AIMS Mathematics, 7(6): 10303–10314.
DOI: 10.3934/math.2022574
Received: 30 December 2021
Revised: 28 February 2022
Accepted: 10 March 2022
Published: 24 March 2022

http://www.aimspress.com/journal/Math

Research article

Finite difference method for transmission dynamics of Contagious Bovine Pleuropneumonia

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Abstract: In this study, the transmission dynamics of Contagious Bovine Pleuropneumonia (CBPP) by finite difference method are presented. This model is made up of sensitive, exposed, vaccinated, infectious, constantly infected, and treated compartments. The model is studied by the finite difference method. Firstly, the finite difference scheme is constructed. Then the stability estimates are proved for this model. As a result, several simulations are given for this model on the verge of antibiotic therapy. From these figures, the supposition that 50% of infectious cattle take antibiotic therapy or the date of infection decrease to 28 days, 50% of susceptible obtain vaccination within 73 days.

Keywords: Contagious Bovine Pleuropneumonia; Caputo differential equation; finite difference method
Mathematics Subject Classification: 34A08; 65N06

1. Introduction

CBPP is a great deal of restriction to cattle augmentation in the vital arcadian territory of Africa (see [1–3]). In [3], mathematical modeling of the transmission dynamics of contagious bovine pleuropneumonia was uncovered aim profiles at a small extent for upgraded vaccines and diagnostic tests. Development of real-time diagnostic analysis specific for Mycoplasma mycoides subspecies mycoides small colony were worked in [4]. It brings about high morbidity and fatality rate damages to cattle which causes economic decline (see [5–8] for more information). They worked Contagious Bovine Pleuropneumonia: Challenges and Prospects Regarding Diagnosis and Control Strategies in Africa [9]. Charge of restrain of CBPP is a big issue in African regions as well [10]. In [11], the model was given with no interference, having the purpose of revealing data that have a crucial part in altering the dynamics of the illness.

In the study [12], the researchers have examined antibiotic treatment and vaccination as a controlling medium of CBPP and given a segmented model with six parts for the transmission dynamics of the CBPP: sensitive, exposed, vaccinated, infectious, often infected, and treated compartments. Antibiotic therapy was taken into consideration in the model by adding the recovery rate of treated cattle to ensure that the treated moved at a rate from the infectious compartment to cured compartment.

The goal of this study [12] was to set up a more efficient handling program out of vaccination, antibiotic care, or both of them. We consider [12]:

$$\frac{dS(t)}{dt} = \mu N + \omega V - \frac{\beta SI}{N} - \rho S - \mu S \tag{2.1}$$

$$\frac{dV(t)}{dt} = \rho S - \omega V - \mu V, \tag{2.2}$$

$$\frac{dE(t)}{dt} = \frac{\beta SI}{N} - \gamma E - \mu E, \tag{2.3}$$

$$\frac{dI(t)}{dt} = \gamma E + kQ - (\alpha_t + \alpha_r)I - \alpha_q I - \mu I, \tag{2.4}$$

$$\frac{dQ(t)}{dt} = \alpha_q I - kQ - \psi Q - \mu Q, \tag{2.5}$$

$$\frac{dR(t)}{dt} = (\alpha_t + \alpha_r)I + \psi Q - \mu Q. \tag{2.6}$$

This system has known well-posedness from [12]. The contagious bovine pleuropneumonia (CBPP) was a respiratory disease of cattle; CBPP was lead to by *Mycoplasma mycoides* subsp, *mycoides* small colony [17]. They gave and analyzed a mathematical model of the transmission dynamics of Contagious Bovine Pleuropneumonia (CBPP) in the presence of antibiotic treatment with limited medical supply [18]. The equilibrium solutions were studied in detail [2]. Using the symbols in this paper are given as:

N: total number
t: time
S: susceptible class
V: vaccinal immune class
E: exposed compartment
I: infectious compartment
R: recovered compartment persistently infected

Local stability analysis and global durability analysis for illness free equipoise the (İFE) were studied in [13]. Examining this model, which is defined by the Caputo derivative, with the finite difference method and obtaining simulations for an approximate solution makes this study different from previous studies.

This paper is constructed as follows. In Section 2, the dynamics of CBPP for a mathematical model with antibiotic interventions and vaccination are demonstrated with Caputo derivative. In Section 3, finite difference method is constructed and the stability estimates of this model is presented. Numerical simulations have been demonstrated in Section 4. In Section 5, conclusion is proposed.

