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Research article

Product evaluation through multi-criteria decision making based on fuzzy parameterized Pythagorean fuzzy hypersoft expert set

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Abstract: In many real-world decision-making situations, uncertain nature of parameters is to be discussed to have unbiased and reliable decisions. Most of the existing literature on fuzzy soft set and its related structures ignored the uncertain parametric attitudes. The concept of fuzzy parameterization is launched to tackle the limitations of existing soft set-like models. Several extensions have already been introduced by using the concept of fuzzy parameterization. In this research, a novel extension, fuzzy parameterized Pythagorean fuzzy hypersoft expert set is aimed to be characterized. This model is more flexible and reliable as compared to existing models because it addresses their insufficiencies for the consideration of multi-argument approximate function. With the entitlement of this function, it tackles the real-life scenarios where each attribute is meant to be further classified into its respective sub-attribute valued disjoint set. The characterization of fuzzy parameterized Pythagorean fuzzy hypersoft expert set is accomplished by employing theoretic, axiomatic and algorithmic approaches. In order to validate the proposed model, an algorithm is proposed to study its role in decision-making while dealing with real-world problem. Moreover, the proposed model is compared with the most relevant existing models to assess its advantageous aspects.

Keywords: soft set; fuzzy soft set; Pythagorean fuzzy soft set; hypersoft set; hypersoft expert set
Mathematics Subject Classification: 03B52, 03E72, 03E75

1. Introduction

Maji et al. [15] introduced the concept of fuzzy soft set (FSS) as a generalization of fuzzy set (FS) [1] and soft set (SS) [2] to adequate the limitations of FS regarding the provision of parameterization tool. The FSS not only validates the FS but also fulfills the characteristics of SS. It uses the collection of fuzzy subsets rather than power set merely as range of single-argument approximate function over the universe of discourse. *Çagman* et al. [3] introduced the structure of intuitionistic fuzzy soft set (IFSS) and provided an application for some decision making methods of this field. Xindong et al. [4] combined the structure of SS and Pythagorean fuzzy set to form Pythagorean fuzzy soft set. The researchers like Feng et al. [5], Chen et al. [6], Liu et al. [7, 8] and Meng et al. [9] made rich contributions regarding the implementation of intuitionistic fuzzy set-like models in various real-world scenarios. The contributions of Athira et al. [10, 11], Feng et al. [12] and Xiao et al. [13] are prominent for the utilization of Pythagorean fuzzy set-like models in several situations. Recently Xiao [14] introduced a complex mass function to predict interference effects. The SS models emphasize the opinion of single expert in a single model. But there are various situations when there is a need of different opinions in different models. Alkhazaleh et al. [16] conceptualized soft expert set (SES) to address the limitations of SS regarding the opinions of different experts in different models. Ihsan et al. [17] conceptualized convexity-cum-concavity on SES and discussed its some properties. Alkhazaleh et al. also extended their work to fuzzy soft expert set (FSES) [18] by introducing its use in decision making problems (DMPs). Ihsan et al. [19] gestated the convexity on FSES and explained its some properties. Broumi et al. [20] conceptualized intuitionistic fuzzy soft expert set and discussed its application in DMPs. In 1998, Smarandache [21] generalized SS to hypersoft set (HsS) by replacing single argument approximate function with multi-argument approximate function. Saeed et al. [22] introduced the fundamentals of HsS and used in DMPs. Ahsan et al. [23] made an analytical cum theoretical approach to composite mappings on fuzzy hypersoft set (FHsS) and discussed its various properties. They also verified several results with examples. Saeed et al. [24] introduced the hybrids of hypersoft graph and discussed its theoretic operations with generalized results. Saeed et al. [25] gave the idea of neutrosophic hypersoft mappings and used in medical diagnosis. Saeed et al. [26] studied certain operations and products of neutrosophic hypersoft graph. Rahman et al. [28] described the structures of HsS like complex FHsS, intuitionistic FHsS and neutrosophic HsS. Rahman et al. [29] gave the idea of convexity on HsS and proved its various properties. Rahman et al. [30] developed the structure of rough HsS and gave an application for the best selection of chemical material in decision-making. Rahman et al. [31] employed a novel approach to neutrosophic hypersoft graph and discussed its certain properties. Rahman et al. [32] introduced the aggregation operations of complex FHsS and applied them in DMPs. They also developed the structure of interval-valued complex FHsS. Rahman et al. [33] conceptualized the bijective HsS and discussed its application in DMPs. Ihsan et al. [34] generalized the HsS to hypersoft expert set (HsES) to acknowledge the opinions of different experts in different models under HsS environment. Ihsan et al. [35] conceptualized the structure FHsES and explained the application of DMPs with the help of proposed algorithm.

1.1. Research gap and motivation

Following points will explain the research gap and motivations behind the choice of proposed structure:

(1) In decision-making process, the choice of parameters plays a vital role for having a reliable decision/ranking/selection. It is commonly observed that importance of parameters varies scenario to scenario basis. The roughness regarding the choice of parameters may lead to biased decision. Therefore the uncertain nature of parameters should be emphasized. In this regards, the concept of fuzzy parameterization is initiated to tackle the uncertain and vague nature of parameters. For example, consider a real-world decision-making scenario, recruitment process, in which some decision-makers consider confidence as a prominent parameter whereas other decision-makers do not take it as an important attribute for recruitment process. Therefore, this issue is tackled by assigning a fuzzy membership to each parameter or its sub-parametric tuple to assess its uncertain behavior. Many researchers have already discussed the extensions of fuzzy parameterization with various fuzzy soft set-like models but the most prominent contributions are being quoted in the next point to make the research gap clear to the readers.

(2) Çağman et al. [36] conceptualized fuzzy parameterized soft set (FPSS) and applied uncertain degree to parameters. They suggested a proposed method to solve the DMPs and gave an application for the best selection of product. Tella et al. [38] introduced the structure of fuzzy parameterized fuzzy soft set (FPFSS) and used for multi criteria decision with the help multi experts assessment. Zhu et al. [39] applied this idea in DMPs. Sulukan et al. [40] conceptualized fuzzy parameterized intuitionistic fuzzy soft set and applied in performance-based value assignment problem. The SS-like structures deal with the opinion of single expert only in a model. But there are various situations, where we need different opinions of different experts in one model. In order to overcome this situation without using any additional operation, Alkhazaleh and Salleh [16] gave the concept of SES and then extended it to FSES [18]. Bashir et al. [41] combined the structures of fuzzy parameterized with SES and introduced the hybrid of fuzzy parameterized soft expert set (FPSES) with application in DMPs. They discussed the application with generalized algorithm of Alkhazaleh & Salleh [16] and compared the results. Ayman et al. [42] conceptualized fuzzy parameterized fuzzy soft expert set and used in decision making. Selvachandran et al. [43] extended the work of Ayman to fuzzy parameterized intuitionistic fuzzy soft expert set. Chinnadurai and Arulselvam [44] conceptualized Q-Pythagorean FSES and applied in multi-criteria decision making. In 2021, Rahman et al. [45] extended the work of FPSS to FP-hypersost set by changing the single set of attribute to multi disjoint attribute-valued sets and discussed the applications in DMPs. Rahman et al. [46] extended their own work and introduced the concept of neutrosophic HsS with applications in DMPs. Rahman et al. [47] added some more work on parametrization in literature by introducing the concept of neutrosophic set under HsS with different settings like fuzzy, intuitionistic fuzzy and neutrosophic sets. Ihsan et al. [34] extended the work of HsS into HsES by giving an application of DMPs.

(3) It can be viewed that the above FPSS-like models deal with opinion of only single expert. But in real-life, there are several situations when we need different opinions of different experts in one model. To tackle this situation, SES has been developed. However, there are also various

situations when features are further classified into their relevant attribute-valued disjoint sets. The Figure 1 presents the analysis of SES and HsES stuctures. It shows the choice of a mobile by using parameters in soft SES and sub-parametric tuples in HsES. Therefore, in need of new structure to handle such situations with multi-decisive opinions under multi-argument SS like environment, HsES has been developed.

(4) Motivating from the above literature in general and specifically from [36–38, 41, 45–47], a novel structure FPPFHsES is developed with various properties. By using the aggregation operations of FPPFHsES, an algorithm is proposed and applied in multi-attribute decision-making problem.

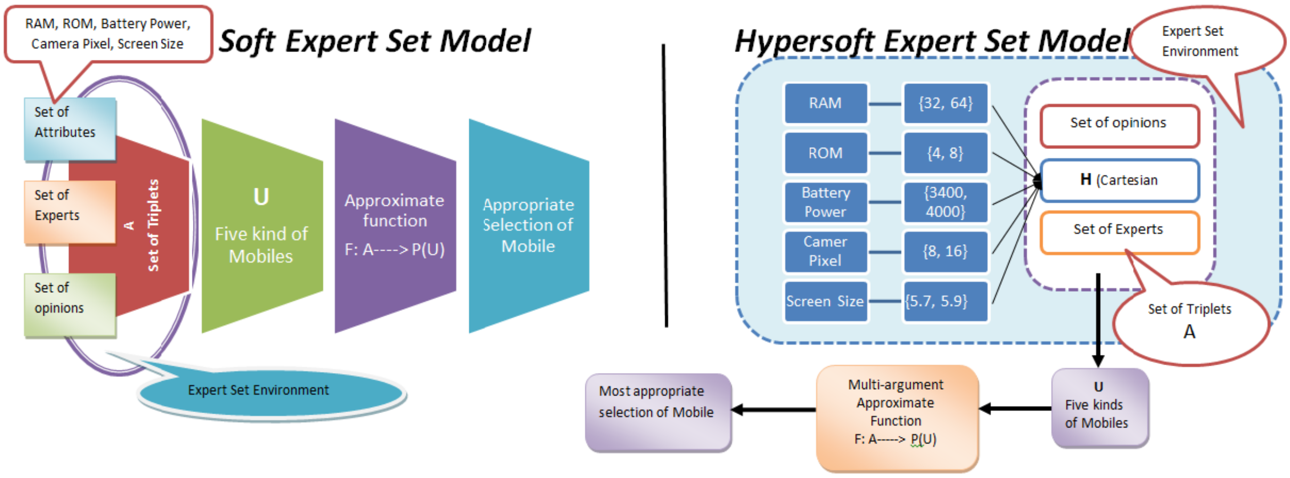

Figure 1. Comparison of soft expert set and hypersoft expert set models.

1.2. Main contributions

Some main contributions of the article are described as:

(1) The essential axiomatic properties and set-theoretic operations of FPPFHsES are characterized to tackle the situations where (i) fuzzy parametric tuples and (ii) the classification of parameters into their respective sub-parametric values in the form of sets are necessary to be considered.

(2) A decision-making based algorithm is proposed to solve the real-world decision-making problems and validated with the help of application-based illustrated example for the optimal selection of laptop. The operational roles of parameters are discussed in detail to judge their significance in the proposed algorithmic model and their uncertain natures are managed by assigning fuzzy membership grades to them and their relevant parametric tuples.

(3) The advantageous aspects of the proposed model are assessed through its vivid comparison with some relevant existing models by considering the most significant evaluating features. Moreover, a rich discussion is provided on the generalization of the proposed model by describing its some particular cases.

1.3. Management of paper

The resting article is structured as presented in Figure 2.

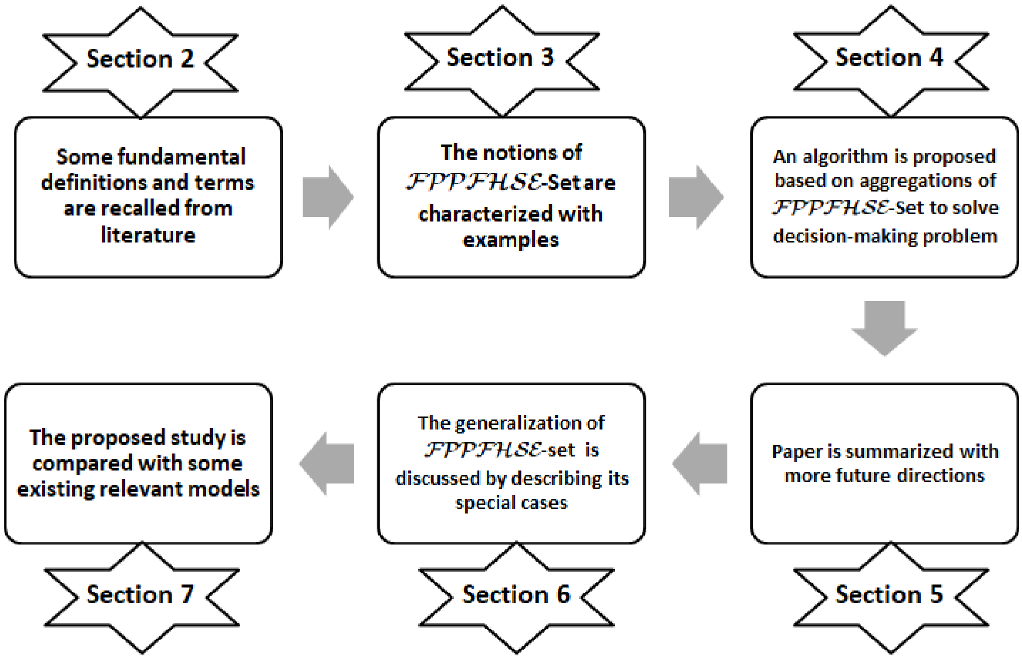

Figure 2. Organization of paper.

2. Preliminaries

This part of the paper reviews the elementary notions regarding fuzzy soft expert sets and hypersoft sets. In this article, $\mathcal{I}$ represents set of specialists(experts) and set of parameters is denoted by $\mathcal{Q}$, $\mathcal{P} = \mathcal{Q}\times\mathcal{I}\times\mathcal{U}$ with $\mathcal{S} \subseteq \mathcal{P}$. While $\mathcal{U}$ represents a set of conclusions i.e, $\mathcal{U}= \{0 = disagree, 1 = agree\}$ and $\hat{\Delta}$ represents the universe with power set $P(\hat{\Delta})$ and $\mathbb{I}$ =[0, 1]. The symbol $\Omega_{Œ}$ will represent the collection of $\mathcal{FPIFH}\int\mathcal{E}$S in this article.

In 1965, Zadeh [1] gave the idea of FS as a generalization of classical set (crisp set) to manage uncertain situations. This set utilizes a membership value (Mv) which maps the set of items to $\mathbb{I}$.

Definition 2.1. [1] A set "F_z" is called a *FS* written as $F_z = \{(\hat{o}, B(\hat{r}))|\hat{o} \in \hat{\Delta}\}$ with $B : \hat{\Delta} \rightarrow \mathbb{I}$ and $B(\hat{o})$ represents the Mv of $\hat{o} \in F_z$.

Definition 2.2. [1] Let $\mathcal{G}$ and $\mathcal{H}$ be two FSs, then following characteristics hold:

(1) $\mathcal{G} \cup \mathcal{H} = \{(\hat{r}, max\{B_{\mathcal{G}}(\hat{o}), B_{\mathcal{H}}(\hat{o})\})|\hat{o} \in \hat{\Delta}\}$;

(2) $\mathcal{G} \cap \mathcal{H} = \{(\hat{r}, min\{B_{\mathcal{G}}(\hat{o}), B_{\mathcal{H}}(\hat{o})\})|\hat{o} \in \hat{\Delta}\}$;

(3) $\mathcal{G}^c = \{\hat{r}, 1 - B_{\mathcal{G}}(\hat{o})|\hat{o} \in \hat{\Delta}\}$.

The FS focuses on Mv just for managing uncertain situations yet there are numerous circumstances where non-membership value (NMv) is important to be considered so Atanassov [2] presented IFS as a an extension of FS.

Definition 2.3. [3] A set "$\mathcal{J}$" is called an *IFS* written as $\mathcal{J} = \{(\check{a}, < Z_{\mathcal{J}}(\check{a}), X_{\mathcal{J}}(\check{a}) >), |(\check{a}) \in \hat{\Delta}\}$ with $Z_{\mathcal{J}} : \mathbb{I} \rightarrow \hat{\Delta}$, $X_{\mathcal{J}} : \mathbb{I} \rightarrow \hat{\Delta}$ and $Z_{\mathcal{J}}(\check{a})$, $X_{\mathcal{J}}(\check{a})$ represent the Mv and the NMv of $\check{a} \in \hat{\Delta}$ satisfying the inequality $0 \leq Z_{\mathcal{J}}(\check{a}) + X_{\mathcal{J}}(\check{a}) \leq 1$.

