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Research article

Hyperbolic function solutions of time-fractional Kadomtsev-Petviashvili equation with variable-coefficients

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Abstract: Based on the variable separation method, the Kadomtsev-Petviashvili equation is transformed into a system of equations, in which one is a fractional ordinary differential equation with respect to time variable t, and the other is an integer order variable coefficients partial differential equation with respect to spatial variables x, y. Assuming that the coefficients of the obtained partial differential equation satisfy certain conditions, the equation is further reduced. The extended homogeneous balance method is used to find the exact solutions of the reduced equation. According to the solutions of some special fractional ordinary differential equations, we obtain some hyperbolic function solutions of time-fractional Kadomtsev-Petviashvili equation with variable coefficients.

Keywords: Riemann-Liouville; variable separation method; time-fractional; exact solution; extended homogeneous balance method
Mathematics Subject Classification: 35N10, 35R11, 83C15

1. Introduction

In recent years, the fractional partial differential equations (FPDEs) have aroused the extensive attention of many scholars because the FPDEs are widely used in many fields, such as fluid mechanics, plasma physics, biology, condensed matter physics, etc. Some effective methods are proposed to find the exact solutions of the FPDEs, such as method of separation variables [1–3], invariant subspace method [4–6], Adomian decomposition method [7–9], dynamical system method [10, 11], Lie group theory method [12–14], and so on. In [15, 16], the variable separation method is successfully used to obtain the exact solutions for the variable-coefficient time fractional partial differential equations with forcing term and (2+1)-dimensional nonlinear time fractional biological population model.

The study of this paper is focused on time-fractional Kadomtsev-Petviashvili equation with variable-coefficients as follows

$$D_t^{\alpha}(u_x) + l(t)(u_x)^2 + l(t)uu_{xx} + g(t)u_{xxxx} + \mu(t)u_x + h(t)u_{yy} = 0, \tag{1.1}$$

where D_t^α $(0 < \alpha < 1)$ is Riemann-Liouville fractional derivative with respect to t.

when $\alpha = 1$, Eq (1.1) is Kadomtsev-Petviashvili equations with variable-coefficients [17, 18] as follows

$$u_{xt} + l(t)(u_x)^2 + l(t)uu_{xx} + g(t)u_{xxxx} + \mu(t)u_x + h(t)u_{yy} = 0,$$

where differentiable function $u(x, y, t)$ represents the wave amplitude; coefficients $l(t)$, $\mu(t)$, $g(t)$, $h(t)$ are all real function; $\mu(t)$ gives the perturbed effect; $l(t)$, $g(t)$ and $h(t)$ represent the coefficients of nonlinearity, dispersion and the disturbed term along the y direction, respectively. In [17], the higher-order rogue waves for Kadomtsev-Petviashvili equations with variable-coefficients have been investigated. In [18], with the help of bilinear BT, the single soliton solution and double soliton solution of the equation and their soliton characteristics are given.

The rest of this paper is arranged as follows. In Section 2, we give some background knowledge which will be used. In Section 3, we apply the variable separation method to construct exact solutions of time-fractional Kadomtsev-Petviashvili equation with variable coefficients. In Section 4, some special solutions in the form of hyperbolic functions are found using the extended homogeneous balance method. Some conclusions are provided at the end of the paper.

2. Preliminaries

Since the definition of fractional derivative was put forward in 1695, there have been many different forms of definition [19–21] such as Riemann-Liouville, Caputo, Grunwald-Letnikov, Riez-Feller, Weyl, etc. However, Riemann-Liouville definition, Caputo definition and Grunwald-Letnikov definition are still the most influential. In this article, we adopt the definition of the left Riemann-Liouville fractional derivative [19–21], which is as follows

$$D_t^\alpha(f(t)) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_a^t (t-\tau)^{n-\alpha-1}(f(\tau))\mathrm{d}\tau,$$

where $t \in [a, b]$, $n = [\alpha + 1]$, $n - 1 \le \alpha < n$, $0 \le a < t$, $\Gamma(\cdot)$ is Gamma function.

