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Research article

Event-triggered bipartite consensus of multi-agent systems in signed networks

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Abstract: This paper focuses on the event-triggered bipartite consensus of multi-agent systems in signed networks, where the dynamics of each agent is assumed to be Lur'e system, and both the cooperative interaction and antagonistic interaction are allowed among neighbor agents. A novel event-triggered communication scheme is presented to save limited network resources, and distributed bipartite control techniques are raised to address the bipartite leaderless consensus and bipartite leader-following consensus respectively. By virtue of the Lyapunov stability theory and algebraic graph theory, bipartite consensus conditions are derived, which can be easily solved by MATLAB. In addition, the upper bounds of the sampling period and triggered parameter can be estimated. Finally, two examples are employed to show the validity and advantage of the proposed transmission scheme.

Keywords: bipartite consensus; Lur'e system; event-triggered communication scheme; signed networks
Mathematics Subject Classification: 93C57, 93C65

1. Introduction

During the past decade, the cooperative control of multi-agent systems has got compelling attentions owing to intensive applications [1–3]. As one of the most important topics of cooperative behaviors, the consensus problem of multi-agent systems has attracted much attention, many fruitful and crucial results have been obtained to build the theoretical consensus framework, see [4–15] and the references therein.

Most of the existing results are focused on the cooperative systems, however, the competitive relationship is very common in practical applications [16, 17]. Therefore, more and more attention has been paid to signed networks with both cooperative and antagonistic links [18–22]. In order to achieve bipartite consensus, the structurally balanced condition is essential [23]. As an extension to the bipartite consensus, the concept of interval bipartite is introduced in [24], and interval bipartite consensus results are derived for structurally balanced networks and structurally unbalanced networks. If the network structure and node function satisfy some given conditions, the synchronization conditions are established for first-order multi-agent systems by utilizing pinning control strategy [25, 26], the above results are extended to second-order nonlinear multi-agent systems [27, 28], where the event-triggered control algorithm and pinning control strategy are used to achieve bipartite consensus. For general linear multi-agent systems, a systematical approach is given in [29] for solving the bipartite consensus problem. In [30], the bipartite tracking consensus problem is investigated for a dynamic leader. The finite-time bipartite consensus is studied in [31]. The adaptive bipartite consensus control is designed for high-order multi-agent systems [32]. Additionally, similar to the bipartite consensus problem, the reverse group consensus is firstly studied in the cooperation-competition network [33], the results show that the couple group consensus can be achieved if the mirror graph is strongly connected, as an extension, the adaptive robust bipartite consensus is considered for high-order uncertain multi-agent systems over the cooperation-competition network [34].

In practical multi-agent systems, each agent is equipped with a micro-processor, it can collect information from neighboring agents, therefore, it is desired to implement the controller updates on a digital platform [35], the scheduling can be done in a time-triggered scheme or an event-triggered scheme, where the time-triggered scheme has a fixed sampling period, which should be selected to guarantee a desired performance under worst conditions, this kind of triggered method will send many unnecessary sampling data to neighboring agents. Considering the limited network resources, the event-triggered scheme is an effective method to improve the resources utilization while ensuring a satisfactory performance [36–38].

The design of event-triggered transmission scheme is crucial to determine whether the sampling data should be transmitted. The event-triggered transmission schemes can be classified into two types, the first case is the absolute error-based event-triggered transmission scheme [38], which is dependent on a pre-specified positive instant and the state error. The second case is the relative error-based event-triggered transmission scheme, which is related to the system state [39, 40], these schemes can reduce the data transmission significantly compared with the periodical transmission scheme. More recently, significant considerations have been focused on the event-triggered control for multi-agent systems [41–51]. Actuator saturation can be observed in many practical systems due to physical or safety constraints, two kinds of distributed event-triggered control schemes are designed for the consensus of nonlinear systems subject to input saturation [41], where event-triggered parameters can be adaptively adjusted by using some adaptive laws. In [42], the distributed protocol is assumed to be subject to saturation, by utilizing the event-triggered control, the group consensus is discussed for a class of multi-agent systems with non-identical dynamics. The general linear models are considered to investigate event-triggered consensus problems in [43], the advantage of the event-based strategy is the significant decrease of the number of controller updates. Three types of event-triggered schemes are proposed for different network topologies [44]. Noted that communication topologies face more risks from cyber attacks, some distributed event-triggered schemes are proposed for linear and nonlinear multi-agent systems with Dos attacks [45, 46]. Based on event-triggered schemes, the consensus problems are discussed for nonlinear systems [47, 48]. Especially, in [48], an integrated sampled-data-based event-triggered communication scheme includes advantages of both absolute and relative error-based event-triggered transmission schemes, thus it can lead to a high efficiency of data transmissions. In [49], considering quantized information, the periodic event-triggered algorithms are proposed for the consensus of multi-agent systems. The results on observer-based event-triggered consensus are also developed in [51, 52]. In addition, switching topologies and stochastic sampling have been also considered for the consensus of multi-agent systems by event-triggered schemes [53–56], where the stochastic sampling interval randomly switches between two given values. The dynamic event-triggered schemes are developed for the consensus of multi-agent systems [57, 58], compared with the traditional static event-triggered schemes, the time-varying threshold can ensure less triggered instants. In [59], the new event-triggered control law and triggering condition are constructed without continuous inter-neighboring communication, the event-triggered control is designed for the prescribed-time bipartite consensus. In [60], a novel observer-based bipartite control scheme is developed on the basis of two event-triggering mechanisms. In [61], a mode-dependent event-triggered transmission strategy is proposed for fixed-time bipartite consensus of a class of nonlinear multi-agent systems. In [62], distributed event-triggered control strategy is proposed for bipartite consensus for high-order multi-agent systems. However, the above schemes still embrace some room for improvement, in [39, 40, 45, 46], the next transmission instant is determined by $t_{k+1}h = t_kh + \min\{e^T(j_kh)\Phi e(j_kh) > \sigma x^T(t_kh)\Phi x(t_kh)\}$, where Φ is a symmetric positive definite matrix, which can be replaced by $t_{k+1}h = t_kh + \min\{e^T(j_kh)\Phi_1 e(j_kh) > \sigma x^T(t_kh)\Phi_2 x(t_kh)\}$, where Φ_1 and Φ_2 are two symmetric positive definite matrices, in simulation examples, it can be seen that this scheme can lead to larger sampling period. Compared with [27, 52, 55], in our paper the form of the event-triggered scheme is relatively simple and easy to be applied in practical applications, the lower bound event interval is the sampling period h, which is strictly larger than zero, therefore, the Zeno behavior is excluded in our proposed scheme. Moreover, in [27, 52, 55], in order to avoid Zeno behavior, the index term $-\beta e^{-\gamma(t-t_0)}$ has been introduced in the event-triggered function, which may bring some trouble in the theoretical analysis or practical application. In addition, the bipartite synchronization of Lur'e network or neural network have been investigated in [25, 26], where the controller designs have been based on the continuous or sampled communication information among the agents. In [63], the consensus problem is discussed for second-order multi-agent systems based on the event-triggered scheme, but the consensus protocol contains the continuous communication information of the leader. To the best of our knowledge, there are few results focusing on bipartite consensus of multi-agent systems with positive and negative communication links via event-triggered strategy, which is the main motivation of this paper.

Motivated by the aforementioned discussion, this paper is devoted to investigating bipartite consensus of Lur'e system in signed networks. The main contributions of this paper can be summarized as follows: (1) Compared with existing communication schemes in the literatures, the proposed event-triggered scheme can ensure the desired performance while further reducing the frequency of data transmission. (2) By using coordinate transform and matrix analysis techniques, sufficient conditions are derived to guarantee that bipartite leaderless consensus and leader-following consensus can be achieved, respectively. (3) The upper bound of the sampling period can be obtained by solving a constrained optimization problem. (4) In the proposed event-triggered communication scheme, the Zeno behavior is excluded owing to the fact that the lower bound of inter event interval is the sampling period.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is a set of real $n \times m$ matrices. $diag\{\cdots\}$ stands for a block-diagonal matrix. The notation with the superscript $1_n(0_n)$ indicates the

n-dimensional column vector with each entry being 1(0). $sgn(\cdot)$ is the standard sign function. Notations $\|.\|$ and $\otimes$ denote the Euclidian norm and the Kronecker product, respectively.

2. Problem formulation and preliminaries

The communication topology of a network can be modeled as a signed graph $g = (v, \varepsilon, A)$, where $v = (v_1, v_2, \ldots, v_N)$ denotes the set of nodes, $\varepsilon \subseteq v \times v$ is the set of edges, $A = (a_{ij})_{N\times N}$ denotes a signed adjacency matrix of g representing the communication topology. If $a_{ij} = 0$, it means that agent i cannot receive information from agent j. The entry a_{ij} can be allowed to be positive or negative, with $a_{ij} > 0$ indicating that node i and node j are cooperative, $a_{ij} < 0$ indicating the node i and node j are competitive. Based on the weighted adjacency matrix A, the Laplacian matrix of the signed graph g can be defined as $L = (l_{ij})_{N\times N}$, where $l_{ii} = \sum_{k=1,k\neq i} |a_{ik}|$, $l_{ij} = -a_{ij}$ $(i \neq j)$. If there exists a rooted node having directed path to all other nodes, the signed graph g contains a directed spanning tree.

Remark 1. *For the above definition of Laplacian matrix L, the sum of the ith row of L is* $\sum_{k=1,k\neq i}^{N} |a_{ik}| - \sum_{k=1,k\neq i}^{N} a_{ik}$*, the element* a_{ij} *can be allowed to be positive or negative for the signed graph g, thus* $\sum_{k=1,k\neq i}^{N} |a_{ik}| = \sum_{k=1,k\neq i}^{N} a_{ik}$ *may not be hold, that is the Laplacian matrix of signed digraph may not be a zero-row-sum matrix, which leads to the consequence that the traditional control protocol design method can not be used directly.*

Consider a class of nonlinear multi-agent systems with the Lur'e system form

$$\dot{x}_i(t) = Ax_i(t) + Bf(Cx_i(t)) + u_i(t), \qquad (i = 1, 2, \ldots, N), \tag{2.1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector of node i, $f(Cx_i(t)) = [f_1(C_1^T x_i(t)), f_2(C_2^T x_i(t)), \ldots, f_m(C_m^T x_i(t))]^T \in \mathbb{R}^m$ is a vector-valued nonlinear function. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C = [C_1, C_2, \ldots, C_i, \ldots, C_m]^T \in \mathbb{R}^{m\times n}$ and $C_i = [C_{i1}, C_{i2}, \ldots, C_{in}]^T$ $(i = 1, 2, \ldots, m)$ are some constant matrices. $u_i(t) \in \mathbb{R}^n$ is the control input.

