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Research article

Improved VIKOR methodology based on q-rung orthopair hesitant fuzzy rough aggregation information: application in multi expert decision making

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Abstract: The main objective of this article is to introduce the idea of a q-rung orthopair hesitant fuzzy rough set (q-ROHFRS) as a robust fusion of the q-rung orthopair fuzzy set, hesitant fuzzy set, and rough set. A q-ROHFRS is a novel approach to uncertainty modelling in multi-criteria decision making (MCDM). Various key properties of q-ROHFRS and some elementary operations on q-ROHFRSs are proposed. Based on the q-ROHFRS operational laws, novel q-rung orthopair hesitant fuzzy rough weighted averaging operators have been developed. Some interesting properties of the proposed operators are also demonstrated. Furthermore, by using the proposed aggregation operator, we develop a modified VIKOR method in the context of q-ROHFRS. The outcome of this research is to rank and select the best alternative with the help of the modified VIKOR method based on aggregation operators for q-ROHFRS. A decision-making algorithm based on aggregation operators and extended VIKOR methodology has been developed to deal with the uncertainty and incompleteness of real-world decision-making. Finally, a numerical illustration of agriculture farming is considered to demonstrate the applicability of the proposed methodology. Also, a comparative study is presented to demonstrate the validity and effectiveness of the proposed approach. The results show that the proposed decision-making methodology is feasible, applicable, and effective to address uncertainty in decision making problems.

Keywords: q-rung orthopair hesitant fuzzy rough sets; q-rung orthopair hesitant fuzzy rough aggregation operators; improved VIKOR method; multi expert decision making
Mathematics Subject Classification: 03B52, 03E72

1. Introduction

Numerous researchers have conducted extensive research on multi-criteria decision making (MCDM) techniques in the real world [5–8, 11, 12]. This pursuit resulted in the development of various industrious techniques for dealing with real-world challenges. The approaches developed to achieve that goal are based entirely on a description of the problem under consideration. Every aspect of practical life contains numerous ambiguities, complexities, and uncertainties, and it is impossible to analyse different types of decision-making information using a single value. Numerous scholars have concentrated on the challenges associated with imprecise, ambiguous, and hazy information throughout the previous few decades. To deal with uncertainties and vagueness, Zadeh [36] pioneered the concept of fuzzy sets (FSs), which has been defined by the element's membership function. Various researchers have shown the fuzzy set's applicability in a variety of fields, including decision-making, medical diagnosis, engineering, socioeconomic, and financial difficulties, etc.

Atanassov [9] introduced the notion of intuitionistic fuzzy sets (IFSs) based on the two characteristic functions, which are membership and non-membership grades, such that their sum is less than or equal to 1. As an efficient mathematical tool, the IFSs have been widely applied to many research fields. However, as the complexity of problems increases, the IFSs cannot depict the fuzzy situations where $ð_F(x) + \mathcal{B}_F(x) > 1$. To address this issue, Yager [33] expanded the restrictive conditions of IFSs to $(ð_F(x))^2 + (\mathcal{B}_F(x))^2 = 1$ and created Pythagorean fuzzy sets (PyFSs). In 2014, Zhang [38] introduced the concept of a scoring function based on Yager's approach and enhanced TOPSIS by utilizing PFS to describe ambiguous information. In 2015, Peng and Yang [26] investigated the basic operational laws under PyFS and discussed their significance in group DM through similarity measures. Khan et al. [16] established the Dombi aggregation operators (AOp) based on Dombi t-norm and t-conorm. Batool et al. [10] introduced the DM methodology using the entropy measure and Pythagorean probabilistic HFSs and discussed their applicability in evaluation of the fog-haze factor. Ashraf et al. [2] presented the sine trigonometric function based novel AOp under PyFS. This good work provides the necessary preparation for the extensive application of PyF numbers. Although PyFSs can express fuzzier information than IFSs, they will also lose efficiency in situations where $(ð_F(x))^2 + (\mathcal{B}_F(x))^2 > 1$. q-Rung orthopair fuzzy sets (q-ROFSs) in which $(ð_F(x))^q + (\mathcal{B}_F(x))^q \leq 1$, introduced by Yager [34], are able to solve the above-mentioned predicament fundamentally. The concept of q-ROFSs provides decision-makers (DMs) with a lot of flexibility and space to explore different alternatives under a set of criteria. In recent years, q-ROFSs have emerged many new methods and techniques in decision-making theory and its application. These achievements can be attributed to two aspects: MADM and multi-attribute group decision making (MAGDM) [19, 22–24]. This is mainly because it is difficult for decision-makers to accurately decide alternatives for multiple attribute indexes when they obtain uncertain or incomplete information. In 2018, Peng et al. [27] developd the novel AOp using exponential function and elaborate their applicability in MADM. Peng and Liu [28] introduced the information measures under q-ROFSs. Khan et al., [17] established the knowledge measures for q-ROFSs. Ali [1] presented the novel DM technique to tackle the uncertainty in the form of q-ROFSs. Liu and Huang [21] established the extended TOPSIS under probabilistic linguistic q-ROFSs using correlation measures and highlighted their applicability in decision making. Afterwards, by taking into consideration the hesitancy, the notion of the hesitant fuzzy set (HFS) was formulated by Torra [29], in which the membership degree is represented by multiple discrete values rather than a single value.

Khan et al. [18] established the technique of similarity measures under probabilistic hesitant fuzzy rough environment. Liu et al. [20] extended the notation of HFSs to q-rung orthopair hesitant fuzzy sets (q-ROHFSs), considering the hesitancy in membership as well as in non-membership grades. Wang et al. [30] proposed the Hamacher norm based AOps under dual hesitant q-ROFSs and discussed their application in decision making problems. Wang et al. [31] developed the AOps based on Muirhead mean under dual hesitant q-rung orthopair fuzzy information. Wang et al. [32] constructed the extended TOPSIS process for q-ROHFSs and discussed their application in DM.

Pawlak [25] is pioneer who studied the dominant concept of rough set (RS) theory. Rough set theory (RS) is an extension of traditional set theory that deals with inconsistencies and uncertainty. In recent decades, research on the rough set has progressed significantly, both in terms of theoretical implementations and theory itself. Numerous scholars from across the globe have broadened the notion of RS in a variety of areas. Using the fuzzy relation rather than the crisp binary relation, Dubois et al. [15] initiated the notion of fuzzy rough sets. The hybrid structure of IFSs and rough sets, intuitionistic fuzzy rough (IFR) sets introduced by Cornelis et al. [13]. IFRSs serve as a crucial link between these two theories. By utilizing IFR approximations, Zhou and Wu [40] established a novel decision making technique under IFR environment to address their constrictive and axiomatic analysis in detail. Zhan et al. [37] presented the DM techniques under IFR environment and explored their application in real word problems. Different extensions of IFRS are being investigated [35, 39] to tackle the uncertainty in MCGDM problems. Chinram et al. [14] developed the algebraic norm based AOs based EDAS technique under IFR settings and discussed their application in MAGDM. In certain real-world situations, decision-makers (DMs) have strong opinions regarding the ranking or rating of a organization's plans, projects, or official statements. For example, let the administration of a university initiate a large-scale project of football ground. The members of the university administration may rate their project highly by assigning positive membership ($\eth = 0.9$), but others may consider the same program as a waste of money and attempt to discredit it by proposing completely contradictory viewpoints. So they assign negative membership ($\mathcal{B} = 0.7$). In this situation, $\eth_F(x) + \mathcal{B}_F(x) > 1$ and $(\eth_F(x))^2 + (\mathcal{B}_F(x))^2 > 1$ but $(\eth_F(x))^q + (\mathcal{B}_F(x))^q < 1$ for $q > 3$. So that $(\eth, \mathcal{B})$ is neither IFN nor PFN but it is q-ROFN. Thus, Yager's q-ROFNs are effective in dealing with data uncertainty. The q-Rung orthopair hesitant fuzzy rough sets (q-ROHFRS), a hybrid intelligent structure of rough sets, and q-ROHFS is a comprehensive classification approach that has attracted researchers for its capacity to cope with ambiguous and imperfect information. The research leads to the fact that AOps are crucial in DM because they provide information from several sources to be integrated into a single value. According to existing literature, the development of AOps following q-ROHFS hybridization with a rough set is not observed in the q-ROF environment. As a consequence of this inspiration, we construct a variety of algebraic aggregation operators for rough information, including q-rung orthopair hesitant fuzzy rough weighted averaging, order weighted averaging, and hybrid weighted averaging, under the algebraic t-norm and t-conorm. The noteworthy contributions of the present article are follows:
(1) To construct new notion of q-rung orthopair hesitant fuzzy rough sets and investigate their basic operational laws.
(2) To compile a list of aggregation operators based on the algebraic t-norm and t-conorm, and analyze their associated features in details.
(3) To develop a DM methodology using proposed aggregation operators to aggregate the uncertain information in real word decision making problems.

(4) A numerical case study of a real-world DM problem in agricultural farming is addressed to demonstrate the methodology's validity.
(5) q-ROHFR-VIKOR method is employed to validate the proposed DM approach.
The remainder of the manuscript is arranged as follows: Section 2 briefly retrospects some basic concepts of q-ROFSs, HFSs and rough set theory. A novel notion of q-rung orthopair hesitant fuzzy sets is presented and also their basic interesting operational laws are defined in Section 3. Section 4 highlights the improved q-ROHFR-VIKOR methodology under q-rung orthopair hesitant fuzzy rough information. Section 5 presents the numerical illustration concerning the agriculture farming. Further this section deals with the applicability of the developed methodology. Section 6 establishes comparison analysis using q-ROHFR weighted averaging aggregation operator to validate the q-ROHFR-VIKOR methodology. Section 7 concludes this manuscript.

2. Preliminaries

This section describes the basic terminologies i.e., intuitionistic fuzzy sets (IFS), q-Rung orthopair fuzzy sets (q-ROFS), hesitant fuzzy sets (HFS), q-rung ortopair hesitant fuzzy sets (q-ROHFS), rough sets (RS) and q-rung orthopair fuzzy rough set (q-ROFRS).

Definition 1. *[9] For a universal set $\aleph$, an IFS F over $\aleph$ is follows as:*

$$F = \{\langle x, ð_F(x), \mathcal{B}_F(x)\rangle | x \in \aleph\},$$

for each $x \in F$ the functions $ð_F : \aleph \to [0\,,1]$ and $\mathcal{B}_F : \aleph \to [0, 1]$ denotes the degree of membership and non membership respectively, which must satisfy the property $0 \le ð_F(x) + \mathcal{B}_F(x) \le 1$.

Definition 2. *[34] For a universal set $\aleph$, a q-ROFS $\mathcal{T}$ over $\aleph$ is defined as:*

$$\mathcal{T} = \{\langle x, ð_{\mathcal{T}}(x), \mathcal{B}_{\mathcal{T}}(x)\rangle | x \in \aleph\}$$

for each $x \in \mathcal{T}$ the functions $ð_{\mathcal{T}} : \aleph \to [0, 1]$ and $\mathcal{B}_{\mathcal{T}} : \aleph \to [0, 1]$ denote the degrees of membership and non membership respectively, which must satisfy $(\mathcal{B}_{\mathcal{T}}(x))^q + (ð_{\mathcal{T}}(x))^q \le 1, (q > 2 \in \mathbb{Z})$.

Definition 3. *[20] For a universal set $\aleph$, a q-rung orthopair hesitant fuzzy set (q-ROHFS) $\mathcal{H}$ is defined as:*

$$\mathcal{H} = \{\left\langle x, ð_{h_{\mathcal{H}}}(x), \mathcal{B}_{h_{\mathcal{H}}}(x)\right\rangle | x \in \aleph\},$$

where $ð_{h_{\mathcal{H}}}(x)$ and $\mathcal{B}_{h_{\mathcal{H}}}(x)$ are sets of some values in $[0, 1]$ denote the membership and non membership grades respectively. It is required to satisfy the following properties: $\forall x \in \aleph$, $\forall \mu_{\mathcal{H}}(x) \in ð_{h_{\mathcal{H}}}(x)$, $\forall \nu_{\mathcal{H}}(x) \in \mathcal{B}_{h_{\mathcal{H}}}(x)$ with $\left(\max\left(ð_{h_{\mathcal{H}}}(x)\right)\right)^q + \left(\min\left(\mathcal{B}_{h_{\mathcal{H}}}(x)\right)\right)^q \le 1$ and $\left(\min\left(ð_{h_{\mathcal{H}}}(x)\right)\right)^q + \left(\max\left(\mathcal{B}_{h_{\mathcal{H}}}(x)\right)\right)^q \le 1$. For simplicity, we will use a pair $\mathcal{H} = (ð_{h_{\mathcal{H}}}, \mathcal{B}_{h_{\mathcal{H}}})$ to mean q-ROHF number (q-ROHFN).

