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Research article

Estimation of finite population mean using dual auxiliary variable for non-response using simple random sampling

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Abstract: This paper addresses the issue of estimating the population mean for non-response using simple random sampling. A new family of estimators is proposed for estimating the population mean with auxiliary information on the sample mean and the rank of the auxiliary variable. Bias and mean square errors of existing and proposed estimators are obtained using the first order of measurement. Theoretical comparisons are made of the performance of the proposed and existing estimators. We show that the proposed family of estimators is more efficient than existing estimators in the literature under the given constraints using these theoretical comparisons.

Keywords: population mean; auxiliary information; mean square error; bias; non-response; numerical comparisons
Mathematics Subject Classification: 65D05

1. Introduction

In survey sampling, the appropriate use of auxiliary information is known to enhance the accuracy of an estimator of the unknown population parameter. This information (auxiliary) can be used to select a random sample using SRSWR or SRSWOR. Auxiliary information gives us a sort of technique in terms of ratio, product, regression, and other methods, it is therefore necessary to have a representative part of the population, when the population of interest is more homogeneous, than simple random sampling can be used to select units. A considerable amount of work was done on estimating the population mean by simple random sampling, a number of important references include [2, 5, 7, 11–13, 16–20, 22–24, 26, 27, 30, 32] and the references cited therein, have suggested different types of estimators to estimate the population mean and population distribution function in the presence of non-response.

As a practical matter, one of the main problems with surveys is that they suffer from non-response, non-response has a lot of ways to happen. Examples are language problems, non-availability of response, incorrect return address and input from another person, censorship or clustering is a problem across several data. The statistician has recognized for quite few time that ignoring the stochastic nature of incompleteness or non-response may change the nature of the data. Several factors affect the non-response rate for a survey, some of these factors are the type of information collected, the official status of the investigating agency, the extent of the publicity, legal requirements of respondents, the duration of the enumerator's visit and the length of withdrawal period etc.

A great deal of work has been done on the estimation of the population mean to check non-response bias and increase efficiency of estimators by different authors. The issue of non-response in sample surveys is more common and prevalent in mail surveys than in special interview surveys. [10] was the first to address the issue of incomplete samples in the postal or telephone surveys. For certain related work, we refer to [1–3, 12–15, 17, 18, 20, 21, 23–26, 28, 31, 32] and the references cited therein.

On the line of [11] a new family of estimators is proposed for the estimation of population mean in the presence of non-response. We will prove theoretically and numerically that the proposed family of estimators is more precise than the existing estimators.

The rest of the paper is set out as follows: In Section 2, some notations are introduced by SRS with non-responding. In Section 3, the existing estimators examined for the two non-response situations. A new family of estimators is presented in Section 4 under both non-response situations using simple random sampling. The existing and proposed estimators are theoretically compared in Section 5. In Section 6, the existing and proposed family of estimators are compared numerically. Section 7 condenses the principal discovery and culminate the document.

2. Notations

Suppose $\Omega = \{U_1, U_2, ..., U_N\}$ denotes be a finite population of N distinct units that is bisect into two groups, respondents and non-respondents, having sizes N_1 and N_2, where $N = N_1 + N_2$. Thus we denote $\Omega_1 = \{U_1, U_2, ..., U_{N_1}\}$ for the response group and $\Omega_2 = \{U_1, U_2, ..., U_{N_2}\}$ for the non-response group. In order to estimate the population mean, a sample of n is taken from the underlying population by simple random sampling without replacement (SRSWOR), and for which units n_1 are responding and $n_2 = n - n_1$ are not responding. It is also assumed that the sample size n_1 is drawn from the response group of Ω_1 and n_2 is drawn from the non-response group of Ω_2. Moreover a sample of size $r = n_2/k$ units, where $k > 1$ is drawn by simple random sampling without replacement from n_2, and the temporal response is obtained from all r units.

Let Y, X, Z, be the study, auxiliary and ranks of the auxiliary variable.

$\bar{Y} = \sum_{i=1}^{N} Y_i/N$, $\hat{\bar{Y}} = \sum_{i=1}^{n} Y_i/n$: The population and sample mean of Y.

$\bar{X} = \sum_{i=1}^{N} X_i/N$, $\hat{\bar{X}} = \sum_{i=1}^{n} X_i/n$: The population and sample mean of X.

$\bar{Z} = \sum_{i=1}^{N} Z_i/N$, $\hat{\bar{Z}} = \sum_{i=1}^{n} Z_i/n$: The population and sample mean of Z.

$\bar{Y}_{(2)} = \sum_{i=1}^{N_2} Y_i/N_2$: The population mean of Y for non-response group.

$\bar{X}_{(2)} = \sum_{i=1}^{N_2} X_i/N_2$: The population mean of X for non-response group.

$\bar{Z}_{(2)} = \sum_{i=1}^{N_2} Z_i/N_2$: The population mean of Z for non-response group.

$\hat{\bar{Y}}_{(1)} = \sum_{i=1}^{n_1} Y_i/n_1$ denote the sample mean based on n_1 responding units out of n units.

$\hat{\bar{X}}_{(1)} = \sum_{i=1}^{n_1} X_i/n_1$ denote the sample mean based on n_1 responding units out of n units.

$\hat{\bar{Z}}_{(1)} = \sum_{i=1}^{n_1} Z_i/n_1$ be the sample mean based on n_1 responding units out of n units.

$\hat{\bar{Y}}_{(2r)} = \sum_{i=1}^{r} Y_i/r$ be the sample mean based on r reacting units out of n_2 non-response units.

$\hat{\bar{X}}_{(2r)} = \sum_{i=1}^{r} X_i/r$ be the sample mean based on r reacting units out of n_2 non-response units.

$\hat{\bar{Z}}_{(2r)} = \sum_{i=1}^{r} Z_i/r$ denote the sample mean based on r reacting units out of n_2 non-response units.

$S_Y^2 = \sum_{i=1}^{N} (Y_i - \bar{Y})^2/(N-1)$, $S_X^2 = \sum_{i=1}^{N} (X_i - \bar{X})^2/(N-1)$. $S_Z^2 = \sum_{i=1}^{N} (Z_i - \bar{Z})^2/(N-1)$: The population variance of Y, X, and Z.

$S_{Y_2}^2 = \sum_{i=1}^{N_2} (Y_i - \bar{Y}_2)^2/(N_2-1)$, $S_{X_2}^2 = \sum_{i=1}^{N_2} (X_i - \bar{X}_2)^2/(N_2-1)$, $S_{Z_2}^2 = \sum_{i=1}^{N_2} (Z_i - \bar{Z}_2)^2/(N_2-1)$: The population variance of Y, X, and Z for non-response group.

$C_Y = S_Y/\bar{Y}$, $C_X = S_X/\bar{X}$, $C_Z = S_Z/\bar{Z}$: The population coefficient of variation of Y, X and Z.

$C_{Y(2)} = S_{Y(2)}/\bar{Y}_{(2)}$, $C_{X(2)} = S_{X(2)}/\bar{X}_{(2)}$, $C_{Z(2)} = S_{Y(2)}/\bar{Z}_{(2)}$: Be the population coefficient of variation of Y, X and Z for non-response group.

$S_{YX} = \sum_{i=1}^{N}(Y_i-\bar{Y})(X_i-\bar{X})/(N-1)$, $S_{YZ} = \sum_{i=1}^{N}(Y_i-\bar{Y})(Z_i-\bar{Z})/(N-1)$, $S_{XZ} = \sum_{i=1}^{N}(X_i-\bar{X})(Z_i-\bar{Z})/(N-1)$: The population covariance between (Y, X), (Y, Z), and (X, Z).

$S_{Y_2X_2} = \sum_{i=1}^{N_2}(Y_i-\bar{Y}_2)(X_i-\bar{X}_2)/(N_2-1)$, $S_{Y_2Z_2} = \sum_{i=1}^{N_2}(Y_i-\bar{Y}_2)(Z_i-\bar{Z}_2)/(N_2-1)$, $S_{X_2Z_2} = \sum_{i=1}^{N_2}(X_i-\bar{X}_2)(Z_i-\bar{Z}_2)/(N_2-1)$: The population covariance between (Y, X), (Y, Z), and (X, Z) for non-response group.

$\rho_{YX} = S_{YX}/(S_Y S_X)$, $\rho_{YZ} = S_{YZ}/(S_Y S_Z)$, $\rho_{XZ} = S_{XZ}/(S_X S_Z)$: Be the population correlation coefficient between (Y, X), (Y, Z), and (X, Z).

$\rho_{Y_2X_2} = S_{Y_2X_2}/(S_{Y_2}S_{X_2})$, $\rho_{Y_2Z_2} = S_{Y_2Z_2}/(S_{Y_2}S_{Z_2})$, $\rho_{X_2Z_2} = S_{X_2Z_2}/(S_{X_2}S_{Z_2})$: The population correlation coefficient between (Y, X), (Y, Z), and (X, Z) for non-response group.

$R^2_{Y.XZ} = (\rho^2_{YX} + \rho^2_{YZ} - 2\rho_{YX}\rho_{YZ}\rho_{XZ})/(1-\rho^2_{XZ})$: The population coefficient of multiple determination of Y on X and Z.

$R^2_{Y.XZ(2)} = (\rho^2_{YX(2)} + \rho^2_{YZ(2)} - 2\rho_{YX(2)}\rho_{YZ(2)}\rho_{XZ(2)})/(1-\rho^2_{XZ(2)})$: The population coefficient of multiple determination of Y on X and Z for non-response group.

The population mean Y may be written as such

$$\bar{Y} = W_1\hat{\bar{Y}}_{(1)} + W_2\bar{Y}_{(2)}, \tag{2.1}$$

$$\bar{X} = W_1\hat{\bar{X}}_{(1)} + W_2\bar{X}_{(2)}, \tag{2.2}$$

$$\bar{Z} = W_1\hat{\bar{Z}}_{(1)} + W_2\bar{Z}_{(2)}, \tag{2.3}$$

where $W_j = N_j/N$, $\bar{Y}_j = \sum_{i=1}^{N_j} Z_i/N_j$, for $j = 1, 2.$, $\bar{X}_j = \sum_{i=1}^{N_j} X_i/N_j$ and $\bar{Z}_j = \sum_{i=1}^{N_j} Z_i/N_j$. Following [10, 12] have suggested an unbiased estimator of $\bar{Y}$ under non-response, which is given by

$$\hat{\bar{Y}}^* = w_1\hat{\bar{Y}}_{(1)} + w_2\hat{\bar{Y}}_{(2r)}$$

and

$$\operatorname{Var}\left(\hat{\bar{Y}}^*\right) = \lambda S_1^2 + \lambda_2 S_{1(2)}^2, \tag{2.4}$$

where $w_j = n_j/n$ for j =1,2, $\lambda = (1/n - 1/N)$ and $\lambda_2 = W_2(k-1)$.

