Mathematics

Research article

MBJ-neutrosophic subalgebras and filters in BE-algebras

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Abstract: The concept of a neutrosophic set, which is a generalization of an intuitionistic fuzzy set and a para consistent set etc., was introduced by F. Smarandache. Since then, it has been studied in various applications. In considering a generalization of the neutrosophic set, Mohseni Takallo et al. used the interval valued fuzzy set as the indeterminate membership function because interval valued fuzzy set is a generalization of a fuzzy set, and introduced the notion of MBJ-neutrosophic sets, and then they applied it to BCK/BCI-algebras. The aim of this paper is to apply the concept of MBJ-neutrosophic sets to a BE-algebra, which is a generalization of a BCK-algebra. The notions of MBJ-neutrosophic subalgebras and MBJ-neutrosophic filters of BE-algebras are introduced and related properties are investigated. The conditions under which the MBJ-neutrosophic set can be a MBJ-neutrosophic subalgebra/filter are searched. Characterizations of MBJ-neutrosophic subalgebras and MBJ-neutrosophic filters are considered. The relationship between an MBJ-neutrosophic subalgebra and an MBJ-neutrosophic filter is established.

1. Introduction

In 2007, Y. H. Kim and H. S. Kim [4] introduced the notion of a BE-algebra, and investigated its several properties. In [1], Ahn and So introduced the notion of an ideal in BE-algebras. They gave several descriptions of ideals in BE-algebras.

Zadeh [10] introduced the degree of a membership/truth (t) in 1965 and defined the fuzzy set. As a generalization of a fuzzy set, Atanassov [2] introduced the degree of nonmembership/falsehood (f) in 1986, and he defined the intuitionistic fuzzy set. Smarandache introduced the degree of

indeterminacy/neutrality (i) as independent component in 1995 (published in 1998) and defined the neutrosophic set on three components (t, i, f) = (truth, indeterminacy, falsehood). In 2015, neutrosophic set theory was applied to *BE*-algebra, and the notion of a neutrosophic filter was introduced [5]. As an extension theory of the neutrosophic set, Singh [7] introduced the notion of a type-2 neutrosophic set that could provide a granular representation of features and help model uncertainties with six different memberships. Singh et al. [8] proposed a novel hybrid time series forecasting model using neutrosophic set theory, artificial neural network and gradient descent algorithm. They dealt with three main problems of time series dataset, viz., representation of time series dataset using neutrosophic set, three degrees of memberships of neutrosophic set together, and generation of the forecasting results. In [9], the notion of MBJ-neutrosophic sets was defined as an another generalization of neutrosophic sets to BCK/BCI-algebras. The concept of MBJ-neutrosophic subalgebras in BCK/BCI-algebras was introduced and some related properties were investigated [9].

In this paper, we introduce the notion of an MBJ-neutrosophic subalgebra of a *BE*-algebra and investigate some related properties of an MBJ-neutrosophic subalgebra. We define the concept of an MBJ-neutrosophic filter of *BE*-algebras. The relationship between MBJ-neutrosophic subalgebras and MBJ-neutrosophic filters is established. We provide some characterizations of MBJ-neutrosophic filter.

2. Preliminaries

By a *BE*-algebra [4] we mean a system $(U; *, 1)$ of type $(2, 0)$ which the following axioms hold:

(BE1) $(\forall x \in U)(x * x = 1)$;
(BE2) $(\forall x \in U)(x * 1 = 1)$;
(BE3) $(\forall x \in U)(1 * x = x)$;
(BE4) $(\forall x, y, z \in U)(x * (y * z) = y * (x * z))$ (exchange).

We introduce a relation " $\leq$ " on U by $x \leq y$ if and only if $x * y = 1$.

A *BE*-algebra $(U; *, 1)$ is said to be transitive if it satisfies that for any $x, y, z \in U$, $y * z \leq (x * y) * (x * z)$. Note that if $(U; *, 1)$ is a transitive *B*-algebra, then the relation " $\leq$ " is a quasi-order on U. A *BE*-algebra $(U; *, 1)$ is said to be self distributive if it satisfies that for any $x, y, z \in U$, $x * (y * z) = (x * y) * (x * z)$. Note that every self distributive *BE*-algebra is transitive, but the converse need not be true in general (see [4]).

Every self distributive *BE*-algebra $(U; *, 1)$ satisfies the following properties:

(2.1) $(\forall x, y, z \in U)(x \leq y \Rightarrow z * x \leq z * y$ and $y * z \leq x * z)$;
(2.2) $(\forall x, y \in U)(x * (x * y) = x * y)$;
(2.3) $(\forall x, y, z \in U)(x * y \leq (z * x) * (z * y))$.

Definition 2.1. *Let* $(U; *, 1)$ *be a BE-algebra and let F be a nonempty subset of U. Then F is called a filter of U [4] if*

(F1) $1 \in F$;
(F2) $(\forall x, y \in U)(x * y, x \in F \Rightarrow y \in F)$.

An interval number is defined to be a closed subinterval $\tilde{a} = [a^-, a^+]$ of $[0, 1]$, where $0 \leq a^- \leq a^+ \leq 1$. Denote by $[I]$ the set of all interval numbers. Let us define what is known as refined

minimum (briefly, rmin) and refined maximum (briefly, rmax) of two elements in $[I]$. We also define the symbols " $\geq$ ", " $\leq$ ", " $=$ " in case of two elements in $[I]$. Given two interval numbers $\tilde{a}_1 = [a_1^-, a_1^+]$ and $\tilde{a}_2 = [a_2^-, a_2^+]$, we define

$$\text{rmin}\{\tilde{a}_1, \tilde{a}_2\} = [\min\{a_1^-, a_2^-\}, \min\{a_1^+, a_2^+\}],$$
$$\text{rmax}\{\tilde{a}_1, \tilde{a}_2\} = [\max\{a_1^-, a_2^-\}, \max\{a_1^+, a_2^+\}],$$
$$\tilde{a}_1 \geq \tilde{a}_2 \Leftrightarrow a_1^- \geq a_2^-, a_1^+ \geq a_2^+,$$

and similarly we may have $\tilde{a}_1 \leq \tilde{a}_2$ and $\tilde{a}_1 = \tilde{a}_2$. Let $\tilde{a}_i \in [I]$, where $i \in \Lambda$. We define

$$\underset{i \in \Lambda}{\text{rinf}}\, \tilde{a}_i = [\, \underset{i \in \Lambda}{\inf}\, a_i^-, \, \underset{i \in \Lambda}{\inf}\, a_i^+] \text{ and } \underset{i \in \Lambda}{\text{rsup}}\, \tilde{a}_i = [\, \underset{i \in \Lambda}{\sup}\, a_i^-, \, \underset{i \in \Lambda}{\sup}\, a_i^+].$$

Let U be a nonempty set. A function $A : U \rightarrow [I]$ is called an interval-valued fuzzy set (briefly, an *IVF* set) in U. Let $[I]^U$ stand for the set of all IVF sets in U. For every $A \in [I]^U$ and $a \in U$, $A(a) = [A^-(a), A^+(a)]$ is called the degree of membership of an element a to A, where $A^- : U \rightarrow I$ and $A^+ : U \rightarrow I$ are fuzzy sets in U which are called a lower fuzzy set and an upper fuzzy set in U, respectively. For simplicity, we denote $A = [A^-, A^+]$.

Let U be a nonempty set. A neutrosophic set (NC) in U (see [6]) is a structure of the form:

$$\mathscr{A} := \{\langle x; A_T(x), A_I(x), A_F(x) \rangle | x \in U\},$$

where $A_T : U \rightarrow [0, 1]$ is a truth membership function, $A_I : U \rightarrow [0, 1]$ is an intermediate membership function, and $A_F : U \rightarrow [0, 1]$ is a false membership function.

Definition 2.2. *Let U be a nonempty set. By an MBJ-neutrosophic set in U, we mean a structure of the form:*

$$\mathscr{A} := \{\langle x; A_M(x), A_{\tilde{B}}(x), A_J(x) \rangle | x \in U\},$$

where A_M and A_J are fuzzy sets in U, which are called a truth membership function and a false membership function, respectively, and $A_{\tilde{B}}$ is an IVF set in U which is called an indeterminate interval-valued membership function.

For the sake of simplicity, we shall use the symbol $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ for the MBJ-neutrosophic set

$$\mathscr{A} := \{\langle x; A_M(x), A_{\tilde{B}}(x), A_J(x) \rangle | x \in U\}.$$

In an MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in U, if we take

$$A_{\tilde{B}} : U \rightarrow [I], x \rightarrow [A_{\tilde{B}}^-(x), A_{\tilde{B}}^+(x)]$$

with $A_{\tilde{B}}^-(x) = A_{\tilde{B}}^+(x)$, then $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is a neutrosophic set in U.

