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Research article

The discrete power-Ailamujia distribution: properties, inference, and applications

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Abstract: In this article, a new two-parameter discrete power-Ailamujia (DsPA) distribution is derived using the survival discretization technique. Some key distributional properties and reliability measures are explored in closed forms, such as probability generating function, first four moments and mean residual life. The DsPA parameters are estimated using the maximum likelihood approach. The performance of this estimation method is assessed via a simulation study. The flexibility of the DsPA distribution is shown using three count datasets. The DsPA distribution provides a better fit than some recent discrete models such as the discrete Burr-XII, uniform Poisson–Ailamujia, Poisson, discrete-Pareto, discrete-Rayleigh, discrete inverse-Rayleigh, and discrete Burr–Hutke distributions.

Keywords: power-Ailamujia distribution; discretization; reliability analysis; parameter estimation; mean residual life
Mathematics Subject Classification: 60E05, 62E10, 62F10, 62N05,62P10

1. Introduction

In numerous practical situations, the datasets are measured in the number of cycles, runs, and/or shocks the device sustains before its failure. For example, the number of voltage fluctuations, the lifetime of a discrete random variable (rv), and frequency of a device switched on/off, the life of a weapon is measured by the number of rounds fired before failure, and the number of completed cycles measures the life of the equipment. Further, the number of patients, number of deaths due to a disease/virus, and number of days a patient stays in a hospital ward. Various discrete probability models can be adopted to analyze such types of datasets.

The well-known traditional discrete probability models, including the negative binomial, geometric, and Poisson distributions have limitations to use due to their specific behavior such as the Poisson distribution that performs better with datasets having dispersion equal to average; the NB distribution is applicable for over-dispersed datasets. The real-life datasets may be over-dispersed or under-dispersed, so there is always a clear need for flexible discrete distributions to have a good resolution.

Several discretized forms of continuous distributions have been derived to model different count datasets in the last few decades. The most notable discretization approach in the literature is the survival discretizing approach which has gained much attention.

Let $rv\ X$ follows a continuous distribution with survival function (sf) $S(x)$. Using the survival discretization approach introduced by Kemp (2004), the probability mass function (pmf) of a discrete rv follows as

$$p(x) = P(X = x) = S\,(x) - S\,(x + 1), \qquad x = 0,1,2,3, \ldots \tag{1}$$

The survival discretization approach has been adopted to develop many discrete models. For example, the discrete normal [1], discrete Rayleigh [2], discrete half-normal [3], discrete Burr and discrete Pareto [4], discrete inverse-Weibull [5], new generalization of the geometric [6], discrete Lindley [7], generalized exponential type II [8], discrete inverse-Rayleigh [9], two-parameter discrete Lindley [10], discrete log-logistic [11], discrete extended Weibull [12], exponentiated discrete-Lindley [13], discrete Burr-Hutke [14], discrete Marshall-Olkin Weibull [15], natural discrete-Lindley [16], discrete Bilal [17], discrete inverted Topp-Leone [18], uniform Poisson–Ailamujia [19], exponentiated discrete Lindley [20], discrete exponentiated Burr–Hatke [21], discrete Ramos-Louzada [22] and [23], and discrete type-II half-logistic exponential [24].

The main goal of the present study is to introduce a new discrete distribution to model over-dispersed as well as under-dispersed datasets. The proposed distribution is called the discrete power-Ailamujia (DsPA) distribution. The mathematical properties of the DsPA distribution are derived and its parameters are estimated using the maximum likelihood method. Three real count datasets are fitted using the DsPA model and other competing discrete distributions. The DsPA distribution provides a better fit to the three datasets than some well-known discrete models according to the results of the simulation.

The paper is organized in the following sections. Section 2 is devoted to the derivation of the new DsPA distribution. Its mathematical properties are explored in Section 3. Section 4 is devoted to estimating the DsPA parameters and providing a comprehensive simulation study. The usefulness of the DsPA distribution is addressed in Section 5. Finally, we conclude the study in Section 6.

2. The DsPA distribution

Jamal et al. [25] proposed a new continuous lifetime distribution called the power-Ailamujia distribution. Its probability density function and sf can be expressed as

$$f(x) = \theta^2 \beta x^{2\beta-1} e^{-\theta x^\beta}, \quad x \geq 0, \theta, \beta > 0 \tag{2}$$

and

$$S(x) = \left(1 + \theta x^\beta\right) e^{-\theta x^\beta}, \quad x \geq 0, \theta, \beta > 0, \tag{3}$$

respectively.

Applying the survival discretization approach in (1), the rv X is said to have the DsPA distribution with parameters $0 < \lambda < 1$ and $\beta > 0$, if its sf takes the form

$$S(x; \lambda, \beta) = \lambda^{(x+1)^\beta}\left[1 - (x+1)^\beta \ln \lambda\right], \quad x \in \mathbb{N}_0, \tag{4}$$

where $\lambda = e^{-\theta}$ and $\mathbb{N}_0 = \{0, 1, 2, \ldots, w\}$ for $0 < w < \infty$.

The corresponding cumulative distribution function (cdf) and pmf can be expressed as

$$F(x; \lambda, \beta) = 1 - \lambda^{(x+1)^\beta}\left[1 - (x+1)^\beta \ln \lambda\right], \quad x \in \mathbb{N}_0 \tag{5}$$

and

$$P_x(x; \lambda) = \lambda^{x^\beta}\left[1 - x^\beta \ ln\, \lambda\right] - \lambda^{(x+1)^\beta}\left[1 - (x+1)^\beta \ln \lambda\right], \quad x \in \mathbb{N}_0. \tag{6}$$

Plots of the DsPA pmf, for various values of the parameters λ and β, are presented in Figure 1.

The hazard rate function (hrf) of the DsPA distribution can be expressed as

$$h(x; \lambda) = \frac{p(x)}{S(x)} = \frac{\lambda^{x^\beta}\left[1 - x^\beta \ ln\, \lambda\right]}{\lambda^{(x+1)^\beta}\left[1 - (x+1)^\beta \ln \lambda\right]} - 1, \quad x \in \mathbb{N}_0 \tag{7}$$

where $h(x; \lambda) = \frac{P(x)}{S(x)}$. Figure 2 shows the DsPA hrf plots for different values of λ and β.

The quantile function of the DsPA distribution reduces to

$$Q(u) = \left\lfloor 1 - \left[1 - (x+1)^\beta \ln \lambda\right] \lambda^{(x+1)^\beta} \right\rfloor, 0 < u < 1,$$

where $\lfloor x \rfloor$ denotes the integer part of x.