Similarly

$$\hat{\bar{X}}^* = w_1\hat{\bar{X}}_{(1)} + w_2\hat{\bar{X}}_{(2r)} \text{ and } \hat{\bar{Z}}^* = w_1\hat{\bar{Z}}_{(1)} + w_2\bar{Z}_{(2r)},$$

are unbiased estimators of $\bar{X}$ and $\bar{Z}$ respectively under non-response with corresponding variances

$$\operatorname{Var}\left(\hat{\bar{X}}^*\right) = \lambda S_2^2 + \lambda_2 S_{2(2)}^2,$$
$$\operatorname{Var}\left(\hat{\bar{Z}}^*\right) = \lambda S_3^2 + \lambda_2 S_{3(2)}^2,$$

respectively.

In order to obtain the properties of the proposed estimator, we consider the following relative error terms.

Let $\xi_0^* = (\hat{\bar{Y}}_H - \bar{Y})/\bar{Y}$, $\xi_1^* = (\hat{\bar{X}}_H - \bar{X})/\bar{X}$, $\xi_2^* = (\hat{\bar{Z}}_H - \bar{Z})/\bar{Z}$, $\xi_1 = (\hat{\bar{X}}_H - \bar{X})/\bar{X}$, and $\xi_2 = (\hat{\bar{Z}}_H - \bar{Z})/\bar{Z}$, such that $E(\xi_i^*) = E(\xi_i) = 0$ for $i = 0, 1, 2$, and for $i = 1, 2$. Where $E(\cdot)$ represents the mathematical expectation of $(\cdot)$. Let

$$V_{rst} = E\left[e_0^r\ e_1^s\ e_2^t\right] \text{ and } V_{rst}^* = E\left[e_0^{*r}\ e_1^{*s}\ e_2^{*t}\right],$$

where $r, s, t, u = 0, 1, 2$. Here,

$$E(\xi_0^{*2}) = (\theta S_Y^2 + \theta_2 S_{Y_2}^2)/(\bar{Y}^2) = V_{200}, \quad E(\xi_0^*\xi_1^*) = (\theta\rho_{YX}S_YS_X + \theta_2\rho_{Y_2X_2}S_{Y_2}S_{X_2})/(\bar{Y}\bar{X}) = V_{110},$$
$$E(\xi_1^{*2}) = (\theta S_X^2 + \theta_2 S_{X_2}^2)/(\bar{X}^2) = V_{020}, \quad E(\xi_0^*\xi_2^*) = (\theta\rho_{YZ}S_YS_Z + \theta_2\rho_{Y_2Z_2}S_{Y_2}S_{Z_2})/(\bar{Y}\bar{Z}) = V_{101},$$
$$E(\xi_2^{*2}) = (\theta S_Z^2 + \theta_2 S_{Z_2}^2)/(\bar{Z}^2) = V_{002}, \quad E(\xi_1^*\xi_2^*) = (\theta\rho_{XZ}S_XS_Z + \theta_2\rho_{X_2Z_2}S_{X_2}S_{Z_2})/(\bar{X}\bar{Z}) = V_{011}$$
$$E(\xi_1^2) = (\theta S_X^2)/(\bar{X}^2) = \Psi_{020}, \quad E(\xi_0^*\xi_1) = (\theta\rho_{YX}S_YS_X)/(\bar{Y}\bar{X}) = \Psi_{110},$$
$$E(\xi_2^2) = (\theta S_Z^2)/(\bar{Z}^2) = \Psi_{002}, \quad E(\xi_0^*\xi_2) = (\theta\rho_{YZ}S_YS_Z)/(\bar{Y}\bar{Z}) = \Psi_{101},$$
$$E(\xi_1\xi_2) = (\theta\rho_{XZ}S_XS_Z)/(\bar{X}\bar{Z}) = \Psi_{011},$$

where $\theta = (1/n - 1/N)$ and $\theta_2 = W_2(k-1)/n$.

Usually in case of non-response, two situations are more likely to happen, namely non-response on Y only (say Situation-I) and non-response on both Y, X and Z (say Situation-II).

3. Existing estimators

In this portion, some existing estimates of the population mean for non-response are briefly reviewed for both situations.

3.1. Situation-I

When non-response occurs in only one study variable, say Y

(1) The estimator of the typical ratio of the $\bar{Y}$ is given as:

$$\hat{\hat{Y}}_R^* = \hat{\hat{Y}}_H^* \left(\frac{\bar{X}}{\hat{\hat{X}}_H} \right). \tag{3.1}$$

The properties of $\hat{\hat{Y}}_R$, are given by:

$$\begin{aligned} \text{Bias}(\hat{\hat{Y}}_R^*) &\cong \bar{Y}\left(V_{020} - V_{110}\right), \\ \text{MSE}(\hat{\hat{Y}}_R^*) &\cong \bar{Y}^2\left(V_{200} + V_{020} - 2V_{110}\right), \end{aligned} \tag{3.2}$$

respectively.

(2) The typical product estimator $\bar{Y}$ is given as:

$$\hat{\hat{Y}}_P^* = \hat{\hat{Y}}_H^* \left(\frac{\hat{\hat{X}}_H}{\bar{X}} \right). \tag{3.3}$$

The properties of $\hat{\hat{Y}}_P^*$, are given as:

$$\begin{aligned} \text{Bias}(\hat{\hat{Y}}_P^*) &= \bar{Y}V_{110}, \\ \text{MSE}(\hat{\hat{Y}}_P^*) &\cong \bar{Y}^2\left(V_{200} + V_{020} + 2V_{110}\right). \end{aligned} \tag{3.4}$$

(3) The typical difference estimator for the $\bar{Y}$ is given as:

$$\hat{\hat{Y}}_D^* = \hat{\hat{Y}}_H^* + d(\bar{X} - \hat{\hat{X}}_H). \tag{3.5}$$

The minimal variance of $\hat{\hat{Y}}_D^*$ at $d_{(\text{opt})} = (\bar{Y}V_{110})/(\bar{X}V_{020})$ is given as:

$$\text{Var}_{\min}(\hat{\hat{Y}}_D^*) \cong \frac{\bar{Y}^2(V_{200}V_{020} - V_{110}^2)}{V_{020}}. \tag{3.6}$$

Here in (3.6) can be written as:

$$\text{Var}_{\min}(\hat{\hat{Y}}_D^*) \cong \bar{Y}^2 V_{200}(1 - \rho_{YX}^2). \tag{3.7}$$

(4) Following [27], a difference-type estimator of $\bar{Y}$ is

$$\hat{\hat{Y}}_{R,D}^* = k_1\hat{\hat{Y}}_H^* + k_2(\bar{X} - \hat{\hat{X}}_H). \tag{3.8}$$

The properties of $\hat{\hat{Y}}_{R,D}^*$, are given by:

$$\text{Bias}(\hat{\hat{Y}}_{R,D}^*) = \bar{Y}(k_1 - 1) \tag{3.9}$$

and

$$\text{MSE}(\hat{\hat{Y}}_{R,D}^*) \cong \bar{Y}^2(k_1 - 1)^2 + \bar{Y}^2 V_{200} k_1^2 + \bar{X}^2 V_{020} k_2^2 - 2\bar{Y}\bar{X}V_{110}k_1k_2. \tag{3.10}$$

By simplify Eq (3.10) the value of k_1 and k_2, are given as:

$$k_{1(\text{opt})} = \frac{V_{020}}{\{V_{020}(1+V_{200}) - V_{110}^2\}},$$
$$k_{2(\text{opt})} = \frac{\bar{Y}V_{110}}{\bar{X}\{V_{020}\,(1+V_{200}) - V_{110}^2\}},$$

respectively. The minimal MSE of $\hat{\bar{Y}}^*_{R,D}$ at the optimal values is given by:

$$\text{MSE}_{\min}(\hat{\bar{Y}}^*_{R,D}) \cong \frac{\bar{Y}^2(V_{200}V_{020} - V_{110}^2)}{\{V_{020}\,(1+V_{200}) - V_{110}^2\}}. \tag{3.11}$$

Equation (3.11) may be written as

$$\text{MSE}_{\min}(\hat{\bar{Y}}^*_{R,D}) \cong \frac{\bar{Y}^2V_{200}(1-\rho_{YX}^2)}{\{1+V_{200}(1-\rho_{YX}^2)\}}. \tag{3.12}$$

(5) Following [4], is given as:

$$\hat{\bar{Y}}_{BT,R^*} = \hat{\bar{Y}}^*_H \exp\left(\frac{\bar{X} - \hat{\bar{X}}_H}{\bar{X} + \hat{\bar{X}}_H}\right), \tag{3.13}$$

$$\hat{\bar{Y}}^*_{BT,P} = \hat{\bar{Y}}^*_H \exp\left(\frac{\hat{\bar{X}}_H - \bar{X}}{\bar{X} + \hat{\bar{X}}_H}\right). \tag{3.14}$$

The biases and MSEs of $\hat{\bar{Y}}^*_{BT,R}$ and $\hat{\bar{Y}}^*_{BT,P}$, are given as:

$$\text{Bias}(\hat{\bar{Y}}^*_{BT,R}) \cong \bar{Y}\left(\frac{3}{8}V_{020} - \frac{1}{2}V_{110}\right),$$
$$\text{MSE}(\hat{\bar{Y}}_{BT,R}) \cong \frac{\bar{Y}^2}{4}\left(4V_{200} + V_{020} - 4V_{110}\right), \tag{3.15}$$

and

$$\text{Bias}(\hat{\bar{Y}}^*_{BT,P}) \cong \bar{Y}\left(\frac{1}{2}V_{110} - \frac{1}{8}V_{020}\right),$$
$$\text{MSE}(\hat{\bar{Y}}^*_{BT,P}) \cong \frac{\bar{Y}^2}{4}\left(4V_{200} + V_{020} + 4V_{110}\right). \tag{3.16}$$

(6) Following [29], a generalized ratio-type exponential estimator of $\bar{Y}$ is

$$\hat{\bar{Y}}^*_S = \hat{\bar{Y}}^*_H \exp\left(\frac{a(\bar{X} - \hat{\bar{X}}_H)}{a(\bar{X} + \hat{\bar{X}}_H) + 2b}\right). \tag{3.17}$$

The properties of $\hat{\bar{Y}}^*_S$, are given as:

$$\text{Bias}(\hat{\bar{Y}}^*_S) \cong \bar{Y}\left(\frac{3}{8}\theta^2V_{020} - \frac{1}{2}\theta V_{110}\right),$$
$$\text{MSE}(\hat{\bar{Y}}^*_S) \cong \frac{\bar{Y}^2}{4}\left(4V_{200} + \theta^2V_{020} - 4\theta V_{110}\right),$$

where $\theta = a\bar{X}/(a\bar{X}+b)$.

