude the paper.

2. Model formulation

Here, we have formulated and analysed a compartmental differential equation model for COVID-19 infection. The model monitors the dynamics of six populations, which are as follows:

Total healthy population (susceptible population) is $S(t)$, infected but not infectious population is $E(t)$, $Q(t)$ is the quarantine population, the infected population is $I(t)$, isolated individuals (those who have developed clinical symptoms and isolated) is $J(t)$, and the recovered population is $R(t)$.

A simple mathematical model for COVID-19 transmission dynamics is formulated as follows:

$$
\begin{aligned}
\frac{dS}{dt} &= r - \eta S I - \mu S, \\
\frac{dE}{dt} &= \eta S I - (c + \mu)E, \\
\frac{dQ}{dt} &= cE - (\gamma + \mu)Q, \\
\frac{dI}{dt} &= p\gamma Q - (\lambda + \mu + d_1)I, \\
\frac{dJ}{dt} &= \lambda I - (d_2 + \mu)J, \\
\frac{dR}{dt} &= (1 - p)\gamma Q + d_1 I + d_2 J - \mu R.
\end{aligned}
\tag{2.1}
$$

The parameters used in the model (2.1) are defined in Table 1.

Table 1. Short description of the parameters of the model (2.1).

Parameter	Short description
r	the recruitment rate of susceptible population including new birth and immigration
β	the disease transmission rate
μ	the natural death rate of the population
c	the rate at which exposed population is quarantined
γ	the rate at which quarantined people are infected
d_1	the rate at which people are recovered from quarantine
λ	the rate at which infected people are isolated
d	disease induced mortality rate
d_1	the disease induced mortality rate
d_2	the rate at which isolated people are recovered
p	portion of quarantine population who become infected

We now impose public awareness on the system (2.3) using the following discussion and example.

Spread of infectious disease is highly reliant on the public's responses and habits. For example, in 1973, the cholera outburst of Southeast Italy was possible to restrict within a few days since the local residents maintained essential activities to avoid further risks of transmission, and the Capasso-Serio epidemic model was established, in which individual behaviour was incorporated using the Monod-Haldane functional response [22]. In [23, 24], the infection rate was assumed as a decreasing function of infected individuals as the susceptible take essential measures to avoid being infected themselves.

Following [23, 24], we take the infection rate as a function of infected human, $I(t)$ as follows:

$$\eta(I) = \beta S I e^{-mI(t-\tau)} \tag{2.2}$$

where τ is the time delay as described in the Introduction. Therefore, the above model (2.1) becomes,

$$\begin{aligned}
\frac{dS}{dt} &= r - \beta e^{-mI(t-\tau)} S I - \mu S, \\
\frac{dE}{dt} &= \beta e^{-mI(t-\tau)} S I - (c + \mu)E, \\
\frac{dQ}{dt} &= cE - (\gamma + \mu)Q, \\
\frac{dI}{dt} &= p\gamma Q - (\lambda + \mu + d_1)I, \\
\frac{dJ}{dt} &= \lambda I - (d_2 + \mu)J, \\
\frac{dR}{dt} &= (1 - p)\gamma Q + d_1 I + d_2 J - \mu R.
\end{aligned} \tag{2.3}$$

As living biological populations always have nonnegative values, the initial functions of model (2.3) are introduced as follows:

$$S(0) = \phi_1(0),\ E(0) = \phi_2(0),\ Q(0) = \phi_3(0),\ I(\zeta) = \phi_4(\zeta),\ J(0) = \phi_5(0),$$

$$R(0) = \phi_6(0), \text{ and } \phi_4(\zeta) \geq 0, \ \zeta \in (-\tau, 0], \ \phi_i(0) > 0, \ i = 1, 2, 3, 4, 5 \tag{2.4}$$

where $\phi = (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6) \in C((-\tau, 0], \mathbb{R}^6)$, and C denotes the Banach space of continuous functions $\phi : (-\tau, 0] \to \mathbb{R}^6_+$ equipped with the sup-norm

$$||\phi|| = \sup_{-\tau \leq \zeta \leq 0} \{|\phi_1(\zeta)|, |\phi_2(\zeta)|, |\phi_3(\zeta)|, |\phi_4(\zeta)|, |\phi_5(\zeta)|, |\phi_6(\zeta)|\}.$$

We now discuss some basic properties of the model (nonnegativity, boundedness, etc.) using the above initial conditions.