Definition 2.4. [3] Let $\mathcal{J}_1$ and $\mathcal{J}_2$ be two IFSs, then following characteristics hold:

(1) $\mathcal{J}_1 \cup \mathcal{J}_2 = \{(\hat{a}, max\{Z_{\mathcal{J}_1}(\hat{a})), Z_{\mathcal{J}_2}(\hat{a}))\}, min\{X_{\mathcal{J}_1}(\hat{a})), X_{\mathcal{J}_2}(\hat{a}))\}\}$;

(2) $\mathcal{J}_1 \cap \mathcal{J}_2 = \{(\hat{a}, min\{Z_{\mathcal{J}_1}(\hat{a})), Z_{\mathcal{J}_2}(\hat{a}))\}, max\{X_{\mathcal{J}_1}(\hat{a})), X_{\mathcal{J}_2}(\hat{a}))\}\}$;

(3) $\mathcal{J}_1^c = \{(\check{a}, < X_{\mathcal{J}}(\check{a}), Z_{\mathcal{J}}(\check{a}) >), |(\check{a}) \in \hat{\triangle}\}$.

Definition 2.5. A set "$\mathcal{J}$" is called *Pythagorean fuzzy set* written as $\mathcal{J} = \{(\check{a}, < Z_{\mathcal{J}}(\check{a}), X_{\mathcal{J}}(\check{a}) >), |(\check{a}) \in \hat{\triangle}\}$ with $Z_{\mathcal{J}} : \mathbb{I} \to \hat{\triangle}$, $X_{\mathcal{J}} : \mathbb{I} \to \hat{\triangle}$ and $Z_{\mathcal{J}}(\check{a})$, $X_{\mathcal{J}}(\check{a})$ represent the Mv and the NMv of $\check{a} \in \hat{\triangle}$ satisfying the inequality $0 \leq Z_{\mathcal{J}}^2(\check{a}) + X_{\mathcal{J}}^2(\check{a}) \leq 1$.

The FS and IFS portray some sort of deficiency in regards to the thought of numerical methods. To deal with this limitation, Molodtsov [2] conceptualized SS as a new mathematical tool to handle data having uncertainties.

Definition 2.6. [2] A pair $(\Upsilon_s, \mathcal{A})$ is named as SS on $\hat{\triangle}$, with $\Upsilon_s : \mathcal{A} \to P(\hat{\triangle})$ and $\mathcal{A}$ is a subset of $\mathcal{Q}$ (a set of parameters).

Definition 2.7. [15] A pair $(\Lambda_L, \mathcal{B})$ is named as FSS on $\hat{\triangle}$, with $\Lambda_L : \mathcal{B} \to FP(\hat{\triangle})$ and $FP(\hat{\triangle})$ is a collection of fuzzy subsets over $\hat{\triangle}$, $\mathcal{B} \subseteq \mathcal{Q}$.

Definition 2.8. [3] A pair $(\Lambda_M, \mathcal{B})$ is named as IFSS on $\hat{\triangle}$, with $\Lambda_M : \mathcal{B} \to IFP(\hat{\triangle})$ and $IFP(\hat{\triangle})$ is a collection of intuitionistic fuzzy subsets over $\hat{\triangle}$.

Definition 2.9. [3] A pair $(\Lambda_P, \mathcal{B})$ is named as *Pythagorean fuzzy soft set* on $\hat{\triangle}$, with $\Lambda_P : \mathcal{B} \to PFP(\hat{\triangle})$ and here $PFP(\hat{\triangle})$ is a collection of Pythagorean fuzzy subsets over $\hat{\triangle}$.

In some real-world situations the classification of attributes into sub-attributive valued sets is essential. The current idea of SS is inadequate and incongruent with such situations so Smarandache [16] acquainted HS which addresses the inadequacy of SS.

Definition 2.10. [21] Suppose $\mho_1, \mho_2, \mho_3, ..., \mho_\alpha$, for $\alpha \geq 1$, be α different parameters and the sets $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, ..., \mathcal{L}_\alpha$ have corresponding parametric values with $\mathcal{L}_m \cap \mathcal{L}_n = \emptyset$, for $m \neq n$, and $m, n \in \{1, 2, 3, ..., \alpha\}$. Then the pair $(\eta, \bigodot)$ is named as a *hypersoft set* on $\hat{\triangle}$ where $\bigodot = \mathcal{L}_1 \times \mathcal{L}_2 \times \mathcal{L}_3 \times ..... \times \mathcal{L}_\alpha$ and $\eta : \bigodot \to P(\hat{\triangle})$.

Definition 2.11. [49] Suppose $\mho_1, \mho_2, \mho_3, ..., \mho_\alpha$, for $\alpha \geq 1$, be α different parameters and the sets $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, ..., \mathcal{L}_\alpha$ have corresponding parametric values with $\mathcal{L}_m \cap \mathcal{L}_n = \emptyset$, for $m \neq n$, and $m, n \in \{1, 2, 3, ..., \alpha\}$. Then the pair $(\eta, \bigodot)$ is named as a *fuzzy hypersoft set* on $\hat{\triangle}$ with $\bigodot = \mathcal{L}_1 \times \mathcal{L}_2 \times \mathcal{L}_3 \times ..... \times \mathcal{L}_\alpha$ and $\eta : \bigodot \to FP(\hat{\triangle})$ and $(\eta, \mathfrak{G}) = \{\langle \beta, (a/(\xi_\eta(\beta)(a)\rangle : a \in \hat{\triangle}, \beta \in \bigodot\}$, ξ is a membership function.

Definition 2.12. [48] Suppose $\mho_1, \mho_2, \mho_3, ..., \mho_\alpha$, for $\alpha \geq 1$, be α different parameters and the sets $\mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3, ..., \mathcal{L}_\alpha$ have corresponding parametric values with $\mathcal{L}_m \cap \mathcal{L}_n = \emptyset$, for $m \neq n$, and $m, n \in \{1, 2, 3, ..., \alpha\}$. Then the pair $(\eta, \bigodot)$ is named as a *Intuitionistic fuzzy hypersoft set* on $\hat{\triangle}$ with $\bigodot = \mathcal{L}_1 \times \mathcal{L}_2 \times \mathcal{L}_3 \times ..... \times \mathcal{L}_\alpha$ and $\eta : \bigodot \to IFP(\hat{\triangle})$ and $(\eta, \bigodot) = \{\langle \bigwedge, (\ddot{n}/(\xi_\eta(\bigwedge)(\ddot{n})(, \theta_\eta(\bigwedge)(\ddot{n}))\rangle : \ddot{n} \in \hat{\triangle}, \bigwedge \in \bigodot\}$, ξ and θ are membership and non membership functions respectively.

Definition 2.13. [34] A pair $(\beta, \check{S})$ is named as a *hypersoft expert set* if $\beta : \check{S} \to P(\hat{\triangle})$ where $\check{S} \subseteq \check{A} = \mathcal{Q} \times \mathcal{I} \times \mathcal{U}$, $\mathcal{Q} = \times_1 \times \times_2 \times \times_3 \times .... \times \times_r$ and $\times_1, \times_2, \times_3, ..., \times_r$ are parametric valued non-overlapping sets corresponding to r different parameters $q_1, q_2, q_3, ..., q_r$.

Definition 2.14. [35] A pair $(\varpi, \breve{S})$ is named as a *fuzzy hypersoft expert set* if $\varpi : \breve{S} \rightarrow FP(\hat{\Delta})$ where $\breve{S} \subseteq \breve{A} = \mathcal{Q} \times \mathcal{I} \times \mathcal{U}$, $\mathcal{Q} = \times_1 \times \times_2 \times \times_3 \times .... \times \times_r$ and $\times_1, \times_2, \times_3, ..., \times_r$ are parametric valued non-overlapping sets corresponding to r different parameters $q_1, q_2, q_3, ..., q_r$.

Definition 2.15. [35] The union between FHsESs $(\mho_1, \Upsilon_1)$ and $(\mho_2, \Upsilon_2)$ is a FHsES $(\mho_3, \Upsilon_3)$; $\Upsilon_3 \doteq \Upsilon_1 \cup \Upsilon_2$, and $\forall\, \imath \in \Upsilon_3$,

$$\mho_3^T(\imath) = \begin{cases} \mho_1(\imath) & ; \imath \in \Upsilon_1 - \Upsilon_2 \\ \mho_2(\imath) & ; \imath \in \Upsilon_2 - \Upsilon_1 \\ max\{\mho_1(\imath),\ \mho_2(\imath)\} & ; \imath \in \Upsilon_1 \cap \Upsilon_2. \end{cases}$$

$$\mho_3^F(\imath) = \begin{cases} \mho_1(\imath) & ; \imath \in \Upsilon_1 - \Upsilon_2 \\ \mho_2(\imath) & ; \imath \in \Upsilon_2 - \Upsilon_1 \\ min\{\mho_1(\imath)\,,\ \mho_2(\imath)\} & ; \imath \in \Upsilon_1 \cap \Upsilon_2. \end{cases}$$

Definition 2.16. [35] The intersection of FHsESs $(\mho_1, \Upsilon_1)$ and $(\mho_2, \Upsilon_2)$ on $\hat{\Delta}$ is a FHsES $(\mho_3, \Upsilon_3)$ where $\Upsilon_3 \doteq \Upsilon_1 \cap \Upsilon_2$,

$$\mho_3(\imath) = \{(\imath, < min\{\mho_1(\imath)\,,\ \mho_2(\imath)\}, max\{\mho_1(\imath)\,,\ \mho_2(\imath)\} >); \imath \in \Upsilon_3\}$$

$$\mho_3(\imath) = \begin{cases} \mho_1(\imath) & ; \imath \in \Upsilon_1 - \Upsilon_2 \\ \mho_2(\imath) & ; \imath \in \Upsilon_2 - \Upsilon_1 \\ min\{\mho_1(\imath)\,,\ \mho_2(\imath)\} & ; \imath \in \Upsilon_1 \cap \Upsilon_2. \end{cases}$$

$$\mho_3(\imath) = \begin{cases} \mho_1(\imath) & ; \imath \in \Upsilon_1 - \Upsilon_2 \\ \mho_2(\imath) & ; \imath \in \Upsilon_2 - \Upsilon_1 \\ max\{\mho_1(\imath)\,,\ \mho_2(\imath)\} & ; \imath \in \Upsilon_1 \cap \Upsilon_2. \end{cases}$$

Definition 2.17. [35] A FHsES $(\mho_1, \mathcal{S})$ is called a fuzzy hypersoft expert subset of another FHsES $(\mho_2, \mathcal{P})$, denoted by $(\mho_1, \mathcal{S}) \subseteq (\mho_2, \mathcal{P})$, on $\hat{\Delta}$, if

(1) $\mathcal{S} \subseteq \mathcal{P}$;

(2) $\forall\, \imath \in \mathcal{S}, \mho_1(\imath) \subseteq \mho_2(\imath)$.

3. Fuzzy parameterized Pythagorean fuzzy hypersoft expert set (FPPFHsES)

In this section, a new structure FPPFHsES is characterized with the help of existing concept of fuzzy parameterized Pythagorean fuzzy soft expert set.

Definition 3.1. A fuzzy parameterized Pythagorean fuzzy hypersoft expert set $\Psi_{\mathcal{F}}$ over $\hat{\Delta}$ is defined as

$$\Psi_{\mathcal{F}} = \left\{\left(\left(\frac{\hat{q}}{\mu_{\mathcal{F}}(\hat{q})}, \hat{E}_i, \hat{O}_i\right), \frac{\hat{\delta}}{\psi_{\mathcal{F}}(\hat{\delta})}\right); \forall \hat{q} \in \mathcal{Q}, \hat{E}_i \in \mathcal{I}, \hat{O}_i \in \mathcal{U}, \hat{\delta} \in \hat{\Delta}\right\}$$

where

(1) $\mu_{\mathcal{F}} : \breve{\mathcal{J}} \rightarrow FP(\hat{\Delta})$;

(2) $\psi_{\mathcal{F}} : \check{\mathcal{J}} \rightarrow PFP(\hat{\Delta})$ is called approximate function of FPPFHsES;

(3) $\check{\mathcal{J}} \subseteq \mathcal{H} = Q \times \mathcal{I} \times \mathcal{U}$;

(4) where $Q_1, Q_2, Q_3, ..., Q_r$ are different sets of parameter corresponding to r different parameters $q_1, q_2, q_3, ..., q_r$.

Example 3.2. Assume that an organization delivers novel brands of items and intends to take the assessment of few specialists about these items. Let $\hat{\Delta} = \{æ_1, æ_2, æ_3, æ_4\}$ be a set of products and $\{\vartheta_1/0.2, \vartheta_2/0.4, \vartheta_3/0.6, \vartheta_4/0.8, \vartheta_5/0.9, \vartheta_6/0.3, \vartheta_7/0.5, \vartheta_8/0.7\}$ be the fuzzy subset and $\mathcal{G}_1 = \{q_{11}, q_{12}\}$, $\mathcal{G}_2 = \{q_{21}, q_{22}\}$, $\mathcal{G}_3 = \{q_{31}, q_{32}\}$, be non-overlapping characteristic sets for different attributes q_1= simple to utilize, q_2= nature, q_3= modest. Now $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \mathcal{G}_3$

$$\mathcal{G} = \begin{Bmatrix} \vartheta_1/0.2 = (q_{11}, q_{21}, q_{31}), \vartheta_2/0.4 = (q_{11}, q_{21}, q_{32}), \vartheta_3/0.6 = (q_{11}, q_{22}, q_{31}), \vartheta_4/0.8 = (q_{11}, q_{22}, q_{32}), \\ \vartheta_5/0.9 = (q_{12}, q_{21}, q_{31}), \vartheta_6/0.3 = (q_{12}, q_{21}, q_{32}), \vartheta_7/0.5 = (q_{12}, q_{22}, q_{31}), \vartheta_8/0.7 = (q_{12}, q_{22}, q_{32}) \end{Bmatrix}$$

Now $\mathcal{H} = \mathcal{G} \times \mathcal{I} \times \mathcal{U}$

$$\mathcal{H} = \begin{Bmatrix} (\vartheta_1/0.2, s, 0), (\vartheta_1/0.2, s, 1), (\vartheta_1/0.2, t, 0), (\vartheta_1/0.2, t, 1), (\vartheta_1/0.2, u, 0), (\vartheta_1/0.2, u, 1), \\ (\vartheta_2/0.4, s, 0), (\vartheta_2/0.4, s, 1), (\vartheta_2/0.4, t, 0), (\vartheta_2/0.4, t, 1), (\vartheta_2/0.4, u, 0), (\vartheta_2/0.4, u, 1), \\ (\vartheta_3/0.6, s, 0), (\vartheta_3/0.6, s, 1), (\vartheta_3/0.6, t, 0), (\vartheta_3/0.6, t, 1), (\vartheta_3/0.6, u, 0), (\vartheta_3/0.6, u, 1), \\ (\vartheta_4/0.8, s, 0), (\vartheta_4/0.8, s, 1), (\vartheta_4/0.8, t, 0), (\vartheta_4/0.8, t, 1), (\vartheta_4/0.8, u, 0), (\vartheta_4/0.8, u, 1), \\ (\vartheta_5/0.9, s, 0), (\vartheta_5/0.9, s, 1), (\vartheta_5/0.9, t, 0), (\vartheta_5/0.9, t, 1), (\vartheta_5/0.9, u, 0), (\vartheta_5/0.9, u, 1), \\ (\vartheta_6/0.3, s, 0), (\vartheta_6/0.3, s, 1), (\vartheta_6/0.3, t, 0), (\vartheta_6/0.3, t, 1), (\vartheta_6/0.3, u, 0), (\vartheta_6/0.3, u, 1), \\ (\vartheta_7/0.5, s, 0), (\vartheta_7/0.5, s, 1), (\vartheta_7/0.5, t, 0), (\vartheta_7/0.5, t, 1), (\vartheta_7/0.5, u, 0), (\vartheta_7/0.5, u, 1), \\ (\vartheta_8/0.7, s, 0), (\vartheta_8/0.7, s, 1), (\vartheta_8/0.7, t, 0), (\vartheta_8/0.7, t, 1), (\vartheta_8/0.7, u, 0), (\vartheta_8/0.7, u, 1) \end{Bmatrix}$$

let

$$\check{\mathcal{J}} = \begin{Bmatrix} (\vartheta_1/0.2, s, 0), (\vartheta_1/0.2, s, 1), (\vartheta_1/0.2, t, 0), (\vartheta_1/0.2, t, 1), (\vartheta_1/0.2, u, 0), (\vartheta_1/0.2, u, 1), \\ (\vartheta_2/0.4, s, 0), (\vartheta_2/0.4, s, 1), (\vartheta_2/0.4, t, 0), (\vartheta_2/0.4, t, 1), (\vartheta_2/0.4, u, 0), (\vartheta_2/0.4, u, 1), \\ (\vartheta_3/0.6, s, 0), (\vartheta_3/0.6, s, 1), (\vartheta_3/0.6, t, 0), (\vartheta_3/0.6, t, 1), (\vartheta_3/0.6, u, 0), (\vartheta_3/0.6, u, 1), \end{Bmatrix}$$

