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Research article

Analysis of dengue transmission using fractional order scheme

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Abstract: In this paper, we will check the existence and stability of the dengue internal transmission model with fraction order derivative as well as analyze it qualitatively. The solution has been determined using Atangana-Baleanu in Caputo sense (ABC) with the help of Sumudu transform (ST). Atangana-Toufik (AT) and fractal fractional operator are used to analyze the dengue transmission which is an advanced approach for such types of biological models. Existence theory and uniqueness for the equilibrium solution are provided via nonlinear functional analysis and fixed point theory. Global stability of the system was also proved by using the Lyapunov function. Such kind of study helps us to analyze dengue transmission which shows the actual effect of dengue transmission in society, also will be helpful in future analysis and control strategies.

Keywords: dengue model; Sumudu transform; Atangana-Toufik scheme; uniqueness; fractal fractional operator
Mathematics Subject Classification: 37C75, 93B05, 65L07

1. Introduction

Dengue is a multifaceted disease with affects human health badly. The dengue fever symptoms start to appear in 2 to 7 days [1–3]. It is during this period that differentiating dengue from other febrile diseases proves troublesome [4]. Most people do not give attention to this disease and do self-medication which does not effective for this disease. Secondary dengue infection increases the risk of disease harshness [5,6]. Unfortunately at a time, there is no specific vaccine exists for dengue treatment [7–9]. Dengue fever is similar to flu which can affect all age groups [10], the fever spread due to a mosquito named Aedes. When an infected mosquito bites a normal person the virus is entered into the body of a human through the skin. Hence, the most severe clinical presentation during the infection course does not correlate with a high viral load [11]. According to WHO, dengue-infected people can be divided into two groups uncomplicated and severe [12]. The people who suffer severe conditions are associated with organ impairment or harsh plasma escape and the left are considered uncomplicated [13].

The generalization of classical calculus is called fractional calculus which is concerned with the operation of integration and differentiation of fractional order. In the 19th century by using fractional calculus mathematicians introduce fractional differential equations, fractional dynamics, and fractional geometry. Fractional calculus is used in almost every field of science. It is used to model physical as well as engineering processes. The standard mathematical model of integer order does not work properly. Due to this reason, fractional calculus made a major contribution to the field of mechanics, chemistry, biology, and image processing. By using fractional calculus several physical problems are solved. By using integer-order derivatives the system shows many problems such as history and nonlocal effect. Primarily, all the studies were depending on Caputo fractional-order and Reimann-Liouville fractional derivatives (RLFD). Now a day it had been highlighted that these derivatives have an issue and the issue is they have a singular kernel. That is the reason so many new definitions were presented in the studies [18–20]. These new definitions were very impactful because they have nonsingular kernels which are according to their needs. Caputo fractional derivatives [21], the Caputo-Fabrizio derivative, and AB [22] fractional derivative e have differed from each other only because Caputo is defined by a power law, Fabrizio is defined by using exponential decay law, and ABC is defined by Mittage-Leffler (ML) law. Tateishi et al. describe the role of fractional time operator derivative in a study of anomalous diffusion [23]. We extended the nutrient-phytoplankton-zooplankton model involving variable-order fractional differential operators in [24]. To analyze the dynamics of the fractional calcium oscillation model, powerful techniques are applied to the governing non-linear fractional studied in [25] and some other applications of real-world problems are also studied in [26–28]. In [27] authors used Caputo derivative which kernel is singular and nonlocal properties. But we used ABC operator which is non-singular and nonlocal kernels.

This paper is organized as follows: In sections 1 and 2 consists of an introduction and basic definitions for analysis. Section 3 is for the stability and uniqueness of the proposed scheme with fixed pint theory. Gloabal stablitity was also proved with Lyapunov function. A numerical algorithm for results is developed with the AT scheme and Fractal fractional operator by using the Mittag Leffler function in sections 4 and 5. Conclusions of results are described in section 6.

2. Basic concepts of fractional order

Definition 2.1. The ABC of function $\phi(t)$ defined as [15, 19, 20]:

$$ {}^{ABC}_{a}D_t^{\alpha}\left(\phi(t)\right) = \frac{AB(\alpha)}{n-\alpha}\int_a^t \frac{d^n}{dw^n}\phi(w)E_\alpha\left\{-\alpha\frac{(t-w)^\alpha}{n-\alpha}\right\}dw,\ n-1<\alpha<n, \tag{1}$$

where E_α is the Mittag-Leffler, $AB(\alpha)$ is normalization function and $AB(0) = AB(1) = 1$. By applying Laplace transform, we have

$$ [{}^{ABC}_{a}D_t^{\alpha}\phi(t)](s) = \frac{AB(\alpha)}{1-\alpha}\frac{s^\alpha L[\phi(t)](s)-s^{\alpha-1}\phi(0)}{s^\alpha+\frac{\alpha}{1-\alpha}}. \tag{2}$$

By using ST for (1), we acquire

$$ ST[{}^{ABC}_{0}D_t^{\alpha}\phi(t)](s) = \frac{B(\alpha)}{1-\alpha}\left\{\alpha\Gamma(\alpha+1)E_\alpha\left(-\frac{1}{1-\alpha}w^\alpha\right)\right\}\times\left[ST\left(\phi(t)\right)-\phi(0)\right]. \tag{3}$$

Definition 2.2. The fractional integral of ABC with order α given by

$$ {}^{ABC}_{a}I_t^{\alpha}\left(\phi(t)\right) = \frac{1-\alpha}{B-\alpha}\phi(t) + \frac{\alpha}{B(\alpha)\Gamma(\alpha)}\int_a^t \phi(s)(t-s)^{\alpha-1}ds. \tag{4}$$

Definition 2.3. For a function $g(t) \in W_2^1(0,1), b > a \ and\ \alpha_1 \in [0,1]$, the definition of ABC is given by

$$ {}^{ABC}_{0}D_t^{\alpha_1}g(t) = \frac{AB(\alpha_1)}{1-\alpha_1}\int_0^t \frac{d}{d\tau}g(\tau)E_{\alpha_1}[-\frac{\alpha_1}{1-\alpha_1}(t-\tau)^{\alpha_1}]d\tau,$$

where

$$ AB(\alpha_1) = 1-\alpha_1+\frac{\alpha_1}{\Gamma(\alpha_1)}.$$

Definition 2.4. Suppose that $g(t)$ is continuous on an open interval (a, b), then the fractal-fractional integral of $g(t)$ of order α_1 having Mittag-Leffler type kernel and given by

$$ {}^{FFM}J_{0,t}^{\alpha_1,\alpha_2}\left(g(t)\right) = \frac{\alpha_1\alpha_2}{\mathrm{AB}(\alpha_1)\Gamma(\alpha_1)}\int_0^t s^{\alpha_2-1}g(s)(t-s)^{\alpha_1}ds + \frac{\alpha_2(1-\alpha_1)t^{\alpha_2-1}g(t)}{\mathrm{AB}(\alpha_1)}.$$

3. Mathematical model

To develop the equation the dengue viruses are virulent and no other microorganism that about the human body. Initially, macrophages, monocytes, and other cells of the reticuloendothelial organ are a major source of dengue. The susceptible cell is denoted by S, the infected cell by I, and the free virus by V is shown in Figure 1.

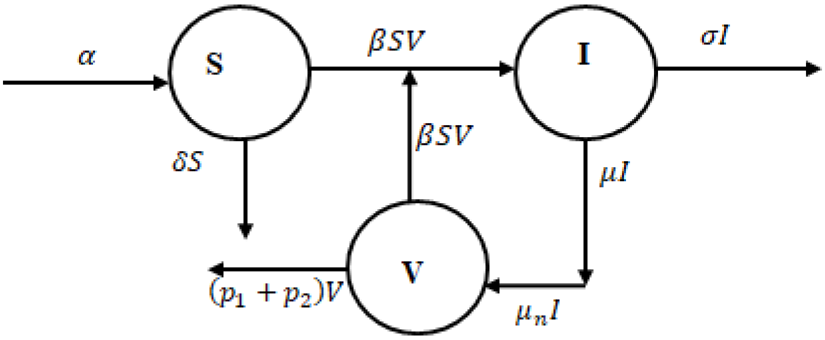

Figure 1: Flow chart of the model.