2.1. Non-negativity of solutions

Using the results in [25], it is easy to show that the solution of the system of Eq (2.3) exists in the region $\mathbb{R}^6_+$. The next task is to show the non-negativity of the solutions. Biologically, non-negativity means the survival of a population. We prove this using the methods as provided by Bodnar [26] and Yang et al. [27]. In this respect we have the following theorem.

Theorem 1. *All the solutions of (2.3) with the positive initial condition (2.4) are positive for all $t > 0$.*

Proof. It is easy to check in system (2.3) that whenever choosing $X(\zeta) \in R_+$ such that $S = 0, E = 0, Q = 0, I = 0, J = 0, R = 0$, then

$$f_i(X)|_{x_i=0, X \in R^6_+} \geq 0, \tag{2.5}$$

where, $x_1(t) = S(t), x_2(t) = E(t), x_3(t) = Q(t), x_4(t) = I(t), x_5(t) = J(t), x_6(t) = R(t)$ and f_i are the right sides of system (2.3), for example, $f_1 = r - \beta S I e^{-mI(t-\tau)} - \mu S$.
Now, using the results in [27] and [26], we conclude that any solution $X(t)$ of (2.3) with $X(\zeta) \in C$ (where $C = ([-\tau, 0], R^6)$ is the Banach space of continuous functions with sup-norm), say $X(t) = X(t, X(\lambda))$, is such that $X(\zeta) \in R^6_+$ for all $t \geq 0$. Therefore, the set $\mathbb{R}^6_+$ is an invariant region for system (2.3). □

We have derived the region of attraction for the model (2.3) and provided using the following set:

$$B = \left\{ (S, E, Q, I, J, R) \in C([-\tau, 0], R^6_+) : 0 \leq S + E + Q + I + J + R \leq \frac{r}{\mu} \right\}.$$

Remark 1. *The recovery class has no impact on other model populations; therefore, we analyse the dynamics such as stability of equilibriums and Hopf bifurcation of the system* (2.3) *without considering the recovery class $R(t)$.*

2.2. Characteristic equation

Linearisation at any equilibrium $E(S, E, Q, I, J)$ gives the following characteristic equation:

$$\triangle(\xi) = | \xi I - \mathbf{A} - e^{-\xi\tau} \mathbf{B} | = 0,$$

where $\mathbf{A} = [a_{ij}]$ and $\mathbf{B} = [b_{ij}]$ are the following 5×5 matrices:

$$\mathbf{A} = \left[a_{ij}\right] = \begin{bmatrix} -\beta e^{-mI} I - \mu & 0 & 0 & 0 & 0 \\ \beta e^{-mI} I & -(c+\mu) & 0 & 0 & 0 \\ 0 & c & -(\gamma+\mu) & 0 & 0 \\ 0 & 0 & p\gamma & -(\lambda+\mu+d_1) & 0 \\ 0 & 0 & 0 & \lambda & -(d_2+\mu) \end{bmatrix},$$

and

$$\mathbf{B} = \left[b_{ij}\right] = \begin{bmatrix} 0 & 0 & 0 & b_{14} & 0 \\ 0 & 0 & 0 & -b_{14} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

where $b_{14} = -\beta e^{-mI} S + m\beta e^{-mI} SI$.

$$\begin{aligned} \sigma_1 &= -a_{11} - a_{22} - a_{33} - a_{44}, \\ \sigma_2 &= a_{11}a_{22} + a_{11}a_{33} + a_{22}a_{33} + a_{11}a_{44} + a_{22}a_{44} + a_{33}a_{44}, \\ \sigma_3 &= -a_{11}a_{22}a_{33} + b_{14}a_{32}a_{43} - a_{11}a_{22}a_{44} - a_{11}a_{33}a_{44} - a_{22}a_{33}a_{44}, \\ \sigma_4 &= -a_{11}b_{14}a_{32}a_{43} - b_{14}a_{21}a_{32}a_{43} + a_{11}a_{22}a_{33}a_{44}, \\ \sigma_5 &= b_{14}a_{32}a_{43}, \\ \sigma_6 &= -a_{11}b_{14}a_{32}a_{43} - b_{14}a_{21}a_{32}a_{43}. \end{aligned}$$