The reverse hrf (rhrf) of the DsPA distribution is defined as

$$r^*(x) = \frac{p(x)}{F(x)} = \frac{\lambda^{x^\beta}\left[1 - x^\beta \ ln\, \lambda\right] - \lambda^{(x+1)^\beta}\left[1 - (x+1)^\beta \ln \lambda\right]}{1 - \lambda^{(x+1)^\beta}\left[1 - (x+1)^\beta \ln \lambda\right]}, \quad x \in \mathbb{N}_0, \tag{8}$$

where $r^*(x) = \frac{P(x)}{F(x)}$. Figure 3 shows the DsPA rhrf plots for several values of λ and β.

The second failure rate of the DsPA distribution is expressed by

$$r^{**}(x) = \log\left\{\frac{S(x)}{S(x+1)}\right\} = \log\left\{\frac{\lambda^{(x+1)^\beta}\left[1 - (x+1)^\beta \ln \lambda\right]}{\lambda^{(x+2)^\beta}\left[1 - (x+2)^\beta \ln \lambda\right]}\right\}, \quad x \in \mathbb{N}_0. \tag{9}$$

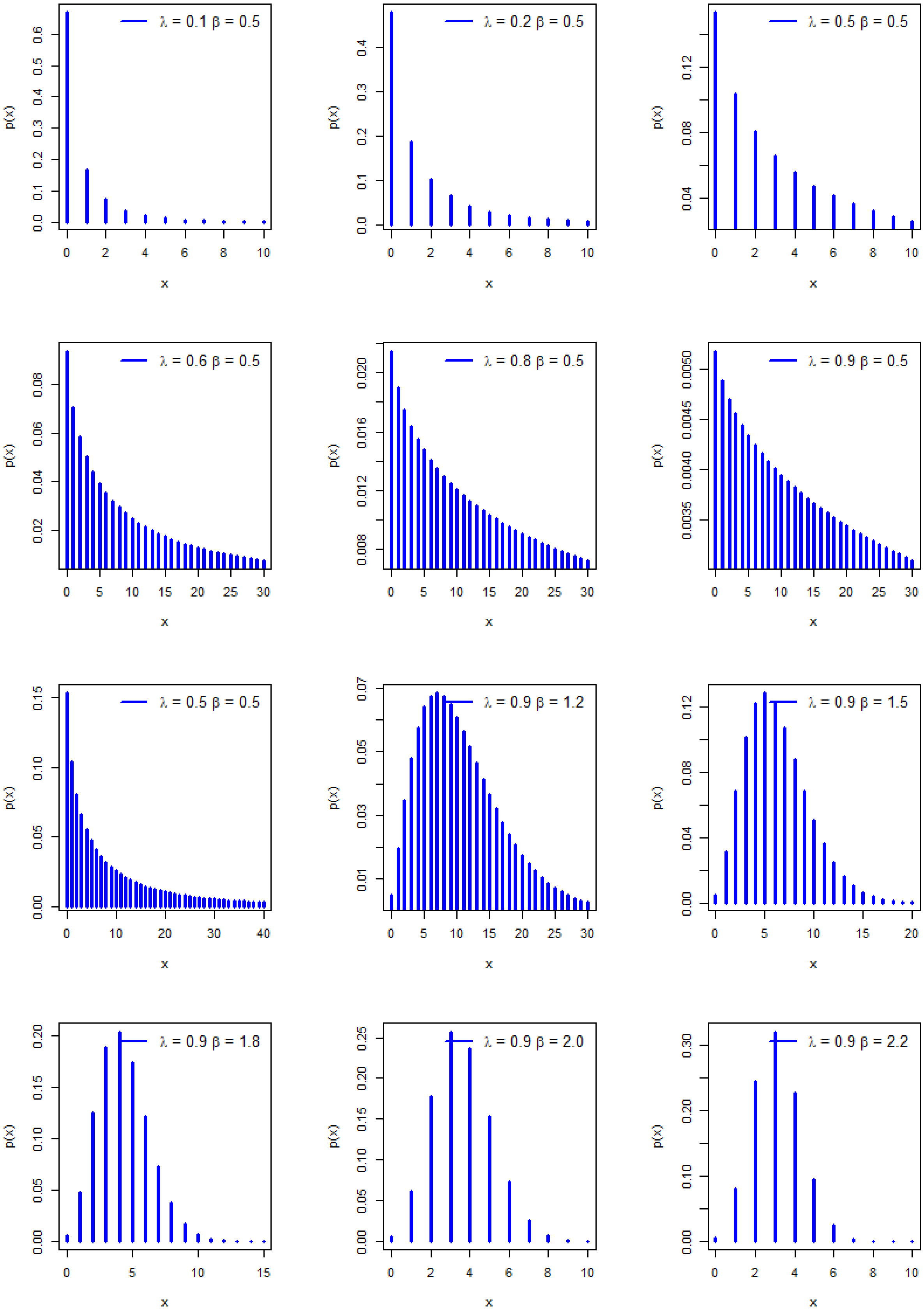

Figure 1. Shapes of the DsPA pmf for some values of λ and β.

The recurrence relation of probabilities from the DsPA distribution has the form

$$\frac{P(x+1)}{P(x)} = \frac{\lambda^{(x+1)^\beta}\left[1-(x+1)^\beta \, ln\,\lambda\right] - \lambda^{(x+2)^\beta}\left[1-(x+2)^\beta \ln\lambda\right]}{\lambda^{x^\beta}\left[1-x^\beta \, ln\,\lambda\right] - \lambda^{(x+1)^\beta}\left[1-(x+1)^\beta \ln\lambda\right]}. \tag{10}$$

Hence,

$$P(x+1) = \frac{\lambda^{(x+1)^\beta}\left[1-(x+1)^\beta \, ln\,\lambda\right] - \lambda^{(x+2)^\beta}\left[1-(x+2)^\beta \ln\lambda\right]}{\lambda^{x^\beta}\left[1-x^\beta \, ln\,\lambda\right] - \lambda^{(x+1)^\beta}\left[1-(x+1)^\beta \ln\lambda\right]} P(x).$$