Definition 4. *[20] Let $\mathcal{H}_1 = (ð_{h_{\mathcal{H}_1}}, \mathcal{B}_{h_{\mathcal{H}_1}})$ and $\mathcal{H}_2 = (ð_{h_{\mathcal{H}_2}}, \mathcal{B}_{h_{\mathcal{H}_2}})$ be two q-ROHFNs. Then the basic set theoretic operations are as follows:*

(1) $$\mathcal{H}_1 \cup \mathcal{H}_2 = \left\{ \bigcup_{\substack{\mu_1 \in ð_{h_{\mathcal{H}_1}} \\ \mu_2 \in ð_{h_{\mathcal{H}_2}}}} \max(\mu_1, \mu_2),\ \bigcup_{\substack{\nu_1 \in \mathcal{B}_{h_{\mathcal{H}_1}} \\ \nu_2 \in \mathcal{B}_{h_{\mathcal{H}_2}}}} \min(\nu_1, \nu_2) \right\};$$

(2) $\mathcal{H}_1 \cap \mathcal{H}_2 = \left\{ \bigcup_{\substack{\mu_1 \in \eth_{h_{\mathcal{H}_1}} \\ \mu_2 \in \eth_{h_{\mathcal{H}_2}}}} \min(\mu_1, \mu_2), \bigcup_{\substack{\nu_1 \in \mathcal{B}_{h_{\mathcal{H}_1}} \\ \nu_2 \in \mathcal{B}_{h_{\mathcal{H}_2}}}} \max(\nu_1, \nu_2) \right\}$;

(3) $\mathcal{H}_1^c = \left\{ \mathcal{B}_{h_{\mathcal{H}_1}}, \eth_{h_{\mathcal{H}_1}} \right\}$.

Definition 5. *[20] Let* $\mathcal{H}_1 = (\eth_{h_{\mathcal{H}_1}}, \mathcal{B}_{h_{\mathcal{H}_1}})$ *and* $\mathcal{H}_2 = (\eth_{h_{\mathcal{H}_2}}, \mathcal{B}_{h_{\mathcal{H}_2}})$ *be two q-ROHFNs and* $q > 2$ *and* $\gamma > 0$ *be any real number. Then the operational laws can be defined as:*

(1) $\mathcal{H}_1 \oplus \mathcal{H}_2 = \left\{ \bigcup_{\substack{\mu_1 \in \eth_{h_{\mathcal{H}_1}} \\ \mu_2 \in \eth_{h_{\mathcal{H}_2}}}} \left\{ \sqrt[q]{\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q} \right\}, \bigcup_{\substack{\nu_1 \in \mathcal{B}_{h_{\mathcal{H}_1}} \\ \nu_2 \in \mathcal{B}_{h_{\mathcal{H}_2}}}} \{\nu_1 \cdot \nu_2\} \right\}$;

(2) $\mathcal{H}_1 \otimes \mathcal{H}_2 = \left\{ \bigcup_{\substack{\mu_1 \in \eth_{h_{\mathcal{H}_1}} \\ \mu_2 \in \eth_{h_{\mathcal{H}_2}}}} \{\mu_1 \cdot \mu_2\}, \bigcup_{\substack{\nu_1 \in \mathcal{B}_{h_{\mathcal{H}_1}} \\ \nu_2 \in \mathcal{B}_{h_{\mathcal{H}_2}}}} \left\{ \sqrt[q]{\nu_1^q + \nu_2^q - \nu_1^q \nu_2^q} \right\} \right\}$;

(3) $\gamma \mathcal{H}_1 = \left\{ \bigcup_{\mu_1 \in \eth_{h_{\mathcal{H}_1}}} \left\{ \sqrt[q]{1 - (1 - \mu_1^q)^\gamma} \right\}, \bigcup_{\nu_1 \in \mathcal{B}_{h_{\mathcal{H}_1}}} \left\{ \nu_1^\gamma \right\} \right\}$;

(4) $\mathcal{H}_1^\gamma = \left\{ \bigcup_{\mu_1 \in \eth_{h_{\mathcal{H}_1}}} \left\{ \mu_1^\gamma \right\}, \bigcup_{\nu_1 \in \mathcal{B}_{h_{\mathcal{H}_1}}} \left\{ \sqrt[q]{1 - (1 - \nu_1^q)^\gamma} \right\} \right\}$.

Definition 6. *[25] Let* $\aleph$ *be the universal set and* $\mathcal{P} \subseteq \aleph \times \aleph$ *be a crisp relation. Then*
(1) $\mathcal{P}$ *is reflexive if* $(\mathfrak{b}, \mathfrak{b}) \in \mathcal{P}$, *for each* $\mathfrak{b} \in \aleph$;
(3) $\mathcal{P}$ *is symmetric if* $\forall \mathfrak{b}, a \in \aleph$, $(\mathfrak{b}, a) \in \mathcal{P}$ *then* $(a, \mathfrak{b}) \in \mathcal{P}$;
(4) $\mathcal{P}$ *is transitive if* $\forall\ \mathfrak{b}, a, b \in \aleph$, $(\mathfrak{b}, a) \in \aleph$ *and* $(a, b) \in \mathcal{P}$ *implies* $(\mathfrak{b}, b) \in \mathcal{P}$.

Definition 7. *[25] Let* $\aleph$ *be a universal set and* $\mathcal{P}$ *be any relation on* $\aleph$. *Define a set valued mapping* $\mathcal{P}^* : \aleph \to M(\aleph)$ *by* $\mathcal{P}^*(\mathfrak{b}) = \{a \in \aleph | (\mathfrak{b}, a) \in \mathcal{P}\}$, *for* $\mathfrak{b} \in \aleph$ *where* $\mathcal{P}^*(\mathfrak{b})$ *is called a successor neighbourhood of the element* $\mathfrak{b}$ *with respect to relation* $\mathcal{P}$. *The pair* $(\aleph, \mathcal{P})$ *is called crisp approximation space. Now for any set* $\mathcal{S} \subseteq \aleph$, *the lower and upper approximation of* $\mathcal{S}$ *with respect to approximations space* $(\aleph, \mathcal{P})$ *is defined as:*

$$\begin{aligned} \underline{\mathcal{P}}(\mathcal{S}) &= \{\mathfrak{b} \in \aleph | \mathcal{P}^*(\mathfrak{b}) \subseteq \mathcal{S}\}; \\ \overline{\mathcal{P}}(\mathcal{S}) &= \{\mathfrak{b} \in \aleph | \mathcal{P}^*(\mathfrak{b}) \cap \mathcal{S} \neq \phi\}. \end{aligned}$$

The pair $\left(\underline{\mathcal{P}}(\mathcal{S}), \overline{\mathcal{P}}(\mathcal{S})\right)$ *is called rough set and both* $\underline{\mathcal{P}}(\mathcal{S}), \overline{\mathcal{P}}(\mathcal{S}) : M(\aleph) \to M(\aleph)$ *are upper and lower approximation operators.*

Definition 8. *[14] Let* $\aleph$ *be the universal set and* $\mathcal{P} \in IFS(\aleph \times \aleph)$ *be an IF relation. Then*
(1) $\mathcal{P}$ *is reflexive if* $\mu_{\mathcal{P}}(\mathfrak{b}, \mathfrak{b}) = 1$ *and* $\nu_{\mathcal{P}}(\mathfrak{b}, \mathfrak{b}) = 0, \forall \mathfrak{b} \in \aleph$;
(2) $\mathcal{P}$ *is symmetric if* $\forall (\mathfrak{b}, a) \in \aleph \times \aleph$, $\mu_{\mathcal{P}}(\mathfrak{b}, a) = \mu_{\mathcal{P}}(a, \mathfrak{b})$ *and* $\nu_{\mathcal{P}}(\mathfrak{b}, a) = \nu_{\mathcal{P}}(a, \mathfrak{b})$;
(3) $\mathcal{P}$ *is transitive if* $\forall (\mathfrak{b}, b) \in \aleph \times \aleph$,

$$\mu_{\mathcal{P}}(\mathfrak{b}, b) \geq \bigvee\nolimits_{a \in \aleph} \left[\mu_{\mathcal{P}}(\mathfrak{b}, a) \wedge \mu_{\mathcal{P}}(a, b)\right];$$

and

$$\nu_{\mathcal{P}}(\mathfrak{b}, b) = \bigwedge\nolimits_{a\in\aleph}\left[\nu_{\mathcal{P}}(\mathfrak{b}, a) \wedge \nu_{\mathcal{P}}(a, b)\right].$$

Definition 9. *[3] Let $\aleph$ be the universal set and for any subset $\mathcal{P} \in q-ROHFS(\aleph\times\aleph)$ is said to be an q-rung hesitant fuzzy relation. The pair $(\aleph,\mathcal{P})$ is said to be q-ROHF approximation space. If for any $\mathcal{S} \subseteq q-ROHFS(\aleph)$, then the upper and lower approximations of $\mathcal{S}$ with respect to q-ROHF approximation space $(\aleph,\mathcal{P})$ are two q-ROHFSs, which are denoted by $\overline{\mathcal{P}}(\mathcal{S})$ and $\underline{\mathcal{P}}(\mathcal{S})$ and defined as:*

$$\begin{aligned}\overline{\mathcal{P}}(\mathcal{S}) &= \left\{\left\langle \mathfrak{b}, \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}), \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b})\right\rangle | \mathfrak{b}\in\aleph\right\};\\ \underline{\mathcal{P}}(\mathcal{S}) &= \left\{\left\langle \mathfrak{b}, \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}), \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b})\right\rangle | \mathfrak{b}\in\aleph\right\};\end{aligned}$$

where

$$\begin{aligned}\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}) &= \bigvee_{k\in\aleph}\left[\eth_{h_{\mathcal{P}}}(\mathfrak{b}, k)\bigvee \eth_{h_{\mathcal{S}}}(k)\right];\\ \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}) &= \bigwedge_{k\in\aleph}\left[\mathcal{B}_{h_{\mathcal{P}}}(\mathfrak{b}, k)\bigwedge \mathcal{B}_{h_{\mathcal{S}}}(k)\right];\\ \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}) &= \bigwedge_{k\in\aleph}\left[\eth_{h_{\mathcal{P}}}(\mathfrak{b}, k)\bigwedge \eth_{h_{\mathcal{S}}}(k)\right];\\ \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}) &= \bigvee_{k\in\aleph}\left[\mathcal{B}_{h_{\mathcal{P}}}(\mathfrak{b}, k)\bigvee \mathcal{B}_{h_{\mathcal{S}}}(k)\right];\end{aligned}$$

such that $0 \le \left(\max(\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}))\right)^q + \left(\min(\mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}))\right)^q \le 1$ *and* $0 \le \left(\min(\eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b})\right)^q + \left(\max(\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}))\right)^q \le 1$. *As $\left(\overline{\mathcal{P}}(\mathcal{S}), \underline{\mathcal{P}}(\mathcal{S})\right)$ are $q-ROHFS\,s$, so $\overline{\mathcal{P}}(\mathcal{S}), \underline{\mathcal{P}}(\mathcal{S}) : q-ROHFS(\aleph) \to q-RFS(\aleph)$ are upper and lower approximation operators. The pair*

$$\mathcal{P}(\mathcal{S}) = \left(\underline{\mathcal{P}}(\mathcal{S}), \overline{\mathcal{P}}(\mathcal{S})\right) = \left\{\left\langle \mathfrak{b}, \left(\eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}), \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b})\right), \left(\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}), \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b})\right)\right\rangle | \mathfrak{b}\in\mathcal{S}\right\}$$

will be called q-rung orthopair hesitent fuzzy rough set. For simplicity

$$\mathcal{P}(\mathcal{S}) = \left\{\left\langle \mathfrak{b}, \left(\eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}), \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b})\right), \left(\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b}), \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\mathfrak{b})\right)\right\rangle | \mathfrak{b}\in\mathcal{S}\right\}$$

is represented as $\mathcal{P}(\mathcal{S}) = \left((\underline{\eth}, \underline{\mathcal{B}}), (\overline{\eth}, \overline{\mathcal{B}})\right)$ and is known as q-ROHFRV.

Definition 10. *[3] Let $\mathcal{P}(\mathcal{S}_1) = (\underline{\mathcal{P}}(\mathcal{S}_1), \overline{\mathcal{P}}(\mathcal{S}_1))$ and $\mathcal{P}(\mathcal{S}_2) = (\underline{\mathcal{P}}(\mathcal{S}_2), \overline{\mathcal{P}}(\mathcal{S}_2))$ be two q-ROHFRSs. Then*
(1) $\mathcal{P}(\mathcal{S}_1)\cup\mathcal{P}(\mathcal{S}_2) = \{(\underline{\mathcal{P}}(\mathcal{S}_1)\cup\underline{\mathcal{P}}(\mathcal{S}_2)), (\overline{\mathcal{P}}(\mathcal{S}_1)\cup\overline{\mathcal{P}}(\mathcal{S}_2))\}$
(2) $\mathcal{P}(\mathcal{S}_1)\cap\mathcal{P}(\mathcal{S}_2) = \{(\underline{\mathcal{P}}(\mathcal{S}_1)\cap\underline{\mathcal{P}}(\mathcal{S}_2)), (\overline{\mathcal{P}}(\mathcal{S}_1)\cap\overline{\mathcal{P}}(\mathcal{S}_2))\}$.

