AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(6): 10245–10255.
DOI: 10.3934/math.2022570
Received: 16 December 2021
Revised: 05 March 2022
Accepted: 14 March 2022
Published: 22 March 2022

Research article

Sieve bootstrap test for multiple change points in the mean of long memory sequence

Wenzhi Zhao*, **Dou Liu and Huiming Wang**

School of Science, Xi'an Polytechnic University, Xi'an, Shaanxi 710048, China

* **Correspondence:** Email: zhaowenzhi1979@126.com.

Abstract: In this paper, the sieve bootstrap test for multiple change points in the mean of long memory sequence is studied. Firstly, the ANOVA test statistics for change points detection is obtained. Secondly, sieve bootstrap statistics is constructed and the consistency under the Mallows measure is proved. Finally, the effectiveness of the method was illustrated by simulation and example analysis. Simulation results show that our method can not only control the empirical size well but also have reasonable good power.

Keywords: long memory; multiple change points; ANOVA; sieve bootstrap; test
Mathematics Subject Classification: 62F05, 62M10

1. Introduction

The presence of change point can easily mislead the conventional time series analysis and result in erroneous conclusions. One of the problems in change point analysis is to detect whether there are change points in the statistical sequence.

The statistical literature on change point problem start with Page (1954) [1], who publish an article on quality inspection, which has attracted the attention of experts in various fields. Horváth and Kokoszka et al. (1997,1998) [2, 3] study the CUSUM estimator of mean change point and obtain the limiting distribution for the estimator. Kuan et al. (1998) [4] propose least square method to estimate mean change point in fractionally integrated process. Shao (2011) [5] use ratio statistics to solve the testing problem of mean change point. In the actual statistical test, the data sequence may have more than one change points. The detection method designed for at-most-one change point problem performs poor under multiple change point detection problem. So the study of the multiple change points detection is of great significance. Among them, Bai et al. (1997, 1998, 2003) [6–9] consider the estimation and detection problem of the multiple change points in linear process. Bardet et al. (2010) [10] and Kejriwal et al. (2013) [11] use different methods to solve the same problem. Lijing et al. (2020) [12] and Macneill et al. (2020) [13, 14] consider the detection problem of multiple change points in linear process. Noriah et al. (2014) [15] propose ANOVA statistics to test the multiple change points in *i.i.d.* sequence. The limiting distribution of test statistics is derived under *i.i.d.* assumption. While, it is not easy to obtain limiting distribution for long range dependence sequence. Fortunately, bootstrap method give a chance to solve this problem more conveniently.

Long memory processes are prevalent in many areas, for example, in geophysical sciences, microeconomic, asset pricing, stock returns and exchange rates. Hidalago and Robinson et al. (1996) [16] propose the Wald method to test mean change point in long memory sequence. Lazarova (2005) [17] study the change point detection problem in linear regression models with long memory error. Wang (2008) [18] give the estimation method for change point in non-parametric regression models with long memory error. The above literatures focus on the single change point of long memory sequence. However, in practice, due to the interference of various factors, the statistical property of long memory sequence may change not only once, but many times. Thus, it is very necessary to study the multiple point problem of long memory sequence. In this paper, we study the multiple change point detection problem for long memory sequence.

Sieve bootstrap method was introduced by Bühlmann (1997) [19] firstly. Alonso et al. (2002, 2003, 2004) [20–22] and Mukhopadhyay et al. (2010) [23] use sieve bootstrap to study the forecasting problems for time series. Poskitt (2008) [24] prove the properties of the sieve bootstrap method and point out that sieve bootstrap method is very useful to analyze long memory sequence. So, in this paper, we consider sieve bootstrap test for multiple change points in the mean of long memory sequence.

2. The model and the sieve bootstrap

We assume that n observations $X_1, X_2, \cdots, X_n$ are given by:

$$X_t = \mu(t) + e_t, t = 1, 2, \cdots, n,$$

$$\mu(t) = \begin{cases} \mu_1, & 1 \le t \le n_1, \\ \mu_2, & n_1 + 1 \le t \le n_2, \\ \cdots & \\ \mu_{k+1}, & n_{k+1} \le t \le n, \end{cases} \tag{2.1}$$