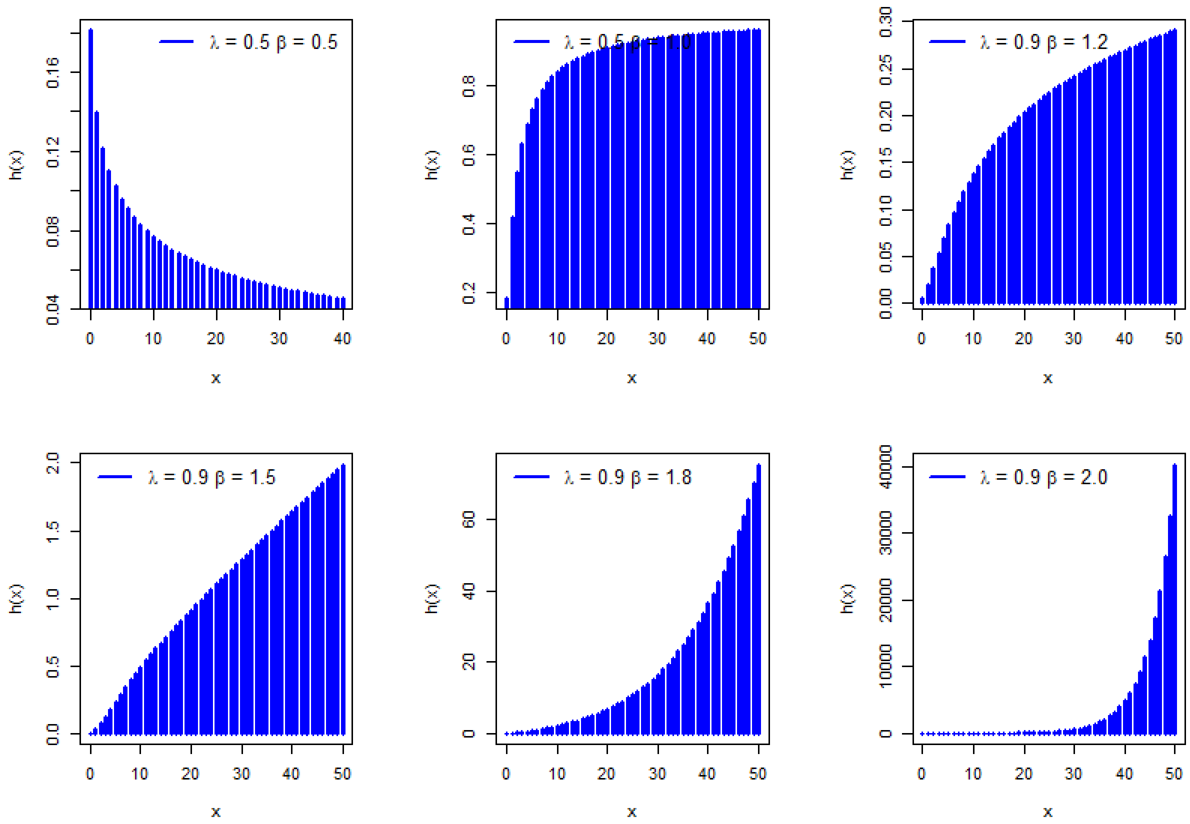

Figure 2. Shapes of the DsPA hrf for some values of λ and β.

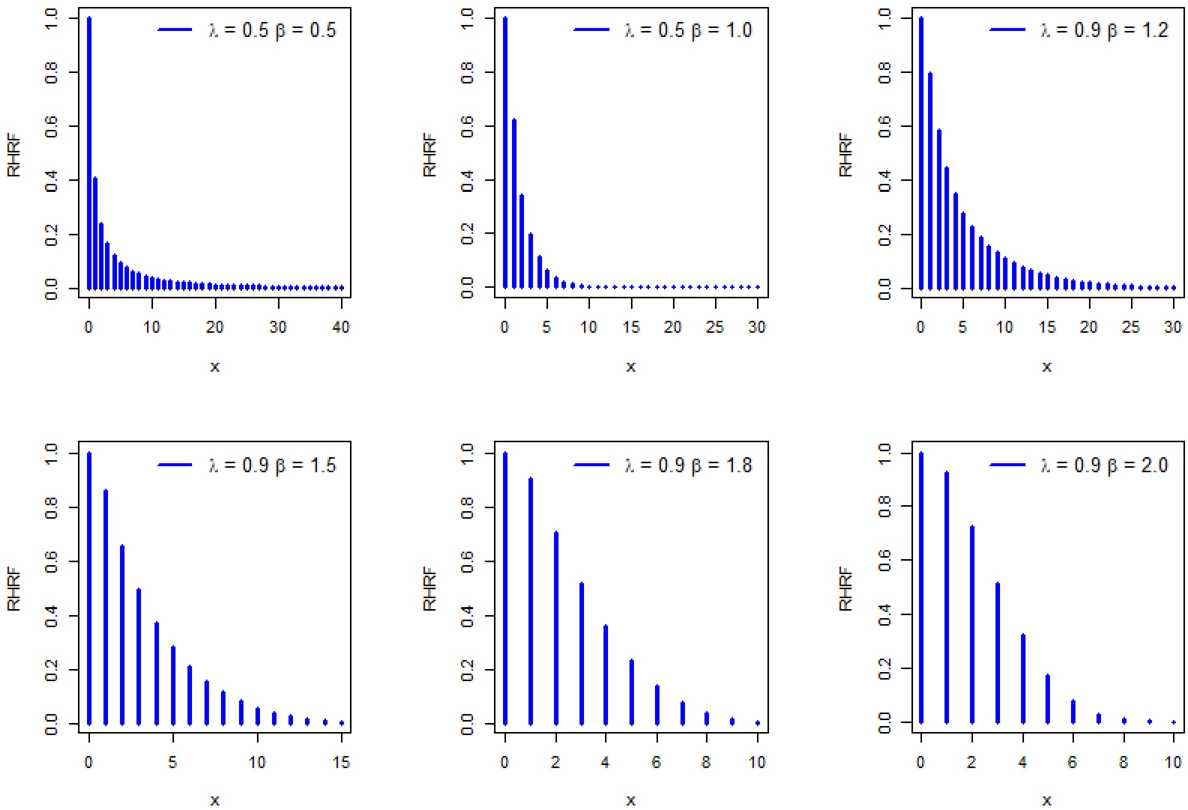

Figure 3. Possible shapes of the DsPA rhrf for several values of λ and β.

3. Some properties of the DsPA distribution

In this section, we studied some mathematical properties of the DsPA distribution. In this section, we studied some mathematical properties of the DsPA distribution.

