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Research article

Modeling and analysis of fractional order Zika model

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Abstract: We propose mathematical model for the transmission of the Zika virus for humans spread by mosquitoes. We construct a scheme for the Zika virus model with Atangna-Baleanue Caputo sense and fractal fractional operator by using generalized Mittag-Leffler kernel. The positivity and boundedness of the model are also calculated. The existence of uniquene solution is derived and stability analysis has been made for the model by using the fixed point theory. Numerical simulations are made by using the Atangana-Toufik scheme and fractal fractional operator with a different dimension of fractional values which support the theoretical outcome of the proposed system. Developed scheme including simulation will provide better understanding in future analysis and for control strategy regarding Zika virus.

Keywords: Zika virus; Atangana-Baleanu in Caputo sense; Atangana-Toufik; fractal fractional; Mittag-Leffler kernel
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1. Introduction

The virus name Zika is first time found in 1947 in monkeys and the first patient was reported in

Uganda in 1952. The genus of Zika is also found in dengue, yellow fever, and West Nile virus [1]. Since the 1950s, it has been found in a very narrow area. During 2007–2016 it spread across the Pacific Ocean to the Americas which cause 2015-16 Zika virus epidemics [2]. Zika virus often causes only mild symptoms which are very similar to dengue. There is no specific treatment for this virus [3,4].

Mathematicians and biologist's main theme is to study disease. Many mathematicians tried to represent the mathematical model in a very natural way such as in the approach of Baleanu et al. [5–7]. In recent years, fractional calculus has fascinated the attention of researchers and the various features of that study under investigation. This is because genetic mutations are an important tool for defining the dynamic function of various body systems. The power of these component operators is their non-local features that are not in the integer separator operator. Separated features of differentiated statistics that define the memory and transfer structures of many mathematical models. As a fact that fractional-order models are more realistic and useful than classical integer-order models. Fractional order findings produce a greater degree of freedom in these models. Unnecessary order outsourcing is a powerful tool for understanding the dynamic behavior of various bio objects and systems. The most repetitive feature of these models is in their global (non-local) features that are not in the old order models. Fractional calculus has acquired great rating and significance over the last few years in various branches of science and engineering. Effective systematic and statistical techniques have been established but they still require special care. This distinctive problem aims to create an assemblage of articles showing the advances in mathematics and the branch of fractional calculus and to explore the applications in applied science [8–10]. Caputo [11] presented from the group that allows for common initial and borderline conditions related to a real-world problem. Baleanu et al. [13] stated advanced techniques in the field of fractional calculus and nanotechnology using monographs. Kailas et al. [14] obtainable basic ideas of equation differences including their uses are explained. Bulut et al. [15] studied the differential measurement of the orderly application of analytical methods and some related details are given in [16–21]. In recent years researchers have been using some mathematical models to simulate the transmission of the Zika virus [22–25].

The common SEIR model is generalized in order to show the dynamics of COVID-19 transmission taking into account the ABO blood group of the infected people. Fractional order Caputo derivative are used in the proposed model [28]. New system is confirmed to have chaotic behaviors by calculating its Lyapunov exponents [29]. the analytical solution using the Reduced differential transforms method (RDTM) for the nonlinear ordinary mathematical smoking model [30]. Complete synchronization between two chaotic systems means complete symmetry between them, but phase synchronization means complete symmetry with a phase shift. In addition, the proposed method is applied to the synchronization of two identical chaotic Lorenz models [31]. Important and adequate conditions to ensure the presence and singularity of the arrangements of the control issue are assumed [32]. The existence and uniqueness of stable solution of the proposed fractional order COVID-19 SEIASqEqHR paradigm are proved. The existence of a stable solution of the fractional order COVID-19 SIDARTHE model is proved and the fractional order necessary conditions of four proposed control strategies are produced [33,34]. SDM is considered as a mixture of Adomian decomposition method and the Sumudu transform method. several vital characteristics and features of this model are investigated, such as its hamiltonian, symmetry, signal flow graph, dissipation, equilibriums and their stability, Lyapunov exponents, Lyapunov dimension, bifurcation diagrams, and chaotic behavior [35–37] and some others applications of fractional order also given in [38–40].