where $e_t = \varphi(B)\varepsilon_t = \sum_{j=0}^{\infty} \varphi_j \varepsilon_{t-j}, \varphi_j \sim c_0 j^{d-1} (0 < c_0 < \infty, j \to \infty, 0 < d < 0.5)$, and $\sum_{j=0}^{\infty} \varphi_j^2 < \infty, \varepsilon_t$ is an *i.i.d.* process with mean zero and finite variance σ^2. The symbol "∼" indicates that the ratio of left- and right-hand sides tends to 1. The sequence $\{e_t, t = 1, 2, \cdots, n\}$ is a linear stationary sequence with long memory, so is $\{X_t, t = 1, 2, \cdots, n\}$. Parameters $\mu_1, \mu_2, \cdots, \mu_{k+1}$ are the finite constant and $n_1, n_2, \cdots, n_k$ are the unknown change point. We consider here the problem of testing the null hypothesis of no change point:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_{k+1} \tag{2.2}$$

against the multiple change points alternative:

$$H_0 : \mu_1 = \mu_2 = \cdots = \mu_{k+1}. \tag{2.3}$$

Let $\tau = (0 < \tau_1 < \tau_2 < \cdots < \tau_k < 1)$ be any partition of [0,1] , $[n\tau_i] \geq [n\tau_{i-1}] + 2, i = 1, 2, \cdots, k+1$ with $\tau_0 = 0, \tau_{k+1} = 1$. Define $d_{i,n} = [n\tau_i] - [n\tau_{i-1}], i = 1, 2, \cdots, k+1$. Let $S_0 = 0, S_r = \sum_{j=1}^{r} X_j$,$r = 1, 2, \cdots, n$ and $\bar{X} = \frac{1}{n} S_n$. For $i = 1, 2, \cdots, k+1$, the mean of $X_{[n\tau_{i-1}]} + 1, \cdots, X_{[n\tau_i]}$ is $\bar{X}_i = \frac{1}{d_{i,n}} \sum_{t=[n\tau_{i-1}]+1}^{[n\tau_i]} X_t$.

The one-way ANOVA-type test statistic proposed by Noriah et al. (2014) [15] is:

$$Z_n(k) := \int \cdots_{\tau} \int V_n(\tau) d\tau, \tag{2.4}$$

where

$$V_n(\tau) = c_0^{-2} n^{-2d} \prod_{i=1}^{k+1} a_{i.n} SSTr(\tau),$$

and

$$SSTr(\tau) = \sum_{i=1}^{k+1} d_{i,n} (\bar{X}_i - \bar{X})^2.$$

Often, the number of change points k is unknown and we assume that it has an upper bound K. Test statistic is defined as $Z_n(K) = \max_{1 \leq k \leq K} Z_n(k)$ and the limiting distribution is approximated by sieve bootstrap method.

$\{X_t\}$ is invertible and has an $AR(\infty)$ representation. The specific steps of the sieve bootstrap method are as follows:

Step 1. Having observed the samples $x_1, x_2, \cdots, x_n$, we start estimating $\hat{d}$. Specific estimation method can be referred to Hurst (1951) [25];

Step 2. Make $\hat{d}$-order difference on $\{x_t\}$ to obtain the sequence $\{y_t\}$. To fit the AR(p) autoregressive process of $\{y_t\}$, that is

$$\hat{y}_t = \hat{\phi}_0 + \hat{\phi}_1 y_1 + \hat{\phi}_2 y_2 + \cdots + \hat{\phi}_p y_{t-p}. \tag{2.5}$$

Given a maximum AR order $p = p(n)$ of the autoregressive approximation, then choose the optimal $\hat{p}$ using the BIC criterion, where $p(n) = o(n)$, and $p(n) \to \infty$ as the sample size $n \to \infty$. We obtain the residuals $\hat{\varepsilon}_1, \hat{\varepsilon}_2, \cdots, \hat{\varepsilon}_t$. The residual is re-sampled to obtain new residual sequence, and the autoregressive process $y_t^* = \hat{y}_t + \hat{\varepsilon}_t^*$ is constructed for the new residual sequence.

Step 3. The new long memory sequence $\{x_t^*\}$ is constructed by $x_t^* = (1 - B)^{-\hat{d}} y_t^*$. Calculates the corresponding sieve bootstrap test statistics value $z_n^*(K)$:

$$z_n^*(K) := \max_{1 \leq k \leq K} \int \cdots_{\tau} \int c_0^{-2} n^{-2d} \prod_{i=1}^{k+1} d_{i.n} \sum_{i=1}^{k+1} d_{i,n} (\bar{x}_i{}^* - \bar{x^*})^2 d\tau. \tag{2.6}$$

Step 4. Repeat Steps 2–3 B times and obtain B values $z_i^*(K)$, $i = 1, 2, \cdots, B$. The sieve bootstrap approximation of the p value is $p^* = \frac{1}{B} \#\{i : z_i^*(K) \geq z_n(K)\}$, where # denotes the number of elements in the set and $z_n(K) := \max_{1 \leq k \leq K} \int \cdots_{\tau} \int c_0^{-2} n^{-2d} \prod_{i=1}^{k+1} d_{i.n} \sum_{i=1}^{k+1} d_{i,n} (\bar{x}_i - \bar{x})^2 d\tau$. We reject H_0 if $p^* < \alpha$.