3.1 Moments and generating functions

The probability generating function (pgf) of the DsPA distribution is given as follows

$$G_x(z) = 1 + (z-1) \sum_{x=1}^{\infty} z^{x-1} \left(1 - x^{\beta} \ln \lambda\right) \lambda^{x^{\beta}}, \tag{11}$$

where $G_x(z) = \sum_{x=0}^{\infty} z^x \, P(x)$. The moment generating function (mgf) can be obtained by replacing z with e^z in Eq (11). Thus, the mgf of the DsPA distribution can be expressed as

$$M_x(z) = 1 + (e^z - 1) \sum_{x=1}^{\infty} (e^z)^{x-1} \left(1 - x^{\beta} \ln \lambda\right) \lambda^{x^{\beta}}. \tag{12}$$

Thus, the first four moments of the DsPA distribution are

$$E(X) = \sum_{x=1}^{\infty} \left(1 - x^{\beta} \ln \lambda\right) \lambda^{x^{\beta}}, \tag{13}$$

$$E(X^2) = \sum_{x=1}^{\infty} (2x - 1) \left(1 - x^{\beta} \ln \lambda\right) \lambda^{x^{\beta}},$$

$$E(X^3) = \sum_{x=1}^{\infty} (3x^2 - 3x + 1) \left(1 - x^{\beta} \ln \lambda\right) \lambda^{x^{\beta}}$$

and

$$E(X^4) = \sum_{x=1}^{\infty} (4x^3 - 6x^2 + 4x - 1) \left(1 - x^{\beta} \ln \lambda\right) \lambda^{x^{\beta}}.$$

Using the above moments, the variance (σ^2), coefficient of skewness (CS), and coefficient of kurtosis (CK) can be presented in closed-form expressions. Further, another classical concept, called dispersion index (DI). The DI is defined as a variance to mean ratio. If the DI value is less than 1, then the model is suitable for under-dispersed datasets. Conversely, if the DI is greater than 1, then it is suitable for over-dispersed datasets. Numerical values of the mean, $E(X)$, σ^2, CS, CK and DI are reported in Tables 1–5.

Table 1. Numerical values for the mean of the DsPA model.

β	λ								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.5	0.7603	1.8909	3.6766	6.5446	10.966	16.848	22.277	22.458	12.187
1.0	0.3954	0.7529	1.1657	1.6848	2.3863	3.4156	5.1075	8.4629	18.452
1.5	0.3415	0.5827	0.8221	1.0907	1.4187	1.8533	2.4910	3.5895	6.2446
2.0	0.3313	0.5338	0.7085	0.8884	1.0967	1.3599	1.7259	2.3141	3.5954
2.5	0.3303	0.5230	0.6698	0.8012	0.9444	1.1259	1.3763	1.7634	2.5557
3.0	0.3303	0.5219	0.6619	0.7720	0.8721	0.9919	1.1725	1.4628	2.0209

Table 2. Numerical values for the variance of the DsPA model.

β	λ								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.5	2.7920	12.243	38.665	103.06	229.88	417.28	621.34	779.36	615.37
1.0	0.3901	0.8140	1.4378	2.4502	4.2371	7.7429	15.802	40.249	177.94
1.5	0.2475	0.3692	0.4988	0.6724	0.9299	1.3485	2.1203	3.8858	10.421
2.0	0.2236	0.2727	0.3017	0.3478	0.4232	0.5408	0.7369	1.1271	2.2935
2.5	0.2213	0.2517	0.2384	0.2287	0.2486	0.3033	0.3864	0.5260	0.8911
3.0	0.2212	0.2496	0.2252	0.1869	0.1626	0.1789	0.2448	0.3256	0.4729

Table 3. Numerical values for the skewness of the DsPA model.

β	λ								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.5	4.6187	4.3533	3.9822	3.2831	2.4657	1.7555	1.2590	1.1461	2.0439
1.0	1.6228	1.4038	1.3664	1.3697	1.3821	1.3943	1.4037	1.4099	1.3511
1.5	0.9461	0.5856	0.5417	0.5744	0.6191	0.6586	0.6895	0.7124	0.7285
2.0	0.7459	0.1163	-0.0092	0.1018	0.2223	0.2912	0.3312	0.3598	0.3836
2.5	0.7226	-0.0655	-0.4988	-0.4984	-0.1012	0.1683	0.1697	0.1591	0.1742
3.0	0.7218	-0.0869	-0.6648	-1.0922	-0.9663	-0.0481	0.3705	0.0738	0.0476

Table 4. Numerical values for the kurtosis of the DsPA model.

β	λ								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.5	41.471	36.784	28.534	18.030	10.188	5.7950	3.7130	3.1485	5.9942
1.0	5.9572	5.6739	5.7391	5.8240	5.8909	5.9380	5.9690	5.9875	5.4737
1.5	2.5630	2.7503	3.1474	3.3764	3.4912	3.5508	3.5857	3.6093	3.6262
2.0	1.6369	1.5891	2.4907	3.0833	3.1595	3.0954	3.0599	3.0514	3.0542
2.5	1.5246	1.0744	1.8108	3.2354	3.9993	3.4076	2.8767	2.8838	2.8977
3.0	1.5211	1.0098	1.4967	2.7867	4.9278	5.5909	3.4605	2.4008	2.8802

Table 5. Numerical values for the DI of the DsPA model.

β	λ								
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.5	3.6720	6.4746	10.516	15.747	20.964	24.768	27.891	34.703	50.494
1.0	0.9866	1.0810	1.2335	1.4543	1.7756	2.2669	3.0939	4.7559	9.6437
1.5	0.7248	0.6337	0.6068	0.6165	0.6555	0.7276	0.8512	1.0826	1.6688
2.0	0.6749	0.5109	0.4258	0.3915	0.3859	0.3977	0.4269	0.4870	0.6379
2.5	0.6699	0.4813	0.3559	0.2854	0.2632	0.2694	0.2808	0.2983	0.3487
3.0	0.6697	0.4782	0.3402	0.2422	0.1865	0.1804	0.2088	0.2226	0.2340

From Tables 1–5, we can conclude that the mean is an increasing function of λ and a decreasing function of β. It is clear that the skewness of the DsPA distribution can be positive or negative. The DI showing increasing behavior for larger values of the parameter λ and small values of β. Further, the DsPA distribution is suitable for over-dispersed and under-dispersed data sets.

3.2. Mean Residual Life (MRL)

The MRL function is a helpful reliability characteristic to model and analyze the burn-in and maintenance policies. Consider the *rv* X that has the cdf $F(.)$. For a discrete *rv*, the MRL function is defined by

$$MRL = \varepsilon(i) = E(X - i|X \geq i) = \frac{1}{1 - F(i-1,\lambda)} \sum_{j=i+1}^{w} [1 - F(j-1,\lambda)], \quad i \in \mathbb{N}_0,$$

where $\mathbb{N}_0 = \{0, 1, 2, \ldots, w\}$ and $0 < w < \infty$.

Then, the MRL of the DsPA model reduces to

$$MRL = \frac{1}{1 - F(i-1,\lambda,\beta)} \sum_{j=i+1}^{w} [1 - F(j-1,\lambda,\beta)]$$

$$= \frac{1}{[1 - (i)^{\beta} \ln \lambda]\lambda^{(i)^{\beta}}} \sum_{j=i+1}^{w} [1 - (j)^{\beta} \ln \lambda]\lambda^{(j)^{\beta}}$$

$$= \frac{1}{[1 - (i)^{\beta} \ln \lambda]\lambda^{(i)^{\beta}}} \left[\sum_{j=i+1}^{w} \lambda^{(j)^{\beta}} - \ln \lambda \sum_{j=i+1}^{w} (j)^{\beta} \lambda^{(j)^{\beta}}\right].$$

4. Parameter estimation

In this section, the parameters λ and β are estimated using the maximum likelihood (ML) method.