Assumption 1. *Suppose that the nonlinear function* $f_k(\cdot)$ $(k = 1, 2, \ldots, m)$ *is an odd function satisfying the following sector condition*

$$0 \leq \frac{f_k(z_2) - f_k(z_1)}{z_2 - z_1} \leq \delta_k, \qquad \forall z_1, z_2 \in \mathbb{R}, z_1 \neq z_2, \tag{2.2}$$

where $\delta_k > 0$ $(k = 1, 2, \ldots, m)$.

Remark 2. *The nonlinear function* $f(\cdot)$ *is assumed to be an odd function due to the fact that the gauge transformation can do work well, which can be found in the proof of Theorem 1. In addition, it is a mild assumption in existing literatures, whether the limitation of nonlinear function can be removed may be discussed in the future.*

Definition 1. *[25] A signed graph g is called structurally balanced if the node set v can be divided into two sub-networks* V_1 *and* V_2*, which satisfy* $V_1 \cup V_2 = v$ *and* $V_1 \cap V_2 = \Phi$*, and* $a_{ij} \geq 0$ *holds for node i and node j which are in the same sub-network, and* $a_{ij} < 0$ *holds for node i and node j which are in different sub-networks. Otherwise, the signed graph g is called structurally unbalanced.*

Lemma 1. *[23] Suppose that the signed network g is structurally balanced, if and only if there is a gauge transformation W such that* $W^TAW = \bar{A}$ *with nonnegative entries, where* $W = diag\{w_1, w_2, \ldots, w_N\}$ *with* $w_i \in \{1, -1\}$*,* $A = (a_{ij})_{N\times N}$ *and* $\bar{A} = (|a_{ij}|)_{N\times N}$ *are the adjacency matrices of the signed and unsigned networks, respectively. In addition, W provides a partition, that is* $V_1 = \{v_i | w_i = 1\}$ *and* $V_2 = \{v_i | w_i = -1\}$*.*

Lemma 2. *[64] If* $x(t) : (a, b) \to \mathbb{R}^n$ *is an absolutely continuous function satisfying* $x(a) = 0$ *and* $M \in \mathbb{R}^{n\times n}$ *is a positive definite matrix, then the following Wirtinger's inequality holds*

$$\int_a^b x^T(s)Mx(s)ds \;\; \le \;\; \frac{4(b-a)^2}{\pi^2}\int_a^b \dot{x}^T(s)M\dot{x}(s)ds. \tag{2.3}$$

Lemma 3. *[65] For any positive definite matrix* $M \in \mathbb{R}^{n\times n}$ *and a vector function* $x(t) : (a, b) \to \mathbb{R}^n$*, such that the following integrations are well defined, then*

$$(b-a)\int_a^b x^T(s)Mx(s)ds \;\; \ge \;\; \int_a^b x^T(s)dsM\int_a^b x(s)ds. \tag{2.4}$$

Lemma 4. *[66] For any two real vector functions* $y_1(t) \in \mathbb{R}^n$ *and* $y_2(t) \in \mathbb{R}^n$*, a positive definite matrix* $R \in \mathbb{R}^{n\times n}$ *and any matrix* $S \in \mathbb{R}^{n\times n}$*, then*

$$\frac{1}{\alpha}y_1^T(t)Ry_1(t) + \frac{1}{1-\alpha}y_2^T(t)Ry_2(t) \;\; \ge \;\; y^T(t)\begin{bmatrix} R & S^T \\ S & R \end{bmatrix}y(t), \tag{2.5}$$

where $y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$, $\begin{bmatrix} R & S^T \\ S & R \end{bmatrix} > 0$ *and* $0 < \alpha < 1$.

Lemma 5. *[67] The following linear matrix inequality*

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} \;\; > \;\; 0, \tag{2.6}$$

where $Q(x) = Q^T(x)$*,*$R(x) = R^T(x)$*, is equivalent to either of the following conditions*

$$(1)\ Q(x) > 0, R(x) - S^T(x)Q^{-1}(x)S(x) > 0,$$
$$(2)\ R(x) > 0, Q(x) - S(x)R^{-1}(x)S^T(x) > 0.$$

3. Main results

3.1. Bipartite leaderless consensus

In this section, it is assumed that the topology is a directed signed graph and structurally balanced. An event-triggered scheme is designed, and by some transformation, the bipartite leaderless consensus problem is converted to the stability of the error system.

Definition 2. *[25] The signed Lur'e network (2.1) is said to achieve bipartite leaderless consensus if the following equation holds*

$$\lim_{t\to\infty} \|w_i x_i(t) - w_j x_j(t)\| = 0, \qquad (i, j = 1, 2, \ldots, N; i \neq j), \tag{3.1}$$

where $w_i = 1, (i \in V_1)$*;* $w_i = -1, (i \in V_2)$*.*

Remark 3. *If the agents $i, j \in V_1$ or $i, j \in V_2$, which means they are in the same sub-network, then $w_i = w_j = 1$ or $w_i = w_j = -1$, the equation (3.1) reduces to $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0$, which is the traditional leaderless consensus of unsigned networks. If the agents $i \in V_1, j \in V_2$ or $i \in V_2, j \in V_1$, it means that the agents i and j are in different sub-networks, the equation (3.1) reduces to $\lim_{t\to\infty} \|x_i(t) + x_j(t)\| = 0$, which can be rewritten as $\lim_{t\to\infty} \|x_i(t) - (-x_j(t))\| = 0$, which represents agent j competes against agent i.*

For reducing the communication burden among the agents, the following sampled control algorithm is designed

$$u_i(t) = -c \sum_{j\in N(i)} |a_{ij}|[x_i(t_k h) - sgn(a_{ij})x_j(t_k h)], \qquad (i = 1, 2, \ldots, N; t \in [t_k h, t_{k+1}h); k \in Z^+), \tag{3.2}$$

where $c > 0$ is the coupling strength, $t_k h$ $(k = 0, 1, 2, \ldots)$ is the release times, $t_0 h = 0$, $\{t_0, t_1, \ldots\} \subset \{0, 1, 2, \ldots\}$, the data will be updated for $t \in [t_k h, t_{k+1}h)$ until $t = t_{k+1}h$, which can effectively reduce the data transmission among the agents. When $a_{ij} > 0$, the coupling information from its neighboring agent can be described as $a_{ij}[x_i(t_k h) - x_j(t_k h)]$, which means agent i and agent j are cooperative, when $a_{ij} < 0$, the coupling information from its neighboring agent is $-a_{ij}[x_i(t_k h) + x_j(t_k h)]$, which represents that interaction is antagonistic.

Applying (3.2) to the system (2.1) yields

$$\dot{x}_i(t) = Ax_i(t) + Bf(Cx_i(t)) - c \sum_{j\in N(i)} |a_{ij}|[x_i(t_k h) - sgn(a_{ij})x_j(t_k h)]. \tag{3.3}$$

Based on the definition of the Laplacian matrix of signed graph, the equation (3.3) can be rewritten as

$$\dot{x}_i(t) = Ax_i(t) + Bf(Cx_i(t)) - c \sum_{j=1}^{N} l_{ij} x_j(t_k h), \qquad (i = 1, 2, \ldots, N), \tag{3.4}$$

where $l_{ii} = \sum_{k=1,k\neq i} |a_{ik}|$, $l_{ij} = -a_{ij}$ $(i \neq j)$.

Considering limited bandwidth of the communication network, to further reduce the data transmission among the agents, an event-triggered communication scheme is proposed for system (2.1), which is utilized to decide whether the current sampled data should be transmitted to neighboring agents or not. Supposed that the latest transmitted data is $x(t_k h)$, then the next transmission instant $t_{k+1}h$ can be determined by

$$\begin{aligned} t_{k+1}h &= t_k h + \min\{lh | [e_i(t_k h + lh) - e_i(t_k h)]^T \Phi_i^{(1)} [e_i(t_k h + lh) - e_i(t_k h)] \\ &> \sigma_i e_i^T(t_k h + lh) \Phi_i^{(2)} e_i(t_k h + lh)\}, \end{aligned} \tag{3.5}$$

where the error $e_i(t) = w_i x_i(t) - w_1 x_1(t)$, $\sigma_i > 0$ are event-triggered parameters, $\Phi_i^{(1)}$ and $\Phi_i^{(2)}$, $(i = 2, 3, \ldots, N)$ are positive definite event-triggered matrices which need to be designed. $t_{k+1}h - t_k h$ denotes the release period which corresponds to the sampling period given by the event-triggered scheme in (3.5).

Remark 4. *Noted that the above event-triggered communication scheme (3.5) including the following special cases: (I) if $\Phi_i^{(1)} = \Phi_i^{(2)} = \Phi_i$ $(i = 2, 3, \ldots, N)$, the scheme (3.5) shrinks to a discrete* *event-triggered transmission scheme [39, 40]. (II) if $\sigma_i = 0$ and $t_{k+1}h = t_k h + h$, then transmission scheme (3.5) shrinks to a periodic transmission scheme. (III) if $\Phi_i^{(1)} = \Phi_i^{(2)} = I_n$ $(i = 2, 3, \ldots, N)$, transmission scheme (3.5) becomes a discrete absolute error-based transmission scheme. Hence the proposed event-triggered communication scheme includes some existing schemes. The proposed transmission scheme (3.5) where only the sampled-data is used is different from a continuous absolute error-based scheme that needs to monitor the continuous measurement. Compared with [27], the form of the event-triggered scheme is relatively simple and easy to be applied in practical applications, moreover in our paper the lower bound event interval is the sampling period h, which is strictly larger than zero, therefore, the Zeno behavior is excluded in our proposed scheme.*

Remark 5. *The bipartite synchronization of Lur'e network or neural network has been investigated in [25,26], where the controller designs have been based on the continuous or sampled communication information among the agents. This paper has been based on the event-triggered scheme, the control signals are only updated at a series of triggered instants. In [63], the consensus problem is discussed for second-order multi-agent systems based on event-triggered scheme, but the consensus protocol contains the continuous communication information of the leader.*

Let $\bar{x}_i(t) = w_i x_i(t)$ $(i = 1, 2, \ldots, N)$, here $w_i = 1$ $(i \in V_1)$ and $w_i = -1$ $(i \in V_2)$, it follows from (3.4) that

$$\dot{\bar{x}}_i(t) = A\bar{x}_i(t) + Bw_i f(Cw_i\bar{x}_i(t)) - c\sum_{j=1}^{N} \bar{l}_{ij}\bar{x}_j(t_k h), \qquad (i = 1, 2, \ldots, N), \tag{3.6}$$

where $\bar{L} = (\bar{l}_{ij})_{N\times N}$ is the Laplacian matrix of the unsigned graph with zero-row-sum, $\bar{l}_{ii} = \sum_{k=1,k\neq i} |a_{ik}|$, $\bar{l}_{ij} = w_i l_{ij} w_j = -|a_{ij}|$ $(i \neq j)$.