2. Mathematical model with Caputo derivative

2.1. The Caputo derivative

Definition 2.1: The definition of the Caputo derivative of α order is given as [6]:

$$D_t^{\alpha} f(t) = \frac{\partial^{\alpha} f(t)}{\partial t^{\alpha}} = \frac{1}{\Gamma(\mathrm{n}-\alpha)} \int_a^t \frac{1}{(t-p)^{\alpha-n+1}} \frac{\partial^{\alpha} f(p)}{\partial p^{\alpha}} dp,$$

where $n-1 < \alpha < n$ and $n = [\alpha] + 1$. The Caputo derivative has some advantages over the Riemann-Liouville derivative. First, the Caputo derivative is frequently used in the solution of fractional differential equations in the Laplace transform method. The Laplace transform of the Riemann-Liouville derivative requires boundary conditions involving the boundary values of the Riemann-Liouville fractional derivatives at the lower bound at $t = a$. Although mathematically such problems are solvable, there is no physical interpretation of such conditions. On the other hand the Laplace transform of the Caputo derivative imposes boundary conditions involving integer-order derivatives at the lower point $t = a$ which usually are acceptable physical conditions. The second advantage is that the Caputo derivative of a constant is zero while the Riemann Liouville derivative is nonzero [14]. The fractional order partial differential equations were studied by many researchers [15,16].

2.2. The constructed finite difference method

In this part, we construct finite difference method for the model of the antibiotic treatment and vaccination as a controlling tool of CBPP and the transmission dynamics of CBPP. The fractional order differential equation model defined by Caputo derivative is given by the following system:

$$ {}_0^C D_t^{\alpha} S(t) = \mu N + \omega V - \frac{\beta SI}{N} - \rho S - \mu S \tag{2.7}$$

$$ {}_0^C D_t^\alpha V(t) = \rho S - \omega V - \mu V, \tag{2.8} $$

$$ {}_0^C D_t^\alpha E(t) = \frac{\beta SI}{N} - \gamma E - \mu E, \tag{2.9} $$

$$ {}_0^C D_t^\alpha I(t) = \gamma E + kQ - (\alpha_t + \alpha_r)I - \alpha_q I - \mu I, \tag{2.10} $$

$$ {}_0^C D_t^\alpha Q(t) = \alpha_q I - kQ - \psi Q - \mu Q, \tag{2.11} $$

$$ {}_0^C D_t^\alpha R(t) = (\alpha_t + \alpha_r)I + \psi Q - \mu Q, \tag{2.12} $$

with initial conditions

$S(0) = S_0,\ V(0) = V_0,\ E(0) = E_0,\ I(0) = I_0,\ Q(0) = Q_0,\ R(0) = R_0.$

3. Finite difference method and stability estimates for mathematical model

We present grids with uniform steps in the domain $[0, \mathrm{T}]$

$$ \mathrm{W}^\tau = \{\mathrm{t_n}: \mathrm{t_n} = \mathrm{n}\tau,\ \mathrm{n} = 0,1, \dots, \mathrm{M}\}, \tau = \frac{\mathrm{T}}{\mathrm{M}}. $$

We use the notation $u_n = \mathrm{u}(\mathrm{t_n})$ for functions defined on the grid (or parts of this grid) W^τ.
For the fractional Caputo derivative operator, difference scheme is known as [13]:

$$ {}_0^C DS(t_n) = \frac{\partial^\alpha s(t_n)}{\partial t^\alpha} \cong \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \sum_{j=0}^{n} w_j^{(\alpha)} (u_{n-j+1} - u_{n-j}) = \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} [S_{n+1} - S_n + \sum_{j=1}^{n} w_j^{(\alpha)} (S_{n-j+1} - S_{n-j}), \tag{2.13} $$

here $w_j^{(\alpha)} = (j+1)^{1-\alpha} - (j)^{1-\alpha},\ S(t_n) = S_n,\ t_n = n\tau.$