By transforming the model in [29], with ABC is as follows:

$$
\begin{aligned}
{}^{ABC}_{\ \ 0}D^{\sigma}_{t} S &= \alpha - \beta S(\tau)V(\tau) - \delta S(\tau),\\
{}^{ABC}_{\ \ 0}D^{\sigma}_{t} I &= \beta S(\tau)V(\tau) - \sigma I(\tau),\\
{}^{ABC}_{\ \ 0}D^{\sigma}_{t} V &= \mu_n I(\tau) - (p_1 + p_2)V(\tau) - \beta S(\tau)V(\tau),
\end{aligned} \tag{5}
$$

with initial conditions

$$S(0) = S_0 \geq 0, \ \ I(0) = I_0 \geq 0, \quad V(0) = V_0 \geq 0. \tag{6}$$

In the given model, we will suppose that all the parameters are taken as positive. By using the greenest supposition that α shows the growth of susceptible cells and at the rate $\delta S(\tau)$ they expire. Free virus particles infect susceptible cells at a rate corresponding to the product of their plenitudes $\beta S(\tau)V(\tau)$. The viability of the process is shown by β. Septic cells yield free virus at a rate proportional to their plenitude $\mu n I(\tau)$, with n being the multiplication rate, and free infection particles are expelled from the system at a rate $(p_1 + p_2)V(\tau)$, where γ_1 is the natural demise rate of the virus and γ_2 is the death rate of the virus by T-cells. The free virus also moves to the susceptible cells compartment as $\beta S(\tau)V(\tau)$ and infected cells bite the dust at a rate $\sigma I(\tau)$.

Equilibrium points of model

The equilibrium point of (5) is attained by cracking the non-linear algebraic equations

$$D^{\gamma 1}S(\tau) = D^{\gamma 2} I(\tau) = D^{\gamma 3} V(\tau) = 0.$$

System (5) has a disease-free equilibrium point $F_0(\alpha/\delta, 0,0)$ if $R_0 < 1$, while if $R_0 > 1$, there is in addition to F_0, a positive endemic equilibrium $F^*(S^*, I^*, V^*)$ and the values of S^*, I^* and V^* are as follows:

$$S^* = \frac{\alpha(p_1 + p_2)}{\beta(\mu_{n\_}\sigma)} = \frac{\alpha}{\delta R_0}$$

$$I^* = \frac{\alpha\beta(\mu_n - \sigma) - \sigma\beta(p_1 + p_2)}{\alpha\beta(\mu_{n\_}\sigma)} = \frac{\alpha(R_0 - 1)}{\alpha R_0}$$

$$V^* = \frac{\alpha\beta(\mu_n - \sigma) - \sigma\beta(p_1 + p_2)}{\alpha\beta(P_1 + P_2)} = \frac{\alpha}{\beta}(R_0 - 1)$$

where R_0 is the basic reproduction number defined in [29] as follows:

$$R_0 = \frac{\alpha\beta(\mu_n - \sigma)}{\alpha\delta(P_2 + P_1)}.$$

The value that R_0 takes can signpost the situations where an epidemic is conceivable. Threshold quantity (R_0) is used to analyze the stability of the system (5) and the values of the following parameters given in Table 1.

Table 1. Parameters value used in model.

Parameters	DFE	EE	Unit
α	0.56	0.56	Day^{-1}
β	0.001	0.1	Day^{-1}
δ	0.1313	0.0041	Day^{-1}
σ	0.5	0.32	Day^{-1}
μ_n	156	175	Day^{-1}
P_1	4	4	Day^{-1}
P_2	25	25	Day^{-1}

Applying ST on the proposed model given in (5) we get

$$\frac{q(\sigma)\sigma\Gamma(\sigma+1)}{1-\sigma} N_\sigma \left(-\frac{1}{1-\sigma}V^\sigma\right) ST\{S(\tau) - S(0)\} = ST[\alpha - \beta S(\tau)V(\tau) - \delta S(\tau)],$$

$$\frac{q(\sigma)\sigma\Gamma(\sigma+1)}{1-\sigma} N_\sigma \left(-\frac{1}{1-\sigma}V^\sigma\right) ST\{I(\tau) - I(0)\} = ST[\beta S(\tau)V(\tau) - \sigma I(\tau)], \quad (7)$$

$$\frac{q(\sigma)\sigma\Gamma(\sigma+1)}{1-\sigma} N_\sigma \left(-\frac{1}{1-\sigma}V^\sigma\right) ST\{V(\tau) - V(0)\} = ST[\mu_n I(\tau) - (p_1 + p_2)V(\tau) - \beta S(\tau)V(\tau)].$$

Rearranging, we get

$$ST\big(S(\tau)\big) = S(0) + \frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST[\alpha - \beta S(\tau)V(\tau) - \delta S(\tau)],$$

$$ST\big(I(\tau)\big) = I(0) + \frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST[\beta S(\tau)V(\tau) - \sigma I(\tau)],$$

$$ST\big(V(\tau)\big) = V(0) + \frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST[\mu_n I(\tau) - (p_1 + p_2)V(\tau) - \beta S(\tau)V(\tau)]. \quad (8)$$

Now using the inverse ST on Eq (8), we have

$$S(\tau) = S(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\alpha - \beta S(\tau)V(\tau) - \delta S(\tau)\}\right],$$

$$I(\tau) = I(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\beta S(\tau)V(\tau) - \sigma I(\tau)\}\right],$$

$$V(\tau) = V(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\mu_n I(\tau) - (p_1+p_2)V(\tau) - \beta S(\tau)V(\tau)\}\right].$$

We next obtain the following recursive formula.

$$S_{(n+1)}(\tau) = S_n(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\alpha - \beta S_n(\tau)V_n(\tau) - \delta S_n(\tau)\}\right],$$

$$I_{(n+1)}(\tau) = I_n(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\beta S_n(\tau)V_n(\tau) - \sigma I_n(\tau)\}\right],$$

$$V_{(n+1)}(\tau) = V_n(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\mu_n I_n(\tau) - (p_1+p_2)V_n(\tau) - \beta S_n(\tau)V_n(\tau)\}\right]. \quad (9)$$

And the solution of (9) is provided by

$$S(\tau) = \lim_{n\to\infty} S_n(\tau), I(\tau) = \lim_{n\to\infty} I_n(\tau), V(\tau) = \lim_{n\to\infty} V_n(\tau)$$

Theorem 3.1. The When the reproductive number $R_0 > 1$, the endemic equilibrium points F^* of the SIV model is globally asymptotically stable.

Proof. The Lyapunov function can be written as

$$M(S^*, I^*, V^*) = \left(S - S^* - S^* \log\frac{S^*}{S}\right) + \left(I - I^* - I^* \log\frac{I^*}{I}\right)$$
$$+\left(V - V^* - V^* \log\frac{V^*}{V}\right). \quad (10)$$

Therefore, applying the derivative respect to t on both sides yields

$$\frac{dM}{dt} = \dot{M} = \left(\frac{S-S^*}{S}\right)\dot{S} + \left(\frac{I-I^*}{I}\right)\dot{I} + \left(\frac{V-V^*}{V}\right)\dot{V}. \quad (11)$$

Now, we can write their values for derivatives as follows

$$\frac{dM}{dt} = \dot{M} = \left(\frac{S-S^*}{S}\right)(\alpha - \beta SV - \delta S) + \left(\frac{I-I^*}{I}\right)(\beta SV - \sigma I)$$
$$+\left(\frac{V-V^*}{V}\right)(\mu_n I - (p_1+p_2)V - \beta SV). \quad (12)$$

Putting $S = S - S^*$, $I = I - I^*$, $L = V - V^*$ leads to

$$\frac{dM}{dt} = \left(\frac{S - S^*}{S}\right)(\alpha - \beta(S - S^*)(V - V^*) - \delta(S - S^*))$$

$$+\left(\frac{I - I^*}{I}\right)(\beta(S - S^*)(V - V^*) - \sigma(I - I^*))$$

$$+\left(\frac{V - V^*}{V}\right)(\mu_n(I - I^*) - (p_1 + p_2)(V - V^*) - \beta(S - S^*)(V - V^*)). \quad (13)$$