By using Assumption 1, it is easy to obtain $w_i f(Cw_i\bar{x}_i(t)) = w_i^2 f(C\bar{x}_i(t)) = f(C\bar{x}_i(t))$ owing to $f(\cdot)$ being an odd function, then (3.6) can be simplified as

$$\dot{\bar{x}}_i(t) = A\bar{x}_i(t) + Bf(C\bar{x}_i(t)) - c\sum_{j=1}^{N} \bar{l}_{ij}\bar{x}_j(t_k h), \qquad (i = 1, 2, \ldots, N). \tag{3.7}$$

In views of (3.3) and $e_i(t) = \bar{x}_i(t) - \bar{x}_1(t)$ $(i = 2, 3, \ldots, N)$, one has

$$\dot{e}_i(t) = Ae_i(t) + Bf(C\bar{x}_i(t)) - Bf(C\bar{x}_1(t)) - c\sum_{j=2}^{N} (\bar{l}_{ij} - \bar{l}_{1j})e_j(t_k h), \qquad (i = 2, 3, \ldots, N). \tag{3.8}$$

Partly inspired by [39, 40], the interval $[t_k h, t_{k+1}h)$ can be expressed as the union of several subintervals

$$[t_k h, t_{k+1}h) = \cup_{l=0}^{t_{k+1}-1}[t_k h + lh, t_k h + lh + h). \tag{3.9}$$

Define $\tau(t) = t - (t_k h + lh)$, for $t \in [t_k h + lh, t_k h + lh + h)$, it can be derived that $0 \leq \tau(t) < h$, let $\bar{e}_i(t_k h + lh) = e_i(t_k h + lh) - e_i(t_k h)$ $(i = 2, 3, \ldots, N)$, we have

$$e_i(t_k h) = e_i(t - \tau(t)) - \bar{e}_i(t_k h + lh). \tag{3.10}$$

Substituting (3.10) into (3.8) yields

$$\dot{e}_i(t) = Ae_i(t) + Bf(C\bar{x}_i(t)) - Bf(C\bar{x}_1(t)) - c\sum_{j=2}^{N}(\bar{l}_{ij} - \bar{l}_{1j})[e_j(t-\tau(t)) - \bar{e}_j(t_kh + lh)]. \quad (3.11)$$

By rewrite (3.11) in compact matrix form, one may further get that

$$\dot{e}(t) = (I_{N-1}\otimes A)e(t) - c(\hat{L}\otimes I_n)e(t-\tau(t)) + c(\hat{L}\otimes I_n)\bar{e}(t_kh+lh) + (I_{N-1}\otimes B)\eta(t), \quad (3.12)$$

where

$$\begin{aligned}
e(t) &= [e_2^T(t), e_3^T(t), \ldots, e_N^T(t)]^T,\\
\eta(t) &= [\eta_2^T(t), \eta_3^T(t), \ldots, \eta_N^T(t)]^T,\\
\eta_i(t) &= [\eta_{i1}(t), \eta_{i2}(t), \ldots, \eta_{im}(t)]^T,\\
\eta_{ik}(t) &= f_k(C_k^T\bar{x}_i(t)) - f_k(C_k^T\bar{x}_1(t)) \quad (k = 1, 2, \ldots, m),\\
\hat{L} &= (\hat{l}_{pq}) \in \mathbb{R}^{(N-1)\times(N-1)},\\
\hat{l}_{pq} &= \bar{l}_{(p+1),(q+1)} - \bar{l}_{1,(q+1)} \quad (p, q = 1, 2, \ldots, N-1).
\end{aligned}$$

Combining (3.5) and (3.10), for $t \in [t_kh, t_{k+1}h)$, the current sampled data $e_i(t_kh + lh)$ will not be sent, then the event-triggered scheme can be rewritten as

$$[e_i(t_kh+lh) - e_i(t_kh)]^T\Phi_i^{(1)}[e_i(t_kh+lh) - e_i(t_kh)] \le \sigma_i e_i^T(t-\tau(t))\Phi_i^{(2)}e_i(t-\tau(t)). \quad (3.13)$$

By using Kronecher product, from (3.13), one get

$$\bar{e}^T(t_kh+lh)\Phi^{(1)}\bar{e}(t_kh+lh) \le e^T(t-\tau(t))(\sigma\otimes I_n)\Phi^{(2)}e(t-\tau(t)), \quad (3.14)$$

where

$$\begin{aligned}
\bar{e}(t_kh+lh) &= [\bar{e}_2^T(t_kh+lh), \bar{e}_3^T(t_kh+lh), \ldots, \bar{e}_N^T(t_kh+lh)]^T,\\
\bar{e}_i(t_kh+lh) &= e_i(t_kh+lh) - e_i(t_kh),\\
\Phi^{(1)} &= diag\{\Phi_2^{(1)}, \Phi_3^{(1)}, \ldots, \Phi_N^{(1)}\},\\
\Phi^{(2)} &= diag\{\Phi_2^{(2)}, \Phi_3^{(2)}, \ldots, \Phi_N^{(2)}\},\\
\sigma &= diag\{\sigma_2, \sigma_3, \ldots, \sigma_N\}.
\end{aligned}$$

Theorem 1. *Suppose that Assumption 1 holds, for given sampled period $h > 0$, trigger parameters $\sigma_i > 0$ $(i = 2, 3, \ldots, N)$ and coupled strength $c > 0$, the bipartite leaderless consensus can be reached in system (2.1) under the control law (3.2) and event-triggered scheme (3.14), if there exist some $(N-1)n\times(N-1)n$ positive definite matrices $P > 0$, $Q > 0$, $R > 0$, $\Omega > 0$ and $T = diag\{\tau_1, \tau_2, \ldots, \tau_m\} > 0$, and matrix $U \in \mathbb{R}^{(N-1)n\times(N-1)n}$, such that the following linear matrix inequalities hold*

$$\begin{bmatrix}
\Gamma_{11} & \Gamma_{12} & U^T & cP(\hat{L}\otimes I_n) & \frac{\pi^2}{4}\Omega & \Gamma_{16} & h(I_{N-1}\otimes A)^TR & h(I_{N-1}\otimes A)^T\Omega\\
* & \Gamma_{22} & -U^T+R & 0 & 0 & 0 & -hc(\hat{L}\otimes I_n)^TR & -hc(\hat{L}\otimes I_n)^T\Omega\\
* & * & -Q-R & 0 & 0 & 0 & 0 & 0\\
* & * & * & -\Phi^{(1)} & 0 & 0 & hc(\hat{L}\otimes I_n)^TR & hc(\hat{L}\otimes I_n)^T\Omega\\
* & * & * & * & -\frac{\pi^2}{4}\Omega & 0 & 0 & 0\\
* & * & * & * & * & -I_{N-1}\otimes T & h(I_{N-1}\otimes B)^TR & h(I_{N-1}\otimes B)^T\Omega\\
* & * & * & * & * & * & -R & 0\\
* & * & * & * & * & * & * & -\Omega
\end{bmatrix}$$

$$< 0, \tag{3.15}$$

$$\begin{bmatrix} R & U^T \\ * & R \end{bmatrix} > 0, \tag{3.16}$$

where

$$\begin{aligned}
\Gamma_{11} &= P(I_{N-1} \otimes A) + (I_{N-1} \otimes A)^T P + Q - R - \frac{\pi^2}{4}\Omega, \\
\Gamma_{12} &= -cP(\hat{L} \otimes I_n) - U^T + R, \\
\Gamma_{16} &= P(I_{N-1} \otimes B) + I_{N-1} \otimes C^T \Delta T, \\
\Gamma_{22} &= -2R + U + U^T + (\sigma \otimes I_n)\Phi^{(2)}, \\
\Delta &= diag\{\delta_1, \delta_2, \ldots, \delta_m\}.
\end{aligned}$$

Proof. Construct the following Lyapunov function candidate

$$V(t) = \sum_{i=1}^{4} V_i(t), \tag{3.17}$$

where

$$\begin{aligned}
V_1(t) &= e^T(t)Pe(t), \\
V_2(t) &= \int_{t-h}^{t} e^T(s)Qe(s)ds, \\
V_3(t) &= h\int_{t-h}^{t}\int_{s}^{t} \dot{e}^T(v)R\dot{e}(v)dvds, \\
V_4(t) &= h^2\int_{t_kh+lh}^{t} \dot{e}^T(s)\Omega\dot{e}(s)ds - \frac{\pi^2}{4}\int_{t_kh+lh}^{t}[e(s) - e(t_kh + lh)]^T\Omega[e(s) - e(t_kh + lh)]ds,
\end{aligned}$$

where $P, Q, R, \Omega \in \mathbb{R}^{(N-1)n\times(N-1)n} > 0$.

Next, some necessary explanations are given to show the Lyapunov function $V_4(t)$ is valid. By using the Wirtinger's inequality in Lemma 2, one has

$$\begin{aligned}
h^2\int_{t_kh+lh}^{t} \dot{e}^T(s)\Omega\dot{e}(s)ds &= h^2\int_{t_kh+lh}^{t}[\dot{e}(s) - \dot{e}(t_kh + lh)]^T\Omega[\dot{e}(s) - \dot{e}(t_kh + lh)]ds \\
&\geq \frac{h^2\pi^2}{4[t - (t_kh + lh)]^2}\int_{t_kh+lh}^{t}[e(s) - e(t_kh + lh)]^T\Omega[e(s) - e(t_kh + lh)]ds \\
&\geq \frac{\pi^2}{4}\int_{t_kh+lh}^{t}[e(s) - e(t_kh + lh)]^T\Omega[e(s) - e(t_kh + lh)]ds,
\end{aligned} \tag{3.18}$$

it can be easily derived that $V_4(t) \geq 0$, thus the Lyapunov function $V_4(t)$ is valid.