Finally, we get the characteristic equation as

$$[\xi + (d_2+\mu)][\xi^4 + \sigma_1\xi^3 + \sigma_2\xi^2 + \sigma_3\xi + \sigma_4 + e^{-\xi\tau}(\sigma_5\xi + \sigma_6)] = 0. \tag{2.6}$$

The stabilities of different equilibria are analysed using the root of this equation at a particular equilibrium point. If the roots of the characteristic Eq (2.3) are negative or containing negative real parts, at an steady point, then the point is stable; otherwise; it is unstable.

3. Equilibria and stability analysis

The system (2.3) has two equilibriums: namely, the disease-free equilibrium $E_0(\frac{r}{\mu}, 0, 0, 0, 0)$ and the endemic equilibrium $E^*(S^*, E^*, Q^*, I^*, J^*)$, given by

$$Q^* = \frac{I^*(\lambda+\mu+d_1)}{p\gamma},\ J^* = \frac{\lambda I^*}{d_2+\mu},\ E^* = \frac{(\gamma+\mu)Q^*}{c}, S^* = \frac{r-(c+\mu)E^*}{\mu}$$

where I^* satisfies the equation

$$f(I) = r - \beta SIe^{-mI} - \mu S = 0. \tag{3.1}$$

Remark 2. *We have shown numerically that there exists a positive root of Eq* (3.1)*. Also, the root is biologically feasible and unique if* $R_0 > 1$ *(see Figures 2 and 3).*

3.1. Stability analysis without delay (i.e. $\tau = 0$)

Using the the characteristic Eq (2.6), we derive that at the disease-free equilibrium E_0, the Jacobian matrix has all eigenvalues negative if and only if

$$\beta(1-m)rcp\gamma - \mu(c+\mu)(\gamma+\mu)(\lambda+\mu+d_1) < 0 \tag{3.2}$$

holds. We derive the basic reproduction number as

$$R_0 = \frac{\beta(1-m)rcp\gamma}{\mu(c+\mu)(\gamma+\mu)(\lambda+\mu+d_1)}.$$

Then, we have the following theorem.

Theorem 2. *The disease-free steady state E_0 is stable if $R_0 < 1$ and unstable if $R_0 > 1$. A forward transcritical bifurcation occurs at $R_0 = 1$.*

Remark 3. *It is important to note that R_0 depends on m (the impact of awareness), and hence it is important how efficiently humans change their behaviour. In light of the fact that R_0 is monotonically decreasing with increasing m, this suggests that eradication of disease, as represented by a stable disease-free steady state E_0, is possible if $R_0 < 1$. The available means to achieve this are quarantine, hand washing, social distancing, etc. [17, 18].*

At the endemic equilibrium E^*, the characteristic equation takes the form

$$[\xi + (d_2+\mu)][\xi^4 + \sigma_1\xi^3 + \sigma_2\xi^2 + (\sigma_3+\sigma_5)\xi + \sigma_4 + \sigma_6] = 0, \tag{3.3}$$

i.e., one eigenvalue is $-(d_2+\mu)$, which is always negative, and the rest of the eigenvalues satisfy,

$$\xi^4 + \sigma_1\xi^3 + \sigma_2\xi^2 + (\sigma_3+\sigma_5)\xi + (\sigma_4+\sigma_6) = 0.$$

If the Routh-Hurwitz criterion [28] is satisfied at E^*, then all the roots of the characteristic equation will be negative or include negative real parts. Using this, we get the stability condition for E^* as:

$$\begin{aligned} &\sigma_1 > 0, \quad (\sigma_4+\sigma_6) > 0, \sigma_1\sigma_2 - (\sigma_3+\sigma_5) > 0, \\ &[\sigma_1\sigma_2 - (\sigma_3+\sigma_5)](\sigma_3+\sigma_5) - \sigma_1^2(\sigma_4+\sigma_6) > 0. \end{aligned} \tag{3.4}$$

From the above discussion, we can write the following theorem.