Definition 11. *[3] Let $\mathcal{P}(\mathcal{S}_1) = (\underline{\mathcal{P}}(\mathcal{S}_1), \overline{\mathcal{P}}(\mathcal{S}_1))$ and $\mathcal{P}(\mathcal{S}_2) = (\underline{\mathcal{P}}(\mathcal{S}_2), \overline{\mathcal{P}}(\mathcal{S}_2))$ be two q-ROHFRSs. Then*
(1) $\mathcal{P}(\mathcal{S}_1)\oplus\mathcal{P}(\mathcal{S}_2) = \{(\underline{\mathcal{P}}(\mathcal{S}_1)\oplus\underline{\mathcal{P}}(\mathcal{S}_2)), (\overline{\mathcal{P}}(\mathcal{S}_1)\oplus\overline{\mathcal{P}}(\mathcal{S}_2))\}$
(2) $\mathcal{P}(\mathcal{S}_1)\otimes\mathcal{P}(\mathcal{S}_2) = \{(\underline{\mathcal{P}}(\mathcal{S}_1)\otimes\underline{\mathcal{P}}(\mathcal{S}_2)), (\overline{\mathcal{P}}(\mathcal{S}_1)\otimes\overline{\mathcal{P}}(\mathcal{S}_2))\}$
(3) $\mathcal{P}(\mathcal{S}_1)\subseteq\mathcal{P}(\mathcal{S}_2) = \{(\underline{\mathcal{P}}(\mathcal{S}_1)\subseteq\underline{\mathcal{P}}(\mathcal{S}_2))$ and $(\overline{\mathcal{P}}(\mathcal{S}_1)\subseteq\overline{\mathcal{P}}(\mathcal{S}_2))\}$
(4) $\gamma\mathcal{P}(\mathcal{S}_1) = (\gamma\underline{\mathcal{P}}(\mathcal{S}_1), \gamma\overline{\mathcal{P}}(\mathcal{S}_1))$ for $\gamma\ge 1$
(5) $(\mathcal{P}(\mathcal{S}_1))^\gamma = ((\underline{\mathcal{P}}(\mathcal{S}_1))^\gamma, (\overline{\mathcal{P}}(\mathcal{S}_1))^\gamma)$ for $\gamma\ge 1$
(6) $\mathcal{P}(\mathcal{S}_1)^c = (\underline{\mathcal{P}}(\mathcal{S}_1)^c, \overline{\mathcal{P}}(\mathcal{S}_1)^c)$ where $\underline{\mathcal{P}}(\mathcal{S}_1)^c$ and $\overline{\mathcal{P}}(\mathcal{S}_1)^c$ shows the complement of q-rung fuzzy rough approximation operators $\underline{\mathcal{P}}(\mathcal{S}_1)$ and $\overline{\mathcal{P}}(\mathcal{S}_1)$,that is $\underline{\mathcal{P}}(\mathcal{S}_1)^c = \left(\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}, \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}\right)$.
(7) $\mathcal{P}(\mathcal{S}_1) = \mathcal{P}(\mathcal{S}_2)$ iff $\underline{\mathcal{P}}(\mathcal{S}_1) = \underline{\mathcal{P}}(\mathcal{S}_2)$ and $\overline{\mathcal{P}}(\mathcal{S}_1) = \overline{\mathcal{P}}(\mathcal{S}_2)$.

For the comparison/ranking of two or more q-ROHFRVs, we will utilize the score function. Superior the score value of q-ROHFRV greater that value is, and smaller the score value inferior that q-ROHFRV is. We will use the accuracy function when the score values are equal.

Definition 12. *[4] The score function for q-ROHFRV* $\mathcal{P}(\mathcal{S}) = (\underline{\mathcal{P}}(\mathcal{S}), \overline{\mathcal{P}}(\mathcal{S})) = ((\underline{\eth}, \underline{\mathcal{B}}), (\overline{\eth}, \overline{\mathcal{B}}))$ *is given as;*

$$\eth(\mathcal{P}(\mathcal{S})) = \frac{1}{4}\begin{pmatrix} 2 + \frac{1}{M_{\mathcal{H}}}\sum_{\underline{\mu_i}\in\eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\left\{\underline{\mu_i}\right\} + \frac{1}{N_{\mathcal{H}}}\sum_{\overline{\mu_i}\in\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}}\{\overline{\mu_i}\} - \\ \frac{1}{M_{\mathcal{H}}}\sum_{\underline{\nu_i}\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}(\underline{\nu_i}) - \frac{1}{M_{\mathcal{H}}}\sum_{\overline{\nu_i}\in\mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}}(\overline{\nu_i}) \end{pmatrix},$$

The accuracy function for q-ROHFRV $\mathcal{P}(\mathcal{S}) = (\underline{\mathcal{P}}(\mathcal{S}), \overline{\mathcal{P}}(\mathcal{S})) = ((\underline{\eth}, \underline{\mathcal{B}}), (\overline{\eth}, \overline{\mathcal{B}}))$ *is given as;*

$$\mathbf{AC}\mathcal{P}(\mathcal{S}) = \frac{1}{4}\begin{pmatrix} \frac{1}{M_{\mathcal{H}}}\sum_{\mu_i\in\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}}(\overline{\mu_i}) + \frac{1}{M_{\mathcal{H}}}\sum_{\mu_i\in\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}}(\overline{\mu_i}) + \\ \frac{1}{M_{\mathcal{H}}}\sum_{\underline{\nu_i}\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}(\underline{\nu_i}) + \frac{1}{M_{\mathcal{H}}}\sum_{\overline{\nu_i}\in\mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}}(\overline{\nu_i}) \end{pmatrix},$$

where $M_{\mathcal{H}}$ *and* $N_{\mathcal{H}}$ *represent the number of elements in* $\eth_{h_g}$ *and* $\mathcal{B}_{h_g}$ *respectively.*

Definition 13. *Suppose* $\mathcal{P}(\mathcal{S}_1) = (\underline{\mathcal{P}}(\mathcal{S}_1), \overline{\mathcal{P}}(\mathcal{S}_1))$ *and* $\mathcal{P}(\mathcal{S}_2) = (\underline{\mathcal{P}}(\mathcal{S}_2), \overline{\mathcal{P}}(\mathcal{S}_2))$ *are two q-ROHFRVs. Then*
(1) If $\eth(\mathcal{P}(\mathcal{S}_1)) > \eth(\mathcal{P}(\mathcal{S}_2))$, *then* $\mathcal{P}(\mathcal{S}_1) > \mathcal{P}(\mathcal{S}_2)$,
(2) If $\eth(\mathcal{P}(\mathcal{S}_1)) \prec \eth(\mathcal{P}(\mathcal{S}_2))$, *then* $\mathcal{P}(\mathcal{S}_1) \prec \mathcal{P}(\mathcal{S}_2)$,
(3) If $\eth(\mathcal{P}(\mathcal{S}_1)) = \eth(\mathcal{P}(\mathcal{S}_2))$, *then*
(a) If $\mathbf{AC}\mathcal{P}(\mathcal{S}_1) > \mathbf{AC}\mathcal{P}(\mathcal{S}_2)$ *then* $\mathcal{P}(\mathcal{S}_1) > \mathcal{P}(\mathcal{S}_2)$,
(b) If $\mathbf{AC}\mathcal{P}(\mathcal{S}_1) \prec \mathbf{AC}\mathcal{P}(\mathcal{S}_2)$ *then* $\mathcal{P}(\mathcal{S}_1) \prec \mathcal{P}(\mathcal{S}_2)$,
(c) If $\mathbf{AC}\mathcal{P}(\mathcal{S}_1) = \mathbf{AC}\mathcal{P}(\mathcal{S}_2)$ *then* $\mathcal{P}(\mathcal{S}_1) = \mathcal{P}(\mathcal{S}_2)$.

3. q-rung orthopair hesitant fuzzy rough aggregation operators

Herein, we introduce new idea of q-ROHF rough aggregation operators by embedding the notions of rough sets and q-ROHF aggregation operators to get aggregation concepts of q-ROHFRWA operators. Some essential features of these concepts are discussed.

Definition 14. *Consider the collection* $\mathcal{P}(\mathcal{S}_i) = (\underline{\mathcal{P}}(\mathcal{S}_i), \overline{\mathcal{P}}(\mathcal{S}_i))$ $(i = 1, 2, 3, ..., n)$ *of q-ROHFRVs along weight vector* $w = (w_1, w_2, ...w_n)^T$ *such that* $\sum_{i=1}^{n} w_i = 1$ *and* $0 \le w_i \le 1$. *The q-ROHFRWA operator is determined as*

$$q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ..., \mathcal{P}(\mathcal{S}_n)\right) = \left(\sum_{i=1}^{n} w_i\underline{\mathcal{P}}(\mathcal{S}_i), \sum_{i=1}^{n} w_i\overline{\mathcal{P}}(\mathcal{S}_i)\right).$$

Theorem 1. *Let* $\mathcal{P}(\mathcal{S}_i) = (\underline{\mathcal{P}}(\mathcal{S}_i), \overline{\mathcal{P}}(\mathcal{S}_i))$ $(i = 1, 2, 3, ...n)$ *be the collection of q-ROHFRVs along weight vector* $w = (w_1, w_2, ...w_n)^T$ *. Then the q-ROHFRWA operator is defined as;*

$$\begin{aligned} & q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_n)\right) \\ = & \left(\sum_{i=1}^{n} w_i\underline{\mathcal{P}}(\mathcal{S}_i), \sum_{i=1}^{n} w_i\overline{\mathcal{P}}(\mathcal{S}_i)\right) \end{aligned}$$

$$= \begin{bmatrix} \bigcup_{\underline{\mu}_i \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n} \left(1 - \left(\underline{\mu i}\right)^q\right)^{w_i}\right)}, & \bigcup_{\underline{\nu}_i \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n} \left(\underline{\nu_i}\right)^{w_i} \\ \bigcup_{\overline{\mu}_i \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n} \left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, & \bigcup_{\overline{\nu}_i \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n} (\overline{\nu_i})^{w_i} \end{bmatrix}$$

Proof. Utilizing mathematical induction to find the the desired proof. Using the operational law, it follows that

$$\mathcal{P}(\mathcal{S}_1) \oplus \mathcal{P}(\mathcal{S}_2) = \left[\underline{\mathcal{P}}(\mathcal{S}_1) \oplus \underline{\mathcal{P}}(\mathcal{S}_2), \overline{\mathcal{P}}(\mathcal{S}_1) \oplus \overline{\mathcal{P}}(\mathcal{S}_2)\right]$$

and

$$\gamma\mathcal{P}(\mathcal{S}_1) = \left(\gamma\underline{\mathcal{P}}(\mathcal{S}_1), \gamma\overline{\mathcal{P}}(\mathcal{S}_1)\right)$$

If $n = 2$, then

$$\begin{aligned} & q - ROHFRWA\,(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2)) \\ = & \left(\sum_{i=1}^{2} w_i\underline{\mathcal{P}}(\mathcal{S}_i), \sum_{i=1}^{2} w_i\overline{\mathcal{P}}(\mathcal{S}_i)\right) \\ = & \left(\begin{array}{c} \left(\bigcup_{\underline{\mu}_i \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{2} \left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\right)}, \bigcup_{\underline{\nu}_i \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{2} \left(\underline{\nu_i}\right)^{w_i} \right) \\ \left(\bigcup_{\overline{\mu}_i \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{2} \left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, \bigcup_{\overline{\nu}_i \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{2} (\overline{\nu_i})^{w_i} \right) \end{array} \right) \end{aligned}$$

The result is true for $n = 2$. Let it is true for $n = k$, that is,

$$\begin{aligned} & q - ROHFRWA\,(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_k)) \\ = & \left(\sum_{i=1}^{k} w_i\underline{\mathcal{P}}(\mathcal{S}_i), \sum_{i=1}^{k} w_i\overline{\mathcal{P}}(\mathcal{S}_i)\right) \\ = & \left(\begin{array}{c} \left(\bigcup_{\underline{\mu}_i \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{k} \left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\right)}, \bigcup_{\underline{\nu}_i \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{k} \left(\underline{\nu_i}\right)^{w_i} \right) \\ \left(\bigcup_{\overline{\mu}_i \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{k} \left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, \bigcup_{\overline{\nu}_i \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{k} (\overline{\nu_i})^{w_i} \right) \end{array} \right) \end{aligned}$$