3. MBJ-neutrosophic subalgebras in BE-algebras

Definition 3.1. *Let U be a BE-algebra. An MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in U is called an MBJ-neutrosophic subalgebra of U if it satisfies:*

(3.1) $(\forall x, y \in U)(A_M(x * y) \geq \min\{A_M(x), A_M(y)\}, A_{\tilde{B}}(x * y) \geq \operatorname{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\},$
$\quad A_J(x * y) \leq \max\{A_J(x), A_J(y)\}$).

Example 3.2. Let $U := \{1, a, b, c\}$ be a *BE*-algebra [3] with a binary operation " $*$ " which is given in Table 1.

Table 1. Cayley table for the binary operation "$*$".

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	a
b	1	1	1	a
c	1	1	a	1

Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set in U defined by Table 2. It is easy to check that $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebra of U.

Table 2. MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$.

U	A_M	$A_{\tilde{B}}$	A_J
1	0.7	$[0.4, 0.9]$	0.2
a	0.5	$[0.2, 0.6]$	0.5
b	0.6	$[0.3, 0.8]$	0.4
c	0.4	$[0.1, 0.5]$	0.7

Proposition 3.3. *If $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebra of a BE-algebra U, then $A_M(1) \geq A_M(x), A_{\tilde{B}}(1) \geq A_{\tilde{B}}(x)$ and $A_J(1) \leq A_J(x)$ for all $x \in U$.*

Proof. For any $x \in U$, we have

$$A_M(1) = A_M(x * x) \geq \min\{A_M(x), A_M(x)\} = A_M(x),$$
$$A_{\tilde{B}}(1) = A_{\tilde{B}}(x * x) \geq \operatorname{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(x)\} = A_{\tilde{B}}(x),$$
$$A_J(1) = A_J(x * x) \leq \max\{A_J(x), A_J(x)\} = A_J(x).$$

This completes the proof. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\square$

Proposition 3.4. *Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic subalgebra of a BE-algebra U. If there exists a sequence $\{x_n\}$ in X such that $\lim\limits_{n\to\infty} A_M(x_n) = 1, \lim\limits_{n\to\infty} A_{\tilde{B}}(x_n) = [1, 1]$ and $\lim\limits_{n\to\infty} A_J(x_n) = 0$, then $A_M(1) = 1, A_{\tilde{B}}(1) = [1, 1]$ and $A_J(1) = 0$.*

Proof. It follows from Proposition 3.3 that $A_M(1) \geq A_M(x_n), A_{\tilde{B}}(1) \geq A_{\tilde{B}}(x_n)$ and $A_J(1) \leq A_J(x_n)$ for all positive integer n. Hence we have

$$1 \geq A_M(1) \geq \lim\limits_{n\to\infty} A_M(x_n) = 1,$$
$$[1, 1] \geq A_{\tilde{B}}(1) \geq \lim\limits_{n\to\infty} A_{\tilde{B}}(x_n) = [1, 1],$$
$$0 \leq A_J(1) \leq \lim\limits_{n\to\infty} A_J(x_n) = 0.$$

Therefore, $A_M(1) = 1, A_{\tilde{B}}(1) = [1, 1]$ and $A_J(1) = 0$. $\qquad\square$

Theorem 3.5. *Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set in a BE-algebra U. If (A_M, A_J) is an intuitionistic fuzzy subalgebra of U and $A_{\tilde{B}}^-, A_{\tilde{B}}^+$ are fuzzy subalgebras of U, then $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebra of U.*

Proof. It is enough to show that $A_{\tilde{B}}$ satisfies:

(3.2) $(\forall x, y \in U)(A_{\tilde{B}}(x * y) \geq \text{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\})$.

For any $x, y \in U$, we obtain

$$
\begin{aligned}
A_{\tilde{B}}(x * y) &= [A_{\tilde{B}}^-(x * y), A_{\tilde{B}}^+(x * y)] \\
&\geq [\min\{A_{\tilde{B}}^-(x), A_{\tilde{B}}^-(y)\}, \min\{A_{\tilde{B}}^+(x), A_{\tilde{B}}^+(y)\}] \\
&= \text{rmin}\{[A_{\tilde{B}}^-(x), A_{\tilde{B}}^+(x)], [A_{\tilde{B}}^-(y), A_{\tilde{B}}^+(y)]\} \\
&= \text{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\}.
\end{aligned}
$$

Therefore, $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ satisfies the condition (3.2). Hence $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebra of U. $\qquad\square$

If $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosohic subalgebra of a *BE*-algebra U, then

$$
\begin{aligned}
{[A_{\tilde{B}}^-(x * y), A_{\tilde{B}}^+(x * y)]} &= A_{\tilde{B}}(x * y) \geq \text{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\} \\
&= \text{rmin}\{[A_{\tilde{B}}^-(x), A_{\tilde{B}}^+(x)], [A_{\tilde{B}}^-(y), A_{\tilde{B}}^+(y)]\} \\
&= [\min\{A_{\tilde{B}}^-(x), A_{\tilde{B}}^-(y)\}, \min\{A_{\tilde{B}}^+(x), A_{\tilde{B}}^+(y)\}]
\end{aligned}
$$

for all $x, y \in U$. It follows that $A_{\tilde{B}}^-(x * y) \geq \min\{A_{\tilde{B}}^-(x), A_{\tilde{B}}^-(y)\}$ and $A_{\tilde{B}}^+(x * y) \geq \min\{A_{\tilde{B}}^+(x), A_{\tilde{B}}^+(y)\}$. Thus, $A_{\tilde{B}}^-$ and $A_{\tilde{B}}^+$ are fuzzy subalgebras of U. But (A_M, A_J) is not an intuitionistic fuzzy subalgebra of U as seen in Example 3.2. This shows that the converse of Theorem 3.5 is not true.

Given an MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in U, we consider the following sets:

$$
\begin{aligned}
U(A_M; t) &:= \{x \in U | A_M(x) \geq t\}, \\
U(A_{\tilde{B}}; [\delta_1, \delta_2]) &:= \{x \in U | A_{\tilde{B}}(x) \geq [\delta_1, \delta_2]\}, \\
L(A_J; s) &:= \{x \in U | A_J(x) \leq s\},
\end{aligned}
$$

where $t, s \in [0, 1]$ and $[\delta_1, \delta_2] \in [I]$.

Theorem 3.6. *An MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in a BE-algebra U is an MBJ-neutrosophic subalgebra of U if and only if the nonempty sets $U(A_M, ; t), U(A_{\tilde{B}}; [\delta_1, \delta_2])$ and $L(A_J; s)$ are subalgebras of U for all $t, s \in [0, 1]$ and $[\delta_1, \delta_2] \in [I]$.*

Proof. Assume that $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebra of U. Let $t, s \in [0, 1]$ and $[\delta_1, \delta_2] \in [I]$ be such that $U(A_M, ; t), U(A_{\tilde{B}}; [\delta_1, \delta_2])$ and $L(A_J; s)$ are nonempty sets. For any $a, b, x, y, u, v \in U$, if $a, b \in U(A_M, ; t), x, y \in U(A_{\tilde{B}}; [\delta_1, \delta_2])\}$ and $u, v \in L(A_J; s)$, then

$$
\begin{aligned}
A_M(a * b) &\geq \min\{A_M(a), A_M(b)\} \geq \min\{t, t\} = t, \\
A_{\tilde{B}}(x * y) &\geq \text{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\} \geq \text{rmin}\{[\delta_1, \delta_2], [\delta_1, \delta_2]\} = [\delta_1, \delta_2], \\
A_J(u * v) &\leq \max\{A_J(u), A_J(v)\} \leq \min\{s, s\} = s,
\end{aligned}
$$

and so $a*b \in U(A_M, ; t)$, $x*y \in U(A_{\tilde{B}}; [\delta_1, \delta_2])$ and $u*v \in L(A_J; s)$. Therefore, $U(A_M, ; t)$, $U(A_{\tilde{B}}; [\delta_1, \delta_2])$ and $L(A_J; s)$ are subalgebras of U.