4.1. *Maximum likelihood estimation*

Suppose $x_1, \ldots, x_n$ be a random sample from the DsPA distribution with pmf (6). Then the log-likelihood function takes the form

$$L = \frac{1}{[1 - (i)^{\beta} \ln \lambda]\lambda^{(i)^{\beta}}} \sum_{j=i+1}^{w} [1 - (j)^{\beta} \ln \lambda]\lambda^{(j)^{\beta}} . \tag{14}$$

Now, by differentiating (14) w.r.t λ and β, we can write

$$\frac{\partial L}{\partial \lambda} = \sum_{i=1}^{n} \frac{\ln \lambda \left[(x_i+1)^{2\beta} \lambda^{(x_i+1)^{\beta}} - {x_i}^{2\beta} \lambda^{{x_i}^{\beta}}\right]}{\lambda\{[1 - {x_i}^{\beta} \ln \lambda]\, \lambda^{{x_i}^{\beta}} - [1 - (x_i+1)^{\beta} \ln \lambda]\lambda^{(x_i+1)^{\beta}}\}} = 0 \tag{15}$$

and

$$\frac{\partial L}{\partial \beta} = \sum_{i=1}^{n} \frac{(\ln \lambda)^2 \left[\lambda^{(x_i+1)^{\beta}} (x_i+1)^{2\beta} \ln(x_i+1) - \lambda^{{x_i}^{\beta}} {x_i}^{2\beta} \ln x_i\right]}{\{[1 - {x_i}^{\beta} \ln \lambda]\, \lambda^{{x_i}^{\beta}} - [1 - (x_i+1)^{\beta} \ln \lambda]\lambda^{(x_i+1)^{\beta}}\}} = 0. \tag{16}$$

The ML estimates (MLEs) of λ and β follow from the above equation. Eqs (15) and (16) can be solved using iterative procedures such as Newton-Raphson. For this purpose, we use the maxLik function of R software [26].

4.2. *Simulation study*

In this section, we carried out a numerical simulation to access the performance of the ML estimation method. This assessment is done by generating $N = 10{,}000$ samples using the qf of the DsPA model for different sample sizes $n = 10, 20, 50,$ and 100 and for several values of the parameters λ and β, where $(\lambda, \beta) = (0.50,\ 0.50), (0.50, 2.0),\ (0.90,\ 1.20), (0.90, 2.0)$. The assessment is completed using absolute bias, mean relative errors (MREs), and mean square errors (MSEs) which are defined by

$$Bias(\boldsymbol{\delta}) = \frac{1}{N}\sum_{i=1}^{N}\left|\widehat{\boldsymbol{\delta}}_i - \boldsymbol{\delta}\right|, MSE(\boldsymbol{\delta}) = \frac{1}{N}\sum_{i=1}^{N}\left(\widehat{\boldsymbol{\delta}}_i - \boldsymbol{\delta}\right)^2 \text{ and } MRE(\boldsymbol{\delta}) = \frac{1}{N}\sum_{i=1}^{N}\frac{\widehat{\boldsymbol{\delta}}_i}{\boldsymbol{\delta}_i},$$

where $\boldsymbol{\delta} = (\lambda,\beta)$.

The simulation results for λ and β are reported in Tables 6–8. The bias, MSE and MRE of the parameters λ and β are computed using the R program using the ML method. For all values of λ and β, the ML estimation approach illustrates the consistency property, that is, the MSEs and MREs decrease as n increases.

From Tables 6–8, we conclude that:

1. The estimates of λ and β close to their true values with the increase of n for all studied cases.
2. The MSEs for λ and β decrease with the increase of n for all studied cases.
3. The MREs for λ and β decrease with the increase of n for all studied cases.

Table 6. Simulation results of the DsPA distribution for $\lambda = 0.5$ and $\beta = 0.5$.

n	$E(\lambda)$	$E(\beta)$	Bias(λ)	Bias(β)	MSE(λ)	MSE(β)	MRE(λ)	MRE(β)
10	0.4605	0.5804	-0.0395	0.0804	0.0483	0.0333	0.0789	0.1609
20	0.4796	0.5370	-0.0204	0.0370	0.0247	0.0111	0.0409	0.0740
50	0.4910	0.5143	-0.0090	0.0143	0.0098	0.0033	0.0180	0.0286
100	0.4966	0.5065	-0.0034	0.0065	0.0049	0.0015	0.0069	0.0130
200	0.4975	0.5037	-0.0025	0.0037	0.0024	0.0007	0.0050	0.0074

Table 7. Simulation results of the DsPA distribution for $\lambda = 0.5$ and $\beta = 2.0$.

n	$E(\lambda)$	$E(\beta)$	Bias(λ)	Bias(β)	MSE(λ)	MSE(β)	MRE(λ)	MRE(β)
10	0.4654	2.3160	-0.0346	0.3160	0.0490	0.5537	0.0692	0.1580
20	0.4809	2.1419	-0.0191	0.1419	0.0243	0.1730	0.0382	0.0709
50	0.4905	2.0571	-0.0095	0.0571	0.0097	0.0514	0.0191	0.0286
100	0.4970	2.0243	-0.0030	0.0243	0.0050	0.0247	0.0060	0.0122
200	0.4984	2.0125	-0.0016	0.0125	0.0024	0.0115	0.0032	0.0062

Table 8. Simulation results of the DsPA distribution for $\lambda = 0.9$ and $\beta = 1.2$.

n	$E(\lambda)$	$E(\beta)$	Bias(λ)	Bias(β)	MSE(λ)	MSE(β)	MRE(λ)	MRE(β)
10	0.8630	1.3915	-0.0370	0.1915	0.0990	0.1973	0.0411	0.1596
20	0.8843	1.2834	-0.0157	0.0834	0.0473	0.0630	0.0174	0.0695
50	0.8926	1.2310	-0.0074	0.0310	0.0181	0.0190	0.0082	0.0258
100	0.8972	1.2154	-0.0028	0.0154	0.0088	0.0086	0.0031	0.0129
200	0.8980	1.2076	-0.0020	0.0076	0.0045	0.0043	0.0022	0.0063

Table 9. Simulation results of the DsPA distribution for $\lambda = 0.9$ and $\beta = 2.0$.

n	$E(\lambda)$	$E(\beta)$	Bias(λ)	Bias(β)	MSE(λ)	MSE(β)	MRE(λ)	MRE(β)
10	0.8690	2.3159	-0.0310	0.3159	0.1005	0.5381	0.0344	0.1580
20	0.8785	2.1443	-0.0215	0.1443	0.0454	0.1699	0.0239	0.0722
50	0.8941	2.0523	-0.0059	0.0523	0.0183	0.0535	0.0065	0.0261
100	0.8957	2.0278	-0.0043	0.0278	0.0089	0.0246	0.0048	0.0139
200	0.8980	2.0139	-0.0020	0.0139	0.0044	0.0117	0.0022	0.0070

5. Three real-life applications

In this section, we illustrate the importance of the newly DsPA distribution by utilizing three real-life datasets. We shall compare the fits of the DsPA distribution with the following competing discrete distributions which are reported in Table 10.