Now, we have to show that it is true for $n = k + 1$, we have

$$\begin{aligned} & q - ROHFRWA\,(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_{k+1})) \\ = & \left(\begin{array}{c} \left(\sum_{i=1}^{k} w_i\underline{\mathcal{P}}\,(\mathcal{S}_i) \oplus w_{k+1}\underline{\mathcal{P}}(\mathcal{S}_{k+1})\right), \\ \left(\sum_{i=1}^{k} w_i\overline{\mathcal{P}}\,(\mathcal{S}_i) \oplus w_{k+1}\overline{\mathcal{P}}(\mathcal{S}_{k+1})\right) \end{array} \right) \\ = & \left(\begin{array}{c} \left(\bigcup_{\underline{\mu}_i \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{k+1} \left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\right)}, \bigcup_{\underline{\nu}_i \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{k+1} \left(\underline{\nu_i}\right)^{w_i} \right) \\ \left(\bigcup_{\overline{\mu}_i \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{k+1} \left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, \bigcup_{\overline{\nu}_i \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{k+1} (\overline{\nu_i})^{w_i} \right) \end{array} \right). \end{aligned}$$

Thus the required result is true for $n = k + 1$. Hence, the result is true for all $n \geq 1$. From the above analysis $\underline{\mathcal{P}}(\mathcal{S})$ and $\overline{\mathcal{P}}(\mathcal{S})$ are q-ROHFRVs. So, $\sum_{i=1}^{k} w_i \underline{\mathcal{P}}(\mathcal{S}_i)$ and $\sum_{i=1}^{k} w_i \overline{\mathcal{P}}(\mathcal{S}_i)$ are also q-ROHFRVs. Therefore, q-ROHFRWA $(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_n))$ is a q-ROHFRV under q-ROHF approximation space $(\aleph, \mathcal{P})$. □

Theorem 2. *Consider the collection* $\mathcal{P}(\mathcal{S}_i) = (\underline{\mathcal{P}}(\mathcal{S}_i), \overline{\mathcal{P}}(\mathcal{S}_i))$ $(i = 1, 2, 3, ..., n)$ *of q-ROHFRVs with weight vectors* $w = (w_1, w_2, ...w_n)^T$ *such that* $\sum_{i=1}^{n} w_i = 1$ *and* $0 \leq w_i \leq 1$. *Then q-ROHFRWA operator satisfy the following properties:*

(1) Idempotency: *If* $\mathcal{P}(\mathcal{S}_i) = \mathcal{G}(\mathcal{S})$ *for* $(i = 1, 2, 3, ..., n)$, *where* $\mathcal{G}(\mathcal{S}) = \left(\underline{\mathcal{G}}(\mathcal{S}), \overline{\mathcal{G}}(\mathcal{S})\right) = \left((\underline{b}, \underline{d}), (\overline{b}, \overline{d})\right)$. *Then*

$$q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_n)\right) = \mathcal{G}(\mathcal{S}).$$

(2) Boundedness: *Let* $(\mathcal{P}(\mathcal{S}))^- = \left(\min_i \underline{\mathcal{P}}(\mathcal{S}_i), \max_i \overline{\mathcal{P}}(\mathcal{S}_i)\right)$ *and* $(\mathcal{P}(\mathcal{S}))^+ = \left(\max_i \underline{\mathcal{P}}(\mathcal{S}_i), \min_i \overline{\mathcal{P}}(\mathcal{S}_i)\right)$. *Then*

$$(\mathcal{P}(\mathcal{S}))^- \leq q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ..., \mathcal{P}(\mathcal{S}_n)\right) \leq (\mathcal{P}(\mathcal{S}))^+.$$

(3) Monotonicity: *Suppose* $\mathcal{G}(\mathcal{S}) = \left(\underline{\mathcal{G}}(\mathcal{S}_i), \overline{\mathcal{G}}(\mathcal{S}_i)\right)$ $(i = i, 2, ..., n)$ *be another collection of q-ROHFRVs such that* $\underline{\mathcal{G}}(\mathcal{S}_i) \leq \underline{\mathcal{P}}(\mathcal{S}_i)$ *and* $\overline{\mathcal{G}}(\mathcal{S}_i) \leq \overline{\mathcal{P}}(\mathcal{S}_i)$. *Then*

$$q - ROHFRWA\left(\mathcal{G}(\mathcal{S}_1), \mathcal{G}(\mathcal{S}_2), ..., \mathcal{G}(\mathcal{S}_n)\right) \leq q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_n)\right).$$

(4) Shiftinvariance: *Consider another q-ROHFRV* $\mathcal{G}(\mathcal{S}) = \left(\underline{\mathcal{G}}(\mathcal{S}), \overline{\mathcal{G}}(\mathcal{S})\right) = \left((\underline{b}, \underline{d}), (\overline{b}, \overline{d})\right)$. *Then*

$$\begin{aligned}&q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1) \oplus \mathcal{G}(\mathcal{S}), \mathcal{P}(\mathcal{S}_2) \oplus \mathcal{G}(\mathcal{S}), ..., \mathcal{P}(\mathcal{S}_n) \oplus \mathcal{G}(\mathcal{S})\right) = \\ &q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_n)\right) \oplus \mathcal{G}(\mathcal{S}).\end{aligned}$$

(5) Homogeneity: *For any real number* $\gamma > 0$;

$$q - ROHFRWA\left(\gamma\mathcal{P}(\mathcal{S}_1), \gamma\mathcal{P}(\mathcal{S}_2), ..., \gamma\mathcal{P}(\mathcal{S}_n)\right) = \gamma \cdot q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ..., \mathcal{P}(\mathcal{S}_n)\right).$$

(6) Commutativity: *Suppose* $\mathcal{P}'(\mathcal{S}_i) = \left(\underline{\mathcal{P}'}(\mathcal{S}_i), \overline{\mathcal{P}'}(\mathcal{S}_i)\right)$ *and* $\mathcal{P}(\mathcal{S}_i) = (\underline{\mathcal{P}}(\mathcal{S}_i), \overline{\mathcal{P}}(\mathcal{S}_i))$, $(i = 1, 2, 3, ..., n)$ *is a collection of q-ROHFRVs. Then*

$$q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ..., \mathcal{P}(\mathcal{S}_n)\right) = q - ROHFRWA\left(\mathcal{P}'(\mathcal{S}_1), \mathcal{P}'(\mathcal{S}_2), ..., \mathcal{P}'(\mathcal{S}_n)\right).$$

Proof. **(1) Idempotency:** As $\mathcal{P}(\mathcal{S}_i) = \mathcal{G}(\mathcal{S})$ (for all $i = 1, 2, 3, ..., n$) where $\mathcal{G}(\mathcal{S}_i) = \left(\underline{\mathcal{G}}(\mathcal{S}), \overline{\mathcal{G}}(\mathcal{S})\right) = \left((\underline{b_i}, \underline{d}_i), (\overline{b_i}, \overline{d}_i)\right)$

$$\begin{aligned}&q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ..., \mathcal{P}(\mathcal{S}_n)\right) \\ = &\left(\sum_{i=1}^{n} w_i \underline{\mathcal{P}}(\mathcal{S}_i), \sum_{i=1}^{n} w_i \overline{\mathcal{P}}(\mathcal{S}_i)\right) \\ = &\left[\begin{array}{cc} \bigcup_{\underline{\mu_i} \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\right)}, & \bigcup_{\underline{\nu_i} \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n}\left(\underline{\nu_i}\right)^{w_i} \\ \bigcup_{\overline{\mu_i} \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, & \bigcup_{\overline{\nu_i} \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n}\left(\overline{\nu_i}\right)^{w_i} \end{array}\right]\end{aligned}$$

for all i, $\mathcal{P}(\mathcal{S}_i) = \mathcal{G}(\mathcal{S}) = \left(\underline{\mathcal{G}}(\mathcal{S}), \overline{\mathcal{G}}(\mathcal{S})\right) = \left((\underline{b_i}, \underline{d_i}), (\overline{d}_i, \overline{e_i})\right)$. Therefore,

$$= \begin{bmatrix} \bigcup_{\underline{b_i} \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n} \left(1 - \left(\underline{b_i}\right)^q\right)^{w_i}\right)}, & \bigcup_{\underline{d_i} \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n} \left(\underline{d_i}\right)^{w_i} \\ \bigcup_{\overline{b_i} \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n} \left(1 - \left(\overline{b_i}\right)^q\right)^{w_i}\right)}, & \bigcup_{\overline{d_i} \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n} \left(\overline{d_i}\right)^{w_i} \end{bmatrix}$$

$$= \left[\left(1 - \left(1 - \underline{b_i}\right), \underline{b_i}\right), \left(1 - \left(1 - \overline{d}_i\right), \overline{b}_i\right)\right] = \left(\underline{\mathcal{G}}(\mathcal{S}), \overline{\mathcal{G}}(\mathcal{S})\right) = \mathcal{G}(\mathcal{S}).$$

Hence q-ROHFRWA$(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_n)) = \mathcal{G}(\mathcal{S})$.

(2) Boundedness: As

$$\left(\underline{\mathcal{P}}(\mathcal{S})\right)^- = \left[\left(\min_i\{\underline{\mu_i}\}, \max_i\left\{\underline{\nu_i}\right\}\right), \left(\min_i\{\overline{\mu_i}\}, \max_i\{\overline{\nu}_i\}\right)\right]$$
$$\left(\underline{\mathcal{P}}(\mathcal{S})\right)^+ = \left[\left(\max_i\{\underline{\mu_i}\}, \min_i\left\{\underline{\nu_i}\right\}\right), \left(\max_i\{\overline{\mu_i}\}, \min_i\{\overline{\nu}_i\}\right)\right]$$

and $\mathcal{P}(\mathcal{S}_i) = \left[\left(\underline{\eth_i}, \underline{\mathcal{B}_i}\right), \left(\overline{\eth}_i, \overline{\mathcal{B}}_i\right)\right]$. To prove that

$$(\mathcal{P}(\mathcal{S}))^- \leq q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ..., \mathcal{P}(\mathcal{S}_n)\right) \leq (\mathcal{P}(\mathcal{S}))^+ .$$

Since for each $i = 1, 2, 3, ..., n$, it follows that

$$\begin{aligned} \min_i\{\underline{\mu_i}\} &\leq \{\underline{\mu_i}\} \leq \max_i\{\underline{\mu_i}\} \Longleftrightarrow 1 - \max_i\{\underline{\mu_i}\} \leq 1 - \{\underline{\mu_i}\} \leq 1 - \{\underline{\mu_i}\} \\ &\Longleftrightarrow \prod_{i=1}^{n}\left(1 - \max_i\{\underline{\mu_i}\}\right)^{w_i} \leq \prod_{i=1}^{n}\left(1 - \{\underline{\mu_i}\}\right)^{w_i} \leq \prod_{i=1}^{n}\left(1 - \min_i\{\underline{\mu_i}\}\right)^{w_i} \\ &\Longleftrightarrow \left(1 - \max_i\{\underline{\mu_i}\}\right) \leq \prod_{i=1}^{n}\left(1 - \{\underline{\mu_i}\}\right)^{w_i} \leq \left(1 - \min_i\{\underline{\mu_i}\}\right) \\ &\Longleftrightarrow 1 - \left(1 - \min_i\{\underline{\mu_i}\}\right) \leq 1 - \prod_{i=1}^{n}\left(1 - \{\underline{\mu_i}\}\right)^{w_i} \leq 1 - \left(1 - \max_i\{\underline{\mu_i}\}\right) \end{aligned}$$

Hence

$$\min_i\{\underline{\mu_i}\} \leq 1 - \prod_{i=1}^{n}\left(1 - \{\underline{\mu_i}\}\right)^{w_i} \leq \max_i\{\underline{\mu_i}\} \tag{3.1}$$

Next for each $i = 1, 2, 3, ..., n$, we have

$$\begin{aligned} \min_i\left\{\underline{\nu_i}\right\} &\leq \left\{\underline{\nu_i}\right\} \leq \max_i\left\{\underline{\nu_i}\right\} \Longleftrightarrow \prod_{i=1}^{n}\left(\min_i\left\{\underline{\nu_i}\right\}\right)^{w_i} \\ &\leq \prod_{i=1}^{n}\left(\underline{\nu_i}\right)^{w_i} \leq \prod_{i=1}^{n}\left(\max_i\left\{\underline{\nu_i}\right\}\right)^{w_i}. \end{aligned}$$

This imples that

$$\min_i\left\{\underline{\nu_i}\right\} \leq \prod_{i=1}^{n}\left\{\underline{\nu_i}\right\}^{w_i} \leq \max_i\left\{\underline{\nu_i}\right\}. \tag{3.2}$$

Likewise, we can present that

$$\min_i \{\overline{\mu_i}\} \le \prod_{i=1}^{n} \{\overline{\mu_i}\}^{w_i} \le \max_i \{\overline{\mu_i}\} \tag{3.3}$$

and

$$\min_i \{\overline{\nu}_i\} \le \prod_{i=1}^{n} \{\overline{\nu}_i\}^{w_i} \le \max_i \{\overline{\nu}_i\}. \tag{3.4}$$