3. Main results

In order to prove that the asymptotic results are consistent, the following assumption and lemmas are required [24].

Assumption 1. Let ξ_t denote the σ-algebra of events determined by ε_s, $s \leq t$. Also, assume that $\{\varepsilon_t\}_{t\in Z}$ are *i.i.d* and that

$$E[\varepsilon_t|\xi_{t-1}] = 0, E[\varepsilon_t^2|\xi_{t-1}] = \sigma^2, t \in Z.$$

Furthermore, assume that $E[\varepsilon_t^4] < \infty, t \in Z$.

Lemma 1. Assume that n observations $X_1, X_2, \cdots, X_n$ satisfy Eq (2.1). Then, for $t \in Z, 0 < d < 0.5, p = p(n) \to \infty$, we have

$$\frac{1}{n}\sum_{t=1}^{n}(\hat{\varepsilon}_t^* - \varepsilon_t)^2 = O_{a.s}\{(\frac{p}{\lambda_{min}(\Gamma_p)})(\frac{logn}{n})^{1-2d}\}, \tag{3.1}$$

where $\lambda_{min}(\Gamma_p) = O\{p^{-q}\}$, $q \geq 0$.

Proof. See Lemma 2 in reference [24], the proof is omitted.

Lemma 2. Assume that n observations $X_1, X_2, \cdots, X_n$ satisfy Eq (2.1). We obtain the represents $e_t^* = \sum_{j=0}^{\infty} \hat{\varphi}_j \hat{\varepsilon}_{t-j}^*$, $e_t = \sum_{j=0}^{\infty} \varphi_j \varepsilon_{t-j}$, then, for $t \in Z, 0 < d < 0.5, p(n) \to \infty$, we have

$$\sum_{j=0}^{\infty} |\hat{\varphi}_j - \varphi_j| = O_{a.s}\{(\frac{p^5}{\lambda_{min}(\Gamma_p)})^{\frac{1}{2}}(\frac{logn}{n})^{\frac{1}{2}-d}\}, \tag{3.2}$$

where $\lambda_{min}(\Gamma_p) = O\{p^{-q}\}$, $q \geq 0$.

Proof. See Lemma 3 in reference [24], the proof is omitted.

Theorem 1. Let $\eta(F_X, F_Y)$ denotes Mallow's measure of the distance between two probability distribution F_X and F_Y, defined as $inf\{E\|X-Y\|^2\}^{\frac{1}{2}}$ where the infimum is taken over all square integrable random variables X and Y in R^m with marginal distributions F_X and F_Y. Assume that n observations $X_1, X_2, \cdots, X_n$ satisfy Eq (2.1). So if the null hypothesis H_0 is true, when $n \to \infty$, for $0 < d < 0.5$ and $p = p(n) \to \infty$. Then with probability one

$$\eta(F_{Z_n^*(k)}, F_{Z_n(k)}) = O\{(\frac{p^5}{\lambda_{min}(\Gamma_p^2)})^{\frac{1}{2}}(\frac{logn}{n})^{\frac{1}{2}-d}\}, \tag{3.3}$$

where $Z_n(k)$ denotes for the test statistic, $Z_n^*(k)$ denotes for the sieve bootstrap test statistic, $F_{Z_n(k)}$ denotes for the true distribution, $F_{Z_n^*(k)}$ denotes for the sieve bootstrap approximate distribution, $\lambda_{min}(\Gamma_p) = O\{p^{-q}\}, q \geq 0$.