Some properties [19] of the Riemann-Liouville fractional derivative are used, which are as follows

$$D_t^\alpha[\lambda u(x) + \nu v(x)] = \lambda D_t^\alpha u(x) + \nu D_t^\alpha v(x), \tag{2.1}$$

where λ and ν are constants.

$$D_t^\alpha t^\gamma = \frac{\Gamma(n+\gamma)}{\Gamma(n+\gamma-\alpha)} t^{\gamma-\alpha}. \tag{2.2}$$

If $0 < \alpha < 1$, (2.2) is reduced into

$$D_t^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)} t^{\gamma-\alpha}, \quad \gamma > -1. \tag{2.3}$$

The solutions of two fractional ordinary differential equations with $0 < \alpha < 1$ are as below [19]. The solution to the Cauchy type problem

$$D_t^\alpha(y(t)) - \lambda y = 0, \quad D_t^{\alpha-1}(y(t))|_{t=0} = b \tag{2.4}$$

with $\lambda,\ \ b \in \mathbf{R}$ is given by

$$y(t) = bt^{\alpha-1}E_{\alpha,\alpha}(\lambda t^{\alpha}), \tag{2.5}$$

where $E_{\alpha,\alpha}$ is Mittag-Leffler function [19].

The equation

$$D_t^{\alpha}(y(t)) - \lambda t^{\beta} y = 0, \ \ (\lambda \in \mathbf{R},\ \ \beta \in \mathbf{R},\ \ \beta > -\{\alpha\}), \tag{2.6}$$

with initial condition $D_t^{\alpha-1}(y(t))|_{t=0} = b,\ \ b \in \mathbf{R}$.

Solution of Eq (2.6) is

$$y(t) = \frac{b}{\Gamma(\alpha)} t^{\alpha-1} E_{\alpha,1+\frac{\beta}{\alpha},1+\frac{\beta-1}{\alpha}}(\lambda t^{\alpha+\beta}), \tag{2.7}$$

where $E_{\alpha,1+\frac{\beta}{\alpha},1+\frac{\beta-1}{\alpha}}$ is generalized Mittag-Leffler function [19].

3. Construction of exact solutions of Eq (1.1) by variable separation method

It is assumed that the solution of Eq (1.1) is in the following form

$$u = H(t)W(x, y). \tag{3.1}$$

Substituting (3.1) into Eq (1.1), we obtain

$$\begin{aligned} &D_t^{\alpha}(H(t))W_x + l(t)(H(t))^2(W_x)^2 + l(t)(H(t))^2 W W_{xx} + g(t)H(t)W_{xxxx} + \mu(t)H(t)W_x \\ &+h(t)H(t)W_{yy} = 0. \end{aligned} \tag{3.2}$$

Letting

$$D_t^{\alpha}(H(t))W_x + \mu(t)H(t)W_x = 0, \tag{3.3}$$

$$l(t)(H(t))^2(W_x)^2 + l(t)(H(t))^2 W W_{xx} + g(t)H(t)W_{xxxx} + h(t)H(t)W_{yy} = 0. \tag{3.4}$$

Equation (3.3) is reduced into an ODE as follows

$$D_t^{\alpha}(H(t)) + \mu(t)H(t) = 0, \tag{3.5}$$

The solution of Eq (3.5) is complicated. Here we consider only two special cases: $\mu(t) = -\lambda$ and $\mu(t) = -\lambda t^{\beta}$, then the solution of Eq (3.5) can refer to Eqs (2.4) and (2.6), respectively.

For Eq (3.4), it is also difficult to find exact solution. It may be assumed that the coefficients of Eq (3.4) satisfy the following relationships

$$g(t) = Al(t)H(t),\ \ h(t) = Bl(t)H(t), \tag{3.6}$$

where A and B are constants. Then Eq (3.4) is reduced into a nonlinear partial differential equation with constant coefficients as follows

$$(W_x)^2 + WW_{xx} + AW_{xxxx} + BW_{yy} = 0. \tag{3.7}$$

There are many methods for exact solutions of nonlinear partial differential equations. Then we use the extended homogeneous balance method [22–24] to find the exact solution of Eq (3.7).

4. Exact solutions of Eq (1.1) by extended homogeneous balance method

According to the idea of extended homogeneous balance method [22–24], we assume that the solution of Eq (3.7) has the following form

$$W(x,y)=\frac{\partial^{(r+s)}f(\phi)}{\partial x^r\partial y^s}+W_0=f^{(r+s)}\phi_x^r\phi_y^s+\cdots, \tag{4.1}$$

where $W_0=W_0(x,y)$ is the arbitrary known seed solution; r, s and function $f(\phi)$ are to be determined later.