Using the formula (2.13), we can obtain the finite difference method for the formulas (2.7)–(2.12)

$$ \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \left[S_{n+1} - S_n + \sum_{j=1}^{n} w_j^{(\alpha)} (S_{n-j+1} - S_{n-j}) \right] = \mu N + \omega V_n - \frac{\beta S_n I_n}{N} - \rho S_n - \mu S_n, \tag{2.14} $$

$$ \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \left[V_{n+1} - V_n + \sum_{j=1}^{k} w_j^{(\alpha)} (V_{n-j+1} - V_{n-j}) \right] = \rho S_n - \omega V_n - \mu V_n, \tag{2.15} $$

$$ \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \left[E_{n+1} - E_n + \sum_{j=1}^{n} w_j^{(\alpha)} (E_{n-j+1} - E_{n-j}) \right] = \frac{\beta S_n I_n}{N} - \gamma E_n - \mu E_n, \tag{2.16} $$

$$ \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \left[I_{n+1} - I_n + \sum_{j=1}^{n} w_j^{(\alpha)} (I_{n-j+1} - I_{n-j}) \right] = \gamma E_n + kQ_n - (\alpha_t + \alpha_r) I_n - \alpha_q I_n - \mu I_n, \tag{2.17} $$

$$ \frac{\tau^{-\alpha}}{\Gamma(2-\alpha)} \left[Q_{n+1} - Q_n + \sum_{j=1}^{n} w_j^{(\alpha)} (Q_{n-j+1} - Q_{n-j}) \right] = \alpha_q I_n - kQ_n - \psi Q_n - \mu Q_n, \tag{2.18} $$

$$\frac{\tau^{-\alpha}}{\Gamma(2-\alpha)}\left[R_{n+1}-R_n+\sum_{j=1}^{n} w_j^{(\alpha)}\left(R_{n-j+1}-R_{n-j}\right)\right]=(\alpha_t+\alpha_r)I_n+\psi Q_n-\mu Q_n. \quad (2.19)$$

Now, we shall prove that these systems are satisfied the stability estimates. For this, the Von-Neuman analysis method will be used as follow:

$$S_n=V_n=E_n=I_n=Q_n=R_n=r^n. \quad (2.20)$$

Taking $\alpha \to 1,\ n=1,$ the formulas (2.14)–(2.19) can be written as:

$$\left(\frac{1}{\tau}+\frac{\beta}{N}\right)r^2+\left(\rho+\mu-\frac{1}{\tau}-\omega\right)r-\mu N=0, \quad (2.21)$$

$$\frac{1}{\tau}r^2+\left(\omega+\mu-\frac{1}{\tau}-\rho\right)r=0, \quad (2.22)$$

$$\left(\frac{1}{\tau}-\frac{\beta}{N}\right)r^2+\left(\gamma+\mu-\frac{1}{\tau}\right)r=0, \quad (2.23)$$

$$\left(\frac{1}{\tau}\right)r^2+\left(\alpha_t+\alpha_r+\alpha_q+\mu-k-\gamma-\frac{1}{\tau}\right)r=0, \quad (2.24)$$

$$\left(\frac{1}{\tau}\right)r^2+\left(\mu+\psi+k-\alpha_q-\frac{1}{\tau}\right)r=0, \quad (2.25)$$

$$\left(\frac{1}{\tau}\right)r^2+(\mu-\psi-(\alpha_t+\alpha_r)-\frac{1}{\tau})r=0. \quad (2.26)$$

These formulas are quadratic equations. For the stability estimates the following conditions have to be satisfied:

i) $a)\ \omega<\gamma+\mu+\frac{\beta}{N},\ b)\ \frac{1}{\tau}+\frac{\beta}{N}>-\mu N,$

ii) $\rho<\omega+\mu,$

iii) $\beta<N(\gamma+\mu),$

iv) $k+\gamma<\alpha_t+\alpha_r+\alpha_q+\mu,$

v) $\alpha_q<\mu+\psi+k,$

vi) $\psi+\alpha_t+\alpha_r<\mu.$

From the Von-Neumann analysis method, it can be seen that the (2.14)–(2.19) system is stable if the conditions (i)–(vi) are satisfied. Because the roots of the quadratic equation satisfying the system are 0 or less than 1.

Table 1. Explanations of the data.