Table 10. The competing discrete models of the DsPA distribution with their pmfs.

Model	**Abbreviation**	**pmf**
Discrete Bur-XII	DsBXII	$P(x) = \lambda^{\ln(1+x^{\alpha})} - \lambda^{\ln(1+(1+x)^{\alpha})}$.
Uniform Poisson–Ailamujia	UPA	$P(x) = 2\,\lambda\,(1+2\,\lambda)^{-x-1}$.
Poisson	Poi	$P(x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$.
Discrete-Pareto	DsPr	$P(x) = e^{-\lambda\ln(1+x)} - e^{-\lambda\ln(2+x)}$.
Discrete-Rayleigh	DsR	$P(x) = e^{-\frac{x^2}{2\lambda^2}} - e^{-\frac{(x+1)^2}{2\lambda^2}}$.
Discrete inverse-Rayleigh	DsIR	$P(x) = e^{-\frac{\lambda}{(1+x)^2}} - e^{-\frac{\lambda}{x^2}}$.
Discrete Burr-Hutke	DsBH	$P(x) = \left(\dfrac{1}{x+1} - \dfrac{\lambda}{x+2}\right)\lambda^{x}$.

The fitted distributions are compared using the negative maximum log-likelihood (-Loglik.), Akaike information criterion (AIC), Bayesian information criterion (BIC), and the p-value of Kolmogorov–Smirnov test (KS p-value).

Dataset I: The first dataset is about the failure times for a sample of 15 electronic components in an acceleration life test [27]. The data observations are: 1.0, 5.0, 6.0, 11.0, 12.0, 19.0, 20.0, 22.0, 23.0, 31.0, 37.0, 46.0, 54.0, 60.0 and 66.0.

Dataset II: The second dataset is about the number of fires in Greece from July 1, 1998 to August 31, 1998. This dataset is studied [28]. The data observations are: 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, 6, 7, 7, 7, 7, 8, 8, 8, 8, 8, 8, 8, 8, 8, 9, 9, 9, 9, 9, 9, 10, 10, 10, 11, 11, 11, 11, 12, 12, 12, 12, 12, 12, 15, 15, 15, 15, 16, 20 and 43.

Dataset III: The third dataset consists of 48 final mathematics examination marks for slow-paced students in the Indian Institute of Technology at Kanpur. The data is analyzed by [29]. The observations are: 29, 25, 50, 15, 13, 27, 15, 18, 7, 7, 8, 19, 12, 18, 5, 21, 15, 86, 21, 15, 14, 39, 15, 14,70, 44, 6, 23, 58, 19, 50, 23, 11, 6, 34, 18, 28, 34, 12, 37, 4, 60, 20, 23, 40, 65, 19 and 31.

The MLEs of the competing discrete models, standard errors (SEs), and goodness-of-fit measures are listed in Tables 11–13 for the three datasets, respectively. For visual comparisons, the P-P (probability–probability) plots of fitted distributions are displayed in Figures 4, 6 and 8 for the analyzed datasets, respectively. Furthermore, the estimated cdf, sf, hrf of the DsPA distribution are depicted in Figures 5, 7 and 9, respectively.

The findings in Tables 11–13 illustrate that the DsPA distribution provides a superior fit over other competing discrete models, since it has the lowest values for all measures and the largest K-S p-value.

Table 11. Findings of the competing discrete distributions to the failure times of electronic components.

Model	λ		α		Measures			
	MLE	SEs	MLE	SEs	-Loglik.	AIC	BIC	KS p-value
DsBXII	0.9839	0.0355	20.868	46.483	75.69	155.38	156.80	0.0150
UPA	0.0182	0.0047	-	-	65.00	132.00	132.71	0.6734
Poi	27.535	1.3548	-	-	151.21	304.41	305.12	0.0180
DsPr	0.3283	0.0848	-	-	77.40	156.80	157.51	0.0097
DsR	24.384	3.1487	-	-	66.39	134.79	135.50	0.4300
DsIR	42.021	11.243	-	-	83.99	169.97	170.68	0.0000
DsBH	0.9992	0.0076	-	-	91.37	184.74	185.44	0.0000
DsPA	0.8886	0.0674	0.8588	0.1738	64.49	131.58	132.97	0.9500