(7) Following [8], a generalized class of ratio-type exponential estimators of $\bar{Y}$ is given as:

$$\hat{\bar{Y}}_{GK}^{*} = \left\{k_1 \hat{\bar{Y}}_H^{*} + k_2(\bar{X} - \hat{\bar{X}}_H)\right\} \exp\left(\frac{a(\bar{X} - \hat{\bar{X}}_H)}{a(\bar{X} + \hat{\bar{X}}_H) + 2b}\right). \tag{3.18}$$

The properties of $\hat{\bar{Y}}_{GK}^{*}$, are given as:

$$\begin{aligned} \text{Bias}(\hat{\bar{Y}}_{GK}^{*}) \cong& \bar{Y}(k_1 - 1) + \frac{3}{8}\theta^2 \bar{Y} V_{200} k_1 + \frac{1}{2}\theta \bar{X} V_{020} k_2 - \frac{1}{2}\theta \bar{Y} V_{110}, \\ \text{MSE}(\hat{\bar{Y}}_{GK}^{*}) \cong& \bar{Y}^2(k_1 - 1)^2 + \bar{Y}^2 V_{200} k_1^2 + \bar{X}^2 V_{020} v_2^2 + \theta^2 \bar{Y}^2 V_{020} v_1^2 \\ &+ 2\theta \bar{Y}\bar{X} V_{020} k_1 k_2 - \frac{3}{4}\theta^2 \bar{Y}^2 V_{020} k_1 - \theta \bar{Y}\bar{X} V_{020} k_2 \\ &+ \theta \bar{Y}^2 V_{110} k_1 - 2\theta \bar{Y}^2 V_{110} v_1^2 - 2\bar{Y}\bar{X} V_{110} k_1 k_2. \end{aligned} \tag{3.19}$$

The optimum values of k_1 and k_2 determined by simplifying (23), are given as:

$$\begin{aligned} k_{1(\text{opt})} &= \frac{V_{020}\left(8 - \theta^2 V_{020}\right)}{8\{V_{020}(1 + V_{200}) - V_{110}^2\}}, \\ k_{2(\text{opt})} &= \frac{\bar{Y}\left[\theta^3 V_{020}^2 - 4V_{110}(-2 + \theta V_{110}) - \theta V_{020}(4 + \theta V_{110} - 4V_{200})\right]}{8\bar{X}\{V_{020}(1 + V_{200}) - V_{110}^2)\}}. \end{aligned}$$

The simplified minimum MSE of $\hat{\bar{Y}}_{GK}^{*}$ at the optimum values of k_1 and k_2 is given by

$$\text{MSE}_{\min}(\hat{\bar{Y}}_{GK}^{*}) \cong \frac{\bar{Y}^2}{64}\left(64 - 16\theta^2 V_{020} - \frac{V_{020}(-8 + \theta^2 V_{020})^2}{V_{020}(1 + V_{200}) - V_{110}^2}\right). \tag{3.20}$$

Here (3.20) may be written as

$$\text{MSE}_{\min}(\hat{\bar{Y}}_{GK}^{*}) \cong \text{Var}_{\min}(\hat{\bar{Y}}_D^{*}) - \frac{\bar{Y}^2(\theta^2 V_{020}^2 - 8V_{110}^2 + 8V_{020}V_{200})^2}{64V_{020}^2\{1 + V_{200}(1 - \rho_{YX}^2)\}}. \tag{3.21}$$

3.2. Situation-II

When non response is occur in both study and auxiliary variables, say Y and X.

(1) The traditional ratio estimator of $\bar{Y}$ is given as:

$$\hat{\bar{Y}}_R^{**} = \hat{\bar{Y}}_H^{**}\left(\frac{\bar{X}}{\hat{\bar{X}}_H^{**}}\right). \tag{3.22}$$

The properties of $\hat{\bar{Y}}_R^{**}$, are given as:

$$\begin{aligned} \text{Bias}(\hat{\bar{Y}}_R^{**}) &\cong \bar{Y}\left(\Psi_{020} - \Psi_{110}\right), \\ \text{MSE}(\hat{\bar{Y}}_R^{**}) &\cong \bar{Y}^2\left(V_{200} + \Psi_{020} - 2\Psi_{110}\right). \end{aligned} \tag{3.23}$$

(2) The traditional product estimator of $\bar{Y}$ is given as:

$$\hat{\hat{Y}}_P^{**} = \hat{\hat{Y}}_H^{**}\left(\frac{\hat{\hat{X}}_H^{**}}{\bar{X}}\right). \tag{3.24}$$

The properties of $\hat{\hat{Y}}_P^{**}$, are given as:

$$\begin{aligned}\text{Bias}(\hat{\hat{Y}}_P^{**}) &= \bar{Y}\Psi_{110},\\ \text{MSE}(\hat{\hat{Y}}_P^{**}) &\cong \bar{Y}^2\left(V_{200}+\Psi_{020}+2\Psi_{110}\right).\end{aligned} \tag{3.25}$$

(3) The traditional difference estimator of $\bar{Y}$ is

$$\hat{\hat{Y}}_D^{**} = \hat{\hat{Y}}_H^{**} + d(\bar{X} - \hat{\hat{X}}_H^{**}). \tag{3.26}$$

The minimal variance of $\hat{\hat{Y}}_D^{**}$ at the optimal value $d_{(\text{opt})} = (\bar{Y}\Psi_{110})/(\bar{X}\Psi_{020})$ is

$$\text{Var}_{\min}(\hat{\hat{Y}}_D^{**}) \cong \frac{\bar{Y}^2(V_{200}\Psi_{020} - \Psi_{110}^2)}{\Psi_{020}}. \tag{3.27}$$

Equation (3.27) may be written as:

$$\text{Var}_{\min}(\hat{\hat{Y}}_D^{**}) \cong \bar{Y}^2 V_{200}(1-\rho_{YX(2)}^2). \tag{3.28}$$

(4) Following [27], a difference-type estimator of $\bar{Y}$ is

$$\hat{\hat{Y}}_{R,D}^{**} = k_1\hat{\hat{Y}}_H^{**} + k_2(\bar{X} - \hat{\hat{X}}_H^{**}). \tag{3.29}$$

The properties of $\hat{\hat{Y}}_{R,D}^{**}$, are given as:

$$\text{Bias}(\hat{\hat{Y}}_{R,D}^{**}) = \bar{Y}(k_1 - 1)$$

and

$$\text{MSE}(\hat{\hat{Y}}_{R,D}^{**}) \cong \bar{Y}^2(k_1-1)^2 + \bar{Y}^2V_{200}k_1^2 + \bar{X}^2\Psi_{020}k_2^2 - 2\bar{Y}\bar{X}\Psi_{110}k_1k_2. \tag{3.30}$$

The optimal values of k_1 and k_2, determined by minimizing (3.30), are given as:

$$\begin{aligned}k_{1(\text{opt})} &= \frac{\Psi_{020}}{\{\Psi_{020}(1+V_{200}) - \Psi_{110}^2\}},\\ k_{2(\text{opt})} &= \frac{\bar{Y}\Psi_{110}}{\bar{X}\{\Psi_{020}\,(1+V_{200}) - \Psi_{110}^2\}}.\end{aligned}$$

The minimal MSE of $\hat{\hat{Y}}_{R,D}^{**}$ at the optimal values is given by:

$$\text{MSE}_{\min}(\hat{\hat{Y}}_{R,D}^{**}) \cong \frac{\bar{Y}^2(V_{200}\Psi_{020} - \Psi_{110}^2)}{\{\Psi_{020}\,(1+V_{200}) - \Psi_{110}^2\}}. \tag{3.31}$$

Equation (3.31) may be written as:

$$\text{MSE}_{\min}(\hat{\hat{Y}}_{R,D}^{**}) \cong \frac{\bar{Y}^2V_{200}(1-\rho_{YX(2)}^2)}{\{1+V_{200}(1-\rho_{YX(2)}^2)\}}. \tag{3.32}$$

(5) Following [4], the ratio and product-type exponential estimators of $\bar{Y}$, are given by:

$$\hat{\bar{Y}}^{**}_{BT,R} = \hat{\bar{Y}}^{**}_{H} \exp\left(\frac{\bar{X} - \hat{\bar{X}}^{**}_{H}}{\bar{X} + \hat{\bar{X}}^{**}_{H}}\right), \tag{3.33}$$

$$\hat{\bar{Y}}^{**}_{BT,P} = \hat{\bar{Y}}^{**}_{H} \exp\left(\frac{\hat{\bar{X}}^{**}_{H} - \bar{X}}{\bar{X} + \hat{\bar{X}}^{**}_{H}}\right). \tag{3.34}$$

The biases and MSEs of $\hat{\bar{Y}}^{**}_{BT,R}$ and $\hat{\bar{Y}}^{**}_{BT,P}$, are given by:

$$\begin{aligned}\text{Bias}(\hat{\bar{Y}}^{**}_{BT,R}) &\cong \bar{Y}\left(\frac{3}{8}\Psi_{020} - \frac{1}{2}\Psi_{110}\right),\\ \text{MSE}(\hat{\bar{Y}}^{**}_{BT,R}) &\cong \frac{\bar{Y}^2}{4}\left(4V_{200} + \Psi_{020} - 4\Psi_{110}\right),\end{aligned} \tag{3.35}$$

and

$$\begin{aligned}\text{Bias}(\hat{\bar{Y}}^{**}_{BT,P}) &\cong \bar{Y}\left(\frac{1}{2}\Psi_{110} - \frac{1}{8}\Psi_{020}\right),\\ \text{MSE}(\hat{\bar{Y}}^{**}_{BT,P}) &\cong \frac{\bar{Y}^2}{4}\left(4V_{200} + \Psi_{020} + 4\Psi_{110}\right).\end{aligned} \tag{3.36}$$

(6) Following [6], a generalized ratio-type exponential estimator of $\bar{Y}$ is given by:

$$\hat{\bar{Y}}^{**}_{S} = \hat{\bar{Y}}^{**}_{H} \exp\left(\frac{a(\bar{X} - \hat{\bar{X}}^{**}_{H})}{a(\bar{X} + \hat{\bar{X}}^{**}_{H}) + 2b}\right). \tag{3.37}$$

The properties of $\hat{\bar{Y}}^{**}_{S}$, are given by:

$$\begin{aligned}\text{Bias}(\hat{\bar{Y}}^{**}_{S}) &\cong \bar{Y}\left(\frac{3}{8}\theta^2\Psi_{020} - \frac{1}{2}\theta\Psi_{110}\right),\\ \text{MSE}(\hat{\bar{Y}}^{**}_{S}) &\cong \frac{\bar{Y}^2}{4}\left(4V_{200} + \theta^2\Psi_{020} - 4\theta\Psi_{110}\right),\end{aligned} \tag{3.38}$$

where $\theta = a\bar{X}/(a\bar{X} + b)$.