Conversely, suppose that the nonempty sets $U(A_M, ; t)$, $U(A_{\tilde{B}}; [\delta_1, \delta_2])$ and $L(A_J; s)$ are subalgebras of U for all $t, s \in [0, 1]$ and $[\delta_1, \delta_2] \in [I]$. If $A_M(x_0 * y_0) < \min\{A_M(x_0), A_M(y_0)\}$ for some $x_0, y_0 \in U$, then $x_0, y_0 \in U(A_M; t_0)$ but $x_0 * y_0 \notin U(A_M; t_0)$ where $t_0 = \min\{A_M(x_0), A_M(y_0)\}$. This is a contradiction. Thus, $A_M(x * y) \geq \min\{A_M(x), A_M(y)\}$ for all $x, y \in U$. By a similar way, we can prove that $A_J(u * v) \leq \max\{A_J(u), A_J(v)\}$ for all $u, v \in U$. Assume that $A_{\tilde{B}}(a_0 * b_0) < \mathrm{rmin}\{A_{\tilde{B}}(a_0), A_{\tilde{B}}(b_0)\}$ for some $a_0, b_0 \in U$. Let $A_{\tilde{B}}(a_0) = [\alpha_1, \alpha_2]$, $A_{\tilde{B}}(b_0) = [\alpha_3, \alpha_4]$ and $A_{\tilde{B}}(a_0 * b_0) = [\delta_1, \delta_2]$. Then

$$[\delta_1, \delta_2] \prec \mathrm{rmin}\{[\alpha_1, \alpha_2], [\alpha_3, \alpha_4]\} = [\min\{\alpha_1, \alpha_3\}, \min\{\alpha_2, \alpha_4\}],$$

and so $\delta_1 < \min\{\alpha_1, \alpha_3\}$ and $\delta_2 < \min\{\alpha_2, \alpha_4\}$. Put $\gamma_1, \gamma_2 \in [0, 1]$ so that

$$[\gamma_1, \gamma_2] = \frac{1}{2}(A_{\tilde{B}}(a_0 * b_0) + \mathrm{rmin}\{A_{\tilde{B}}(a_0), A_{\tilde{B}}(b_0)\}).$$

Then we have

$$[\gamma_1, \gamma_2] = \frac{1}{2}([\delta_1, \delta_2] + [\min\{\alpha_1, \alpha_3\}, \min\{\alpha_2, \alpha_4\}])$$
$$= [\frac{1}{2}(\delta_1 + \min\{\alpha_1, \alpha_3\}), \frac{1}{2}(\delta_2 + \min\{\alpha_2, \alpha_4\})],$$

which shows $\min\{\alpha_1, \alpha_3\} > \gamma_1 = \frac{1}{2}(\delta_1 + \min\{\alpha_1, \alpha_3\}) > \delta_1$ and $\min\{\alpha_2, \alpha_4\} > \gamma_2 = \frac{1}{2}(\delta_2 + \min\{\alpha_2, \alpha_4\}) > \delta_2$. Thus, $[\min\{\alpha_1, \alpha_3\}, \min\{\alpha_2, \alpha_4\}] \succ [\gamma_1, \gamma_2] \succ [\delta_1, \delta_2] = A_{\tilde{B}}(a_0 * b_0)$, and therefore $a_0 * b_0 \notin U(A_{\tilde{B}}; [\gamma_1, \gamma_2])$. On the other hand,

$$A_{\tilde{B}}(a_0) = [\alpha_1, \alpha_2] \succeq [\min\{\alpha_1, \alpha_3\}, \min\{\alpha_2, \alpha_4\}] \succ [\gamma_1, \gamma_2]$$

and

$$A_{\tilde{B}}(b_0) = [\alpha_3, \alpha_4] \succeq [\min\{\alpha_1, \alpha_3\}, \min\{\alpha_2, \alpha_4\}] \succ [\gamma_1, \gamma_2],$$

that is, $a_0, b_0 \in U(A_{\tilde{B}}; [\gamma_1, \gamma_2])$. This is a contradiction. Therefore, $A_{\tilde{B}}(x * y) \succeq \mathrm{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\}$ for all $x, y \in U$. Thus, $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in X is an MBJ-neutrosophic subalgebra of U. $\square$

By Proposition 3.3 and Theorem 3.6, we obtain the following corollary.

Corollary 3.7. *If $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebras of a BE-algebra U, then the sets $U_{A_M} := \{x \in U | A_M(x) = A_M(1)\}$, $U_{A_{\tilde{B}}} := \{x \in U | A_{\tilde{B}}(x) = A_{\tilde{B}}(1)\}$ and $U_{A_J} := \{x \in U | A_J(x) = A_J(1)\}$ are subalgebras of U.*

We say that the subalgebras $U(A_M, ; t)$, $U(A_{\tilde{B}}; [\delta_1, \delta_2])$ and $L(A_J; s)$ of U are *MBJ-subalgebras* of $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$.

Theorem 3.8. *Every subalgebra of a BE-algebra U can be realized as MBJ-subalgebras of an MBJ-neutrosophic subalgebra of U.*

Proof. Let S be a subalgebra of U and let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set in U defined by

$$A_M(x) := \begin{cases} a & \text{if } x \in S, \\ 0 & \text{otherwise,} \end{cases}$$

$$A_{\tilde{B}}(x) := \begin{cases} [\alpha_1, \alpha_2] & \text{if } x \in S, \\ [0, 0] & \text{otherwise,} \end{cases} \qquad (3.3)$$

$$A_J(x) := \begin{cases} b, & \text{if } x \in S, \\ 1 & \text{otherwise,} \end{cases}$$

where $a \in (0, 1]$, $b \in [0, 1)$ and $\alpha_1, \alpha_2 \in (0, 1]$ with $\alpha_1 < \alpha_2$. It is clear that $U(A_M; a) = S$, $U(A_{\tilde{B}}; [\alpha_1, \alpha_2]) = S$ and $L(A_J; b) = S$. Let $x, y \in U$. If $x, y \in S$, then $x * y \in S$ and so

$$A_M(x * y) = a = \min\{A_M(x), A_M(y)\},$$
$$A_{\tilde{B}}(x * y) = [\alpha_1, \alpha_2] = \text{rmin}\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = \text{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\},$$
$$A_J(x * y) = b = \max\{A_J(x), A_J(y)\}.$$

If any one of x, y is contained in S, say $x \in S$, then $A_M(x) = a$, $A_{\tilde{B}}(x) = [\alpha_1, \alpha_2]$, $A_J(x) = b$, $A_M(y) = 0$, $A_{\tilde{B}}(y) = [0, 0]$ and $A_J(y) = 1$. Hence we have

$$A_M(x * y) \geq 0 = \min\{a, 0\} = \min\{A_M(x), A_M(y)\},$$
$$A_{\tilde{B}}(x * y) \geq [0, 0] = \text{rmin}\{[\alpha_1, \alpha_2], [0, 0]\} = \text{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\},$$
$$A_J(x * y) \leq 1 = \max\{b, 1\} = \max\{A_J(x), A_J(y)\}.$$

If $x, y \notin S$, then $A_M(x) = 0 = A_M(y)$, $A_{\tilde{B}}(x) = [0, 0] = A_{\tilde{B}}(y)$ and $A_J(x) = 1 = A_J(y)$. It follows that

$$A_M(x * y) \geq 0 = \min\{0, 0\} = \min\{A_M(x), A_M(y)\},$$
$$A_{\tilde{B}}(x * y) \geq [0, 0] = \text{rmin}\{[0, 0], [0, 0]\} = \text{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\},$$
$$A_J(x * y) \leq 1 = \max\{1, 1\} = \max\{A_J(x), A_J(y)\}.$$

Therefore, $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebra of U. $\square$

Theorem 3.9. *For any nonempty subset S of a BE-algera U, let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic set in U which is given in (3.3). If $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebra of U, then S is a subalgebra of U.*

Proof. Let $x, y \in S$. Then $A_M(x) = a = A_M(y)$, $A_{\tilde{B}}(x) = [\alpha_1, \alpha_2] = A_{\tilde{B}}(y)$ and $A_J(x) = b = A_J(y)$. Thus, we obtain

$$A_M(x * y) \geq \min\{A_M(x), A_M(y)\} = a,$$
$$A_{\tilde{B}}(x * y) \geq \text{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\} = [\alpha_1, \alpha_2],$$
$$A_J(x * y) \leq \max\{A_J(x), A_J(y)\} = b.$$

Hence $x * y \in S$. Therefore, S is a subalgebra of U. $\square$

Let $f : U \to V$ be a homomorphism of *BE*-algebras. For any MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in V, we define a new MBJ-neutrosophic set $A^f := (A_M^f, A_{\tilde{B}}^f, A_J^f)$ in U, which is called the induced MBJ-neutrosophic set, by

(3.4) $(\forall x, y \in U)(A_M^f(x) = A_M(f(x)),\ A_{\tilde{B}}^f(x) = A_{\tilde{B}}(f(x)),\ A_J^f(x) = A_J(f(x))\ ).$

Theorem 3.10. *Let $f : U \to V$ be a homomorphism of BE-algebras. If $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in V is an MBJ-neutrosophic set, then the induced MBJ-neutrosophic set $\mathscr{A} = (A_M^f, A_{\tilde{B}}^f, A_J^f)$ in U is an MBJ-neutrosophic subalgebra of U.*