By using Assumption 1, we can obtain

$$\eta_{ik}(t)(\eta_{ik}(t) - \delta_k C_k^T e_i(t)) \leq 0, \tag{3.19}$$

for $\tau_k > 0$ $(k = 1, 2, \ldots, m)$, we have

$$\sum_{k=1}^{m} \tau_k \eta_{ik}(t)(\eta_{ik}(t) - \delta_k C_k^T e_i(t)) \quad \leq \quad 0. \tag{3.20}$$

The above equation (3.20) can be rewritten in the compact matrix form as follows

$$\eta^T(t)(I_{N-1} \otimes T)\eta(t) - e^T(t)(I_{N-1} \otimes C^T \Delta T)\eta(t) \quad \leq \quad 0, \tag{3.21}$$

where

$$\begin{aligned} T &= diag\{\tau_1, \tau_2, \ldots, \tau_m\}, \\ C &= [C_1, C_2, \ldots, C_m]^T, \\ \Delta &= diag\{\delta_1, \delta_2, \ldots, \delta_m\}. \end{aligned}$$

For $t \in [t_k h, t_{k+1} h)$, taking the derivative of $V_i(t)$ $(i = 1, 2, 3, 4)$ along the trajectories (3.12), one gets

$$\begin{aligned} \dot{V}_1(t) &= 2e^T(t)P[(I_{N-1} \otimes A)e(t) - c(\hat{L} \otimes I_n)e(t - \tau(t)) + c(\hat{L} \otimes I_n)\bar{e}(t_k h + lh) + (I_{N-1} \otimes B)\eta(t)] \\ &= e^T(t)[P(I_{N-1} \otimes A) + (I_{N-1} \otimes A)^T P]e(t) - 2ce^T(t)P(\hat{L} \otimes I_n)e(t - \tau(t)) \\ &\quad +2ce^T(t)P(\hat{L} \otimes I_n)\bar{e}(t_k h + lh) + 2e^T(t)P(I_{N-1} \otimes B)\eta(t). \end{aligned} \tag{3.22}$$

$$\dot{V}_2(t) = e^T(t)Qe(t) - e^T(t - h)Qe(t - h). \tag{3.23}$$

$$\dot{V}_3(t) = h^2 \dot{e}^T(t)R\dot{e}(t) - h\int_{t-h}^{t} \dot{e}^T(s)R\dot{e}(s)ds. \tag{3.24}$$

$$\dot{V}_4(t) = h^2 \dot{e}^T(t)\Omega\dot{e}(t) - \frac{\pi^2}{4}[e(t) - e(t_k h + lh)]^T \Omega[e(t) - e(t_k h + lh)]. \tag{3.25}$$

For any $\tau(t) \in [0, h)$, it follows from Jensen's inequality in Lemma 3 and the reciprocally convex approach in Lemma 4 that

$$\begin{aligned} & -h\int_{t-h}^{t} \dot{e}^T(s)R\dot{e}(s)ds \\ = & -h\int_{t-\tau(t)}^{t} \dot{e}^T(s)R\dot{e}(s)ds - h\int_{t-h}^{t-\tau(t)} \dot{e}^T(s)R\dot{e}(s)ds \\ \leq & -\frac{h}{\tau(t)}[e(t) - e(t-\tau(t))]^T R[e(t) - e(t-\tau(t))] \\ & -\frac{h}{h-\tau(t)}[e(t-\tau(t)) - e(t-h)]^T R[e(t-\tau(t)) - e(t-h)] \\ \leq & -\begin{bmatrix} e(t) - e(t-\tau(t)) \\ e(t-\tau(t)) - e(t-h) \end{bmatrix}^T \begin{bmatrix} R & U^T \\ U & R \end{bmatrix} \begin{bmatrix} e(t) - e(t-\tau(t)) \\ e(t-\tau(t)) - e(t-h) \end{bmatrix} \\ = & -\begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ e(t-h) \end{bmatrix}^T \begin{bmatrix} I & 0 \\ -I & I \\ 0 & -I \end{bmatrix} \begin{bmatrix} R & U^T \\ U & R \end{bmatrix} \begin{bmatrix} I & -I & 0 \\ 0 & I & -I \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ e(t-h) \end{bmatrix} \\ = & -\begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ e(t-h) \end{bmatrix}^T \begin{bmatrix} R & U^T - R & -U^T \\ -R + U & 2R - U - U^T & U^T - R \\ -U & U - R & R \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ e(t-h) \end{bmatrix}. \end{aligned} \tag{3.26}$$

It can be derived from (3.25) that

$$-\frac{\pi^2}{4}[e(t)-e(t_kh+lh)]^T\Omega[e(t)-e(t_kh+lh)] = -\frac{\pi^2}{4}\begin{bmatrix} e(t) \\ e(t_kh+lh)\end{bmatrix}^T\begin{bmatrix}\Omega & -\Omega \\ -\Omega & \Omega\end{bmatrix}\begin{bmatrix} e(t) \\ e(t_kh+lh)\end{bmatrix}. \tag{3.27}$$

From (3.12), (3.22)–(3.27), we can obtain

$$\begin{aligned}\dot V(t) \le\ & e^T(t)[P(I_{N-1}\otimes A)+(I_{N-1}\otimes A)^TP]e(t)-2ce^T(t)P(\hat L\otimes I_n)e(t-\tau(t)) \\ &+2ce^T(t)P(\hat L\otimes I_n)\bar e(t_kh+lh)+2e^T(t)P(I_{N-1}\otimes B)\eta(t) \\ &+e^T(t)Qe(t)-e^T(t-h)Qe(t-h)+h^2\dot e^T(t)(R+\Omega)\dot e(t) \\ &-\begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ e(t-h)\end{bmatrix}^T\begin{bmatrix} R & U^T-R & -U^T \\ -R+U & 2R-U-U^T & U^T-R \\ -U & U-R & R\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau(t)) \\ e(t-h)\end{bmatrix} \\ &-\frac{\pi^2}{4}\begin{bmatrix} e(t) \\ e(t_kh+lh)\end{bmatrix}^T\begin{bmatrix}\Omega & -\Omega \\ -\Omega & \Omega\end{bmatrix}\begin{bmatrix} e(t) \\ e(t_kh+lh)\end{bmatrix} \\ &-\bar e^T(t_kh+lh)\Phi^{(1)}\bar e(t_kh+lh)+e^T(t-\tau(t))(\sigma\otimes I_n)\Phi^{(2)}e(t-\tau(t)) \\ &-\eta^T(t)(I_{N-1}\otimes T)\eta(t)+e^T(t)(I_{N-1}\otimes C^T\Delta T)\eta(t) \\ =\ & \xi^T(t)\Sigma\xi(t)+h^2\dot e^T(t)(R+\Omega)\dot e(t) \\ =\ & \xi^T(t)[\Sigma+h^2\Pi^T(R+\Omega)\Pi]\xi(t),\end{aligned} \tag{3.28}$$

where

$$\xi(t) = \begin{bmatrix} e^T(t) & e^T(t-\tau(t)) & e^T(t-h) & \bar e^T(t_kh+lh) & e^T(t_kh+lh) & \eta^T(t)\end{bmatrix}^T,$$

$$\Sigma = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & U^T & cP(\hat L\otimes I_n) & \frac{\pi^2}{4}\Omega & \Gamma_{16} \\ * & \Gamma_{22} & -U^T+R & 0 & 0 & 0 \\ * & * & -Q-R & 0 & 0 & 0 \\ * & * & * & -\Phi^{(1)} & 0 & 0 \\ * & * & * & * & -\frac{\pi^2}{4}\Omega & 0 \\ * & * & * & * & * & -I_{N-1}\otimes T\end{bmatrix},$$

$$\Pi = \begin{bmatrix} I_N\otimes A & c(\hat L\otimes I_n) & 0 & c(\hat L\otimes I_n) & 0 & I_N\otimes B\end{bmatrix},$$

using Schur complement in Lemma 5 with (3.15), we can obtain $\Sigma+h^2\Pi^T(R+\Omega\Pi)<0$, then

$$\dot V(t) < 0, \tag{3.29}$$

thus system (3.12) is globally asymptotically stable at the origin, then we have $\lim_{t\to\infty}\|w_ix_i(t)-w_1x_1(t)\| = 0$ $(i=2,3,\ldots,N)$, which means $\lim_{t\to\infty}\|w_ix_i(t)-w_jx_j(t)\|=0$ $(i=2,3,\ldots,N; i\neq j)$, by using Definition 1, it can be derived that bipartite leaderless consensus is achieved. This completes the proof. □

Remark 6. *As a by-product, given the triggered parameters $\sigma_i>0$ $(i=2,3,\ldots,N)$ and the coupling strength $c>0$, the following constrained optimization problem can be employed to find the maximum allowable sampling period h, that is*

$$Max\ \ h$$

$$Subject\ to : (3.15)\ and\ (3.16).$$

Remark 7. *In Theorem 1, the triggered parameters $\sigma_i > 0$ $(i = 2, 3, \ldots, N)$ depend on the variation of the system state, if the parameters σ_i keep a big value, then the network resources can be further saved, for given the coupling strength c and the sampling period h, when $\Phi^{(1)} = \Phi^{(2)}$, the maximum values of trigger parameters $\sigma_i > 0$ $(i = 2, 3, \ldots, N)$ can be obtained by the following constrained optimization problem*

$$\begin{aligned} & Max\ \ \sigma \\ & Subject\ to : (3.15)\ and\ (3.16). \end{aligned}$$

Notice that if $\sigma_i = 0$ $(i = 2, 3, \ldots, N)$, the event-triggered scheme will reduce to the time-triggered scheme. Let $\sigma_i \to 0^+$ in Theorem 1, we will obtain the bipartite leaderless consensus under the sampled-data control.

Corollary 1. *Suppose that Assumption 1 holds, for given sampled period $h > 0$ and coupled strength $c > 0$, the bipartite leaderless consensus under the sampled-data control can be reached in system (2.1) under the control law (3.2), if there exist some $(N-1)n \times (N-1)n$ positive definite matrices $P > 0$, $Q > 0$, $R > 0$, $\Omega > 0$ and $T = diag\{\tau_1, \tau_2, \ldots, \tau_m\} > 0$, and matrix $U \in \mathbb{R}^{(N-1)n \times (N-1)n}$, such that the following linear matrix inequalities hold*

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} & U^T & \frac{\pi^2}{4}\Omega & \Gamma_{16} & h(I_{N-1} \otimes A)^T R & h(I_{N-1} \otimes A)^T \Omega \\ * & \bar{\Gamma}_{22} & -U^T + R & 0 & 0 & -hc(\hat{L} \otimes I_n)^T R & -hc(\hat{L} \otimes I_n)^T \Omega \\ * & * & -Q - R & 0 & 0 & 0 & 0 \\ * & * & * & -\frac{\pi^2}{4}\Omega & 0 & 0 & 0 \\ * & * & * & * & -I_{N-1} \otimes T & h(I_{N-1} \otimes B)^T R & h(I_{N-1} \otimes B)^T \Omega \\ * & * & * & * & * & -R & 0 \\ * & * & * & * & * & * & -\Omega \end{bmatrix} < 0, \quad (3.30)$$

$$\begin{bmatrix} R & U^T \\ * & R \end{bmatrix} > 0, \quad (3.31)$$

where

$$\begin{aligned} \Gamma_{11} &= P(I_{N-1} \otimes A) + (I_{N-1} \otimes A)^T P + Q - R - \frac{\pi^2}{4}\Omega, \\ \Gamma_{12} &= -cP(\hat{L} \otimes I_n) - U^T + R, \\ \Gamma_{16} &= P(I_{N-1} \otimes B) + I_{N-1} \otimes C^T \Delta T, \\ \bar{\Gamma}_{22} &= -2R + U + U^T, \\ \Delta &= diag\{\delta_1, \delta_2, \ldots, \delta_m\}. \end{aligned}$$

Remark 8. *When $\sigma_i \to 0^+$, it can be concluded that (3.14) can not be satisfied, the data of each sampling time will be transmitted to the neighbor agents, that is, the event-triggered scheme reduces to the time-triggered scheme, thus bipartite leaderless consensus can be derived based on the sampled control law in [25].*

3.2. Bipartite leader-following consensus

In this section, the bipartite leader-following consensus for Lur'e network is considered under the event-triggered communication scheme.