(7) Following [8], estimators of $\bar{Y}$ is given by:

$$\hat{\bar{Y}}^{**}_{GK} = \left\{k_1\hat{\bar{Y}}^{**}_{H} + k_2(\bar{X} - \hat{\bar{X}}^{**}_{H})\right\} \exp\left(\frac{a(\bar{X} - \hat{\bar{X}}^{**}_{H})}{a(\bar{X} + \hat{\bar{X}}^{**}_{H}) + 2b}\right). \tag{3.39}$$

The properties of $\hat{\bar{Y}}^{**}_{GK}$, are given by:

$$\text{Bias}(\hat{\bar{Y}}^{**}_{GK}) \cong \bar{Y}(k_1 - 1) + \frac{3}{8}\theta^2\bar{Y}V_{200}k_1 + \frac{1}{2}\theta\bar{X}\Psi_{020}k_2 - \frac{1}{2}\theta\bar{Y}\Psi_{110},$$

$$\begin{aligned}\mathrm{MSE}(\hat{\hat{Y}}^{**}_{GK}) \cong{}& \bar{Y}^2(k_1-1)^2+\bar{Y}^2V_{200}k_1^2+\bar{X}^2\Psi_{020}v_2^2+\theta^2\bar{Y}^2\Psi_{020}v_1^2\\ &+2\theta\bar{Y}\bar{X}\Psi_{020}k_1k_2-\frac{3}{4}\theta^2\bar{Y}^2\Psi_{020}k_1-\theta\bar{Y}\bar{X}\Psi_{020}k_2\\ &+\theta\bar{Y}^2\Psi_{110}k_1-2\theta\bar{Y}^2\Psi_{110}v_1^2-2\bar{Y}\bar{X}\Psi_{110}k_1k_2. \end{aligned} \tag{3.40}$$

The ideal values of k_1 and k_2 is expressing by (3.40),

$$\begin{aligned} k_{1(\text{opt})} &= \frac{\Psi_{020}\left(8-\theta^2\Psi_{020}\right)}{8\{\Psi_{020}(1+V_{200})-\Psi_{110}^2\}},\\ k_{2(\text{opt})} &= \frac{\bar{Y}\left[\theta^3\Psi_{020}^2-4\Psi_{110}(-2+\theta\Psi_{110})-\theta\Psi_{020}(4+\theta\Psi_{110}-4V_{200})\right]}{8\bar{X}\{\Psi_{020}(1+V_{200})-\Psi_{110}^2)\}}. \end{aligned}$$

The minimal MSE of $\hat{\hat{Y}}_{GK}$ at the optimal values of k_1 and k_2 is given by:

$$\mathrm{MSE}_{\min}(\hat{\hat{Y}}^{**}_{GK}) \cong \frac{\bar{Y}^2}{64}\left(64-16\theta^2\Psi_{020}-\frac{\Psi_{020}(-8+\theta^2\Psi_{020})^2}{\Psi_{020}(1+V_{200})-\Psi_{110}^2}\right). \tag{3.41}$$

Equation (3.41) may be written as:

$$\mathrm{MSE}_{\min}(\hat{\hat{Y}}^{**}_{GK}) \cong \mathrm{Var}_{\min}(\hat{\hat{Y}}^{**}_{D})-\frac{\bar{Y}^2(\theta^2\Psi_{020}^2-8\Psi_{110}^2+8\Psi_{020}V_{200})^2}{64\Psi_{020}^2\{1+V_{200}(1-\rho^2_{YX(2)})\}}. \tag{3.42}$$

4. Proposed estimator in non-response using simple random sampling

The proper use of ancillary variable improve the accuracy of estimator in the design and estimation stages. Complete auxiliary information is frequently supplied along with the sample frame for social, economic, and natural surveys. When the study variable and the auxiliary variable have a sufficient amount of connection, the rankings of the auxiliary variable are also correlated with the values of the auxiliary variable. Consequently, The categorised auxiliary variable (which includes the auxiliary variable's rank) can be treated as a new auxiliary variable, and this information can help an estimator perform better. Because of We present an improved family of estimators for predicting the population mean that requires additional information on the study and auxiliary variable sample means, as well as the ranks of the auxiliary variable under non-response using simple random sampling.

4.1. Situation-I

When non-response occur only in study variable. On the lines of [11], the proposed improved estimator of $\bar{Y}$ in the presence of non-response using SRS, say $\hat{\hat{Y}}^{*}_{Suggested}$ is given as:

$$\hat{\hat{Y}}^{*}_{Suggested} = \{w_1\hat{\hat{Y}}^{*}_{H}+w_2(\bar{X}-\hat{\hat{X}}_{H})+w_3(\bar{Z}-\hat{\hat{Z}}_{H})\}\exp\left(\frac{a(\bar{X}-\hat{\hat{X}}_{H})}{a(\bar{X}+\hat{\hat{X}}_{H})+2b}\right), \tag{4.1}$$

where w_1, w_2, and w_3 are unknown constant. The proposed estimator $\hat{\hat{Y}}_{Suggested}j$ can be rewritten as

$$\hat{\hat{Y}}^{*}_{Suggested} = \{w_1\bar{Y}(1+\xi_0^*)-w_2\bar{X}\xi_1-w_3\bar{Z}\xi_2\}\left\{1-\frac{\theta\xi_1}{2}+\frac{3\theta^2\xi_1^2}{8}+\cdots\right\}. \tag{4.2}$$

Simplifying (4.2), we have

$$\left(\hat{\bar{Y}}^{*}_{Suggested}-\bar{Y}\right)\cong-\bar{Y}+w_1\bar{Y}+w_1\bar{Y}\xi_0^{*}-\frac{1}{2}w_1\theta\bar{Y}\xi_1-w_2\bar{X}\xi_1-w_3\bar{Z}\xi_2$$
$$+\frac{3}{8}w_1\theta^2\bar{Y}\xi_1^2+\frac{1}{2}w_2\theta\bar{X}\xi_1^2-\frac{1}{2}w_1\theta\bar{Y}\xi_0^{*}\xi_1+\frac{1}{2}w_3\theta\bar{Z}\xi_1\xi_2. \quad (4.3)$$

The properties of $\hat{\bar{Y}}^{*}_{Suggested}$, are given as:

$$\begin{aligned}\text{Bias}(\hat{\bar{Y}}^{*}_{Suggested}) &\cong \bar{Y}(w_1-1)+\frac{3}{8}\theta^2\bar{Y}V_{020}w_1+\frac{1}{2}\theta\bar{X}V_{020}w_2-\frac{1}{2}\theta\bar{Y}V_{110}w_1+\frac{1}{2}\theta\bar{Z}V_{011},\\ \text{MSE}(\hat{\bar{Y}}^{*}_{Suggested}) &\cong \bar{Y}^2(w_1-1)^2+\bar{Y}^2V_{200}w_1^2+\bar{X}^2V_{020}w_2+\bar{Z}^2V_{002}w_3+\theta^2\bar{Y}^2V_{020}w_1^2\\ &-\theta\bar{Y}\bar{X}V_{020}w_2+2\theta\bar{Y}\bar{X}V_{020}w_1w_2-\frac{3}{4}\theta^2\bar{Y}^2V_{020}w_1+\theta\bar{Y}^2V_{110}w_1\\ &-2\theta\bar{Y}^2V_{110}w_1^2-2\bar{Y}\bar{X}V_{110}w_1w_2-2\bar{Y}\bar{Z}V_{101}w_1w_3-\theta\bar{Y}\bar{Z}V_{011}w_3\\ &+2\theta\bar{Y}\bar{Z}V_{011}w_1w_3-2\bar{X}\bar{Z}V_{011}w_2w_3. \end{aligned}\quad (4.4)$$

The optimal values of w_1, w_2, and w_3 determined by minimizing (4.4), are

$$w_{1(\text{opt})}=\frac{(\theta^2V_{020}-8)(V_{110}^2-V_{002}V_{020})}{8\left[-V_{020}V_{101}^2+2V_{011}V_{101}V_{110}-V_{011}^2(1+V_{200})+V_{002}\{-V_{110}^2+V_{020}(1+V_{200})\}\right]},$$

$$w_{2(\text{opt})}=-\frac{\bar{Y}\begin{bmatrix}4\theta V_{020}V_{101}^2-V_{011}V_{101}(-8+\theta^2V_{020}+8\theta V_{110})+V_{002}\{-\theta^3V_{020}^2+4V_{110}(-2+\theta V_{110})\\ +\theta V_{020}(4+\theta V_{110}-4V_{200})\}+\theta V_{011}^2(-4+\theta^2V_{020}+4V_{200})\end{bmatrix}}{8\bar{X}\left[-V_{020}V_{101}^2+2V_{011}V_{101}V_{110}-V_{011}^2(1+V_{200})+V_{002}\{-V_{110}^2+V_{020}(1+V_{200})\}\right]},$$

$$w_{3(\text{opt})}=-\frac{\bar{Y}(\theta^2V_{020}-8)(V_{020}V_{101}-V_{110}V_{011})}{8\bar{Z}\left[-V_{020}V_{101}^2+2V_{011}V_{101}V_{110}-V_{011}^2(1+V_{200})+V_{002}\{-V_{110}^2+V_{020}(1+V_{200})\}\right]}.$$