In this paper, Section 1 consists an introduction and some basic definition of fractional-order derivatives to solve the epidemiological model respectively. Sections 3 and 4 consists of the generalized solution of the fractional-order model, consist of the uniqueness and stability of the model. Fractal fractional techniques with exponential decay kernel and Mittag-Leffler kernel are applied for suitable results in Section 5. Results and conclusion are discussed in Sections 6 and 7 respectively.

2. Basic concepts of fractional operators

Definition 2.1. For a function $g(t) \in W_2^1(0,1)$, $b > a \; and \; \sigma \in [0,1]$, the definition of Atangana-Baleanu derivative in the Caputo sense is given by

$$ {}^{ABC}_{\;\;0}D_t^{\sigma} g(t) = \frac{AB(\sigma)}{1-\sigma}\int_0^t \frac{d}{d\tau} g(\tau) M_\sigma\left[-\frac{\sigma}{1-\sigma}(t-\tau)^\sigma\right] d\tau, \quad n-1<\sigma<n \tag{1}$$

where

$$AB(\sigma) = 1-\sigma+\frac{\sigma}{\Gamma(\sigma)}.$$

By using Sumudu transform (ST) for (1), we obtain

$$ST[{}^{ABC}_{\;\;0}D_t^{\sigma} g(t)](s) = \frac{q(\sigma)}{1-\sigma}\left\{\sigma\Gamma(\sigma+1)M_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)\right\} \times \left[ST\big(g(t)\big) - g(0)\right]. \tag{2}$$

Definition 2.2. The Laplace transform of the Caputo fractional derivative of a function $g(t)$ of order $\sigma > 0$ is defined as

$$L[{}^{C}_{0}D_t^{\sigma} g(t)] = s^\sigma g(s) - \sum_{\sigma=0}^{n-1} g^{(\sigma)}(0)\, s^{\sigma-v-1}. \tag{3}$$

Definition 2.3. The Laplace transform of the function $t^{\sigma_1-1}E_{\sigma,\sigma_1}(\pm\mu t^\sigma)$ is defined as

$$L[t^{\sigma_1-1}E_{\sigma,\sigma_1}(\pm\mu t^\sigma)] = \frac{s^{\sigma-\sigma_1}}{s^\sigma \mp \mu}, \tag{4}$$

Where E_{σ,σ_1} is the two-parameter Mittag-Leffler function with $\sigma, \sigma_1 > 0$. Further, the Mittag-Leffler function satisfies the following equation [17].

$$E_{\sigma,\sigma_1}(f) = fE_{\sigma,\sigma+\sigma_1}(f) + \frac{1}{\Gamma(\sigma_1)}. \tag{5}$$

Definition 2.4. Suppose that $g(t)$ is continuous on an open interval (a,b), then the fractal-fractional integral of $g(t)$ of order σhaving Mittag-Leffler type kernel and given by

$${}^{FFM}J_{0,t}^{\sigma,\sigma_1}\big(g(t)\big) = \frac{\sigma\sigma_1}{\mathrm{AB}(\sigma)\Gamma(\sigma_1)}\int_0^t s^{\sigma_1-1} g(s)(t-s)^\sigma ds + \frac{\sigma_1(1-\sigma)t^{\sigma_1-1}g(t)}{\mathrm{AB}(\sigma)}. \tag{6}$$

3. Material and method

In this portion, we give a mathematical model for the transmission of the Zika virus using the

Atangna-Baleanue in Caputo sense of fractional order. We make two portions of the Human population: Susceptible people Sp and infected people Ip so that $Np = Sp + Ip$. Same as we make two portions of a total number of mosquitoes Nqinto two groups: Susceptible mosquitoes Sq and infected mosquitoes Iq, so that $Nq = Sq + Iq$. To explain the method of the spread of the Zika virus given in [26], we consider the compartmental mathematical model as follows:

$$
\begin{aligned}
{}^{ABC}_{0}D^{\sigma}_{t} S_p &= \Lambda_p - \beta_1 S_p I_p - \beta_2 S_p I_q - \kappa_1 S_p, \\
{}^{ABC}_{0}D^{\sigma}_{t} I_p &= \beta_1 S_p I_p + \beta_2 S_p I_q - \kappa_1 I_p, \\
{}^{ABC}_{0}D^{\sigma}_{t} S_q &= \Lambda_q - \mu S_q I_p - \kappa_2 S_q, \\
{}^{ABC}_{0}D^{\sigma}_{t} I_q &= \mu S_q I_p - \kappa_2 I_q.
\end{aligned}
\tag{7}
$$

with the initial conditions

$$S_p(0) \geq 0, I_p(0) \geq 0\ , \ S_q(0) \geq 0, I_q(0)\ \geq 0. \tag{8}$$

The model parameters are: The recruitment rate of human population p, the recruitment rate of mosquito population q, the effective contact rate human to humanβ_1, the effective contact rate of mosquitoes to humanβ_2, the effective contact rate human to mosquito's μ, the natural death rate of human k_1, the natural death rate of mosquitoes k_2.