Theorem 3. *The endemic equilibrium point E^* is stable if the conditions (3.4) are satisfied.*

3.2. Stability analysis of the system with delay

Suppose that the endemic equilibrium E^* is stable, i.e., the conditions in (3.4) are satisfied. Now, a necessary condition that E^* will change its stability is that Eq (2.6) possesses a purely imaginary roots.

Let $i\theta$, $\theta \in \mathbb{R}$, be a purely imaginary root of Eq (2.6). Putting $\xi = i\theta$, $\theta \in \mathbb{R}$, in (2.6) and splitting the real and imaginary portions, one can obtain

$$\sigma_5 \cos\theta\tau - \sigma_6 \sin\theta\tau = \sigma_1\theta^3 - \sigma_3\theta, \tag{3.5}$$

$$\sigma_5 \sin\theta\tau + \sigma_6 \cos\theta\tau = -\theta^4 + \sigma_2\theta^2 - \sigma_4. \tag{3.6}$$

First, squaring and adding the above equations and then taking $\theta^2 = l$, we obtain

$$l^4 + \omega_1 l^3 + \omega_2 l^2 + \omega_3 l + \omega_4 = 0, \tag{3.7}$$

where,

$$\omega_1 = {\sigma_1}^2 - 2\sigma_2, \quad \omega_2 = \sigma_2^2 + 2\sigma_4 - 2\sigma_1\sigma_3,$$
$$\omega_3 = -2\sigma_2\sigma_4 + \sigma_3^2 - \sigma_5^2, \omega_4 = \sigma_4^2 - (\sigma_5^2 + \sigma_6^2).$$

If the coefficients of (3.7) satisfy the Routh-Hurwitz criterion [28], then the roots of Eq (3.7) will contains negative real parts. In that case, the characteristic Eq (2.6) does not possess purely imaginary roots.

We summarise the above discussion as a proposition.

Proposition 1. *Suppose that the system* (2.3) *without delay (i.e. $\tau = 0$) is stable. Now, if conditions*

$$\omega_1 > 0, \quad \omega_4 > 0, \quad \omega_1\omega_2 - \omega_3 > 0, \quad (\omega_1\omega_2 - \omega_3)\omega_3 - \omega_1^2\omega_4 > 0.$$

are satisfied, then E^* is locally asymptotically stable (LAS) for all $\tau > 0$.

If $\omega_4 < 0$ holds for a set of parameters, then Eq (3.7) will have at least one positive root. Suppose that θ_0^2 is the smallest positive root of (3.7). This implies that $\pm i\theta_0$ is a purely imaginary root related to $\tau = \tau_0$. Using Butler's lemma [29], there is a threshold value of τ, denoted as τ^* below, for which the equilibrium E^* will remain stable and unstable for $\tau > \tau^*$, and the stability change occurs through Hopf bifurcation. We have calculated the critical value τ^* using Eq (3.5) as

$$\tau^* = \frac{1}{\theta_0} cos^{-1}\left[\frac{\sigma_6(-\theta_0^4 + \sigma_2\theta_0^2 - \sigma_4) + \sigma_5\sigma_1\theta_0^3}{\sigma_5^2 + \sigma_6^2}\right] + \frac{2\pi n}{\theta_0}, \quad n = 0, 1, 2, 3, \ldots$$

The following theorem can be written using the above discussion.

Theorem 4. *Suppose that the conditions in* (3.4) *hold, and $\omega_4 < 0$ is satisfied; then, for $\tau < \tau^*$, the endemic equilibrium E^* is stable and unstable for $\tau > \tau^*$. Moreover, Hopf bifurcation occurs at $\tau = \tau^*$, providing $4\theta_0^6 + A_1\theta_0^4 + A_2\theta_0^2 + A_3 \neq 0$, where*

$$A_1 = 3\sigma_1 - 6\sigma_2, \quad A_2 = 2\sigma_2 + 4\sigma_4 - 4\sigma_1\sigma_3, \quad A_3 = \sigma_3^2 - 2\sigma_2\sigma_4 - \sigma_5^2.$$