So from Equations (3.1), (3.2), (3.3) and (3.4) we have

$$\left(\underline{\mathcal{P}}(\mathcal{S})\right)^{-} = \left[\left(\min_i\{\underline{\mu_i}\}, \max_i\left\{\underline{\nu_i}\right\}\right), \left(\min_i\{\overline{\mu_i}\}, \max_i\{\overline{\nu}_i\}\right)\right].$$

(3) Monotonicity: Since $\mathcal{G}(\mathcal{S}) = \left(\underline{\mathcal{G}}(\mathcal{S}_i), \overline{\mathcal{G}}(\mathcal{S}_i)\right) = \left(\left(\underline{b}, \underline{d}\right), \left(\overline{b}, \overline{d}\right)\right)$ and $\mathcal{P}(\mathcal{S}_i) = \left(\underline{\mathcal{P}}(\mathcal{S}_i), \overline{\mathcal{P}}(\mathcal{S}_i)\right)$ to show that $\underline{\mathcal{G}}(\mathcal{S}_i) \le \underline{\mathcal{P}}(\mathcal{S}_i)$ and $\overline{\mathcal{G}}(\mathcal{S}_i) \le \overline{\mathcal{P}}(\mathcal{S}_i)$ (for $i = 1, 2, 3, ..., n$), so

$$\begin{aligned} \underline{b_i} &\le \underline{\mu_i} \Rightarrow 1 - \underline{b_i} \le 1 - \underline{\mu_i} \Rightarrow \prod_{i=1}^{n}\left(1 - \underline{\mu_i}\right)^{w_i} \le \prod_{i=1}^{n}\left(1 - \underline{b_i}\right)^{w_i} \\ &\Rightarrow 1 - \prod_{i=1}^{n}\left(1 - \underline{b_i}\right)^{w_i} \le 1 - \prod_{i=1}^{n}\left(1 - \underline{\mu_i}\right)^{w_i} \end{aligned} \tag{3.5}$$

next

$$\underline{d}_i \ge \underline{\nu_i} \Rightarrow \prod_{i=1}^{n} \underline{d}_i^{w_i} \ge \prod_{i=1}^{n} \underline{\nu_i}^{w_i}. \tag{3.6}$$

Likewise, we can show that

$$1 - \prod_{i=1}^{n}\left(1 - \overline{b}_i\right)^{w_i} \le 1 - \prod_{i=1}^{n}(1 - \overline{\mu_i})^{w_i} \tag{3.7}$$

$$\prod_{i=1}^{n}\left(\overline{b}_{ij}\right)^{w_i} \ge \prod_{i=1}^{n}\left(\overline{\nu}_{ij}\right)^{w_i} \tag{3.8}$$

Hence from Equations (3.5), (3.6), (3.7) and (3.8), we get $\underline{\mathcal{G}}(\mathcal{S}_i) \le \underline{\mathcal{P}}(\mathcal{S}_i)$ and $\overline{\mathcal{G}}(\mathcal{S}_i) \le \overline{\mathcal{P}}(\mathcal{S}_i)$. Therefore,

$$q - ROHFRWA\left(\mathcal{G}(\mathcal{S}_1), \mathcal{G}(\mathcal{S}_2), ..., \mathcal{G}(\mathcal{S}_n)\right) \le q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_n)\right).$$

(4) Shiftinvariance: As $\mathcal{G}(\mathcal{S}) = \left(\underline{\mathcal{G}}(\mathcal{S}), \overline{\mathcal{G}}(\mathcal{S})\right) = \left((\underline{b_i}, \underline{d_i}), (\overline{b}_i, \overline{d}_i)\right)$ is a q-ROHFRV and $\mathcal{P}(\mathcal{S}_i) = \left(\underline{\mathcal{P}}(\mathcal{S}_i), \overline{\mathcal{P}}(\mathcal{S}_i)\right) = \left[\left(\underline{ð_i}, \underline{\mathcal{B}_i}\right), \left(\overline{ð}_i, \overline{\mathcal{B}}_i\right)\right]$ is the collection of q-ROHFRVs, so

$$\mathcal{P}(\mathcal{S}_1) \oplus \mathcal{G}(\mathcal{S}) = \left[\underline{\mathcal{P}}(\mathcal{S}_1) \oplus \underline{\mathcal{G}}(\mathcal{S}), \overline{\mathcal{P}}(\mathcal{S}_i) \oplus \overline{\mathcal{G}}(\mathcal{S})\right].$$

As

$$\left(\left(1 - \left(1 - \underline{\mu_i}\right)\left(1 - \underline{d_i}\right), \underline{\nu_i d_i}\right), \left(1 - (1 - \overline{\mu_i})\left(1 - \overline{d_i}\right), \overline{\nu}_i\overline{d_i}\right)\right).$$

Thus, q-ROHFRV $\mathcal{G}(\mathcal{S}) = \left(\underline{\mathcal{G}}(\mathcal{S}), \overline{\mathcal{G}}(\mathcal{S})\right) = \left((\underline{b_i}, \underline{d_i}), (\overline{b_i}, \overline{d_i})\right)$. It follows that

$$
\begin{aligned}
& q-ROHFRWA\left(\mathcal{P}(\mathcal{S}_1) \oplus \mathcal{G}(\mathcal{S}), \mathcal{P}(\mathcal{S}_2) \oplus \mathcal{G}(\mathcal{S}), ..., \mathcal{P}(\mathcal{S}_n) \oplus \mathcal{G}(\mathcal{S})\right) \\
= & \left[\sum_{i=1}^{n} w_i \underline{\mathcal{P}}(\mathcal{S}_i) \oplus \mathcal{G}(\mathcal{S}), \sum_{i=1}^{n} w_i \left(\underline{\mathcal{P}}(\mathcal{S}_i) \oplus \mathcal{G}(\mathcal{S})\right)\right] \\
= & \left[\begin{array}{c}
\left\{\begin{array}{c}
\bigcup_{\underline{\mu_i} \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\left(1 - \underline{b_i}\right)^{w_i}\right)}, \\
\bigcup_{\underline{\nu_i} \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n}\left(\underline{\nu_i}\right)^{w_i} \underline{d}_i^{w_i}
\end{array}\right\}, \\
\left\{\begin{array}{c}
\bigcup_{\overline{\mu_i} \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)\left(1 - \overline{b_i}\right)^{w_i}}, \\
\bigcup_{\overline{\nu_i} \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \underline{d}_i \prod_{i=1}^{n}\left(\overline{\nu}_i\right)^{w_i}
\end{array}\right\}
\end{array}\right]
\end{aligned}
$$

$$
= \left[\begin{array}{c}
\left\{\begin{array}{c}
\bigcup_{\underline{\mu_i} \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \left(1 - \underline{b}\right)\prod_{i=1}^{n}\left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\right)}, \\
\bigcup_{\underline{\nu_i} \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \underline{d}\prod_{i=1}^{n}\left(\underline{\nu_i}\right)^{w_i}
\end{array}\right\}, \\
\left\{\begin{array}{c}
\bigcup_{\overline{\mu_i} \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \overline{b}\right)\left(1 - \prod_{i=1}^{n}\left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, \\
\bigcup_{\overline{\nu_i} \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \overline{d}\prod_{i=1}^{n}\left(\overline{\nu}_i\right)^{w_i}
\end{array}\right\}
\end{array}\right]
$$

$$
= \left[\begin{array}{c}
\left\{\left(\begin{array}{c}
\bigcup_{\underline{\mu_i} \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\right)}, \\
\bigcup_{\underline{\nu_i} \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n}\left(\underline{\nu_i}\right)^{w_i}
\end{array}\right) \oplus \left(\underline{b_i}, \underline{d}_i\right)\right\}, \\
\left\{\left(\begin{array}{c}
\bigcup_{\overline{\mu_i} \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, \\
\bigcup_{\overline{\nu_i} \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n}\left(\overline{\nu}_i\right)^{w_i}
\end{array}\right) \oplus \left(\overline{b_i}, \overline{d_i}\right)\right\}
\end{array}\right]
$$

$$
= \left[\begin{array}{c}
\left(\begin{array}{c}
\bigcup_{\underline{\mu_i} \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\right)}, \\
\bigcup_{\underline{\nu_i} \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n}\left(\underline{\nu_i}\right)^{w_i}
\end{array}\right), \\
\left(\begin{array}{c}
\bigcup_{\overline{\mu_i} \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, \\
\bigcup_{\overline{\nu_i} \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n}\left(\overline{\nu}_i\right)^{w_i}
\end{array}\right)
\end{array}\right] \oplus \left[\left(\underline{b_i}, \underline{d_i}\right), \left(\overline{b_i}, \overline{d_i}\right)\right]
$$

$$= \quad q-ROHFRWA\left(\mathcal{P}(\mathcal{S}_1),\mathcal{P}(\mathcal{S}_2),...,\mathcal{P}(\mathcal{S}_n)\right)\oplus\mathcal{G}(\mathcal{S}).$$

(5) Homogeneity: For real number $\gamma > 0$ and $\mathcal{P}(\mathcal{S}_i) = \left(\underline{\mathcal{P}}(\mathcal{S}_i), \overline{\mathcal{P}}(\mathcal{S}_i)\right)$ be a q-ROHFRVs. Consider

$$\begin{aligned}
\gamma\mathcal{P}(\mathcal{S}_i) &= \left(\gamma\underline{\mathcal{P}}(\mathcal{S}_i), \gamma\overline{\mathcal{P}}(\mathcal{S}_i)\right) \\
&= \left[\begin{array}{c}
\left\{\bigcup_{\underline{\mu}_i\in\eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\left(\sqrt[q]{\left(1-\left(1-\underline{\mu}_i{}^q\right)^{\gamma}\right)}\right), \bigcup_{\underline{\nu}_i\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\left(\underline{\nu}_i{}^{\gamma}\right)\right\}, \\
\left\{\bigcup_{\overline{\mu}_i\in\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}}\left(\sqrt[q]{\left(1-(1-\overline{\mu}_i{}^q)^{\gamma}\right)}\right), \bigcup_{\overline{\nu}_i\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\left(\overline{\nu}_i^{\gamma}\right)\right\}
\end{array}\right]
\end{aligned}$$

Now

$$\begin{aligned}
& q-ROHFRWA\left(\gamma\mathcal{P}(\mathcal{S}_1),\gamma\mathcal{P}(\mathcal{S}_2),...,\gamma\mathcal{P}(\mathcal{S}_n)\right) \\
&= \left[\begin{array}{c}
\left(\bigcup_{\underline{\mu}_i\in\eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\mu}_i\right)^q\right)^{\gamma}\right)}, \bigcup_{\underline{\nu}_i\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\prod_{i=1}^{n}\left(\underline{\nu}_i\right)^{\gamma}\right), \\
\left(\bigcup_{\overline{\mu}_i\in\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}(1-(\overline{\mu}_i)^q)^{\gamma}\right)}, \bigcup_{\overline{\nu}_i\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\prod_{i=1}^{n}(\overline{\nu}_i)^{\gamma}\right)
\end{array}\right] \\
&= \gamma q-ROHFRWA\left(\mathcal{P}(\mathcal{S}_1),\mathcal{P}(\mathcal{S}_2),...,\mathcal{P}(\mathcal{S}_n)\right).
\end{aligned}$$

(6) Commutativity: Suppose

$$\begin{aligned}
& q-ROHFRWA\left(\mathcal{P}(\mathcal{S}_1),\mathcal{P}(\mathcal{S}_2),...,\mathcal{P}(\mathcal{S}_n)\right) \\
&= \left[\sum_{i=1}^{n}\gamma_i\underline{\mathcal{P}}(\mathcal{S}_i), \sum_{i=1}^{n}\gamma_i\overline{\mathcal{P}}(\mathcal{S}_i)\right] \\
&= \left[\begin{array}{c}
\left(\bigcup_{\underline{\mu}_i\in\eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\mu}_i\right)^q\right)^{\gamma_i}\right)}, \bigcup_{\underline{\nu}_i\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\prod_{i=1}^{n}\left(\underline{\nu}_i\right)^{\gamma_i}\right), \\
\left(\bigcup_{\overline{\mu}_i\in\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}(1-(\overline{\mu}_i)^q)^{\gamma_i}\right)}, \bigcup_{\overline{\nu}_i\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\prod_{i=1}^{n}(\overline{\nu}_i)^{\gamma_i}\right)
\end{array}\right],
\end{aligned}$$