Equilibrium points

In this section, we will discuss the equilibrium points of the given Zika Virus model (7). Equilibrium points have two types namely disease-free equilibrium and endemic equilibrium. We obtained these points by putting the right-hand side of the system (7) is zero. We suppose that E' represents disease-free equilibrium and endemic equilibrium is represented by E*. We take our both equilibriums by, we have

$$E' = \left(S_p{}', I_p{}', S_q{}', S_q{}'\right) = \left(\frac{\lambda}{d+\mu}, \frac{\alpha}{d+\mu}, 0{,}0\right),$$

$$S^*{}_p = \frac{\mathrm{k2k1}}{(\beta 2 \mu \mathrm{S}{*}\mathrm{q} + \mathrm{k2}\beta 1)}, I^*{}_p = \frac{\Lambda_p \beta_2 \mu S^*{}_q + \Lambda_p \kappa_2 \beta_1 - \kappa_2 \kappa_1{}^2}{\kappa_1\left(\beta_2 \mu S^*{}_q + \kappa_2 \beta_1\right)}, I^*{}_q = \frac{\mu\left(\Lambda_p \beta_2 \mu S^*{}_q + \Lambda_p \kappa_2 \beta_1 - \kappa_2 \kappa_1{}^2\right) S^*{}_q}{\kappa_1\left(\beta_2 \mu S^*{}_q + \kappa_2 \beta_1\right)\kappa_1}.$$

Reproductive number R_0 given in [26], we have

$$R_0 = \frac{\beta_1 \kappa_2 \Lambda_p + \sqrt{\beta_1{}^2 \kappa_2{}^2 \Lambda_p{}^2} + 4\kappa_1{}^2 \beta_2 \mu \Lambda_p \Lambda_q}{2\kappa_2 \kappa_1{}^2}.$$

Theorem 3.1. The solution of the proposed fractional-order model (7) along initial conditions is unique and bounded in R^4_+.

Proof.

The existence and uniqueness of the solution of the system (7) on the time interval $(0,\infty)$ can be obtained. Subsequently, we have to explain the non-negative region R_+^4 is a positively invariant region. From model (7) we find

$$ {}_{0}^{ABC}D_t^{\alpha} S_p|_{S_p=0} = \Lambda_p \geq 0, $$

$$ {}_{0}^{ABC}D_t^{\alpha} I_p|_{I_p=0} = \beta_2 S_p(\mathrm{t}) \geq 0, $$

$$ {}_{0}^{ABC}D_t^{\alpha} S_q|_{S_q=0} = \Lambda_q \geq 0, $$

$$ {}_{0}^{ABC}D_t^{\alpha} I_q|_{I_q=0} = \mu S_q(\mathrm{t}) \geq 0. $$

If $(S_p(0), I_p(0), S_q(0), I_q(0)) \in R_+^4$, then according to Eq (7) the solution $[S_p(t), I_p(t), S_q(t), I_q(t)]$ cannot escape from the hyperplanes $S_p = 0,\ I_p = 0,\ S_q = 0\ and\ I_q = 0$. Also on each hyperplane bounding the non-negative orthant, the vector field points into R_+^4, i.e., the domain R_+^4 is a positively invariant set.

Theorem 3.2. The region $A = \{(S_p(t), I_p(t), S_q(t), I_q(t)) \in R_+^4 | 0 < S_p(t) + I_p(t) \leq \frac{\Lambda_p}{\kappa_1}, S_q(t) + I_q(t) \leq \frac{\Lambda_q}{\kappa_1}\}$ is a positively invariant set for the system (7).

