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Research article

Global dynamics analysis of a Zika transmission model with environment transmission route and spatial heterogeneity

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Abstract: Zika virus, a recurring mosquito-borne flavivirus, became a global public health agency in 2016. It is mainly transmitted through mosquito bites. Recently, experimental result demonstrated that *Aedes* mosquitoes can acquire and transmit Zika virus by breeding in contaminated aquatic environments. The environmental transmission route is unprecedented discovery for the Zika virus. Therefore, it is necessary to introduce environment transmission route into Zika model. Furthermore, we consider diffusive terms in order to capture the movement of humans and mosquitoes. In this paper, we propose a novel reaction-diffusion Zika model with environment transmission route in a spatial heterogeneous environment, which is different from all Zika models mentioned earlier. We introduce the basic offspring number R_0^m and basic reproduction number R_0 for this spatial model. By using comparison arguments and the theory of uniform persistence, we prove that disease free equilibrium with the absence of mosquitoes is globally attractive when $R_0^m < 1$, disease free equilibrium with the presence of mosquitoes is globally attractive when $R_0^m > 1$ and $R_0 < 1$, the model is uniformly persistent when $R_0^m > 1$ and $R_0 > 1$. Finally, numerical simulations conform these analytical results.

Keywords: Zika model; environment transmission route; spatial heterogeneity; reproduction number; global dynamics
Mathematics Subject Classification: 35Q80, 35Q99

1. Introduction

Zika virus, a mosquito-borne flavivirus, was first isolated in monkeys a rhesus in Uganda in 1947. Later, it was detected in humans in Uganda and the United Republic of Tanzania in 1952 [1]. From the

1960s to 1980s, rare sporadic cases of human infections were found throughout Africa and Asia. The first recorded outbreak was reported from the Island of Yap in 2007 [2]. In March 2015, Brazil reported a large outbreak of rash illness, soon identified as Zika virus infection, and later found to be associated with Guillain-Barr*é* syndrome and microcephaly [3]. On February 1, 2016, WHO declared Zika as a "Public Health Emergency of International Concern" [4]. Since the outbreak of Zika in Brazil, the expansion of the Zika outbreak has seemed unstoppable. It spread rapidly from Brazil to northern Europe [5], Australia [6], through Canada [7], the USA [8], subsequently, arrived to reach Japan [9], China [10]. Zika cases have been reported in 90 countries and territories by November 4, 2019 [11]. Recently, November 6, 2021, 13 new cases were reported in Uttar Pradesh's Kanpur district, which took the case tally to 79 in the state during the past two weeks. Authorities in the Indian capital region said they were on alert in the wake of a spike in Zika virus cases in the neighbouring state of Uttar Pradesh. It is natural to ask how the previously unknown Zika virus spreads rapidly in the short term.

Since the outbreak of Zika in Brazil, many models have been proposed to study spread, impact, and control of Zika disease and dynamic behaviors. Zhang et al. [12] employed a SEIR (Susceptible-Exposed-Infected-Removed)-SEI (Susceptible-Exposed-Infected) human-vector model to estimate the time of first introduction of Zika to Brazil. Zhao et al. [13] considered the limited medical resources in Zika model and obtained rich bifurcation phenomena, such as, backward bifurcation, Hopf bifurcation, Bogdanov-Takens bifurcation of codimension 2 and discontinuous bifurcation. Various ordinary differential equations (ODEs) models and dynamics analyses had been applied to study Zika outbreak [14–20]. However, the above models ignore the effects of the spatial factors. In fact, the spread of the disease concerns not only the time, but also the spatial location. For this purpose, some researchers began to describe spatio-temporal transmission of Zika disease through partial differential equations (PDEs) [21, 22]. Miyaoka et al. [21] considered spatial movement of humans and vectors and formulated a reaction diffusion model to research the effect of vaccination on the transmission and control of Zika disease. Yamazaki [22] added diffusive terms in Zika model in order to capture the movement of human hosts and mosquitoes, considering the unique threat of the sexual transmission of Zika disease. In the above PDEs models, all the coefficients are positive constants. That is, the dynamics of humans and vectors are described in spatially homogeneous environments. However, the diffusion dynamics of the disease is affected by the natural landscapes, the urban and rural distribution, even cultural geographical factors [23]. To make the model more consistent with the spread laws of the disease, the spatial heterogeneity must be considered. Hence, it is necessary to understand the transmission dynamics of the Zika disease influenced by the spatial heterogeneity [24]. However, in the above studies, the contaminated aquatic environments, an important transmission route for Zika virus, seem to have received little attention.

Recently, experimental result [25] demonstrated that *Aedes* mosquitoes can acquire and transmit Zika virus by breeding in contaminated aquatic environments. It implies that *Aedes* mosquitoes are infected by Zika virus not only through biting infectious hosts but also through urine excreted by Zika patients. This new transmission route makes the transmission cycle of Zika virus much shorter. It may be one of the major causes of rapid spread of Zika virus in nature. Therefore, it is more reasonable to introduce environment transmission route (That is, human-environment-mosquito-human transmission route) into Zika model [26]. However, few Zika models incorporate environment transmission route and spatial heterogeneity simultaneously.

The paper is organized as follows. In Section 2, we propose a novel reaction-diffusion Zika model with environment transmission route in a spatial heterogeneous environment, which is different from all Zika models mentioned earlier. In Section 3, the well-posedness and some properties of the model are also discussed. The basic offspring number R_0^m and basic reproduction number R_0 for our spatial model are established in Section 4. In Section 5, by using comparison arguments and the theory of uniform persistence, the threshold dynamics for the model in terms of R_0^m and R_0 are analysed. A brief conclusion is given in Section 7.

2. Model formulation

In this section, we propose a reaction-diffusion Zika model with environment transmission route. Considering the effects of individual mobility, we assume that a host population lives in a spatial heterogeneous environment, which is represented with a bounded domain Ω with smooth boundary $\partial\Omega$. Mosquitoes are classified in aquatic and adult mosquitoes. Here we combine the egg, larval and pupal stages as one aquatic stage. Aquatic mosquitoes are divided into susceptible and infectious compartments, and their spatial densities at location x and time t are represented by $S_a(x,t)$ and $I_a(x,t)$, respectively. Adult mosquitoes are divided into susceptible and infectious compartments with spatial densities $S_m(x,t)$ and $I_m(x,t)$, respectively. We divide the density of total human population at location x and time t, denoted by $N_h(x,t)$, into three categories: susceptible humans $S_h(x,t)$, infectious humans $I_h(x,t)$ and recovered humans $R_h(x,t)$. So $N_h(x,t) = S_h(x,t) + I_h(x,t) + R_h(x,t)$. $V(x,t)$ represents the density at location x and time t of Zika virus within the contaminated aquatic environments. Implication of $V(x,t)$ is similar to avian influenza virus concentration in water [27,28].

In this paper, we extend our previous model [26] to consider mosquitoes and humans in spatially heterogeneous environments. So, the transmission path of Zika virus is similar to literature [26]. In order to incorporate the multiple factors of diffusion and spatial heterogeneity in the spatial domain Ω, we assume that the parameters $K(x)$, $\vartheta(x)$, $\omega(x)$, $\mu_a(x)$, $\mu_m(x)$, $\Lambda(x)$, $\mu_h(x)$, $r(x)$, $a(x)$, $\beta_1(x)$, $\beta_2(x)$, $\beta_v(x)$, $\theta(x)$, $\delta(x)$ are functions of the spatial location x where the contact occurs, and these space dependent parameters are continuous and strictly positive. Mathematically, we assume that all aquatic mosquitoes do not diffuse, and all adult mosquitoes have the same diffusion rate, denoted by $d_m > 0$, while all humans have the same diffusion rate, denoted by $d_h > 0$. The biological meanings of all parameters are shown in Table 1.

Table 1. Parameters description.

Parameter	Description
$K(x)$	The environment carrying capacity of aquatic mosquitoes at location x
l	The fraction of hatched female mosquitoes from all eggs
s	Probability of mosquito egg-to-adult survival
$\vartheta(x)$	Intrinsic oviposition rate at location x
$\omega(x)$	Maturation rate from aquatic stages to adult mosquitoes at location x
$\mu_a(x)$	Death rate of aquatic mosquitoes at location x
$\mu_m(x)$	Death rate of adult mosquitoes at location x
$\Lambda(x)$	Recruitment rate of humans at location x
$\mu_h(x)$	Natural death rate of humans at location x
$r(x)$	Recovery rate of humans at location x
$a(x)$	Biting rate of mosquitoes at location x
$\beta_1(x)$	Transmission probability from infectious humans to susceptible mosquitoes at location x
$\beta_2(x)$	Transmission probability from infectious mosquitoes to susceptible humans at location x
$\beta_v(x)$	Transmission rate from contaminated aquatic environments to aquatic mosquitoes at location x
$\theta(x)$	Excretion rate for each infected individual at location x
$\delta(x)$	Clearance rate of Zika virus in contaminated aquatic environments at location x
d_m	Adult mosquito diffusion rate
d_h	Human diffusion rate

On the basis of above assumptions, following the flow diagram in Figure 1, we will focus on the spatiotemporal reaction-diffusion Zika model with environment transmission route as follows:

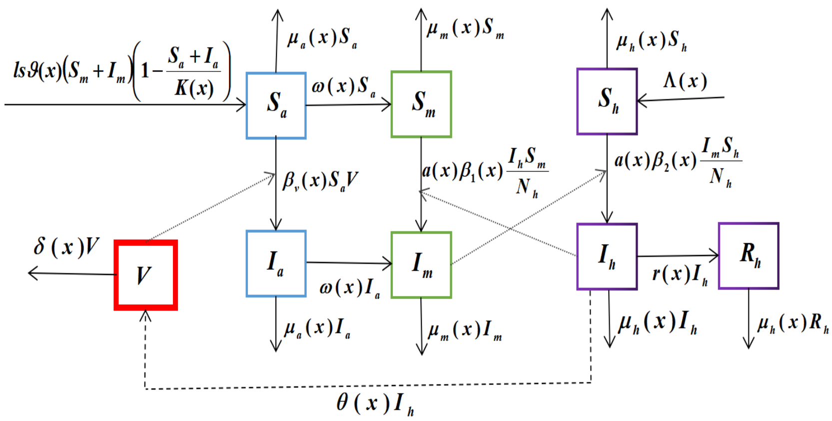

Figure 1. Flow diagram of Zika transmission. The parameters are given in Table 1.

$$
\begin{cases}
\dfrac{\partial S_a}{\partial t} = ls\vartheta(x)(S_m + I_m)\left(1 - \dfrac{S_a + I_a}{K(x)}\right) - \beta_v(x)S_a V - \omega(x)S_a - \mu_a(x)S_a, & x \in \Omega,\ t > 0,\\
\dfrac{\partial I_a}{\partial t} = \beta_v(x)S_a V - \omega(x)I_a - \mu_a(x)I_a, & x \in \Omega,\ t > 0,\\
\dfrac{\partial S_m}{\partial t} = d_m \triangle S_m + \omega(x)S_a - a(x)\beta_1(x)\dfrac{I_h S_m}{N_h} - \mu_m(x)S_m, & x \in \Omega,\ t > 0,\\
\dfrac{\partial I_m}{\partial t} = d_m \triangle I_m + \omega(x)I_a + a(x)\beta_1(x)\dfrac{I_h S_m}{N_h} - \mu_m(x)I_m, & x \in \Omega,\ t > 0,\\
\dfrac{\partial S_h}{\partial t} = d_h \triangle S_h + \Lambda(x) - a(x)\beta_2(x)\dfrac{I_m S_h}{N_h} - \mu_h(x)S_h, & x \in \Omega,\ t > 0,\\
\dfrac{\partial I_h}{\partial t} = d_h \triangle I_h + a(x)\beta_2(x)\dfrac{I_m S_h}{N_h} - r(x)I_h - \mu_h(x)I_h, & x \in \Omega,\ t > 0,\\
\dfrac{\partial R_h}{\partial t} = d_h \triangle R_h + r(x)I_h - \mu_h(x)R_h, & x \in \Omega,\ t > 0,\\
\dfrac{\partial V}{\partial t} = \theta(x)I_h - \delta(x)V, & x \in \Omega,\ t > 0,\\
\dfrac{\partial S_m}{\partial n} = \dfrac{\partial I_m}{\partial n} = \dfrac{\partial S_h}{\partial n} = \dfrac{\partial I_h}{\partial n} = \dfrac{\partial R_h}{\partial n} = 0, & x \in \partial\Omega,\ t > 0,\\
S_a(x,0) = S_{a0}(x) \geq 0,\ I_a(x,0) = I_{a0}(x) \geq 0,\ S_m(x,0) = S_{m0}(x) \geq 0, & x \in \Omega,\\
I_m(x,0) = I_{m0}(x) \geq 0,\ S_h(x,0) = S_{h0}(x) \geq 0,\ I_h(x,0) = I_{h0}(x) \geq 0, & x \in \Omega,\\
R_h(x,0) = R_{h0}(x) \geq 0,\ V(x,0) = V_0(x) \geq 0, & x \in \Omega,
\end{cases}
\tag{2.1}
$$

where $\triangle$ represents the Laplacian operator. The density of total human population $N_h(x,t)$ can be determined by the following equation

$$
\begin{cases}
\dfrac{\partial N_h}{\partial t} = d_h \triangle N_h + \Lambda(x) - \mu_h(x)N_h,\ x \in \Omega,\ t > 0,\\
\dfrac{\partial N_h}{\partial n} = 0,\ x \in \partial\Omega,\ t > 0.
\end{cases}
\tag{2.2}
$$