$\subseteq \mathcal{H}$ and $\mathcal{I} = \{s, t, u, \}$ represents a collection of specialists.
Following analysis characterizes the options of three specialists:

$$\mho_1 = \mho(\vartheta_1/0.2, s, 1) = \left\{ \frac{æ_1}{< 0.01, 0.02 >}, \frac{æ_2}{< 0.02, 0.07 >}, \frac{æ_3}{< 0.03, 0.05 >}, \frac{æ_4}{< 0.06, 0.01 >} \right\},$$

$$\mho_2 = \mho(\vartheta_1/0.2, t, 1) = \left\{ \frac{æ_1}{< 0.02, 0.04 >}, \frac{æ_2}{< 0.01, 0.08 >}, \frac{æ_3}{< 0.03, 0.04 >}, \frac{æ_4}{< 0.01, 0.02 >} \right\},$$

$$\mho_3 = \mho(\vartheta_1/0.2, u, 1) = \left\{ \frac{æ_1}{< 0.01, 0.07 >}, \frac{æ_2}{< 0.02, 0.05 >}, \frac{æ_3}{< 0.03, 0.06 >}, \frac{æ_4}{< 0.02, 0.03 >} \right\},$$

$$\mho_4 = \mho(\vartheta_2/0.4, s, 1) = \left\{ \frac{æ_1}{< 0.01, 0.09 >}, \frac{æ_2}{< 0.05, 0.04 >}, \frac{æ_3}{< 0.02, 0.07 >}, \frac{æ_4}{< 0.04, 0.03 >} \right\},$$

$$\mho_5 = \mho(\vartheta_2/0.4, t, 1) = \left\{\frac{\text{æ}_1}{< 0.02, 0.04 >}, \frac{\text{æ}_2}{< 0.02, 0.08 >}, \frac{\text{æ}_3}{< 0.03, 0.06 >}, \frac{\text{æ}_4}{< 0.02, 0.06 >}\right\},$$

$$\mho_6 = \mho(\vartheta_2/0.4, u, 1) = \left\{\frac{\text{æ}_1}{< 0.05, 0.04 >}, \frac{\text{æ}_2}{< 0.03, 0.06 >}, \frac{\text{æ}_3}{< 0.06, 0.02 >}, \frac{\text{æ}_4}{< 0.08, 0.01 >}\right\},$$

$$\mho_7 = \mho(\vartheta_3/0.6, s, 1) = \left\{\frac{\text{æ}_1}{< 0.02, 0.07 >}, \frac{\text{æ}_2}{< 0.06, 0.01 >}, \frac{\text{æ}_3}{< 0.04, 0.05 >}, \frac{\text{æ}_4}{< 0.05, 0.04 >}\right\},$$

$$\mho_8 = \mho(\vartheta_3/0.6, t, 1) = \left\{\frac{\text{æ}_1}{< 0.04, 0.03 >}, \frac{\text{æ}_2}{< 0.06, 0.03 >}, \frac{\text{æ}_3}{< 0.07, 0.02 >}, \frac{\text{æ}_4}{< 0.09, 0.01 >}\right\},$$

$$\mho_9 = \mho(\vartheta_3/0.6, u, 1) = \left\{\frac{\text{æ}_1}{< 0.07, 0.02 >}, \frac{\text{æ}_2}{< 0.03, 0.05 >}, \frac{\text{æ}_3}{< 0.05, 0.04 >}, \frac{\text{æ}_4}{< 0.02, 0.07 >}\right\},$$

$$\mho_{10} = \mho(\vartheta_1/0.2, s, 0) = \left\{\frac{\text{æ}_1}{< 0.03, 0.02 >}, \frac{\text{æ}_2}{< 0.02, 0.04 >}, \frac{\text{æ}_3}{< 0.04, 0.05 >}, \frac{\text{æ}_4}{< 0.01, 0.08 >}\right\},$$

$$\mho_{11} = \mho(\vartheta_1/0.2, t, 0) = \left\{\frac{\text{æ}_1}{< 0.01, 0.08 >}, \frac{\text{æ}_2}{< 0.09, 0.01 >}, \frac{\text{æ}_3}{< 0.06, 0.03 >}, \frac{\text{æ}_4}{< 0.02, 0.07 >}\right\},$$

$$\mho_{12} = \mho(\vartheta_1/0.2, u, 0) = \left\{\frac{\text{æ}_1}{< 0.02, 0.07 >}, \frac{\text{æ}_2}{< 0.01, 0.08 >}, \frac{\text{æ}_3}{< 0.03, 0.05 >}, \frac{\text{æ}_4}{< 0.05, 0.04 >}\right\},$$

$$\mho_{13} = \mho(\vartheta_2/0.4, s, 0) = \left\{\frac{\text{æ}_1}{< 0.08, 0.01 >}, \frac{\text{æ}_2}{< 0.03, 0.06 >}, \frac{\text{æ}_3}{< 0.05, 0.04 >}, \frac{\text{æ}_4}{< 0.07, 0.02 >}\right\},$$

$$\mho_{14} = \mho(\vartheta_2/0.4, t, 0) = \left\{\frac{\text{æ}_1}{< 0.07, 0.02 >}, \frac{\text{æ}_2}{< 0.02, 0.06 >}, \frac{\text{æ}_3}{< 0.09, 0.01 >}, \frac{\text{æ}_4}{< 0.04, 0.05 >}\right\},$$

$$\mho_{15} = \mho(\vartheta_2/0.4, u, 0) = \left\{\frac{\text{æ}_1}{< 0.06, 0.02 >}, \frac{\text{æ}_2}{< 0.07, 0.02 >}, \frac{\text{æ}_3}{< 0.03, 0.05 >}, \frac{\text{æ}_4}{< 0.02, 0.07 >}\right\},$$

$$\mho_{16} = \mho(\vartheta_3/0.6, s, 0) = \left\{\frac{\text{æ}_1}{< 0.01, 0.07 >}, \frac{\text{æ}_2}{< 0.04, 0.05 >}, \frac{\text{æ}_3}{< 0.07, 0.02 >}, \frac{\text{æ}_4}{< 0.08, 0.02 >}\right\},$$

$$\mho_{17} = \mho(\vartheta_3/0.6, t, 0) = \left\{\frac{\text{æ}_1}{< 0.02, 0.07 >}, \frac{\text{æ}_2}{< 0.09, 0.01 >}, \frac{\text{æ}_3}{< 0.08, 0.02 >}, \frac{\text{æ}_4}{< 0.03, 0.05 >}\right\},$$

$$\mho_{18} = \mho(\vartheta_3/0.6, u, 0) = \left\{\frac{\text{æ}_1}{< 0.05, 0.04 >}, \frac{\text{æ}_2}{< 0.03, 0.06 >}, \frac{\text{æ}_3}{< 0.06, 0.03 >}, \frac{\text{æ}_4}{< 0.01, 0.08 >}\right\}.$$

The FPPFHsES can be described as $(\mho, \check{\mathcal{J}}) =$

$$\left\{\begin{array}{l}
\left((\vartheta_1/0.2, s, 1), \left\{\frac{\text{æ}_1}{<0.01,0.02>}, \frac{\text{æ}_2}{<0.02,0.07>}, \frac{\text{æ}_3}{<0.03,0.05>}, \frac{\text{æ}_4}{<0.06,0.1>}\right\}\right), \left((\vartheta_1/0.2, t, 1), \left\{\frac{\text{æ}_1}{<0.01,0.07>}, \frac{\text{æ}_2}{<0.02,0.05>}, \frac{\text{æ}_3}{<0.03,0.06>}, \frac{\text{æ}_4}{<0.02,0.03>}\right\}\right), \\
\left((\vartheta_1/0.2, u, 1), \left\{\frac{\text{æ}_1}{<0.07,0.02>}, \frac{\text{æ}_2}{<0.05,0.04>}, \frac{\text{æ}_3}{<0.06,0.03>}, \frac{\text{æ}_4}{<0.03,0.05>}\right\}\right), \left((\vartheta_2/0.4, s, 1), \left\{\frac{\text{æ}_1}{<0.09,0.05>}, \frac{\text{æ}_2}{<0.04,0.05>}, \frac{\text{æ}_3}{<0.07,0.02>}, \frac{\text{æ}_4}{<0.03,0.06>}\right\}\right), \\
\left((\vartheta_2/0.4, t, 1), \left\{\frac{\text{æ}_1}{<0.04,0.06>}, \frac{\text{æ}_2}{<0.08,0.02>}, \frac{\text{æ}_3}{<0.03,0.06>}, \frac{\text{æ}_4}{<0.02,0.06>}\right\}\right), \left((\vartheta_2/0.4, u, 1), \left\{\frac{\text{æ}_1}{<0.05,0.05>}, \frac{\text{æ}_2}{<0.03,0.06>}, \frac{\text{æ}_3}{<0.06,0.02>}, \frac{\text{æ}_4}{<0.08,0.02>}\right\}\right), \\
\left((\vartheta_3/0.6, s, 1), \left\{\frac{\text{æ}_1}{<0.02,0.07>}, \frac{\text{æ}_2}{<0.09,0.01>}, \frac{\text{æ}_3}{<0.04,0.05>}, \frac{\text{æ}_4}{<0.05,0.05>}\right\}\right), \left((\vartheta_3/0.6, t, 1), \left\{\frac{\text{æ}_1}{<0.04,0.05>}, \frac{\text{æ}_2}{<0.06,0.03>}, \frac{\text{æ}_3}{<0.07,0.02>}, \frac{\text{æ}_4}{<0.09,0.01>}\right\}\right), \\
\left((\vartheta_3/0.6, u, 1), \left\{\frac{\text{æ}_1}{<0.07,0.03>}, \frac{\text{æ}_2}{<0.03,0.06>}, \frac{\text{æ}_3}{<0.05,0.04>}, \frac{\text{æ}_4}{<0.02,0.07>}\right\}\right), \left((\vartheta_1/0.2, s, 0), \left\{\frac{\text{æ}_1}{<0.03,0.04>}, \frac{\text{æ}_2}{<0.02,0.07>}, \frac{\text{æ}_3}{<0.04,0.05>}, \frac{\text{æ}_4}{<0.01,0.08>}\right\}\right), \\
\left((\vartheta_1/0.2, t, 0), \left\{\frac{\text{æ}_1}{<0.01,0.04>}, \frac{\text{æ}_2}{<0.09,0.01>}, \frac{\text{æ}_3}{<0.06,0.02>}, \frac{\text{æ}_4}{<0.02,0.07>}\right\}\right), \left((\vartheta_1/0.2, u, 0), \left\{\frac{\text{æ}_1}{<0.02,0.07>}, \frac{\text{æ}_2}{<0.01,0.06>}, \frac{\text{æ}_3}{<0.03,0.04>}, \frac{\text{æ}_4}{<0.05,0.05>}\right\}\right), \\
\left((\vartheta_2/0.4, s, 0), \left\{\frac{\text{æ}_1}{<0.08,0.02>}, \frac{\text{æ}_2}{<0.03,0.01>}, \frac{\text{æ}_3}{<0.05,0.04>}, \frac{\text{æ}_4}{<0.07,0.02>}\right\}\right), \left((\vartheta_2/0.4, t, 0), \left\{\frac{\text{æ}_1}{<0.07,0.02>}, \frac{\text{æ}_2}{<0.02,0.07>}, \frac{\text{æ}_3}{<0.09,0.01>}, \frac{\text{æ}_4}{<0.04,0.05>}\right\}\right), \\
\left((\vartheta_2/0.4, u, 0), \left\{\frac{\text{æ}_1}{<0.06,0.02>}, \frac{\text{æ}_2}{<0.07,0.01>}, \frac{\text{æ}_3}{<0.03,0.07>}, \frac{\text{æ}_4}{<0.02,0.07>}\right\}\right), \left((\vartheta_3/0.6, s, 0), \left\{\frac{\text{æ}_1}{<0.01,0.08>}, \frac{\text{æ}_2}{<0.04,0.05>}, \frac{\text{æ}_3}{<0.07,0.03>}, \frac{\text{æ}_4}{<0.08,0.02>}\right\}\right), \\
\left((\vartheta_3/0.6, t, 0), \left\{\frac{\text{æ}_1}{<0.02,0.07>}, \frac{\text{æ}_2}{<0.09,0.01>}, \frac{\text{æ}_3}{<0.08,0.02>}, \frac{\text{æ}_4}{<0.03,0.04>}\right\}\right), \left((\vartheta_3/0.6, u, 0), \left\{\frac{\text{æ}_1}{<0.05,0.04>}, \frac{\text{æ}_2}{<0.03,0.06>}, \frac{\text{æ}_3}{<0.06,0.03>}, \frac{\text{æ}_4}{<0.01,0.08>}\right\}\right),
\end{array}\right\}.$$

Definition 3.3. Let $(\mho_1, \check{\mathcal{J}}_1)$, $(\mho_2, \check{\mathcal{J}}_2) \in \Omega_{\Xi}$, then $(\mho_1, \check{\mathcal{J}}_1) \subseteq (\mho_2, \check{\mathcal{J}}_2)$ if
(i) $\check{\mathcal{J}}_1 \subseteq \check{\mathcal{J}}_2$, (ii) $\forall\, \alpha \in \check{\mathcal{J}}_1, \mho_1(\alpha) \subseteq \mho_2(\alpha)$.

Example 3.4. Considering Example 3.2, suppose

$$\mho_1 = \Big\{(\vartheta_1/0.2, s, 1), (\vartheta_3/0.6, s, 0), (\vartheta_1/0.2, t, 1), (\vartheta_3/0.6, t, 1), (\vartheta_3/0.6, t, 0), (\vartheta_1/0.2, u, 0), (\vartheta_3/0.6, u, 1)\Big\}$$

$$\mho_2 = \begin{Bmatrix} (\vartheta_1/0.3, s, 1), (\vartheta_3/0.7, s, 0), (\vartheta_3/0.6, s, 1), (\vartheta_1/0.4, t, 1), (\vartheta_3/0.9, t, 1), \\ (\vartheta_1/0.2, t, 0), (\vartheta_3/0.8, t, 0), (\vartheta_1/0.4, u, 0), (\vartheta_3/0.6, u, 1), (\vartheta_1/0.2, u, 1) \end{Bmatrix}.$$

It is clear that $\mho_1 \subset \mho_2$. Suppose $(\mho_1, \check{\mathcal{J}}_1)$ and $(\mho_2, \check{\mathcal{J}}_2)$ be defined as following
$(\mho_1, \check{\mathcal{J}}_1) =$

$$\left\{ \begin{array}{l} \left((\vartheta_1/0.2, s, 1), \left\{\frac{æ_1}{<0.01,0.06>}, \frac{æ_2}{<0.06,0.05>}, \frac{æ_3}{<0.04,0.06>}, \frac{æ_4}{<0.01,0.08>}\right\}\right), \\ \left((\vartheta_1/0.2, t, 1), \left\{\frac{æ_1}{<0.03,0.04>}, \frac{æ_2}{<0.06,0.04>}, \frac{æ_3}{<0.02,0.05>}, \frac{æ_4}{<0.01,0.05>}\right\}\right), \\ \left((\vartheta_3/0.6, t, 1), \left\{\frac{æ_1}{<0.02,0.06>}, \frac{æ_2}{<0.05,0.04>}, \frac{æ_3}{<0.06,0.05>}, \frac{æ_4}{<0.08,0.06>}\right\}\right), \\ \left((\vartheta_3/0.6, u, 1), \left\{\frac{æ_1}{<0.06,0.04>}, \frac{æ_2}{<0.02,0.07>}, \frac{æ_3}{<0.04,0.05>}, \frac{æ_4}{<0.01,0.07>}\right\}\right), \\ \left((\vartheta_1/0.2, u, 0), \left\{\frac{æ_1}{<0.01,0.06>}, \frac{æ_2}{<0.01,0.07>}, \frac{æ_3}{<0.02,0.07>}, \frac{æ_4}{<0.01,0.06>}\right\}\right), \\ \left((\vartheta_3/0.6, s, 0), \left\{\frac{æ_1}{<0.01,0.08>}, \frac{æ_2}{<0.03,0.06>}, \frac{æ_3}{<0.06,0.03>}, \frac{æ_4}{<0.07,0.02>}\right\}\right), \\ \left((\vartheta_3/0.6, t, 0), \left\{\frac{æ_1}{<0.01,0.07>}, \frac{æ_2}{<0.06,0.03>}, \frac{æ_3}{<0.07,0.02>}, \frac{æ_4}{<0.02,0.07>}\right\}\right) \end{array} \right\}.$$