The minimal MSE of $\hat{\bar{Y}}^{*}_{Suggested}$ at optimal values of w_1, w_2 and w_3 is given by:

$$\text{MSE}\left(\hat{\bar{Y}}^{*}_{Suggested}\right)\cong\frac{\bar{Y}^2}{64}\left[64-16\theta^2\Psi_{020}+\frac{(\theta^2\Psi_{020}-8)^2(\Psi_{011}^2-\Psi_{002}\Psi_{020})}{\begin{bmatrix}-\Psi_{020}\Psi_{101}^2+2\Psi_{011}\Psi_{101}\Psi_{110}-\Psi_{011}^2(1+\Psi_{200}^2)\\ +\Psi_{020}\left\{-\Psi_{110}^2+\Psi_{020}\left(1+\Psi_{200}\right)\right\}\end{bmatrix}}\right]. \quad (4.5)$$

Equation (4.5) me be written as

$$\text{MSE}_{\min}(\hat{\bar{Y}}^{*}_{Suggested})\cong\text{Var}_{\min}(\hat{\bar{Y}}^{*}_{D})-A_1-A_2, \quad (4.6)$$

where

$$A_1=\frac{\bar{Y}^2(\theta^2V_{020}^2-8V_{110}^2+8V_{020}V_{200})^2}{64V_{020}^2\{1+V_{200}(1-\rho_{YX}^2)\}}, \quad (4.7)$$
$$A_2=\frac{\bar{Y}^2(\theta^2V_{020}-8)^2(V_{020}V_{101}-V_{011}V_{110})^2}{64V_{020}^2V_{002}(1-\rho_{XZ}^2)\{1+V_{200}(1-\rho_{YX}^2)\}\{1+V_{200}(1-R_{Y.XZ(1)}^2)\}}.$$

4.2. Situation-II

When non-response are in both study and auxiliary variable. Taking motivation on the lines of [11], we proposed a family of estimators of $\bar{Y}$ in the presence of non-response say $\hat{\bar{Y}}^{**}_{Suggested}$, is given by:

$$\hat{\bar{Y}}^{**}_{Suggested} = \{w_1\hat{\bar{Y}}^{**}_H + w_2(\bar{X} - \hat{\bar{X}}^{**}_H) + w_3(\bar{Z} - \hat{\bar{Z}}^{**}_H)\} \exp\left(\frac{a(\bar{X} - \hat{\bar{X}}^{**}_H)}{a(\bar{X} + \hat{\bar{X}}^{**}_H) + 2b}\right), \tag{4.8}$$

where w_1, w_2, and w_3 are unknown constants. The proposed estimator $\hat{\bar{Y}}^{**}_{Suggested}$ can be rewritten as:

$$\hat{\bar{Y}}^{**}_{Suggested} = \{w_1\bar{Y}(1+\xi_0^*) - w_2\bar{X}\xi_1^* - w_3\bar{Z}\xi_2^*\}\left\{1 - \frac{\theta\xi_1^*}{2} + \frac{3\theta^2\xi_1^{*2}}{8} + \cdots\right\}. \tag{4.9}$$

Simplifying (4.9), we can write

$$\begin{aligned}\left(\hat{\bar{Y}}^{**}_{Suggested}\right) &\cong -\bar{Y} + w_1\bar{Y} + w_1\bar{Y}\xi_0^* - \frac{1}{2}w_1\theta\bar{Y}\xi_1^* - w_2\bar{X}\xi_1^* - w_3\bar{Z}\xi_2^* \\ &\quad + \frac{3}{8}w_1\theta^2\bar{Y}\xi_1^{*2} + \frac{1}{2}w_2\theta\bar{X}\xi_1^{*2} - \frac{1}{2}w_1\theta\bar{Y}\xi_0^*\xi_1^* + \frac{1}{2}w_3\theta\bar{Z}\xi_1^*\xi_2^*.\end{aligned} \tag{4.10}$$

The properties of $\hat{\bar{Y}}^{**}_{Suggested}$, are given by:

$$\begin{aligned}\text{Bias}(\hat{\bar{Y}}^{**}_{Suggested}) &\cong \bar{Y}(w_1 - 1) + \frac{3}{8}\theta^2\bar{Y}\Psi_{020}w_1 + \frac{1}{2}\theta\bar{X}\Psi_{020}w_2 - \frac{1}{2}\theta\bar{Y}\Psi_{110}w_1 + \frac{1}{2}\theta\bar{Z}\Psi_{011}, \text{ and} \\ \text{MSE}(\hat{\bar{Y}}^{**}_{Suggested}) &\cong \bar{Y}^2(w_1-1)^2 + \bar{Y}^2V_{200}w_1^2 + \bar{X}^2\Psi_{020}w_2 + \bar{Z}^2\Psi_{002}w_3 + \theta^2\bar{Y}^2\Psi_{020}w_1^2 \\ &\quad -\theta\bar{Y}\bar{X}\Psi_{020}w_2 + 2\theta\bar{Y}\bar{X}\Psi_{020}w_1w_2 - \frac{3}{4}\theta^2\bar{Y}^2\Psi_{020}w_1 + \theta\bar{Y}^2\Psi_{110}w_1 \\ &\quad -2\theta\bar{Y}^2\Psi_{110}w_1^2 - 2\bar{Y}\bar{X}\Psi_{110}w_1w_2 - 2\bar{Y}\bar{Z}\Psi_{101}w_1w_3 - \theta\bar{Y}\bar{Z}\Psi_{011}w_3 \\ &\quad +2\theta\bar{Y}\bar{Z}\Psi_{011}w_1w_3 - 2\bar{X}\bar{Z}\Psi_{011}w_2w_3.\end{aligned} \tag{4.11}$$

The optimal values of w_1, w_2, and w_3 determined by minimizing (4.11), are given as:

$$w_{1(\text{opt})} = \frac{(\theta^2\Psi_{020} - 8)(\Psi_{110}^2 - \Psi_{002}\Psi_{020})}{8\left[-\Psi_{020}\Psi_{101}^2 + 2\Psi_{011}\Psi_{101}\Psi_{110} - \Psi_{011}^2(1+V_{200}) + \Psi_{002}\{-\Psi_{110}^2 + \Psi_{020}(1+V_{200})\}\right]},$$

$$w_{2(\text{opt})} = -\frac{\bar{Y}\left[\begin{array}{l}4\theta\Psi_{020}\Psi_{101}^2 - \Psi_{011}\Psi_{101}(-8+\theta^2\Psi_{020}+8\theta\Psi_{110}) + \Psi_{002}\{-\theta^3\Psi_{020}^2 + 4\Psi_{110}(-2+\theta\Psi_{110}) \\ +\theta\Psi_{020}(4+\theta\Psi_{110}-4V_{200})\} + \theta\Psi_{011}^2(-4+\theta^2\Psi_{020}+4V_{200})\end{array}\right]}{8\bar{X}\left[-\Psi_{020}\Psi_{101}^2 + 2\Psi_{011}\Psi_{101}\Psi_{110} - \Psi_{011}^2(1+V_{200}) + \Psi_{002}\{-\Psi_{110}^2 + \Psi_{020}(1+V_{200})\}\right]},$$

$$w_{3(\text{opt})} = -\frac{\bar{Y}(\theta^2\Psi_{020} - 8)(\Psi_{020}\Psi_{101} - \Psi_{110}\Psi_{011})}{8\bar{Z}\left[-\Psi_{020}\Psi_{101}^2 + 2\Psi_{011}\Psi_{101}\Psi_{110} - \Psi_{011}^2(1+V_{200}) + \Psi_{002}\{-\Psi_{110}^2 + \Psi_{020}(1+V_{200})\}\right]},$$

The minimal MSE of the $\hat{\bar{Y}}^{**}_{Suggested}$ at optimal values of w_1, w_2 and w_3 is given by:

$$\mathrm{MSE}\left(\hat{\bar{Y}}^{**}_{Suggested}\right) \cong \frac{\bar{Y}^2}{64}\left[64 - 16\theta^2\Psi_{020} + \frac{(\theta^2\Psi_{020} - 8)^2(\Psi^2_{011} - \Psi_{002}\Psi_{020})}{\begin{bmatrix} -\Psi_{020}\Psi^2_{101} + 2\Psi_{011}\Psi_{101}\Psi_{110} - \Psi^2_{011}(1 + V^2_{200}) \\ + \Psi_{020}\left\{-\Psi^2_{110} + \Psi_{020}\left(1 + V_{200}\right)\right\} \end{bmatrix}}\right]. \quad (4.12)$$

Here (4.12) me be written as:

$$\mathrm{MSE}_{\min}(\hat{\bar{Y}}^{**}_{Suggested}) \cong \mathrm{Var}_{\min}(\hat{\bar{Y}}^{**}_{D}) - B_1 - B_2, \quad (4.13)$$

where

$$B_1 = \frac{\bar{Y}^2(\theta^2\Psi^2_{020} - 8\Psi^2_{110} + 8\Psi_{020}V_{200})^2}{64\Psi^2_{020}\{1 + V_{200}(1 - \rho^2_{YX(2)})\}}, \quad (4.14)$$

$$B_2 = \frac{\bar{Y}^2(\theta^2\Psi_{020} - 8)^2(\Psi_{020}\Psi_{101} - \Psi_{011}\Psi_{110})^2}{64\Psi^2_{020}\Psi_{002}(1 - \rho^2_{XZ(2)})\{1 + V_{200}(1 - \rho^2_{YX(2)})\}\{1 + V_{200}(1 - R^2_{Y.XZ(2)})\}}.$$

In Table 1, we put some members of the [8, 29], and proposed families of estimators with selected choices of a and b.