From (4.1), we obtain

$$W_{xxxx}=f^{(r+s+4)}\phi_x^{r+4}\phi_y^s+\cdots, \tag{4.2}$$
$$W_x^2=f^{(r+s+1)}f^{(r+s+1)}\phi_x^{2r+2}\phi_y^{2s}+\cdots, \tag{4.3}$$
$$WW_{xx}=f^{(r+s)}f^{(r+s+2)}\phi_x^{2r+2}\phi_y^{2s}+\cdots. \tag{4.4}$$

Balancing the highest order derivative term W_{xxxx} and the highest order nonlinear term $W_x^2,\ WW_{xx}$ can be obtained

$$2r+2=2r+2=r+4,\quad s=2s=2s,$$

which gives

$$r=2,\quad s=0.$$

Thus, (4.1) can be rewritten as follows

$$W(x,y)=\frac{\partial^2 f(\phi)}{\partial x^2}+W_0=f''\phi_x^2+f'\phi_{xx}+W_0. \tag{4.5}$$

From (4.5), it is easy to deduce that

$$\begin{aligned}W_{yy}&=f^{(4)}\phi_y^2\phi_x^2+f^{(3)}\phi_{yy}\phi_x^2+4f^{(3)}\phi_y\phi_x\phi_{xy}+2f''\phi_{xy}^2+2f''\phi_x\phi_{xyy}\\&\quad+f^{(3)}\phi_y^2\phi_{xx}+f''\phi_{xx}\phi_{yy}+f''\phi_y\phi_{xxy}+f'\phi_{xxyy}+W_{0yy}.\end{aligned} \tag{4.6}$$

$$W_{xx}=f^{(4)}\phi_x^4+6f^{(3)}\phi_x^2\phi_{xx}+3f''\phi_{xx}^2+4f''\phi_x\phi_{xxx}+f'\phi_{xxxx}+W_{0xx}. \tag{4.7}$$

$$\begin{aligned}W_x^2&=(f^{(3)})^2\phi_x^6+9(f'')^2\phi_x^2\phi_{xx}^2+(f')^2\phi_{xxx}^2+u_{0x}^2+6f^{(3)}f''\phi_x^4\phi_{xx}\\&\quad+2f^{(3)}f'\phi_x^3\phi_{xxx}+2W_{0x}f^{(3)}\phi_x^3+6f''f'\phi_x\phi_{xx}\phi_{xxx}+6W_{0x}f''\phi_x\phi_{xx}+2W_{0x}f'\phi_{xxx}.\end{aligned} \tag{4.8}$$

$$\begin{aligned}WW_{xx}&=f''f^{(4)}\phi_x^6+6f''f^{(3)}\phi_x^4\phi_{xx}+3(f'')^2\phi_x^2\phi_{xx}^2+4(f'')^2\phi_x^3\phi_{xxx}+f'f''\phi_x^2\phi_{xxxx}+f''\phi_x^2W_{0xx}\\&\quad+f'f^{(4)}\phi_x^4\phi_{xx}+6f'f^{(3)}\phi_x^2\phi_{xx}^2+3f'f''\phi_{xx}^3+4f'f''\phi_x\phi_{xx}\phi_{xxx}+(f')^2\phi_{xx}\phi_{xxxx}+f'\phi_{xx}W_{0xx}\\&\quad+f^{(4)}W_0\phi_x^4+6f^{(3)}W_0\phi_x^2\phi_{xx}+3f''W_0\phi_{xx}^2+4f''W_0\phi_x\phi_{xxx}+f'W_0\phi_{xxxx}+W_0W_{0xx}.\end{aligned} \tag{4.9}$$

$$\begin{aligned}W_{xxxx}&=f^{(6)}\phi_x^6+15f^{(5)}\phi_x^4\phi_{xx}+45f^{(4)}\phi_x^2\phi_{xx}^2+20f^{(4)}\phi_x^3\phi_{xxx}+15f^{(3)}\phi_{xx}^3\\&\quad+60f^{(3)}\phi_x\phi_{xx}\phi_{xxx}+15f^{(3)}\phi_x^2\phi_{xxxx}+10f''\phi_{xx}^2+15f''\phi_{xx}\phi_{xxxx}+6f''\phi_x\phi_{xxxx}\end{aligned}$$