$(\mho_2, \check{\mathcal{J}}_2) =$

$$\left\{ \begin{array}{l} \left((\vartheta_1/0.3, s, 1), \left\{\frac{æ_1}{<0.02,0.03>}, \frac{æ_2}{<0.07,0.04>}, \frac{æ_3}{<0.05,0.04>}, \frac{æ_4}{<0.02,0.04>}\right\}\right), \\ \left((\vartheta_1/0.4, t, 1), \left\{\frac{æ_1}{<0.04,0.03>}, \frac{æ_2}{<0.08,0.03>}, \frac{æ_3}{<0.04,0.03>}, \frac{æ_4}{<0.02,0.06>}\right\}\right), \\ \left((\vartheta_3/0.6, s, 1), \left\{\frac{æ_1}{<0.01,0.03>}, \frac{æ_2}{<0.09,0.01>}, \frac{æ_3}{<0.04,0.05>}, \frac{æ_4}{<0.05,0.03>}\right\}\right), \\ \left((\vartheta_3/0.9, t, 1), \left\{\frac{æ_1}{<0.04,0.02>}, \frac{æ_2}{<0.06,0.03>}, \frac{æ_3}{<0.07,0.04>}, \frac{æ_4}{<0.09,0.05}\right\}\right), \\ \left((\vartheta_1/0.2, u, 1), \left\{\frac{æ_1}{<0.07,0.02>}, \frac{æ_2}{<0.05,0.02>}, \frac{æ_3}{<0.06,0.02>}, \frac{æ_4}{<0.03,0.05}\right\}\right), \\ \left((\vartheta_3/0.6, u, 1), \left\{\frac{æ_1}{<0.07,0.03>}, \frac{æ_2}{<0.03,0.05>}, \frac{æ_3}{<0.05,0.04>}, \frac{æ_4}{<0.02,0.06>}\right\}\right), \\ \left((\vartheta_1/0.4, u, 0), \left\{\frac{æ_1}{<0.02,0.05>}, \frac{æ_2}{<0.02,0.06>}, \frac{æ_3}{<0.03,0.05>}, \frac{æ_4}{<0.05,0.03>}\right\}\right), \\ \left((\vartheta_1/0.2, t, 0), \left\{\frac{æ_1}{<0.01,0.06>}, \frac{æ_2}{<0.09,0.01>}, \frac{æ_3}{<0.06,0.03>}, \frac{æ_4}{<0.02,0.06>}\right\}\right), \\ \left((\vartheta_3/0.7, s, 0), \left\{\frac{æ_1}{<0.02,0.07>}, \frac{æ_2}{<0.04,0.05>}, \frac{æ_3}{<0.07,0.02>}, \frac{æ_4}{<0.08,0.01>}\right\}\right), \\ \left((\vartheta_3/0.6, t, 0), \left\{\frac{æ_1}{<0.02,0.05>}, \frac{æ_2}{<0.07,0.02>}, \frac{æ_3}{<0.08,0.02>}, \frac{æ_4}{<0.03,0.05>}\right\}\right) \end{array} \right\}$$

$\Rightarrow (\mho_1, \check{\mathcal{J}}_1) \subseteq (\mho_2, \check{\mathcal{J}}_2)$.

Definition 3.5. Let $(\mho_1, \check{\mathcal{J}}_1)$, $(\mho_2, \check{\mathcal{J}}_2) \in \Omega_{Œ}$. Then these are equal if $(\mho_1, \check{\mathcal{J}}_1) \subseteq (\mho_2, \check{\mathcal{J}}_2)$ and $(\mho_2, \check{\mathcal{J}}_2)$ $\subseteq (\mho_1, \check{\mathcal{J}}_1)$.

Definition 3.6. Let $(\mho, \check{\mathcal{J}}) \in \Omega_{Œ}$, then its complement $(\mho, \check{\mathcal{J}})^c$, is defined as
$(\mho, \check{\mathcal{J}})^c = \tilde{c}(\mho(\imath))\ \forall\, \imath \in \hat{\Delta}$ while $\tilde{c}$ is a PF complement.

Definition 3.7. An Agree-FPPFHsES $(\mho, \check{\mathcal{J}})_{ag}$ is a $\mathcal{FPPFH}\int\mathcal{E}$-subset of $(\mho, \check{\mathcal{J}})$ expressed by as $(\mho, \check{\mathcal{J}})_{ag} = \{\mho_{ag}(\varrho) : \varrho \in \mathcal{I} \times \boldsymbol{\mathcal{U}} \times \{1\}\}$.

Example 3.8. Finding Agree-FPPFHsES determined in Example 3.2, we get

$$(\mho, \check{\mathcal{J}}) = \left\{ \begin{array}{l} \left((\vartheta_1/0.2, s, 1), \left\{\frac{æ_1}{<0.02,0.07>}, \frac{æ_2}{<0.07,0.02>}, \frac{æ_3}{<0.05,0.04>}, \frac{æ_4}{<0.01,0.05}\right\}\right), \\ \left((\vartheta_1/0.2, t, 1), \left\{\frac{æ_1}{<0.04,0.05>}, \frac{æ_2}{<0.08,0.01>}, \frac{æ_3}{<0.04,0.05>}, \frac{æ_4}{<0.02,0.04>}\right\}\right), \\ \left((\vartheta_1/0.2, u, 1), \left\{\frac{æ_1}{<0.07,0.02>}, \frac{æ_2}{<0.05,0.04>}, \frac{æ_3}{<0.06,0.03>}, \frac{æ_4}{<0.03,0.05>}\right\}\right), \\ \left((\vartheta_2/0.4, s, 1), \left\{\frac{æ_1}{<0.09,0.05>}, \frac{æ_2}{<0.04,0.05>}, \frac{æ_3}{<0.07,0.02>}, \frac{æ_4}{<0.03,0.06>}\right\}\right), \\ \left((\vartheta_2/0.4, t, 1), \left\{\frac{æ_1}{<0.04,0.06>}, \frac{æ_2}{<0.08,0.02>}, \frac{æ_3}{<0.03,0.06>}, \frac{æ_4}{<0.02,0.06>}\right\}\right), \\ \left((\vartheta_2/0.4, u, 1), \left\{\frac{æ_1}{<0.05,0.05>}, \frac{æ_2}{<0.03,0.06>}, \frac{æ_3}{<0.06,0.02>}, \frac{æ_4}{<0.08,0.02>}\right\}\right), \\ \left((\vartheta_3/0.6, s, 1), \left\{\frac{æ_1}{<0.02,0.07>}, \frac{æ_2}{<0.09,0.01>}, \frac{æ_3}{<0.04,0.05>}, \frac{æ_4}{<0.05,0.05>}\right\}\right), \\ \left((\vartheta_3/0.6, t, 1), \left\{\frac{æ_1}{<0.04,0.05>}, \frac{æ_2}{<0.06,0.03>}, \frac{æ_3}{<0.07,0.02>}, \frac{æ_4}{<0.09,0.01>}\right\}\right), \\ \left((\vartheta_3/0.6, u, 1), \left\{\frac{æ_1}{<0.07,0.03>}, \frac{æ_2}{<0.03,0.06>}, \frac{æ_3}{<0.05,0.04>}, \frac{æ_4}{<0.02,0.07>}\right\}\right), \end{array} \right\}.$$

Definition 3.9. A disagree-FPPFHsES $(\mho, \check{\mathcal{J}})_{ag}$ over $\hat{\Delta}$, is a $\mathcal{FPPFH\int E}$-subset of $(\mho, \check{\mathcal{J}})$ and is characterized as $(\mho, \check{\mathcal{J}})_{ag} = \{\mho_{ag}(\varrho) : \varrho \in \mathcal{I} \times \mathcal{U} \times \{0\}\}$.

Example 3.10. Getting disagree-FPPFHsES determined in Example3.2, $(\mho, \check{\mathcal{J}})$ =

$$\left\{ \begin{array}{l} \left((\vartheta_1/0.2, s, 0), \left\{\frac{æ_1}{<0.03,0.04>}, \frac{æ_2}{<0.02,0.07>}, \frac{æ_3}{<0.04,0.05>}, \frac{æ_4}{<0.01,0.08>}\right\}\right), \\ \left((\vartheta_1/0.2, t, 0), \left\{\frac{æ_1}{<0.01,0.04>}, \frac{æ_2}{<0.09,0.01>}, \frac{æ_3}{<0.06,0.02>}, \frac{æ_4}{<0.02,0.07>}\right\}\right), \\ \left((\vartheta_1/0.2, u, 0), \left\{\frac{æ_1}{<0.02,0.07>}, \frac{æ_2}{<0.01,0.06>}, \frac{æ_3}{<0.03,0.04>}, \frac{æ_4}{<0.05,0.05>}\right\}\right), \\ \left((\vartheta_2/0.4, s, 0), \left\{\frac{æ_1}{<0.08,0.02>}, \frac{æ_2}{<0.03,0.01>}, \frac{æ_3}{<0.05,0.04>}, \frac{æ_4}{<0.07,0.02>}\right\}\right), \\ \left((\vartheta_2/0.4, t, 0), \left\{\frac{æ_1}{<0.07,0.02>}, \frac{æ_2}{<0.02,0.07>}, \frac{æ_3}{<0.09,0.01>}, \frac{æ_4}{<0.04,0.05>}\right\}\right), \\ \left((\vartheta_2/0.4, u, 0), \left\{\frac{æ_1}{<0.06,0.02>}, \frac{æ_2}{<0.07,0.01>}, \frac{æ_3}{<0.03,0.07>}, \frac{æ_4}{<0.02,0.07>}\right\}\right), \\ \left((\vartheta_3/0.6, s, 0), \left\{\frac{æ_1}{<0.01,0.08>}, \frac{æ_2}{<0.04,0.05>}, \frac{æ_3}{<0.07,0.03>}, \frac{æ_4}{<0.08,0.02>}\right\}\right), \\ \left((\vartheta_3/0.6, t, 0), \left\{\frac{æ_1}{<0.02,0.07>}, \frac{æ_2}{<0.09,0.01>}, \frac{æ_3}{<0.08,0.02>}, \frac{æ_4}{<0.03,0.04>}\right\}\right), \\ \left((\vartheta_3/0.6, u, 0), \left\{\frac{æ_1}{<0.05,0.02>}, \frac{æ_2}{<0.03,0.02>}, \frac{æ_3}{<0.06,0.03>}, \frac{æ_4}{<0.01,0.02>}\right\}\right), \end{array} \right\}.$$

Proposition 3.11. *Suppose* $(\mho, \check{\mathcal{J}}) \in \Omega_{Œ}$, *then*

(1) $((\mho, \check{\mathcal{J}})^c)^c = (\mho, \check{\mathcal{J}})$;

(2) $(\mho, \check{\mathcal{J}})^c_{ag} = (\mho, \check{\mathcal{J}})_{dag}$;

(3) $(\mho, \check{\mathcal{J}})^c_{dag} = (\mho, \check{\mathcal{J}})_{ag}$.

Definition 3.12. Let $(\mho_1, \check{\mathcal{J}}_1), (\mho_2, \check{\mathcal{J}}_2) \in \Omega_{Œ}$, then their union is $(\mho_3, \check{\mathcal{J}}_3)$ with $\check{\mathcal{J}}_3 = \check{\mathcal{J}}_1 \cup \check{\mathcal{J}}_2$, defined as

$$\mho_3(\imath) = \begin{cases} \mho_1(\imath) & ; \imath \in \check{\mathcal{J}}_1 \setminus \check{\mathcal{J}}_2 \\ \mho_2(\imath) & ; \imath \in \check{\mathcal{J}}_2 \setminus \check{\mathcal{J}}_1 \\ s(\mho_1(\imath), \mho_2(\imath)) & ; \imath \in \check{\mathcal{J}}_1 \cap \check{\mathcal{J}}_2. \end{cases}$$

where s is s-norm.

Example 3.13. Reconsidering the Example 3.2 with following two sets

$$\check{\mathcal{J}}_1 = \left\{ \begin{array}{l} (\vartheta_1/0.2, s, 1), (\vartheta_3/0.6, s, 0), (\vartheta_1/0.2, t, 1), (\vartheta_3/0.6, t, 1), (\vartheta_3/0.6, t, 0), \\ (\vartheta_1/0.2, u, 0), (\vartheta_3/0.6, u, 1) \end{array} \right\}$$

$$\check{\mathcal{J}}_2 = \begin{Bmatrix} (\vartheta_1/0.2, s, 1), (\vartheta_3/0.6, s, 0), (\vartheta_3/0.6, s, 1), (\vartheta_1/0.2, t, 1), (\vartheta_3/0.6, t, 1), \\ (\vartheta_1/0.2, u, 1), (\vartheta_3/0.6, t, 0), (\vartheta_1/0.2, u, 0), (\vartheta_3/0.6, u, 1), (\vartheta_1/0.2, t, 0) \end{Bmatrix}.$$

Suppose $(\mho_1, \check{\mathcal{J}}_1)$, $(\mho_2, \check{\mathcal{J}}_2) \in \Omega_{\OE}$ such that $(\mho_1, \check{\mathcal{J}}_1) =$

$$\left\{ \begin{array}{l} \left((\vartheta_1/0.2, s, 1), \left\{\frac{\ae_1}{<0.01,0.06>}, \frac{\ae_2}{<0.06,0.05>}, \frac{\ae_3}{<0.04,0.06>}, \frac{\ae_4}{<0.01,0.08>}\right\}\right), \\ \left((\vartheta_1/0.2, t, 1), \left\{\frac{\ae_1}{<0.03,0.04>}, \frac{\ae_2}{<0.06,0.04>}, \frac{\ae_3}{<0.02,0.05>}, \frac{\ae_4}{<0.01,0.05>}\right\}\right), \\ \left((\vartheta_3/0.6, t, 1), \left\{\frac{\ae_1}{<0.02,0.06>}, \frac{\ae_2}{<0.05,0.04>}, \frac{\ae_3}{<0.06,0.05>}, \frac{\ae_4}{<0.08,0.06>}\right\}\right), \\ \left((\vartheta_3/0.6, u, 1), \left\{\frac{\ae_1}{<0.06,0.04>}, \frac{\ae_2}{<0.02,0.07>}, \frac{\ae_3}{<0.04,0.05>}, \frac{\ae_4}{<0.01,0.07>}\right\}\right), \\ \left((\vartheta_1/0.2, u, 0), \left\{\frac{\ae_1}{<0.01,0.06>}, \frac{\ae_2}{<0.01,0.07>}, \frac{\ae_3}{<0.02,0.07>}, \frac{\ae_4}{<0.01,0.06>}\right\}\right), \\ \left((\vartheta_3/0.6, s, 0), \left\{\frac{\ae_1}{<0.01,0.08>}, \frac{\ae_2}{<0.03,0.06>}, \frac{\ae_3}{<0.06,0.03>}, \frac{\ae_4}{<0.07,0.02>}\right\}\right), \\ \left((\vartheta_3/0.6, t, 0), \left\{\frac{\ae_1}{<0.01,0.07>}, \frac{\ae_2}{<0.06,0.03>}, \frac{\ae_3}{<0.07,0.02>}, \frac{\ae_4}{<0.02,0.07>}\right\}\right) \end{array} \right\}.$$