Table 1. Components of our proposed and existing estimates.

a	b	$\hat{\bar{Y}}_S$	$\hat{\bar{Y}}_{G.K}$	$\hat{\bar{Y}}_{Suggested}$
1	C_x	$\hat{\bar{Y}}^{(1)}_S$	$\hat{\bar{Y}}^{(1)}_{G.K}$	$\hat{\bar{Y}}^{(1)}_{Suggested}$
1	β_2	$\hat{\bar{Y}}^{(2)}_S$	$\hat{\bar{Y}}^{(2)}_{G.K}$	$\hat{\bar{Y}}^{(2)}_{Suggested}$
β_2	C_x	$\hat{\bar{Y}}^{(3)}_S$	$\hat{\bar{Y}}^{(3)}_{G.K}$	$\hat{\bar{Y}}^{(3)}_{Suggested}$
C_x	β_2	$\hat{\bar{Y}}^{(4)}_S$	$\hat{\bar{Y}}^{(4)}_{G.K}$	$\hat{\bar{Y}}^{(4)}_{Suggested}$
1	ρ_{yx}	$\hat{\bar{Y}}^{(5)}_S$	$\hat{\bar{Y}}^{(5)}_{G.K}$	$\hat{\bar{Y}}^{(5)}_{Suggested}$
C_x	ρ_{yx}	$\hat{\bar{Y}}^{(6)}_S$	$\hat{\bar{Y}}^{(6)}_{G.K}$	$\hat{\bar{Y}}^{(6)}_{Suggested}$
ρ_{yx}	C_x	$\hat{\bar{Y}}^{(7)}_S$	$\hat{\bar{Y}}^{(7)}_{G.K}$	$\hat{\bar{Y}}^{(7)}_{Suggested}$
β_2	ρ_{yx}	$\hat{\bar{Y}}^{(8)}_S$	$\hat{\bar{Y}}^{(8)}_{G.K}$	$\hat{\bar{Y}}^{(8)}_{Suggested}$
ρ_{yx}	β_2	$\hat{\bar{Y}}^{(9)}_S$	$\hat{\bar{Y}}^{(9)}_{G.K}$	$\hat{\bar{Y}}^{(9)}_{Suggested}$
1	$N\bar{X}$	$\hat{\bar{Y}}^{(10)}_S$	$\hat{\bar{Y}}^{(10)}_{G.K}$	$\hat{\bar{Y}}^{(10)}_{Suggested}$

5. Efficiency comparisons for both condition

5.1. Situation-I

In this section, we performed a comparison of the adapted and proposed estimators, when non-response is available in the study variable.

(i) By taking (2.3) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{Suggested}\right) < \mathrm{Var}\left(\hat{\bar{Y}}^{*}_{SRS}\right) \text{ if } \bar{Y}^2 V_{200}^2 \rho_{yx}^2 + A_1 + A_2 > 0.$$

(ii) By taking (2.4) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{Suggested}\right) < \mathrm{MSE}\left(\hat{\bar{Y}}^{*}_{R}\right) \text{ if } \frac{\bar{Y}^2}{V_{200}}\left(V_{020} - V_{110}\right)^2 + A_1 + A_2 > 0.$$

(iii) By taking (3.2) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{Suggested}\right) < \mathrm{MSE}\left(\hat{\bar{Y}}^{*}_{P}\right) \text{ if } \frac{\bar{Y}^2}{V_{200}}\left(V_{020} + V_{110}\right)^2 + A_1 + A_2 > 0.$$

(iv) By taking (3.5) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{Suggested}\right) < \mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{Reg}\right) \text{ if } A_1 + A_2 > 0. \tag{5.1}$$

(v) By taking (3.10) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{Suggested}\right) < \mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{R,D}\right) \text{ if } \frac{\bar{Y}^2\theta^2 V_{020}\left\{\theta^2 V_{020} + 16V_{200}\left(1-\rho_{yx}^2\right)\right\}}{64\left\{1 + V_{200}\left(1-\rho_{yx}^2\right)\right\}} + A_2 > 0.$$

(vi) By taking (3.12) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{Suggested}\right) < \mathrm{MSE}\left(\hat{\bar{Y}}^{*}_{S}\right) \text{ if } \frac{\bar{Y}^2}{V_{200}}\left(\frac{\theta V_{020}}{2} - V_{110}\right)^2 + A_1 + A_2 > 0.$$

(vii) By taking (3.16) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{Suggested}\right) < \mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{*}_{G.K}\right) \text{ if } A_2 > 0.$$

5.2. Situation-II

In this section, we made efficiency comparison of all estimator, when non-response occur in both the study and auxiliary variables.

(i) By taking (2.3) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{Suggested}\right) < \mathrm{Var}\left(\hat{\bar{Y}}^{**}_{SRS}\right) \text{ if } \bar{Y}^2 V_{200}^2 \rho_{yx(2)}^2 + B_1 + B_2 > 0.$$

(ii) By taking (2.4) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{Suggested}\right) < \mathrm{MSE}\left(\hat{\bar{Y}}^{**}_{R}\right) \text{ if } \frac{\bar{Y}^2}{V_{200}}\left(\Psi_{020} - \Psi_{110}\right)^2 + B_1 + B_2 > 0.$$

(iii) By taking (3.2) and (3.21),

$$\mathrm{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{Suggested}\right) < \mathrm{MSE}\left(\hat{\bar{Y}}^{**}_{P}\right) \text{ if } \frac{\bar{Y}^2}{V_{200}}\left(\Psi_{020} + \Psi_{110}\right)^2 + B_1 + B_2 > 0.$$

(iv) By taking (3.5) and (3.21),

$$\text{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{Suggested}\right) < \text{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{Reg}\right) \text{ if } B_1 + B_2 > 0.$$

(v) By taking (3.10) and (3.21),

$$\text{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{Suggested}\right) < \text{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{R,D}\right) \text{ if } \frac{\bar{Y}^2\theta^2\Psi_{020}\left\{\theta^2\Psi_{020} + 16V_{200}\left(1-\rho^2_{yx(2)}\right)\right\}}{64\left\{1 + V_{200}\left(1-\rho^2_{yx(2)}\right)\right\}} + B_2 > 0.$$

(vi) By taking (3.12) and (3.21),

$$\text{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{Suggested}\right) < \text{MSE}\left(\hat{\bar{Y}}^{**}_{S}\right) \text{ if } \frac{\bar{Y}^2}{V_{200}}\left(\frac{\theta\Psi_{020}}{2} - \Psi_{110}\right)^2 + B_1 + B_2 > 0.$$

(vii) By taking (3.16) and (3.21),

$$\text{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{Suggested}\right) < \text{MSE}_{\min}\left(\hat{\bar{Y}}^{**}_{G.K}\right) \text{ if } B_2 > 0.$$

6. Numerical investigation

In this section, the mathematical result is shown to verify the effectiveness of all estimators as compared to existing estimators. Four data sets are under consideration. The data description and mean square error are listed in Tables 2 and 3. The percent efficiency of estimator $\hat{\bar{Y}}_i$ w.r.t $\hat{\bar{Y}}_{SRS}$:

$$\text{PRE}\left(\hat{\bar{Y}}_i, \hat{\bar{Y}}_{SRS}\right) = \frac{\text{Var}\left(\hat{\bar{Y}}_{SRS}\right)}{\text{MSE}_{\min}\left(\hat{\bar{Y}}_i\right)} \times 100,$$

where $i = R, P, \ldots, Sugeested$.

The MSEs and PREs of mean estimators, computed from two populations, are given in Tables 4–11.
Population I. (Source: [9]) Y: The egg assemble in 1990, X: Value per dozen in 1991.

Table 2. Description for Population I.

Parameter	Value	Parameter	Value
N	50	S_Y	1661.242
n	15	S_X	22.18052
λ	0.046670	S_Z	14.57598
$\bar{Y}$	1357.622	ρ_{YX}	−0.3022287
$\bar{X}$	75.8720	ρ_{YZ}	−0.2662075
$\bar{Z}$	25.5000	ρ_{XZ}	0.9574204
Non-response			
Parameter	Value	Parameter	Value
N_2	12	$\rho_{YX(2)}$	1.0000000
w_2	0.240000	$\rho_{YZ(2)}$	−0.3376224
λ_2	0.016000	$\rho_{XZ(2)}$	0.9871019
$S_{Y(2)}$	940.7629		
$S_{X(2)}$	19.53920		
$S_{Z(2)}$	3.605551		

Population II. (Source: [9]) Y: Eggs assemble in 1990, X: Value per dozen in 1990.

Table 3. Description for Population II.

Parameter	Value	Parameter	Value
N	50	S_Y	1661.242
n	15	S_X	21.31747
λ	0.046670	S_Z	14.57563
$\bar{Y}$	1357.622	ρ_{YX}	−0.2888328
$\bar{X}$	78.29000	ρ_{YZ}	−0.2469467
$\bar{Z}$	25.50000	ρ_{XZ}	0.9467713
Non-response			
Parameter	Value	Parameter	Value
N_2	12	$\rho_{YX(2)}$	1.0000000
w_2	0.240000	$\rho_{YZ(2)}$	−0.2066633
λ_2	0.016000	$\rho_{XZ(2)}$	0.9795199
$S_{Y(2)}$	940.7629		
$S_{X(2)}$	18.25925		
$S_{Z(2)}$	3.605551		

Table 4. MSEs using Population I Situation-I.