Proof. Let $x, y \in U$. Then

$$A_M^f(x * y) = A_M(f(x * y)) = A_M(f(x) * f(y))$$
$$\geq \min\{A_M(f(x)), A_M(f(y))\} = \min\{A_M^f(x), A_M^f(y)\},$$
$$A_{\tilde{B}}^f(x * y) = A_{\tilde{B}}(f(x * y)) = A_{\tilde{B}}(f(x) * f(y))$$
$$\geq \mathrm{rmin}\{A_{\tilde{B}}(f(x)), A_{\tilde{B}}(f(y))\} = \mathrm{rmin}\{A_{\tilde{B}}^f(x), A_{\tilde{B}}^f(y)\},$$
$$A_J^f(x * y) = A_J(f(x * y)) = A_J(f(x) * f(y))$$
$$\leq \max\{A_J(f(x)), A_J(f(y))\} = \max\{A_J^f(x), A_J^f(y)\}.$$

Therefore, $\mathscr{A} = (A_M^f, A_{\tilde{B}}^f, A_J^f)$ is an MBJ-neutrosophic subalgebra of U. □

4. MBJ-neutrosophic filters in *BE*-algebras

Definition 4.1. *Let U be a BE-algebra. An MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in U is called an MBJ-neutrosophic filter of U if it satisfies:*

(4.1) $(\forall x \in U)(A_M(1) \geq A_M(x), A_{\tilde{B}}(1) \geq A_{\tilde{B}}(x), A_J(1) \leq A_J(x)$);
(4.2) $(\forall x, y \in U)(A_M(y) \geq \min\{A_M(x * y), A_M(x)\}, A_{\tilde{B}}(y) \geq \mathrm{rmin}\{A_{\tilde{B}}(x * y), A_{\tilde{B}}(x)\}, A_J(y) \leq \max\{A_J(x * y), A_J(x)\}$).

Example 4.2. Let $V := \{1, a, b, c\}$ be a *BE*-algebra [3] with a binary operation " $*$ " which is given in Table 3.

Table 3. Cayley table for the binary operation "$*$".

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	a
b	1	1	1	a
c	1	a	a	1

Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set in V defined by Table 4.

Table 4. MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$.

V	A_M	$A_{\tilde{B}}$	A_J
1	0.8	[0.4, 0.9]	0.1
a	0.5	[0.2, 0.6]	0.5
b	0.4	[0.1, 0.5]	0.7
c	0.7	[0.3, 0.8]	0.2

It is routine to verify that $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of V.

Proposition 4.3. *Every MBJ-neutrosophic filter of a BE-algebra U is an MBJ-neutrosophic subalgebra of U.*

Proof. Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic filter of U. Then we have

$$
\begin{aligned}
\min\{A_M(x), A_M(y)\} &\leq \min\{A_M(1), A_M(y)\} \\
&= \min\{A_M(y * (x * y)), A_M(y)\} \leq A_M(x * y), \\
\mathrm{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(y)\} &\leq \mathrm{rmin}\{A_{\tilde{B}}(1), A_{\tilde{B}}(y)\} \\
&= \mathrm{rmin}\{A_{\tilde{B}}(y * (x * y)), A_{\tilde{B}}(y)\} \leq A_{\tilde{B}}(x * y), \\
\max\{A_J(x), A_J(y)\} &\geq \max\{A_J(1), A_J(y)\} \\
&= \max\{A_J(y * (x * y)), A_J(y)\} \geq A_J(x * y)
\end{aligned}
$$

for any $x, y \in U$. Hence $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebra of U. $\qquad\square$

The converse of Proposition 4.3 may not be true in general (see the following example).

Example 4.4. Consider $U = \{1, a, b, c\}$ and $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ as in Example 3.2. Then $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic subalgebra of U (see Example 3.2), but it is not an MBJ-neutrosophic filter of U, since

$$
\begin{aligned}
A_M(a) = 0.5 &\ngeq \min\{A_M(b * a), A_M(b)\} = \min\{A_M(1), A_M(b)\} = A_M(b) = 0.6, \\
A_{\tilde{B}}(a) = [0.2, 0.6] &\ngeq \mathrm{rmin}\{A_{\tilde{B}}(b * a), A_{\tilde{B}}(b)\} = \mathrm{rmin}\{A_{\tilde{B}}(1), A_{\tilde{B}}(b)\} = A_{\tilde{B}}(b) = [0.3, 0.8], \\
A_J(a) = 0.5 &\nleq \max\{A_J(b * a), A_J(b)\} = \max\{A_J(1), A_J(b)\} = A_J(b) = 0.4.
\end{aligned}
$$

Proposition 4.5. *Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic filter of a BE-algebra U. Then the following assertions are valid:*

(i) $(\forall x, y \in U)(x \leq y \Rightarrow A_M(x) \leq A_M(y), A_{\tilde{B}}(x) \leq A_{\tilde{B}}(y), A_J(x) \geq A_J(y))$;
(ii) $(\forall x, y, z \in U)(A_M(x * z) \geq \min\{A_M(x * (y * z)), A_M(y)\}, A_{\tilde{B}}(x * z) \geq \mathrm{rmin}\{A_{\tilde{B}}(x * (y * z)), A_M(y)\}, A_J(x * z) \leq \max\{A_J(x * (y * z)), A_J(y)\})$;
(iii) $(\forall a, x \in U)(A_M(a) \leq A_M((a * x) * x), A_{\tilde{B}}(a) \leq A_{\tilde{B}}((a * x) * x), A_J(a) \geq A_J((a * x) * x))$.

Proof. (i) Let $x, y \in U$ be such that $x \leq y$, then $x * y = 1$. It follows from (4.1) and (4.2) that

$$
\begin{aligned}
A_M(x) &= \min\{A_M(1), A_M(x)\} = \min\{A_M(x * y), A_M(x)\} \leq A_M(y), \\
A_{\tilde{B}}(x) &= \mathrm{rmin}\{A_{\tilde{B}}(1), A_{\tilde{B}}(x)\} = \mathrm{rmin}\{A_{\tilde{B}}(x * y), A_{\tilde{B}}(x)\} \leq A_{\tilde{B}}(y), \\
A_J(x) &= \max\{A_J(1), A_J(x)\} = \max\{A_J(x * y), A_J(x)\} \geq A_J(y).
\end{aligned}
$$

(ii) Using (BE4) and (4.2), we obtain

$$
\begin{aligned}
A_M(x * z) &\geq \min\{A_M(y * (x * z)), A_M(y)\} = \min\{A_M(x * (y * z)), A_M(y)\}, \\
A_{\tilde{B}}(x * z) &\geq \mathrm{rmin}\{A_{\tilde{B}}(y * (x * z)), A_{\tilde{B}}(y)\} = \mathrm{rmin}\{A_{\tilde{B}}(x * (y * z)), A_{\tilde{B}}(y)\}, \\
A_J(x * z) &\leq \max\{A_J(y * (x * z)), A_J(y)\} = \max\{A_J(x * (y * z)), A_J(y)\}
\end{aligned}
$$

for all $x, y, z \in U$.

(iii) Taking $y := (a * x) * x$ and $x := a$ in (4.2), we have

$$
\begin{aligned}
A_M((a * x) * x) &\geq \min\{A_M(a * ((a * x) * x)), A_M(a)\} \\
&= \min\{A_M((a * x) * (a * x)), A_M(a)\} \\
&= \min\{A_M(1), A_M(a)\} = A_M(a),
\end{aligned}
$$

$$
\begin{aligned}
A_{\tilde{B}}((a * x) * x) &\geq \mathrm{rmin}\{A_{\tilde{B}}(a * ((a * x) * x)), A_{\tilde{B}}(a)\} \\
&= \mathrm{rmin}\{A_{\tilde{B}}((a * x) * (a * x)), A_{\tilde{B}}(a)\} \\
&= \mathrm{rmin}\{A_{\tilde{B}}(1), A_{\tilde{B}}(a)\} = A_{\tilde{B}}(a),
\end{aligned}
$$

$$
\begin{aligned}
A_J((a * x) * x) &\leq \max\{A_J(a * ((a * x) * x)), A_J(a)\} \\
&= \max\{A_J((a * x) * (a * x)), A_J(a)\} \\
&= \max\{A_J(1), A_J(a)\} = A_J(a)
\end{aligned}
$$

by using (BE1), (BE4), (4.2) and (4.2), proving the proposition. □

Corollary 4.6. *Every MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ of a BE-algebra U satisfying (4.1) and Proposition 4.5(ii) is an MBJ-neutrosophic filter of U.*