Then $(\mho_1, \check{\mathcal{J}}_1) \cup (\mho_2, \check{\mathcal{J}}_2) = (\mho_3, \check{\mathcal{J}}_3) =$

$$\left\{ \begin{array}{l} \left((\vartheta_1/0.2, s, 1), \left\{\frac{\ae_1}{<0.02,0.03>}, \frac{\ae_2}{<0.07,0.03>}, \frac{\ae_3}{<0.05,0.04>}, \frac{\ae_4}{<0.02,0.04>}\right\}\right), \\ \left((\vartheta_1/0.2, t, 1), \left\{\frac{\ae_1}{<0.03,0.04>}, \frac{\ae_2}{<0.08,0.02>}, \frac{\ae_3}{<0.04,0.03>}, \frac{\ae_4}{<0.02,0.05>}\right\}\right), \\ \left((\vartheta_3/0.6, s, 1), \left\{\frac{\ae_1}{<0.02,0.05>}, \frac{\ae_2}{<0.02,0.05>}, \frac{\ae_3}{<0.02,0.05>}, \frac{\ae_4}{<0.02,0.05>}\right\}\right), \\ \left((\vartheta_3/0.6, t, 1), \left\{\frac{\ae_1}{<0.04,0.02>}, \frac{\ae_2}{<0.06,0.02>}, \frac{\ae_3}{<0.07,0.03>}, \frac{\ae_4}{<0.09,0.05>}\right\}\right), \\ \left((\vartheta_1/0.2, u, 1), \left\{\frac{\ae_1}{<0.07,0.02>}, \frac{\ae_2}{<0.03,0.05>}, \frac{\ae_3}{<0.05,0.03>}, \frac{\ae_4}{<0.01,0.05>}\right\}\right), \\ \left((\vartheta_3/0.6, u, 1), \left\{\frac{\ae_1}{<0.02,0.05>}, \frac{\ae_2}{<0.02,0.05>}, \frac{\ae_3}{<0.02,0.05>}, \frac{\ae_4}{<0.02,0.05>}\right\}\right), \\ \left((\vartheta_1/0.2, u, 0), \left\{\frac{\ae_1}{<0.01,0.03>}, \frac{\ae_2}{<0.01,0.06>}, \frac{\ae_3}{<0.03,0.05>}, \frac{\ae_4}{<0.05,0.03>}\right\}\right), \\ \left((\vartheta_1/0.2, t, 0), \left\{\frac{\ae_1}{<0.02,0.05>}, \frac{\ae_2}{<0.02,0.05>}, \frac{\ae_3}{<0.02,0.05>}, \frac{\ae_4}{<0.02,0.05>}\right\}\right), \\ \left((\vartheta_3/0.6, s, 0), \left\{\frac{\ae_1}{<0.01,0.06>}, \frac{\ae_2}{<0.01,0.05>}, \frac{\ae_3}{<0.07,0.01>}, \frac{\ae_4}{<0.08,0.01>}\right\}\right), \\ \left((\vartheta_3/0.6, t, 0), \left\{\frac{\ae_1}{<0.01,0.05>}, \frac{\ae_2}{<0.08,0.01>}, \frac{\ae_3}{<0.08,0.02>}, \frac{\ae_4}{<0.03,0.05>}\right\}\right), \end{array} \right\}.$$

Proposition 3.14. *Suppose* $(\mho_1, \check{\mathcal{J}}_1)$,$(\mho_2, \check{\mathcal{J}}_2)$ *and* $(\mho_3, \check{\mathcal{J}}_3) \in \Omega_{\OE}$*, then*

(1) $(\mho_1, \check{\mathcal{J}}_1) \cup (\mho_2, \check{\mathcal{J}}_2) = (\mho_2, \check{\mathcal{J}}_2) \cup (\mho_1, \check{\mathcal{J}}_1)$*;*

(2) $((\mho_1, \check{\mathcal{J}}_1) \cup (\mho_2, \check{\mathcal{J}}_2)) \cup (\mho_3, \check{\mathcal{J}}_2) = (\mho_1, \check{\mathcal{J}}_1) \cup ((\mho_2, \check{\mathcal{J}}_2) \cup (\mho_3, \check{\mathcal{J}}_3))$.

Definition 3.15. Suppose $(\mho_1, \check{\mathcal{J}}_1)$, $(\mho_2, \check{\mathcal{J}}_2) \in \Omega_{\OE}$, then their intersection is $(\mho_3, \check{\mathcal{J}}_3)$ with $\check{\mathcal{J}}_3 = \check{\mathcal{J}}_1 \cap \check{\mathcal{J}}_2$, defined as

$$\mho_3(\imath) = \begin{cases} \mho_1(\imath) & ; \imath \in \check{\mathcal{J}}_1 \setminus \check{\mathcal{J}}_2 \\ \mho_2(\imath) & ; \imath \in \check{\mathcal{J}}_2 \setminus \check{\mathcal{J}}_1 \\ t(\mho_1(\imath), \mho_2(\imath)) & ; \imath \in \check{\mathcal{J}}_1 \cap \check{\mathcal{J}}_2. \end{cases}$$

where t is a t-norm.

Example 3.16. Reconsidering Example 3.2 with the following two sets

$$\check{\mathcal{J}}_1 = \begin{Bmatrix} (\vartheta_1/0.2, s, 1), (\vartheta_3/0.6, s, 0), (\vartheta_1/0.2, t, 1), (\vartheta_3/0.6, t, 1), \\ (\vartheta_3/0.6, t, 0), (\vartheta_1/0.2, u, 0), (\vartheta_3/0.6, u, 1) \end{Bmatrix}$$

$$\check{\mathcal{J}}_2 = \begin{Bmatrix} (\vartheta_1/0.2, s, 1), (\vartheta_3/0.6, s, 0), (\vartheta_3/0.6, s, 1), (\vartheta_1/0.2, t, 1), (\vartheta_3/0.6, t, 1), \\ (\vartheta_1/0.2, t, 0), (\vartheta_3/0.6, t, 0), (\vartheta_1/0.2, u, 0), (\vartheta_1/0.2, u, 1), , (\vartheta_1/0.2, u, 1) \end{Bmatrix}.$$

Suppose $(\mho_1, \check{\mathcal{J}}_1)$, $(\mho_2, \check{\mathcal{J}}_2) \in \Omega_{Œ}$ such that $(\mho_1, \check{\mathcal{J}}_1)$ =

$$\left\{\begin{array}{l}
\left((\vartheta_1/0.2, s, 1), \left\{\frac{æ_1}{<0.01,0.06>}, \frac{æ_2}{<0.06,0.05>}, \frac{æ_3}{<0.04,0.06>}, \frac{æ_4}{<0.01,0.08>}\right\}\right), \\
\left((\vartheta_1/0.2, t, 1), \left\{\frac{æ_1}{<0.03,0.04>}, \frac{æ_2}{<0.06,0.04>}, \frac{æ_3}{<0.02,0.05>}, \frac{æ_4}{<0.01,0.05>}\right\}\right), \\
\left((\vartheta_3/0.6, t, 1), \left\{\frac{æ_1}{<0.02,0.06>}, \frac{æ_2}{<0.05,0.04>}, \frac{æ_3}{<0.06,0.05>}, \frac{æ_4}{<0.08,0.06>}\right\}\right), \\
\left((\vartheta_3/0.6, u, 1), \left\{\frac{æ_1}{<0.06,0.04>}, \frac{æ_2}{<0.02,0.07>}, \frac{æ_3}{<0.04,0.05>}, \frac{æ_4}{<0.01,0.07>}\right\}\right), \\
\left((\vartheta_1/0.2, u, 0), \left\{\frac{æ_1}{<0.01,0.06>}, \frac{æ_2}{<0.01,0.07>}, \frac{æ_3}{<0.02,0.07>}, \frac{æ_4}{<0.01,0.06>}\right\}\right), \\
\left((\vartheta_3/0.6, s, 0), \left\{\frac{æ_1}{<0.01,0.08>}, \frac{æ_2}{<0.03,0.06>}, \frac{æ_3}{<0.06,0.03>}, \frac{æ_4}{<0.07,0.02>}\right\}\right), \\
\left((\vartheta_3/0.6, t, 0), \left\{\frac{æ_1}{<0.01,0.07>}, \frac{æ_2}{<0.06,0.03>}, \frac{æ_3}{<0.07,0.02>}, \frac{æ_4}{<0.02,0.07>}\right\}\right)
\end{array}\right\},$$

$$(\mho_2, \check{\mathcal{J}}_2) = \left\{\begin{array}{l}
\left((\vartheta_1/0.3, s, 1), \left\{\frac{æ_1}{<0.02,0.03>}, \frac{æ_2}{<0.07,0.04>}, \frac{æ_3}{<0.05,0.04>}, \frac{æ_4}{<0.02,0.04>}\right\}\right), \\
\left((\vartheta_1/0.4, t, 1), \left\{\frac{æ_1}{<0.04,0.03>}, \frac{æ_2}{<0.08,0.03>}, \frac{æ_3}{<0.04,0.03>}, \frac{æ_4}{<0.02,0.06>}\right\}\right), \\
\left((\vartheta_3/0.6, s, 1), \left\{\frac{æ_1}{<0.01,0.03>}, \frac{æ_2}{<0.09,0.01>}, \frac{æ_3}{<0.04,0.05>}, \frac{æ_4}{<0.05,0.03>}\right\}\right), \\
\left((\vartheta_3/0.9, t, 1), \left\{\frac{æ_1}{<0.04,0.02>}, \frac{æ_2}{<0.06,0.03>}, \frac{æ_3}{<0.07,0.04>}, \frac{æ_4}{<0.09,0.05}\right\}\right), \\
\left((\vartheta_1/0.2, u, 1), \left\{\frac{æ_1}{<0.07,0.02>}, \frac{æ_2}{<0.05,0.02>}, \frac{æ_3}{<0.06,0.02>}, \frac{æ_4}{<0.03,0.05}\right\}\right), \\
\left((\vartheta_3/0.6, u, 1), \left\{\frac{æ_1}{<0.07,0.03>}, \frac{æ_2}{<0.03,0.05>}, \frac{æ_3}{<0.05,0.04>}, \frac{æ_4}{<0.02,0.06>}\right\}\right), \\
\left((\vartheta_1/0.4, u, 0), \left\{\frac{æ_1}{<0.02,0.05>}, \frac{æ_2}{<0.02,0.06>}, \frac{æ_3}{<0.03,0.05>}, \frac{æ_4}{<0.05,0.03>}\right\}\right), \\
\left((\vartheta_1/0.2, t, 0), \left\{\frac{æ_1}{<0.01,0.06>}, \frac{æ_2}{<0.09,0.01>}, \frac{æ_3}{<0.06,0.03>}, \frac{æ_4}{<0.02,0.06>}\right\}\right), \\
\left((\vartheta_3/0.7, s, 0), \left\{\frac{æ_1}{<0.02,0.07>}, \frac{æ_2}{<0.04,0.05>}, \frac{æ_3}{<0.07,0.02>}, \frac{æ_4}{<0.08,0.01>}\right\}\right), \\
\left((\vartheta_3/0.6, t, 0), \left\{\frac{æ_1}{<0.02,0.05>}, \frac{æ_2}{<0.07,0.02>}, \frac{æ_3}{<0.08,0.02>}, \frac{æ_4}{<0.03,0.05>}\right\}\right)
\end{array}\right\}.$$

Then $(\mho_1, \check{\mathcal{J}}_1) \cap (\mho_2, \check{\mathcal{J}}_2) = (\mho_3, \check{\mathcal{J}}_3)$=

$$\left\{\begin{array}{l}
\left((\vartheta_1/0.2, s, 1), \left\{\frac{æ_1}{<0.01,0.06>}, \frac{æ_2}{<0.06,0.04>}, \frac{æ_3}{<0.04,0.05>}, \frac{æ_4}{<0.01,0.08>}\right\}\right), \\
\left((\vartheta_1/0.2, t, 1), \left\{\frac{æ_1}{<0.03,0.04>}, \frac{æ_2}{<0.06,0.02>}, \frac{æ_3}{<0.02,0.05>}, \frac{æ_4}{<0.02,0.05>}\right\}\right), \\
\left((\vartheta_3/0.6, t, 1), \left\{\frac{æ_1}{<0.04,0.06>}, \frac{æ_2}{<0.05,0.02>}, \frac{æ_3}{<0.06,0.03>}, \frac{æ_4}{<0.01,0.05>}\right\}\right), \\
\left((\vartheta_3/0.6, u, 1), \left\{\frac{æ_1}{<0.06,0.02>}, \frac{æ_2}{<0.02,0.05>}, \frac{æ_3}{<0.04,0.03>}, \frac{æ_4}{<0.01,0.06>}\right\}\right), \\
\left((\vartheta_1/0.2, u, 0), \left\{\frac{æ_1}{<0.01,0.05>}, \frac{æ_2}{<0.01,0.07>}, \frac{æ_3}{<0.02,0.07>}, \frac{æ_4}{<0.04,0.06>}\right\}\right), \\
\left((\vartheta_3/0.6, s, 0), \left\{\frac{æ_1}{<0.01,0.07>}, \frac{æ_2}{<0.03,0.06>}, \frac{æ_3}{<0.06,0.02>}, \frac{æ_4}{<0.07,0.02>}\right\}\right), \\
\left((\vartheta_3/0.6, t, 0), \left\{\frac{æ_1}{<0.01,0.07>}, \frac{æ_2}{<0.07,0.02>}, \frac{æ_3}{<0.07,0.02>}, \frac{æ_4}{<0.03,0.07>}\right\}\right),
\end{array}\right\}.$$

Proposition 3.17. *Suppose* $(\mho_1, \check{\mathcal{J}}_1)$,$(\mho_2, \check{\mathcal{J}}_2)$ *and* $(\mho_3, \check{\mathcal{J}}_3) \in \Omega_{Œ}$*, then*

(1) $(\mho_1, \check{\mathcal{J}}_1) \cap (\mho_2, \check{\mathcal{J}}_2) = (\mho_2, \check{\mathcal{J}}_2) \cap (\mho_1, \check{\mathcal{J}}_1)$*;*

(2) $((\mho_1, \check{\mathcal{J}}_1) \cap (\mho_2, \check{\mathcal{J}}_2)) \cap (\mho_3, \check{\mathcal{J}}_3) = (\mho_1, \check{\mathcal{J}}_1) \cap ((\mho_2, \check{\mathcal{J}}_2) \cap (\mho_3, \check{\mathcal{J}}_3))$.

Proposition 3.18. *Suppose* $(\mho_1, \check{\mathcal{J}}_1)$,$(\mho_2, \check{\mathcal{J}}_2)$ *and* $(\mho_3, \check{\mathcal{J}}_3) \in \Omega_{Œ}$*, then*

(1) $(\mho_1, \check{\mathcal{J}}_1) \cup ((\mho_2, \check{\mathcal{J}}_2) \cap (\mho_3, \check{\mathcal{J}}_3)) = ((\mho_1, \check{\mathcal{J}}_1) \cup ((\mho_2, \check{\mathcal{J}}_2)) \cap ((\mho_1, \check{\mathcal{J}}_1) \cup (\mho_3, \check{\mathcal{J}}_3))$*;*

(2) $(\mho_1, \check{\mathcal{J}}_1) \cap ((\mho_2, \check{\mathcal{J}}_2) \cup (\mho_3, \check{\mathcal{J}}_3)) = ((\mho_1, \check{\mathcal{J}}_1) \cap ((\mho_2, \check{\mathcal{J}}_2)) \cup ((\mho_1, \check{\mathcal{J}}_1) \cap (\mho_3, \check{\mathcal{J}}_3))$.

Definition 3.19. Suppose $(\mho_1, \check{\mathcal{J}}_1)$, $(\mho_2, \check{\mathcal{J}}_2) \in \Omega_{Œ}$, then $(\mho_1, \check{\mathcal{J}}_1)$ AND $(\mho_2, \check{\mathcal{J}}_2)$ denoted by $(\mho_1, \check{\mathcal{J}}_1) \wedge (\mho_2, \check{\mathcal{J}}_2)$ is defined by $(\mho_1, \check{\mathcal{J}}_1) \wedge (\mho_2, \check{\mathcal{J}}_2) = (\mho_3, \check{\mathcal{J}}_1 \times \check{\mathcal{J}}_2)$, while $\mho_3(b, c) = \mho_1(b) \cap \mho_2(c), \forall (b, c) \in \check{\mathcal{J}}_1 \times \check{\mathcal{J}}_2$.

Definition 3.20. Suppose $(\mho_1, \check{\mathcal{J}}_1)$, $(\mho_2, \check{\mathcal{J}}_2) \in \Omega_{Œ}$, then $(\mho_1, \check{\mathcal{J}}_1)$ OR $(\mho_2, \check{\mathcal{J}}_2)$ denoted by $(\mho_1, \check{\mathcal{J}}_1) \vee (\mho_2, \check{\mathcal{J}}_2)$ is defined by $(\mho_1, \check{\mathcal{J}}_1) \vee (\mho_2, \check{\mathcal{J}}_2) = (\mho_3, \check{\mathcal{J}}_1 \times \check{\mathcal{J}}_2)$, while $\mho_3(b, c) = \mho_1(b) \cup \mho_2(c), \forall (b, c) \in \check{\mathcal{J}}_1 \times \check{\mathcal{J}}_2$.

Proposition 3.21. *Suppose* $(\mho_1, \check{\mathcal{J}}_1)$,$(\mho_2, \check{\mathcal{J}}_2)$ *and* $(\mho_3, \check{\mathcal{J}}_3) \in \Omega_{Œ}$, *then*

(1) $((\mho_1, \check{\mathcal{J}}_1) \wedge (\mho_2, \check{\mathcal{J}}_2))^c = ((\mho_1, \check{\mathcal{J}}_1))^c \vee ((\mho_2, \check{\mathcal{J}}_2))^c$;

(2) $((\mho_1, \check{\mathcal{J}}_1) \vee (\mho_2, \check{\mathcal{J}}_2))^c = ((\mho_1, \check{\mathcal{J}}_1))^c \wedge ((\mho_2, \check{\mathcal{J}}_2))^c$.