Suppose that there is only one virtual leader, and the dynamics can be described by

$$\dot{x}_0(t) = Ax_0(t) + Bf(Cx_0(t)), \tag{3.32}$$

where $x_0(t) \in R^n$ is the position state of the virtual leader, the other notations and the dynamics of the followers are the same as those in (2.1).

Definition 3. *[25] Under Assumption 1, the signed Lur'e network (2.1) is said to achieve bipartite leader-following consensus if* $\lim_{t\to\infty} \|x_i(t) - w_i x_0(t)\| = 0$ *holds for all* $i = 1, 2, \ldots, N$, *where* $w_i = 1$ $(i \in V_1)$ *and* $w_i = -1$ $(i \in V_2)$.

Remark 9. *In Definition 3, if* $i \in V_1$, *we have* $w_i = 1$, *then* $\lim_{t\to\infty} \|x_i(t) - w_i x_0(t)\| = 0$ *reduces to* $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0$. *If* $i \in v_2$, *we have* $w_i = -1$, *then* $\lim_{t\to\infty} \|x_i(t) - w_i x_0(t)\| = 0$ *reduces to* $\lim_{t\to\infty} \|x_i(t) + x_0(t)\| = 0$, *which means* $x_i(t) \to -x_0(t)$ $(t \to \infty)$, *in other words, one part are synchronized to the state of the leader, the other part is synchronized to the opposite state of the leader, the structural balanced is critical to achieve the bipartite leader-following consensus.*

The following sampled control law is proposed to solve the bipartite leader-following consensus problem for Lur'e network (2.1) and (3.32) as follows

$$\begin{aligned} u_i(t) &= -c \sum_{j \in N(i)} |a_{ij}|[x_i(t_k h) - sgn(a_{ij}) x_j(t_k h)] - c d_i [x_i(t_k h) - w_i x_0(t_k h)] \\ &\quad (i = 1, 2, \ldots, N; t \in [t_k h, t_{k+1} h)), \end{aligned} \tag{3.33}$$

where $c > 0$ is the coupling strength, $t_k h$ $(k = 0, 1, 2, \ldots)$ is the transmitted instants, $t_0 h = 0$, $\{t_0, t_1, \ldots\} \subset \{0, 1, 2, \ldots\}$. d_i $(i = 1, 2, \ldots, N)$ is the pining feedback gain, $d_i > 0$ if agent i is pinned, otherwise $d_i = 0$.

Applying (3.33) to system (2.1), one has

$$\dot{x}_i(t) = Ax_i(t) + Bf(Cx_i(t)) - c \sum_{j=1}^{N} l_{ij} x_j(t_k h) - c d_i [x_i(t_k h) - w_i x_0(t_k h)]. \tag{3.34}$$

Let $\bar{x}_i(t) = w_i x_i(t)$ and $e_i(t) = \bar{x}_i(t) - x_0(t)$ $(i = 1, 2, \ldots, N)$, it follows from (3.34) that

$$\dot{e}_i(t) = Ae_i(t) + Bf(C\bar{x}_i(t)) - Bf(Cx_0(t)) - c \sum_{j=1}^{N} \bar{l}_{ij} e_j(t_k h) - c d_i e_i(t_k h). \tag{3.35}$$

The event-triggered communication scheme is proposed as follows

$$\begin{aligned} t_{k+1} h &= t_k h + \min\{lh | [e_i(t_k h + lh) - e_i(t_k h)]^T \Phi_i^{(1)} [e_i(t_k h + lh) - e_i(t_k h)] \\ &> \sigma_i e_i^T (t_k h + lh) \Phi_i^{(2)} e_i(t_k h + lh)\}, \end{aligned} \tag{3.36}$$

where $\sigma_i > 0$ are event trigger parameters, $\Phi_i^{(1)}$ and $\Phi_i^{(2)}$ $(i = 1, 2, \ldots, N)$ are positive definite matrices to be designed.

The interval $[t_k h, t_{k+1} h)$ can be expressed as follows

$$[t_k h, t_{k+1} h) \;=\; \cup_{l=0}^{t_{k+1}-1} [t_k h + lh, t_k h + lh + h). \tag{3.37}$$

Define $\tau(t) = t - (t_k h + lh)$, for $t \in [t_k h + lh, t_k h + lh + h)$, one has $0 \leq \tau(t) < h$, let $\bar{e}_i(t_k h + lh) = e_i(t_k h + lh) - e_i(t_k h)$ $(i = 2, 3, \ldots, N)$, it can be derived that

$$e_i(t_k h) \;=\; e_i(t - \tau(t)) - \bar{e}_i(t_k h + lh), \tag{3.38}$$

then for $t \in [t_k h, t_{k+1} h)$, the event-triggered scheme can be rewritten as follows

$$\bar{e}^T(t_k h + lh)\Phi^{(1)}\bar{e}(t_k h + lh) \;\leq\; e^T(t - \tau(t))(\sigma \otimes I_n)\Phi^{(2)} e(t - \tau(t)), \tag{3.39}$$

where

$$\begin{aligned} \Phi^{(1)} &= diag\{\Phi_1^{(1)}, \Phi_2^{(1)}, \ldots, \Phi_N^{(1)}\}, \\ \Phi^{(2)} &= diag\{\Phi_1^{(2)}, \Phi_2^{(2)}, \ldots, \Phi_N^{(2)}\}, \\ \sigma &= diag\{\sigma_1, \sigma_2, \ldots, \sigma_N\}. \end{aligned}$$

From the above analysis, it can be known from (3.39) that the current sampled data $e_i(t_k h + lh)$ will not be sent, which will be employed in the following consensus analysis.

For convenience, some notations are given as

$$\begin{aligned} e(t) &= [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T, \\ \eta(t) &= [\eta_1^T(t), \eta_2^T(t), \ldots, \eta_N^T(t)]^T, \\ \eta_i(t) &= [\eta_{i1}(t), \eta_{i2}(t), \ldots, \eta_{im}(t)]^T, \\ \eta_{ik}(t) &= f_k(C_k^T \bar{x}_i(t)) - f_k(C_k^T x_0(t)) \quad (k = 1, 2, \ldots, m). \end{aligned}$$

Rewrite (3.35) in compact matrix form as

$$\dot{e}(t) \;=\; (I_N \otimes A)e(t) - c(\hat{L} \otimes I_n)e(t - \tau(t)) + c(\hat{L} \otimes I_n)\bar{e}(t_k h + lh) + (I_{N-1} \otimes B)\eta(t), \tag{3.40}$$

where $\hat{L} = (\hat{l}_{ij})_{N\times N}$, $\hat{l}_{ij} = \bar{l}_{ij} (i \neq j)$, $\hat{l}_{ii} = \bar{l}_{ii} + d_i$.

Theorem 2. *Suppose that Assumption 1 holds, for given sampled period $h > 0$, triggered parameters $\sigma_i > 0$ $(i = 1, 2, \ldots, N)$, pinning feedback gain d_i and coupled strength $c > 0$, the bipartite leader-following consensus can be reached in system (2.1) under the control law (39) and event-triggered scheme (3.39), if there exist some $Nn \times Nn$ definite matrices $P > 0$, $Q > 0$, $R > 0$, $\Omega > 0$ and $T = diag\{\tau_1, \tau_2, \ldots, \tau_m\} > 0$, and matrix $U \in \mathbb{R}^{Nn\times Nn}$, such that the following matrix inequalities hold*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & U^T & cP(\hat{L}\otimes I_n) & \frac{\pi^2}{4}\Omega & \Pi_{16} & h(I_N\otimes A)^T R & h(I_N\otimes A)^T\Omega \\ * & \Pi_{22} & -U^T + R & 0 & 0 & 0 & -hc(\hat{L}\otimes I_n)^T R & -hc(\hat{L}\otimes I_n)^T\Omega \\ * & * & -Q - R & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\Phi^{(1)} & 0 & 0 & hc(\hat{L}\otimes I_n)^T R & hc(\hat{L}\otimes I_n)^T\Omega \\ * & * & * & * & -\frac{\pi^2}{4}\Omega & 0 & 0 & 0 \\ * & * & * & * & * & -I_{N-1}\otimes T & h(I_N\otimes B)^T R & h(I_N\otimes B)^T\Omega \\ * & * & * & * & * & * & -R & 0 \\ * & * & * & * & * & * & * & -\Omega \end{bmatrix}$$

$$< 0, \tag{3.41}$$

$$\begin{bmatrix} R & U^T \\ * & R \end{bmatrix} > 0, \tag{3.42}$$

where

$$\begin{aligned}
\Pi_{11} &= P(I_N \otimes A) + (I_N \otimes A)^T P + Q - R - \frac{\pi^2}{4}\Omega, \\
\Pi_{12} &= -cP(\hat{L} \otimes I_n) - U^T + R, \\
\Pi_{16} &= P(I_N \otimes B) + I_N \otimes C^T \Delta T, \\
\Pi_{22} &= -2R + U + U^T + (\sigma \otimes I_n)\Phi^{(2)}, \\
\Delta &= diag\{\delta_1, \delta_2, \ldots, \delta_m\}.
\end{aligned}$$

Proof. Construct the following Lyapunov function candidate

$$V(t) = \sum_{i=1}^{4} V_i(t), \tag{3.43}$$

where

$$\begin{aligned}
V_1(t) &= e^T(t)Pe(t), \\
V_2(t) &= \int_{t-h}^{t} e^T(s)Qe(s)ds, \\
V_3(t) &= h\int_{t-h}^{t}\int_{s}^{t} \dot{e}^T(v)R\dot{e}(v)dvds, \\
V_4(t) &= h^2\int_{t_kh+lh}^{t} \dot{e}^T(s)\Omega\dot{e}(s)ds - \frac{\pi^2}{4}\int_{t_kh+lh}^{t}[e(s) - e(t_kh + lh)]^T\Omega[e(s) - e(t_kh + lh)]ds,
\end{aligned}$$

where $P, Q, R, \Omega \in \mathbb{R}^{Nn\times Nn} > 0$.

The proof method is similar to Theorem 1, the detailed process is omitted to save space. □

Notice that if $\sigma_i = 0$ $(i = 1, 2, \ldots, N)$, the event-triggered scheme will reduce to the time-triggered scheme. In Theorem 2, let $\sigma_i \to 0^+$, we will obtain bipartite leader-following consensus under the sampled-data control.