Let $\left(\mathcal{P}'(\mathcal{S}_1),\mathcal{P}'(\mathcal{S}_2),...,\mathcal{P}'(\mathcal{S}_n)\right)$ be a permutation of $(\mathcal{P}(\mathcal{S}_1),\mathcal{P}(\mathcal{S}_2),...,\mathcal{P}(\mathcal{S}_n))$. Then we have $\mathcal{P}(\mathcal{S}_i) = \mathcal{P}'(\mathcal{S}_i)(i = 1,2,3,...,n)$

$$\begin{aligned}
&= \left[\begin{array}{c}
\left(\bigcup_{\underline{\mu}_i\in\eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\underline{\mu}'_i\right)^q\right)^{\gamma_i}\right)}, \bigcup_{\underline{\nu}_i\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\prod_{i=1}^{n}\left(\underline{\nu}'_i\right)^{\gamma_i}\right), \\
\left(\bigcup_{\overline{\mu}_i\in\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}}\sqrt[q]{\left(1-\prod_{i=1}^{n}\left(1-\left(\overline{\mu}'_i\right)^q\right)^{\gamma_i}\right)}, \bigcup_{\overline{\nu}_i\in\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}}\prod_{i=1}^{n}\left(\overline{\nu}'_i\right)^{\gamma_i}\right)
\end{array}\right], \\
&= \left[\sum_{i=1}^{n}\gamma_i\underline{\mathcal{P}}'(\mathcal{S}_i), \sum_{i=1}^{n}\gamma_i\overline{\mathcal{P}'}(\mathcal{S}_i)\right] \\
&= q-ROHFRWA\left(\mathcal{P}'(\mathcal{S}_1),\mathcal{P}'(\mathcal{S}_2),...,\mathcal{P}'(\mathcal{S}_n)\right).
\end{aligned}$$

Proved. □

4. Improved q-ROHFR-VIKOR methodology

Here, we developed an algorithm for addressing uncertainty in multi-attribute group decision making (MAGDM) under q-rung orthopair hesitant fuzzy rough information. Consider a DM problem with $\{\flat_1, \flat_2, ..., \flat_n\}$ a set of n alternatives and a set of attributes $\{c_1, c_2, ..., c_n\}$ with $(w_1, w_2, ..., w_n)^T$ the weights, that is, $w_i \in [0, 1]$, $\sum_{i=1}^{n} w_i = 1$. To test the reliability of kth alternative $\flat_i$ under the the attribute c_i, let be a set of decision makers (DMs) $\left\{\mathring{D}_1, \mathring{D}_2, ..., \mathring{D}_{\hat{j}}\right\}$ and $(\varrho_1, \varrho_2, ..., \varrho_i)^T$ be DMs weights such that $\varrho_i \in [0, 1]$, $\sum_{i=1}^{n} \varrho_i = 1$. The expert evaluation matrix is described as:

$$
\begin{aligned}
M &= \left[\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{ij})\right]_{m\times n} \\
&= \begin{bmatrix}
\left(\overline{\mathcal{P}}(\mathcal{S}_{11}), \underline{\mathcal{P}}(\mathcal{S}_{11})\right) & \left(\overline{\mathcal{P}}(\mathcal{S}_{12}), \underline{\mathcal{P}}(\mathcal{S}_{12})\right) & \cdots & \left(\overline{\mathcal{P}}(\mathcal{S}_{1j}), \underline{\mathcal{P}}(\mathcal{S}_{1j})\right) \\
\left(\overline{\mathcal{P}}(\mathcal{S}_{21}), \underline{\mathcal{P}}(\mathcal{S}_{21})\right) & \left(\overline{\mathcal{P}}(\mathcal{S}_{22}), \underline{\mathcal{P}}(\mathcal{S}_{22})\right) & \cdots & \left(\overline{\mathcal{P}}(\mathcal{S}_{2j}), \underline{\mathcal{P}}(\mathcal{S}_{2j})\right) \\
\left(\overline{\mathcal{P}}(\mathcal{S}_{31}), \underline{\mathcal{P}}(\mathcal{S}_{31})\right) & \left(\overline{\mathcal{P}}(\mathcal{S}_{32}), \underline{\mathcal{P}}(\mathcal{S}_{32})\right) & \cdots & \left(\overline{\mathcal{P}}(\mathcal{S}_{3j}), \underline{\mathcal{P}}(\mathcal{S}_{3j})\right) \\
\vdots & \vdots & \ddots & \vdots \\
\left(\overline{\mathcal{P}}(\mathcal{S}_{i1}), \underline{\mathcal{P}}(\mathcal{S}_{i1})\right) & \left(\overline{\mathcal{P}}(\mathcal{S}_{i2}), \underline{\mathcal{P}}(\mathcal{S}_{i2})\right) & \cdots & \left(\overline{\mathcal{P}}(\mathcal{S}_{ij}), \underline{\mathcal{P}}(\mathcal{S}_{ij})\right)
\end{bmatrix},
\end{aligned}
$$

where $\overline{\mathcal{P}}(\mathcal{S}_{ij}) = \left\{\left\langle \flat, \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\flat), \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\flat)\right\rangle | \flat \in \aleph\right\}$ and $\underline{\mathcal{P}}(\mathcal{S}) = \left\{\left\langle \flat, \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\flat), \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\flat)\right\rangle | \flat \in \aleph\right\}$ such that $0 \leq \left(\max(\eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\flat))\right)^q + \left(\min(\mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}(\flat))\right)^q \leq 1$ and $0 \leq \left(\min(\eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\flat)\right)^q + \left(\max(\mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}(\flat))\right)^q \leq 1$ are the q-ROHF rough values. The main steps for MAGDM are as follows:

Step-1 Construct the experts evaluation matrices as:

$$
(E)^{\hat{j}} = \begin{bmatrix}
\left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{11}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{11})\right) & \left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{12}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{12})\right) & \cdots & \left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{1j}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{1j})\right) \\
\left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{21}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{21})\right) & \left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{22}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{22})\right) & \cdots & \left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{2j}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{2j})\right) \\
\left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{31}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{31})\right) & \left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{32}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{32})\right) & \cdots & \left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{3j}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{3j})\right) \\
\vdots & \vdots & \ddots & \vdots \\
\left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{i1}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{i1})\right) & \left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{i2}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{i2})\right) & \cdots & \left(\overline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{ij}), \underline{\mathcal{P}}(\mathcal{S}^{\hat{j}}_{ij})\right)
\end{bmatrix}
$$

where $\hat{j}$ represents the number of expert.

Step-2 Evaluate the normalized experts matices $(N)^{\hat{j}}$, as

$$
(N)^{\hat{j}} = \begin{cases}
\mathcal{P}(\mathcal{S}_{ij}) = \left(\underline{\mathcal{P}}\left(\mathcal{S}_{ij}\right), \overline{\mathcal{P}}\left(\mathcal{S}_{ij}\right)\right) = \left(\left(\underline{\mu_{ij}}, \underline{\nu_{ij}}\right), \left(\overline{\mu_{ij}}, \overline{\nu_{ij}}\right)\right) & \text{if For benefit} \\
\left(\mathcal{P}(\mathcal{S}_{ij})\right)^c = \left(\left(\underline{\mathcal{P}}\left(\mathcal{S}_{ij}\right)\right)^c, \left(\overline{\mathcal{P}}\left(\mathcal{S}_{ij}\right)\right)^c\right) = \left(\left(\underline{\nu_{ij}}, \underline{\mu_{ij}}\right), \left(\overline{\nu_{ij}}, \overline{\mu_{ij}}\right)\right) & \text{if For cost}
\end{cases}
$$

Step-3 Compute the collected q-rung orthopair hesitant fuzzy rough information of decision makers using the q-ROHFRWA aggregation operators.

$$
\begin{aligned}
&q - ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_n)\right) \\
&= \left(\sum_{i=1}^{n} w_i \underline{\mathcal{P}}(\mathcal{S}_i), \sum_{i=1}^{n} w_i \overline{\mathcal{P}}(\mathcal{S}_i)\right)
\end{aligned}
$$

$$= \begin{bmatrix} \bigcup_{\underline{\mu_i} \in \eth_{h_{\underline{\mathcal{P}}(S)}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n} \left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\right)}, & \bigcup_{\underline{\nu_i} \in \mathcal{B}_{h_{\underline{\mathcal{P}}(S)}}} \prod_{i=1}^{n} \left(\underline{\nu_i}\right)^{w_i} \\ \bigcup_{\overline{\mu_i} \in \eth_{h_{\overline{\mathcal{P}}(S)}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n} \left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, & \bigcup_{\overline{\nu_i} \in \mathcal{B}_{h_{\overline{\mathcal{P}}(S)}}} \prod_{i=1}^{n} (\overline{\nu_i})^{w_i} \end{bmatrix}$$

Step-4 Determine the q-ROHFR positive ideal solutions I^+ and the q-ROHFR negative ideal solutions I^-as follows:

$$\begin{aligned} I^+ &= (\varpi_1^+, \varpi_2^+, \varpi_3^+, ..., \varpi_n^+) = \left(\max_i \varpi_{i1}, \max_i \varpi_{i2}, \max_i \varpi_{i3}, ..., \max_i \varpi_{in}.\right), \\ I^+ &= (\varpi_1^-, \varpi_2^-, \varpi_3^-, ..., \varpi_n^-) = \left(\min_i \varpi_{i1}, \min_i \varpi_{i2}, \min_i \varpi_{i3}, ..., \min_i \varpi_{in}.\right) \end{aligned}$$

Step-5 Our next goal is to calculate the q-ROHFR group utility measure $S_i (i = 1, 2, 3, 3, ..., n)$ and the regret measure $R_i(i = 1, 2, 3, 3, ..., n)$ of all alternatives $L = (A_1, A_2, A_3, ..., A_n)$ by using the following formulas.

$$\begin{aligned} S_i &= \sum_{j=1}^{n} \frac{w_j d\left(\varpi_{ij}, \varpi_j^+\right)}{d\left(\varpi_j^+, \varpi_j^-\right)}, \ i = 1, 2, 3, 3, ...m. \\ R_i &= \max \frac{w_j d\left(\varpi_{ij}, \varpi_j^+\right)}{d\left(\varpi_j^+, \varpi_j^-\right)}, \ i = 1, 2, 3, 3, ...m \end{aligned}$$

Step-6 Now, we identify the S and R maximum and minimum values as follows:

$$S^{\#} = \min_i S_i, \ S^* = \max_i S_i, \ R^{\#} = \min_i R_i, \ R^* = \max_i R_i, \ i = 1, 2, 3, ...n,$$

Finally, to evaluate the ranking measure Q_i for the alternative $L = (A_1, A_2, A_3, ..., A_n)$, we combine the feature of the group utility S_i and individual regret R_i as follows:

$$Q_i = \tau \frac{S_i - S^{\#}}{S^* - S^{\#}} + (1 - \tau) \frac{R_i - R^{\#}}{R^* - R^{\#}}, R_i, \ i = 1, 2, 3, 3, ...n,$$

where τ is the strategy weight of most of the parameters (maximum group utility) and plays a crucial role in the evaluation of the compromise solution. The value is taken from the uni interval [0, 1], but 0.5 is generally taken.

Step-7 Further, with respect to group utility measure S_i, individual regret measure R_i and ranking measure Q_i, the alternatives are ranked in descending order. Here, we get three ranking lists that are useful in determining the compromise solution.

5. Numerical application

In this section, we propose the numerical application related to agriculture robotics to improve the ability and capability of agriculture and presented to verifying the reliability, superiority and scientific correctness of the established aggregation operators.

5.1. Case study (robotic agri-farming)

Farming is the practice of cultivating crops and raising livestock. It entails both raising livestock and cultivating crops for agricultural goods. Farming has been practised for thousands of years. Agriculture is a way of life rather than a profession. Without this, we would be unable to survive in this environment, and it was also essential for the birth of human civilization. Agriculture is the most advantageous, helpful, and dignified activity available to mankind. Human beings are born farmers who take great pleasure in nurturing their farm, whether in their home gardens or out in the fields. Inside houses, plants are cultivated in little mud pots; however, in the fields, individuals may grow any kind of plant or crop. Our nation is now being dismantled under the pretence of economic advancement, industrialisation, and the development of housing projects. and we will be required to pay much more for our daily nutritional requirements. With increasing population, people need more food to survive word population rate grows rapidly. Farmers have to use agriculture robots for improving the yield of crops. Robotics in agriculture is an example of innovation that transcends invention. Agriculture is a business, and it is anticipated to expand into a high-tech enterprise in the contemporary era. Agricultural capabilities of farmers are increasingly growing in accordance with technological advancements. Robotics and automation are now helping to improve yields of crops. Robots may be used for harvesting, weeding, trimming, sowing, spraying, sorting, and packing in agriculture. The agriculture robots are called agri-robots or agribots. An vital purpose for Agribots will be performed in agriculture in the future. In this area, we will concentrate on a specific application: the employment of robots in horticulture. Horticulture is the agricultural practice of cultivating material, food, decorative, and comforting plants. A Terra Sentia is a new generation robot which is the smallest robot having 12.5 inches height and 12.5 inches width with weight of 30 pounds. It looks like a lawnmower, with high-resolution cameras on each side, and it navigates a field by scanning it with laser pulses. There are several ways to visualize a field in terms of stem diameters, fruit-producing plants, and general health and size of the plants. Additionally, it may be employed in plant breeding research. The robot has been used in a variety of areas, including maize, wheat, strawberries, citrus, cotton, soybean, sorghum, and almond grapes. We will examine the agricultural productivity of robots. The attributes of robotic farming are described below:

(c_1) Automation of manual tasks: When farmers adopt automation, they increase their production by spending less time on routine tasks and more time on improvements.