Proof. The proof of the first part of the theorem follows from the above arguments. Thus, we only provide the proof of the last part of the theorem, as follows:
Differentiating (2.6) with respect to τ, one can get

$$\frac{d\tau}{d\xi} = \frac{4\xi^3 + 3\sigma_1\xi^2 + 2\sigma_2\xi + \sigma_3}{\sigma_5\xi^2 + \sigma_6} e^{\xi\tau} + \frac{\sigma_5}{\sigma_5\xi^2 + \sigma_6\xi} - \frac{\tau}{\xi}.$$

Using Eq (3.5), after some simple calculation, we finally get

$$\left[\frac{d(Re\xi)}{d\tau}\right]_{\tau=\tau^*} = \left[Re\left(\frac{d\xi}{d\tau}\right)^{-1}\right]_{\xi=i\theta_0} = \left[\frac{4\theta_0^6 + A_1\theta_0^4 + A_2\theta_0^2 + A_3}{\sigma_5\theta_0^2 + \sigma_6^2}\right]_{\theta=\theta_0}. \tag{3.8}$$

Since $4\theta_0^6 + A_1\theta_0^4 + A_2\theta_0^2 + A_3 \neq 0$, we have

$$\left[\frac{d(Re\xi}{d\tau}\right]_{\tau=\tau^*} \neq 0.$$

Thus, the transversality condition is satisfied and consequently Hopf bifurcation occurs at $\tau = \tau^*$. □

4. Sensitivity analysis

This section provides the sensitivity analysis of the parameters of the system (2.3). Sensitivity analysis tells about the importance of a parameter on the dynamics of a system. The methodology provided in [30, 31] is followed for this analysis. A short description of this method is as follows [32].

Suppose that we want to analyse whether the generic parameter ς is significant with respect to the population X. For this we use the original system (2.3) with the additional differential equations for the sensitivities. We differentiate partially the original differential equations with respect to the parameter ς to get the auxiliary equations. The partial derivative of each model population with respect to the desired parameter, i.e., $\frac{\partial X}{\partial \varsigma}$, is known as the sensitivity index of that parameter.

5. Numerical results and discussion

To study the dynamic behavior, extensive numerical simulations were carried out for no time delay and then for various values of τ. The numerical experiments were performed on the systems (with delay and without delay) to confirm our theoretical findings. Parameters values were taken as follows:

$$\begin{aligned} &r = 40,\ \beta = 0.0025,\ m = 0.2,\ \mu = 0.01,\ d_1 = 0.1,\ d_2 = 0.1,\\ &c = 0.6,\ p = 0.6,\ \gamma = 0.02,\ \lambda = 0.1. \end{aligned} \tag{5.1}$$

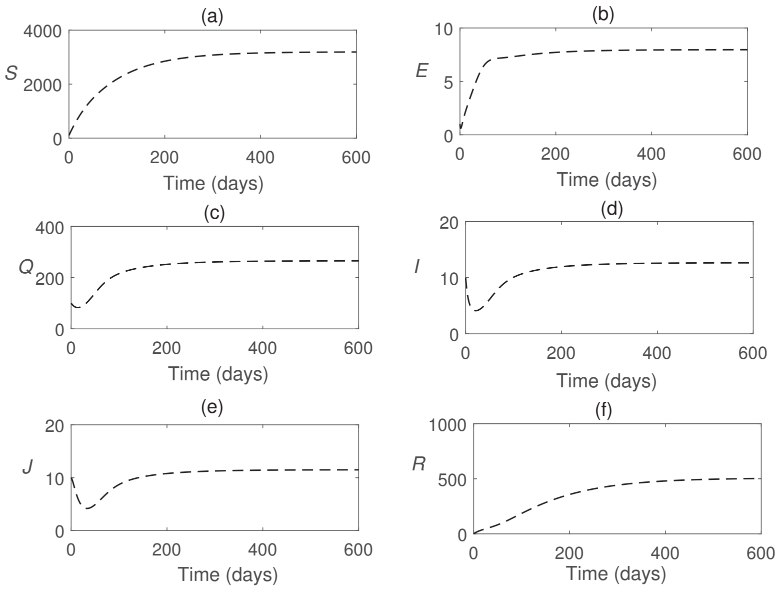

Figure 1. Numerical simulation of model (2.3) is plotted taking the set of parameters as in (5.1).