Corollary 2. *Suppose that Assumption 1 holds, for given sampled period $h > 0$, pinning feedback gain d_i and coupled strength $c > 0$, the bipartite leader-following consensus under the sampled-data control can be reached in system (2.1) under the control law (3.33), if there exist some $Nn\times Nn$ definite matrices $P > 0$, $Q > 0$, $R > 0$, $\Omega > 0$ and $T = diag\{\tau_1, \tau_2, \ldots, \tau_m\} > 0$, and matrix $U \in \mathbb{R}^{Nn\times Nn}$, such that the following matrix inequalities hold*

$$\begin{bmatrix}
\Pi_{11} & \Pi_{12} & U^T & \frac{\pi^2}{4}\Omega & \Pi_{16} & h(I_N \otimes A)^T R & h(I_N \otimes A)^T\Omega \\
* & \bar{\Pi}_{22} & -U^T + R & 0 & 0 & -hc(\hat{L} \otimes I_n)^T R & -hc(\hat{L} \otimes I_n)^T\Omega \\
* & * & -Q - R & 0 & 0 & 0 & 0 \\
* & * & * & -\frac{\pi^2}{4}\Omega & 0 & 0 & 0 \\
* & * & * & * & -I_{N-1} \otimes T & h(I_N \otimes B)^T R & h(I_N \otimes B)^T\Omega \\
* & * & * & * & * & -R & 0 \\
* & * & * & * & * & * & -\Omega
\end{bmatrix} < 0, \tag{3.44}$$

$$\begin{bmatrix} R & U^T \\ * & R \end{bmatrix} > 0, \tag{3.45}$$

where

$$\begin{aligned}
\Pi_{11} &= P(I_N \otimes A) + (I_N \otimes A)^T P + Q - R - \frac{\pi^2}{4}\Omega, \\
\Pi_{12} &= -cP(\hat{L} \otimes I_n) - U^T + R, \\
\Pi_{16} &= P(I_N \otimes B) + I_N \otimes C^T \Delta T, \\
\bar{\Pi}_{22} &= -2R + U + U^T, \\
\Delta &= diag\{\delta_1, \delta_2, \ldots, \delta_m\}.
\end{aligned}$$

Remark 10. *Similar to Remark 6 and Remark 7, the constrained optimization problem can be employed to find the maximum allowable sampling period h and the maximum values of the trigger parameters* σ_i $(i = 1, 2, \ldots, N)$.

Remark 11. *For structurally balanced topology, by gauge transformation, the leader-following bipartite consensus of signed network will be converted to the leader-following consensus of unsigned network, then the selected pinning nodes and controller gains can be designed according to the scheme in [68].*

4. Numerical examples

This section provides two simulation examples to demonstrate the effectiveness of the proposed event-triggered communication scheme. The first example is for the bipartite leaderless consensus issue, and the second example is for the leader-following consensus issue.

Consider a Lur'e network consisting of seven agents

$$\dot{x}_i(t) = Ax_i(t) + Bf(Cx_i(t)) + u_i(t), \qquad (i = 1, 2, \ldots, 7), \tag{4.1}$$

where

$$\begin{aligned}
x_i(t) &= \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \\ x_{i3}(t) \end{bmatrix}, \quad A = \begin{bmatrix} 0.5598 & -1.3018 & 0 \\ 1 & -1 & 1 \\ 0 & 0.0135 & 0.0297 \end{bmatrix}, \quad B = \begin{bmatrix} 0.8841 \\ 0 \\ 0 \end{bmatrix}, \quad C^T = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \\
f(Cx_i(t)) &= \frac{1}{2}(|x_{i1}(t) + 1| - |x_{i1}(t) - 1|),
\end{aligned}$$

it is easy to verify that the nonlinear function $f(\cdot)$ is an odd function with $\delta_k = 1$, which satisfies Assumption 1. The directed communication topology is shown in Figure 1, the solid lines denote the cooperative interactions, the dashed lines denote the antagonistic interactions, obviously, which can be divided into two sub-networks $V_1 = \{1, 2, 3, 4\}$ and $V_2 = \{5, 6, 7\}$, the agents are cooperative in V_1 or V_2, the agents are competitive between V_1 and V_2, by using Lemma 1, we can obtain the gauge transformation $W = diag\{1, 1, 1, 1, -1, -1, -1\}$.

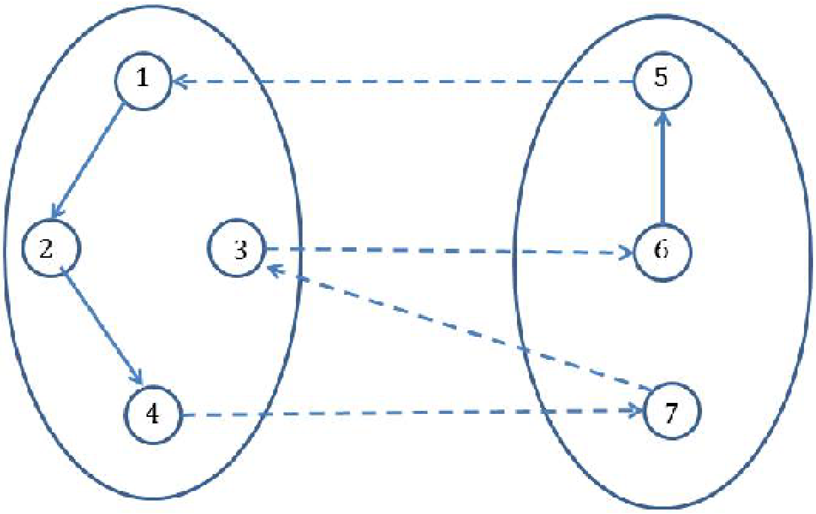

Figure 1. Illustration for the communication topology.

Case 1: Bipartite leaderless consensus

Let the coupling strength $c = 10$, the sampling period $h = 0.01$, the event-triggered parameters are given as $\sigma_i = 0.1$ $(i = 2, 3, \ldots, 7)$. By using Theorem 1, the related matrices can be obtained, which means bipartite leaderless consensus can be achieved, the event-triggered matrices are given as follows

$$\Phi_2^{(1)} = \begin{bmatrix} 72.9635 & -6.8829 & -1.4110 \\ -6.8829 & 60.4371 & 1.3122 \\ -1.4110 & 1.3122 & 61.1014 \end{bmatrix} \quad \Phi_2^{(2)} = \begin{bmatrix} 6.1201 & -4.5902 & 0.8825 \\ -4.5902 & 28.7651 & -2.0352 \\ 0.8825 & -2.0352 & 27.5160 \end{bmatrix},$$

$$\Phi_3^{(1)} = \begin{bmatrix} 76.8473 & -8.0696 & -1.5618 \\ -8.0696 & 58.8743 & 1.3443 \\ -1.5618 & 1.3443 & 59.4599 \end{bmatrix} \quad \Phi_3^{(2)} = \begin{bmatrix} 1.4753 & -2.3764 & 0.7966 \\ -2.3764 & 15.0325 & -2.9694 \\ 0.7966 & -2.9694 & 12.3513 \end{bmatrix},$$

$$\Phi_4^{(1)} = \begin{bmatrix} 72.9517 & -7.1222 & -1.4862 \\ -7.1222 & 57.8293 & 1.1390 \\ -1.4862 & 1.1390 & 58.2693 \end{bmatrix} \quad \Phi_4^{(2)} = \begin{bmatrix} 2.2950 & -3.2744 & 0.9581 \\ -3.2744 & 19.8816 & -3.1954 \\ 0.9581 & -3.1954 & 17.2817 \end{bmatrix},$$

$$\Phi_5^{(1)} = \begin{bmatrix} 89.5063 & -8.0961 & -1.5371 \\ -8.0961 & 66.9095 & 1.3660 \\ -1.5371 & 1.3660 & 68.3950 \end{bmatrix} \quad \Phi_5^{(2)} = \begin{bmatrix} 5.0561 & -3.1629 & 0.8159 \\ -3.1629 & 24.5482 & -2.1727 \\ 0.8159 & -2.1727 & 22.4386 \end{bmatrix},$$

$$\Phi_6^{(1)} = \begin{bmatrix} 82.2557 & -8.1110 & -1.5933 \\ -8.1110 & 60.8831 & 1.4077 \\ -1.5933 & 1.4077 & 61.9600 \end{bmatrix} \quad \Phi_6^{(2)} = \begin{bmatrix} 2.0090 & -2.5110 & 0.7739 \\ -2.5110 & 16.7158 & -2.6963 \\ 0.7739 & -2.6963 & 14.1200 \end{bmatrix},$$

$$\Phi_7^{(1)} = \begin{bmatrix} 73.6601 & -7.5971 & -1.4995 \\ -7.5971 & 57.9647 & 1.2207 \\ -1.4995 & 1.2207 & 58.3560 \end{bmatrix} \quad \Phi_7^{(2)} = \begin{bmatrix} 1.5471 & -2.6018 & 0.8665 \\ -2.6018 & 16.0262 & -3.1921 \\ 0.8665 & -3.1921 & 13.2993 \end{bmatrix},$$

taking $t \in [0, 2)$, the simulation results show that only 54 sampled data is used, which takes 27% of the whole sampled signals. Moreover, it can be computed that the average sampling period is 0.0369, which is 3.69 times of the sampling period $h = 0.01$. The release instants and release intervals are illustrated in Figure 2. The curve of $x_{ij}(t)$ $(i = 1, 2, \ldots, 7; j = 1, 2, 3)$ are presented in Figure 3, it can be seen the bipartite leaderless consensus is achieved. In order to show the benefits of our proposed event-triggered scheme than [39, 40, 45, 46], some comparisons are given in Table 1, it can be seen that our event-triggered scheme can lead to larger sampling period h, that is, our results are less conservative than [39, 40, 45, 46]. In addition, by using Remark 7, the upper bound of σ_i is shown in Table 2 for given coupling strength $c = 10$ and sampled period $h = 0.01$, which plays an important role owing to a big value can further save the network resources.

Table 1. The upper bound of sampling period h for bipartite leaderless consensus.

Trigger scheme	Trigger matrix	Coupling strength	Trigger parameter	Upper bound of h
Our scheme	$\Phi_i^{(1)} \neq \Phi_i^{(2)}$	$c = 10$	$\sigma_i = 0.01$	0.0227
[41,42,43,44]	$\Phi_i^{(1)} = \Phi_i^{(2)}$	$c = 10$	$\sigma_i = 0.01$	0.0089

Table 2. The upper bound of triggered parameter σ_i.