(c_2) High-quality production and reduced production costs: There are certain farming factors which have an influence on the improvement of the quality of products for example, (soil, climate, ripening period, fertilizer etc). The level of maturity and dryness are important in the production of agriculture (barley, wheat, rice etc.). Agriculture has made a significant contribution to cost reduction via the use of robots. We must maintain certain uncontrollable factors that reduce production costs, such as weather conditions, seed purchases from various brands, and chemical consumption.

(c_3) Completing a challenging task: Using automation to complete the challenging activity in an easy and plain way, scientists, researchers, technologists, and farmers all agreed that the use of automation will simplify and streamline the process.

(c_4) Consistent role to complete a task/placement perfection and accuracy: In order to play a systematic role, the farm must be operated by artificial intelligence (automate the complete agricultural process) from sowing through harvesting. In the field, plant placement is extremely important. The accuracy will contribute to perfection. The automation of nursing operations includes grafting, propagation, and spacing.

The evaluation procedure for selection of robotic agri-farming: suppose an organization wants to assess the procedure of selecting a robotic agri-farming. They invite a panel of experts to analyze an appropriate robotic farming. Let $\{A_1, A_2, A_3, A_4\}$ be four alternatives for robotic farming, and the penal select ideal one. Let $\{c_1, c_2, c_3, c_4\}$ be the attributes of each alternative based on the influencing factors determined as follows: automation of manual tasks (c_1), high-quality production and reduced production costs (c_2), completing a challenging task (c_3) and consistent role to complete a task/placement perfection and accuracy (c_4) of robotic agri-farming. Because of the uncertainty, the DMs' selection information is presented as q-ROHFR information. The considered attribute weight vector is $w = (0.18, 0.25, 0.31, 0.26)^T$ and DM experts weight vector is $w = (0.23, 0.38, 0.39)^T$. The following computations are carried out in order to evaluate the MCDM problem utilizing the defined approach for evaluating alternatives:

Step-1 The information of three professional experts are evaluated in Table-2 to Table-4 using q-ROHFRVs.

Table-2(a): Expert-1 information

	c_1	c_2
A_1	$\begin{pmatrix} \{(0.10,0.20,0.50),(0.30,0.40)\}, \\ \{(0.30,0.80,0.90),(0.40,0.60)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.5,0.7,0.9),(0.5,0.6,0.8)\}, \\ \{(0.3,0.5,0.6),(0.7,0.9)\} \end{pmatrix}$
A_2	$\begin{pmatrix} \{(0.50,0.60,0.70),(0.70,0.90)\}, \\ \{(0.30,0.50,0.70),(0.60,0.70)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.2,0.4,0.5),(0.5)\}, \\ \{(0.6,0.7),(0.3,0.5,0.9)\} \end{pmatrix}$
A_3	$\begin{pmatrix} \{(0.40,0.50,0.60),(0.60,0.70,0.80)\}, \\ \{(0.70,0.80),(0.10,0.40,0.70)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.1),(0.5,0.6)\}, \\ \{(0.4,0.6,0.7),(0.5,0.7)\} \end{pmatrix}$
A_4	$\begin{pmatrix} \{(0.6,0.7,0.9),(0.3,0.4,0.6)\}, \\ \{(0.2,0.7),(0.7,0.8,0.9)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.3,0.4,0.5),(0.4,0.7,0.9)\}, \\ \{(0.1,0.2),(0.2,0.3)\} \end{pmatrix}$

Table-2(b): Expert-1 information

	c_3	c_4
A_1	$\begin{pmatrix} \{(0.2,0.3,0.4),(0.3,0.4,0.7)\}, \\ \{(0.1,0.5),(0.3,0.5)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.5,0.6),(0.4,0.5,0.7)\}, \\ \{(0.6,0.8,0.9),(0.6,0.7,0.9)\} \end{pmatrix}$
A_2	$\begin{pmatrix} \{(0.4,0.5,0.8),(0.4,0.5,0.7)\}, \\ \{(0.2,0.5),(0.4,0.5)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.4,0.6,0.8),(0.3,0.5)\}, \\ \{(0.7),(0.1,0.3,0.4)\} \end{pmatrix}$
A_3	$\begin{pmatrix} \{(0.3,0.6,0.7),(0.5,0.7,0.8)\}, \\ \{(0.5,0.9),(0.5,0.8)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.3,0.6),(0.5,0.6,0.8)\}, \\ \{(0.1,0.3,0.7),(0.3,0.4)\} \end{pmatrix}$
A_4	$\begin{pmatrix} \{(0.3,0.4,0.5),(0.7,0.8,0.9)\}, \\ \{(0.6,0.7),(0.4,0.7)\} \end{pmatrix}$	$\begin{pmatrix} \{(0.2,0.3,0.4),(0.5,0.6,0.9)\}, \\ \{(0.3,0.4),(0.7,0.8)\} \end{pmatrix}$

Table-3(a): Expert-2 information

	c_1	c_2
A_1	$\begin{pmatrix} (\{0.2,0.3,0.4\},\{0.2,0.5\}), \\ (\{0.4,0.6\},\{0.2,0.5\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.4,0.5,0.6\},\{0.3,0.7,0.8\}), \\ (\{0.2,0.7,0.8\},\{0.2,0.8,0.9\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.1,0.3,0.4\},\{0.5,0.8\}), \\ (\{0.5,0.6\},\{0.8,0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.3,0.4,0.6\},\{0.7,0.8\}), \\ (\{0.1,0.5\},\{0.3,0.7,0.8\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.6,0.7,0.8\},\{0.3,0.4\}), \\ (\{0.3,0.8,0.9\},\{0.2,0.5,0.7\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.3,0.4,0.5\},\{0.4,0.7,0.9\}), \\ (\{0.4,0.6,0.7\},\{0.7,0.8,0.9\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.1,0.2,0.3\},\{0.5,0.7\}), \\ (\{0.2,0.4\},\{0.7,0.8\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.5,0.7,0.8\},\{0.3,0.5,0.7\}), \\ (\{0.3,0.4,0.6\},\{0.4,0.5,0.7\}) \end{pmatrix}$

Table-3(b): Expert-2 information

	c_3	c_4
A_1	$\begin{pmatrix} (\{0.2,0.4\},\{0.3,0.5\}), \\ (\{0.4,0.7,0.8\},\{0.2,0.6\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.1,0.2\},\{0.4,0.6\}), \\ (\{0.2,0.5\},\{0.7,0.9\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.3,0.5,0.7\},\{0.2,0.6\}), \\ (\{0.6,0.7,0.8\},\{0.2,0.8\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2,0.3\},\{0.4,0.6,0.7\}), \\ (\{0.1,0.3,0.5\},\{0.2,0.3,0.5\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.5,0.6,0.7\},\{0.3,0.5\}), \\ (\{0.7,0.8,0.9\},\{0.2,0.3,0.5\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2,0.7,0.8\},\{0.2,0.7\}), \\ (\{0.1,0.2\},\{0.5,0.6,0.7\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.6,0.7,0.9\},\{0.2,0.5\}), \\ (\{0.6,0.9\},\{0.2,0.5\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.3,0.5\},\{0.4,0.6,0.7\}), \\ (\{0.2,0.3,0.6\},\{0.4,0.5,0.7\}) \end{pmatrix}$

Table-4(a): Expert-3 information

	c_1	c_2
A_1	$\begin{pmatrix} (\{0.4,0.7,0.9\},\{0.3,0.6,0.8\}), \\ (\{0.2,0.3,0.8\},\{0.7,0.8,0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.4,0.7,0.8\},\{0.7,0.8\}), \\ (\{0.3,0.5,0.6\},\{0.7,0.8\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.1,0.3,0.4\},\{0.5,0.6,0.9\}), \\ (\{0.2,0.3,0.7\},\{0.2,0.6,0.8\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2,0.3,0.7\},\{0.3,0.8,0.9\}), \\ (\{0.1,0.5,0.8\},\{0.2,0.7,0.8\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.2,0.3,0.5\},\{0.4,0.8,0.9\}), \\ (\{0.1,0.8,0.9\},\{0.4,0.7\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2,0.3,0.8\},\{0.2,0.8\}), \\ (\{0.5,0.8,0.9\},\{0.2,0.9\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.1,0.5,0.7\},\{0.5,0.8\}), \\ (\{0.3,0.5,0.7\},\{0.4,0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2,0.3\},\{0.5,0.6\}), \\ (\{0.3,0.8,0.9\},\{0.7,0.8,0.9\}) \end{pmatrix}$

Table-4(b): Expert-3 information

	c_3	c_4
A_1	$\begin{pmatrix} (\{0.2,0.3,0.8\},\{0.5,0.6,0.7\}), \\ (\{0.3,0.5,0.6\},\{0.2,0.8,0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2,0.3,0.7\},\{0.2,0.3,0.7\}), \\ (\{0.2,0.3,0.8\},\{0.5,0.7\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.1,0.3,0.6\},\{0.4,0.6,0.8\}), \\ (\{0.6,0.7,0.9\},\{0.3,0.8,0.9\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.1,0.2,0.3\},\{0.2,0.5\}), \\ (\{0.3,0.4,0.6\},\{0.1,0.2\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.1,0.2,0.3\},\{0.3,0.5,0.9\}), \\ (\{0.2,0.3,0.4\},\{0.2,0.4,0.6\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2,0.8,0.9\},\{0.1,0.2\}), \\ (\{0.2,0.4,0.5\},\{0.7,0.8\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.2,0.3,0.7\},\{0.8,0.9\}), \\ (\{0.2,0.3,0.8\},\{0.1,0.2,0.3\}) \end{pmatrix}$	$\begin{pmatrix} (\{0.2,0.3,0.8\},\{0.2,0.3\}), \\ (\{0.3,0.5,0.8\},\{0.4,0.5\}) \end{pmatrix}$

Step-2 All the experts information are benefit type. So in this case, we do not need to normalize the q-ROHFRVs.

Step-3 Cumulative the collective information of three professional expert using q-ROHFRWA aggregation operator is evaluated in Table-5;

Table-5(a): Collective aggregated q-ROHFR information

	c_1	c_2
A_1	$\left(\begin{pmatrix}\{0.2402, 0.5445, 0.7558\},\\ \{0.2572, 0.5100, 0.8403\}\end{pmatrix}, \begin{pmatrix}\{0.3239, 0.6173, 0.7607\},\\ \{0.2748, 0.6263, 0.9597\}\end{pmatrix}\right)$	$\left(\begin{pmatrix}\{0.4280, 0.6443, 0.6630\},\\ \{0.4695, 0.7117, 0.9500\}\end{pmatrix}, \begin{pmatrix}\{0.2706, 0.5993, 0.7281\},\\ \{0.3823, 0.7758, 0.9378\}\end{pmatrix}\right)$
A_2	$\left(\begin{pmatrix}\{0.3141, 0.4201, 0.3675\},\\ \{0.5214, 0.6935, 0.9211\}\end{pmatrix}, \begin{pmatrix}\{0.3880, 0.5004, 0.6121\},\\ \{0.4361, 0.8113, 0.9597\}\end{pmatrix}\right)$	$\left(\begin{pmatrix}\{0.2481, 0.3678, 0.6295\},\\ \{0.4656, 0.8421, 0.9597\}\end{pmatrix}, \begin{pmatrix}\{0.3807, 0.4606, 0.6249\},\\ \{0.2561, 0.6479, 0.8220\}\end{pmatrix}\right)$
A_3	$\left(\begin{pmatrix}\{0.4716, 0.5668, 0.6816\},\\ \{0.3936, 0.5962, 0.9211\}\end{pmatrix}, \begin{pmatrix}\{0.4670, 0.7515, 0.9194\},\\ \{0.2235, 0.5416, 0.8045\}\end{pmatrix}\right)$	$\left(\begin{pmatrix}\{0.2391, 0.3276, 0.6554\},\\ \{0.3213, 0.8005, 0.9608\}\end{pmatrix}, \begin{pmatrix}\{0.4453, 0.7036, 0.8086\},\\ \{0.3975, 0.8123, 0.9608\}\end{pmatrix}\right)$
A_4	$\left(\begin{pmatrix}\{0.3807, 0.5202, 0.6726\},\\ \{0.4446, 0.6484, 0.8891\}\end{pmatrix}, \begin{pmatrix}\{0.2491, 0.4204, 0.5326\},\\ \{0.5627, 0.8376, 0.9761\}\end{pmatrix}\right)$	$\left(\begin{pmatrix}\{0.3880, 0.5532, 0.6396\},\\ \{0.3912, 0.5800, 0.8523\}\end{pmatrix}, \begin{pmatrix}\{0.2762, 0.6417, 0.7634\},\\ \{0.4242, 0.5340, 0.8381\}\end{pmatrix}\right)$