Proof. From the definition of $e_t^* = \sum_{j=0}^{\infty} \hat{\varphi}_j \hat{\varepsilon}_{t-j}^*$ and $e_t = \sum_{j=0}^{\infty} \varphi_j \varepsilon_{t-j}$, we have

$$\bar{X}_i = \frac{1}{d_{i,n}} \sum_{t=[n\tau_{i-1}]+1}^{[n\tau_i]} X_t = \frac{1}{d_{i,n}} \sum_{t=[n\tau_{i-1}]+1}^{[n\tau_i]} (\mu_t + e_t)$$

and

$$\bar{X}_i^* = \frac{1}{d_{i,n}} \sum_{t=[n\tau_{i-1}]+1}^{[n\tau_i]} X_t^* = \frac{1}{d_{i,n}} \sum_{t=[n\tau_{i-1}]+1}^{[n\tau_i]} (\mu_t + e_t^*). \tag{3.4}$$

It follows that

$$|\bar{X}_i - \bar{X}_i^*|^2 \leq \frac{1}{d_{i,n}} \sum_{t=[n\tau_{i-1}]+1}^{[n\tau_i]} |X_t - X_t^*|^2 = \frac{1}{d_{i,n}} \sum_{t=[n\tau_{i-1}]+1}^{[n\tau_i]} |e_t - e_t^*|^2. \tag{3.5}$$

Where $Z_n^*(k)$ denotes for the sieve bootstrap test statistic, that is

$$Z_n{}^*(k) := \int_{\tau}^{\cdots}\int c_0^{-2} n^{-2d} \prod_{i=1}^{k+1} d_{i.n} \sum_{i=1}^{k+1} d_{i,n} (\bar{X}_i{}^* - \bar{X}^*)^2 d\tau.$$

So applying the mean value theorem of calculus, we have

$$Z_n(k) - Z_n^*(k) = \sum_{t=1}^{n} \frac{\partial Z_i(k)}{\partial X_t}(X_t - X_t^*) = \sum_{t=1}^{n} \frac{\partial Z_i(k)}{\partial X_t}(e_t - e_t^*),$$

and

$$\|Z_n(k) - Z_n^*(k)\| \leq \sum_{t=1}^{n} \|\frac{\partial Z_i(k)}{\partial X_t}\| |X_t - X_t^*| = \sum_{t=1}^{n} \|\frac{\partial Z_i(k)}{\partial X_t}\| |e_t - e_t^*|. \tag{3.6}$$

By assumption, $n\frac{\partial Z_i(k)}{\partial X_t}$ are continuous on the domain R and hence uniformly bounded, so there is $M > 0$. We can therefore conclude that $n\|\frac{\partial Z_i(k)}{\partial X_t}\| \leq M$ and $Z_n(k)$ will satisfy the Lipschitz condition, that is

$$\|Z_n(k) - Z_n^*(k)\|^2 \leq \frac{1}{n} \sum_{t=1}^{n} M^2 |e_t - e_t^*|^2. \tag{3.7}$$

Similar to the proof of Theorem 1 in literature [22], from the definition of Mallows metric and applying the Cauchy-Schwartz inequality, we have

$$\begin{aligned}
\eta(F_{Z_n^*(k)}, F_{Z_n(k)})^2 &\leq E[E^*[\|Z_n^*(k) - Z_n(k)\|^2] \\
&\leq E[E^*[\frac{1}{n} \sum_{t=1}^{n} M^2 |e_t - e_t^*|^2] \\
&\leq \frac{1}{n} \sum_{t=1}^{n} E[E^*[M^2]] \frac{1}{n} \sum_{t=1}^{n} E[E^*[(e_t - e_t^*)^2]].
\end{aligned} \tag{3.8}$$

We obtain the representation $e_t^* = \sum_{j=0}^{\infty} \hat{\varphi}_j \hat{\varepsilon}_{t-j}^*, e_t = \sum_{j=0}^{\infty} \varphi_j \varepsilon_{t-j}$, from which it follows that

$$e_t - e_t^* = \sum_{j=0}^{\infty} (\varphi_j - \hat{\varphi}_j) \varepsilon_{t-j} + \sum_{j=0}^{\infty} \hat{\varphi}_j (\varepsilon_{t-j} - \hat{\varepsilon}_{t-j}^*) = u(t) + v(t).$$

Where $\frac{1}{n} \sum_{t=1}^{n} E[E^*[M^2]] \leq \infty$, thus we are faced with the task of evaluating $E[E^*[(u(t)+v(t))^2]$. Consider $E[E^*[v(t)^2]$ firstly. By construction, $\varepsilon_t - \hat{\varepsilon}_t^*$ are *i.i.d* with respect to all the observations $X_1, X_2, \cdots, X_t$. Hence

$$E^*[v(t)^2] = \frac{1}{n} \sum_{t=1}^{n} (\hat{\varepsilon}_t^* - \varepsilon_t)^2 \sum_{j=0}^{\infty} |\hat{\varphi}_j|^2. \tag{3.9}$$