Type	Trigger matrix	Coupling strength	Sampled period	Upper bound of σ_i
Leaderless	$\Phi_i^{(1)} = \Phi_i^{(2)}$	$c = 10$	$h = 0.01$	0.0086
Leader-following	$\Phi_i^{(1)} = \Phi_i^{(2)}$	$c = 10$	$h = 0.01$	0.4681

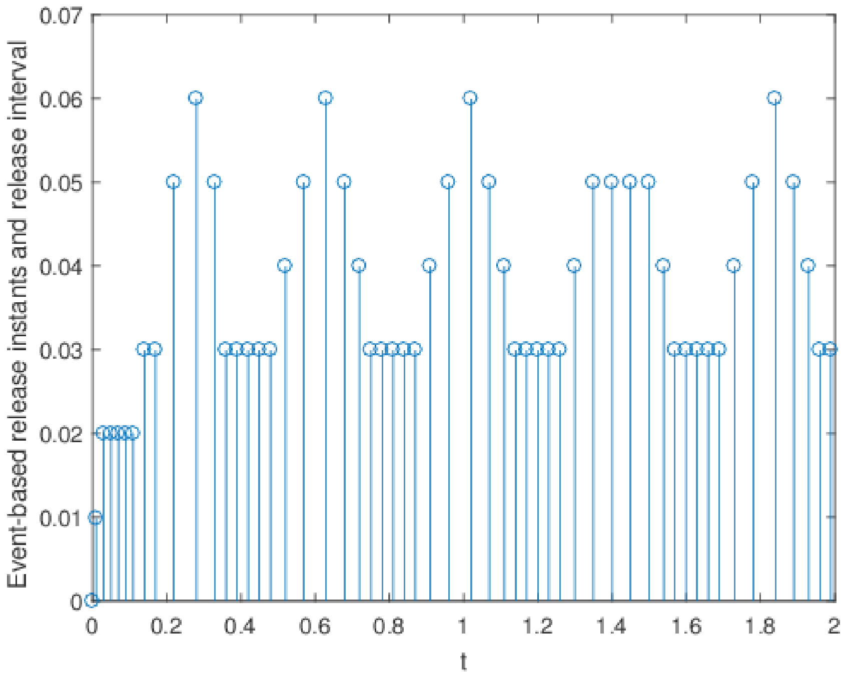

Figure 2. The release instants and release intervals under the event-triggered scheme.

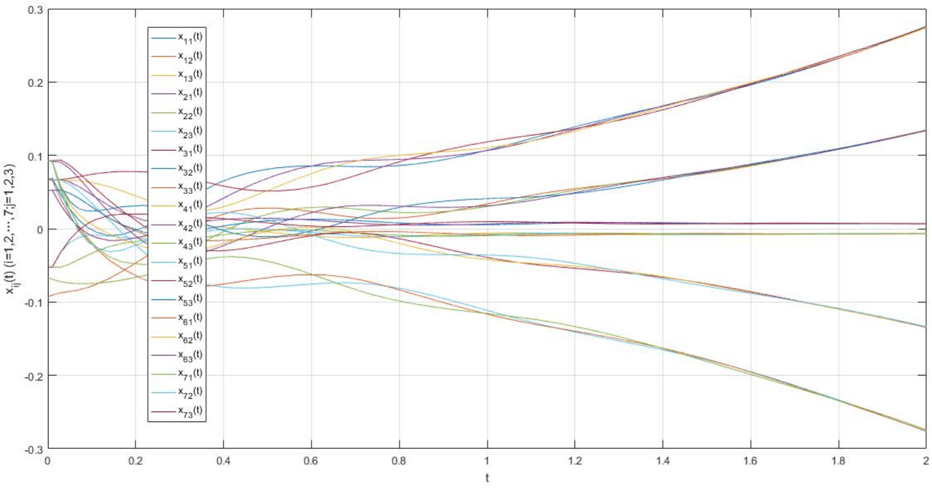

Figure 3. The trajectories of the state $x_{ij}(t)(i = 1, 2, \ldots, 7; j = 1, 2, 3)$ for bipartite leaderless consensus of signed Lur'e network.

If the sampling period takes it's upper bound, that is $h = 0.0227$, taking $t \in [0, 2)$, the simulation results show that only 53 sampled data is used. The release instants and release intervals are illustrated in Figure 4. The curve of $x_{ij}(t)$ $(i = 1, 2, \ldots, 7; j = 1, 2, 3)$ are presented in Figure 5, it can be seen the bipartite leaderless consensus is achieved, the event-triggered matrices are given as follows

$$\Phi_2^{(1)} = \begin{bmatrix} 11.5443 & -4.1383 & -1.8645 \\ -4.1383 & 4.4236 & 0.9458 \\ -1.8645 & 0.9458 & 3.5124 \end{bmatrix} \quad \Phi_2^{(2)} = \begin{bmatrix} 0.0001 & -0.0003 & 0.0036 \\ -0.0003 & 0.0011 & -0.0174 \\ 0.0036 & -0.0174 & 1.0540 \end{bmatrix},$$

$$\Phi_3^{(1)} = \begin{bmatrix} 13.0083 & -4.4585 & -1.9485 \\ -4.4585 & 4.3301 & 0.8827 \\ -1.9485 & 0.8827 & 3.5738 \end{bmatrix} \quad \Phi_3^{(2)} = \begin{bmatrix} 0 & -0.0001 & 0.0018 \\ -0.0001 & 0.0004 & -0.0076 \\ 0.0018 & -0.0076 & 0.3599 \end{bmatrix},$$

$$\Phi_4^{(1)} = \begin{bmatrix} 12.1986 & -4.3612 & -1.9434 \\ -4.3612 & 4.4044 & 0.9609 \\ -1.9434 & 0.9609 & 3.4542 \end{bmatrix} \quad \Phi_4^{(2)} = \begin{bmatrix} 0 & -0.0001 & 0.0022 \\ -0.0001 & 0.0005 & -0.0100 \\ 0.0022 & -0.0100 & 0.5528 \end{bmatrix},$$

$$\Phi_5^{(1)} = \begin{bmatrix} 14.7671 & -4.7152 & -2.0267 \\ -4.7152 & 4.6674 & 0.7678 \\ -2.0267 & 0.7678 & 4.3585 \end{bmatrix} \quad \Phi_5^{(2)} = \begin{bmatrix} 0.0141 & -0.0429 & 0.0269 \\ -0.0429 & 0.1311 & -0.0859 \\ 0.0269 & -0.0859 & 0.7288 \end{bmatrix},$$

$$\Phi_6^{(1)} = \begin{bmatrix} 14.0597 & -4.6911 & -2.0240 \\ -4.6911 & 4.4159 & 0.8550 \\ -2.0240 & 0.8550 & 3.8012 \end{bmatrix} \quad \Phi_6^{(2)} = \begin{bmatrix} 0.0001 & -0.0003 & 0.0027 \\ -0.0003 & 0.0010 & -0.0108 \\ 0.0027 & -0.0108 & 0.4249 \end{bmatrix},$$

$$\Phi_7^{(1)} = \begin{bmatrix} 12.4379 & -4.3538 & -1.9247 \\ -4.3538 & 4.3454 & 0.9195 \\ -1.9247 & 0.9195 & 3.4819 \end{bmatrix} \quad \Phi_7^{(2)} = \begin{bmatrix} 0 & -0.0001 & 0.0017 \\ -0.0001 & 0.0004 & -0.0076 \\ 0.0017 & -0.0076 & 0.3942 \end{bmatrix}.$$

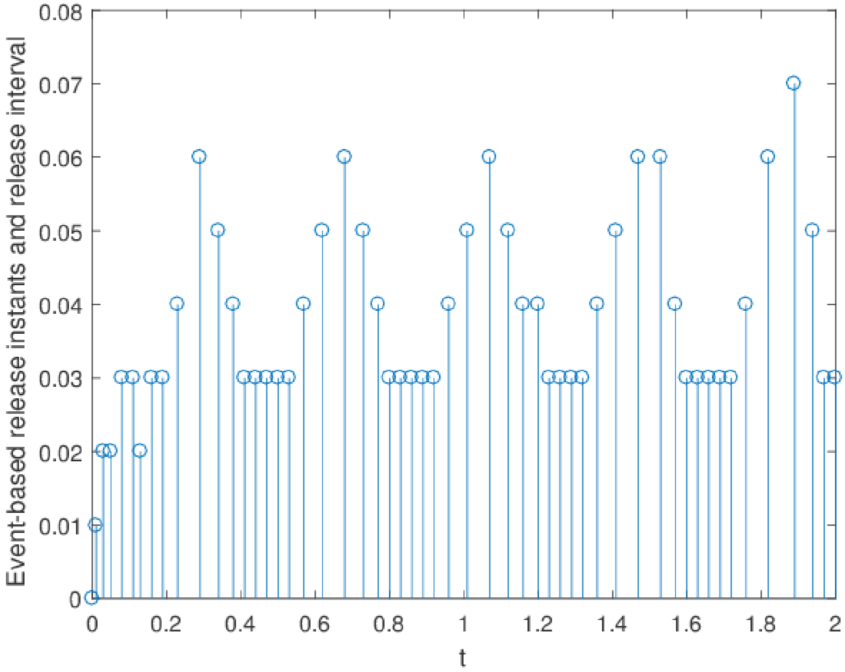

Figure 4. The release instants and release intervals under the event-triggered scheme.

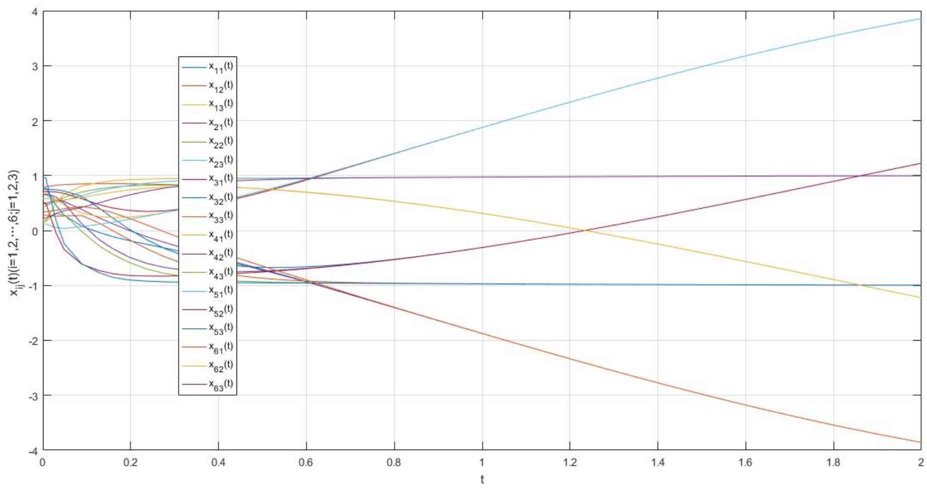

Figure 5. The trajectories of the state $x_{ij}(t)(i = 1, 2, \ldots, 7; j = 1, 2, 3)$ for bipartite leaderless consensus of signed Lur'e network.