From Lemma 1 in [29], system (2.2) admits a globally attractive positive steady state $H(x)$ in $C(\bar{\Omega},\ \mathbb{R}_+)$. For simplicity, we assume that the density of total human population at location x and time t stabilizes at $H(x)$. That is, $N_h(x,t) \equiv H(x)$, $\forall\, t \geq 0, x \in \Omega$. Therefore, it suffices to consider the following reduced system:

$$
\begin{cases}
\dfrac{\partial S_a}{\partial t} = \rho(x)(S_m + I_m)\left(1 - \dfrac{S_a + I_a}{K(x)}\right) - \beta_v(x)S_aV - \omega(x)S_a - \mu_a(x)S_a, & x \in \Omega,\ t > 0,\\
\dfrac{\partial I_a}{\partial t} = \beta_v(x)S_aV - \omega(x)I_a - \mu_a(x)I_a, & x \in \Omega,\ t > 0,\\
\dfrac{\partial S_m}{\partial t} = d_m\triangle S_m + \omega(x)S_a - \dfrac{a(x)\beta_1(x)}{H(x)}I_hS_m - \mu_m(x)S_m, & x \in \Omega,\ t > 0,\\
\dfrac{\partial I_m}{\partial t} = d_m\triangle I_m + \omega(x)I_a + \dfrac{a(x)\beta_1(x)}{H(x)}I_hS_m - \mu_m(x)I_m, & x \in \Omega,\ t > 0,\\
\dfrac{\partial S_h}{\partial t} = d_h\triangle S_h + \Lambda(x) - \dfrac{a(x)\beta_2(x)}{H(x)}I_mS_h - \mu_h(x)S_h, & x \in \Omega,\ t > 0,\\
\dfrac{\partial I_h}{\partial t} = d_h\triangle I_h + \dfrac{a(x)\beta_2(x)}{H(x)}I_mS_h - r(x)I_h - \mu_h(x)I_h, & x \in \Omega,\ t > 0,\\
\dfrac{\partial V}{\partial t} = \theta(x)I_h - \delta(x)V, & x \in \Omega,\ t > 0,\\
\dfrac{\partial S_m}{\partial n} = \dfrac{\partial I_m}{\partial n} = \dfrac{\partial S_h}{\partial n} = \dfrac{\partial I_h}{\partial n} = 0, & x \in \partial\Omega,\ t > 0,\\
S_a(x,0) = S_{a0}(x) \ge 0,\ I_a(x,0) = I_{a0}(x) \ge 0, & x \in \Omega,\\
S_m(x,0) = S_{m0}(x) \ge 0,\ I_m(x,0) = I_{m0}(x) \ge 0, & x \in \Omega,\\
S_h(x,0) = S_{h0}(x) \ge 0,\ I_h(x,0) = I_{h0}(x) \ge 0,\ V(x,0) = V_0(x) \ge 0, & x \in \Omega,
\end{cases} \tag{2.3}
$$

where $\rho(x) = ls\vartheta(x)$.

3. Well-posedness

In this section, we will study the well-posedness of system (2.3). Let $\mathbb{X} := C(\bar{\Omega},\ \mathbb{R}^7)$ be the Banach space with the supremum norm $\|\cdot\|$. Define $\mathbb{X}_+ := C(\bar{\Omega},\ \mathbb{R}^7_+)$, then $(\mathbb{X},\ \mathbb{X}_+)$ is a strongly ordered Banach space. Let $\mathbb{X}_K$ be the subset in $\mathbb{X}$ defined by

$$
\mathbb{X}_K := \left\{\phi = (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7)^T \in \mathbb{X}^+ : 0 \le \phi_1(x) + \phi_2(x) \le K(x),\ \forall\ x \in \bar{\Omega}\right\}.
$$

In order to simplify notations, we set $u = (u_1, u_2, u_3, u_4, u_5, u_6, u_7)^T = (S_a, I_a, S_m, I_m, S_h, I_h, V)^T$, and the initial data satisfies $u^0 = (u^0_1, u^0_2, u^0_3, u^0_4, u^0_5, u^0_6, u^0_7)^T = (S_{a0}, I_{a0}, S_{m0}, I_{m0}, S_{h0}, I_{h0}, V_0)^T$. Throughout, for any $w \in C(\bar{\Omega},\ \mathbb{R})$, we denote $\overline{w} := \max\limits_{x\in\bar{\Omega}} w(x)$, $\underline{w} := \min\limits_{x\in\bar{\Omega}} w$.

We define $T_i(t),\ T_j(t),\ T_5(t),\ T_6(t),\ T_7(t) : C(\bar{\Omega},\ \mathbb{R}) \to C(\bar{\Omega},\ \mathbb{R})$ as the C_0 semigroups associated with

$$
-(\omega(\cdot) + \mu_a(\cdot)),\ d_m\triangle - \mu_m(\cdot),\ d_h\triangle - \mu_h(\cdot),\ d_h\triangle - (r(\cdot) + \mu_h(\cdot)),\ -\sigma(\cdot)
$$

subject to the Neumann boundary condition, respectively, $i = 1,\ 2,\ j = 3,\ 4$. Then it follows that for any $\psi \in C(\bar{\Omega},\ \mathbb{R}),\ t \ge 0$,

$$
(T_k(t)\psi)(x) = \int_{\Omega} \Gamma_k(x, y, t)\psi(y)dy,\ k = 1,\ 2,\ 3,\ 4,\ 5,\ 6, 7, \tag{3.1}
$$

in which Γ_i, Γ_j, Γ_6, Γ_7 and Γ_7 are the Green functions associated with

$$-(\omega(\cdot)+\mu_a(\cdot)),\ d_m\triangle-\mu_m(\cdot),\ d_h\triangle-\mu_h(\cdot),\ d_h\triangle-(r(\cdot)+\mu_h(\cdot)),\ -\sigma(\cdot)$$

subject to the Neumann boundary condition, respectively, $i=1,\ 2,\ j=3,\ 4$.

It is well-known that for all $t>0$ and $k=1,\ 2,\ 3,\ 4,\ 5,\ 6,7$, T_k is compact and strongly positive (see [30], Section 7.1 and Corollary 7.2.3). Moreover, $T(t)=(T_1(t),\ T_2(t),\ T_3(t),\ T_4(t),\ T_5(t),\ T_6(t),\ T_7(t))^T$, $t\geq 0$, is a C_0 semigroup. For $\forall\ x\in\partial\Omega$ and $u=(u_1,\ u_2,\ u_3,\ u_4,\ u_5,\ u_6,\ u_7)^T\in\mathbb{X}_K$, the nonlinear operator $F=(F_1,\ F_2,\ F_3,\ F_4,\ F_5,\ F_6,\ F_7)^T:\mathbb{X}_K\to\mathbb{X}$ is defined by

$$\begin{cases} F_1(u)(x)=\rho(x)(u_3+u_4)\left(1-\dfrac{u_1+u_2}{K(x)}\right)-\beta_v(x)u_1u_7,\\ F_2(u)(x)=\beta_v(x)u_1u_7,\\ F_3(u)(x)=\omega(x)u_1-\dfrac{a(x)\beta_1(x)}{H(x)}u_6u_3,\\ F_4(u)(x)=\omega(x)u_2+\dfrac{a(x)\beta_1(x)}{H(x)}u_6u_3,\\ F_5(u)(x)=\Lambda(x)-\dfrac{a(x)\beta_2(x)}{H(x)}u_4u_5,\\ F_6(u)(x)=\dfrac{a(x)\beta_2(x)}{H(x)}u_4u_5,\\ F_7(u)(x)=\theta(x)u_6.\end{cases} \tag{3.2}$$

Then system (2.3) can be rewritten as the following integral equation

$$u_k(t)=T_i(t)u_k^0+\int_0^t\Gamma_i(t-s)F_k(u(\cdot,s))ds,\ \ k=1,\ 2,\ 3,\ 4,\ 5,\ 6,7. \tag{3.3}$$

For any $\phi\in\mathbb{X}_K$ and $h\geq 0$, then we have

$$\phi+hF(\phi)\ =\ \begin{bmatrix}\phi_1+h\left(\rho(x)(\phi_3+\phi_4)\left(1-\dfrac{\phi_1+\phi_2}{K(x)}\right)-\beta_v(x)\phi_1\phi_7\right)\\ \phi_2+h\beta_v(x)\phi_1\phi_7\\ \phi_3+h\left(\omega(x)\phi_1-\dfrac{a(x)\beta_1(x)}{H(x)}\phi_6\phi_3\right)\\ \phi_4+h\left(\omega(x)\phi_2+\dfrac{a(x)\beta_1(x)}{H(x)}\phi_6\phi_3\right)\\ \phi_5+h\left(\Lambda(x)-\dfrac{a(x)\beta_2(x)}{H(x)}\phi_4\phi_5\right)\\ \phi_6+h\dfrac{a(x)\beta_2(x)}{H(x)}\phi_4\phi_5\\ \phi_7+h\theta(x)\phi_6\end{bmatrix}\geq\begin{bmatrix}\phi_1\left(1-h\overline{\beta_v}\phi_7\right)\\ \phi_2\\ \phi_3\left(1-h\dfrac{\overline{a\beta_1}}{\underline{H}}\phi_6\right)\\ \phi_4\\ \phi_5\left(1-h\dfrac{\overline{a\beta_2}}{\underline{H}}\phi_4\right)\\ \phi_6\\ \phi_7\end{bmatrix},$$

and

$$K(x) - (\phi_1 + hF_1(\phi) + \phi_2 + hF_2(\phi)) = K(x) - \left((\phi_1 + \phi_2) + h\rho(x)(\phi_3 + \phi_4)\left(1 - \frac{\phi_1 + \phi_2}{K(x)}\right)\right)$$
$$= (K(x) - (\phi_1 + \phi_2))\left[1 - h\frac{\rho(x)}{K(x)(\phi_3 + \phi_4)}\right]. \tag{3.4}$$

This means that

$$\lim_{h \to 0^+} \frac{1}{h} dist\,(\phi + hF(\phi),\ \mathbb{X}_K) = 0,\ \forall\ \phi \in \mathbb{X}_K.$$

It then follows from Corollary 8.1.3 in [31] (see also Corollary 4 in [32]), we have the following result.

Lemma 3.1. *For every initial value function $\phi \in \mathbb{X}_K$, system (2.3) admits a unique mild solution, denoted by*

$$u(\cdot, t, \phi) = (S_a(\cdot, t, \phi),\ I_a(\cdot, t, \phi),\ S_m(\cdot, t, \phi),\ I_m(\cdot, t, \phi),\ S_h(\cdot, t, \phi),\ I_h(\cdot, t, \phi),\ V(\cdot, t, \phi))^T$$

on its maximal existence interval $[0,\ b_\phi)$ with $u^0 = \phi$, where $b_\phi \le \infty$. Moreover, $u(\cdot, t, \phi) \in \mathbb{X}_K$ for $\forall\ t \in (0,\ b_\phi)$ and $u(\cdot, t, \phi)$ is a classical solution of system (2.3).

Next, we will show that solutions of system (2.3) exist globally on $[0,\ \infty)$, and admit a global compact attractor on $\mathbb{X}_K$.