Estimator	Value	Estimator	Value	Estimator	Value	Estimator	Value
$\hat{\bar{Y}}_{SRS}$	142947.8	$\hat{\bar{Y}}_S^{(1)}$	148429.5	$\hat{\bar{Y}}_{G.K}^{(1)}$	131185.2	$\hat{\bar{Y}}_{Suggested}^{(1)}$	109348.7
$\hat{\bar{Y}}_R$	157953.8	$\hat{\bar{Y}}_S^{(2)}$	148093.3	$\hat{\bar{Y}}_{G.K}^{(2)}$	131197.8	$\hat{\bar{Y}}_{Suggested}^{(2)}$	109370.1
$\hat{\bar{Y}}_P$	143904.1	$\hat{\bar{Y}}_S^{(3)}$	148449.0	$\hat{\bar{Y}}_{G.K}^{(3)}$	131184.4	$\hat{\bar{Y}}_{Suggested}^{(3)}$	109347.5
$\hat{\bar{Y}}_{Reg}$	141402.0	$\hat{\bar{Y}}_S^{(4)}$	147314.0	$\hat{\bar{Y}}_{G.K}^{(4)}$	131226.0	$\hat{\bar{Y}}_{Suggested}^{(4)}$	109417.5
$\hat{\bar{Y}}_{R,D}$	131326.8	$\hat{\bar{Y}}_S^{(5)}$	148483.3	$\hat{\bar{Y}}_{G.K}^{(5)}$	131183.1	$\hat{\bar{Y}}_{Suggested}^{(5)}$	109345.3
		$\hat{\bar{Y}}_S^{(6)}$	148558.9	$\hat{\bar{Y}}_{G.K}^{(6)}$	131180.2	$\hat{\bar{Y}}_{Suggested}^{(6)}$	109340.4
		$\hat{\bar{Y}}_S^{(7)}$	148547.2	$\hat{\bar{Y}}_{G.K}^{(7)}$	131180.7	$\hat{\bar{Y}}_{Suggested}^{(7)}$	109341.1
		$\hat{\bar{Y}}_S^{(8)}$	148462.4	$\hat{\bar{Y}}_{G.K}^{(8)}$	131183.9	$\hat{\bar{Y}}_{Suggested}^{(8)}$	109346.6
		$\hat{\bar{Y}}_S^{(9)}$	150174.0	$\hat{\bar{Y}}_{G.K}^{(9)}$	131115.3	$\hat{\bar{Y}}_{Suggested}^{(9)}$	109230.8
		$\hat{\bar{Y}}_S^{(10)}$	143017.4	$\hat{\bar{Y}}_{G.K}^{(10)}$	131326.8	$\hat{\bar{Y}}_{Suggested}^{(10)}$	109586.3

Table 5. MSEs using Population I Situation-II.

Estimator	Value	Estimator	Value	Estimator	Value	Estimator	Value
$\hat{\bar{Y}}_{SRS}$	142947.8	$\hat{\bar{Y}}_S^{(1)}$	153113.7	$\hat{\bar{Y}}_{G.K}^{(1)}$	122477.1	$\hat{\bar{Y}}_{Suggested}^{(1)}$	113173.2
$\hat{\bar{Y}}_R$	166549.9	$\hat{\bar{Y}}_S^{(2)}$	152581.3	$\hat{\bar{Y}}_{G.K}^{(2)}$	122486.5	$\hat{\bar{Y}}_{Suggested}^{(2)}$	113190.8
$\hat{\bar{Y}}_P$	132099.8	$\hat{\bar{Y}}_S^{(3)}$	153144.4	$\hat{\bar{Y}}_{G.K}^{(3)}$	122476.5	$\hat{\bar{Y}}_{Suggested}^{(3)}$	113172.2
$\hat{\bar{Y}}_{Reg}$	131316.2	$\hat{\bar{Y}}_S^{(4)}$	151320.3	$\hat{\bar{Y}}_{G.K}^{(4)}$	122507.5	$\hat{\bar{Y}}_{Suggested}^{(4)}$	113229.8
$\hat{\bar{Y}}_{R,D}$	122582.7	$\hat{\bar{Y}}_S^{(5)}$	153198.3	$\hat{\bar{Y}}_{G.K}^{(5)}$	122475.5	$\hat{\bar{Y}}_{Suggested}^{(5)}$	113170.4
		$\hat{\bar{Y}}_S^{(6)}$	153317.0	$\hat{\bar{Y}}_{G.K}^{(6)}$	122473.4	$\hat{\bar{Y}}_{Suggested}^{(6)}$	113166.3
		$\hat{\bar{Y}}_S^{(7)}$	153298.6	$\hat{\bar{Y}}_{G.K}^{(7)}$	122473.7	$\hat{\bar{Y}}_{Suggested}^{(7)}$	113167.0
		$\hat{\bar{Y}}_S^{(8)}$	153165.4	$\hat{\bar{Y}}_{G.K}^{(8)}$	122476.1	$\hat{\bar{Y}}_{Suggested}^{(8)}$	113171.5
		$\hat{\bar{Y}}_S^{(9)}$	155785.1	$\hat{\bar{Y}}_{G.K}^{(9)}$	122425.0	$\hat{\bar{Y}}_{Suggested}^{(9)}$	113076.2
		$\hat{\bar{Y}}_S^{(10)}$	143117.2	$\hat{\bar{Y}}_{G.K}^{(10)}$	122582.6	$\hat{\bar{Y}}_{Suggested}^{(10)}$	113369.2

Table 6. PREs using Population I Situation-I.

Estimator	Value	Estimator	Value	Estimator	Value	Estimator	Value
$\hat{\bar{Y}}_{SRS}$	100.00	$\hat{\bar{Y}}_S^{(1)}$	96.31	$\hat{\bar{Y}}_{G.K}^{(1)}$	108.97	$\hat{\bar{Y}}_{Suggested}^{(1)}$	130.73
$\hat{\bar{Y}}_R$	90.500	$\hat{\bar{Y}}_S^{(2)}$	96.53	$\hat{\bar{Y}}_{G.K}^{(2)}$	108.96	$\hat{\bar{Y}}_{Suggested}^{(2)}$	130.70
$\hat{\bar{Y}}_P$	99.340	$\hat{\bar{Y}}_S^{(3)}$	96.29	$\hat{\bar{Y}}_{G.K}^{(3)}$	108.97	$\hat{\bar{Y}}_{Suggested}^{(3)}$	130.73
$\hat{\bar{Y}}_{Reg}$	101.09	$\hat{\bar{Y}}_S^{(4)}$	97.04	$\hat{\bar{Y}}_{G.K}^{(4)}$	108.93	$\hat{\bar{Y}}_{Suggested}^{(4)}$	130.64
$\hat{\bar{Y}}_{R,D}$	108.85	$\hat{\bar{Y}}_S^{(5)}$	96.27	$\hat{\bar{Y}}_{G.K}^{(5)}$	108.97	$\hat{\bar{Y}}_{Suggested}^{(5)}$	130.73
		$\hat{\bar{Y}}_S^{(6)}$	96.22	$\hat{\bar{Y}}_{G.K}^{(6)}$	108.97	$\hat{\bar{Y}}_{Suggested}^{(6)}$	130.74
		$\hat{\bar{Y}}_S^{(7)}$	96.23	$\hat{\bar{Y}}_{G.K}^{(7)}$	108.97	$\hat{\bar{Y}}_{Suggested}^{(7)}$	130.74
		$\hat{\bar{Y}}_S^{(8)}$	96.29	$\hat{\bar{Y}}_{G.K}^{(8)}$	108.97	$\hat{\bar{Y}}_{Suggested}^{(8)}$	130.73
		$\hat{\bar{Y}}_S^{(9)}$	95.19	$\hat{\bar{Y}}_{G.K}^{(9)}$	109.02	$\hat{\bar{Y}}_{Suggested}^{(9)}$	130.87
		$\hat{\bar{Y}}_S^{(10)}$	99.95	$\hat{\bar{Y}}_{G.K}^{(10)}$	108.85	$\hat{\bar{Y}}_{Suggested}^{(10)}$	130.44

Table 7. PREs using Population I Situation-II.

Estimator	Value	Estimator	Value	Estimator	Value	Estimator	Value
$\hat{\hat{Y}}_{SRS}$	100.00	$\hat{\hat{Y}}_S^{(1)}$	93.36	$\hat{\hat{Y}}_{G.K}^{(1)}$	116.71	$\hat{\hat{Y}}_{Suggested}^{(1)}$	126.31
$\hat{\hat{Y}}_R$	85.830	$\hat{\hat{Y}}_S^{(2)}$	93.69	$\hat{\hat{Y}}_{G.K}^{(2)}$	116.70	$\hat{\hat{Y}}_{Suggested}^{(2)}$	126.29
$\hat{\hat{Y}}_P$	108.21	$\hat{\hat{Y}}_S^{(3)}$	93.34	$\hat{\hat{Y}}_{G.K}^{(3)}$	116.71	$\hat{\hat{Y}}_{Suggested}^{(3)}$	126.31
$\hat{\hat{Y}}_{Reg}$	108.86	$\hat{\hat{Y}}_S^{(4)}$	94.47	$\hat{\hat{Y}}_{G.K}^{(4)}$	116.68	$\hat{\hat{Y}}_{Suggested}^{(4)}$	126.25
$\hat{\hat{Y}}_{R,D}$	116.61	$\hat{\hat{Y}}_S^{(5)}$	93.31	$\hat{\hat{Y}}_{G.K}^{(5)}$	116.72	$\hat{\hat{Y}}_{Suggested}^{(5)}$	126.31
		$\hat{\hat{Y}}_S^{(6)}$	93.24	$\hat{\hat{Y}}_{G.K}^{(6)}$	116.72	$\hat{\hat{Y}}_{Suggested}^{(6)}$	126.32
		$\hat{\hat{Y}}_S^{(7)}$	93.25	$\hat{\hat{Y}}_{G.K}^{(7)}$	116.72	$\hat{\hat{Y}}_{Suggested}^{(7)}$	126.32
		$\hat{\hat{Y}}_S^{(8)}$	93.33	$\hat{\hat{Y}}_{G.K}^{(8)}$	116.71	$\hat{\hat{Y}}_{Suggested}^{(8)}$	126.31
		$\hat{\hat{Y}}_S^{(9)}$	91.76	$\hat{\hat{Y}}_{G.K}^{(9)}$	116.76	$\hat{\hat{Y}}_{Suggested}^{(9)}$	126.42
		$\hat{\hat{Y}}_S^{(10)}$	99.88	$\hat{\hat{Y}}_{G.K}^{(10)}$	116.61	$\hat{\hat{Y}}_{Suggested}^{(10)}$	126.09

Table 8. MSEs using Population II Situation-I.