A time series solution of system (2.3) has been shown in Figure 1 taking parameters as in (5.1). For this set of parameters, the system solution converges to the endemic equilibrium E^*. Also, the conditions of Theorem 3.4 are satisfied, i.e., endemic equilibrium is stable. Figures 2 and 3 show that system (2.3) possesses a unique endemic equilibrium, as at $R_0 = 1$ forword bifurcation occurs.

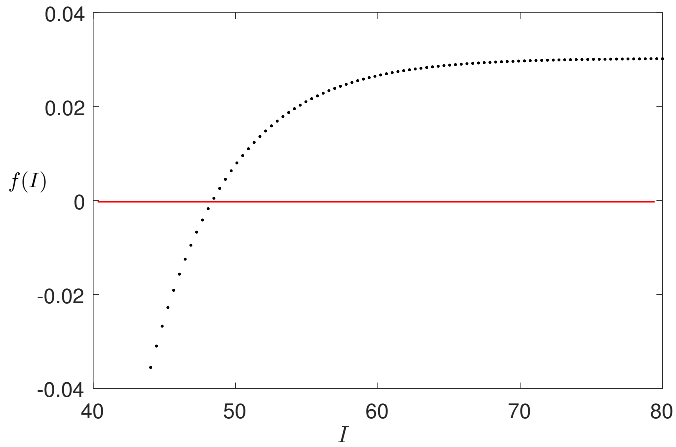

Figure 2. $f(I)$ is plotted versus I. I is varied between 40 and 80. Parameters' values are as given in Figure 1.

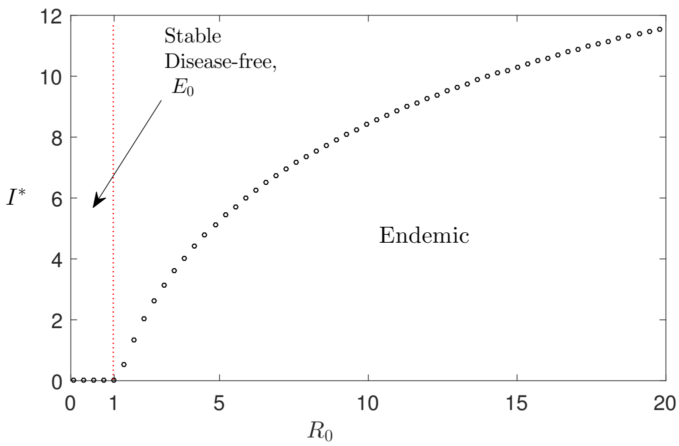

Figure 3. Forward bifurcation at $R_0 = 1$. Force of infection β has been varied, and all other parameters' values are as given in Figure 1.

In Figure 4, it is observed that if the rate of awareness (i.e., m) increases, the equilibrium numbers of infected, exposed, etc. individuals decreased, except for susceptible ones. When it crosses some threshold value, the system becomes disease-free. Basic reproduction number R_0 is a decreasing function of m. As m increases, R_0 decreases. For some higher value of m, R_0 is below unity. Thus, awareness in the form of lockdowns, social distancing, use of masks, etc., has the ability to yield a disease-free system.