Variables	Definitions	Baseline	references
ω	vaccinal immunity loss rate	$\frac{1}{3\times 365}$	0.00078–0.0011 [11]
P_e	Vaccination success rate	0.65	0.5–0.8 [11]
P_v	Vaccination rate	0.5	[11]
$\in$	Vaccination efficacy	0.8	[11]
p	Ratio of immunization	$P_e \times P_v \times \in$	[11]
β	Rate of contact efficiency	0.126	0.07–0.13 [10]
ρ	Ratio of vaccination	$\frac{p}{73}$	assumed
γ	Transition rate from exposed to contagious compartment	0.0238	0.0179–0.0357 [11]
a_r	Natural recuperation rate of contagious cattle	0.0045	0.0060–0.0036 [11]
a_q	Rate of sequestrum formation of contagious cattle	$3a_r$	[11]
a_t	Rate of recovery of treated cattle	0.0179	0.0119–0.0214 assumed
k	Rate of sequestrum re-initiate	0.00009	0.00007–0.00011 [11]
ψ	Rate of sequestrum resolution	0.0075	0.0068 to 0.0079 [11]
μ	death rate	$\frac{1}{5\times 365}$	$\frac{1}{6\times 365}$ to $\frac{1}{20\times 365}$ [11]
B	Birth rate	$\frac{1}{5\times 365}$	$\frac{1}{6\times 365}$ to $\frac{1}{20\times 365}$ [11] and estimated

4. Numerical simulations

Firstly, we investigated 500 bovine populations consisting of an infectious cattle and 499 susceptible cattle with individual animals as epidemiological units. Consistent with the conclusion of [7], we suggested that the best way to control the disease is vaccination with antibiotic therapy. Because the proportion to be vaccinated p_v and t are not dependent variables of ρ, a given value of ρ can have many practical interpretation. Therefore, practical application of the value of ρ can be adjusted based on cost of control, availability, and time value.

Numerical simulations are obtained using MATLAB in Figures 1–7.

We did not treat any of the infected cattle in the 49-day period. We can control by vaccinating 80% of susceptible cattle in see Figure 2.

On the condition that the other values in Figure 6 and Figure 7 are the same, the values of $\alpha = 0.01$ and $\alpha = 0.50$ are compared.

Finally, for the parametric values in Table 1, assuming 50% of susceptible people are vaccinated a period of 73 days and 50% of infected cattle appear to be cured.

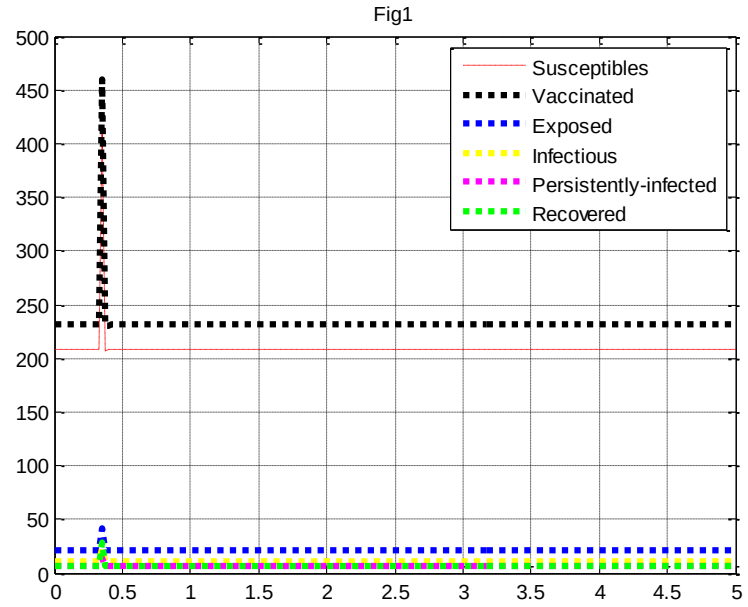

Figure 1. Using the supposition that 50% of infectious cattle take antibiotic therapy or the interval of infection is decreased to 28 days (a_t = 1/28−1/56), 50% of credulous obtain vaccination within 73 days (ρ = (0.5×0.8×0.65)/73), $I_0 = 1$, $S_0 = 499$ and $V_0 = E_0 = Q_0 = R_0 = 0$.