Table-5(b): Collective aggregated q-ROHFR information

	c_3	c_4
A_1	$\left(\begin{pmatrix}\{0.2000, 0.3455, 0.6345\},\\ \{0.3661, 0.5100, 0.8016\}\end{pmatrix}, \begin{pmatrix}\{0.3283, 0.5993, 0.6738\},\\ \{0.2195, 0.6437, 0.9597\}\end{pmatrix}\right)$	$\left(\begin{pmatrix}\{0.3228, 0.4068, 0.5780\},\\ \{0.2329, 0.5359, 0.9212\}\end{pmatrix}, \begin{pmatrix}\{0.3921, 0.5873, 0.7607\},\\ \{0.5925, 0.7701, 0.9761\}\end{pmatrix}\right)$
A_2	$\left(\begin{pmatrix}\{0.2950, 0.4448, 0.6998\},\\ \{0.3074, 0.5754, 0.8445\}\end{pmatrix}, \begin{pmatrix}\{0.6105, 0.7212, 0.8155\},\\ \{0.2748, 0.7180, 0.9597\}\end{pmatrix}\right)$	$\left(\begin{pmatrix}\{0.2644, 0.4064, 0.5441\},\\ \{0.2857, 0.5359, 0.8732\}\end{pmatrix}, \begin{pmatrix}\{0.4674, 0.3288, 0.5137\},\\ \{0.1301, 0.2561, 0.6224\}\end{pmatrix}\right)$
A_3	$\left(\begin{pmatrix}\{0.3880, 0.5279, 0.7000\},\\ \{0.4946, 0.6794, 0.9500\}\end{pmatrix}, \begin{pmatrix}\{0.5603, 0.7618, 0.8142\},\\ \{0.1884, 0.3209, 0.4805\}\end{pmatrix}\right)$	$\left(\begin{pmatrix}\{0.2315, 0.7310, 0.8155\},\\ \{0.1884, 0.4145, 0.9500\}\end{pmatrix}, \begin{pmatrix}\{0.1552, 0.3256, 0.5170\},\\ \{0.5069, 0.6115, 0.8732\}\end{pmatrix}\right)$
A_4	$\left(\begin{pmatrix}\{0.4593, 0.5532, 0.7930\},\\ \{0.4581, 0.7006, 0.9761\}\end{pmatrix}, \begin{pmatrix}\{0.4064, 0.5657, 0.8142\},\\ \{0.1790, 0.3779, 0.6253\}\end{pmatrix}\right)$	$\left(\begin{pmatrix}\{0.2481, 0.4030, 0.6345\},\\ \{0.3213, 0.4579, 0.8523\}\end{pmatrix}, \begin{pmatrix}\{0.2706, 0.4212, 0.6774\},\\ \{0.4549, 0.5571, 0.8732\}\end{pmatrix}\right)$

Step-4 The q-ROHFR positive ideal solutions I^+ and the q-ROHFR negative ideal solutions I^- are calculated in Table-6:

Table-6: Ideal solutions

Criteria	I^+	I^-
c_1	$\left(\left(\begin{array}{c}\{0.3807, 0.5202, 0.6726\},\\ \{0.4446, 0.6484, 0.8891\}\end{array}\right), \left(\begin{array}{c}\{0.2491, 0.4204, 0.5326\},\\ \{0.5627, 0.8376, 0.9761\}\end{array}\right)\right)$	$\left(\left(\begin{array}{c}\{0.3141, 0.4201, 0.3675\},\\ \{0.5214, 0.6935, 0.9211\}\end{array}\right), \left(\begin{array}{c}\{0.3880, 0.5004, 0.6121\},\\ \{0.4361, 0.8113, 0.9597\}\end{array}\right)\right)$
c_2	$\left(\left(\begin{array}{c}\{0.2481, 0.3678, 0.6295\},\\ \{0.4656, 0.8421, 0.9597\}\end{array}\right), \left(\begin{array}{c}\{0.3807, 0.4606, 0.6249\},\\ \{0.2561, 0.6479, 0.8220\}\end{array}\right)\right)$	$\left(\left(\begin{array}{c}\{0.2391, 0.3276, 0.6554\},\\ \{0.3213, 0.8005, 0.9608\}\end{array}\right), \left(\begin{array}{c}\{0.4453, 0.7036, 0.8086\},\\ \{0.3975, 0.8123, 0.9608\}\end{array}\right)\right)$
c_3	$\left(\left(\begin{array}{c}\{0.4593, 0.5532, 0.7930\},\\ \{0.4581, 0.7006, 0.9761\}\end{array}\right), \left(\begin{array}{c}\{0.4064, 0.5657, 0.8142\},\\ \{0.1790, 0.3779, 0.6253\}\end{array}\right)\right)$	$\left(\left(\begin{array}{c}\{0.2950, 0.4448, 0.6998\},\\ \{0.3074, 0.5754, 0.8445\}\end{array}\right), \left(\begin{array}{c}\{0.6105, 0.7212, 0.8155\},\\ \{0.2748, 0.7180, 0.9597\}\end{array}\right)\right)$
c_4	$\left(\left(\begin{array}{c}\{0.2644, 0.4064, 0.5441\},\\ \{0.2857, 0.5359, 0.8732\}\end{array}\right), \left(\begin{array}{c}\{0.4674, 0.3288, 0.5137\},\\ \{0.1301, 0.2561, 0.6224\}\end{array}\right)\right)$	$\left(\left(\begin{array}{c}\{0.3228, 0.4068, 0.5780\},\\ \{0.2329, 0.5359, 0.9212\}\end{array}\right), \left(\begin{array}{c}\{0.3921, 0.5873, 0.7607\},\\ \{0.5925, 0.7701, 0.9761\}\end{array}\right)\right)$

Step-5 The q-ROHFR group utility measure $S_i(i = 1, 2, 3, 4)$ and the regret measure $R_i(i = 1, 2, 3, 3, 4)$ of consider alternatives are evaluated in Table-7.

Table-7: S_i, R_i, Q_i for each alternatives

alternatives	S_i	R_i	Q_i
A_1	1.0520	0.3599	0.9918
A_2	0.4900	0.3099	0.5000
A_3	1.3689	0.4050	1.2319
A_4	0.7422	0.3421	0.6588

Step-6 & 7 Alternative ranking with respect to group utility measure S_i, individual regret measure R_i and ranking measure Q_i is given in Table-8.

Table-8: Ranking of alternative based on S_i, R_i, Q_i

alternatives	Ranking position by S_i	Ranking position by R_i	Ranking position by Q_i
A_1	2	2	2
A_2	4	4	4
A_3	1	1	1
A_4	3	3	3

6. Comparison analysis

In this section, we intend to aggregate decision making information using q-ROHFR weighted average aggregation operators.

6.1. q-ROHFR weighted averaging aggregation operator

The decision making methodology is thoroughly outlined as follows:

Step-1 Construct the experts evaluation matrices in the form of q-ROHFRVs.

Step-2 Computed the collected information of decision makers against their weight vector and get the aggregated decision matrix utilizing q-ROHFRWA aggregation operator.

$$\begin{aligned} & q-ROHFRWA\left(\mathcal{P}(\mathcal{S}_1), \mathcal{P}(\mathcal{S}_2), ...\mathcal{P}(\mathcal{S}_n)\right) \\ = & \left(\sum_{i=1}^{n} w_i \underline{\mathcal{P}}(\mathcal{S}_i), \sum_{i=1}^{n} w_i \overline{\mathcal{P}}(\mathcal{S}_i)\right) \\ = & \begin{bmatrix} \bigcup_{\underline{\mu_i} \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - \left(\underline{\mu_i}\right)^q\right)^{w_i}\right)}, & \bigcup_{\underline{\nu_i} \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n} \left(\underline{\nu_i}\right)^{w_i} \\ \bigcup_{\overline{\mu_i} \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \sqrt[q]{\left(1 - \prod_{i=1}^{n}\left(1 - (\overline{\mu_i})^q\right)^{w_i}\right)}, & \bigcup_{\overline{\nu_i} \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} \prod_{i=1}^{n} (\overline{\nu_i})^{w_i} \end{bmatrix} \end{aligned}$$

Step-3 Evaluate the aggregated q-ROHFRVs for each considered alternative with respect to the given list of criteria/attributes by utilizing the proposed aggregation information.

Step-4 Find the ranking of alternatives based on score function as,

$$\eth(\mathcal{P}(\mathcal{S})) = \frac{1}{4}\left(\begin{array}{c} 2 + \frac{1}{M_{\mathcal{H}}} \sum_{\mu_i \in \eth_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} (\underline{\mu_i}) + \frac{1}{N_{\mathcal{H}}} \sum_{\mu_i \in \eth_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} (\overline{\mu_i}) \\ \frac{1}{M_{\mathcal{H}}} \sum_{\underline{\nu_i} \in \mathcal{B}_{h_{\underline{\mathcal{P}}(\mathcal{S})}}} (\underline{\nu_i}) - \frac{1}{M_{\mathcal{H}}} \sum_{\overline{\nu_i} \in \mathcal{B}_{h_{\overline{\mathcal{P}}(\mathcal{S})}}} (\overline{\nu_i}) \end{array}\right),$$

Step-5 Rank all the alternative scores in descending order. The alternative having larger value will be superior/best.

6.2. Numerical Example of decision making methodology

Here, we apply our proposed weighted average operator to MAGDM problem to determined the best agriculture robots from the list of four reborts under four attributes given in above numerical example.

Step-1 The collective expert information utilizing the q-ROHFRWA aggregation operator is given in Table-5:

Step-2 Aggregation information of the alternative under the given list of attributes are evaluated using proposed aggregation operators as follows;

Aggregation information using q-ROHFRWA operator presented in Table-9:

Table-9: Aggregated information using q-ROHFRWA

A_1	$\begin{pmatrix} (\{0.7104, 0.8112, 0.8931\}, \{0.3022, 0.5443, 0.7961\}), \\ (\{0.7397, 0.8775, 0.9408\}, \{0.3305, 0.6940, 0.8864\}) \end{pmatrix}$
A_2	$\begin{pmatrix} (\{0.6701, 0.7732, 0.8724\}, \{0.3456, 0.6056, 0.8199\}), \\ (\{0.8202, 0.8277, 0.8949\}, \{0.2181, 0.5111, 0.7816\}) \end{pmatrix}$
A_3	$\begin{pmatrix} (\{0.7030, 0.9075, 0.9504\}, \{0.3050, 0.5818, 0.8738\}), \\ (\{0.75920.8800, 0.9351\}, \{0.2928, 0.5132, 0.6773\}) \end{pmatrix}$
A_4	$\begin{pmatrix} (\{0.7264, 0.8142, 0.9152\}, \{0.3774, 0.5674, 0.8395\}), \\ (\{0.6954, 0.8271, 0.9299\}, \{0.3344, 0.5108, 0.7683\}) \end{pmatrix}$

Step-3 & 4 Score values of all alternatives under established aggregation operators presented in Table-10.

Table-10: Ranking of alternative

Proposed operators	Score values of alternatives				Ranking
	A_1	A_2	A_3	A_4	
q-ROHFRWA	0.6183	0.6314	0.6576	0.6259	$A_3 > A_2 > A_4 > A_1$

7. Conclusions

In this research work, q-rung orthopair hesitant fuzzy rough rough set is presented as a new hybrid structure of the q-rung orthopair fuzzy set, the hesitant fuzzy set, and the rough set.The incorporation of rough set theory makes this approach more flexible and effective for modelling fuzzy systems and vital decision making under uncertainty. The algebraic t-norm and t-conorm are used to introduce a list of aggregation operators such as q-ROHFR weighted averaging operators. Additionally, the essential characteristics of evolved operators are described in detail. A decision-making algorithm was developed to solve real-world decision-making problems concerning agri-farming involving imprecise and insufficient information. The suggested technique is ideal for determining the most appropriate kind of robotic agri-farming from a variety of possible types of agri-farming. Numerical examples highlighted the potential applications of the MCDM technique. To demonstrate the capability, superiority, and reliability of the suggested approaches, a comparative study of the final ranking and best choice in robotic agri-farming determined by the proposed techniques and the q-ROHFR-VKOR method is also provided. The established approach can be used to effectively tackle DM challenges. In terms of future study, the innovative notion of q-ROHFRSs might be extended to establish the Yager and Dombi t-norm and t-conorm using the generalised aggregation information.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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