We can organize the above as follows

$$\frac{dM}{dt} = \alpha - \beta SV + \beta S^*V + \beta SV^* - \beta S^*V^* - \alpha\left(\frac{S^*}{S}\right) + \beta SV\left(\frac{S^*}{S}\right)$$

$$-\beta S^*V\left(\frac{S^*}{S}\right) - \beta SV^*\left(\frac{S^*}{S}\right) + \beta S^*V^*\left(\frac{S^*}{S}\right) - \frac{\delta}{S}(S - S^*)^2$$

$$+\beta SV - \beta S^*V - \beta SV^* + \beta S^*V^* - \beta SV\left(\frac{I^*}{I}\right) + \beta S^*V\left(\frac{I^*}{I}\right)$$

$$+\beta SV^*\left(\frac{I^*}{I}\right) - \beta S^*V^*\left(\frac{I^*}{I}\right) - \frac{\sigma}{I}(I - I^*)^2 + \mu_n I - \mu_n I^*$$

$$-\beta SV + \beta S^*V + \beta SV^* - \beta S^*V^* - \mu_n I\left(\frac{V^*}{V}\right) + \mu_n I^*\left(\frac{V^*}{V}\right)$$

$$+\beta SV\left(\frac{V^*}{V}\right) - \beta S^*V\left(\frac{V^*}{V}\right) + \beta SV^*\left(\frac{V^*}{V}\right) + \beta S^*V^*\left(\frac{V^*}{V}\right)$$

$$-\frac{(p_1 + p_2)}{V}(V - V^*)^2. \quad (14)$$

To avoid the complexity, the above can be written as

$$\frac{dM}{dt} = \Sigma - \Omega \quad (15)$$

where

$$\Sigma = \alpha + \beta S^*V + \beta SV^* + \beta SV\left(\frac{S^*}{S}\right) + \beta S^*V^*\left(\frac{S^*}{S}\right) + \beta SV + \beta S^*V^*$$

$$+\beta S^*V\left(\frac{I^*}{I}\right) + \beta SV^*\left(\frac{I^*}{I}\right) + \mu_n I + \beta S^*V + \beta SV^* + \mu_n I^*\left(\frac{V^*}{V}\right)$$

$$+\beta SV\left(\frac{V^*}{V}\right) + \beta SV^*\left(\frac{V^*}{V}\right) + \beta S^*V^*\left(\frac{V^*}{V}\right),$$

$$\Omega = \beta SV + \beta S^*V^* + \alpha\left(\frac{S^*}{S}\right) + \beta S^*V\left(\frac{S^*}{S}\right) + \beta SV^*\left(\frac{S^*}{S}\right) + \frac{\delta}{S}(S - S^*)^2$$

$$+\beta S^*V + \beta SV^* + \beta SV\left(\frac{I^*}{I}\right) + \beta S^*V^*\left(\frac{I^*}{I}\right) + \frac{\sigma}{I}(I - I^*)^2$$

$$+\beta SV + \beta S^*V^* + \mu_n I\left(\frac{V^*}{V}\right) + \beta S^*V\left(\frac{V^*}{V}\right) + \frac{(p_1 + p_2)}{V}(V - V^*)^2.$$

It is concluded that if $\Sigma < \Omega$, this yields, $\frac{dM}{dt} < 0$, however when, $S = S^*, I = I^*, V = V^*$

$$0 = \Sigma - \Omega \Rightarrow \frac{dM}{dt} = 0. \tag{16}$$

We can see that the largest compact invariant set for the suggested model in

$$\left\{(S^*, I^*, V^*) \in \Gamma; \frac{dM}{dt} = 0\right\} \tag{17}$$

is the point $\{F^*\}$ the endemic equilibrium of the considered model. By the help of the Lasalles invariance concept, it follows that F^* is globally asymptotically stable in Γ if $\Sigma < \Omega$.

Theorem 3.2. Let $(X, |.|)$ be a Banach space and H a self-map of X satisfying

$$\|H_x - H_r\| \le \theta\|X - H_x\| + \theta\|x - r\|,$$

$$S_{(n+1)}(\tau) = S_n(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\alpha - \beta S_n(\tau)V_n(\tau) - \delta S_n(\tau)\}\right],$$

$$I_{(n+1)}(\tau) = I_n(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\beta S_n(\tau)V_n(\tau) - \sigma I_n(\tau)\}\right],$$

$$V_{(n+1)}(\tau) = V_n(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\mu_n I_n(\tau) - (p_1 + p_2)V_n(\tau) - \beta S_n(\tau)V_n(\tau)\}\right],$$

where $\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}$ is the fractional Lagrange multiplier.

Proof. Define K be a self-map [30] is given by

$$K\left[S_{(n+1)}(\tau)\right] = S_{(n+1)}(\tau) = S_n(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} ST\{\alpha - \beta S_n(\tau)V_n(\tau) - \delta S_n(\tau)\}\right],$$

$$K\left[I_{(n+1)}(\tau)\right] = I_{(n+1)}(\tau) = I_n(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\beta S_n(\tau)V_n(\tau) - \sigma I_n(\tau)\}\right], \tag{18}$$

$$K\left[V_{(n+1)}(\tau)\right] = V_{(n+1)}(\tau) = V_n(0) + ST^{-1}$$

$$\times\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\mu_n I_n(\tau) - (p_1 + p_2)V_n(\tau) - \beta S_n(\tau)V_n(\tau)\}\right].$$

Applying the properties of the norm and triangular inequality, we get

$$\|K[S_n(\tau)] - K[S_m(\tau)]\| \le \|S_n(\tau) - S_m(\tau)\|$$

$$+ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\alpha + \beta\|S_n(\tau)V_n(\tau) - S_m(\tau)V_m(\tau)\| + \delta\|S_n(\tau) - S_m(\tau)\|\}\right]$$

$$\|K[I_n(\tau)] - K[I_m(\tau)]\| \le \|I_n(\tau) - I_m(\tau)\| + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} \times ST\{\beta\|S_n(\tau)V_n(\tau) - S_m(\tau)V_m(\tau)\| + \sigma\|I_n(\tau) - I_m(\tau)\|\}\right], \quad (19)$$

$$\begin{aligned}\|K[V_n(\tau)] - K[V_m(\tau)]\| &\le \|V_n(\tau) - V_m(\tau)\| \\ &+ ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}\right. \\ &\left.\times ST\{\mu_n\|I_n(\tau) - I_m(\tau)\| + (p_1+p_2)\|V_n(\tau) - V_m(\tau)\| + \beta\|S_n(\tau)V_n(\tau) - S_m(\tau)V_m(\tau)\|\}\right].\end{aligned}$$

K fulfills the conditions associated with Theorem 3.2 when

$$\theta = (0,0,0,0,0), \theta = \begin{cases} \|S_n(t) - S_m(t)\| \times \|-(S_n(t) + S_m(t))\| + \alpha - \beta\|S_n(\tau)V_n(\tau) - S_m(\tau)V_m(\tau)\| - \delta\|S_n(\tau) - S_m(\tau)\| \\ \times \|(I_n(t) - I_m(t))\| \times \|-(I_n(t) + I_m(t))\| + \beta\|S_n(\tau)V_n(\tau) - S_m(\tau)V_m(\tau)\| - \sigma\|I_n(\tau) - I_m(\tau)\| \\ \times \|(V_n(t) - V_m(t))\| \times \|-(V_n(t) + V_m(t))\| + \mu_n\|I_n(\tau) - I_m(\tau)\| - (p_1+p_2)\|V_n(\tau) - V_m(\tau)\| \\ -\beta\|S_n(\tau)V_n(\tau) - S_m(\tau)V_m(\tau)\| \end{cases}.$$

Hence K is Picard K-stable.

Theorem 3.3. The special solution of Eq (5) using the iteration method is the unique singular solution.