Using Lemma 1 we get that,

$$\frac{1}{n} \sum_{t=1}^{n} (\hat{\varepsilon}_t^* - \varepsilon_t)^2 = O_{a.s}\{(\frac{p}{\lambda_{min}(\Gamma_p)})(\frac{logn}{n})^{1-2d}\}. \tag{3.10}$$

Since $\sum_{j=0}^{\infty}|\varphi_j| \leq \infty$, using Lemma 2 we can conclude that

$$\sum_{j=0}^{\infty}|\hat{\varphi}_j| \leq \sum_{j=0}^{\infty}|\varphi_j| + \sum_{j=0}^{\infty}|\hat{\varphi}_j - \varphi_j| = O(1) + O\{(\frac{p^5}{\lambda_{min}(\Gamma_p^2)})^{\frac{1}{2}}(\frac{logn}{n})^{\frac{1}{2}-d}\}.$$

Thus,

$$E[E^*[v(t)^2]] = O\{(\frac{p}{\lambda_{min}(\Gamma_p)})(\frac{logn}{n})^{1-2d}\}. \tag{3.11}$$

Now consider $E[E^*[u(t)^2]]$. Since $u(t) = \sum_{j=0}^{\infty}(\varphi_j - \hat{\varphi}_j)\varepsilon_{t-j}$ is a constant relative to all the observations $X_1, X_2, \cdots, X_t$, we have

$$E[E^*[u(t)^2]] = E[u(t)^2 E^*(1)].$$

According to Poskitt (2008) [24], under H_0, X_t has the following spectral density

$$f(\omega) = \frac{\sigma^2|\varphi(e^{i\omega})|^2}{2\phi}.$$

Thus,

$$E[u(t)^2] = \frac{\sigma^2}{2\pi}\int_{-\pi}^{\pi}|\varphi(e^{i\omega}) - \hat{\varphi}(e^{i\omega})|^2|\phi(e^{i\omega})\varphi(e^{i\omega})|^2 d\omega. \tag{3.12}$$

For any $\delta > 0, \omega \in (-\pi, \pi]$, from Lemma 4 in literature [24], we have

$$|\phi(e^{i\omega}) - \varphi(e^{i\omega})| \leq 1 + |\phi(e^{i\omega})\varphi(e^{i\omega}) - 1| \leq 1 + \delta.$$

Hence

$$E[u(t)^2] \leq \frac{\sigma^2}{2\pi}\int_{-\pi}^{\pi}|\varphi(e^{i\omega}) - \hat{\varphi}(e^{i\omega})|^2(1+\delta)^2 d\omega = \frac{[\delta(1+\delta)]^2}{2\pi}\sum_{j=0}^{\infty}|\hat{\varphi}_j - \varphi_j|^2.$$

And by Lemma 2 this equals

$$\sum_{j=0}^{\infty}|\hat{\varphi}_j - \varphi_j| = O_{a.s}\{(\frac{p^5}{\lambda_{min}(\Gamma_p)})^{\frac{1}{2}}(\frac{logn}{n})^{\frac{1}{2}-d}\}.$$

We can therefore conclude that

$$E[u(t)^2] = O\{(\frac{p^5}{\lambda_{min}(\Gamma_p)})(\frac{logn}{n})^{1-2d}\}. \tag{3.13}$$

To sum up, we have

$$\eta(F_{Z_n^*(k)}, F_{Z_n(k)}) = O\{(\frac{p^5}{\lambda_{min}(\Gamma_p^2)})^{\frac{1}{2}}(\frac{logn}{n})^{\frac{1}{2}-d}\}.$$

Which completes the proof of the theorem.

4. Simulation and application

4.1. Simulation

In this section, we evaluate the performance of the test statistics trough a simulation. Experiments are conducted based on sample sizes $n = 400$ and $n = 600$ with 1000 replications. Consider the following data generation process:

$$X_t = \mu(t) + e_t, t = 1, 2, \cdots, n,$$

$$\mu(t) = \begin{cases} \mu_1, & 1 \leq t \leq n_1, \\ \mu_2, & n_1 + 1 \leq t \leq n_2, \\ \cdots & \\ \mu_{k+1}, & n_{k+1} \leq t \leq n, \end{cases}$$