Lemma 3.2. *For every initial value function $\phi \in \mathbb{X}_K$, system (2.3) has a unique solution, denoted by*

$$u(\cdot, t, \phi) = (S_a(\cdot, t, \phi),\ I_a(\cdot, t, \phi),\ S_m(\cdot, t, \phi),\ I_m(\cdot, t, \phi),\ S_h(\cdot, t, \phi),\ I_h(\cdot, t, \phi),\ V(\cdot, t, \phi))^T$$

on $[0,\ \infty)$ with $u^0 = \phi$. Moreover, define the semiflow $\Phi(t) : \mathbb{X}_K \to \mathbb{X}_K$ associated with system (2.3) by

$$\Phi(t)\phi = u(\cdot, t, \phi),\ \forall \phi = u^0 \in \mathbb{X}_K,\ t \ge 0.$$

Then the semiflow $\Phi(t) : \mathbb{X}_K \to \mathbb{X}_K$ admits a global compact attractor on $\mathbb{X}_K$, $\forall t \ge 0$.

Proof. Clearly, for $\forall\ \phi \in \mathbb{X}_K$, we have $0 \le S_h(\cdot, t, \phi),\ I_h(\cdot, t, \phi) \le H(\cdot)$ for all $t \ge 0$. The comparison principle ([30], Theorem 7.3.4) implies that $S_h(x, t, \phi)$ and $I_h(x, t, \phi)$ are uniformly bounded and ultimately bounded. It then follows from the seventh equation of (2.3) that

$$\frac{\partial V(x,t)}{\partial t} \le \theta(x)H(x) - \delta(x)V(x,t),$$
$$\le \overline{\theta H} - \underline{\delta} V(x,t)\ t > 0. \tag{3.5}$$

Thus, the comparison principle shows that $V(\cdot, t, \phi)$ is uniformly bounded on $[0, b_\phi)$, and

$$\lim_{t \to \infty} V(x,t) \le \frac{\overline{\theta H}}{\underline{\delta}},\ uniformly\ in\ x \in \overline{\Omega}. \tag{3.6}$$

More precisely, there exists a $t_1 > 0$ such that

$$V(\cdot, t) \le 2\frac{\overline{\theta H}}{\underline{\delta}},\ \forall\ t \ge t_1.$$

Letting $N_1 = \max\left\{\max_{t\in[0,t_1],\ x\in\bar{\Omega}} V(x,t),\ 2^{\frac{\overline{\theta H}}{\underline{\delta}}}\right\} < \infty$, we deduce

$$V(x,t) \le N_1 \ for\ all\ x \in \Omega,\ t > 0. \tag{3.7}$$

Let $A(x,t) = S_a(x,t) + I_a(x,t)$, $M(x,t) = S_m(x,t) + I_m(x,t)$. Then it follows from the first four equations of (2.3) that $(A(x,t),\ M(x,t))$ satisfies

$$\begin{cases} \dfrac{\partial A}{\partial t} = \rho(x)\left(1 - \dfrac{A}{K(x)}\right)M - (\omega(x) + \mu_a(x))A, & x \in \Omega,\ t > 0, \\ \dfrac{\partial M}{\partial t} = d_m \triangle M + \omega(x)A - \mu_m(x)M, & x \in \Omega,\ t > 0, \\ \dfrac{\partial M}{\partial n} = 0, & x \in \partial\Omega,\ t > 0, \\ A(x,0) = A_0(x),\ M(x,0) = M_0(x), & x \in \Omega, \end{cases} \tag{3.8}$$

where $A_0(x) = \phi_1(x) + \phi_2(x)$, $M_0(x) = \phi_3(x) + \phi_4(x)$. It is easy to see that there exists a positive vector $\nu = (\nu_1,\ \nu_2) := \left(\overline{K},\ \frac{f\overline{\omega}\overline{K}}{\underline{\mu_m}}\right)$ such that

$$\rho(x)\left(1 - \frac{\nu_1}{K(x)}\right)\nu_2 - (\omega(x) + \mu_a(x))\nu_1 \ge 0,\ \ \omega(x)\nu_1 - \mu_m(x)\nu_2 \le 0.$$

Thus, ν is an upper solution of (3.8). The comparison principle implies that solutions of (3.8) are uniformly bounded on $[0, b_\phi)$. Hence, so are $S_a(x,t)$, $I_a(x,t)$, $S_m(x,t)$ and $I_m(x,t)$. Then, we can extend the local unique solution from Lemma 3.1 to global in time via a standard a priori estimates and continuation of local theory. That is, solutions of (2.3) exist on $[0,\ \infty)$. Next, we show that $S_a(x,t)$, $I_a(x,t)$, $S_m(x,t)$ and $I_m(x,t)$ are ultimately bounded.

From Lemma 3.1, we have $S_a(x,t) + I_a(x,t) \le \overline{K}$ for all $x \in \bar{\Omega},\ t \ge 0$. This implies that $S_a(x,t)$ and $I_a(x,t)$ are ultimately bounded and

$$S_a(x,t) \le \overline{K},\ I_a(x,t) \le \overline{K},\ \ \forall\ x \in \bar{\Omega},\ t \ge 0. \tag{3.9}$$

It then follows from the third equation of (2.3) that

$$\begin{cases} \dfrac{\partial S_m}{\partial t} \le d_m \triangle S_m + \omega(x)\overline{K} - \mu_m(x)S_m \\ \qquad\quad \le d_m \triangle S_m + \overline{\omega}\overline{K} - \underline{\mu_m} S_m, & x \in \Omega,\ t > 0, \\ \dfrac{\partial S_m}{\partial n} = 0, & x \in \partial\Omega,\ t > 0, \\ S_m(x,0) = \phi_3(x), & x \in \Omega. \end{cases} \tag{3.10}$$

Consider

$$\begin{cases} \dfrac{\partial W}{\partial t} = d_m \triangle W + \overline{\omega}\overline{K} - \underline{\mu_m} W, & x \in \Omega,\ t > 0, \\ \dfrac{\partial W}{\partial n} = 0, & x \in \partial\Omega,\ t > 0, \\ W(x,0) = \phi_3(x), & x \in \Omega. \end{cases} \tag{3.11}$$

From Lemma 1 in [29], $\frac{\overline{\omega}\overline{K}}{\underline{\mu_m}}$ is a unique positive steady state that is globally attractive in $C(\bar{\Omega},\ R_+)$. Hence there exists $t_2 \geq 0$ such that $W(x,t) \leq 2\frac{\overline{\omega}\overline{K}}{\underline{\mu_m}}$. By comparison principle, $S_m(\cdot,t) \leq 2\frac{\overline{\omega}\overline{K}}{\underline{\mu_m}}$ when $t > t_2$.

Then,

$$\begin{cases} \dfrac{\partial I_m}{\partial t} \leq d_m \triangle I_m + \overline{\omega}\overline{K} + 2\dfrac{\overline{a}\overline{\beta_1}\overline{\omega}\overline{K}}{\underline{\mu_m}} - \underline{\mu_m} I_m, & x \in \Omega,\ t > t_2, \\ \dfrac{\partial I_m}{\partial n} = 0, & x \in \partial\Omega,\ t > t_2, \\ I_m(x,t_2) := I_{m2}(x), & x \in \Omega. \end{cases} \tag{3.12}$$

Similarly, $I_m(x,t)$ is ultimately bounded. More precisely, there exists a $t_3 > 0$ such that $I_m(\cdot,t) \leq \frac{2\overline{\omega}\overline{K}}{\underline{\mu_m}}\left(1 + 2\frac{\overline{a}\overline{\beta_1}}{\underline{\mu_m}}\right)$, for $\forall t > t_3$. Thus, we can obtain

$$I_m(\cdot,t) \leq N_2,\ for\ \forall t \geq 0, \tag{3.13}$$

where $N_2 = \max\left\{\max\limits_{t\in[0,t_3],\ x\in\bar{\Omega}} I_m(x,t),\ \frac{2\overline{\omega}\overline{K}}{\underline{\mu_m}}\left(1 + 2\frac{\overline{a}\overline{\beta_1}}{\underline{\mu_m}}\right)\right\} < \infty$.

In addition, since the first two equations and the last equation of (2.3) have no diffusion term, the solution semiflow $\Phi(t)$ is not compact. However, due to $-\omega(x) - \mu_a(x) < 0, -\delta(x) < 0, \forall x \in \bar{\Omega}$, using similar arguments from Theorem 4.1 in [33] (also Lemma 4.1 in [34] and Theorem 2.6 in [35]) that the semiflow $\Phi(t) : \mathbb{X}_K \to \mathbb{X}_K$ has a global compact attractor on $\mathbb{X}_K,\ \forall t \geq 0$. This completes the proof of Lemma 3.2. □

The following result shows that the solution of system (2.3) is strictly positive.

Lemma 3.3. *Let* $(S_a(\cdot,t,\phi), I_a(\cdot,t,\phi), S_m(\cdot,t,\phi), I_m(\cdot,t,\phi), S_h(\cdot,t,\phi), I_h(\cdot,t,\phi), V(\cdot,t,\phi))^T$ *be the solution of system (2.3) with the initial value* $\phi \in \mathbb{X}_K$. *If there exists some* $t_0 \geq 0$ *such that* $I_a(\cdot,t_0,\phi) \not\equiv 0,\ I_m(\cdot,t_0,\phi) \not\equiv 0,\ I_h(\cdot,t_0,\phi) \not\equiv 0,\ V(\cdot,t_0,\phi) \not\equiv 0,$ *then the solution of system (2.3) satisfies*

$$\begin{aligned} &S_a(x,t,\phi) > 0, I_a(x,t,\phi) > 0, S_m(x,t,\phi) > 0, I_m(x,t,\phi) > 0, \\ &S_h(x,t,\phi) > 0, I_h(x,t,\phi) > 0, V(x,t,\phi) > 0,\ \ \forall\ t > t_0,\ x \in \bar{\Omega}. \end{aligned}$$

Moreover, for any initial value $\phi \in \mathbb{X}_K$, *there exists some positive constant* ζ_0 *such that*

$$\liminf_{t\to\infty} S_h(x,t,\phi) \geq \zeta_0,\ uniformly\ for\ x \in \bar{\Omega}. \tag{3.14}$$

Proof. For a give $\phi \in \mathbb{X}_K$, it is easy to see that $I_a(x,t,\phi)$, $I_m(x,t,\phi)$, $I_h(x,t,\phi)$ and $V(x,t,\phi)$ satisfy

$$\begin{cases} \dfrac{\partial I_a}{\partial t} \geq -(\overline{\omega} + \overline{\mu_a})I_a, & x \in \Omega,\ t > 0, \\ \dfrac{\partial I_m}{\partial t} \geq d_m \triangle I_m - \overline{\mu_m} I_m, & x \in \Omega,\ t > 0, \\ \dfrac{\partial I_h}{\partial t} \geq d_h \triangle I_h - (\overline{r} + \overline{\mu_h})I_h, & x \in \Omega,\ t > 0, \\ \dfrac{\partial V}{\partial t} \geq -\bar{\delta} V, & x \in \Omega,\ t > 0, \\ \dfrac{\partial I_a}{\partial n} = \dfrac{\partial I_m}{\partial n} = \dfrac{\partial I_h}{\partial n} = \dfrac{\partial V}{\partial n} = 0, & x \in \partial\Omega. \end{cases}$$

If there exists some $t_0 \geq 0$ such that $I_a(\cdot, t_0, \phi) \not\equiv 0$, $I_m(\cdot, t_0, \phi) \not\equiv 0$, $I_h(\cdot, t_0, \phi) \not\equiv 0$, $V(\cdot, t_0, \phi) \not\equiv 0$, it then follows from the strong maximum principle (see Proposition 13.1 in [33]) that $I_a(x, t, \phi) > 0, I_m(x, t, \phi) > 0, I_h(x, t, \phi) > 0, V(x, t, \phi) > 0, \ \forall\ t > t_0,\ x \in \bar{\Omega}$.