Proof. Take into consideration the following Hilbert space $H = L^2\big((p,q) \times (0,T)\big)$ which can be defined as

$$h: (p,q) \times (0,T) \to \mathbb{R}, \iint ghdgdh < \infty.$$

In this regard, the following operators are considered

$$\theta(0,0,0,0,0), \theta = \begin{cases} \alpha - \beta S(\tau)V(\tau) - \delta S(\tau) \\ \beta S(\tau)V(\tau) - \sigma I(\tau) \\ \mu_n I(\tau) - (p_1+p_2)V(\tau) - \beta S(\tau)V(\tau) \end{cases}.$$

We establish that the inner product of

$$\big(T(S_{11} - S_{12}, I_{21} - I_{22}, V_{31} - V_{32},), (V_1, V_2, V_3)\big).$$

Where $(S_{11} - S_{12}, I_{21} - I_{22}, V_{31} - V_{32})$, are the special solutions of the system. Taking into account the inner function and the norm, we have

$$\{\alpha - \beta(S_{11} - S_{12})(V_{31} - V_{32}) - \delta(S_{11} - S_{12}), V_1\} \le \alpha\|V_1\| + \beta\|S_{11} - S_{12}\|\|V_{31} - V_{32}\|\|V_1\| + \delta\|S_{11} - S_{12}\|\|V_1\|,$$

$$\{\beta(S_{11} - S_{12})(V_{31} - V_{32}) - \sigma(I_{21} - I_{22}), V_2\} \le \beta\|S_{11} - S_{12}\|\|V_{31} - V_{32}\|\|V_2\| + \sigma\|I_{21} - I_{22}\|\|V_2\|,$$

$$\{\mu_n(I_{21}-I_{22})-(p_1+p_2)(V_{31}-V_{32})-\beta(S_{11}-S_{12})(V_{31}-V_{32}),V_3\}$$
$$\leq \mu_n\|I_{21}-I_{22}\|\|V_3\|+(p_1+p_2)\|V_{31}-V_{32}\|\|V_3\|$$
$$+\beta\|S_{11}-S_{12}\|\|V_{31}-V_{32}\|\|V_3\|.$$

In the case of a large number $e_1,\ e_2\ and\ e_3$, both solutions happen to be converged to the exact solution. Employing the topology concept, we can obtain five positive very small parameters $(\chi_{e_1},\ \chi_{e_2}\ and\ \chi_{e_3})$.

$$\|S-S_{11}\|,\|S-S_{12}\|\leq\frac{\chi_{e_1}}{\varpi},\|I-I_{21}\|,\|I-I_{22}\|\leq\frac{\chi_{e_2}}{\varsigma},\|V-V_{31}\|,\|V-V_{32}\|\leq\frac{\chi_{e_3}}{\upsilon},$$

where

$$\varpi=3(\alpha+\beta\|S_{11}-S_{12}\|\|V_{31}-V_{32}\|+\delta\|S_{11}-S_{12}\|)\|V_1\|$$

$$\varsigma=3(\beta\|S_{11}-S_{12}\|\|V_{31}-V_{32}\|+\sigma\|I_{21}-I_{22}\|)\|V_2\|$$

$$\upsilon=3(\mu_n\|I_{21}-I_{22}\|+(p_1+p_2)\|V_{31}-V_{32}\|+\beta\|S_{11}-S_{12}\|\|V_{31}-V_{32}\|)\|V_3\|.$$

But, it is obvious that

$$(\alpha+\beta\|S_{11}-S_{12}\|\|V_{31}-V_{32}\|+\delta\|S_{11}-S_{12}\|)\neq 0$$

$$(\beta\|S_{11}-S_{12}\|\|V_{31}-V_{32}\|+\sigma\|I_{21}-I_{22}\|)\neq 0$$

$$(\mu_n\|I_{21}-I_{22}\|+(p_1+p_2)\|V_{31}-V_{32}\|+\beta\|S_{11}-S_{12}\|\|V_{31}-V_{32}\|)\neq 0,$$

where $\|V_1\|,\|V_2\|,\|V_3\|\neq 0$.

Therefore, we have

$$\|S_{11}-S_{12}\|=0,\|I_{21}-I_{22}\|=0,\|V_{31}-V_{32}\|=0,$$

which yields that

$$S_{11}=S_{12},I_{21}=I_{22},V_{31}=V_{32}.$$

This completes the proof of uniqueness.

4. Advanced numerical scheme (AT method)

Applying the advanced numerical scheme AT method on system given in Eq (5) and fundamental theorem of fractional calculus, we get

$$S(\tau)-S(0)=\frac{(1-\sigma)}{ABC(\sigma)}\{\alpha-\beta S(\tau)V(\tau)-\delta S(\tau)\}+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\int_0^{t}\{\alpha-\beta S(t)V(t)-\delta S(t)\}(\tau-t)^{\sigma-1}dt,$$

$$I(\tau)-I(0)=\frac{(1-\sigma)}{ABC(\sigma)}\{\beta S(\tau)V(\tau)-\sigma I(\tau)\}+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\int_0^{\tau}\{\beta S(t)V(t)-\sigma I(t)\}(\tau-t)^{\sigma-1}dt,$$

$$V(\tau)-V(0)=\frac{(1-\sigma)}{ABC(\sigma)}\left\{\mu_n I(\tau)-\left(p_1+p_2\right)V(\tau)-\beta S(\tau)V(\tau)\right\}$$

$$+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\int_0^{\tau}\left\{\mu_n I(\tau)-\left(p_1+p_2\right)V(\tau)-\beta S(\tau)V(\tau)\right\}(\tau-t)^{\sigma-1}dt. \qquad (20)$$

At a given point τ_{n+1}, $n=0,1,2,3,...,$ the above equation is reformulated as

$$S(\tau_{n+1})-S(0)$$
$$=\frac{(1-\sigma)}{ABC(\sigma)}\{\alpha-\beta S(\tau_n)V(\tau_n)-\delta S(\tau_n)\}$$
$$+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\int_0^{t}\{\alpha-\beta S(t)V(t)-\delta S(t)\}(\tau_{n+1}-t)^{\sigma-1}dt,$$

$$I(\tau_{n+1})-I(0)=\frac{(1-\sigma)}{ABC(\sigma)}\{\beta S(\tau_n)V(\tau_n)-\sigma I(\tau_n)\}$$
$$+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\int_0^{\tau_{n+1}}\{\beta S(t)V(t)-\sigma I(t)\}(\tau_{n+1}-t)^{\sigma-1}dt, \qquad (21)$$

$$V(\tau_{n+1})-V(0)$$
$$=\frac{(1-\sigma)}{ABC(\sigma)}\left\{\mu_n I(\tau)-\left(p_1+p_2\right)V(\tau)-\beta S(\tau)V(\tau)\right\}$$
$$+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\int_0^{\tau_{n+1}}\left\{\mu_n I(\tau)-\left(p_1+p_2\right)V(\tau)-\beta S(\tau)V(\tau)\right\}(\tau_{n+1}-t)^{\sigma-1}dt.$$

Also, we have

$$S(\tau_{n+1})-S(0)$$
$$=\frac{(1-\sigma)}{ABC(\sigma)}\{\alpha-\beta S(\tau_n)V(\tau_n)-\delta S(\tau_n)\}$$
$$+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\sum_{k=0}^{n}\int_{\tau_k}^{\tau_{k+1}}\{\alpha-\beta S(t)V(t)-\delta S(t)\}(\tau_{n+1}-t)^{\sigma-1}dt,$$

$$I(\tau_{n+1})-I(0)=\frac{(1-\sigma)}{ABC(\sigma)}\{\beta S(\tau_n)V(\tau_n)-\sigma I(\tau_n)\}$$

$$+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\sum_{k=0}^{n}\int_{\tau_k}^{\tau_{k+1}}\{\beta S(t)V(t)-\sigma I(t)\}(\tau_{n+1}-t)^{\sigma-1}dt,$$

$$V(\tau_{n+1})-V(0)=\frac{(1-\sigma)}{ABC(\sigma)}\left\{\mu_n I(\tau)-\left(p_1+p_2\right)V(\tau)-\beta S(\tau)V(\tau)\right\}$$

$$+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\sum_{k=0}^{n}\int_{\tau_k}^{\tau_{k+1}}\left\{\mu_n I(\tau)-\left(p_1+p_2\right)V(\tau)-\beta S(\tau)V(\tau)\right\}(\tau_{n+1}-t)^{\sigma-1}dt. \qquad (22)$$