Proof. For the proof of the theorem, firstly we use the first to equations of system (7). So

$$ {}_{0}^{ABC}D_t^{\alpha} \mathrm{N}_p(t) = \Lambda_p - \kappa_1 \mathrm{N}_p(t), $$

where

$$ \mathrm{N}_p(t) = S_p(t) + I_p(t), $$

we get

$$ s^{\alpha}\mathrm{N}_p(s) - s^{\alpha-1}\mathrm{N}_p(0) = \frac{\Lambda_p}{s} - \kappa_1 \mathrm{N}_p(s), $$

which further gives

$$ \mathrm{N}_p(s) = \frac{s^{-1}}{s^{\alpha}+\kappa_1}\Lambda_p + \frac{s^{\alpha-1}}{s^{\alpha}+\kappa_1}\mathrm{N}_p(0). $$

We infer that if $(S_{0p}, I_{0p}) \in R_+^4$, then

$$ \begin{aligned} \mathrm{N}_p(t) &= \Lambda_p \mathrm{t}^{\alpha} E_{\alpha,\alpha+1}(-\kappa_1 \mathrm{t}^{\alpha}) + E_{\alpha,1}(-\kappa_1 \mathrm{t}^{\alpha}) \\ &= \frac{(\Omega-\delta)}{\kappa_1}\left(\kappa_1 \mathrm{t}^{\alpha} E_{\alpha,\alpha+1}(-\kappa_1 \mathrm{t}^{\alpha})\right) + E_{\alpha,1}(-\kappa_1 \mathrm{t}^{\alpha}) \\ &= \frac{\Lambda_p}{\kappa_1}\frac{1}{\Gamma(1)} \\ &= \frac{\Lambda_p}{\kappa_1}. \end{aligned} $$

Similarly, we can prove for $\mathrm{N}_q(t) = S_q(t) + I_q(t)$ that if $\mathrm{N}_q(t) = \frac{\Lambda_q}{\kappa_1}$.

4. Atangana-Baleanu Caputo sense

In this section, coniseder the system with Atangana-Baleanu fractional derivative (ABC) of orderσ and $\sigma \in (0,1]$ for sytem (7), we

$$
\begin{aligned}
{}^{ABC}_{0}D_t^{\sigma} S_p &= \Lambda_p - \beta_1 S_p I_p - \beta_2 S_p I_q - \kappa_1 S_p, \\
{}^{ABC}_{0}D_t^{\sigma} I_p &= \beta_1 S_p I_p + \beta_2 S_p I_q - \kappa_1 I_p, \\
{}^{ABC}_{0}D_t^{\sigma} S_q &= \Lambda_q - \mu S_q I_p - \kappa_2 S_q, \\
{}^{ABC}_{0}D_t^{\sigma} I_q &= \mu S_q I_p - \kappa_2 I_q.
\end{aligned} \tag{9}
$$

By applying the definition (2) of sumudu transform in ABC sense, we have

$$\frac{q(\sigma)\sigma\Gamma(\sigma+1)}{1-\sigma} N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right) ST\{S_p(t) - S_p(0)\} = ST\left[\Lambda_p - \beta_1 S_p I_p - \beta_2 S_p I_q - \kappa_1 S_p\right],$$

$$\frac{q(\sigma)\sigma\Gamma(\sigma+1)}{1-\sigma} N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right) ST\{I_p(t) - I_p(0)\} = ST\left[\beta_1 S_p I_p + \beta_2 S_p I_q - \kappa_1 I_p\right],$$

$$\frac{q(\sigma)\sigma\Gamma(\sigma+1)}{1-\sigma} N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right) ST\{S_q(t) - S_q(0)\} = ST\left[\Lambda_q - \mu S_q I_p - \kappa_2 S_q\right],$$

$$\frac{q(\sigma)\sigma\Gamma(\sigma+1)}{1-\sigma} N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right) ST\{I_q(t) - I_q(0)\} = ST\left[\mu S_q I_p - \kappa_2 I_q\right].$$