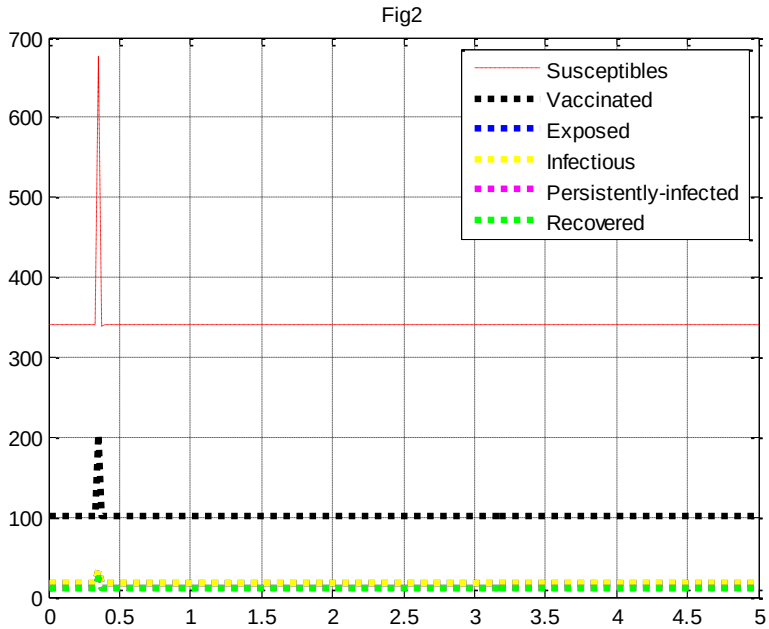

Figure 2. Using the supposition that 80% of sensitive cattle are vaccinated within 49 days (ρ = (0.65 ∗ 0.8 ∗ 0.8)/49) with no treating infectious cattle ($a_t = 0$), $I_0 = 1$, $S_0 = 499$ and $V_0 = E_0 = Q_0 = R_0 = 0$.

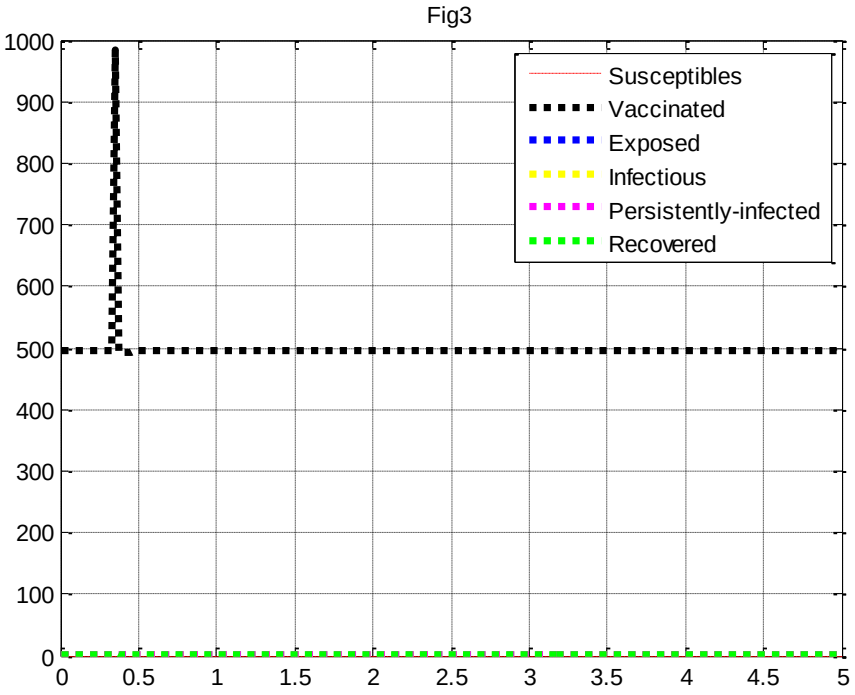

Figure 3. Using the Table 1 with the supposition that 85.7% of infectious cattle give antibiotic treatment within 8 days (a_t =1/8−1/56) with no vaccinating healthy cattle ($\rho = 0$), $I_0 = 1$, $S_0 = 499$ and $V_0 = E_0 = Q_0 = R_0 = 0$.