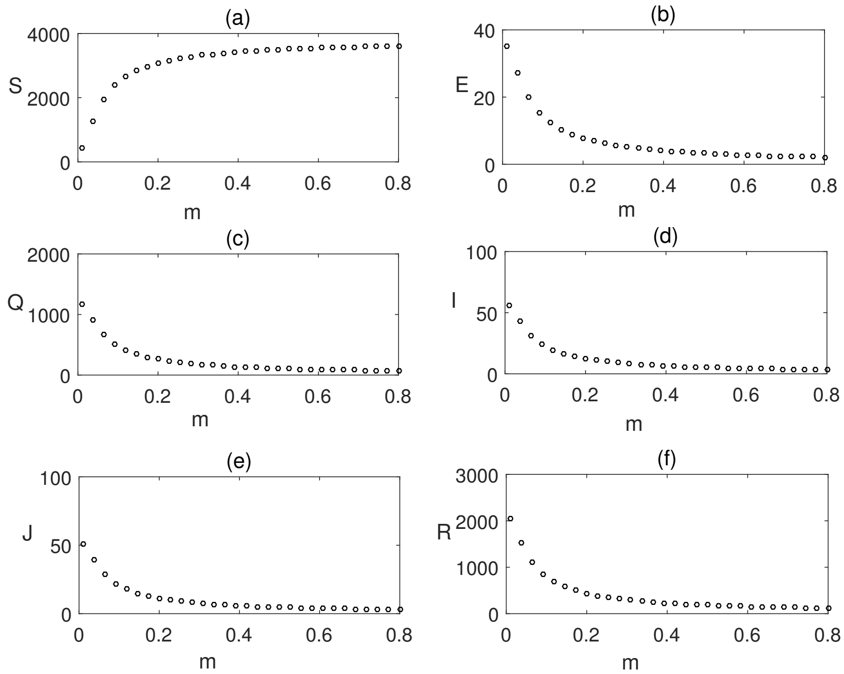

Figure 4. Effect of awareness, *m*, on the steady states of the model populations. Parameters' values are taken as in Figure 1.

Effects of time delay τ have been shown in Figures 5 and 6, showing that as the value of delay τ exceeds its critical value τ^*, all variables bifurcate into periodic solutions at the endemic equilibria E^*. This specifies that the number of infective cases will be oscillating, which makes difficulties in forecasting the size of the epidemic. The endemic equilibrium takes a longer time to become steady. These figures indicate that equilibrium E^* of model system (2.3) is stable for $\tau < \tau^*$ and unstable if the inequality is reversed.

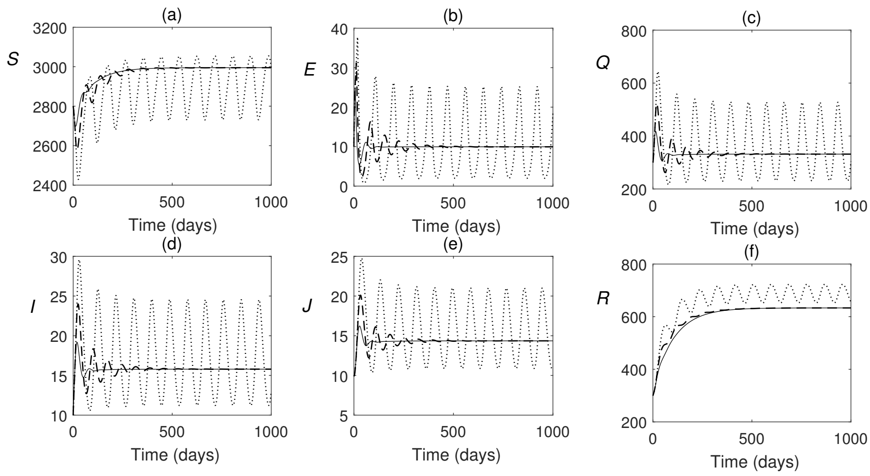

Figure 5. Numerical solutions of the system with different values of delay τ. Parameters' values are the same as in Figure 2.

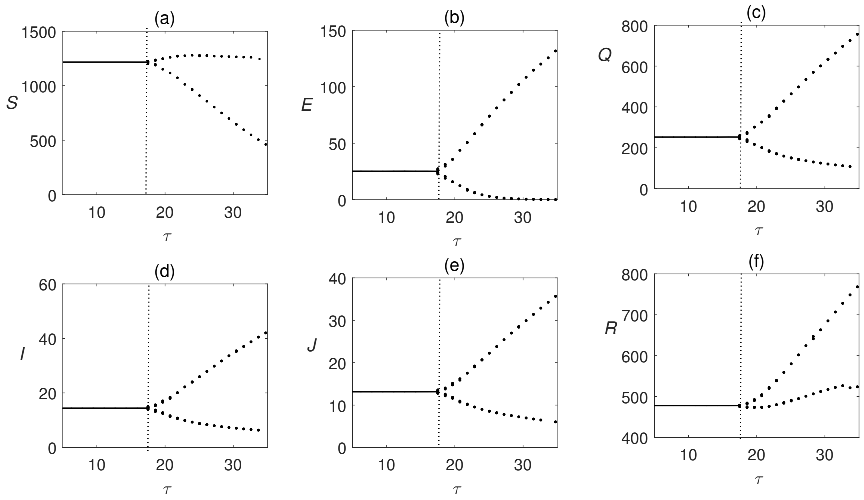

Figure 6. Bifurcation diagram of endemic system taking τ as main parameter. The minimum/maximum values of the periodic oscillations are plotted whenever they exist. Solid lines indicate stable endemic equilibrium.