where e_t is a FARIMA(0,d,0) process. Simulation studies are based on B=5000 and $\alpha = 0.05$. Take $d = 0.1, 0.2, 0.3, 0.4$. Under H_0, $\mu_1 = \mu_2 = \cdots = \mu_{k+1} = 0$, the empirical sizes of $Z_n^*(K)$ are summarized in Table 1. Under H_1, the number of change points $K = 2$ and 3. When $K = 2$, the change point combinations (n_1, n_2) are divided into three situations: $(\frac{1}{8}n, \frac{3}{8}n)$, $(\frac{3}{8}n, \frac{5}{8}n)$ and $(\frac{5}{8}n, \frac{7}{8}n)$ and $(\mu_1, \mu_2, \mu_3) = (0, 1, 2)$. When $K = 3$, the change point combinations (n_1, n_2, n_3) are divided into two situations: $(\frac{1}{8}n, \frac{3}{8}n, \frac{5}{8}n)$ and $(\frac{3}{8}n, \frac{5}{8}n, \frac{7}{8}n)$. The mean parameters $(\mu_1, \mu_2, \mu_3, \mu_4)$ are taken as $(0, 1, 2, 3)$ and $(0, 1, -1, 2)$. The empirical powers of $Z_n^*(K)$ are shown from Table 2 to Table 5.

Table 1. The empirical size of $Z_n^*(K)$.

d	$n = 400$	$n = 600$
0.1	0.025	0.037
0.2	0.036	0.045
0.3	0.048	0.051
0.4	0.056	0.055

Table 1 displays the empirical size of $Z_n^*(K)$ under H_0. With the increase of sample size n, the empirical size is close to the significant level $\alpha = 0.05$.

Table 2. The empirical power with $n = 400$.

d	$(\frac{1}{8}n, \frac{3}{8}n)$	$(\frac{3}{8}n, \frac{5}{8}n)$	$(\frac{5}{8}n, \frac{7}{8}n)$
0.1	0.699	0.721	0.704
0.2	0.686	0.709	0.737
0.3	0.715	0.753	0.828
0.4	0.853	0.714	0.895

Table 3. The empirical power with $n = 600$.

d	$(\frac{1}{8}n, \frac{3}{8}n)$	$(\frac{3}{8}n, \frac{5}{8}n)$	$(\frac{5}{8}n, \frac{7}{8}n)$
0.1	0.684	0.643	0.776
0.2	0.688	0.715	0.675
0.3	0.722	0.825	0.870
0.4	0.885	0.909	0.916

Table 4. The empirical power with $n = 400$.

d	$(\mu_1, \mu_2, \mu_3, \mu_4)$	$(\frac{1}{8}n, \frac{3}{8}n, \frac{5}{8}n)$	$(\frac{3}{8}n, \frac{5}{8}n, \frac{7}{8}n)$
0.1	(0,1,2,3)	0.614	0.693
	(0,1,-1,2)	0.559	0.685
0.2	(0,1,2,3)	0.640	0.708
	(0,1,-1,2)	0.668	0.690
0.3	(0,1,2,3)	0.713	0.734
	(0,1,-1,2)	0.682	0.699
0.4	(0,1,2,3)	0.775	0.877
	(0,1,-1,2)	0.717	0.850

Table 5. The empirical power with $n = 600$.

d	$(\mu_1, \mu_2, \mu_3, \mu_4)$	$(\frac{1}{8}n, \frac{3}{8}n, \frac{5}{8}n)$	$(\frac{3}{8}n, \frac{5}{8}n, \frac{7}{8}n)$
0.1	(0,1,2,3)	0.702	0.685
	(0,1,-1,2)	0.694	0.696
0.2	(0,1,2,3)	0.698	0.690
	(0,1,-1,2)	0.709	0.689
0.3	(0,1,2,3)	0.732	0.877
	(0,1,-1,2)	0.786	0.792
0.4	(0,1,2,3)	0.814	0.901
	(0,1,-1,2)	0.827	0.874

It can be seen from Tables 2–5 that the empirical power of $Z_n^*(K)$ under H_1 increases significantly when the sample size n increases. The larger d is, the bigger the value of the empirical power.

4.2. Application

In this subsection, the method in this paper is used to detect mean change in monthly average temperature of Northern Hemisphere (1854–1989). The data comes from Beran (1994) [26] and $\hat{d} = 0.37$.

Wang (2008) [18] suggests that there is one change point in the data. Thus, we take $K = 1$ and repeat Steps 1–4 to generate 5000 values of sieve bootstrap statistics. Figure 1 shows these values. The horizontal solid line is significant level $\alpha = 0.05$. $p^* = \frac{1}{5000}\{\#i : z_i^*(K) \geq z_n(K)\} = 0.0020 < 0.05$. Thus, we reject H_0.