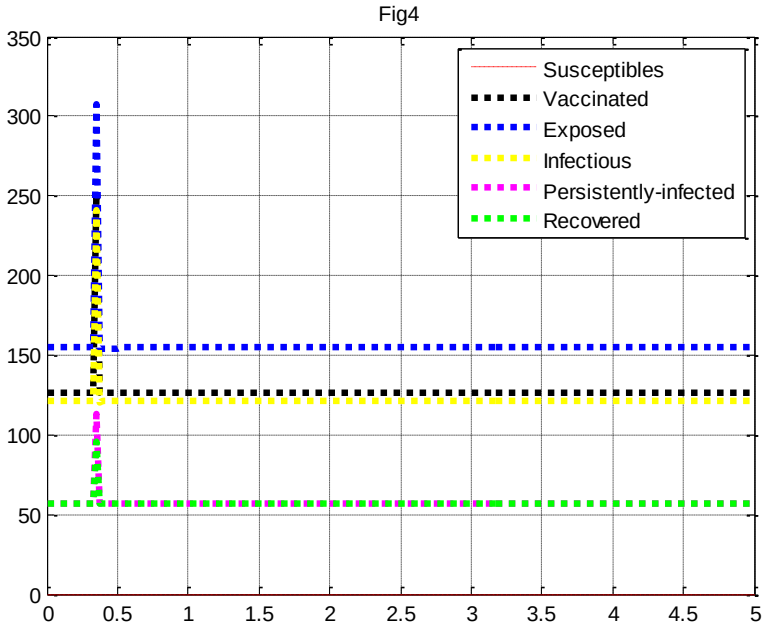

Figure 4. Using the Table 1 with $\rho = a_t = 0$, $I_0 = 1$, $S_0 = 499$ and $V_0 = E_0 = Q_0 = R_0 = 0$.

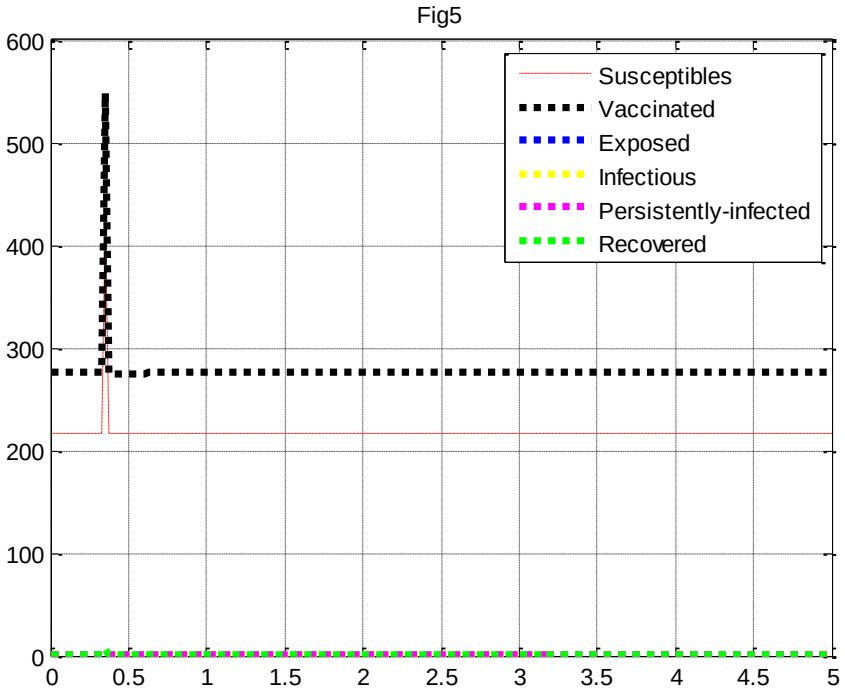

Figure 5. Using the Table 1 with a_r= 1/56 (as in [14]) and $I_0 = 1$, $S_0 = 499$ and $V_0 = E_0 = Q_0 = R_0 = 0$.