By using equation above equations, we have

$$S_{n+1} = S_0 + \frac{(1-\sigma)}{ABC(\sigma)}\{\alpha - \beta S(\tau_n)V(\tau_n) - \delta S(\tau_n)\} + \frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\sum_{k=0}^{n}\left\{\frac{\alpha - \beta S_k V_k - \delta S_k}{h} \times \int_{\tau_k}^{\tau_{k+1}}\{(t-\tau_{k-1})\}(\tau_{n+1}-t)^{\alpha-1}dt - \frac{\alpha - \beta S_{k-1}V_{k-1} - \delta S_{k-1}}{h} \times \int_{\tau_k}^{\tau_{k+1}}\{(t-\tau_{k-1})\}(\tau_{n+1}-t)^{\sigma-1}dt\right\},$$

$$I(\tau_{n+1}) - I(0) = \frac{(1-\sigma)}{ABC(\sigma)}\{\beta S(\tau_n)V(\tau_n) - \sigma I(\tau_n)\} + \frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\sum_{k=0}^{n}\left\{\frac{\beta S_k V_k - \sigma I_k}{h} \times \int_{\tau_k}^{\tau_{k+1}}\{(t-\tau_{k-1})\}(\tau_{n+1}-t)^{\sigma-1}dt - \frac{\beta S_{k-1}V_{k-1} - \sigma I_{k-1}}{h} \times \int_{\tau_k}^{\tau_{k+1}}\{(t-\tau_{k-1})\}(\tau_{n+1}-t)^{\sigma-1}dt\right\},$$

$$V(\tau_{n+1}) - V(0) = \frac{(1-\sigma)}{ABC(\sigma)}\left\{\mu_n I(\tau) - \left(p_1+p_2\right)V(\tau) - \beta S(\tau)V(\tau)\right\} + \frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)} \times \sum_{k=0}^{n}\left\{\frac{\mu_n I_k - (p_1+p_2)V_k - \beta S_k V_k}{h} \times \frac{\mu_n I_{k-1} - (p_1+p_2)V_{k-1} - \beta S_{k-1}V_{k-1}}{h} \times \int_{\tau_k}^{\tau_{k+1}}\{(t-\tau_{k-1})\}(\tau_{n+1}-t)^{\sigma-1}dt\right\}. \quad (23)$$

Thus integrating equation (21) and Eq (22), and replacing them in equations of the system (23) we get

$$S_{n+1} = S_0 + \frac{(1-\sigma)}{ABC(\sigma)}\{\alpha - \beta S(\tau_n)V(\tau_n) - \delta S(\tau_n)\} + \frac{\sigma}{ABC(\sigma)}\sum_{k=0}^{n}\left(\frac{h^{\sigma}\{\alpha - \beta S_k V_k - \delta S_k\}}{\Gamma(\sigma+2)} \times \{(n+1-k)^{\alpha}(n-k+2+\alpha) - (n-k)^{\alpha}(n-k+2+2\alpha)\} - \frac{h^{\alpha}\{\alpha - \beta S_{k-1}V_{k-1} - \delta S_{k-1}\}}{\Gamma(\alpha+2)} \times \{(n+1-k)^{\alpha+1} - (n-k)^{\alpha}(n-k+1+\alpha)\}\right),$$

$$I(\tau_{n+1}) - I(0) = \frac{(1-\sigma)}{ABC(\sigma)}\{\beta S(\tau_n)V(\tau_n) - \sigma I(\tau_n)\} + \frac{\sigma}{ABC(\sigma)}\sum_{k=0}^{n}\left(\frac{h^{\alpha}\{\beta S_k V_k - \sigma I_k\}}{\Gamma(\sigma+2)} \times \{(n+1-k)^{\alpha}(n-k+2+\alpha) - (n-k)^{\alpha}(n-k+2+2\alpha)\} - \frac{h^{\alpha}\{\beta S_{k-1}V_{k-1} - \sigma I_{k-1}\}}{\Gamma(\sigma+2)} \times \{(n+1-k)^{\alpha+1} - (n-k)^{\alpha}(n-k+1+\alpha)\}\right), \quad (24)$$

$$
\begin{aligned}
&V(\tau_{n+1}) - V(0) \\
&\quad = \frac{(1-\sigma)}{ABC(\sigma)}\left\{\mu_n I(\tau) - \left(p_1 + p_2\right)V(\tau) - \beta S(\tau)V(\tau)\right\} \\
&\quad + \frac{\sigma}{ABC(\sigma)}\sum_{k=0}^{n}\left(\frac{h^{\sigma}\left\{\mu_n \mathrm{I}_k - \left(p_1 + p_2\right)\mathrm{V}_k - \beta \mathrm{S}_k \mathrm{V}_k\right\}}{\Gamma(\sigma+2)}\right. \\
&\quad \times \{(n+1-k)^{\alpha}(n-k+2+\alpha) - (n-k)^{\alpha}(n-k+2+2\alpha)\} \\
&\quad - \frac{h^{\alpha}\left\{\mu_n \mathrm{I}_{k-1} - \left(p_1 + p_2\right)\mathrm{V}_{k-1} - \beta \mathrm{S}_{k-1}\mathrm{V}_{k-1}\right\}}{\Gamma(\alpha+2)} \\
&\quad \left.\times \{(n+1-k)^{\alpha+1} - (n-k)^{\alpha}(n-k+1+\alpha)\}\right).
\end{aligned}
$$

5. Dengue model fractal-fractional

We have the following model with fractal fraction operator in ABC sense is given as

$$
\begin{aligned}
{}^{FF}D_{0,\tau}^{\alpha_1,\alpha_2} S &= \alpha - \beta S(\tau)V(\tau) - \delta S(\tau), \\
{}^{FF}D_{0,\tau}^{\alpha_1,\alpha_2} I &= \beta S(\tau)V(\tau) - \sigma I(\tau), \\
{}^{FF}D_{0,\tau}^{\alpha_1,\alpha_2} V &= \mu_n I(\tau) - (p_1 + p_2)V(\tau) - \beta S(\tau)V(\tau),
\end{aligned}
\tag{25}
$$

with initial conditions (6).

Numerical procedure with fractal fractional

We present the numerical algorithm for the fractal-fractional Dengue model (5). The following is obtained by integrating the system (5).

$$
\begin{aligned}
S(\tau) - S(0) &= \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2\tau^{\alpha_2-1}\{\alpha - \beta S(\tau)V(\tau) - \delta S(\tau)\} \\
&\quad + \frac{\alpha_1\alpha_2}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{\tau} t^{\alpha_2-1}\{\alpha - \beta S(t)V(t) - \delta S(t)\}(\tau - t)^{\alpha_1-1}dt,
\end{aligned}
$$

$$
\begin{aligned}
I(\tau) - I(0) &= \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2\tau^{\alpha_2-1}\{\beta S(\tau)V(\tau) - \sigma I(\tau)\} \\
&\quad + \frac{\alpha_1\alpha_2}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{\tau} t^{\alpha_2-1}\{\beta S(t)V(t) - \sigma I(t)\}(\tau - t)^{\alpha_1-1}dt,
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
V(\tau) - V(0) &= \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2\tau^{\alpha_2-1}\{\mu_n I(\tau) - (p_1 + p_2)V(\tau) - \beta S(\tau)V(\tau)\} \\
&\quad + \frac{\alpha_1\alpha_2}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{\tau} t^{\alpha_2-1}\{\mu_n I(t) - (p_1 + p_2)V(t) - \beta S(\tau)V(t)\}(\tau - t)^{\alpha_1-1}dt.
\end{aligned}
$$

Let

$$
\begin{aligned}
k\big(\tau, S(\tau)\big) &= \alpha_2\tau^{\alpha_2-1}\{\alpha - \beta S(\tau)V(\tau) - \delta S(\tau)\}, \\
k\big(\tau, I(\tau)\big) &= \alpha_2\tau^{\alpha_2-1}\{\beta S(\tau)V(\tau) - \sigma I(\tau)\},
\end{aligned}
$$

$$k\big(\tau,V(\tau)\big)=\alpha_2\tau^{\alpha_2-1}\{\mu_n I(\tau)-(p_1+p_2)V(\tau)-\beta S(\tau)V(\tau)\}.$$