The sensitivity indices of the model parameters are plotted in Figure 7 with respect to the population $J(t)$. We have seen that each parameter is significant for the dynamics of the system. Some are positively, sensitive and some are negatively sensitive for the isolated population. For example, the awareness parameter m and infection rate β are negatively sensitive, but λ (rate of isolation) is positively sensitive throughout the time interval.

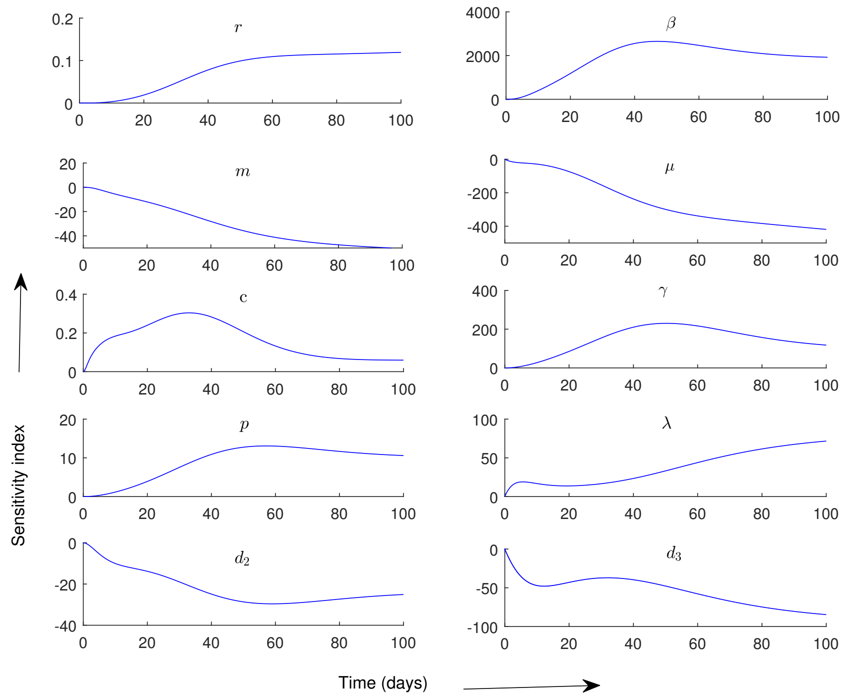

Figure 7. Sensitivity indices of the model parameters are plotted with time.

6. Conclusions

In this article, a mathematical model has been proposed and analysed using a set of delay differential equations with an aim to study the dynamics of COVID-19 transmission in a region. Impact of media awareness and a time delay due to the lag in reporting the infected cases are included in the model by considering the infection transmission rate as a function of the infected human population $I(t)$ and by further assuming that the progress of public awareness depends on the number of infected people.

The proposed model possesses two equilibria, namely, the disease-free and the coexisting equilibria. We have derived the basic reproduction number R_0 for this pandemic. We have seen that it depends on the awareness rate m. The disease-free equilibrium is stable when R_0 is below unity (Remark 3). Numerical simulation shows that the endemic equilibrium exists when R_0 crosses unity and is stable depending on the parameters of the model system. We have also seen the effects of time delay in the system. The coexisting equilibrium is stable when the delay τ is below its critical value τ^*. The sensitivity analysis of the model parameters demonstrates that each parameter is sensitive for this pandemic, but the media impact rate m and the force of infection β are the most important ones.

In summary, media awareness in the form of social distancing, lockdown, testing, etc., can reduce the pandemic with sufficiently tolerable time lag. A rapid social awareness can help us currently fight back this highly contagious disease. Good knowledge on COVID-19 transmission is associated with hopeful attitudes and appropriate practices towards it. On the other hand, health education programs for improving knowledge are helpful for encouraging a positive attitude and maintaining safe practices. In this way, scientists will be able to have some time in discovering the appropriate medicine. Optimistically, with everyone's combined efforts, we will win the fight against COVID-19 in the near future.

Conflict of interest

Authors declare that there is no conflict of interest.

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