Estimator	Value	Estimator	Value	Estimator	Value	Estimator	Value
$\hat{\hat{Y}}_{SRS}$	142947.8	$\hat{\hat{Y}}_S^{(1)}$	149277.7	$\hat{\hat{Y}}_{G.K}^{(1)}$	130985.1	$\hat{\hat{Y}}_{Suggested}^{(1)}$	111745.4
$\hat{\hat{Y}}_R$	160327.6	$\hat{\hat{Y}}_S^{(2)}$	148916.6	$\hat{\hat{Y}}_{G.K}^{(2)}$	130998.8	$\hat{\hat{Y}}_{Suggested}^{(2)}$	111769.0
$\hat{\hat{Y}}_P$	144181.5	$\hat{\hat{Y}}_S^{(3)}$	149301.8	$\hat{\hat{Y}}_{G.K}^{(3)}$	130984.2	$\hat{\hat{Y}}_{Suggested}^{(3)}$	111743.8
$\hat{\hat{Y}}_{Reg}$	141197.0	$\hat{\hat{Y}}_S^{(4)}$	148128.4	$\hat{\hat{Y}}_{G.K}^{(4)}$	131027.6	$\hat{\hat{Y}}_{Suggested}^{(4)}$	111818.8
$\hat{\hat{Y}}_{R,D}$	131150.0	$\hat{\hat{Y}}_S^{(5)}$	149345.8	$\hat{\hat{Y}}_{G.K}^{(5)}$	130982.5	$\hat{\hat{Y}}_{Suggested}^{(5)}$	111740.8
		$\hat{\hat{Y}}_S^{(6)}$	149431.5	$\hat{\hat{Y}}_{G.K}^{(6)}$	130979.2	$\hat{\hat{Y}}_{Suggested}^{(6)}$	111735.1
		$\hat{\hat{Y}}_S^{(7)}$	149423.6	$\hat{\hat{Y}}_{G.K}^{(7)}$	130979.5	$\hat{\hat{Y}}_{Suggested}^{(7)}$	111735.6
		$\hat{\hat{Y}}_S^{(8)}$	149320.5	$\hat{\hat{Y}}_{G.K}^{(8)}$	130983.4	$\hat{\hat{Y}}_{Suggested}^{(8)}$	111742.5
		$\hat{\hat{Y}}_S^{(9)}$	151041.8	$\hat{\hat{Y}}_{G.K}^{(9)}$	130914.4	$\hat{\hat{Y}}_{Suggested}^{(9)}$	111623.0
		$\hat{\hat{Y}}_S^{(10)}$	143027.8	$\hat{\hat{Y}}_{G.K}^{(10)}$	131150.0	$\hat{\hat{Y}}_{Suggested}^{(10)}$	112028.6

Table 9. MSEs using Population II Situation-II.

Estimator	Value	Estimator	Value	Estimator	Value	Estimator	Value
$\hat{\bar{Y}}_{SRS}$	142947.8	$\hat{\bar{Y}}_S^{(1)}$	154034.9	$\hat{\bar{Y}}_{G.K}^{(1)}$	122345.9	$\hat{\bar{Y}}_{Suggested}^{(1)}$	121502.7
$\hat{\bar{Y}}_R$	168897.1	$\hat{\bar{Y}}_S^{(2)}$	153492.4	$\hat{\bar{Y}}_{G.K}^{(2)}$	122356.0	$\hat{\bar{Y}}_{Suggested}^{(2)}$	121522.9
$\hat{\bar{Y}}_P$	131700.4	$\hat{\bar{Y}}_S^{(3)}$	154071.0	$\hat{\bar{Y}}_{G.K}^{(3)}$	122345.2	$\hat{\bar{Y}}_{Suggested}^{(3)}$	121501.4
$\hat{\bar{Y}}_{Reg}$	131184.1	$\hat{\bar{Y}}_S^{(4)}$	152286.3	$\hat{\bar{Y}}_{G.K}^{(4)}$	122377.3	$\hat{\bar{Y}}_{Suggested}^{(4)}$	121565.4
$\hat{\bar{Y}}_{R,D}$	122467.5	$\hat{\bar{Y}}_S^{(5)}$	154136.6	$\hat{\bar{Y}}_{G.K}^{(5)}$	122344.0	$\hat{\bar{Y}}_{Suggested}^{(5)}$	121498.9
		$\hat{\bar{Y}}_S^{(6)}$	154264.3	$\hat{\bar{Y}}_{G.K}^{(6)}$	122341.6	$\hat{\bar{Y}}_{Suggested}^{(6)}$	121494.0
		$\hat{\bar{Y}}_S^{(7)}$	154252.5	$\hat{\bar{Y}}_{G.K}^{(7)}$	122341.8	$\hat{\bar{Y}}_{Suggested}^{(7)}$	121494.4
		$\hat{\bar{Y}}_S^{(8)}$	154098.8	$\hat{\bar{Y}}_{G.K}^{(8)}$	122344.7	$\hat{\bar{Y}}_{Suggested}^{(8)}$	121500.3
		$\hat{\bar{Y}}_S^{(9)}$	156610.1	$\hat{\bar{Y}}_{G.K}^{(9)}$	122293.9	$\hat{\bar{Y}}_{Suggested}^{(9)}$	121398.4
		$\hat{\bar{Y}}_S^{(10)}$	143130.8	$\hat{\bar{Y}}_{G.K}^{(10)}$	122467.5	$\hat{\bar{Y}}_{Suggested}^{(10)}$	121745.2

Table 10. PREs using Population II Situation-I.

Estimator	Value	Estimator	Value	Estimator	Value	Estimator	Value
$\hat{\bar{Y}}_{SRS}$	100.00	$\hat{\bar{Y}}_S^{(1)}$	95.76	$\hat{\bar{Y}}_{G.K}^{(1)}$	109.13	$\hat{\bar{Y}}_{Suggested}^{(1)}$	127.92
$\hat{\bar{Y}}_R$	89.160	$\hat{\bar{Y}}_S^{(2)}$	95.99	$\hat{\bar{Y}}_{G.K}^{(2)}$	109.12	$\hat{\bar{Y}}_{Suggested}^{(2)}$	127.90
$\hat{\bar{Y}}_P$	99.140	$\hat{\bar{Y}}_S^{(3)}$	95.74	$\hat{\bar{Y}}_{G.K}^{(3)}$	109.13	$\hat{\bar{Y}}_{Suggested}^{(3)}$	127.92
$\hat{\bar{Y}}_{Reg}$	101.24	$\hat{\bar{Y}}_S^{(4)}$	96.50	$\hat{\bar{Y}}_{G.K}^{(4)}$	109.10	$\hat{\bar{Y}}_{Suggested}^{(4)}$	127.84
$\hat{\bar{Y}}_{R,D}$	109.00	$\hat{\bar{Y}}_S^{(5)}$	95.72	$\hat{\bar{Y}}_{G.K}^{(5)}$	109.14	$\hat{\bar{Y}}_{Suggested}^{(5)}$	127.93
		$\hat{\bar{Y}}_S^{(6)}$	95.66	$\hat{\bar{Y}}_{G.K}^{(6)}$	109.14	$\hat{\bar{Y}}_{Suggested}^{(6)}$	127.93
		$\hat{\bar{Y}}_S^{(7)}$	95.67	$\hat{\bar{Y}}_{G.K}^{(7)}$	109.14	$\hat{\bar{Y}}_{Suggested}^{(7)}$	127.93
		$\hat{\bar{Y}}_S^{(8)}$	95.73	$\hat{\bar{Y}}_{G.K}^{(8)}$	109.13	$\hat{\bar{Y}}_{Suggested}^{(8)}$	127.93
		$\hat{\bar{Y}}_S^{(9)}$	94.64	$\hat{\bar{Y}}_{G.K}^{(9)}$	109.19	$\hat{\bar{Y}}_{Suggested}^{(9)}$	128.06
		$\hat{\bar{Y}}_S^{(10)}$	99.94	$\hat{\bar{Y}}_{G.K}^{(10)}$	109.00	$\hat{\bar{Y}}_{Suggested}^{(10)}$	127.60

Table 11. PREs using Population II Situation-II.

Estimator	Value	Estimator	Value	Estimator	Value	Estimator	Value
$\hat{\bar{Y}}_{SRS}$	100.00	$\hat{\bar{Y}}_{S}^{(1)}$	92.80	$\hat{\bar{Y}}_{G.K}^{(1)}$	116.84	$\hat{\bar{Y}}_{Suggested}^{(1)}$	117.65
$\hat{\bar{Y}}_{R}$	84.640	$\hat{\bar{Y}}_{S}^{(2)}$	93.13	$\hat{\bar{Y}}_{G.K}^{(2)}$	116.83	$\hat{\bar{Y}}_{Suggested}^{(2)}$	117.63
$\hat{\bar{Y}}_{P}$	108.54	$\hat{\bar{Y}}_{S}^{(3)}$	92.78	$\hat{\bar{Y}}_{G.K}^{(3)}$	116.84	$\hat{\bar{Y}}_{Suggested}^{(3)}$	117.65
$\hat{\bar{Y}}_{Reg}$	108.97	$\hat{\bar{Y}}_{S}^{(4)}$	93.87	$\hat{\bar{Y}}_{G.K}^{(4)}$	116.81	$\hat{\bar{Y}}_{Suggested}^{(4)}$	117.59
$\hat{\bar{Y}}_{R,D}$	116.72	$\hat{\bar{Y}}_{S}^{(5)}$	92.74	$\hat{\bar{Y}}_{G.K}^{(5)}$	116.84	$\hat{\bar{Y}}_{Suggested}^{(5)}$	117.65
		$\hat{\bar{Y}}_{S}^{(6)}$	92.66	$\hat{\bar{Y}}_{G.K}^{(6)}$	116.84	$\hat{\bar{Y}}_{Suggested}^{(6)}$	117.66
		$\hat{\bar{Y}}_{S}^{(7)}$	92.67	$\hat{\bar{Y}}_{G.K}^{(7)}$	116.84	$\hat{\bar{Y}}_{Suggested}^{(7)}$	117.66
		$\hat{\bar{Y}}_{S}^{(8)}$	92.76	$\hat{\bar{Y}}_{G.K}^{(8)}$	116.84	$\hat{\bar{Y}}_{Suggested}^{(8)}$	117.65
		$\hat{\bar{Y}}_{S}^{(9)}$	91.28	$\hat{\bar{Y}}_{G.K}^{(9)}$	116.89	$\hat{\bar{Y}}_{Suggested}^{(9)}$	117.75
		$\hat{\bar{Y}}_{S}^{(10)}$	99.87	$\hat{\bar{Y}}_{G.K}^{(10)}$	116.72	$\hat{\bar{Y}}_{Suggested}^{(10)}$	117.42

7. Conclusions

In this paper, a new family of estimators for estimating the population mean with information on the auxiliary variable in the form of the sample mean and ranks of the auxiliary variable in the presence of non-response has been devised. The suggested family of estimators a mathematical expressions for biases and minimum MSEs have been generated up to the first order of approximation and compared both theoretically and numerically with the [6,10,22], the conventional difference, [8,27,29] estimators under Situation-I and Situation-II. It has been observed that the proposed family of estimators is more efficient in both non-response situations.

Conflict of interest

The authors declare no conflict of interest.

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