Then, we have

$$S(\tau)-S(0)=\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau,S(\tau)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{\tau}k\big(t,S(t)\big)(\tau-t)^{\alpha_1-1}dt,$$

$$I(\tau)-I(0)=\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau,I(\tau)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{\tau}k\big(t,S(t)\big)(\tau-t)^{\alpha_1-1}dt, \tag{27}$$

$$V(\tau)-V(0)=\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau,V(\tau)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{\tau}k\big(t,S(t)\big)(\tau-t)^{\alpha_1-1}dt.$$

At $\tau_{n+1}=(n+1)\Delta\tau,$ we have

$$S(\tau_{n+1})-S(0)=\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,S(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{\tau_{n+1}}k\big(t,S(t)\big)(\tau_{n+1}-t)^{\alpha_1-1}dt,$$

$$I(\tau_{n+1})-I(0)=\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,I(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{\tau_{n+1}}k\big(t,I(t)\big)(\tau_{n+1}-t)^{\alpha_1-1}dt, \tag{28}$$

$$V(\tau_{n+1})-V(0)=\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,V(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{\tau_{n+1}}k\big(t,V(t)\big)(\tau_{n+1}-t)^{\alpha_1-1}dt.$$

Also, we have

$$S(t_{n+1})=S(0)+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,S(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\int_{\tau_j}^{\tau_{j+1}}k\big(t,S(t)\big)(\tau_{n+1}-t)^{\alpha_1-1}dt,$$

$$I(t_{n+1})=I(0)+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,I(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\int_{\tau_j}^{\tau_{j+1}}k\big(t,I(t)\big)(\tau_{n+1}-t)^{\alpha_1-1}dt,$$

$$V(t_{n+1})=V(0)+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,V(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\int_{\tau_j}^{\tau_{j+1}}k\big(t,V(t)\big)(\tau_{n+1}-t)^{\alpha_1-1}dt.$$

In general, approximating the function $k\big(t,y(t)\big),$ using the Newton polynomial, we have

$$P_n(t)=k\big(\tau_{n-2},y(\tau_{n-2})\big)+\frac{k\big(\tau_{n-1},y(\tau_{n-1})\big)-k\big(\tau_{n-2},y(\tau_{n-2})\big)}{\Delta\tau}(t-\tau_{n-2})$$
$$+\frac{k\big(\tau_n,y(\tau_n)\big)-2k\big(\tau_{n-1},y(\tau_{n-1})\big)+k\big(\tau_{n-2},y(\tau_{n-2})\big)}{2(\Delta\tau)^2}(t-\tau_{n-2})(t-\tau_{n-1}). \tag{29}$$

Using Eq (29) into the above system, we have

$$S^{n+1}=S^0+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,S(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\int_{\tau_j}^{\tau_{j+1}}\Big\{k\big(\tau_{j-2},S^{j-2}\big)+\frac{k\big(\tau_{j-1},S^{j-1}\big)-k\big(\tau_{j-2},S^{j-2}\big)}{\Delta\tau}\big(t-\tau_{j-2}\big)+\frac{k\big(\tau_j,S^j\big)-2k\big(\tau_{j-1},S^{j-1}\big)+k\big(\tau_{j-2},S^{j-2}\big)}{2(\Delta\tau)^2}\big(t-\tau_{j-2}\big)\big(t-\tau_{j-1}\big)\Big\}(\tau_{n+1}-t)^{\alpha_1-1}dt,$$

$$I^{n+1}=I^0+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,I(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\int_{\tau_j}^{\tau_{j+1}}\Big\{k\big(\tau_{j-2},I^{j-2}\big)+\frac{k\big(\tau_{j-1},I^{j-1}\big)-k\big(\tau_{j-2},I^{j-2}\big)}{\Delta\tau}\big(t-\tau_{j-2}\big)+\frac{k\big(\tau_j,I^j\big)-2k\big(\tau_{j-1},I^{j-1}\big)+k\big(\tau_{j-2},I^{j-2}\big)}{2(\Delta\tau)^2}\big(t-\tau_{j-2}\big)\big(t-\tau_{j-1}\big)\Big\}(\tau_{n+1}-t)^{\alpha_1-1}dt, \tag{30}$$

$$V^{n+1}=V^0+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,V(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\int_{\tau_j}^{\tau_{j+1}}\Big\{k\big(\tau_{j-2},V^{j-2}\big)+\frac{k\big(\tau_{j-1},V^{j-1}\big)-k\big(\tau_{j-2},V^{j-2}\big)}{\Delta\tau}\big(t-\tau_{j-2}\big)+\frac{k\big(\tau_j,V^j\big)-2k\big(\tau_{j-1},V^{j-1}\big)+k\big(\tau_{j-2},V^{j-2}\big)}{2(\Delta\tau)^2}\big(t-\tau_{j-2}\big)\big(t-\tau_{j-1}\big)\Big\}(\tau_{n+1}-t)^{\alpha_1-1}dt.$$

Rearranging the above system, we have

$$S^{n+1}=S^0+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,S(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\left\{\int_{\tau_j}^{\tau_{j+1}}k\big(\tau_{j-2},S^{j-2}\big)(\tau_{n+1}-t)^{\alpha_1-1}dt+\int_{\tau_j}^{\tau_{j+1}}\frac{k\big(\tau_{j-1},S^{j-1}\big)-k\big(\tau_{j-2},S^{j-2}\big)}{\Delta\tau}(t-\tau_{j-2})(\tau_{n+1}-t)^{\alpha_1-1}dt+\int_{\tau_j}^{\tau_{j+1}}\frac{k\big(\tau_j,S^j\big)-2k\big(\tau_{j-1},S^{j-1}\big)+k\big(\tau_{j-2},S^{j-2}\big)}{2(\Delta\tau)^2}\big(t-\tau_{j-2}\big)\big(t-\tau_{j-1}\big)(\tau_{n+1}-t)^{\alpha_1-1}dt,\right\}$$

$$I^{n+1}=I^0+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(\tau_n,I(\tau_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\left\{\int_{\tau_j}^{\tau_{j+1}}k\big(\tau_{j-2},I^{j-2}\big)(\tau_{n+1}-t)^{\alpha_1-1}dt+\int_{\tau_j}^{\tau_{j+1}}\frac{k\big(\tau_{j-1},I^{j-1}\big)-k\big(\tau_{j-2},I^{j-2}\big)}{\Delta\tau}(t-\tau_{j-2})(\tau_{n+1}-t)^{\alpha_1-1}dt+\int_{\tau_j}^{\tau_{j+1}}\frac{k\big(\tau_j,I^j\big)-2k\big(\tau_{j-1},I^{j-1}\big)+k\big(\tau_{j-2},I^{j-2}\big)}{2(\Delta\tau)^2}\big(t-\tau_{j-2}\big)\big(t-\tau_{j-1}\big)(\tau_{n+1}-t)^{\alpha_1-1}dt\right\}, \tag{31}$$

$$V^{n+1} = V^0 + \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)} k\big(\tau_n, V(\tau_n)\big) + \frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} \left\{ \int_{\tau_j}^{\tau_{j+1}} k\big(\tau_{j-2}, V^{j-2}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt + \int_{\tau_j}^{\tau_{j+1}} \frac{k\big(\tau_{j-1}, V^{j-1}\big) - k\big(\tau_{j-2}, V^{j-2}\big)}{\Delta\tau} \big(t-\tau_{j-2}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt + \int_{\tau_j}^{\tau_{j+1}} \frac{k\big(\tau_j, V^j\big) - 2k\big(\tau_{j-1}, V^{j-1}\big) + k\big(\tau_{j-2}, V^{j-2}\big)}{2(\Delta\tau)^2} \big(t-\tau_{j-2}\big)\big(t-\tau_{j-1}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt \right\}.$$