Proof. Setting $x := 1$ in Proposition 4.5(ii) and (BE2), we obtain

$$
\begin{aligned}
A_M(z) &= A_M(1 * z) \geq \min\{A_M(1 * (y * z)), A_M(y)\} = \min\{A_M(y * z), A_M(y)\}, \\
A_{\tilde{B}}(z) &= A_{\tilde{B}}(1 * z) \geq \mathrm{rmin}\{A_{\tilde{B}}(1 * (y * z)), A_{\tilde{B}}(y)\} = \mathrm{rmin}\{A_{\tilde{B}}(y * z), A_{\tilde{B}}(y)\}, \\
A_J(z) &= A_J(1 * z) \leq \max\{A_J(1 * (y * z)), A_J(y)\} = \max\{A_J(y * z), A_J(y)\}
\end{aligned}
$$

for all $y, z \in U$. Hence $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U. □

Theorem 4.7. *An MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ of a BE-algebra U is an MBJ-neutrosophic filter of U if and only if it satisfies the following conditions:*

(i) $(\forall x, y \in U)(A_M(y * x) \geq A_M(x), A_{\tilde{B}}(y * x) \geq A_{\tilde{B}}(x), A_J(y * x) \leq A_J(x))$;
(ii) $(\forall x, a, b \in U)(A_M((a * (b * x)) * x) \geq \min\{A_M(a), A_M(b)\}, A_{\tilde{B}}((a * (b * x)) * x) \geq \mathrm{rmin}\{A_{\tilde{B}}(a), A_{\tilde{B}}(b)\}, A_J((a * (b * x)) * x) \leq \max\{A_J(a)), A_J(b)\})$.

Proof. (i) Assume that $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U. Using (BE2), (BE4), (4.1) and (4.2) we have

$$
\begin{aligned}
A_M(y * x) &\geq \min\{A_M(x * (y * x)), A_M(x)\} = \min\{A_M(y * (x * x)), A_M(x)\} \\
&= \min\{A_M(1), A_M(x)\} = A_M(x),
\end{aligned}
$$

$$
\begin{aligned}
A_{\tilde{B}}(y * x) &\geq \mathrm{rmin}\{A_{\tilde{B}}(x * (y * x)), A_{\tilde{B}}(x)\} = \mathrm{rmin}\{A_{\tilde{B}}(y * (x * x)), A_{\tilde{B}}(x)\} \\
&= \mathrm{rmin}\{A_{\tilde{B}}(1), A_{\tilde{B}}(x)\} = A_{\tilde{B}}(x),
\end{aligned}
$$

$$
\begin{aligned}
A_J(y * x) &\leq \max\{A_J(x * (y * x)), A_J(x)\} = \max\{A_J(y * (x * x)), A_J(x)\} \\
&= \max\{A_J(1), A_J(x)\} = A_J(x)
\end{aligned}
$$

for all $x, y \in U$. It follows from Proposition 4.5 that

$$A_M((a * (b * x)) * x) \geq \min\{A_M((a * (b * x)) * (b * x)), A_M(b)\}$$
$$\geq \min\{A_M(a), A_M(b)\},$$

$$A_{\tilde{B}}((a * (b * x)) * x) \geq \text{rmin}\{A_{\tilde{B}}((a * (b * x)) * (b * x)), A_{\tilde{B}}(b)\}$$
$$\geq \text{rmin}\{A_{\tilde{B}}(a), A_{\tilde{B}}(b)\},$$

$$A_J((a * (b * x)) * x) \leq, \max\{A_J((a * (b * x)) * (b * x)), A_J(b)\}$$
$$\leq \max\{A_J(a), A_J(b)\}$$

for all $a, b, x \in U$.

Conversely, let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set of U satisfying conditions (i) and (ii). Taking $y := x$ in (i), we obtain $A_M(1) = A_M(x * x) \geq A_M(x)$, $A_{\tilde{B}}(1) = A_{\tilde{B}}(x * x) \geq A_{\tilde{B}}(x)$, $A_J(1) = A_J(x * x) \leq A_J(x)$ for all $x \in U$. Using (ii), we get

$$A_M(y) = A_M(1 * y) = A_M(((x * y) * (x * y)) * y)$$
$$\geq \min\{A_M(x * y), A_M(x)\},$$

$$A_{\tilde{B}}(y) = A_{\tilde{B}}(1 * y) = A_{\tilde{B}}(((x * y) * (x * y)) * y)$$
$$\geq \text{rmin}\{A_{\tilde{B}}(x * y), A_{\tilde{B}}(x)\},$$

$$A_J(y) = A_J(1 * y) = A_J(((x * y) * (x * y)) * y)$$
$$\leq \max\{A_J(x * y), A_J(x)\}$$

for all $x, y \in U$. Hence $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U. □

Proposition 4.8. *Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set of a BE-algebra U. Then $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U if and only if*

(4.3) $(\forall x, y, z \in U)(z \leq x * y \Rightarrow A_M(y) \geq \min\{A_M(x), A_M(z)\}, A_{\tilde{B}}(y) \geq \text{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(z)\}$ *and* $A_J(y) \leq \max\{A_J(x), A_J(z)\})$.

Proof. Assume that $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U. Let $x, y, z \in U$ be such that $z \leq x * y$. By Proposition 4.5(i) and (4.2), we have

$$A_M(y) \geq \min\{A_M(x * y), A_M(x)\} \geq \min\{A_M(z), A_M(x)\},$$
$$A_{\tilde{B}}(y) \geq \text{rmin}\{A_{\tilde{B}}(x * y)A_{\tilde{B}}(x)\} \geq \text{rmin}\{A_{\tilde{B}}(z), A_{\tilde{B}}(x)\},$$
$$A_J(y) \leq \max\{A_J(x * y), A_J(x)\} \leq \max\{A_J(z), A_J(x)\}.$$

Conversely, suppose that $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ satisfies (4.3). By (BE2), we have $x \leq x * 1 = 1$. Using (4.3), we have $A_M(1) \geq A_M(x)$, $A_{\tilde{B}}(1) \geq A_{\tilde{B}}(x)$ and $A_J(1) \leq A_J(x)$ for all $x \in U$. It follows from (BE1) and (BE4) that $x \leq (x * y) * y$ for all $x, y \in U$. Using (4.3), we have

$$A_M(y) \geq \min\{A_M(x * y), A_M(x)\},$$
$$A_{\tilde{B}}(y) \geq \mathrm{rmin}\{A_{\tilde{B}}(x * y), A_{\tilde{B}}(x)\},$$
$$A_J(y) \leq \max\{A_J(x * y), A_J(x)\}$$

for all $x, y \in U$. Therefore, $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U. $\qquad\square$

As a generalization of Proposition 4.8, we get the following results.

Theorem 4.9. *Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic filter of a BE-algebra U. Then*

(4.4) $(\forall x, w_1, \cdots, w_n \in U)(\prod_{i=1}^{n} w_i * x = 1 \Rightarrow A_M(x) \geq \min_{i=1}^{n}\{A_M(w_i)\}, A_{\tilde{B}}(x) \geq \mathrm{rmin}_{i=1}^{n}\{A_{\tilde{B}}(w_i)\}$ *and* $A_J(x) \leq \max_{i=1}^{n}\{A_J(w_i)\})$,

where $\prod_{i=1}^{n} w_i * x = w_n * (w_{n-1} * (\cdots w_1 * x) \cdots))$.