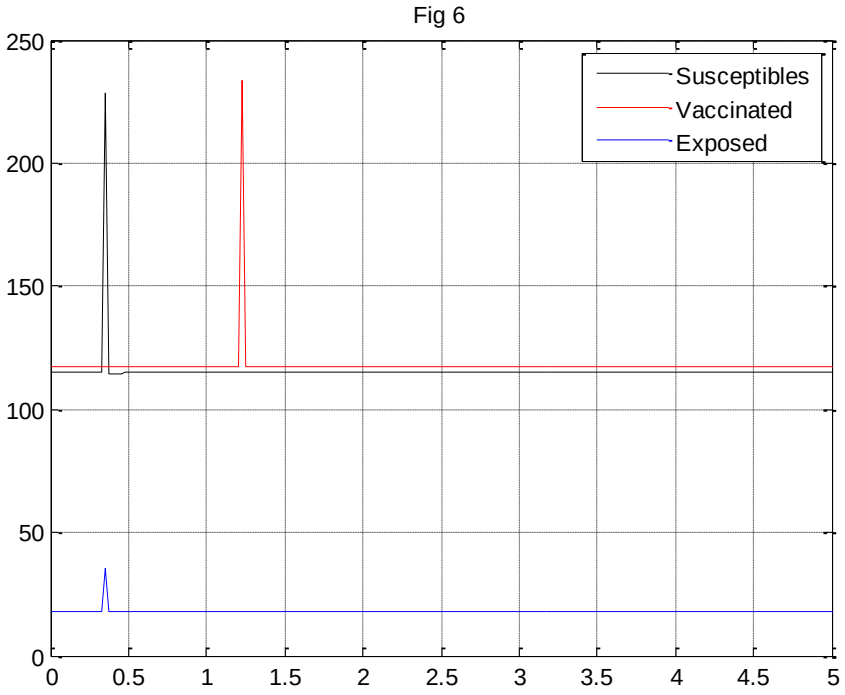

Figure 6. Using the Table 1 with $\alpha = 0.01$ a_r= 1/56, $a_t = 0.1049$ and $S_0 = 499$ and $V_0 = E_0 = 0$.

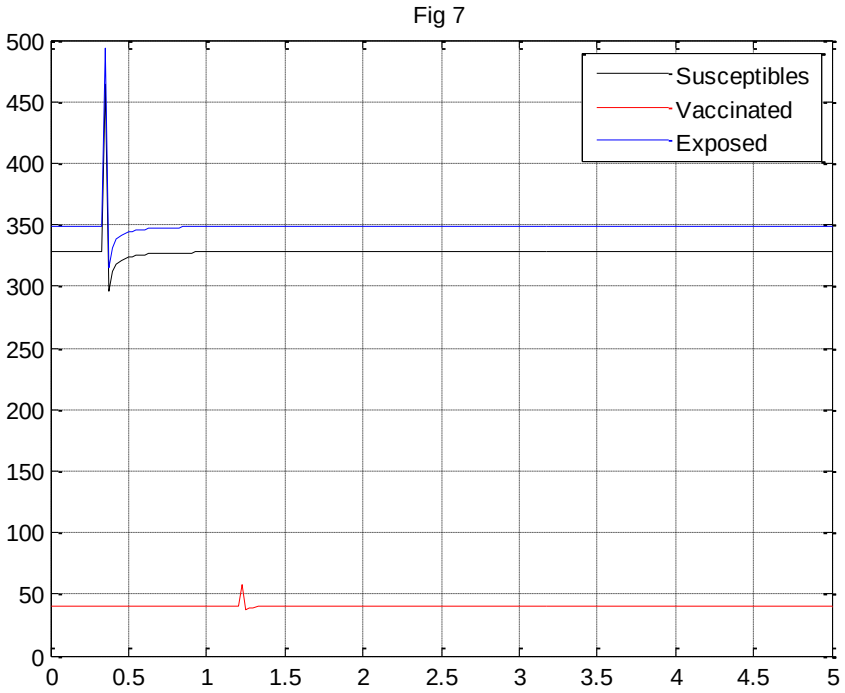

Figure 7. Using the Table 1 with $\alpha = 0.50$ a_r= 1/56, $a_t = 0.1049$ and $S_0 = 499$ and $V_0 = E_0 = 0$.

5. Conclusions

The simulations obtained using the Matlab program and the results obtained from these simulations were given in the main text. In this paper, we presented differential equations defined by Caputo derivative for the transmission aspects of CBPP with intercession. We constructed finite difference scheme for this equation. The stability estimates are proved for this difference method. Consequently, the verge of the antibiotic therapy is $\mathbf{a_t = 0.1049}$. The values of $\boldsymbol{\alpha = 0.01}$ and $\boldsymbol{\alpha = 0.50}$ are compared and showed by Figures 6 and 7. This fractional order model defined by the Atangana-Baleanu derivative can be compared with the Caputo derivative by applying the finite difference method.

Conflict of interest

There is no conflict of interest declared by the authors.

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