Writing further above system, we have

$$S^{n+1} = S^0 + \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)} k\big(\tau_n, S(\tau_n)\big) + \frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} k\big(\tau_{j-2}, S^{j-2}\big) \int_{\tau_j}^{\tau_{j+1}} (\tau_{n+1}-t)^{\alpha_1-1} dt + \frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} \frac{k\big(\tau_{j-1}, S^{j-1}\big) - k\big(\tau_{j-2}, S^{j-2}\big)}{\Delta\tau} \int_{\tau_j}^{\tau_{j+1}} \big(t-\tau_{j-2}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt + \frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} \frac{k\big(\tau_j, S^j\big) - 2k\big(\tau_{j-1}, S^{j-1}\big) + k\big(\tau_{j-2}, S^{j-2}\big)}{2(\Delta\tau)^2} \int_{\tau_j}^{\tau_{j+1}} \big(t-\tau_{j-2}\big)\big(t-\tau_{j-1}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt,$$

$$I^{n+1} = I^0 + \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)} k\big(\tau_n, I(\tau_n)\big) + \frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} k\big(t_{j-2}, I^{j-2}\big) \int_{\tau_j}^{\tau_{j+1}} (\tau_{n+1}-t)^{\alpha_1-1} dt + \frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} \frac{k\big(\tau_{j-1}, I^{j-1}\big) - k\big(\tau_{j-2}, I^{j-2}\big)}{\Delta\tau} \int_{\tau_j}^{\tau_{j+1}} \big(t-\tau_{j-2}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt + \frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} \frac{k\big(\tau_j, I^j\big) - 2k\big(\tau_{j-1}, I^{j-1}\big) + k\big(\tau_{j-2}, I^{j-2}\big)}{2(\Delta\tau)^2} \int_{\tau_j}^{\tau_{j+1}} \big(t-\tau_{j-2}\big)\big(t-\tau_{j-1}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt,$$

$$V^{n+1} = V^0 + \frac{(1-\alpha_1)}{C(\alpha_1)} k\big(\tau_n, V(\tau_n)\big) + \frac{\alpha_1}{C(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} k\big(t_{j-2}, V^{j-2}\big) \int_{\tau_j}^{\tau_{j+1}} (\tau_{n+1}-t)^{\alpha_1-1} dt + \frac{\rho_1}{C(\rho_1)\Gamma(\rho_1)} \sum_{j=2}^{n} \frac{k\big(\tau_{j-1}, V^{j-1}\big) - k\big(\tau_{j-2}, V^{j-2}\big)}{\Delta\tau} \int_{\tau_j}^{\tau_{j+1}} \big(t-\tau_{j-2}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt + \frac{\rho_1}{C(\rho_1)\Gamma(\rho_1)} \sum_{j=2}^{n} \frac{k\big(\tau_j, V^j\big) - 2k\big(\tau_{j-1}, V^{j-1}\big) + k\big(\tau_{j-2}, V^{j-2}\big)}{2(\Delta\tau)^2} \int_{\tau_j}^{\tau_{j+1}} \big(t-\tau_{j-2}\big)\big(t - \tau_{j-1}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt.$$

Now, calculating the integrals in above system, we get

$$\int_{\tau_j}^{\tau_{j+1}} (\tau_{n+1}-t)^{\alpha_1-1} dt = \frac{(\Delta\tau)^{\alpha_1}}{\alpha_1}\left[(n-j+1)^{\alpha_1} - (n-j)^{\alpha_1}\right],$$

$$\int_{\tau_j}^{\tau_{j+1}} \big(t-\tau_{j-2}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt = \frac{(\Delta\tau)^{\alpha_1+1}}{\alpha_1(\alpha_1+1)}\left[(n-j+1)^{\alpha_1}(n-j+3+2\alpha_1) - (n-j+1)^{\alpha_1}(n-j+3+3\alpha_1)\right],$$

$$\int_{\tau_j}^{\tau_{j+1}} \big(t-\tau_{j-2}\big)\big(t-\tau_{j-1}\big)(\tau_{n+1}-t)^{\alpha_1-1} dt = \frac{(\Delta\tau)^{\alpha_1+2}}{\alpha_1(\alpha_1+1)(\alpha_1+2)}\left[(n-j+1)^{\alpha_1}\{2(n-j)^2 + (3\alpha_1+10)(n-j) + 2{\alpha_1}^2 + 9\alpha_1 + 12\} - (n-j)^{\alpha_1}\{2(n-j)^2 + (5\alpha_1+10)(n-j) + 6{\alpha_1}^2 + 18\alpha_1 + 12\}\right].$$

Inserting them into a system (31), we get

$$S^{n+1} = S^0 + \frac{(1-\alpha_1)}{C(\alpha_1)} k\big(\tau_n, S(\tau_n)\big) + \frac{\alpha_1(\Delta\tau)^{\alpha_1}}{C(\alpha_1)\Gamma(\alpha_1+1)} \sum_{j=2}^{n} k\big(\tau_{j-2}, S^{j-2}\big)\left[(n-j+1)^{\alpha_1} - (n-j)^{\alpha_1}\right] + \frac{\alpha_1(\Delta\tau)^{\alpha_1}}{C(\alpha_1)\Gamma(\alpha_1+2)} \sum_{j=2}^{n} \left[k\big(\tau_{j-1}, S^{j-1}\big) - k\big(\tau_{j-2}, S^{j-2}\big)\right]\left[(n-j+1)^{\alpha_1}(n-j+3+2\alpha_1) - (n-j+1)^{\alpha_1}(n-j+3+3\alpha_1)\right] + \frac{\alpha_1(\Delta\tau)^{\alpha_1}}{C(\alpha_1)\Gamma(\alpha_1+3)} \sum_{j=2}^{n} \left[k\big(\tau_j, S^j\big) - 2k\big(\tau_{j-1}, S^{j-1}\big) + k\big(\tau_{j-2}, S^{j-2}\big)\right]\left[(n-j+1)^{\alpha_1}\{2(n-j)^2 + (3\alpha_1+10)(n-j) + 2{\alpha_1}^2 + 9\alpha_1 + 12\} - (n-j)^{\alpha_1}\{2(n-j)^2 + (5\alpha_1+10)(n-j) + 6{\alpha_1}^2 + 18\alpha_1 + 12\}\right],$$

$$I^{n+1} = I^0 + \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)} k\big(\tau_n, I(\tau_n)\big) + \frac{\alpha_1 (\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+1)} \sum_{j=2}^{n} k\big(\tau_{j-2}, I^{j-2}\big)[(n-j+1)^{\alpha_1} - (n-j)^{\alpha_1}]$$
$$+ \frac{\alpha_1 (\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+2)} \sum_{j=2}^{n} \big[k\big(\tau_{j-1}, I^{j-1}\big)$$
$$- k\big(\tau_{j-2}, I^{j-2}\big)\big][(n-j+1)^{\alpha_1}(n-j+3+2\alpha_1)$$
$$- (n-j+1)^{\alpha_1}(n-j+3+3\alpha_1)]$$
$$+ \frac{\alpha_1 (\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+3)} \sum_{j=2}^{n} \big[k\big(\tau_j, I^j\big) - 2k\big(\tau_{j-1}, I^{j-1}\big)$$
$$+ k\big(\tau_{j-2}, I^{j-2}\big)\big][(n-j+1)^{\alpha_1}\{2(n-j)^2 + (3\alpha_1+10)(n-j) + 2{\alpha_1}^2 + 9\alpha_1 + 12\}$$
$$- (n-j)^{\alpha_1}\{2(n-j)^2 + (5\alpha_1+10)(n-j) + 6{\alpha_1}^2 + 18\alpha_1 + 12\}], \tag{32}$$

$$V^{n+1} = V^0 + \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)} k\big(\tau_n, V(\tau_n)\big)$$
$$+ \frac{\alpha_1 (\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+1)} \sum_{j=2}^{n} k\big(\tau_{j-2}, V^{j-2}\big)[(n-j+1)^{\alpha_1} - (n-j)^{\alpha_1}]$$
$$+ \frac{\alpha_1 (\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+2)} \sum_{j=2}^{n} \big[k\big(\tau_{j-1}, V^{j-1}\big) - k\big(\tau_{j-2}, V^{j-2}\big)\big][(n-j+1)^{\alpha_1}(n-j+3+2\alpha_1)$$
$$- (n-j+1)^{\alpha_1}(n-j+3+3\alpha_1)]$$
$$+ \frac{\alpha_1 (\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+3)} \sum_{j=2}^{n} \big[k\big(\tau_j, V^j\big) - 2k\big(\tau_{j-1}, V^{j-1}\big)$$
$$+ k\big(\tau_{j-2}, V^{j-2}\big)\big][(n-j+1)^{\alpha_1}\{2(n-j)^2 + (3\alpha_1+10)(n-j) + 2{\alpha_1}^2 + 9\alpha_1 + 12\}$$
$$- (n-j)^{\alpha_1}\{2(n-j)^2 + (5\alpha_1+10)(n-j) + 6{\alpha_1}^2 + 18\alpha_1 + 12\}].$$