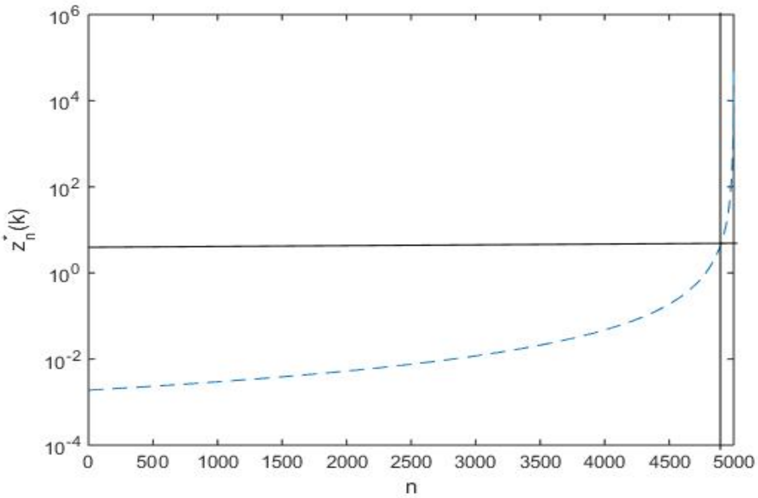

Figure 1. The value of 5000 sieve bootstrap test statistics.

Further, the least square method in Kuan (1998) [4] is used to estimate location of change point. The change point estimator is in the 860 observation, corresponding to July 1925. Figure 2 shows these results. The vertical line in the figure indicates the position of the change point. The two horizontal lines in the figure indicate the mean value before and after the change point. The mean value before the change point is -0.34, and the mean value after the change point is 0.02, which means that the monthly average temperature in the Northern Hemisphere has increased.

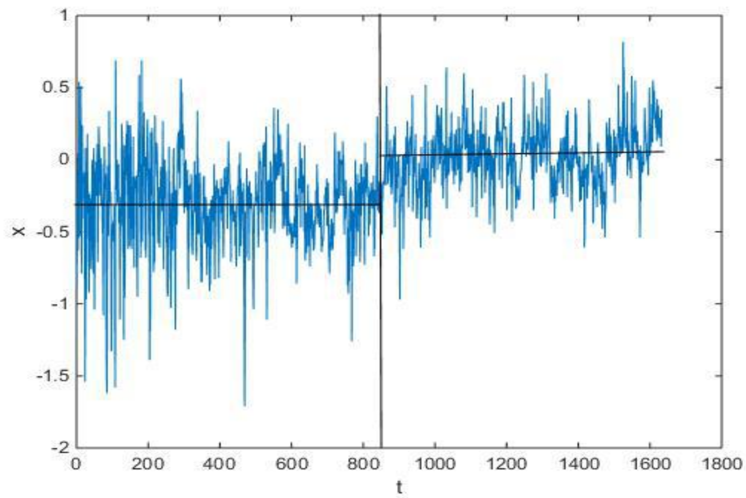

Figure 2. Monthly average temperature data of the Northern Hemisphere from 1854 to 1989.

5. Conclusions

This paper considers the sieve bootstrap test for multiple change points in the mean of long memory sequence. The consistency of sieve bootstrap approximation is proved. Numerical simulation and application results support the conclusion.

Acknowledgments

This work was supported by the National Natural Foundation of China (Grant nos. 11771353, 12171391) and the Natural Science Basic Research Program of Shaanxi (No. 2022JM-024).

Conflict of interest

The authors declare that they have no conflicts of interest.

References

1. E. S. Page, Continuous inspection schemes, *Biometrika,* **41** (1954), 100–115. https://doi.org/10.1093/biomet/41.1-2.100
2. L. Horváth, P. Kokoszka, The effect of long-range dependence on change point estimators, *J. Stat. Plan. Infer.,* **64** (1997), 57–81. https://doi.org/10.1016/S0378-3758(96)00208-X
3. P. Kokoszka, R. Leipus, Change point in the mean of dependent observations, *Stat. Probabil. Lett.,* **40** (1998), 385–393. https://doi.org/10.1016/S0167-7152(98)00145-X
4. C. M. Kuan, C. C. Hus, Change-point estimation of fractionally integrated process, *J. Time Ser. Anal.,* **19** (1998), 693–708. https://doi.org/10.1111/1467-9892.00117
5. X. Shao, A simple test of changes in mean in the possible presence of long-range dependence, *J. Time Ser. Anal.,* **32** (2011), 598–606. https://doi.org/10.1111/j.1467-9892.2010.00717.x
6. J. S. Bai, Estimation of a change point in multiple regression models, *Rev. Econ. Stat.,* **79** (1997), 551–563. https://doi.org/10.1162/003465397557132
7. J. S. Bai, P. Perron, Estimating and testing linear models with multiple structural changes, *Econometrica,* **66** (1998), 47–78. https://doi.org/10.2307/2998540
8. J. S. Bai, P. Perron, Critical values for multiple structural change tests, *Economet. J.,* **6** (2003), 72–78. http://dx.doi.org/10.1111/1368-423x.00102
9. J. S. Bai, P. Perron, Multiple structural change models: A simulation analysis, *J. Appl. Economet.,* **18** (2003), 1–22. https://doi.org/10.1017/CBO9781139164863.010
10. J. M. Bardet, W. C. Kengne, O. Wintenberger, Detecting multiple change-points in general causal time series using penalized quasi-likelihood, *Electron. J. Stat.,* **6** (2010), 435–477. https://doi.org/10.48550/arXiv.1008.0054
11. M. Kejriwal, P. Perron, J. Zhou, Wald tests for detecting multiple structural changes in persistence, *Economet. Theor.,* **29** (2013), 289–323. http://dx.doi.org/10.1017/S0266466612000357