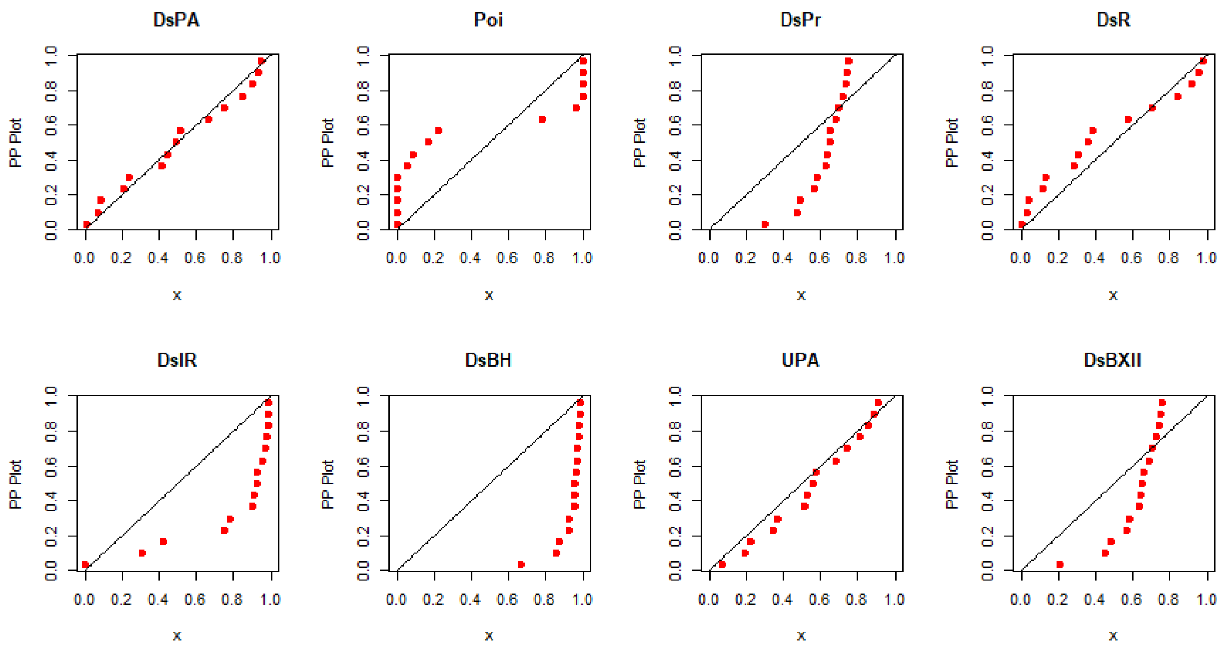

Figure 4. The P-P plots of the competing discrete models for dataset I.

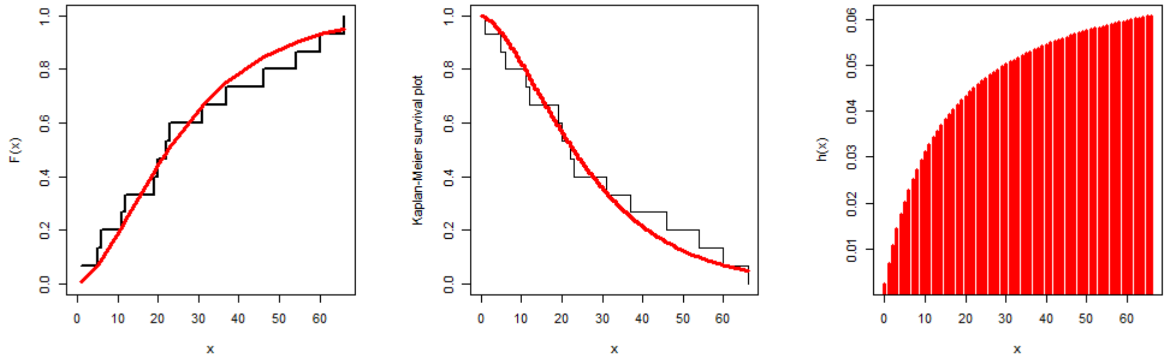

Figure 5. The fitted cdf, sf, and hrf plots for dataset I.

Table 12. Findings of the competing discrete distributions to the number of fires in Greece.

Model	λ		α		Measures			
	MLE	SE	MLE	SE	-Loglik.	AIC	BIC	KS p-value
DsBXII	0.7612	0.0427	2.5026	0.4870	373.39	750.79	756.41	0.0000
UPA	0.0926	0.0090	-	-	341.14	684.28	687.09	0.0028
Poi	5.3988	0.2095	-	-	467.83	937.65	940.47	0.0000
DsPr	0.6046	0.0546	-	-	389.64	781.27	784.08	0.0000
DsR	5.6792	0.2567	-	-	385.25	772.49	775.31	0.0000
DsIR	3.9959	0.3995	-	-	412.72	827.44	830.25	0.0000
DsBH	0.9836	0.0127	-	-	407.16	816.31	819.12	0.0000
DsPA	0.5812	0.0407	0.7709	0.0562	340.33	684.67	690.29	0.2484

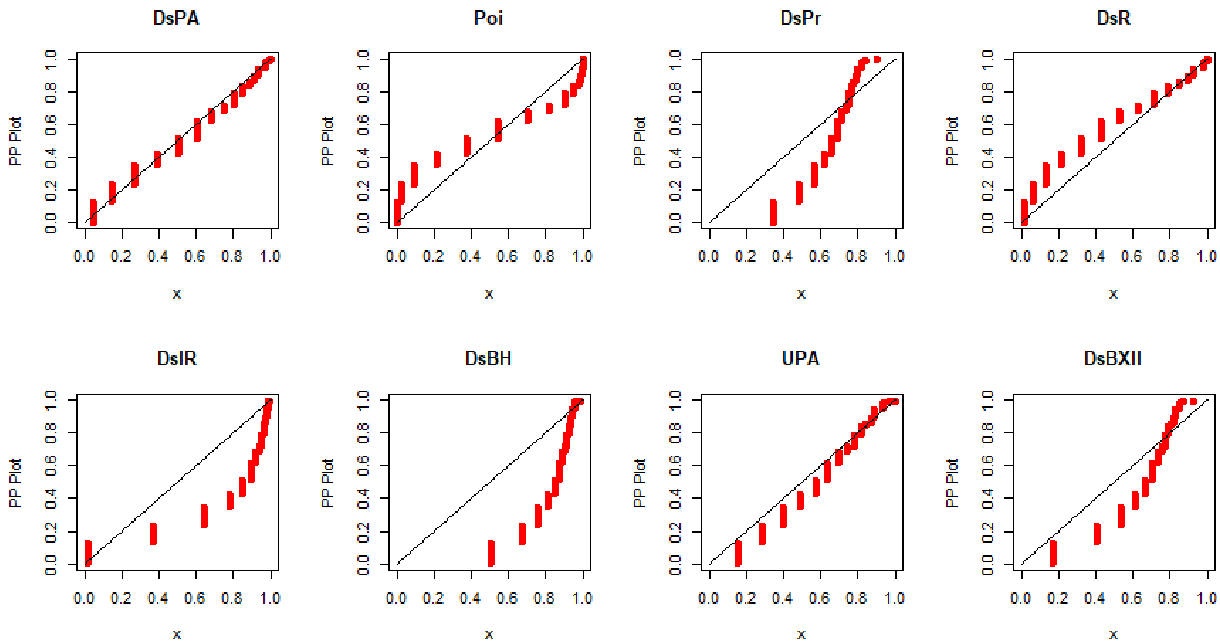

Figure 6. The P-P plots of the competing discrete models for dataset II.