Next, we will prove $S_a(x, t, \phi) > 0,\ \forall t > t_0,\ x \in \bar{\Omega}$. To this end, we first show $A(x, t) = S_a(x, t) + I_a(x, t) < K(x)$ for all $x \in \bar{\Omega},\ t \geq 0$. If not, then there exists $x_1 \in \bar{\Omega},\ t_1 \geq 0$, such that $A(x_1, t_1) = K(x_1)$. Since $A(x, t)$ satisfies

$$\begin{cases} \dfrac{\partial A}{\partial t} = \rho(x)\left(1 - \dfrac{A}{K(x)}\right)M - (\omega(x) + \mu_a(x))A, & x \in \Omega,\ t > 0, \\ A(x, 0) = A_0(x), & x \in \Omega, \end{cases}$$

$$0 = \frac{\partial A(x_1, t_1)}{\partial t} = \rho(x)\left(1 - \frac{A(x_1, t_1)}{K(x_1)}\right)M(x_1, t_1) - (\omega(x_1) + \mu_a(x_1))A(x_1, t_1).$$

It implies that $A(x_1, t_1) = 0$. Then $K(x_1) = 0$ which contradicts that $K(x)$ is strictly positive. So, $S_a(x, t), I_a(x, t) < K(x)$ for all $x \in \bar{\Omega},\ t \geq 0$. Next, we show $S_a(x, t, \phi) > 0,\ \forall t > t_0,\ x \in \bar{\Omega}$. Suppose not, there exists $x_2 \in \bar{\Omega},\ t_2 > t_0$, such that $S_a(x_2, t_2) = 0$. From the first equation of system (2.3), we have

$$0 = \frac{\partial S_a(x_2, t_2)}{\partial t} = \rho(x_2)(S_m(x_2, t_2) + I_m(x_2, t_2))\left(1 - \frac{I_a(x_2, t_2)}{K(x_2)}\right) > 0.$$

It is contradictory. Thus, $S_a(x, t, \phi) > 0,\ \forall t > t_0,\ x \in \bar{\Omega}$. Similarly, we can show $S_m(x, t, \phi) > 0, S_h(x, t, \phi) > 0,\ \forall t > t_0,\ x \in \bar{\Omega}$.

Moreover, for a give $\phi \in \mathbb{X}_K$, it follows from the fifth equation of system (2.3) and (3.13) that $S_h(x, t, \phi)$ satisfies

$$\begin{cases} \dfrac{\partial S_h}{\partial t} \geq d_h \Delta S_h + \underline{\Lambda} - \left(\dfrac{\overline{a}\overline{\beta_2} N_2}{\underline{H}} + \overline{\mu_h}\right) S_h, & x \in \Omega,\ t > 0, \\ \dfrac{\partial S_h}{\partial n} = 0, & x \in \partial\Omega. \end{cases}$$

According to the comparison principle, we have

$$\liminf_{t \to \infty} S_h(x, t, \phi) \geq \frac{\underline{\Lambda}}{\frac{\overline{a}\overline{\beta_2} N_2}{\underline{H}} + \overline{\mu_h}},\ uniformly\ for\ x \in \bar{\Omega}.$$

This completes the proof of Lemma 3.3.

□

4. Threshold index

One of the most important concepts in epidemiology is the basic reproduction number R_0 which is a threshold index to determine the disease invasion. It is defined to be the average number of secondary cases produced in a completely susceptible population, by a typical infective individual, during its lifetime as infectious. By using the concept of next generation operators, Diekmann, Heesterbeek and Metz [36] presented a general approach to R_0 for autonomous epidemic models. Van den Driessche and Watmough [37] gave a computation formula of R_0 for compartmental models of ordinary differential equations with constant coefficients. For the reaction-diffusion system with spatially dependent coefficients, Wang and Zhao [38] defined the basic reproduction ratio as the spectral radius of the next infection operator. According to the above methods, in this section, in order to obtain the basic reproduction number, we should first find the disease free equilibrium (DFE) (infection-free steady state). System (2.3) admits two possible DFEs: $E_{01}(x) = (0,0,0,0,H(x),0,0)$ and $E_{02}(x) = (A(x)^*, 0, M^*(x), 0, H(x), 0, 0)$. $E_{01}(x)$ is characterized by the absence of mosquitoes. $E_{02}(x)$ represents an eradication of Zika in the presence of mosquito population. Firstly, we give the basic offspring number which determines whether mosquito population persists, corresponds to the stability of $E_{01}(x)$.

4.1. Basic offspring number R_0^m

Since $(A(x,t),\ M(x,t))$ satisfies system (3.8), it suffices to consider system (3.8). Obviously, system (3.8) always admits a DFE $(0,0)$. Linearizing system (3.8) at $(0,0)$, we have

$$\begin{cases} \dfrac{\partial A}{\partial t} = \rho(x)M - (\omega(x) + \mu_a(x))A, & x \in \Omega,\ t > 0, \\ \dfrac{\partial M}{\partial t} = d_m \triangle M + \omega(x)A - \mu_m(x)M, & x \in \Omega,\ t > 0, \\ \dfrac{\partial M}{\partial n} = 0, & x \in \partial\Omega,\ t > 0, \\ A(x,0) = A_0(x),\ M(x,0) = M_0(x), & x \in \Omega. \end{cases} \tag{4.1}$$

The eigenvalue problem associated with (4.1) is as follows

$$\begin{cases} \rho(x)\psi_2 - (\omega(x) + \mu_a(x))\psi_1 = \lambda_1\psi_1, & x \in \Omega, \\ d_m\triangle\psi_2 + \omega(x)\psi_1 - \mu_m(x)\psi_2 = \lambda_1\psi_2, & x \in \Omega, \\ \dfrac{\partial\psi_2}{\partial n} = 0, & x \in \partial\Omega, \end{cases} \tag{4.2}$$

where ψ_1 and ψ_2 are both positive for $x \in \Omega$. Let B_m be defined as follows

$$B_m = \begin{bmatrix} -(\omega(x) + \mu_a(x)) & \rho(x) \\ \omega(x) & d_m\triangle - \mu_m(x) \end{bmatrix}. \tag{4.3}$$

Denote the basic offspring number as R_0^m. According to Lemma 4.2 in [38], $\frac{1}{R_0^m}$ is the unique positive eigenvalue of the eigenvalue problem

$$\begin{cases} -d_m\triangle\varphi + \mu_m(x)\varphi = \lambda_2 \dfrac{\omega(x)\rho(x)}{\omega(x) + \mu_a(x)}\varphi, & x \in \Omega, \\ \dfrac{\partial\phi}{\partial n} = 0, & x \in \partial\Omega. \end{cases} \tag{4.4}$$

By using the variational characterization of principal eigenvalue [39], we can obtain

$$R_0^m = \sup_{\varphi \in H_0^1(\Omega),\ \phi \neq 0} \left\{ \frac{\int_\Omega \frac{\omega(x)\rho(x)}{\omega(x)+\mu_a(x)}\varphi^2}{\int_\Omega d_m|\nabla\varphi|^2 + \mu_m(x)\varphi^2} \right\}. \tag{4.5}$$

By Lemma 2.2 and Lemma 2.3 in [40], we have the following observation.

Lemma 4.1. *Let $\lambda_m^* = s(B_m)$ be the spectral bound of B_m.*

(A1) *If $\lambda_m^* \geq 0$, then λ_m^* is the principal eigenvalue of (4.2) with a strongly positive eigenfunction.*

(A2) *$R_0^m - 1$ and λ_m^* have the same sign.*

Let $\mathbb{Y}_K := \{(A_0, M_0)^T \in C(\bar{\Omega}, \mathbb{R}_+^2) : 0 \leq A_0(x) \leq K(x)), \ \forall\ x \in \bar{\Omega}\}$. The following result is concerned with the global dynamics of system (3.8).

Lemma 4.2. *([40], Lemma 2.5) Suppose that $R_0^m > 1$. System (3.8) admits a unique steady state $(A^*(x), M^*(x))$ which is globally asymptotically stable in $\mathbb{Y}_K \backslash \{(0,0)\}$. Moreover, $0 < A^*(x) < K(x),\ \forall\ x \in \bar{\Omega}$.*

Below we use the method proposed in Wang and Zhao [38] to introduce the basic reproduction number.

4.2. Basic reproduction number R_0

This sub-section is devoted to formulate of the reproduction number for system (2.3) that determines invasion of Zika disease. So, it is the essential condition that guarantees the persistence of mosquito population. From Lemma 4.2, we assume that $R_0^m > 1$, and then $E_{02}(x)$ exists. Linearizing system (2.3) at $E_{02}(x)$, and then considering only the equations of infective compartments, we have

$$\begin{cases} \dfrac{\partial I_a}{\partial t} = \beta_v(x)A^*(x)V - (\omega(x) + \mu_a(x))I_a, & x \in \Omega,\ t > 0, \\ \dfrac{\partial I_m}{\partial t} = d_m \triangle I_m + \omega(x)I_a + \dfrac{a(x)\beta_1(x)M^*(x)}{H(x)} I_h - \mu_m(x)I_m, & x \in \Omega,\ t > 0, \\ \dfrac{\partial I_h}{\partial t} = d_h \triangle I_h + a(x)\beta_2(x)I_m - (r(x) + \mu_h(x))I_h, & x \in \Omega,\ t > 0, \\ \dfrac{\partial V}{\partial t} = \theta(x)I_h - \delta(x)V, & x \in \Omega,\ t > 0, \\ \dfrac{\partial I_m}{\partial n} = \dfrac{\partial I_h}{\partial n} = 0, & x \in \partial\Omega,\ t > 0, \\ I_a(x,0) = I_{a0}(x),\ I_m(x,0) = I_{m0}(x),\ I_h(x,0) = I_{h0}(x),\ V(x,0) = V_0(x), & x \in \Omega. \end{cases} \tag{4.6}$$

Let $\bar{T}(t) : C(\bar{\Omega},\ \mathbb{R}^4) \to C(\bar{\Omega},\ \mathbb{R}^4)$ be the solution semigroup generated by system (4.6). It is easy to see that $\bar{T}(t)$ is a positive C_0 semigroup, and its generator B can be written as

$$B = \begin{bmatrix} -(\omega(x) + \mu_a(x)) & 0 & 0 & \beta_v(x)A^*(x) \\ \omega(x) & d_m\triangle - \mu_m(x) & \frac{a(x)\beta_1(x)M^*(x)}{H(x)} & 0 \\ 0 & a(x)\beta_2(x) & d_h\triangle - (r(x) + \mu_h(x)) & 0 \\ 0 & 0 & \theta(x) & -\delta(x) \end{bmatrix}.$$

Further, B is a closed and resolvent positive operator (see Theorem 3.12 in [41]). The eigenvalue problem associated with (4.6) is as follows

$$\begin{cases} \lambda_3\varphi_1 = \beta_v(x)A^*(x)\varphi_4 - (\omega(x)+\mu_a(x))\varphi_1, & x\in\Omega,\ t>0,\\ \lambda_3\varphi_2 = d_m\triangle\varphi_2 + \omega(x)\varphi_1 + \dfrac{a(x)\beta_1(x)M^*(x)}{H(x)}\varphi_3 - \mu_m(x)\varphi_2, & x\in\Omega,\ t>0,\\ \lambda_3\varphi_3 = d_h\triangle\varphi_3 + a(x)\beta_2(x)\varphi_2 - (r(x)+\mu_h(x))\varphi_3, & x\in\Omega,\ t>0,\\ \lambda_3\varphi_4 = \theta(x)\varphi_3 - \delta(x)\varphi_4, & x\in\Omega,\ t>0,\\ \dfrac{\partial\varphi_2}{\partial n} = \dfrac{\partial\varphi_3}{\partial n} = 0, & x\in\partial\Omega,\ t>0. \end{cases} \tag{4.7}$$

By a similar argument as Theorem 7.6.1 in [30], we have the following observation.

Lemma 4.3. *Let $\lambda^* = s(B)$ be the spectral bound of B. If $\lambda^* \geq 0$, then λ^* is the principal eigenvalue of the eigenvalue problem (4.7) with a strongly positive eigenfunction.*

In the following, we will use the ideas in [38] to define the basic reproduction number. Let $\mathbb{T}(t)$: $C(\bar{\Omega},\ \mathbb{R}^4) \to C(\bar{\Omega},\ \mathbb{R}^4)$ be the solution semigroup generated by the following linear system

$$\begin{cases} \dfrac{\partial I_a}{\partial t} = -(\omega(x)+\mu_a(x))I_a, & x\in\Omega,\ t>0,\\ \dfrac{\partial I_m}{\partial t} = d_m\triangle I_m + \omega(x)I_a - \mu_m(x)I_m, & x\in\Omega,\ t>0,\\ \dfrac{\partial I_h}{\partial t} = d_h\triangle I_h - (r(x)+\mu_h(x))I_h, & x\in\Omega,\ t>0,\\ \dfrac{\partial V}{\partial t} = \theta(x)I_h - \delta(x)V, & x\in\Omega,\ t>0,\\ \dfrac{\partial I_m}{\partial n} = \dfrac{\partial I_h}{\partial n} = 0, & x\in\partial\Omega,\ t>0,\\ I_a(x,0)=I_{a0}(x),\ I_m(x,0)=I_{m0}(x),\ I_h(x,0)=I_{h0}(x),\ V(x,0)=V_0(x), & x\in\Omega. \end{cases} \tag{4.8}$$

It is easy to see that $\mathbb{T}(t)$ is a C_0 semigroup on $C(\bar{\Omega},\ \mathbb{R}^4)$.