Proof. The proof is by an induction on n. Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic filter of U. By Propositions 4.5(i) and 4.8, we know that the condition (4.4) is true for $n = 1, 2$. Assume that $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ satisfies the condition (4.4) for $n = k$, i.e., $\prod_{i=1}^{k} w_i * x = 1 \Rightarrow A_M(x) \geq \min_{i=1}^{k}\{A_M(w_i)\}$, $A_{\tilde{B}}(x) \geq \mathrm{rmin}_{i=1}^{k}\{A_{\tilde{B}}(w_i)\}$ and $A_J(x) \leq \max_{i=1}^{k}\{A_J(w_i)\}$ for all $x, w_1, \cdots, w_k \in U$. Suppose that $\prod_{i=1}^{k+1} w_i * x = 1$ for all $x, w_1, \cdots, w_k, w_{k+1} \in U$. Then $A_M(w_1 * x) \geq \min_{i=2}^{k+1}\{A_M(w_i)\}$, $A_{\tilde{B}}(w_1 * x) \geq \mathrm{rmin}_{i=2}^{k+1}\{A_{\tilde{B}}(w_i)\}$ and $A_J(w_1 * x) \leq \max_{i=2}^{k+1}\{A_J(w_i)\}$. Since $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U, it follows from (4.2) that

$$A_M(x) \geq \min\{A_M(w_1 * x), A_M(w_1)\} \geq \min\{\min_{i=2}^{k+1} A_M(w_i), A_M(w_1)\}$$
$$= \min_{i=1}^{k+1}\{A_M(w_i)\},$$

$$A_{\tilde{B}} \geq \mathrm{rmin}\{A_{\tilde{B}}(w_1 * x), A_{\tilde{B}}(w_1)\} \geq \mathrm{rmin}\{\mathrm{rmin}_{i=2}^{k+1} A_{\tilde{B}}(w_i), A_{\tilde{B}}(w_1)\}$$
$$= \mathrm{rmin}_{i=1}^{k+1}\{A_{\tilde{B}}(w_i)\},$$

$$A_J(x) \leq \max\{A_M(w_1 * x), A_M(w_1)\} \leq \max\{\min_{i=2}^{k+1} A_J(w_i), A_J(w_1)\}$$
$$= \min_{i=1}^{k+1}\{A_J(w_i)\}.$$

This completes the proof. $\qquad\square$

Theorem 4.10. *Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set of a BE-algebra U satisfying (4.4). Then $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U.*

Proof. Let $x, y, z \in U$ be such that $z \leq x * y$. Then $z * (x * y) = 1$ and so $A_M(y) \geq \min\{A_M(x), A_M(z)\}, A_{\tilde{B}}(y) \geq \mathrm{rmin}\{A_{\tilde{B}}(x), A_{\tilde{B}}(z)\}, A_J(y) \leq \max\{A_J(x), A_J(z)\}$ by (4.4). Using Proposition 4.8, $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U. $\qquad\square$

Let U be a BE-algebra. For two elements $a, b \in U$, we consider an MBJ-neutrosophic set

$$\mathscr{A}^{a,b} = (A_M^{a,b}, A_{\tilde{B}}^{a,b}, A_J^{a,b}),$$

where

$$A_M^{a,b} : U \to [0,1], \quad x \to \begin{cases} \alpha_1 & \text{if } a * (b * x) = 1, \\ \alpha_2 & \text{otherwise} \end{cases}$$

with $\alpha_2 \leq \alpha_1$,

$$A_{\tilde{B}}^{a,b} : U \to [0,1], \quad x \to \begin{cases} \tilde{a}_1 = [a_1^-, a_1^+] & \text{if } a * (b * x) = 1, \\ \tilde{a}_2 = [a_2^-, a_2^+] & \text{otherwise} \end{cases}$$

with $\tilde{a}_1 \geq \tilde{a}_2$,

$$A_J^{a,b} : U \to [0,1], \quad x \to \begin{cases} \delta_1 & \text{if } a * (b * x) = 1, \\ \delta_2 & \text{otherwise} \end{cases}$$

with $\delta_1 \leq \delta_2$.

In the following, we know that there exist $a, b \in U$ such that $\mathscr{A}^{a,b}$ is not an MBJ-neutrosophic filter of U.

Example 4.11. Let $U := \{1, a, b, c\}$ be a BE-algebra [3] with a binary operation " $*$ " which is given in Table 5.

Table 5. Cayley table for the binary operation "$*$".

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	c
b	1	1	1	c
c	1	a	b	1

Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set in U defined by Table 6.

Table 6. MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$.

U	A_M	$A_{\tilde{B}}$	A_J
1	0.7	$[0.1, 0.9]$	0.3
a	0.4	$[0.2, 0.25]$	0.7
b	0.6	$[0.25, 0.75]$	0.4
c	0.5	$[0.75, 0.9]$	0.5

Then $\mathscr{A}^{a,b} = (A_M^{a,b}, A_{\tilde{B}}^{a,b}, A_J^{a,b})$ is not an MBJ-neutrosophic filter of U, since $\min\{A^{1,a}(a * b)_M, A_M^{1,a}(a)\} = 0.7 \nleq A_M^{1,a}(b) = 0.4$, $\text{rmin}\{A_{\tilde{B}}^{1,a}(a * b), A_{\tilde{B}}^{1,a}(a)\} = [0.1, 0.9] \nleq A_{\tilde{B}}^{1,a}(b) = [0.2, 0.25]$, $\max\{A_J^{1,a}(a * b), A_J^{1,a}(a)\} = 0.3 \ngeq A_J^{1,a}(b) = 0.7$.

Theorem 4.12. *Let U be a self distributive BE-algebra. Then every MBJ-neutrosopshic set $\mathscr{A}^{a,b}$ of U is an MBJ-neutrosophic filter of U.*

Proof. Let $a, b \in U$. Obviously, $A_M^{a,b}(1) \geq A_M^{a,b}(x)$, $A_{\tilde{B}}^{a,b}(1) \geq A_{\tilde{B}}^{a,b}(x)$ and $A_J^{a,b}(1) \leq A_J^{a,b}(x)$ for all $x \in U$.

Let $x, y \in U$ be such that $a * (b * (x * y)) \neq 1$ or $a * (b * x) \neq 1$. Then $A_M^{a,b}(x * y) = \alpha_2$ or $A_M^{a,b}(x) = \alpha_2$. Hence $\min\{A_M^{a,b}(x * y), A_M^{a,b}(x)\} = \alpha_2 \leq A_M^{a,b}(y)$. Assume that $a * (b * (x * y)) = 1$ and $a * (b * x) = 1$. Then

$$
\begin{aligned}
1 &= a * (b * (x * y)) = a * ((b * x) * (b * y)) \\
&= (a * (b * x)) * (a * (b * y)) = 1 * (a * (b * y)) = a * (b * y).
\end{aligned}
$$

Hence $\min\{A_M^{a,b}(x * y), A_M^{a,b}(x)\} = \alpha_1 = A_M^{a,b}(y)$.

By a similar way, we prove that $\operatorname{rmin}\{A_{\tilde{B}}^{a,b}(x * y), A_{\tilde{B}}^{a,b}(x)\} \leq A_{\tilde{B}}^{a,b}(y)$.

Let $x, y \in U$ be such that $a * (b * (x * y)) \neq 1$ or $a * (b * x) \neq 1$. Then $A_J^{a,b}(x * y) = \delta_2$ or $A_J^{a,b}(x) = \delta_2$. Hence $\max\{A_J^{a,b}(x * y), A_J^{a,b}(x)\} = \delta_2 \geq A_J^{a,b}(x)$. Assume that $a * (b * (x * y)) = 1$ and $a * (b * x) = 1$. Then $1 = a * (b * y)$. Hence $\max\{A_J^{a,b}(x * y), A_J^{a,b}(x)\} = \delta_1 = A_J^{a,b}(y)$. Therefore, $\mathscr{A}^{a,b} = (A_M^{a,b}, A_{\tilde{B}}^{a,b}, A_J^{a,b})$ is an MBJ-neutrosophic filter of U. $\qquad\square$

Theorem 4.13. *Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic filter of a BE-algebra U. Let $a \in U$. Consider the set*

$$
\mathscr{A}_a := (A_{M,a}, A_{\tilde{B},a}, A_{J,a}),
$$

where

$$
\begin{aligned}
A_{M,a} &:= \{x \in U | A_M(a) \leq A_M(x)\}, \\
A_{\tilde{B},a} &:= \{x \in U | A_{\tilde{B}}(a) \leq A_{\tilde{B}}(x)\}, \\
A_{J,a} &:= \{x \in U | A_J(a) \geq A_J(x)\}.
\end{aligned}
$$

Then $A_{M,a}, A_{\tilde{B},a}, A_{J,a}$ are filters of U for all $a \in U$.

Proof. Let $x, y \in U$ be such that $x * y \in A_{M,a}$ and $x \in A_{M,a}$. Then $A_M(a) \leq A_M(x * y)$, $A_M(a) \leq A_M(x)$. Using (4.1) and (4.2), we have $A_M(a) \leq \min\{A_M(x * y), A_M(x)\} \leq A_M(y) \leq A_M(1)$. Hence $1, y \in A_{M,a}$.

Let $x, y \in U$ be such that $x * y \in A_{\tilde{B},a}$ and $x \in A_{\tilde{B},a}$. Then $A_{\tilde{B}}(a) \leq A_{\tilde{B}}(x * y)$, $A_{\tilde{B}}(a) \leq A_{\tilde{B}}(x)$. Using (4.1) and (4.2), we have $A_{\tilde{B}}(a) \leq \operatorname{rmin}\{A_{\tilde{B}}(x * y), A_{\tilde{B}}(x)\} \leq A_{\tilde{B}}(y) \leq A_{\tilde{B}}(1)$. Hence $1, y \in A_{\tilde{B},a}$.