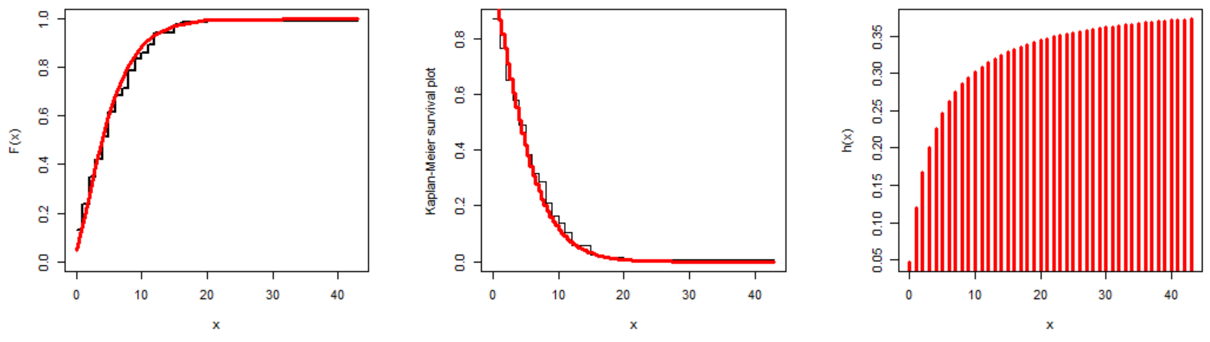

Figure 7. The fitted cdf, sf, and hrf plots for dataset II.

Table 13. Findings of the competing discrete distributions to the the examination marks in mathematics.

Model	λ		α		Measures			
	MLE	SE	MLE	SE	-Loglik.	AIC	BIC	KS p-value
DsBXII	0.9382	0.1926	5.1500	16.5597	247.48	498.97	502.71	0.0000
UPA	0.0193	0.0028	-	-	205.11	412.22	414.09	0.0174
Poi	25.8950	0.7345	-	-	396.59	795.18	797.05	0.0000
DsPr	0.3225	0.0466	-	-	215.18	504.36	506.23	0.0000
DsR	22.7562	1.6427	-	-	201.89	405.79	407.66	0.0460
DsIR	177.56	26.02	-	-	205.13	412.27	414.14	0.0000
DsBH	0.9990	0.0046	-	-	297.68	597.35	599.22	0.0000
DsPA	0.9409	0.0231	1.0621	0.1113	197.44	398.88	402.62	0.8102

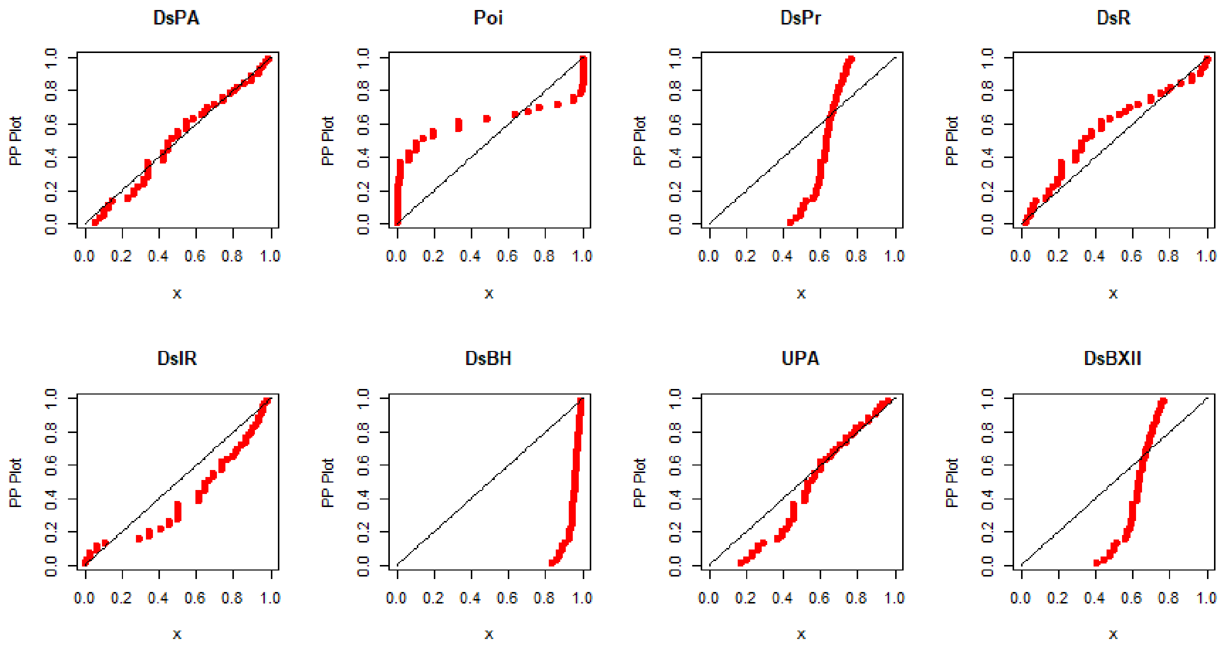

Figure 8. The P-P plots of the competing discrete models for dataset III.

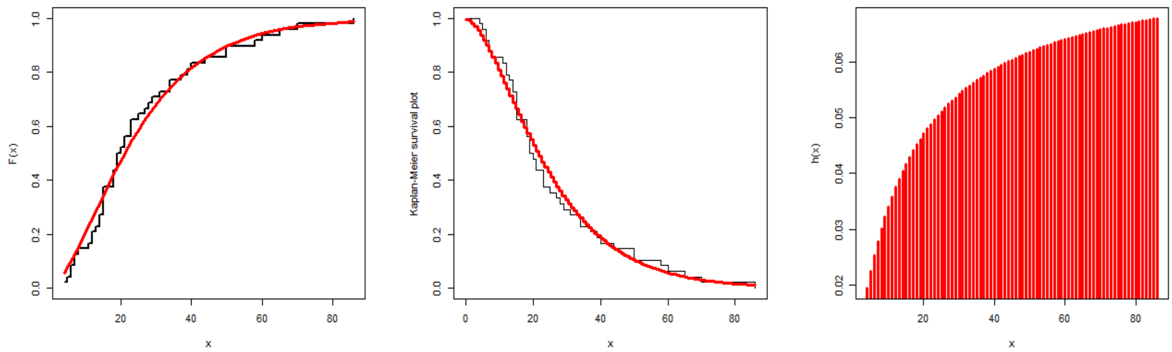

Figure 9. The fitted cdf, sf, and hrf plots for dataset III.

6. Conclusions

In this study, a new one-parameter discrete model is proposed as a good alternative to some well-known discrete distributions. The newly introduced model is called the discrete-power-Ailamujia (DsPA) distribution. Some statistical properties of the DsPA distribution are derived. Its parameters are estimated by the maximum likelihood method. A simulation study is carried out to check the performance of the estimators. It is observed that the maximum likelihood method is efficient in estimating the DsPA parameters for large samples. Finally, three real-world datasets are analyzed to check the usefulness and applicability of the DsPA distribution. The goodness-of-fit measures and figures show that the DsPA distribution is a useful attractive alternative for competing discrete models.

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Conflicts of interest

The authors declare no conflict of interest.

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