$$+f'\phi_{xxxxxx}+W_{0xxxx}. \tag{4.10}$$

Now substituting (4.6)–(4.10) into Eq (3.7) and simplifying, it yields

$$\begin{aligned}&\phi_x^6[(f^{(3)})^2+f^{(4)}f''+Af^{(6)}]+\phi_x^4[(12f''f^{(3)}+f'f^{(4)}+15Af^{(5)})\phi_{xx}+W_0f^{(4)}]\\&+\phi_x^3[(2f'f^{(3)}+4(f')^2+20Af^{(4)})\phi_{xxx}+2f^{(3)}W_{0x}]+\phi_x^2[Bf^{(4)}\phi_y^2+Bf^{(3)}\phi_{yy}+6W_0f^{(3)}\phi_{xx}\\&+(12(f'')^2+6f'f^{(3)}+45Af^{(4)})\phi_{xx}^2+(f'f''+15Af^{(3)})\phi_{xxxx}+f''W_{0xx}]+\phi_x[4Bf^{(3)}\phi_y\phi_{xy}\\&+2Bf''\phi_{xyy}+(10f'f''\phi_{xx}+60Af^{(3)}\phi_{xx}+4W_0f'')\phi_{xxx}+6Af''\phi_{xxxx}+6W_{0x}f''\phi_{xx}]\\&+\phi_{xx}^3(3f'f''+15Af^{(3)})+3W_0f''\phi_{xx}^2+\phi_{xx}[Bf^{(3)}\phi_y^2+Bf''\phi_{yy}+(15Af''+(f')^2)\phi_{xxxx}+W_{0xx}]\\&+\phi_{xxx}^2[(f')^2+10Af'']+2W_{0x}f'\phi_{xxx}+W_0f'\phi_{xxxx}+2Bf''\phi_{xy}^2+Bf'\phi_{xxyy}+Bf''\phi_y\phi_{xxy}\\&+Af'\phi_{xxxxxx}+AW_{0xxxx}+W_{0x}^2+W_0W_{0xx}+BW_{0yy}=0.\end{aligned} \tag{4.11}$$

Setting the coefficient of the term ϕ_x^6 in Eq (4.11) to zero, we obtain a nonlinear ordinary differential equation for function $f(\phi)$

$$(f^{(3)})^2+f^{(4)}f''+Af^{(6)}=0. \tag{4.12}$$

Integrating Eq (4.12) with respective to x twice and letting the constant of integration be zero yields

$$(f'')^2+Af^{(4)}=0, \tag{4.13}$$

which has particular solution

$$f(\phi)=12A\ln\phi. \tag{4.14}$$

According to (4.14), we get the following results

$$\begin{aligned}&(f')^2=-rf'',\ \ f''f'=-\frac{r}{2}f^{(3)},\ \ f^{(3)}f'=-\frac{r}{3}f^{(4)},\\&f^{(4)}f'=-\frac{r}{4}f^{(5)},\ \ (f'')^2=-\frac{r}{6}f^{(4)},\ \ f''f^{(3)}=-\frac{r}{12}f^{(5)}.\end{aligned} \tag{4.15}$$

By (4.5) and (4.14), we obtain the auto-Bäcklund transformation of Eq (3.7) as follows

$$W(x,y)=-\frac{12A\phi_x^2}{\phi^2}+\frac{12A\phi_{xx}}{\phi}+W_0. \tag{4.16}$$

Letting the seed solution

$$W_0(x,y)=0, \tag{4.17}$$

and using results (4.15), then Eq (4.11) can be simplified as

$$\begin{aligned}&f^{(4)}[4A\phi_x\phi_{xxx}+B\phi_y^2-3A\phi_{xx}^2]\phi_x^2+f^{(3)}[(B\phi_{yy}+9A\phi_{xxxx})\phi_x^2+4B\phi_x\phi_y\phi_{xy}-3A\phi_{xx}^3\\&+B\phi_{xx}\phi_y^2]+f''[6A\phi_x\phi_{xxxx}+2B\phi_x\phi_{xyy}+3A\phi_{xx}\phi_{xxxx}+B\phi_{yy}\phi_{xx}-2A\phi_{xxx}^2\\&+2B\phi_{xy}^2+2B\phi_y\phi_{xxy}]+f'[B\phi_{xxyy}+A\phi_{xxxxxx}]=0.\end{aligned} \tag{4.18}$$

Setting the coefficients of $f^{(4)}, f^{(3)}, f'', f'$ in Eq (4.18) to zero yields a set of partial differential equations for $\phi(x,t)$ as follows

$$[4A\phi_x\phi_{xxx}+B\phi_y^2-3A\phi_{xx}^2]\phi_x^2=0, \tag{4.19}$$

$$(B\phi_{yy} + 9A\phi_{xxxx})\phi_x^2 + 4B\phi_x\phi_y\phi_{xy} - 3A\phi_{xx}^3 + B\phi_{xx}\phi_y^2 = 0, \tag{4.20}$$
$$6A\phi_x\phi_{xxxx} + 2B\phi_x\phi_{xyy} + 3A\phi_{xx}\phi_{xxxx} + B\phi_{yy}\phi_{xx} - 2A\phi_{xxx}^2 + 2B\phi_{xy}^2 + 2B\phi_y\phi_{xxy} = 0, \tag{4.21}$$
$$(B\phi_{yy} + A\phi_{xxxx})_{xx} = 0. \tag{4.22}$$