12. L. J. Ma, J. G. Andrew, S. Georgy, Multiple change point detection and validation in autoregressive time series data, *Stat. Pap.,* **61** (2020), 1507–1528. http://dx.doi.org/10.1007/s00362-020-01198-w

13. I. B. Macnelill, V. K. Jandhyala, A. Kaul, S. B. Fotopoulos, Multiple change-point models for time series, *Environmetrics,* **31** (2020), 1–15. https://doi.org/10.1002/env.2593

14. S. Bouzebda, A. A. Ferfache, Asymptotic properties of M-estimators based on estimating equations and censored data in semi-parametric models with multiple change points, *J. Math. Anal. Appl.,* **497** (2021), 297–318. http://dx.doi.org/10.1016/j.jmaa.2020.124883

15. M. A. K. Noriah, A. A. A. Emad-eldin, An ANOVA-type test for multiple change points, *Stat. Pap.,* **55** (2014), 1159–1178. http://dx.doi.org/10.1007/s00362-013-0559-1

16. J. Hidalgo, P. M. Robinson, Testing for structural change in a long-memory environment, *J. Econometrics,* **70** (1996), 159–174. http://dx.doi.org/10.1016/0304-4076(94)01687-9

17. S. Lazarova, Testing for structural change in regression with long memory processes, *J. Econometrice,* **129** (2005), 329–372. http://dx.doi.org/10.1016/j.jeconom.2004.09.011

18. L. Wang, Change point estimation in long memory nonparametric models with applications, *Commun. Stat.-Simul. C.,* **37** (2008), 48–61. http://dx.doi.org/10.1080/03610910701723583

19. P. Bühlmann, Sieve bootstrap for time series, *Bernoulli,* **3** (1997), 123–148. https://doi.org/10.2307/3318584

20. A. M. Alonso, D. Peña, J. Romo, Forecasting time series with sieve bootstrap, *J. Stat. Plan. Infer.,* **100** (2002), 1–11. https://doi.org/10.1016/s0378-3758(01)00092-1

21. A. M. Alonso, D. Peña, J. Romo, On sieve bootstrap prediction intervals, *Stat. Probabil. Lett.,* **65** (2003), 13–20. https://doi.org/10.1016/S0167-7152(03)00214-1

22. A. M. Alonso, D. Peña, J. Romo, Introducing model uncertainty in time series bootstrap, *Stat. Sinica,* **14** (2004), 155–174. https://doi.org/10.1007/s00440-003-0309-8

23. P. Mukhopadhyay, V. A. Samaranayake, Prediction intervals for time series: A modified sieve bootstrap approach, *Commun. Stat.-Simul. C.,* **39** (2010), 517–538. https://doi.org/10.1080/03610910903506521

24. D. S. Poskitt, Properties of the sieve bootstrap for fractionally integrated and non-invertible processes, *J. Time Ser. Anal.,* **29** (2008), 224–250. https://doi.org/10.1111/j.1467-9892.2007.00554.x

25. H. E. Hurst, Long-term storage capacity of reservoirs, *Trans. Am. Soc. Civ. Eng. Tans.,* **116** (1951), 770–799. https://doi.org/10.1016/0013-4694(51)90043-0

26. J. Beran, *Statistics for long-memory process*, New York: Chapman and Hall, 1994. http://dx.doi.org/10.2307/2983481

AIMS Press

© 2022 Author(s), licensee AIMS Press. This is an open access article distributed under the terms of the Creative Commons Attribution License (http://creativecommons.org/licenses/by/4.0)