We define

$$\mathbb{F}(x) = \begin{bmatrix} 0 & 0 & 0 & \beta_v(x)A^*(x)\\ 0 & 0 & \frac{a(x)\beta_1(x)M^*(x)}{H(x)} & 0\\ 0 & a(x)\beta_2(x) & 0 & 0\\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbb{V}(x) = \begin{bmatrix} (\omega(x)+\mu_a(x)) & 0 & 0 & 0\\ -\omega(x) & \mu_m(x) & 0 & 0\\ 0 & 0 & (r(x)+\mu_h(x)) & 0\\ 0 & 0 & -\theta(x) & \delta(x) \end{bmatrix}.$$

In order to define the basic reproduction number for system (2.3), we assume that the state variables are near DFE E_{02}, and introduce the distribution of initial infective individuals described by $\phi(x) \in C(\bar{\Omega},\ \mathbb{R}^4)$. Thus, it is easy to see that $\mathbb{T}(t)\phi(x)$ represents the distribution of those infective individuals at time t. Thus, $\mathbb{F}(x)\mathbb{T}(t)\phi(x)$ represents the distribution of new infective individuals at time t.

Define $\mathbb{L} : C(\bar{\Omega}, \mathbb{R}^4) \to C(\bar{\Omega}, \mathbb{R}^4)$ as follows

$$\mathbb{L}(\phi)(x) := \int_0^\infty \mathbb{F}(x)\mathbb{T}(t)\phi(x)dt. \tag{4.9}$$

It then follows that $\mathbb{L}(\phi)(x)$ represents the distribution of the total new population generated by initial infective individuals $\phi(x)$ during their infection period. So, $\mathbb{L}$ is the next generation operator. We define the spectral radius of $\mathbb{L}$ as the basic reproduction number of system (2.3). That is,

$$R_0 := r(\mathbb{L}). \tag{4.10}$$

From [38], we have the following observation.

Lemma 4.4. *$R_0 - 1$ and λ^* have the same sign.*

The following result indicates that basic offspring number R_0^m is a threshold index for eradication or persistence of the Zika disease.

5. Global dynamic behavior

We firstly focus on the global dynamic behaviors of the DFEs E_{01} and E_{02} of system (2.3).

Theorem 5.1. *If $R_0^m < 1$, then the DFE $E_{01}(x)$ is globally attractive in $\mathbb{X}_K$ for system (2.3).*

Proof. Assume $R_0^m < 1$. It follows from Lemma 4.1 that $\lambda_m^* > 0$. λ_m^* is the principal eigenvalue of eigenvalue problem (4.2) with a strongly positive eigenfunction (ψ_1, ψ_2). Since $(A(x,t), M(x,t))$ satisfies system (3.8), it follows that

$$\begin{cases} \dfrac{\partial A}{\partial t} \leq \rho(x)M - (\omega(x) + \mu_a(x))A, & x \in \Omega,\ t > 0, \\ \dfrac{\partial M}{\partial t} = d_m \triangle M + \omega(x)A - \mu_m(x)M, & x \in \Omega,\ t > 0, \\ \dfrac{\partial M}{\partial n} = 0, & x \in \partial\Omega,\ t > 0. \end{cases} \tag{5.1}$$

For any given $\phi = (\phi_1, \phi_2) \in \mathbb{Y}_K$, there exists some $q > 0$ such that

$$(A(x, 0, \phi),\ M(x, 0, \phi)) \leq q(\psi_1,\ \psi_2),\ \forall\ x \in \Omega.$$

Note that the linear system (4.1) admits a solution $qe^{\lambda_m^* t}(\psi_1,\ \psi_2)$, $\forall\ t \geq 0$. Then the comparison principle implies that

$$(A(x, t, \phi),\ M(x, t, \phi)) \leq qe^{\lambda_m^* t}(\psi_1,\ \psi_2),\ \forall\ t \geq 0,\ \forall\ x \in \bar{\Omega}.$$

Hence, $\lim_{t\to\infty}(A(x, t, \phi), M(x, t, \phi)) = (0, 0)$, uniformly for all $x \in \bar{\Omega}$. Then, from $A(x, t) = S_a(x, t) + I_a(x, t)$ and $M(x, t) = S_m(x, t) + I_m(x, t)$, together with positivity of solutions, for system (2.3), we have for every initial value function $\phi = (\phi_1,\ \phi_2,\ \phi_3,\ \phi_4,\ \phi_5,\ \phi_6,\ \phi_7)^T \in \mathbb{X}_K$,

$$\lim_{t\to\infty} S_a(x, t, \phi) = 0,\ \ \lim_{t\to\infty} I_a(x, t, \phi) = 0,\ \ \lim_{t\to\infty} S_m(x, t, \phi) = 0,\ \ \lim_{t\to\infty} I_m(x, t, \phi) = 0,\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

Then, $S_h(\cdot,t)$ in system (2.3) is asymptotic to the following system

$$\begin{cases} \dfrac{\partial S_h}{\partial t} = d_h \triangle S_h + \Lambda(x) - \mu_h(x) S_h, & x \in \Omega,\ t > 0, \\ \dfrac{\partial S_h}{\partial n} = 0, & x \in \partial\Omega,\ t > 0, \\ S_h(x,0) = \phi_5, & x \in \Omega. \end{cases} \tag{5.2}$$

By the theory for asymptotically autonomous semiflows (see Corollary 4.3 in [42]), together with Lemma 1 in [29], it follows that

$$\lim_{t\to\infty} S_h(x,t,\phi) = H(x),\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

Similarly, $I_h(\cdot,t)$ in system (2.3) is asymptotic to the following system

$$\begin{cases} \dfrac{\partial I_h}{\partial t} = d_h \triangle I_h - (r(x) + \mu_h(x)) I_h, & x \in \Omega,\ t > 0, \\ \dfrac{\partial I_h}{\partial n} = 0, & x \in \partial\Omega,\ t > 0, \\ I_h(x,0) = \phi_6, & x \in \Omega. \end{cases} \tag{5.3}$$

Therefore,

$$\lim_{t\to\infty} I_h(x,t,\phi) = 0,\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

Then $V(\cdot,t)$ in system (2.3) is asymptotic to the following system

$$\begin{cases} \dfrac{\partial V(x,t)}{\partial t} = -\delta(x) V(x,t), & x \in \Omega,\ t > 0, \\ V(x,0) = \phi_7, & x \in \Omega. \end{cases} \tag{5.4}$$

Thus,

$$\lim_{t\to\infty} V(x,t,\phi) = 0,\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

This completes the proof of Theorem 5.1. □

Remark 1. Biologically, Theorem 5.1 shows that the basic offspring number R_0^m can be used as a control parameter which determines whether mosquito population is absent or not. It means that mosquito population can be vanished, and the Zika virus will eradicate in human population and contaminated aquatic environment by reducing R_0^m below 1.

Theorem 5.2. *Let $u(x,t,\phi)$ be the solution of system (2.3) with $u(\cdot,0,\phi) = \phi \in \mathbb{X}_K$. If $R_0^m > 1$, $R_0 < 1$, then the DFE $E_{02}(x)$ is globally attractive for system (2.3). That is, for any $\phi \in \mathbb{X}_K$, if $(\phi_1, \phi_3) \not\equiv (0,0)$, then*

$$\lim_{t\to\infty} u(x,t,\phi) = E_{02}(x),\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

Proof. Suppose $R_0 < 1$. By Lemma 4.4, we have $\lambda^* < 0$. So, there exists a sufficiently small positive number ϵ_0 such that $\lambda^*_{\epsilon_0} < 0$, where $\lambda^*_{\epsilon_0} < 0$ is the principal eigenvalue of the following eigenvalue problem

$$\begin{cases} \lambda\varphi_2^{\epsilon_0} = \beta_v(x)(A^*(x)+\epsilon_0)\varphi_7^{\epsilon_0} - (\omega(x)+\mu_a(x))\varphi_2^{\epsilon_0}, & x\in\Omega,\ t>0,\\ \lambda\varphi_4^{\epsilon_0} = d_m\triangle\varphi_4^{\epsilon_0} + \omega(x)\varphi_2^{\epsilon_0} + \dfrac{a(x)\beta_1(x)(M^*(x)+\epsilon_0)}{H(x)}\varphi_6^{\epsilon_0} - \mu_m(x)\varphi_4^{\epsilon_0}, & x\in\Omega,\ t>0,\\ \lambda\varphi_6^{\epsilon_0} = d_h\triangle\varphi_6^{\epsilon_0} + a(x)\beta_2(x)\varphi_4^{\epsilon_0} - (r(x)+\mu_h(x))\varphi_6^{\epsilon_0}, & x\in\Omega,\ t>0,\\ \lambda\varphi_7^{\epsilon_0} = \theta(x)\varphi_6^{\epsilon_0} - \delta(x)\varphi_7^{\epsilon_0}, & x\in\Omega,\ t>0,\\ \dfrac{\partial\varphi_4^{\epsilon_0}}{\partial n} = \dfrac{\partial\varphi_6^{\epsilon_0}}{\partial n} = 0, & x\in\partial\Omega,\ t>0, \end{cases} \tag{5.5}$$

with a strongly positive eigenfunction $(\varphi_2^{\epsilon_0}, \varphi_4^{\epsilon_0}, \varphi_6^{\epsilon_0}, \varphi_7^{\epsilon_0})$. It follows from the condition $R_0^m > 1$, Lemma 4.2 and the positivity of solutions that there exists a $t_0 > 0$ such that

$$S_a(x,t) \le A^*(x) + \epsilon_0,\ \ S_m(x,t) \le M^*(x) + \epsilon_0,$$

for all $x \in \bar{\Omega}$, $t \ge t_0$. Hence, by the I_a, I_m, I_h and V equations of system (2.3), it follows that

$$\begin{cases} \dfrac{\partial I_a}{\partial t} \le \beta_v(x)(A^*(x)+\epsilon_0)V - (\omega(x)+\mu_a(x))I_a, & x\in\Omega,\ t\ge t_0,\\ \dfrac{\partial I_m}{\partial t} \le d_m\triangle I_m + \omega(x)I_a + \dfrac{a(x)\beta_1(x)(M^*(x)+\epsilon_0)}{H(x)}I_h - \mu_m(x)I_m, & x\in\Omega,\ t\ge t_0,\\ \dfrac{\partial I_h}{\partial t} \le d_h\triangle I_h + a(x)\beta_2(x)I_m - (r(x)+\mu_h(x))I_h, & x\in\Omega,\ t\ge t_0,\\ \dfrac{\partial V}{\partial t} = \theta(x)I_h - \delta(x)V, & x\in\Omega,\ t\ge t_0,\\ \dfrac{\partial I_m}{\partial n} = \dfrac{\partial I_h}{\partial n} = 0, & x\in\partial\Omega,\ t\ge t_0. \end{cases} \tag{5.6}$$

For any given $\phi \in \mathbb{X}_K$, there exists some $q_1 > 0$ such that

$$(I_a(x,t_0,\phi),\ I_m(x,t_0,\phi),\ I_h(x,t_0,\phi),\ V(x,t_0,\phi)) \le q_1(\varphi_2^{\epsilon_0}, \varphi_4^{\epsilon_0}, \varphi_6^{\epsilon_0}, \varphi_7^{\epsilon_0}),\ \forall\ x\in\bar{\Omega}.$$

Note that the following linear system

$$\begin{cases} \dfrac{\partial I_a}{\partial t} = \beta_v(x)(A^*(x)+\epsilon_0)V - (\omega(x)+\mu_a(x))I_a, & x\in\Omega,\ t\ge t_0,\\ \dfrac{\partial I_m}{\partial t} = d_m\triangle I_m + \omega(x)I_a + \dfrac{a(x)\beta_1(x)(M^*(x)+\epsilon_0)}{H(x)}I_h - \mu_m(x)I_m, & x\in\Omega,\ t\ge t_0,\\ \dfrac{\partial I_h}{\partial t} = d_h\triangle I_h + a(x)\beta_2(x)I_m - (r(x)+\mu_h(x))I_h, & x\in\Omega,\ t\ge t_0,\\ \dfrac{\partial V}{\partial t} = \theta(x)I_h - \delta(x)V, & x\in\Omega,\ t\ge t_0,\\ \dfrac{\partial I_m}{\partial n} = \dfrac{\partial I_h}{\partial n} = 0, & x\in\partial\Omega,\ t\ge t_0, \end{cases} \tag{5.7}$$

admits a solution $q_1 e^{\lambda^*_{\epsilon_0}(t-t_0)}(\varphi_2^{\epsilon_0}, \varphi_4^{\epsilon_0}, \varphi_6^{\epsilon_0}, \varphi_7^{\epsilon_0}),\ \forall\ t \geq t_0$. Then the comparison principle implies that

$$(I_a(x,t,\phi),\ I_m(x,t,\phi),\ I_h(x,t,\phi),\ V(x,t,\phi)) \leq q_1 e^{\lambda^*_{\epsilon_0}(t-t_0)}(\varphi_2^{\epsilon_0}, \varphi_4^{\epsilon_0}, \varphi_6^{\epsilon_0}, \varphi_7^{\epsilon_0}),\ \forall\ t \geq t_0,\ \forall\ x \in \bar{\Omega}.$$