Case 2: Bipartite leader-following consensus

Suppose that the dynamics of nonlinear multi-agent systems is the same as **Case 1**, and the communication topology is shown in Figure 6, which is structurally balanced. Let $V_1 = \{1, 2, 3\}$ and $V_2 = \{4, 5, 6\}$, then the agents are cooperative in V_1 or V_2, the agents are competitive between V_1 and V_2, then we can obtain $W = diag\{1, 1, 1, -1, -1, -1\}$. Take $c = 10$, the sampling period $h = 0.01$, the event-triggered parameters $\sigma_i = 0.1$ $(i = 1, 2, \ldots, 6)$, the pinning feedback gain

$D = diag\{2,0,0,0,2,0\}$, By using Theorem 2, the event-triggered matrices can be obtained as follows

$$\Phi_1^{(1)} = \begin{bmatrix} 3.1511 & -0.0393 & 0.0019 \\ -0.0393 & 2.9465 & 0.0269 \\ 0.0019 & 0.0269 & 3.0081 \end{bmatrix} \quad \Phi_1^{(2)} = \begin{bmatrix} 1.7909 & -0.0006 & 0.0008 \\ -0.0006 & 1.8601 & -0.0101 \\ 0.0008 & -0.0101 & 1.8412 \end{bmatrix},$$

$$\Phi_2^{(1)} = \begin{bmatrix} 3.0666 & -0.0575 & 0.0040 \\ -0.0575 & 2.9073 & 0.0366 \\ 0.0040 & 0.0366 & 2.9949 \end{bmatrix} \quad \Phi_2^{(2)} = \begin{bmatrix} 1.6182 & -0.0092 & 0.0034 \\ -0.0092 & 1.7934 & -0.0223 \\ 0.0034 & -0.0223 & 1.7562 \end{bmatrix},$$

$$\Phi_3^{(1)} = \begin{bmatrix} 2.5729 & -0.0388 & 0.0055 \\ -0.0388 & 2.6022 & 0.0011 \\ 0.0055 & 0.0011 & 2.6193 \end{bmatrix} \quad \Phi_3^{(2)} = \begin{bmatrix} 1.4908 & -0.0164 & 0.0051 \\ -0.0164 & 1.7276 & -0.0295 \\ 0.0051 & -0.0295 & 1.6795 \end{bmatrix},$$

$$\Phi_4^{(1)} = \begin{bmatrix} 3.0666 & -0.0575 & 0.0040 \\ -0.0575 & 2.9073 & 0.0366 \\ 0.0040 & 0.0366 & 2.9949 \end{bmatrix} \quad \Phi_4^{(2)} = \begin{bmatrix} 1.6182 & -0.0092 & 0.0034 \\ -0.0092 & 1.7934 & -0.0223 \\ 0.0034 & -0.0223 & 1.7562 \end{bmatrix},$$

$$\Phi_5^{(1)} = \begin{bmatrix} 3.1511 & -0.0393 & 0.0019 \\ -0.0393 & 2.9465 & 0.0269 \\ 0.0019 & 0.0269 & 3.0081 \end{bmatrix} \quad \Phi_5^{(2)} = \begin{bmatrix} 1.7909 & -0.0006 & 0.0008 \\ -0.0006 & 1.8601 & -0.0101 \\ 0.0008 & -0.0101 & 1.8412 \end{bmatrix},$$

$$\Phi_6^{(1)} = \begin{bmatrix} 2.5729 & -0.0388 & 0.0055 \\ -0.0388 & 2.6022 & 0.0011 \\ 0.0055 & 0.0011 & 2.6193 \end{bmatrix} \quad \Phi_6^{(2)} = \begin{bmatrix} 1.4908 & -0.0164 & 0.0051 \\ -0.0164 & 1.7276 & -0.0295 \\ 0.0051 & -0.0295 & 1.6795 \end{bmatrix},$$

taking $t \in [0,1)$, the simulation results show that only 33 sampled data is sent out, which takes 33% of the sampled signals. Moreover, it can be computed that our event-triggered scheme can obtain an average sampling period of 0.03, the release instants and release intervals are illustrated in Figure 7. The curve of $x_{ij}(t)$ $(i = 1,2,\dots,6; j = 1,2,3)$ are presented in Figure 8, which can be seen the bipartite leader-following consensus is achieved. In order to show the benefits of our proposed triggered mechanism than [39, 40, 45, 46], some comparisons are given in Table 3, it can be seen that our event-triggered scheme can lead to larger sampling period h, that is, our results are less conservative than [39, 40, 45, 46]. In addition, by using Theorem 2 and the similar method of Remark 7, the upper bound of σ_i is shown in Table 2 given the coupling strength $c = 10$ and the sampled period $h = 0.01$.

Table 3. The upper bound of sampling period h for bipartite leader-following consensus.

Trigger scheme	Trigger matrix	Coupling strength	Trigger parameter	Upper bound of h
Our scheme	$\Phi_i^{(1)} \neq \Phi_i^{(2)}$	$c = 10$	$\sigma_i = 0.01$	0.0612
[41,42,43,44]	$\Phi_i^{(1)} = \Phi_i^{(2)}$	$c = 10$	$\sigma_i = 0.01$	0.0552

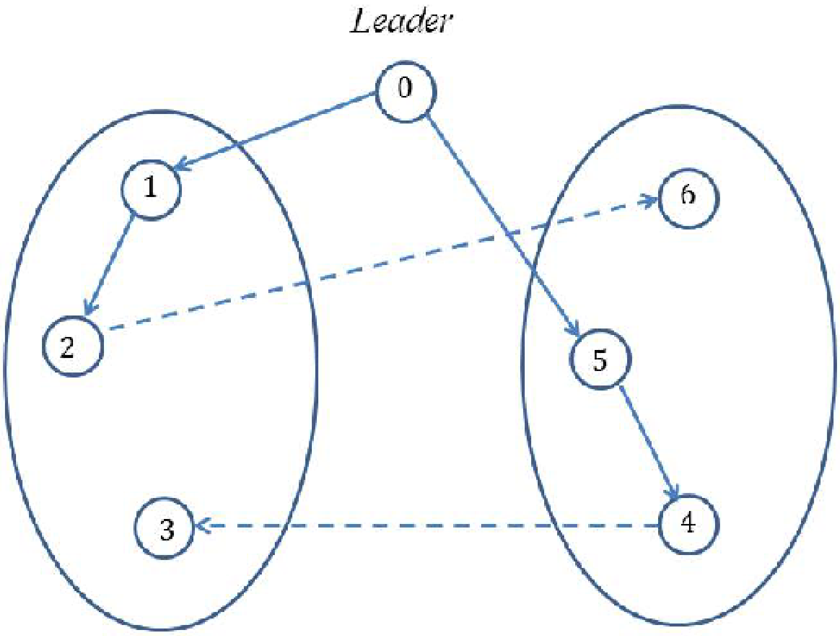

Figure 6. Illustration for the communication topology.

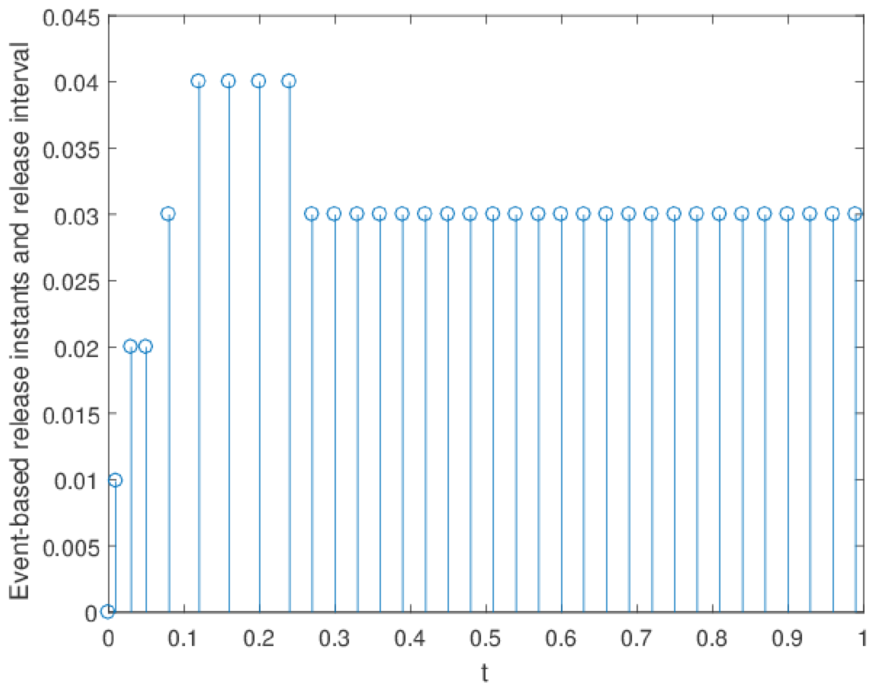

Figure 7. The release instants and release intervals under the event-triggered scheme.

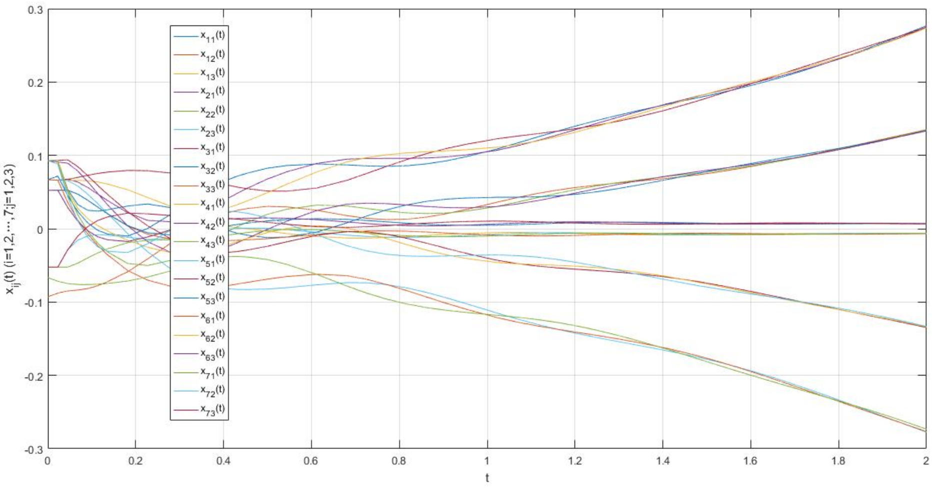

Figure 8. The trajectories of the state $x_{ij}(t)(i = 1, 2, \ldots, 6; j = 1, 2, 3)$ for bipartite leader-following consensus of signed Lur'e network.

5. Conclusions

The event-triggered communication scheme has been proposed to study the bipartite consensus of Lur'e system in the signed networks. By utilizing matrix transformation techniques and algebraic graph theories, some sufficient conditions in terms of LMIs have been established to ensure that both bipartite leaderless and leader-following consensus can be achieved. Compared with some existing event-triggered results, the simulation examples have shown that the proposed event-triggered communication scheme has the advantage to achieve a better performance . It should be emphasized that the sub-network topology is not required to be connected. Notice that the above results are based on the event-triggered parameters that are given as positive constants, it would be interesting to further investigate adaptive event-triggered communication scheme for bipartite consensus, where the triggered parameters can be adjusted with respect to the dynamic errors. The related results can also be extended to the cases with switching topology and time-delay and so on.

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Conflict of interest

The author declare no conflicts of interest in this paper.

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