Let $x, y \in U$ be such that $x * y \in A_{J,a}$ and $y \in A_{J,a}$. Then $A_J(x * y) \leq A_J(y) \leq A_J(a)$. It follows from (4.1) and (4.2) that $A_J(1) \leq A_J(x) \leq \max\{A_J(x * y), A_J(y)\} \leq A_J(a)$. Hence $1, x \in A_{J,a}$. Therefore, $A_{M,a}, A_{\tilde{B},a}, A_{J,a}$ are filters of U for all $a \in U$. $\qquad\square$

We say that the filters $A_{M,a}, A_{\tilde{B},a}, A_{J,a}$ of U are *MBJ-filters* of $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$.

Theorem 4.14. *Let $a \in U$ and $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set of a BE-algebra U. Then the following assertions are valid:*

(i) *If $A_{M,a}, A_{\tilde{B},a}, A_{J,a}$ are MBJ-filters of $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$, then $\mathscr{A}_a$ satisfies:*

$$
\begin{aligned}
(\forall x, y \in U)(A_M(a) &\leq \min\{A_M(x * y), A_M(x)\} \Rightarrow A_M(a) \leq A_M(y), \\
A_{\tilde{B}}(a) &\leq \operatorname{rmin}\{A_{\tilde{B}}(x * y), A_{\tilde{B}}(x)\} \Rightarrow A_{\tilde{B}}(a) \leq A_{\tilde{B}}(y), \\
A_J(a) &\geq \max\{A_J(x * y), A_J(x)\} \Rightarrow A_J(a) \geq A_J(y));
\end{aligned} \quad (4.5)
$$

(ii) *If $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ satisfies (4.1) and (4.5), then $A_{M,a}, A_{\tilde{B},a}, A_{J,a}$ are MBJ-filters of $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$.*

Proof. (i) Assume that $A_{M,a}, A_{\tilde{B},a}, A_{J,a}$ are MBJ-filters of $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$. Let $x, y \in U$ be such that $A_M(a) \leq \min\{A_M(x * y), A_M(x)\}$. Then $x * y, x \in A_{M,a}$. Since $A_{M,a}$ is a filter of U, $y \in A_{M,a}$ and so $A_M(a) \leq A_M(y)$.

Let $u, v \in U$ be such that $A_{\tilde{B}}(a) \leq \text{rmin}\{A_{\tilde{B}}(u * v), A_{\tilde{B}}(u)\}$. Then $u * v, u \in A_{\tilde{B},a}$. Since $A_{\tilde{B},a}$ is a filter of U, we have $v \in A_{\tilde{B},a}$. Hence $A_{\tilde{B}}(a) \leq A_{\tilde{B}}(v)$.

Let $c, d \in U$ be such that $A_J(a) \leq \min\{A_J(c * d), A_J(c)\}$. Then $c * d, c \in A_{J,a}$. Since $A_{J,a}$ is a filter of U, $d \in A_{J,a}$ and so $A_J(a) \geq A_J(d)$.

(ii) Let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set of U which the conditions (4.1) and (4.5) hold. Then $1 \in A_{M,a}$, $1 \in A_{\tilde{B},a}$, $1 \in A_{J,a}$.

Let $u, v \in U$ be such that $u * v, u \in A_{M,a} \cap A_{\tilde{B},a} \cap A_{J,a}$. Then $A_M(a) \leq A_M(u * v)$, $A_M(a) \leq A_M(u)$, $A_{\tilde{B}}(a) \leq A_{\tilde{B}}(u * v)$, $A_{\tilde{B}}(a) \leq A_{\tilde{B}}(u)$ and $A_J(a) \geq A_J(u * v)$, $A_J(a) \geq A_J(u)$. Hence we have

$$A_M(a) \leq \min\{A_M(u * v), A_M(u)\},$$
$$A_{\tilde{B}}(a) \leq \text{rmin}\{A_{\tilde{B}}(u * v), A_{\tilde{B}}(u)\},$$
$$A_J(a) \geq \max\{A_J(u * v), A_J(u)\}.$$

By (4.5), we get $A_M(a) \leq A_M(v)$, $A_{\tilde{B}}(a) \leq A_{\tilde{B}}(v)$ and $A_J(a) \geq A_J(v)$. Therefore $v \in A_{M,a}$, $v \in A_{\tilde{B},a}$ and $v \in A_{J,a}$. Thus, $A_{M,a}, A_{\tilde{B},a}, A_{J,a}$ are MBJ-filters of $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$. $\quad\square$

Theorem 4.15. *An MBJ-neutrosophic set $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in a BE-algebra U is an MBJ-neutrosophic filter of U if and only if the nonempty sets $U(A_M; t)$, $U(A_{\tilde{B}}; [\alpha_1, \alpha_2])$ and $L(A_J; s)$ are filters of U for all $t, s \in [0, 1]$ and $[\alpha_1, \alpha_2] \in I$.*

Proof. Assume that $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U. Let $t, s \in [0, 1]$ and $[\alpha_1, \alpha_2] \in I$ be such that $U(A_M; t)$, $U(A_{\tilde{B}}; [\alpha_1, \alpha_2])$ and $L(A_J; s)$ are the nonempty sets. Obviously, $1 \in U(A_M; t) \cap U(A_{\tilde{B}}; [\alpha_1, \alpha_2]) \cap L(A_J; s)$. For any $a, b, u, v, x, y \in U$, if $a * b, a \in U(A_M; t)$, $x * y, x \in U(A_{\tilde{B}}; [\alpha_1, \alpha_2])$ and $u * v, u \in L(A_J; s)$, then we have

$$A_M(b) \geq \min\{A_M(a * b), A_M(a)\} \geq \min\{t, t\} = t,$$
$$A_{\tilde{B}}(y) \geq \text{rmin}\{A_{\tilde{B}}(x * y), A_{\tilde{B}}(x)\} \geq \min\{[\alpha_1, \alpha_2], [\alpha_1, \alpha_2]\} = [\alpha_1, \alpha_2],$$
$$A_J(v) \leq \max\{A_J(u * v), A_J(v)\} \leq \max\{s, s\} = s,$$

and so $b \in U(A_M; t), y \in U(A_{\tilde{B}}; [\alpha_1, \alpha_2]), v \in L(A_J; s)$. Therefore, $U(A_M; t)$, $U(A_{\tilde{B}}; [\alpha_1, \alpha_2])$ and $L(A_J; s)$ are filters of U.

Conversely, suppose that $U(A_M; t), U(A_{\tilde{B}}; [\alpha_1, \alpha_2])$ and $L(A_J; s)$ are filters of U for all $t, s \in [0, 1]$ and $[\alpha_1, \alpha_2] \in I$. Assume that $A_M(1) < A_M(u)$, $A_{\tilde{B}}(1) < A_{\tilde{B}}(u)$ and $A_J(1) > A_J(u)$ for some $u \in U$. Then $1 \notin U(A_M; t) \cap U(A_{\tilde{B}}; [\alpha_1, \alpha_2]) \cap L(A_J; s)$. This is a contradiction. Hence $A_M(1) \geq A_M(x)$, $A_{\tilde{B}}(1) \geq (x)$ and $A_J(1) \leq A_J(x)$ for all $x \in U$. If $A_M(b_0) < \min\{A_M(a_0 * b_0), A_M(a_0)\}$ for some $a_0, b_0 \in U$, then $a_0 * b_0, a_0 \in U(A_M; t_0)$ but $b_0 \notin U(A_M; t_0)$ for some $t_0 = \min\{A_M(a_0 * b_0), A_M(a_0)\}$, which is a contradiction. Hence $A_M(y) \geq \min\{A_M(x * y), A_M(x)\}$ for all $x, y \in U$. Similarly, we can prove that $A_J(y) \leq \max\{A_J(x * y), A_J(x)\}$ for all $x, y \in U$. Suppose that $A_{\tilde{B}}(y_0) < \text{rmin}\{A_{\tilde{B}}(x_0 * y_0), A_{\tilde{B}}(x_0)\}$ for some $x_0, y_0 \in U$. Let $A_{\tilde{B}}(x_0 * y_0) = [\beta_1, \beta_2]$, $A_{\tilde{B}}(x_0) = [\beta_3, \beta_4]$ and $A_{\tilde{B}}(y_0) = [\delta_1, \delta_2]$. Then

$$[\delta_1, \delta_2] < \text{rmin}\{[\beta_1, \beta_2], [\beta_3, \beta_4]\} = [\min\{\beta_1, \beta_3\}, \min\{\beta_2, \beta_4\}]$$

and so $\delta_1 < \min\{\beta_1, \beta_3\}$ and $\delta_2 < \min\{\beta_2, \beta_4\}$. Set $\gamma_1, \gamma_2 \in [0, 1]$ so that

$$[\gamma_1, \gamma_2] := \frac{1}{2}(A_{\tilde{B}}(y_0) + \mathrm{rmin}\{A_{\tilde{B}}(x_0 * y_0), A_{\tilde{B}}(x_0)\}).$$