Hence, $\lim_{t\to\infty}(I_a(x,t,\phi),\ I_m(x,t,\phi),\ I_h(x,t,\phi),\ V(x,t,\phi)) = (0,0,0,0)$, uniformly for all $x \in \bar{\Omega}$. Then, $(S_a(\cdot,t), S_m(\cdot,t))$ in system (2.3) is asymptotic to system (3.8). By the theory for asymptotically autonomous semiflows, together with Lemma 1 in [29], it follows that

$$\lim_{t\to\infty}(S_a(x,t,\phi),\ S_m(x,t,\phi)) = (A^*(x),\ M^*(x)),\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

Similarly, $S_h(\cdot,t)$ in system (2.3) is asymptotic to system (5.2). That is,

$$\lim_{t\to\infty} S_h(x,t,\phi) = H(x),\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

This completes the proof of Theorem 5.2. □

Remark 2. Biologically, Theorem 5.2 shows that mosquito population is present when the basic offspring number $R_0^m > 1$. Under this premise, the basic reproduction number R_0 can be used as a control parameter which determines whether the disease will eventually die out or not. It means that the disease can be eradicated by reducing R_0 below 1.

Before giving the disease persistence, we first give the following lemma.

Lemma 5.1. *Suppose that $R_m > 1$, and $\phi_i \equiv 0, i = 2, 4$. If there exists some $\zeta_1 > 0$ such that*

$$\liminf_{t\to+\infty} I_h(x,t,\phi) \geq \zeta_1,\ uniformly\ for\ all\ x \in \bar{\Omega},$$

then there exists some $\zeta_2 > 0$ such that

$$\begin{aligned}
&\liminf_{t\to+\infty} S_a(x,t,\phi) \geq \zeta_2, \liminf_{t\to+\infty} I_a(x,t,\phi) \geq \zeta_2, \liminf_{t\to+\infty} S_m(x,t,\phi) \geq \zeta_2,\\
&\liminf_{t\to+\infty} I_m(x,t,\phi) \geq \zeta_2, \liminf_{t\to+\infty} S_h(x,t,\phi) \geq \zeta_2, \liminf_{t\to+\infty} I_h(x,t,\phi) \geq \zeta_2,\\
&\liminf_{t\to+\infty} V(x,t,\phi) \geq \zeta_2,\ uniformly\ for\ all\ x \in \bar{\Omega}.
\end{aligned} \tag{5.8}$$

Proof. From $\liminf_{t\to+\infty} I_h(x,t,\phi) \geq \zeta_1,\ uniformly\ for\ all\ x \in \bar{\Omega}$, we have that there exists $t_{11} > 0$ such that

$$I_h(x,t) \geq \frac{1}{3}\zeta_1,\ \forall t \geq t_{11},\ x \in \bar{\Omega}.$$

It follows from the last equation of system (2.3) that $V(x,t)$ satisfies

$$\begin{cases}
\dfrac{\partial V}{\partial t} \geq \dfrac{1}{3}\zeta_1 \underline{\theta} - \bar{\delta} V, & x \in \Omega,\ t \geq t_{11},\\
\dfrac{\partial V}{\partial n} = 0, & x \in \partial\Omega.
\end{cases}$$

By comparison principle, we have

$$\liminf_{t\to+\infty} V(x,t) \geq \frac{\zeta_1 \underline{\theta}}{3\bar{\delta}} := e_1,\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

Thus, there is a $t_{12} > t_{11}$ such that

$$V(x,t) \geq \frac{1}{3}e_1,\ \forall t \geq t_{12},\ x \in \bar{\Omega}.$$

Due to $R_m > 1$ and $\phi_i \equiv 0, i = 2, 4$, and from Lemma 4.2, we can obtain that there exists $t_{13} > t_{12}$ such that

$$S_a(x,t) + I_a(x,t) \leq A^*(x) + \frac{1}{3}(K(x) - A^*(x)) = \frac{1}{3}K(x) + \frac{2}{3}A^*(x),$$
$$S_m(x,t) + I_m(x,t) \geq \frac{1}{3}M^*(x).$$

From the first equation of system (2.3) and (3.7), one has

$$\begin{cases} \dfrac{\partial S_a}{\partial t} \geq \dfrac{1}{3}M^*(x)\rho(x)\left(1 - \dfrac{\frac{1}{3}K(x) + \frac{2}{3}A^*(x)}{K(x)}\right) - (\beta_v(x)N_1 + \omega(x) + \mu_a(x))S_a \\ \quad \geq \dfrac{2}{9}\underline{M^*\rho}\left(1 - \dfrac{\overline{A^*}}{\underline{K}}\right) - (\overline{\beta_v}N_1 + \overline{\omega} + \overline{\mu_a})S_a, & x \in \Omega,\ t > t_{13}, \\ \dfrac{\partial S_a}{\partial n} = 0, & x \in \partial\Omega. \end{cases}$$

Then, we can obtain

$$\liminf_{t \to +\infty} S_a(x,t) \geq \frac{\frac{2}{9}\underline{M^*\rho}\left(1 - \frac{\overline{A^*}}{\underline{K}}\right)}{\overline{\beta_v}N_1 + \overline{\omega} + \overline{\mu_a}} := e_2,\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

Thus, there is a $t_{14} > t_{13}$ such that

$$S_a(x,t) \geq \frac{1}{3}e_2,\ \forall t \geq t_{14},\ x \in \bar{\Omega}.$$

From the second equation of system (2.3), we have

$$\begin{cases} \dfrac{\partial I_a}{\partial t} \geq \underline{\beta_v} \times \dfrac{1}{3}e_1 \times \dfrac{1}{3}e_2 - (\overline{\omega} + \overline{\mu_a})I_a, & x \in \Omega,\ t \geq t_{14}, \\ \dfrac{\partial I_a}{\partial n} = 0, & x \in \partial\Omega. \end{cases}$$

Then

$$\liminf_{t \to +\infty} I_a(x,t) \geq \frac{\underline{\beta_v}e_1e_2}{9(\overline{\omega} + \overline{\mu_a})} := e_3,\ uniformly\ for\ all\ x \in \bar{\Omega},$$

which implies that there is a $t_{15} > t_{14}$ such that

$$I_a(x,t) \geq \frac{1}{3}e_3,\ \forall t \geq t_{15},\ x \in \bar{\Omega}.$$

Similarly, it follows from the third and fourth equations of system (2.3) that

$$\liminf_{t \to +\infty} S_m(x,t) \geq \frac{f\underline{\omega}e_2}{3(\overline{b\beta_1} + \overline{\mu_m})} := e_4,\ \liminf_{t \to +\infty} I_m(x,t) \geq \frac{f\underline{\omega}e_3}{3\overline{\mu_m}} := e_5,\ uniformly\ for\ all\ x \in \bar{\Omega}.$$

So, there is a $t_{16} > t_{15}$ such that

$$S_m(x,t) \geq \frac{1}{3}e_4,\ I_m(x,t) \geq \frac{1}{3}e_5,\ \forall t \geq t_{16},\ x \in \bar{\Omega}.$$

By Lemma 3.3, we know

$$\liminf_{t\to\infty} S_h(x,t,\phi) \geq \zeta_0,\ uniformly\ for\ x \in \bar{\Omega}.$$

Hence, there is a $t_{17} > t_{16}$ such that

$$S_h(x,t) \geq \frac{1}{3}\zeta_0,\ \forall t \geq t_{17},\ x \in \bar{\Omega}.$$

Therefore, letting $\zeta_2 = \frac{1}{3}\max\{\zeta_0, \zeta_1, e_1, e_2, e_3, e_4, e_5\}$, we can obtain that (5.8) holds. This completes the proof of Lemma 5.1.

□

Theorem 5.3. *Let*

$$\begin{aligned} &\mathbb{M}_0 := \{(S_a, I_a, S_m, I_m, S_h, I_h, V)^T \in \mathbb{X}_K : I_a(\cdot) \not\equiv 0,\ I_m(\cdot) \not\equiv 0,\ I_h(\cdot) \not\equiv 0,\ V(\cdot) \not\equiv 0\},\\ &\partial\mathbb{M}_0 := \mathbb{X}_K \backslash \mathbb{M}_0. \end{aligned}$$

If $R_0^m > 1$ and $R_0 > 1$, then system (2.3) is uniformly persistent, i.e., and there is a constant $\varsigma > 0$ such that, for any initial value $\phi \in \mathbb{M}_0$, we can obtain

$$\begin{aligned} &\liminf_{t\to+\infty} S_a(x,t,\phi) \geq \varsigma, \liminf_{t\to+\infty} I_a(x,t,\phi) \geq \varsigma, \liminf_{t\to+\infty} S_m(x,t,\phi) \geq \varsigma,\\ &\liminf_{t\to+\infty} I_m(x,t,\phi) \geq \varsigma, \liminf_{t\to+\infty} S_h(x,t,\phi) \geq \varsigma, \liminf_{t\to+\infty} I_h(x,t,\phi) \geq \varsigma,\\ &\liminf_{t\to+\infty} V(x,t,\phi) \geq \varsigma,\ uniformly\ for\ all\ x \in \bar{\Omega}. \end{aligned} \tag{5.9}$$

Proof. The following four steps are taken to prove this result.

Step I M_0 is invariant under $\Psi(t)$.

For any initial value $\phi \in M_0$, from Lemma 3.3, we can obtain

$$I_a(x,t,\phi) > 0, I_m(x,t,\phi) > 0, I_h(x,t,\phi) > 0, V(x,t,\phi) > 0,\ \ \forall\ t > 0,\ x \in \bar{\Omega}.$$

Then $\Psi(t)\phi \in M_0$. So M_0 is invariant under $\Psi(t)$.

Step II For any $\phi \in \partial M_0$, one obtains the $\omega-$limit set $\omega(\phi)$=$\{E_{01}\} \cup \{E_{02}\}$, where $\omega(\phi)$ is the omega limit set of the forward orbit $\gamma^+ := \{\Psi(t)\phi : t \geq 0\}$.

Define

$$\Gamma_\partial := \{\phi \in \partial M_0\ :\ \Psi(t)\phi \in \partial M_0, \forall\ t \geq 0\}.$$

For any given $\phi \in \Gamma_\partial$, we have $\Psi(t)\phi \in \partial M_0, \forall\ t \geq 0$. That is, for every $t \geq 0$, we have

$$I_a(\cdot,t,\phi) \equiv 0\ or\ I_m(\cdot,t,\phi) \equiv 0\ or\ I_h(\cdot,t,\phi) \equiv 0\ or\ V(\cdot,t,\phi) \equiv 0.$$

We first consider the case $I_m(\cdot,t,\phi) \equiv 0$ for all $t \geq 0$. From the sixth equation of system (2.3), we can get that $I_h(x,t,\phi)$ satisfies system (5.3). Thus,

$$\lim_{t\to\infty} I_h(x,t,\phi) = 0,\ \text{uniformly}\quad \text{for all}\ x \in \bar{\Omega}.$$

From the seventh equation of system (2.3), and according to Corollary 4.3 in [42], one has

$$\lim_{t\to\infty} V(x,t,\phi) = 0, \ uniformly\ for\ all\ x \in \bar{\Omega}.$$

Similarly, we can obtain

$$\lim_{t\to\infty} I_a(x,t,\phi) = 0, \ uniformly\ for\ all\ x \in \bar{\Omega}.$$

In addition, $S_h(\cdot,t,\phi)$ in system (2.3) is asymptotic to system (5.2). Thus,

$$\lim_{t\to\infty} S_h(x,t,\phi) = H(x), \ uniformly\ for\ all\ x \in \bar{\Omega}.$$

In the case $I_m(\cdot,t,\phi) \not\equiv 0$ for all $t \geq 0$, $S_a(\cdot,t,\phi)$ and $S_m(\cdot,t,\phi)$ are as follows.