Here, Eqs (4.20) and (4.21) can be rewritten as

$$\begin{aligned}&\phi_x^2[B\phi_{yy} + A\phi_{xxxx}] + \phi_{xx}[4A\phi_x\phi_{xxx} + B\phi_y^2 - 3A\phi_{xx}^2]\\ &+2\phi_x[4A\phi_x\phi_{xxx} + B\phi_y^2 - 3A\phi_{xx}^2]_x = 0.\end{aligned} \tag{4.23}$$
$$\begin{aligned}&\phi_{xx}[B\phi_{yy} + A\phi_{xxxx}] + 2\phi_x[B\phi_{yy} + A\phi_{xxxx}]_x\\ &+[4A\phi_x\phi_{xxx} + B\phi_y^2 - 3A\phi_{xx}^2]_{xx} = 0.\end{aligned} \tag{4.24}$$

By analysis Eqs (4.23) and (4.24), we find that the Eqs (4.19)–(4.22) are satisfied automatical under the conditions

$$B\phi_{yy} + A\phi_{xxxx} = 0, \tag{4.25}$$
$$4A\phi_x\phi_{xxx} + B\phi_y^2 - 3A\phi_{xx}^2 = 0. \tag{4.26}$$

To obtain some exact solutions of Eqs (4.25) and (4.26), we assume the solutions of the form

$$\phi(x, y) = M + N\sinh(\xi)\exp(\eta), \tag{4.27}$$

where $\xi = Kx + \Omega y + \xi_0, \eta = kx + \omega y + \eta_0$; $M, N, K, \Omega, \xi_0, k, \omega, \eta_0$ are constants which are to be determined.

Substituting (4.27) into Eqs (4.25) and (4.26) and simplifying, it leads to a system of nonlinear algebraic equations with respect to K, Ω, k, ω as follows

$$\begin{aligned}&AK^4 + 6AK^2k^2 + Ak^4 + B\Omega^2 + B\omega^2 = 0,\\ &4AKk^3 + B\Omega^2 = 0,\\ &2AK^3k + 2AKk^3 + B\Omega\omega = 0,\\ &6AK^2k^2 + Ak^4 - 3AK^4 + B\omega^2 = 0.\end{aligned}$$

By solving the above equations, the following solutions are obtained

Case 1: $K = k,\ \ \Omega = \omega = \sqrt{\frac{-4A}{B}}k^2$, where $A \cdot B < 0$.

Case 2: $K = k,\ \ \Omega = \omega = -\sqrt{\frac{-4A}{B}}k^2$, where $A \cdot B < 0$.

For case 1, according to (4.16), (4.17) and (4.27), the solutions of Eq (3.7) are as follows

$$\begin{aligned}W(x, y) = &-\frac{12AN^2k^2\exp(2\eta)[\cosh(\xi) + \sinh(\xi)]^2}{[M + N\sinh(\xi)\exp(\eta)]^2}\\ &+ \frac{24ANk^2\exp(\eta)[\sinh(\xi) + \cosh(\xi)]}{M + N\sinh(\xi)\exp(\eta)},\end{aligned}$$

where $\xi = kx + \sqrt{\frac{-4A}{B}}k^2y + \xi_0, \eta = kx + \sqrt{\frac{-4A}{B}}k^2y + \eta_0$.

When $\mu(t) = -\lambda$, the exact explicit solutions to time-fractional Kadomtsev-Petviashvili equations with variable-coefficient are obtained as follows

$$\begin{aligned} u(x,y,t) = b\Big[&- \frac{12AN^2k^2\exp(2\eta)[\cosh(\xi)+\sinh(\xi)]^2}{[M+N\sinh(\xi)\exp(\eta)]^2} \\ &+ \frac{24ANk^2\exp(\eta)[\sinh(\xi)+\cosh(\xi)]}{M+N\sinh(\xi)\exp(\eta)}\Big] t^{\alpha-1}E_{\alpha,\alpha}(\lambda t^{\alpha}), \end{aligned} \tag{4.28}$$

where $\xi = kx + \sqrt{\frac{-4A}{B}}k^2y + \xi_0, \eta = kx + \sqrt{\frac{-4A}{B}}k^2y + \eta_0$.