Then we have

$$[\gamma_1, \gamma_2] = \frac{1}{2}([\delta_1, \delta_2] + [\min\{\beta_1, \beta_3\}, \min\{\beta_2, \beta_4\}])$$
$$= [\frac{1}{2}(\delta_1 + \min\{\beta_1, \beta_3\}), \frac{1}{2}(\delta_2 + \min\{\beta_2, \beta_4\})].$$

Hence $\min\{\beta_1, \beta_3\} > \gamma_1 = \frac{1}{2}(\delta_1 + \min\{\beta_1, \beta_3\}) > \delta_1$ and $\min\{\beta_2, \beta_4\} > \gamma_2 = \frac{1}{2}(\delta_2 + \min\{\beta_2, \beta_4\}) > \delta_2$. Thus $[\min\{\beta_1, \beta_3\}, \min\{\beta_2, \beta_4\}] > [\gamma_1, \gamma_2] > [\delta_1, \delta_2] = A_{\tilde{B}}(y_0)$, and therefore $y_0 \notin U(A_{\tilde{B}}; [\gamma_1, \gamma_2])$. On the other hand,

$$A_{\tilde{B}}(x_o * y_0) = [\beta_1, \beta_2] \geq [\min\{\beta_1, \beta_3\}, \min\{\beta_2, \beta_4\}] > [\gamma_1, \gamma_2]$$

and

$$A_{\tilde{B}}(x_0) = [\beta_3, \beta_4] \geq [\min\{\beta_1, \beta_3\}, \min\{\beta_2, \beta_4\}] > [\gamma_1, \gamma_2],$$

that is, $x_0 * y_0, x_0 \in U(A_{\tilde{B}}; [\gamma_1, \gamma_2])$, which is a contradiction. Therefore $A_{\tilde{B}}(y) \geq \mathrm{rmin}\{A_{\tilde{B}}(x * y), A_{\tilde{B}}(x)\}$ for all $x, y \in U$. Thus, $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ in U is an MBJ-neutrosophic filter of U. $\qquad\square$

Theorem 4.16. *Given a filter F of a BE-algebra U, let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ be an MBJ-neutrosophic set in U defined by*

$$A_M(x) = \begin{cases} t & \text{if } x \in F, \\ 0 & \text{otherwise}, \end{cases}$$

$$A_{\tilde{B}}(x) = \begin{cases} [\beta_1, \beta_2] & \text{if } x \in F, \\ [0, 0] & \text{otherwise}, \end{cases} \qquad (4.5)$$

$$A_J(x) = \begin{cases} s, & \text{if } x \in F, \\ 1 & \text{otherwise}, \end{cases}$$

where $t \in (0, 1]$, $s \in [0, 1)$ and $\beta_1, \beta_2 \in (0, 1]$ with $\beta_1 < \beta_2$. Then $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U such that $U(A_M; t) = U(A_{\tilde{B}}; [\beta_1, \beta_2]) = L(A_J; s) = F$.

Proof. It is obviously that $A_M(1) \geq A_M(x)$, $A_{\tilde{B}}(1) \geq A_{\tilde{B}}(x)$ and $A_J(1) \leq A_J(x)$ for all $x \in U$. Let $a, b \in U$. If $a * b \in F$ and $a \in F$, then $b \in F$ and so

$$A_M(b) = a = \min\{A_M(a * b), A_M(a)\},$$
$$A_{\tilde{B}}(b) = [\beta_1, \beta_2] = \mathrm{rmin}\{[\beta_1, \beta_2], [\beta_1, \beta_2]\} = \mathrm{rmin}\{A_{\tilde{B}}(a * b), A_{\tilde{B}}(a)\},$$
$$A_J(b) = s = \max\{A_J(a * b), A_J(a)\}.$$

If any one of $a*b$ and a is contained in F, say $a*b \in S$, $A_M(a*b) = t$, $A_{\tilde{B}}(a*b) = [\beta_1, \beta_2]$, $A_J(a*b) = s$, $A_M(a) = 0$, $A_{\tilde{B}}(a) = [0, 0]$ and $A_J(a) = 1$. Hence we get

$$A_M(b) \geq 0 = \min\{t, 0\} = \min\{A_M(a * b), A_M(a)\},$$
$$A_{\tilde{B}}(b) \geq [0, 0] = \mathrm{rmin}\{[\beta_1, \beta_2], [0, 0]\} = \mathrm{rmin}\{A_{\tilde{B}}(a * b), A_{\tilde{B}}(a)\},$$
$$A_J(b) \leq 1 = \max\{s, 1\} = \max\{A_J(a * b), A_J(a)\}.$$

If $a * b, a \notin S$, then $A_M(a * b) = 0 = A_M(a)$, $A_{\tilde{B}}(a * b) = [0, 0] = A_{\tilde{B}}(a)$ and $A_J(a * b) = 1 = A_J(a)$. It follows that

$$A_M(b) \geq 0 = \min\{0, 0\} = \min\{A_M(a * b), A_M(a)\},$$
$$A_{\tilde{B}}(b) \geq [0, 0] = \mathrm{rmin}\{[0, 0], [0, 0]\} = \mathrm{rmin}\{A_{\tilde{B}}(a * b), A_{\tilde{B}}(a)\},$$
$$A_J(b) \leq 1 = \max\{1, 1\} = \max\{A_J(a * b), A_J(a)\}.$$

Therefore, $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U. Obviously, $U(A_M; t) = U(A_{\tilde{B}}; [\beta_1, \beta_2]) = L(A_J; s) = F$. □

Theorem 4.17. *For any nonempty subset F of a BE-algebra U, let $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic set of U which is given in (4.5). If $\mathscr{A} = (A_M, A_{\tilde{B}}, A_J)$ is an MBJ-neutrosophic filter of U, then F is a filter of U.*

Proof. Obviously, $1 \in F$. Let $x, y \in U$ be such that $x * y \in F$ and $x \in F$. Then $A_M(x * y) = t = A_M(x)$, $A_{\tilde{B}}(x * y) = [\beta_1, \beta_2] = A_{\tilde{B}}(x)$ and $A_J(x * y) = s = A_J(x)$. Thus

$$A_M(y) \geq \min\{A_M(x * y), A_M(x)\} = t,$$
$$A_{\tilde{B}}(y) \geq \mathrm{rmin}\{A_{\tilde{B}}(x * y), A_{\tilde{B}}(x)\} = [\beta_1, \beta_2],$$
$$A_J(y) \leq \max\{A_J(x * y), A_M(y)\} = s,$$

and hence $y \in F$. Therefore, F is a filter of U.

5. Conclusions

The neutrosophic set is a generalized concept of the intuitionistic fuzzy set (IFS), paraconsistent set and intuitionistic set, and was introduced by Smarandache. The neutrosophic set has a significant role for denoising, clustering, segmentation and classification in numerous medical image-processing applications. Mohseni Takallo et al. introduced the notion of MBJ-neutrosophic sets based on the need for a tool that can deal with the uncertainty problem in the case of partially including information expressed by interval values with a neutrosophic concept. The MBJ-neutrosophic set was created by using interval-valued fuzzy set instead of fuzzy set in the indeterminate membership function of neutrosophic set. By applying the MBJ-neutrosophic set to *BE*-algebras, we introduced the concept of MBJ-neutrosophic subalgebra in *BE*-algebras, and investigated some its related properties. We provided some characterizations of MBJ-neutrosophic subalgebras in *BE*-algebras. Also we defined the concept of MBJ-neutrosophic filter in *BE*-algebras and dicussed its related properties. We investigated relationships between MBJ-neutrosophic subalgebras and MBJ-neutrosophic filters. We provided an example which shows that an MBJ-neutrosophic subalgebra need not be an MBJ-neutrosophic filter. We established some charactizations of an MBJ-neutrosophic filter.

Based on the ideas and results of this paper, we will study MBJ-neutrosophic normal filters, MBJ-neutrosophic mighty filters, MBJ-neutrosophic medial filters and MBJ-neutrosophic regular filters in *BE*-algebras, and compare them with the results of this study. Also, our future work involves applications of the MBJ-neutrosophic set to substructures of various algebraic structures, for example, GE-algebra, hoop algebra, equality algebra, EQ-algebra, BL-algebra, group, (near, semi)-ring etc. Moreover, we will find ways and technologies to apply the MBJ-neutrosophic set to decision-making theory, computer science and medical science etc. in the future.

Conflict of interest

All authors declare no conflicts of interest in this paper.

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