(i) $S_a(\cdot,t,\phi) \equiv 0$ and $S_m(\cdot,t,\phi) \equiv 0$ for all $t \geq 0$.

(ii) $S_a(\cdot,t,\phi) \equiv 0$ for all $t \geq 0$, and $S_m(\cdot,t_{21},\phi) \not\equiv 0$ for some $t_{21} > 0$.
In this case, from the third equation of system (2.3), we can get

$$\lim_{t\to\infty} S_m(x,t,\phi) = 0, \ uniformly\ for\ all\ x \in \bar{\Omega}.$$

(iii) $S_m(\cdot,t,\phi) \equiv 0$ for all $t \geq 0$, and $S_a(\cdot,t_{22},\phi) \not\equiv 0$ for some $t_{22} > 0$.
In this case, from the first equation of system (2.3), we can obatin

$$\lim_{t\to\infty} S_a(x,t,\phi) = 0, \ uniformly\ for\ all\ x \in \bar{\Omega}.$$

(iv) $S_a(\cdot,t_{23},\phi) \not\equiv 0$ and $S_m(\cdot,t_{23},\phi) \not\equiv 0$ for some $t_{23} > 0$.
In this case, $(S_a(x,t,\phi), S_m(x,t,\phi))$ in system (2.3) is asymptotic to system (3.8). From Lemma 4.2, we have

$$\lim_{t\to\infty}(S_a(x,t,\phi),\ S_m(x,t,\phi)) = (A^*(x),\ M^*(x)), \ uniformly\ for\ all\ x \in \bar{\Omega}.$$

Thus, we obtain $\omega(\phi)$=$\{E_{01}\} \cup \{E_{02}\}$.

Next, we assume $I_m(\cdot,t_{24},\phi) \not\equiv 0$ for some $t_{24} > 0$. Form Lemma 4.2, one has $I_m(\cdot,t,\phi) > 0$ for all $t > t_{24}$. Then, we get

$$I_a(\cdot,t,\phi) \equiv 0\ or\ I_h(\cdot,t,\phi) \equiv 0\ or\ V(\cdot,t,\phi) \equiv 0, \ for\ all\ t > t_{24}.$$

Here, we only show the case $I_a(\cdot,t,\phi) \equiv 0$ for all $t > t_{24}$. It follows from the second equation of system (2.3) that

$$S_a(\cdot,t,\phi) \equiv 0\ or\ V(\cdot,t,\phi) \equiv 0, \ for\ all\ t > t_{24}.$$

If $S_a(\cdot,t,\phi) \equiv 0,\ for\ all\ t > t_{24}$, then, from the first equation of system (2.3), we have $I_m(\cdot,t,\phi) \equiv 0,\ for\ all\ t > t_{24}$, which contradicts our assumption.

If $V(\cdot,t,\phi) \equiv 0,\ for\ all\ t > t_{24}$, then, from the sixth equation of system (2.3), one has $I_m(\cdot,t,\phi) \equiv 0,\ for\ all\ t > t_{24}$, which contradicts our assumption. Thus, the **step II** is proved.

Step III $W^s(\{E_{01}(x)\}) \cap M_0 = \emptyset$ and $W^s(\{E_{02}(x)\}) \cap M_0 = \emptyset$. In this step, we will show the following two claims.

Claim 1 $E_{01}(x)$ is a uniform weak repeller for M_0. That is, there exists $\varepsilon_1 > 0$ such that

$$\limsup_{t\to+\infty} \|\Psi(t)\phi - E_{01}(x)\| \geq \varepsilon_1, \ for\ all\ \phi \in M_0. \tag{5.10}$$

Claim 2 $E_{02}(x)$ is a uniform weak repeller for M_0. That is, there exists $\varepsilon_2 > 0$ such that

$$\limsup_{t\to+\infty} \|\Psi(t)\phi - E_{02}(x)\| \geq \varepsilon_2, \ for\ all\ \phi \in M_0. \tag{5.11}$$

Here, we just prove *Claim 1*. *Claim 2* can be similarly proven.

If (5.10) does not hold, then

$$\limsup_{t\to+\infty} \|\Psi(t)\tilde{\phi} - E_{01}(x)\| < \varepsilon_1, \ for\ some\ \tilde{\phi} \in M_0. \tag{5.12}$$

That is, there exists $t_{25} > 0$ such that

$$\begin{aligned}
&0 < S_a(x,t,\tilde{\phi}) < \varepsilon_1, \ 0 < I_a(x,t,\tilde{\phi}) < \varepsilon_1, \ 0 < S_m(x,t,\tilde{\phi}) < \varepsilon_1, \ 0 < I_m(x,t,\tilde{\phi}) < \varepsilon_1, \\
&H(x) - \varepsilon_1 < S_h(x,t,\tilde{\phi}) < H(x) + \varepsilon_1, \ 0 < I_h(x,t,\tilde{\phi}) < \varepsilon_1, \ 0 < V(x,t,\tilde{\phi}) < \varepsilon_1, \ for\ \forall\ t \geq t_{25}, \ x \in \Omega.
\end{aligned} \tag{5.13}$$

Thus, $S_a(x,t,\tilde{\phi})$ and $S_m(x,t,\tilde{\phi})$ satisfy

$$\begin{cases}
\dfrac{\partial S_a}{\partial t} \geq \rho(x)\left(1 - \dfrac{2\varepsilon_1}{K(x)}\right) S_m - (\beta_v(x)\varepsilon_1 + \omega(x) + \mu_a(x)) S_a, & x \in \Omega, \ t \geq t_{25}, \\
\dfrac{\partial S_m}{\partial t} \geq d_m \triangle S_m + \omega(x) S_a - \left(\mu_m(x) + \dfrac{\alpha(x)\beta_1(x)}{H(x)}\varepsilon_1\right) S_m, & x \in \Omega, \ t \geq t_{25}, \\
\dfrac{\partial M}{\partial n} = 0, & x \in \partial\Omega, \ t \geq t_{25}.
\end{cases}$$

Consider the following auxiliary linear system

$$\begin{cases}
\dfrac{\partial v}{\partial t} = \tilde{B}_m(\varepsilon_1) v, & x \in \Omega, \ t \geq t_{25}, \\
\dfrac{\partial v_2}{\partial n} = 0, & x \in \partial\Omega, \ t \geq t_{25},
\end{cases} \tag{5.14}$$

where $v = (v_1, v_2)^T$, and

$$\tilde{B}_m(\varepsilon_1) = \begin{bmatrix} -(\beta_v(x)\varepsilon_1 + \omega(x) + \mu_a(x)) & \rho(x)\left(1 - \frac{2\varepsilon_1}{K(x)}\right) \\ \omega(x) & d_m\triangle - \left(\mu_m(x) + \frac{\alpha(x)\beta_1(x)}{H(x)}\varepsilon_1\right) \end{bmatrix}.$$

We know $\tilde{B}_m(0) = B_m$, where B_m is defined by (4.3). From Lemma 4.1, if $R_0^m > 1$, then $\lambda_m^* = s(B_m) > 0$. $\tilde{B}_m(\varepsilon_1)$ is continuous for small ε_1. So, when ε_1 is small enough, we have $s(\tilde{B}_m(\varepsilon_1)) > 0$. Denote $\lambda_{m\varepsilon_1}^* := s(\tilde{B}_m(\varepsilon_1))$. Obviously, $\lambda_{m\varepsilon_1}^* > 0$.

Let $(\tilde{\varphi}_1, \tilde{\varphi}_2)$ be the positive eigenfunction corresponding to $\lambda_{m\varepsilon_1}^*$. Then auxiliary system (5.14) admits a solution $(v_1(x,t), v_2(x,t)) = e^{\lambda_{m\varepsilon_1}^*}(\tilde{\varphi}_1, \tilde{\varphi}_2)$. Due to $S_a(x,t,\tilde{\phi}) > 0$, $S_m(x,t,\tilde{\phi}) > 0$ for $\forall\ t \geq t_{25}$, there exists $\varrho_1 > 0$ such that

$$(S_a(x,t_{25},\tilde{\phi}), S_m(x,t_{25},\tilde{\phi})) \geq \varrho_1(\tilde{\varphi}_1, \tilde{\varphi}_2).$$

According to the comparison principle, we can obtain

$$(S_a(x,t,\tilde{\phi}), S_m(x,t,\tilde{\phi})) \geq \varrho_1 e^{\lambda^*_{m\varepsilon_1}(t-t_{25})}(\tilde{\varphi}_1, \tilde{\varphi}_2), \ for\ \forall\ t \geq t_{25},\ x \in \bar{\Omega}.$$

Since $\lambda^*_{m\varepsilon_1} > 0$, we get

$$\lim_{t\to\infty} S_a(\cdot,t,\tilde{\phi}) = +\infty,\ \ \lim_{t\to\infty} S_m(\cdot,t,\tilde{\phi}) = +\infty,$$

which contradicts with (5.13).

The above discussion implies that $\{E_{01}(x)\}$ and $\{E_{02}(x)\}$ are isolated invariant sets in M_0, and $W^s(\{E_{01}(x)\}) \cap M_0 = \emptyset$, $W^s(\{E_{02}(x)\}) \cap M_0 = \emptyset$. Clearly, every orbit in Γ_∂ converges to either $E_{01}(x)$ or $E_{02}(x)$, and there are no subsets of $\{E_{01}(x), E_{02}(x)\}$ forms a cycle in Γ_∂. From Theorem 4.1 in [43], system (2.3) is uniformly persistent if $R_0^m > 1$ and $R_0 > 1$.

Step IV Define a continues function $\mathbf{g}:\ \mathbb{X}_K \to [0,+\infty)$ with

$$\mathbf{g}(\phi) := \min\left\{\min_{x\in\bar{\Omega}}\phi_2(x), \min_{x\in\bar{\Omega}}\phi_4(x), \min_{x\in\bar{\Omega}}\phi_6(x), \min_{x\in\bar{\Omega}}\phi_7(x)\right\},\ \forall\ \phi \in \mathbb{X}_K.$$

It follows from Lemma 3.3 that $\mathbf{g}^{-1}(0,+\infty) \subseteq M_0$. If $\mathbf{g}(\phi) > 0$, or $\phi \in M_0$ with $\mathbf{g}(\phi) = 0$, then $\mathbf{g}(\Psi(t)\phi) > 0$, $\forall\ t > 0$. That is, $\mathbf{g}$ is a generalized distance fuction for the semiflow $\Psi(t)$ (see [44]). From Lemma 3.2 and according to Theorem 3 in [44], there exists a $\varrho_2 > 0$ such that

$$\min_{\psi\in\omega(\phi)} > \varrho_2,\ \forall\ \phi \in M_0.$$

Thus, $\liminf_{t\to+\infty} I_h(\cdot,t,\phi) \geq \varrho_2, \forall\ \phi \in M_0$. From Lemma 5.1, there exists some $\varsigma > 0$ such that (5.9) holds. This completes the proof.

□

Remark 3. Biologically, Theorem 5.3 shows that mosquito population and the disease will persist when the basic offspring number $R_0^m > 1$ and basic reproduction number $R_0 > 1$.

6. Numerical simulations

In this section, we implement numerical simulations to confirm the analytic results. For the sake of convenience, we concentrate on one dimensional domain Ω, which can be taken, without loss of generality, to be $(0,\pi)$. For the sake of simplicity, we focus on model (2.3) and fix some coefficients and functions as follows: $H(x) = 100$, $K(x) = 500$, $\vartheta(x) = 5$, $l = 1/2$, $\omega(x) = 0.05$, $\mu_a(x) = 0.15$, $\mu_m(x) = 0.05$, $\mu_h(x) = \frac{1}{75\times365}$, $\Lambda(x) = H(x)\times\mu_h(x)$, $r(x) = \frac{1}{7}$, $a(x) = 0.3$, $\theta(x) = 0.1$, $\delta(x) = 0.3$, $d_m = 0.01$, $d_h = 0.1$,

$$\beta_1(x) = 0.12(1+\cos(2x)),\quad \beta_2(x) = 0.15(1+\cos(2x)).$$

We set initial data

$$\begin{aligned}
&S_a(x,0) = 200 + \sin(x) - 200\cos(4x),\ I_a(x,0) = 5 + 0.1\sin(x) - 5\cos(4x),\\
&S_h(x,0) = 21 + 0.5\sin(x) - 20\cos(4x),\ I_h(x,0) = 0.5 + 0.01\sin(x) - 0.5\cos(4x),\\
&S_m(x,0) = \omega(x)S_a(x,0),\quad I_m(x,0) = \omega(x)I_a(x,0),\quad V(x,0) = \theta(x)I_h(x,0),\ x \in (0,\pi).
\end{aligned}\tag{6.1}$$

Next, we will change the parameters s and $\beta_v(x)$ and then observe the longtime behavior of the solution to model (2.3).