Proof. (1) Suppose $((\mho_1, \check{\mathcal{J}}_1)$ and $((\mho_2, \check{\mathcal{J}}_2)$ are defined as
$((\mho_1, \check{\mathcal{J}}_1) = (\mho_1(j_1)$, for all $j_1 \in \check{\mathcal{J}}_1)$ and $((\mho_2, \check{\mathcal{J}}_2) = (\mho_2(j_2)$, for all $j_2 \in \check{\mathcal{J}}_2)$,
then using the definitions of AND and OR, we have $((\mho_1, \check{\mathcal{J}}_1) \wedge (\mho_2, \check{\mathcal{J}}_2))^c = (\mho_1(j_1) \wedge (\mho_2(j_2))^c$
$= ((\mho_1(j_1) \cap (\mho_2(j_2))^c = (c(\mho_1(j_1) \cap (\mho_2(j_2)) = (c(\mho_1(j_1) \cup c(\mho_2(j_2))$
$= ((\mho_1(j_1)^c \vee (\mho_2(j_2)^c)$
$= (\mho_1, \check{\mathcal{J}}_1)^c \vee (\mho_2, \check{\mathcal{J}}_2)^c$.
(2) This can be proved is similar to 1. □

Proposition 3.22. *Suppose* $(\mho_1, \check{\mathcal{J}}_1)$,$(\mho_2, \check{\mathcal{J}}_2)$ *and* $(\mho_3, \check{\mathcal{J}}_3) \in \Omega_{Œ}$, *then*

(1) $((\mho_1, \check{\mathcal{J}}_1) \wedge (\mho_2, \check{\mathcal{J}}_2)) \wedge (\mho_3, \check{\mathcal{J}}_3) = (\mho_1, \check{\mathcal{J}}_1) \wedge ((\mho_2, \check{\mathcal{J}}_2) \wedge (\mho_3, \check{\mathcal{J}}_3))$;

(2) $((\mho_1, \check{\mathcal{J}}_1) \vee (\mho_2, \check{\mathcal{J}}_2)) \vee (\mho_3, \check{\mathcal{J}}_3) = (\mho_1, \check{\mathcal{J}}_1) \vee ((\mho_2, \check{\mathcal{J}}_2) \vee (\mho_3, \check{\mathcal{J}}_3))$;

(3) $(\mho_1, \check{\mathcal{J}}_1) \vee ((\mho_2, \check{\mathcal{J}}_2) \wedge (\mho_3, \check{\mathcal{J}}_3) = ((\mho_1, \check{\mathcal{J}}_1) \vee ((\mho_2, \check{\mathcal{J}}_2)) \wedge ((\mho_1, \check{\mathcal{J}}_1) \vee (\mho_3, \check{\mathcal{J}}_3))$;

(4) $(\mho_1, \check{\mathcal{J}}_1) \wedge ((\mho_2, \check{\mathcal{J}}_2) \vee (\mho_3, \check{\mathcal{J}}_3)) = ((\mho_1, \check{\mathcal{J}}_1) \wedge ((\mho_2, \check{\mathcal{J}}_2)) \vee ((\mho_1, \check{\mathcal{J}}_1) \wedge (\mho_3, \check{\mathcal{J}}_3))$.

4. Application to fuzzy parameterized Pythagorean fuzzy hypersoft expert set

In this section, an application of fuzzy parameterized Pythagorean fuzzy hypersoft expert set theory in a decision making problem, is presented.

Statement of the problem

In product selection scenario, the purchase of an electronics device has become a challenging problem for an individual as well as for an organization. Many adults purchase a laptop numerous instances in the course of their lifetimes. A laptop is a prime purchase. Its charge may be as lac as or multiple year's disposable income. With such countless PCs to browse, choosing the best one to accommodate your financial plan can resemble exploring a minefield. In any event, figuring out the always changing rundown of item determinations is no simple accomplishment. Workstations fluctuate extraordinarily by CPU speed, designs ability, size, drive stockpiling, and RAM, in addition to other things. In addition, your PC needs might be totally unique to another person's, just adding to the disarray. Most human beings need their laptop to offer personal tasks, office work and business point of view, however additionally comforts and conveniences. John is looking to buy a laptop. He has no idea where to begin, so he consults his friends, Stephen, Thomas and Umar. Each of the three experts has a different opinion on which laptop to purchase, and they give their recommendation to John accordingly.

Proposed algorithm

We use an algorithm to select the product descriptions (purchase).

Proposed algorithm: Selection of laptop

▷ **Start**:

▷ **Construction**:

———1. Construct FPPFHsES (ξ, K)

▷ **Computation**:

———2. Determine Agree-FPPFHsES and Disagree-FPPFHsES.

———3. Calculation of Upper and Lower Evaluation Values for Agree and Disagree-FPPFHsESs.

———4. Formation of Evaluation Interval.

———5. Computation of Evaluation scores.

———6. Computation of An Evaluation.

———7. Determine $\Pi_i = \Upsilon_i - \Gamma_i$ for each element $c_i \in \hat{\Delta}$.

▷ **Output**:

———8. Select the alternative with max Π_i.

▷ **End**:

The pictorial representation of this algorithm is expressed in Figure 3. In above algorithm, Υ_i, Γ_i and Π_i represents agree-based score value, disagree-based score value and difference of score values corresponding to each alternative respectively.

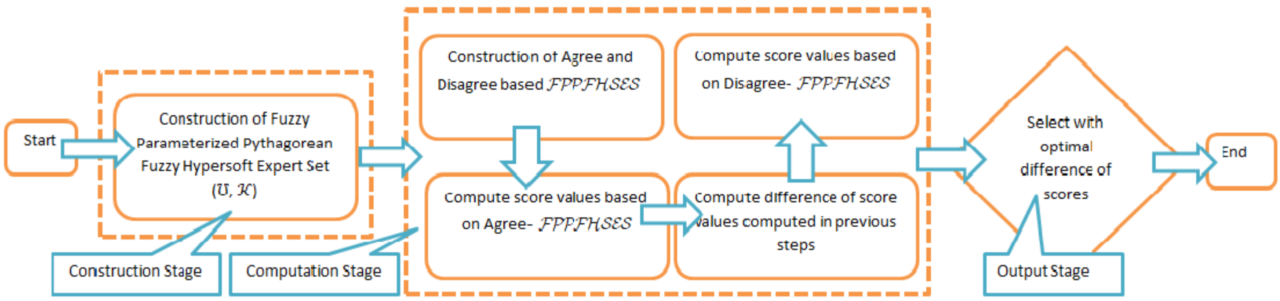

Figure 3. Flow diagram for algorithm.

4.1. Operational role of selected parameters

(1) **Size**: Versatility is probably the best element of a laptop. After all, the capacity to slip it into a backpack and go about your day is very useful. If portability is your main concern, your best bet is to look at laptops with a smaller screen size and light, slim design. These types of laptops are often

marketed as Ultra books, so pay attention for that word. Or, more specifically, aim for a machine with a screen between 12 and 13.3 inches and weighs less than 1.5kg. Different sizes have been shown in Figures 4 and 5.

(2) **RAM**: RAM is critical to PC execution, particularly on the off chance that you do heaps of multitasking on your PC - for example alter photographs, compose word docs, and surf the web in the meantime. The more RAM you have, the quicker your PC will actually want to get to information, and the more applications you can run simultaneously. 4GB of RAM is the base. On the off chance that you utilize heaps of high-power programming, search for 8GB or more. The picture of RAM is given in Figure 6.

(3) **Screen quality**: Assuming you're like the greater part of us, you'll presumably be gazing into your PC screen for a really long time ordinary. So ensure you pick a PC with a screen that is good looking. Glossier screens will generally mirror encompassing light, so remember that. Likewise note that touch screen PCs will have a gleaming screen, so weigh up the upsides and downside. Different versions of screen quality are presented in Figure 7.

(4) **Battery life**: Maker cited battery duration is never characteristic of what this present reality experience of utilizing a PC is like. There are just an excessive number of factors that influence battery duration. There is the screen splendor, the screen goal, the quantity of uses you have running behind the scenes in addition to whether or not you effectively stay associated with Wi Fi organizations or Bluetooth gadgets. Rather than zeroing in on the quantity of hours the producer quotes, check out the rating of the battery in Watt-hours (Wh) or milliamp-hours (mAh). The greater the number, the more extended the battery will endure. The battery life of different laptops is presented in Figure 8.

(5) **Storage**: Not exclusively will you really want to think about how much stockpiling, yet in addition the sort of capacity. Once upon a time, hard plate drives were the top pick. With slimmer, lighter workstations in style, hard drives are not as well known. All things being equal, numerous PC proprietors are choosing strong state drives, which are quicker, calmer, and you got it, more costly. The picture of laptop storage is given in Figure 9.

Screen Resolution (in Pixels)	Possible LCD sizes (diagonal)	Viewable Mega pixels
800 × 600 (SVGA-standard)	12″	0.48
1024 × 768 (XGA-standard)	12″, 13.3″, 14″, 15″	0.79
1280 × 800 (WXGA-wide)	15.4″, 14.1″, 13.3″, 12.1″	1.02
1440 × 900 (WXGA+-wide)	14″	1.30
1280 × 1024 (SXGA-standard)	14″, 15″, 15.7″	1.31
1400 × 1050 (SXGA+-standard)	12.1″, 14″, 15″	1.47
1680 × 1050 (WSXGA+-wide)	15.4″	1.76
1600 × 1200 (UXGA-standard)	14″, 15″, 16″	1.92
1920 × 1200 (WUXGA-wide)	17″, 15.4″	2.30

Figure 4. Size of different laptops (source: https://www.quora.com).

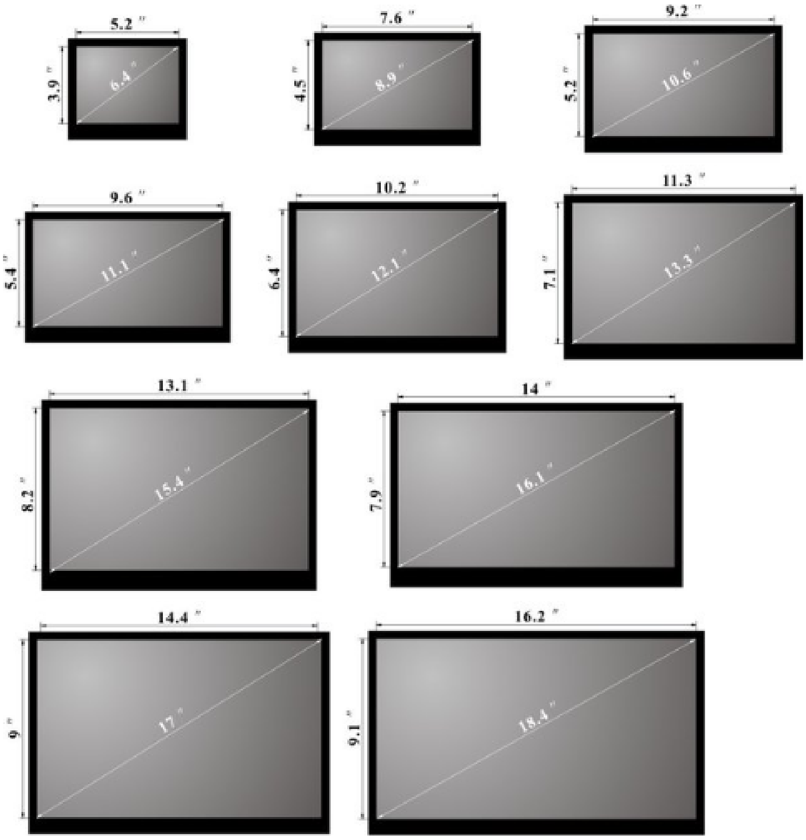

Figure 5. Size of different laptops (source: https://www.quora.com).

Figure 6. RAM of laptop.

1366 x 768	HD (not Full HD)	
1600 x 900	HD+	
1920 x 1080	Full HD	1080p
2304×1440	Retina (Apple only)	
2560 x 1440	QHD / WQHD	2K
2560×1600	Retina (Apple only)	
2880×1800	Retina (Apple only)	
3000 x 2000	PixelSense (MS Only)	
3200 x 1800	QHD+	3K
3840 x 2160	UHD	4K

Figure 7. Screen quality of different laptops.

#	Name	Hours:Minutes	Minutes
1	Asus ExpertBook B9450F	16:29	989
2	LG Gram 14	16:03	963
3	LG Gram 17	16:17	779
4	Samsung Notebook 9 Pro	12:30	750
5	Dell XPS 13 9300	11:26	686
6	HP Elite Dragonfly	11:26	686
7	Lenovo Yoga C940-14IIL	11:15	675
8	Lenovo C740-14IML	11:12	672
9	HP Spectre x360 13-AW0013dx	10:38	638
10	Apple MacBook Pro 13-inch 2019	10:32	632
11	Dell XPS 13 7390 2-in-1	10:07	607
12	Acer Swift SF714-52T	9:50	590
13	Lenovo Yoga C940-15IRH	9:44	584
14	Huawei MateBook Pro X	9:31	571
15	Acer Aspire 1 A115-31	9:05	545
16	Microsoft Surface Laptop 3	8:58	538
17	Acer Aspire 5 A515-54	8:38	518
18	Microsoft Surface Pro 7	8:34	514
19	HP Spectre x360 15 (OLED)	8:34	514
20	Dell XPS 15 9500 (2020)	8:14	494
21	Dell G5 5590	8:10	490
22	Lenovo IdeaPad 730S	8:08	488
23	Apple MacBook Pro (16-inch, 2019)	8:02	482
24	MSI Prestige 15 A10SC	7:57	477
25	Acer Aspire 7 A715-74G-71WS	7:56	476

Figure 8. Battery life of different laptops.

Figure 9. Storage of laptop.

4.2. Application

Example 4.1. Step-1.

Let $\hat{\Delta} = \{z_1, z_2, z_3\}$ represents the three categories of laptops forming the universe of discourse and $X = \{\mathcal{E}_1 = Stephen, \mathcal{E}_2 = Thomas, \mathcal{E}_3 = Asad\}$ is being used as a set of experts. The characteristics-valued sets for recommended features are: $\top_1 = Size = \{15inches = \curlyvee_1, 15.3inches = \curlyvee_2\}$, $\top_2 = Ram = \{4GB = \curlyvee_3, 8GB = \curlyvee_4\}$, $\top_3 = ScreenQuality = \{3200 \times 1800 = \curlyvee_5, 3000 \times 2000 = \curlyvee_6\}$, $\top_4 = BatteryLife = \{16hours = \curlyvee_7, 12hours = \curlyvee_8\}$, $\top_5 = Storage = \{16GB = \curlyvee_9, 512GB = \curlyvee_{10}\}$, and then $\top = \top_1 \times \top_2 \times \top_3 \times \top_4 \times \top_5$, and $R =$

$$\left\{\begin{array}{l} (\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_9), (\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_{10}), (\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_8, \curlyvee_9), (\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_8, \curlyvee_{10}), (\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_9), \\ (\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_{10}), (\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_9), (\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_{10}), (\curlyvee_1, \curlyvee_4, \curlyvee_5, \curlyvee_7, \curlyvee_9), (\curlyvee_1, \curlyvee_4, \curlyvee_5, \curlyvee_7, \curlyvee_{10}), \\ (\curlyvee_1, \curlyvee_4, \curlyvee_5, \curlyvee_8, \curlyvee_9), (\curlyvee_1, \curlyvee_4, \curlyvee_5, \curlyvee_8, \curlyvee_{10}), (\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_9), (\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_{10}), (\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_9), \\ (\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_{10}), (\curlyvee_2, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_9), (\curlyvee_2, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_{10}), (\curlyvee_2, \curlyvee_3, \curlyvee_5, \curlyvee_8, \curlyvee_9), (\curlyvee_2, \curlyvee_3, \curlyvee_5, \curlyvee_8, \curlyvee_{10}), \\ (\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_9), (\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_{10}), (\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_9), (\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_{10}), (\curlyvee_2, \curlyvee_4, \curlyvee_5, \curlyvee_7, \curlyvee_9), \\ (\curlyvee_2, \curlyvee_4, \curlyvee_5, \curlyvee_7, \curlyvee_{10}), (\curlyvee_2, \curlyvee_4, \curlyvee_5, \curlyvee_8, \curlyvee_9), (\curlyvee_2, \curlyvee_4, \curlyvee_5, \curlyvee_8, \curlyvee_{10}), (\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_9), (\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_{10}), \\ (\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_9), (\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_{10}) \end{array}\right\},$$