Finally, we have the following approximation:

$$S^{n+1} = S^0 + \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)} \alpha_2 \tau^{\alpha_2-1}\{\alpha - \beta S(\tau_n)V(\tau_n) - \delta S(\tau_n)\}$$
$$+ \frac{\alpha_1 \alpha_2 (\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+1)} \sum_{j=2}^{n} \tau^{\alpha_2-1}\big\{\alpha - \beta S^{j-2}V^{j-2} - \delta S^{j-2}\big\}[(n-j+1)^{\alpha_1} - (n-j)^{\alpha_1}]$$
$$+ \frac{\alpha_1 \alpha_2 (\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+2)} \sum_{j=2}^{n} \big[\tau^{\alpha_2-1}\big\{\alpha - \beta S^{j-1}V^{j-1} - \delta S^{j-1}\big\}$$
$$- \tau^{\alpha_2-1}\big\{\alpha - \beta S^{j-2}V^{j-2} - \delta S^{j-2}\big\}\big][(n-j+1)^{\alpha_1}(n-j+3+2\alpha_1)$$
$$- (n-j+1)^{\alpha_1}(n-j+3+3\alpha_1)]$$
$$+ \frac{\alpha_1 \alpha_2 (\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+3)} \sum_{j=2}^{n} \big[\tau^{\alpha_2-1}\big\{\alpha - \beta S^j V^j - \delta S^j\big\} - 2\tau^{\alpha_2-1}\big\{\alpha - \beta S^{j-1}V^{j-1} - \delta S^{j-1}\big\}$$
$$+ \tau^{\alpha_2-1}\big\{\alpha - \beta S^{j-2}V^{j-2} - \delta S^{j-2}\big\}\big][(n-j+1)^{\alpha_1}\{2(n-j)^2 + (3\alpha_1+10)(n-j) + 2{\alpha_1}^2$$
$$+ 9\alpha_1 + 12\} - (n-j)^{\alpha_1}\{2(n-j)^2 + (5\alpha_1+10)(n-j) + 6{\alpha_1}^2 + 18\alpha_1 + 12\}],$$

$$
\begin{aligned}
I^{n+1} = I^0 + &\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2\tau^{\alpha_2-1}\{\beta S(\tau_n)V(\tau_n)-\sigma I(\tau_n)\} \\
&+\frac{\alpha_1\alpha_2(\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+1)}\sum_{j=2}^{n}\tau^{\alpha_2-1}\{\beta S^{j-2}V^{j-2}-\sigma I^{j-2}\}[(n-j+1)^{\alpha_1}-(n-j)^{\alpha_1}] \\
&+\frac{\alpha_1\alpha_2(\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+2)}\sum_{j=2}^{n}\big[\tau^{\alpha_2-1}\{\beta S^{j-1}V^{j-1}-\sigma I^{j-1}\} \\
&-\tau^{\alpha_2-1}\{\beta S^{j-2}V^{j-2}-\sigma I^{j-2}\}\big][(n-j+1)^{\alpha_1}(n-j+3+2\alpha_1) \\
&-(n-j+1)^{\alpha_1}(n-j+3+3\alpha_1)] \\
&+\frac{\alpha_1\alpha_2(\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+3)}\sum_{j=2}^{n}\big[\tau^{\alpha_2-1}\{\beta S^{j}V^{j}-\sigma I^{j}\}-2\tau^{\alpha_2-1}\{\beta S^{j-1}V^{j-1}-\sigma I^{j-1}\} \\
&+\tau^{\alpha_2-1}\{\beta S^{j-2}V^{j-2}-\sigma I^{j-2}\}\big][(n-j+1)^{\alpha_1}\{2(n-j)^2+(3\alpha_1+10)(n-j) \\
&+2{\alpha_1}^2+9\alpha_1+12\} \\
&-(n-j)^{\alpha_1}\{2(n-j)^2+(5\alpha_1+10)(n-j)+6{\alpha_1}^2+18\alpha_1+12\}],
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
V^{n+1} = V^0 + &\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2\tau^{\alpha_2-1}\{\mu_n I(\tau_n)-(p_1+p_2)V(\tau_n)-\beta S(\tau_n)V(\tau_n)\} \\
&+\frac{\alpha_1\alpha_2(\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+1)}\sum_{j=2}^{n}\tau^{\alpha_2-1}\{\mu_n I^{j-2}-(p_1+p_2)V^{j-2}-\beta S^{j-2}V^{j-2}\}[(n-j+1)^{\alpha_1} \\
&-(n-j)^{\alpha_1}] \\
&+\frac{\alpha_1\alpha_2(\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+2)}\sum_{j=2}^{n}\big[\tau^{\alpha_2-1}\{\mu_n I^{j-1}-(p_1+p_2)V^{j-1}-\beta S^{j-1}V^{j-1}\} \\
&-\tau^{\alpha_2-1}\{\mu_n I^{j-2}-(p_1+p_2)V^{j-2}-\beta S^{j-2}V^{j-2}\}\big][(n-j+1)^{\alpha_1}(n-j+3+2\alpha_1) \\
&-(n-j+1)^{\alpha_1}(n-j+3+3\alpha_1)] \\
&+\frac{\alpha_1\alpha_2(\Delta\tau)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+3)}\sum_{j=2}^{n}\big[\tau^{\alpha_2-1}\{\mu_n I^{j}-(p_1+p_2)V^{j}-\beta S^{j}V^{j}\} \\
&-2\tau^{\alpha_2-1}\{\mu_n I^{j-1}-(p_1+p_2)V^{j-1}-\beta S^{j-1}V^{j-1}\} \\
&+\tau^{\alpha_2-1}\{\mu_n I^{j-2}-(p_1+p_2)V^{j-2}-\beta S^{j-2}V^{j-2}\}\big][(n-j+1)^{\alpha_1}\{2(n-j)^2 \\
&+(3\alpha_1+10)(n-j)+2{\alpha_1}^2+9\alpha_1+12\} \\
&-(n-j)^{\alpha_1}\{2(n-j)^2+(5\alpha_1+10)(n-j)+6{\alpha_1}^2+18\alpha_1+12\}].
\end{aligned}
$$

6. Results and discussion

To identify the potential effectiveness of the dengue transmission in the Community, the dengue fractional-order model is presented to analyze its effect in society. That's why; we have used ABC with ST, advanced AT scheme, and fractal fractional derivative techniques for fractional-order dengue model. The various numerical methods identify the mechanical features of the fractional-order model with the time-fractional parameters. The results of the nonlinear system memory were also detected with the help of fractional values.

7. Conclusions

In this manuscript, we have investigated a non-integer order dengue internal transmission model with the help of the Picard successive approximation technique and Banach contraction theorem. The arbitrary derivative of fractional order γ and α has been taken ABC with MLK. The concerned results have been handled by coupling ST with mentioned iterative techniques. Also, an algorithm was developed with AT and fractal fractional techniques for numerical results. Existence theory and uniqueness for the equilibrium solution are provided via nonlinear functional analysis and fixed point theory. Global stability of the system was also proved by using the Lyapunov function. The arbitrary derivative of fractional order has been taken in the ABC and fractal fractional Atangana-Baleanu with Mittag-Leffler kernel. Non-linear fractional differential equations were raised from the derivative with the help of a non-singular and non-local kernel within the fractional derivative framework. This kind of study is helpful for a physician for planning and decision-making for the treatment of the disease.

Conflict of interest

Authors has no conflicts of interest.

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