Example 6.1. Choose $s = 0.04$ and $\beta_v(x) = 0.00035(1 + \cos(2x))$. It follows from (4.5) that $R_0^m = 0.52 < 1$. Theorem 5.1 shows that the disease free equilibrium $E_{01}(x)$ is globally attractive when $R_0^m < 1$. We can confirm this in Figure 2. Figure 2 shows that the evolutions of $S_a(x,t), S_m(x,t), I_a(x,t), I_m(x,t), I_h(x,t), V(x,t)$ decay to zero. Biologically, mosquito population will vanish, and Zika virus will eradicate in human population and contaminated aquatic environment.

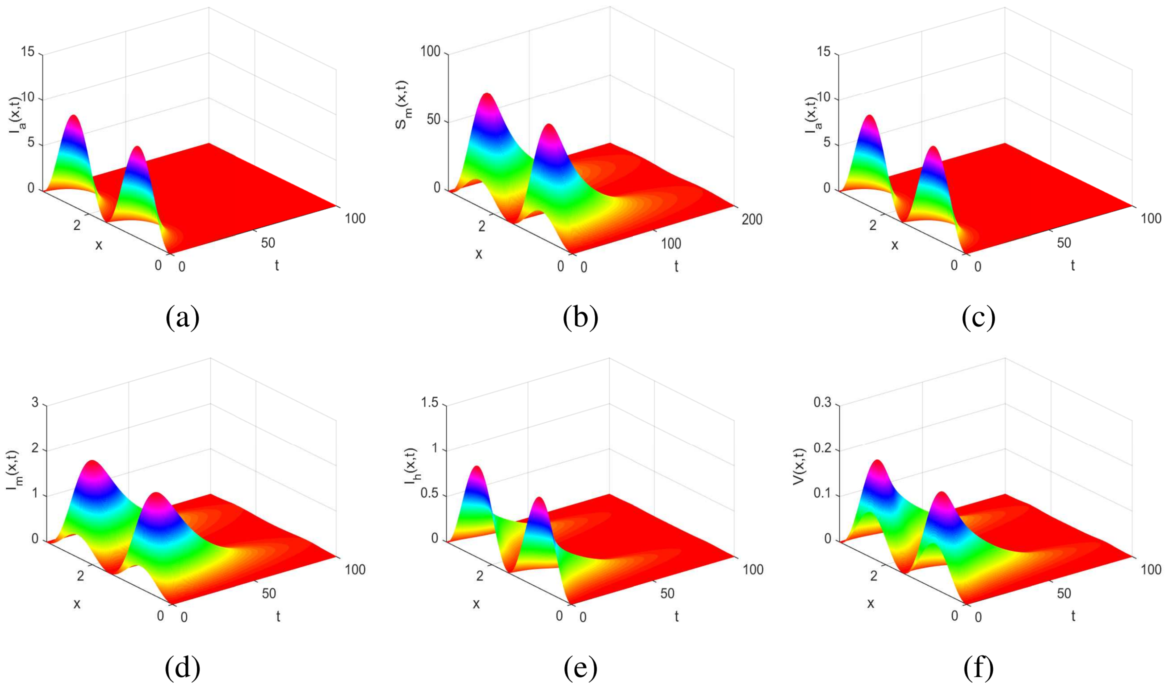

Figure 2. The evolution diagram of numerical solutions of model (2.3) for $R_0^m < 1$. Diagrams (a)-(f) show that the evolutions of $S_a(x,t), S_m(x,t), I_a(x,t), I_m(x,t), I_h(x,t), V(x,t)$ decay to zero. It mean that mosquito population will vanish, and Zika virus will eradicate in human population and contaminated aquatic environment.

Example 6.2. Choose $s = 0.4$ and $\beta_v(x) = 0.00035(1 + \cos(2x))$. It follows from (4.5) that $R_0^m = 5 > 1$. According to the method of calculating the regeneration numbers in [38], we can get $R_0 = 0.98 < 1$. Theorem 5.2 shows that the disease free equilibrium $E_{02}(x)$ is globally attractive when $R_0^m > 1$ and $R_0 < 1$. We can confirm this in Figure 3. Figure 3 shows that the evolutions of $S_a(x,t), S_m(x,t)$ tend to steady state $A^*(x), M^*(x)$, and the evolutions of diseased compartments $I_a(x,t), I_m(x,t), I_h(x,t), V(x,t)$ decay to zero. Biologically, mosquito population is present, and Zika virus will eradicate in mosquito population, human population and contaminated aquatic environment.

Example 6.3. Choose $s = 0.4$ and $\beta_v(x) = 0.035(1+\cos(2x))$. It follows from (4.5) that $R_0^m = 5 > 1$. According to the method of calculating the regeneration numbers in [38], we can get $R_0 = 2.8 > 1$. Theorem 5.3 shows that system (2.3) is uniformly persistent when $R_0^m > 1$ and $R_0 > 1$. We can confirm this in Figure 4. Figure 4 shows that the evolutions of $S_a(x,t), S_m(x,t), I_a(x,t), I_m(x,t), I_h(x,t), V(x,t)$ keep positive. Biologically, mosquito population is present, and Zika virus will persist in mosquito population, human population and contaminated aquatic environment.

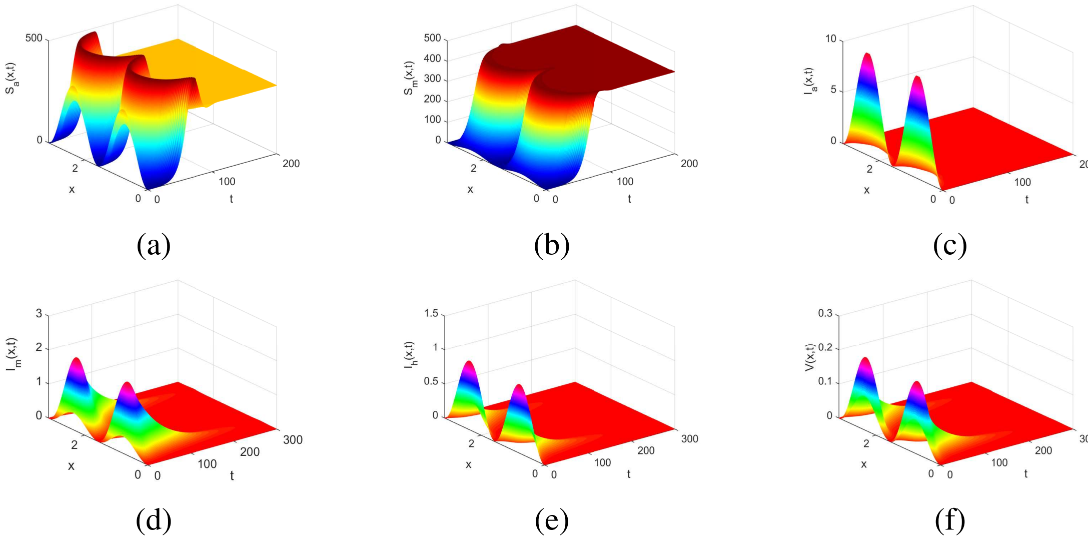

Figure 3. The evolution diagram of numerical solutions of model (2.3) for $R_0^m > 1$ and $R_0 < 1$. Diagrams (a) and (b) show that the evolutions of $S_a(x,t), S_m(x,t)$ tend to steady state $A^*(x), M^*(x)$, respectively. Diagrams (c)–(f) show that the evolutions of diseased compartments $I_a(x,t), I_m(x,t), I_h(x,t), V(x,t)$ decay to zero. It mean that mosquito population is present, and Zika virus will eradicate in mosquito population, human population and contaminated aquatic environment.

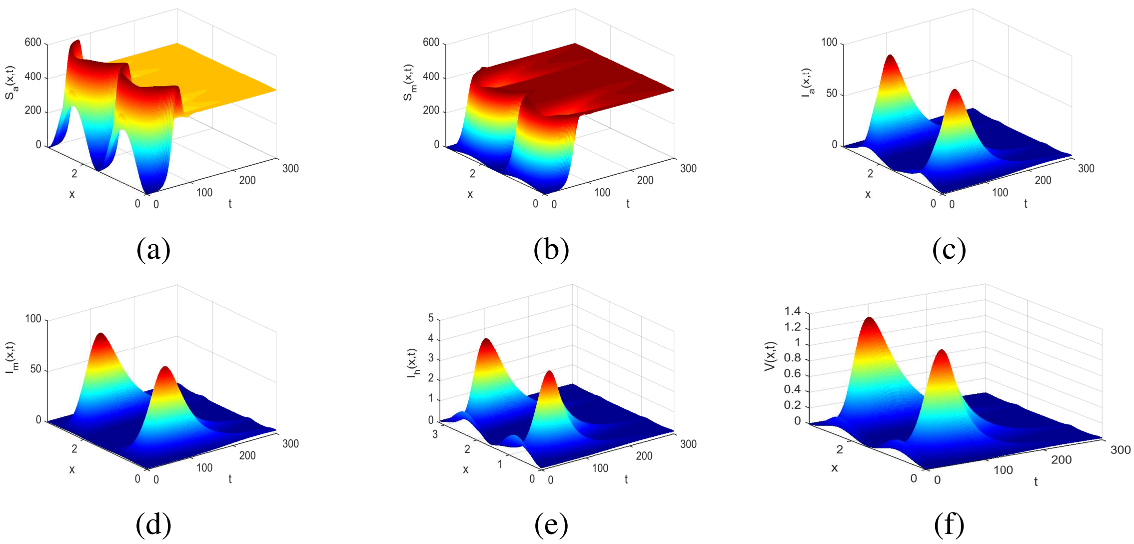

Figure 4. The evolution diagram of numerical solutions of model (2.3) for $R_0^m > 1$ and $R_0 > 1$. Diagrams (a)–(f) show that the evolutions of $S_a(x,t), S_m(x,t), I_a(x,t), I_m(x,t), I_h(x,t), V(x,t)$ keep positive. It mean that mosquito population is present, and Zika virus will persist in mosquito population, human population and contaminated aquatic environment.

7. Conclusions

The main contribution of this study, based on experimental proving evidence [25], is that we propose a new Zika model by introducing the environment transmission route in a spatial heterogeneous environment. In contrast to [12, 14, 24], we consider environment transmission (human-environment-mosquito-human) route in Zika model. From Figures 3 and 4, we can get that increasing environment transmission rate $\beta_v(x)$ can induce the outbreak of Zika. Therefore, the environment transmission route is indispensable. Our work is an extension of previous mathematical Zika models that the transmission of Zika virus involves both mosquito-borne transmission (human-mosquito-human) routes [12, 14, 24]. In fact, environmental transmission route in our model is similar to other waterborne disease models, such as cholera [45]. So, our model analysis can also be applied to other waterborne diseases.

We derive a biologically meaningful threshold indexes, the basic offspring number R_0^m and basic reproduction number R_0 by the theory developed by Wang and Zhao [38], which is characterized as the spectral radius of the next generation operator. Then, we prove that if $R_0^m < 1$, then both mosquitoes and Zika virus will become vanish. If $R_0^m > 1$ and $R_0 < 1$, then mosquitoes will persist and Zika virus will die out. If $R_0^m > 1$ and $R_0 > 1$, then mosquitoes and Zika virus are persistent presences. Finally, numerical simulations conform these results.

Our current study has some limitations. In our model, we do not consider sexual transmission route, but it is indeed an important route of spreading of Zika virus [46]. Future, we will study a Zika model which incorporates mosquito-borne transmission, sexual transmission and environment transmission threes routes. In addition, in our model, we assume same diffusive coefficients d_m for both S_m and I_m, same d_h for all of S_h, I_h and R_h. However, due to mobility for S_m and I_m, S_h, I_h and R_h is different, studies of different diffusion coefficients have more realistic implications. Yin [47] studied a mathematical model for an infectious disease such as Cholera with different diffusion coefficients. By the delicate theory of elliptic and parabolic equations, global asymptotic behavior of the solution was obtained. This work makes progress toward the case of different diffusive coefficients. Therefore, it motivates us to consider different diffusive coefficients into our model. We leave it for future investigation.

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Conflict of interest

The authors declare that they have no competing interests.

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