and now take $K \subseteq R$ as

$K = \{k_1/0.2 = (\curlyvee_1, \curlyvee_3, \curlyvee_5, \curlyvee_7, \curlyvee_9), k_2/0.3 = (\curlyvee_1, \curlyvee_3, \curlyvee_6, \curlyvee_7, \curlyvee_{10}), k_3/0.5 = (\curlyvee_1, \curlyvee_4, \curlyvee_6, \curlyvee_8, \curlyvee_9),$
$k_4/0.4 = (\curlyvee_2, \curlyvee_3, \curlyvee_6, \curlyvee_8, \curlyvee_9), k_5/0.7 = (\curlyvee_2, \curlyvee_4, \curlyvee_6, \curlyvee_7, \curlyvee_{10})\}$ and $(\xi, \mathrm{K}) =$

$$\left\{\begin{array}{l}
\Big((k_1/0.2,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.09,0.01\rangle},\frac{z_2}{\langle 0.03,0.04\rangle},\frac{z_3}{\langle 0.06,0.02\rangle}\Big\}\Big),\Big((k_1/0.2,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.08,0.02\rangle},\frac{z_2}{\langle 0.01,0.02\rangle},\frac{z_3}{\langle 0.06,0.02\rangle}\Big\}\Big),\\
\Big((k_1/0.2,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.07,0.03\rangle},\frac{z_2}{\langle 0.03,0.07\rangle},\frac{z_3}{\langle 0.03,0.01\rangle}\Big\}\Big),\Big((k_2/0.3,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.06,0.04\rangle},\frac{z_2}{\langle 0.04,0.02\rangle},\frac{z_3}{\langle 0.07,0.01\rangle}\Big\}\Big),\\
\Big((k_2/0.3,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.05,0.02\rangle},\frac{z_2}{\langle 0.06,0.04\rangle},\frac{z_3}{\langle 0.03,0.06\rangle}\Big\}\Big),\Big((k_2/0.3,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.04,0.03\rangle},\frac{z_2}{\langle 0.03,0.02\rangle},\frac{z_3}{\langle 0.03,0.02\rangle}\Big\}\Big),\\
\Big((k_3/0.5,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.02,0.04\rangle},\frac{z_2}{\langle 0.05,0.02\rangle},\frac{z_3}{\langle 0.06,0.02\rangle}\Big\}\Big),\Big((k_3/0.5,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.02,0.03\rangle},\frac{z_2}{\langle 0.04,0.02\rangle},\frac{z_3}{\langle 0.06,0.02\rangle}\Big\}\Big),\\
\Big((k_3/0.5,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.03,0.04\rangle},\frac{z_2}{\langle 0.06,0.03\rangle},\frac{z_3}{\langle 0.04,0.02\rangle}\Big\}\Big),\Big((k_4/0.4,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.09,0.01\rangle},\frac{z_2}{\langle 0.01,0.03\rangle},\frac{z_3}{\langle 0.05,0.01\rangle}\Big\}\Big),\\
\Big((k_4/0.4,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.08,0.01\rangle},\frac{z_2}{\langle 0.04,0.03\rangle},\frac{z_3}{\langle 0.02,0.07\rangle}\Big\}\Big),\Big((k_4/0.4,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.06,0.02\rangle},\frac{z_2}{\langle 0.01,0.03\rangle},\frac{z_3}{\langle 0.03,0.05\rangle}\Big\}\Big),\\
\Big((k_5/0.7,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.06,0.03\rangle},\frac{z_2}{\langle 0.02,0.08\rangle},\frac{z_3}{\langle 0.01,0.02\rangle}\Big\}\Big),\Big((k_5/0.7,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.05,0.03\rangle},\frac{z_2}{\langle 0.06,0.02\rangle},\frac{z_3}{\langle 0.04,0.03\rangle}\Big\}\Big),\\
\Big((k_5/0.7,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.04,0.03\rangle},\frac{z_2}{\langle 0.06,0.01\rangle},\frac{z_3}{\langle 0.03,0.05\rangle}\Big\}\Big),\Big((k_1/0.2,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.04,0.03\rangle},\frac{z_2}{\langle 0.09,0.01\rangle},\frac{z_3}{\langle 0.08,0.02\rangle}\Big\}\Big),\\
\Big((k_1/0.2,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.07,0.02\rangle},\frac{z_2}{\langle 0.06,0.04\rangle},\frac{z_3}{\langle 0.02,0.07\rangle}\Big\}\Big),\Big((k_1/0.2,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.02,0.03\rangle},\frac{z_2}{\langle 0.06,0.02\rangle},\frac{z_3}{\langle 0.01,0.07\rangle}\Big\}\Big),\\
\Big((k_2/0.3,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.01,0.04\rangle},\frac{z_2}{\langle 0.04,0.03\rangle},\frac{z_3}{\langle 0.02,0.07\rangle}\Big\}\Big),\Big((k_2/0.3,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.02,0.05\rangle},\frac{z_2}{\langle 0.07,0.02\rangle},\frac{z_3}{\langle 0.06,0.03\rangle}\Big\}\Big),\\
\Big((k_2/0.3,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.07,0.02\rangle},\frac{z_2}{\langle 0.03,0.05\rangle},\frac{z_3}{\langle 0.01,0.07\rangle}\Big\}\Big),\Big((k_3/0.5,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.09,0.01\rangle},\frac{z_2}{\langle 0.04,0.02\rangle},\frac{z_3}{\langle 0.05,0.01\rangle}\Big\}\Big),\\
\Big((k_3/0.5,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.08,0.02\rangle},\frac{z_2}{\langle 0.02,0.07\rangle},\frac{z_3}{\langle 0.03,0.06\rangle}\Big\}\Big),\Big((k_3/0.5,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.06,0.02\rangle},\frac{z_2}{\langle 0.06,0.01\rangle},\frac{z_3}{\langle 0.05,0.04\rangle}\Big\}\Big),\\
\Big((k_4/0.4,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.06,0.03\rangle},\frac{z_2}{\langle 0.03,0.05\rangle},\frac{z_3}{\langle 0.02,0.06\rangle}\Big\}\Big),\Big((k_4/0.4,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.05,0.04\rangle},\frac{z_2}{\langle 0.05,0.02\rangle},\frac{z_3}{\langle 0.03,0.01\rangle}\Big\}\Big),\\
\Big((k_4/0.4,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.04,0.05\rangle},\frac{z_2}{\langle 0.07,0.02\rangle},\frac{z_3}{\langle 0.06,0.01\rangle}\Big\}\Big),\Big((k_5/0.7,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.02,0.05\rangle},\frac{z_2}{\langle 0.06,0.02\rangle},\frac{z_3}{\langle 0.09,0.01\rangle}\Big\}\Big),\\
\Big((k_5/0.7,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.03,0.06\rangle},\frac{z_2}{\langle 0.02,0.08\rangle},\frac{z_3}{\langle 0.03,0.06\rangle}\Big\}\Big),\Big((k_5/0.7,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.01,0.07\rangle},\frac{z_2}{\langle 0.05,0.02\rangle},\frac{z_3}{\langle 0.05,0.01\rangle}\Big\}\Big)
\end{array}\right\},$$

is a FPPFHsES.

Step-2. Construction of Agree and Disagree-FPPFHsESs

$$\left\{\begin{array}{l}
\Big((k_1/0.2,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.09,0.01\rangle},\frac{z_2}{\langle 0.03,0.04\rangle},\frac{z_3}{\langle 0.06,0.02\rangle}\Big\}\Big),\Big((k_1/0.2,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.08,0.02\rangle},\frac{z_2}{\langle 0.01,0.02\rangle},\frac{z_3}{\langle 0.06,0.02\rangle}\Big\}\Big),\\
\Big((k_1/0.2,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.07,0.03\rangle},\frac{z_2}{\langle 0.03,0.07\rangle},\frac{z_3}{\langle 0.03,0.01\rangle}\Big\}\Big),\Big((k_2/0.3,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.06,0.04\rangle},\frac{z_2}{\langle 0.04,0.02\rangle},\frac{z_3}{\langle 0.07,0.01\rangle}\Big\}\Big),\\
\Big((k_2/0.3,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.05,0.02\rangle},\frac{z_2}{\langle 0.06,0.04\rangle},\frac{z_3}{\langle 0.03,0.06\rangle}\Big\}\Big),\Big((k_2/0.3,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.04,0.03\rangle},\frac{z_2}{\langle 0.03,0.02\rangle},\frac{z_3}{\langle 0.03,0.02\rangle}\Big\}\Big),\\
\Big((k_3/0.5,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.02,0.04\rangle},\frac{z_2}{\langle 0.05,0.02\rangle},\frac{z_3}{\langle 0.06,0.02\rangle}\Big\}\Big),\Big((k_3/0.5,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.02,0.03\rangle},\frac{z_2}{\langle 0.04,0.02\rangle},\frac{z_3}{\langle 0.06,0.02\rangle}\Big\}\Big),\\
\Big((k_3/0.5,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.03,0.04\rangle},\frac{z_2}{\langle 0.06,0.03\rangle},\frac{z_3}{\langle 0.04,0.02\rangle}\Big\}\Big),\Big((k_4/0.4,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.09,0.01\rangle},\frac{z_2}{\langle 0.01,0.03\rangle},\frac{z_3}{\langle 0.05,0.01\rangle}\Big\}\Big),\\
\Big((k_4/0.4,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.08,0.01\rangle},\frac{z_2}{\langle 0.04,0.03\rangle},\frac{z_3}{\langle 0.02,0.07\rangle}\Big\}\Big),\Big((k_4/0.4,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.06,0.02\rangle},\frac{z_2}{\langle 0.01,0.03\rangle},\frac{z_3}{\langle 0.03,0.05\rangle}\Big\}\Big),\\
\Big((k_5/0.7,\mathcal{E}_1,1),\Big\{\frac{z_1}{\langle 0.06,0.03\rangle},\frac{z_2}{\langle 0.02,0.08\rangle},\frac{z_3}{\langle 0.01,0.02\rangle}\Big\}\Big),\Big((k_5/0.7,\mathcal{E}_2,1),\Big\{\frac{z_1}{\langle 0.05,0.03\rangle},\frac{z_2}{\langle 0.06,0.02\rangle},\frac{z_3}{\langle 0.04,0.03\rangle}\Big\}\Big),\\
\Big((k_5/0.7,\mathcal{E}_3,1),\Big\{\frac{z_1}{\langle 0.04,0.03\rangle},\frac{z_2}{\langle 0.06,0.01\rangle},\frac{z_3}{\langle 0.03,0.05\rangle}\Big\}\Big),
\end{array}\right\}$$

which is an Agree-FPPFHsES.

$$\left\{\begin{array}{l}
\Big((k_1/0.2,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.04,0.03\rangle},\frac{z_2}{\langle 0.09,0.01\rangle},\frac{z_3}{\langle 0.08,0.02\rangle}\Big\}\Big),\\
\Big((k_1/0.2,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.07,0.02\rangle},\frac{z_2}{\langle 0.06,0.04\rangle},\frac{z_3}{\langle 0.02,0.07\rangle}\Big\}\Big),\Big((k_1/0.2,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.02,0.03\rangle},\frac{z_2}{\langle 0.06,0.02\rangle},\frac{z_3}{\langle 0.01,0.07\rangle}\Big\}\Big),\\
\Big((k_2/0.3,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.01,0.04\rangle},\frac{z_2}{\langle 0.04,0.03\rangle},\frac{z_3}{\langle 0.02,0.07\rangle}\Big\}\Big),\Big((k_2/0.3,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.02,0.05\rangle},\frac{z_2}{\langle 0.07,0.02\rangle},\frac{z_3}{\langle 0.06,0.03\rangle}\Big\}\Big),\\
\Big((k_2/0.3,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.07,0.02\rangle},\frac{z_2}{\langle 0.03,0.05\rangle},\frac{z_3}{\langle 0.01,0.07\rangle}\Big\}\Big),\Big((k_3/0.5,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.09,0.01\rangle},\frac{z_2}{\langle 0.04,0.02\rangle},\frac{z_3}{\langle 0.05,0.01\rangle}\Big\}\Big),\\
\Big((k_3/0.5,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.08,0.02\rangle},\frac{z_2}{\langle 0.02,0.07\rangle},\frac{z_3}{\langle 0.03,0.06\rangle}\Big\}\Big),\Big((k_3/0.5,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.06,0.02\rangle},\frac{z_2}{\langle 0.06,0.01\rangle},\frac{z_3}{\langle 0.05,0.04\rangle}\Big\}\Big),\\
\Big((k_4/0.4,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.06,0.03\rangle},\frac{z_2}{\langle 0.03,0.05\rangle},\frac{z_3}{\langle 0.02,0.06\rangle}\Big\}\Big),\Big((k_4/0.4,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.05,0.04\rangle},\frac{z_2}{\langle 0.05,0.02\rangle},\frac{z_3}{\langle 0.03,0.01\rangle}\Big\}\Big),\\
\Big((k_4/0.4,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.04,0.05\rangle},\frac{z_2}{\langle 0.07,0.02\rangle},\frac{z_3}{\langle 0.06,0.01\rangle}\Big\}\Big),\Big((k_5/0.7,\mathcal{E}_1,0),\Big\{\frac{z_1}{\langle 0.02,0.05\rangle},\frac{z_2}{\langle 0.06,0.02\rangle},\frac{z_3}{\langle 0.09,0.01\rangle}\Big\}\Big),\\
\Big((k_5/0.7,\mathcal{E}_2,0),\Big\{\frac{z_1}{\langle 0.03,0.06\rangle},\frac{z_2}{\langle 0.02,0.08\rangle},\frac{z_3}{\langle 0.03,0.06\rangle}\Big\}\Big),\Big((k_5/0.7,\mathcal{E}_3,0),\Big\{\frac{z_1}{\langle 0.01,0.07\rangle},\frac{z_2}{\langle 0.05,0.02\rangle},\frac{z_3}{\langle 0.05,0.01\rangle}\Big\}\Big),
\end{array}\right\}$$

which is a Disagree-FPPFHsES.

Step-3. Calculation of upper and lower evaluation values of Agree and Disagree-FPPFHsESs

The formulation of evaluation interval, first we have to find the upper and lower evaluation values of z_i defined by as $z_i^- = \mu_z(\text{Mv})$, $z_i^+ = 1 - \nu_z(\text{NMv})$ and closed interval $[z_i^-, z_i^+]$ is called evaluation interval.

The lower and upper evaluation values of z_i for Agree and Disagree-FPPFHsESs are calculated in the following Tables 1 and 2.

Table 1. Upper and lower evaluation values of agree-FPPFHsES.

Z	z_1	z_2	z_3
$(k_1/0.2, \mathcal{E}_1, 1)$	0.09, 0.99	0.03, 0.96	0.06, 0.98
$(k_1/0.2, \mathcal{E}_2, 1)$	0.08, 0.98	0.01, 0.98	0.06, 0.98
$(k_1/0.2, \mathcal{E}_3, 1)$	0.07, 0.97	0.03, 0.93	0.03, 0.99
$(k_2/0.3, \mathcal{E}_1, 1)$	0.06, 0.96	0.04, 0.98	0.07, 0.99
$(k_2/0.3, \mathcal{E}_2, 1)$	0.05, 0.98	0.06, 0.96	0.03, 0.94
$(k_2/0.3, \mathcal{E}_3, 1)$	0.04, 0.97	0.03, 0.98	0.03, 0.98
$(k_3/0.5, \mathcal{E}_1, 1)$	0.02, 0.96	0.05, 0.98	0.06, 0.98
$(k_3/0.5, \mathcal{E}_2, 1)$	0.02, 0.97	0.04, 0.98	0.06, 0.98
$(k_3/0.5, \mathcal{E}_3, 1)$	0.03, 0.96	0.06, 0.97	0.04, 0.98
$(k_4/0.4, \mathcal{E}_1, 1)$	0.09, 0.99	0.01, 0.97	0.05, 0.99
$(k_4/0.4, \mathcal{E}_2, 1)$	0.08, 0.99	0.04, 0.97	0.02, 0.93
$(k_4/0.4, \mathcal{E}_3, 1)$	0.06, 0.98	0.01, 0.97	0.03, 0.95
$(k_5/0.7, \mathcal{E}_1, 1)$	0.06, 0.97	0.02, 0.99	0.01, 0.95
$(k_5/0.7, \mathcal{E}_2, 1)$	0.05, 0.97	0.06, 0.98	0.04, 0.95
$(k_5/0.7, \mathcal{E}_3, 1)$	0.04, 0.97	0.06, 0.99	0.03, 0.95

Table 2. Upper and lower evaluation values of Disagree-FPPFHsES.