When $\mu(t) = -\lambda t^{\beta}$, the exact explicit solutions to time-fractional Kadomtsev-Petviashvili equations with variable-coefficient are obtained as follows

$$\begin{aligned} u(x,y,t) = \frac{b}{\Gamma(\alpha)}\Big[&- \frac{12AN^2k^2\exp(2\eta)[\cosh(\xi)+\sinh(\xi)]^2}{[M+N\sinh(\xi)\exp(\eta)]^2} \\ &+ \frac{24ANk^2\exp(\eta)[\sinh(\xi)+\cosh(\xi)]}{M+N\sinh(\xi)\exp(\eta)}\Big] t^{\alpha-1}E_{\alpha,1+\frac{\beta}{\alpha},1+\frac{\beta-1}{\alpha}}(\lambda t^{\alpha+\beta}), \end{aligned} \tag{4.29}$$

where $\xi = kx + \sqrt{\frac{-4A}{B}}k^2y + \xi_0, \eta = kx + \sqrt{\frac{-4A}{B}}k^2y + \eta_0$.

For case 2, according to (4.16), (4.17) and (4.27), the solutions of Eq (3.7) are as follows

$$\begin{aligned} W(x,y) = &- \frac{12AN^2k^2\exp(2\eta)[\cosh(\xi)+\sinh(\xi)]^2}{[M+N\sinh(\xi)\exp(\eta)]^2} \\ &+ \frac{24ANk^2\exp(\eta)[\sinh(\xi)+\cosh(\xi)]}{M+N\sinh(\xi)\exp(\eta)}, \end{aligned}$$

where $\xi = kx - \sqrt{\frac{-4A}{B}}k^2y + \xi_0, \eta = kx - \sqrt{\frac{-4A}{B}}k^2y + \eta_0$.

When $\mu(t) = -\lambda$, the exact explicit solutions to time-fractional Kadomtsev-Petviashvili equations with variable-coefficient are obtained as follows

$$\begin{aligned} u(x,y,t) = b\Big[&- \frac{12AN^2k^2\exp(2\eta)[\cosh(\xi)+\sinh(\xi)]^2}{[M+N\sinh(\xi)\exp(\eta)]^2} \\ &+ \frac{24ANk^2\exp(\eta)[\sinh(\xi)+\cosh(\xi)]}{M+N\sinh(\xi)\exp(\eta)}\Big] t^{\alpha-1}E_{\alpha,\alpha}(\lambda t^{\alpha}), \end{aligned} \tag{4.30}$$

where $\xi = kx - \sqrt{\frac{-4A}{B}}k^2y + \xi_0, \eta = kx - \sqrt{\frac{-4A}{B}}k^2y + \eta_0$.

When $\mu(t) = -\lambda t^{\beta}$, the exact explicit solutions to time-fractional Kadomtsev-Petviashvili equations with variable-coefficient are obtained as follows

$$\begin{aligned} u(x,y,t) = \frac{b}{\Gamma(\alpha)}\Big[&- \frac{12AN^2k^2\exp(2\eta)[\cosh(\xi)+\sinh(\xi)]^2}{[M+N\sinh(\xi)\exp(\eta)]^2} \\ &+ \frac{24ANk^2\exp(\eta)[\sinh(\xi)+\cosh(\xi)]}{M+N\sinh(\xi)\exp(\eta)}\Big] t^{\alpha-1}E_{\alpha,1+\frac{\beta}{\alpha},1+\frac{\beta-1}{\alpha}}(\lambda t^{\alpha+\beta}), \end{aligned} \tag{4.31}$$

where $\xi = kx - \sqrt{\frac{-4A}{B}}k^2y + \xi_0, \eta = kx - \sqrt{\frac{-4A}{B}}k^2y + \eta_0$.

5. Conclusions

The variable separation method is an effective method to construct exact solutions of fractional differential equations. In this paper, we try to use the extended homogeneous balance method to find the exact solutions of time-fractional Kadomtsev-Petviashvili equation with variable coefficients. The some hyperbolic function solutions are obtained. Clearly, this method is effective to obtain exact solutions for some time-fractional nonlinear partial differential equations with Riemann-Liouville derivative. Other methods for solving the exact solutions of integer order partial differential equations can also be used to construct the exact solutions of fractional order differential equations.

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