Z	z_1	z_2	z_3
$(k_1/0.2, \mathcal{E}_1, 0)$	0.04, 0.97	0.09, 0.99	0.08, 0.98
$(k_1/0.2, \mathcal{E}_2, 0)$	0.07, 0.98	0.06, 0.96	0.02, 0.93
$(k_1/0.2, \mathcal{E}_3, 0)$	0.02, 0.97	0.06, 0.98	0.01, 0.93
$(k_2/0.3, \mathcal{E}_1, 0)$	0.01, 0.96	0.04, 0.97	0.02, 0.93
$(k_2/0.3, \mathcal{E}_2, 0)$	0.02, 0.95	0.07, 0.98	0.06, 0.97
$(k_2/0.3, \mathcal{E}_3, 0)$	0.07, 0.98	0.03, 0.95	0.01, 0.93
$(k_3/0.5, \mathcal{E}_1, 0)$	0.09, 0.99	0.04, 0.98	0.05, 0.99
$(k_3/0.5, \mathcal{E}_2, 0)$	0.08, 0.98	0.02, 0.93	0.03, 0.94
$(k_3/0.5, \mathcal{E}_3, 0)$	0.06, 0.98	0.06, 0.99	0.05, 0.96
$(k_4/0.4, \mathcal{E}_1, 0)$	0.06, 0.97	0.03, 0.95	0.02, 0.94
$(k_4/0.4, \mathcal{E}_2, 0)$	0.05, 0.96	0.05, 0.98	0.01, 0.99
$(k_4/0.4, \mathcal{E}_3, 0)$	0.04, 0.95	0.07, 0.98	0.06, 0.99
$(k_5/0.7, \mathcal{E}_1, 0)$	0.02, 0.95	0.06, 0.98	0.09, 0.99
$(k_5/0.7, \mathcal{E}_2, 0)$	0.03, 0.94	0.02, 0.92	0.03, 0.94
$(k_5/0.7, \mathcal{E}_3, 0)$	0.01, 0.93	0.05, 0.98	0.05, 0.99

Step-4. Formulation of evaluation interval

Evaluation intervals of Agree and Disagree-FPPFHsESs is calculated in Tables 3 and 4 .

Table 3. Evaluation intervals of Agree-FPPFHsES.

Z	z_1	z_2	z_3
$(k_1/0.2, \mathcal{E}_1, 1)$	[0.09, 0.99]	[0.03, 0.96]	[0.06, 0.98]
$(k_1/0.2, \mathcal{E}_2, 1)$	[0.08, 0.98]	[0.01, 0.98]	[0.06, 0.98]
$(k_1/0.2, \mathcal{E}_3, 1)$	[0.07, 0.97]	[0.03, 0.93]	[0.03, 0.99]
$(k_2/0.3, \mathcal{E}_1, 1)$	[0.06, 0.96]	[0.04, 0.98]	[0.07, 0.99]
$(k_2/0.3, \mathcal{E}_2, 1)$	[0.05, 0.98]	[0.06, 0.96]	[0.03, 0.94]
$(k_2/0.3, \mathcal{E}_3, 1)$	[0.04, 0.97]	[0.03, 0.98]	[0.03, 0.98]
$(k_3/0.5, \mathcal{E}_1, 1)$	[0.02, 0.96]	[0.05, 0.98]	[0.06, 0.98]
$(k_3/0.5, \mathcal{E}_2, 1)$	[0.02, 0.97]	[0.04, 0.98]	[0.06, 0.98]
$(k_3/0.5, \mathcal{E}_3, 1)$	[0.03, 0.96]	[0.06, 0.97]	[0.04, 0.98]
$(k_4/0.4, \mathcal{E}_1, 1)$	[0.09, 0.99]	[0.01, 0.97]	[0.05, 0.99]
$(k_4/0.4, \mathcal{E}_2, 1)$	[0.08, 0.99]	[0.04, 0.97]	[0.02, 0.93]
$(k_4/0.4, \mathcal{E}_3, 1)$	[0.06, 0.98]	[0.01, 0.97]	[0.03, 0.95]
$(k_5/0.7, \mathcal{E}_1, 1)$	[0.06, 0.97]	[0.02, 0.99]	[0.01, 0.95]
$(k_5/0.7, \mathcal{E}_2, 1)$	[0.05, 0.97]	[0.06, 0.98]	[0.04, 0.95]
$(k_5/0.7, \mathcal{E}_3, 1)$	[0.04, 0.97]	[0.06, 0.99]	[0.03, 0.95]

Table 4. Evaluation intervals of Disagree-FPPFHsES.

Z	z_1	z_2	z_3
$(k_1/0.2, \mathcal{E}_1, 0)$	[0.04, 0.97]	[0.09, 0.99]	[0.08, 0.98]
$(k_1/0.2, \mathcal{E}_2, 0)$	[0.07, 0.98]	[0.06, 0.96]	[0.02, 0.93]
$(k_1/0.2, \mathcal{E}_3, 0)$	[0.02, 0.97]	[0.06, 0.98]	[0.01, 0.93]
$(k_2/0.3, \mathcal{E}_1, 0)$	[0.01, 0.96]	[0.04, 0.97]	[0.02, 0.93]
$(k_2/0.3, \mathcal{E}_2, 0)$	[0.02, 0.95]	[0.07, 0.98]	[0.06, 0.97]
$(k_2/0.3, \mathcal{E}_3, 0)$	[0.07, 0.98]	[0.03, 0.95]	[0.01, 0.93]
$(k_3/0.5, \mathcal{E}_1, 0)$	[0.09, 0.99]	[0.04, 0.98]	[0.05, 0.99]
$(k_3/0.5, \mathcal{E}_2, 0)$	[0.08, 0.98]	[0.02, 0.93]	[0.03, 0.94]
$(k_3/0.5, \mathcal{E}_3, 0)$	[0.06, 0.98]	[0.06, 0.99]	[0.05, 0.96]
$(k_4/0.4, \mathcal{E}_1, 0)$	[0.06, 0.97]	[0.03, 0.95]	[0.02, 0.94]
$(k_4/0.4, \mathcal{E}_2, 0)$	[0.05, 0.96]	[0.05, 0.98]	[0.01, 0.99]
$(k_4/0.4, \mathcal{E}_3, 0)$	[0.04, 0.95]	[0.07, 0.98]	[0.06, 0.99]
$(k_5/0.7, \mathcal{E}_1, 0)$	[0.02, 0.95]	[0.06, 0.98]	[0.09, 0.99]
$(k_5/0.7, \mathcal{E}_2, 0)$	[0.03, 0.94]	[0.02, 0.92]	[0.03, 0.94]
$(k_5/0.7, \mathcal{E}_3, 0)$	[0.01, 0.93]	[0.05, 0.98]	[0.05, 0.99]

Step-5. Calculation of numerical grades

We calculate the sum of μ_{z_i}(Mv) and μ_{z_i} (NMv) of each z_i and have been represented in Tables 5 and 6.

Table 5. Sum of the membership values of Agree-FPPFHsES.

Z_i	$\sum \mu_{z_i}$	$\sum \nu_{z_i}$
Z_1	0.84	14.61
Z_2	0.55	14.59
Z_3	0.62	14.52

Table 6. Sum of the membership values of Disagree-FPPFHsES.

Z_i	$\sum \mu_{z_i}$	$\sum \nu_{z_i}$
Z_1	0.66	14.46
Z_2	0.75	14.52
Z_3	0.59	14.4

Step-6. Calculation of evaluation scores for Agree and Disagree-FPPFHsESs

In this step, we find evaluation score for Agree-FPPFHsES by as

$$S(z_1) = ([\mu_{z_1}-\mu_{z_2}]-[\nu_{z_1}-\nu_{z_2}]-[\mu_{z_1}-\mu_{z_3}]-[\nu_{z_1}-\nu_{z_3}]) = ([0.84-0.55]+[14.61-14.59]+[0.84-0.62]+[14.61-14.52]) = 0.62$$

$$S(z_2) = ([\mu_{z_2}-\mu_{z_1}]-[\nu_{z_2}-\nu_{z_1}]-[\mu_{z_2}-\mu_{z_3}]-[\nu_{z_2}-\nu_{z_3}]) = ([0.55-0.84]+[14.59-14.61]+[0.55-0.62]+[14.59-14.52]) = -0.31$$

$$S(z_3) = ([\mu_{z_3}-\mu_{z_1}]-[\nu_{z_3}-\nu_{z_1}]-[\mu_{z_3}-\mu_{z_2}]-[\nu_{z_3}-\nu_{z_2}]) = ([0.62-0.84]+[14.52-14.61]+[0.62-0.55]+[14.52-14.59]) = -0.24$$

Now the evaluation score for Disagree-FPPFHsES is calculated as

$$S(z_1) = ([\mu_{z_1}-\mu_{z_2}]-[\nu_{z_1}-\nu_{z_2}]-[\mu_{z_1}-\mu_{z_3}]-[\nu_{z_1}-\nu_{z_3}]) = ([0.66-0.75]+[14.46-14.52]+[0.66-0.59]+[14.46-14.4]) = -0.02$$

$$S(z_2) = ([\mu_{z_2}-\mu_{z_1}]-[\nu_{z_2}-\nu_{z_1}]-[\mu_{z_2}-\mu_{z_3}]-[\nu_{z_2}-\nu_{z_3}]) = ([0.75-0.66]+[14.52-14.46]+[14.52-14.4]+[0.75-0.59]) = 0.43$$

$$S(z_3) = ([\mu_{z_3}-\mu_{z_1}]-[\nu_{z_3}-\nu_{z_1}]-[\mu_{z_3}-\mu_{z_2}]-[\nu_{z_3}-\nu_{z_2}]) = ([0.59-0.66]+[14.4-14.46]+[0.66-0.59]+[14.4-14.52]) = -0.18$$

Step-7. Decision

Ranking has been shown in Table 7.

Table 7. Numerical values of $\Pi_i = \Upsilon_i - \Gamma_i$.

Υ_i	Γ_i	$\Pi_i = \Upsilon_i - \Gamma_i$
$S(z_1) = 0.62$	$S(z_1) = -0.02$	$\Pi_1 = 0.64$
$S(z_2) = -0.31$	$S(z_2) = 0.43$	$\Pi_2 = 0.12$
$S(z_3) = -0.24$	$S(z_3) = -0.18$	$\Pi_3 = -0.06$

As Υ_1 is showing best, so kind z_1 is adopted. Graphical representation of ranking of alternatives has been shown in Figure 10.

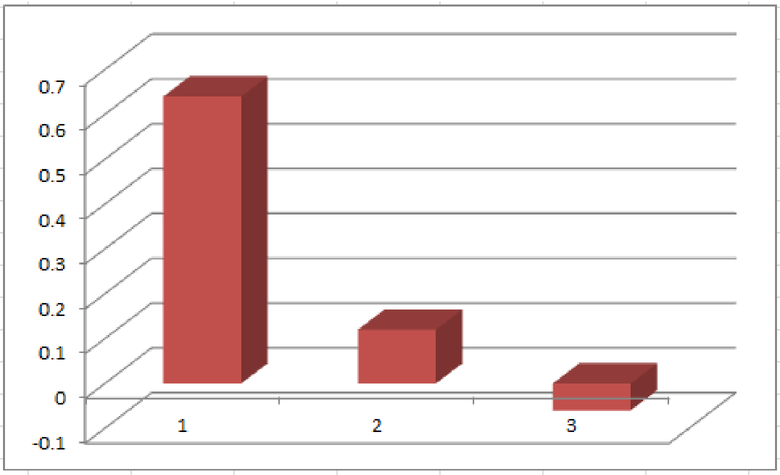

Figure 10. Ranking of alternative for algorithm.

5. Comparative analysis

A $\mathcal{FPIFH\int E}$-model gives best rapport, accurateness and agreeability than existing soft set like models. This can be seen by comparing FPPFHsES with the others models like FPIFSS, FPIFSES, FPIFHsS, FPPFHsS. This proposed model is more useful to others as it contains the multi argument approximate function, which is highly effective in decision making problems. Comparison analysis has been shown in Table 8.

Table 8. Comparison analysis.

Features	FPIFSS	FPIFSES	FPIFHsS	FPPFHsS	Pro. Structure
Multi Decisive Opinion	No	No	No	No	Yes
Multi Argument Apro.Function	No	No	Yes	Yes	Yes
Single Argument Apro. Function	Yes	Yes	Yes	Yes	Yes
Ranking	No	Yes	No	No	Yes

6. Discussion

Here a useful discussion has been made about this structure i.e., fuzzy parameterized Pythagorean fuzzy hypersoft expert set.

(1) It takes the form of fuzzy parameterized hypersoft expert set if membership and non membership values are excluded.

(2) It changes into fuzzy parameterized fuzzy hypersoft expert set if non-membership values are excluded.

(3) It changes into fuzzy parameterized fuzzy hypersoft set if non-membership values and expert set are excluded.

(4) It converts into fuzzy parameterized hypersoft set if membership, non-membership values and expert set are excluded.

(5) It reduces to fuzzy parameterized Pythagorean fuzzy soft expert set if single argument approximate functions are used instead of multi-argument approximate functions.

(6) It reduces to fuzzy parameterized fuzzy soft expert set if single argument approximate functions are used instead of multi-argument approximate functions and non membership values are excluded.

(7) It reduces to fuzzy parameterized soft expert set if single argument approximate functions are used instead of multi-argument approximate functions and membership, non membership values are excluded.

(8) It becomes fuzzy parameterized fuzzy soft set if multi-argument approximate functions, experts set and membership, non membership values are excluded.

(9) It takes the form of fuzzy parameterized soft set when membership, non membership values, expert set and multi-argument approximate functions are excluded.

(10) Fuzzy parameterized soft set becomes the soft set when fuzzy parameterization is excluded.

Advantages

Following are the advantages of FPPFHsES.

(1) The proposed method took the importance of the concept of parameterization along the $\mathcal{PFH}\int\mathcal{E}S$ to cope with real-life decision-making issues. The parameterization taken into consideration, depicts the opportunity of the lifestyles to the extent of support and excusal; alongside these lines, this affiliation has excellent capability within side the authentic depiction in the area of computational incursions.

(2) The present model points up the main study of parameters along with sub parameters under the multi-decisive opinions, it makes the decision-making best, soft and extra stable.

(3) The proposed structure holds all the aspects and features of existing models like FPPFHsS, IFPHsS, IFPSS, IFPSES, IFHsES.

The following table shows the advantages of the this structure i.e., FPPFHsES. In this table, FPPFHsES is compared with some characteristics of already existing structures which are Membership value (Mv), Non-membership value (NMVv), Degree of parameterization (DOP), Single argument approximate function (SAAF), Multi-argument approximate function (MAAF) and Multi-decisive opinion (MDO). In the following table sign $\uparrow$ will be used for Yes and $\downarrow$ for No. From the Table 9, it is clear that our proposed model is more generalized than the above described models as it not only satisfies the essential properties of these existing models but has capability to tackle their insufficiencies. It is much suitable to say that the existing models (mentioned in Subsection 1.1. Research gap and motivation and Table 9) are considered as its particular cases by waiving off some conditions (mentioned in Section 6. Discussion) from the proposed model.

Table 9. Comparison with some structures under particular characteristics.

Authors	Models	Mv	NMv	DOP	SAAF	MAAF	MDO
Molodtsov [2]	SS	↓	↓	↓	↑	↓	↓
Maji et al. [15]	FSS	↑	↓	↓	↑	↓	↓
Çağman et al. [15]	IFSS	↑	↑	↓	↑	↓	↓
Çağman et al. [36]	FPIFSS	↑	↑	↑	↑	↓	↓
Bashir et al. [41]	FPIFSES	↑	↑	↑	↑	↓	↑
Rahman et al. [45]	FPHS	↑	↓	↑	↑	↑	↓
Ihsan et al. [34]	HsES	↓	↓	↓	↑	↑	↑
Ihsan et al. [35]	FHsES	↑	↓	↓	↑	↑	↑
Proposed Structure	FPPFHsES	↑	↑	↑	↑	↑	↑

7. Conclusions

The aim of this paper is to describe and discuss the fundamentals of FPPFHsES. Some basic properties and laws pertaining this concept are obtained for FPPFHsES in a formal way. This article presents a decision-making methodology to select the best product in terms of multi-criteria by using FPPFHsES. This new work will give an outstanding expansion to existing theories for dealing with truthness, falsity and motivates more improvements of additional research and relevant applications. As the focus is laid on fuzzy membership in domain and range of multi-argument approximate function while characterizing FPPFHsES therefore the proposed model has some limitations regarding the scenarios having entitlement of non-membership and indeterminacy grades in domain and range of above mentioned approximate function. The addressal of such limitations may be taken as future task for characterization. Moreover, various other real-world decision-making problems can be resolved by using this proposed model through the employment of various decision-making techniques like MCDM, MCGDM, TOPSIS and PROMETHEE.

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Conflict of interest

The authors declare no conflict of interest.

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