Rearranging, we get

$$
\begin{aligned}
ST\left(S_p(t)\right) &= S_p(0) + \frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} ST\left[\Lambda_p - \beta_1 S_p I_p - \beta_2 S_p I_q - \kappa_1 S_p\right], \\
ST\left(I_p(t)\right) &= I_p(0) + \frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} ST\left[\beta_1 S_p I_p + \beta_2 S_p I_q - \kappa_1 I_p\right], \\
ST\left(S_q(t)\right) &= S_q(0) + \frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} ST\left[\Lambda_q - \mu S_q I_p - \kappa_2 S_q\right], \\
ST\left(I_q(t)\right) &= I_q(0) + \frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)} ST\left[\mu S_q I_p - \kappa_2 I_q\right].
\end{aligned} \tag{10}
$$

Now taking the inverse Sumudu transform on both sides of the Eq (10) we get

$$S_p(t) = S_p(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left[\Lambda_p - \beta_1 S_p I_p - \beta_2 S_p I_q - \kappa_1 S_p\right]\right],$$

$$I_p(t) = I_p(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left[\beta_1 S_p I_p + \beta_2 S_p I_q - \kappa_1 I_p\right]\right],$$

$$S_q(t) = S_q(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left[\Lambda_q - \mu S_q I_p - \kappa_2 S_q\right]\right],$$

$$I_q(t) = I_q(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left[\mu S_q I_p - \kappa_2 I_q\right]\right],.$$

We next attain the following recursive formula:

$$S_{p_{(n+1)}}(t) = S_{p_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\{\Lambda_p - \beta_1 S_{p_n} I_{p_n} - \beta_2 S_{p_n} I_{q_n} - \kappa_1 S_{p_n}\right],$$

$$I_{p_{(n+1)}}(t) = I_{p_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\beta_1 S_{p_n} I_{p_n} + \beta_2 S_{p_n} \mathrm{I}_{q_n} - \kappa_1 I_{p_n}\right\}\right],$$

$$S_{q_{n+1}}(t) = S_{q_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\Lambda_q - \mu S_{q_n} I_{p_n} - \kappa_1 S_{q_n}\right\}\right], \qquad (11)$$

$$I_{q_{(n+1)}}(t) = I_{q_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\mu S_{q_n} I_{p_n} - \kappa_1 I_{q_n}\right\}\right].$$

And solution of above is

$$S_p(t) = \lim_{n\to\infty} S_{p_n}(t),\ I_p(t) = \lim_{n\to\infty} I_{p_n}(t),$$

$$S_q(t) = \lim_{n\to\infty} S_{q_n}(t),\ I_q(t) = \lim_{n\to\infty} I_{q_n}(t).$$

Uniqueness and stability of the iterative scheme

Theorem 4.1. Let $(X_2, ||.||)$ be a Banach space and M be a self-map of X_2 satisfying $\|K_X - K_Y\| \leq$ C‖X−K_X‖+C‖X-Y‖ for all x, y $\in X_2$ where $0\leq C, 0 \leq c < 1$.

Let consider that M is P-stable. Let us take into account the following recursive formula:

$$S_{p_{(n+1)}}(t) = S_{p_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\{\Lambda_p - \beta_1 S_{p_n} I_{p_n} - \beta_2 S_{p_n} I_{q_n} - \kappa_1 S_{p_n}\right],$$

$$I_{p_{(n+1)}}(t) = I_{p_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\beta_1 S_{p_n} I_{p_n} + \beta_2 S_{p_n} \mathrm{I}_{q_n} - \kappa_1 I_{p_n}\right\}\right],$$

$$S_{q_{n+1}}(t) = S_{q_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\Lambda_q - \mu S_{q_n} I_{p_n} - \kappa_1 S_{q_n}\right\}\right],$$

$$I_{q_{(n+1)}}(t) = I_{q_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\mu S_{q_n} I_{p_n} - \kappa_1 I_{q_n}\right\}\right],$$

$$I_{q_{(n+1)}}(t) = I_{q_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\mu S_{q_n} I_{p_n} - \kappa_1 I_{q_n}\right\}\right],$$

where $\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\alpha+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}$ is the fractional Lagrange multiplier.

Theorem 4.2. Define M be a self-map is given by

$$M[S_{p_{(n+1)}}(t)] = S_{p_n}(0)$$
$$+ ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\{\Lambda_p - \beta_1 S_{p_n} I_{p_n} - \beta_2 S_{p_n} I_{q_n} - \kappa_1 S_{p_n}\right],$$

$$M[I_{p_{(n+1)}}(t)] = I_{p_n}(0)$$
$$+ ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\beta_1 S_{p_n} I_{p_n} + \beta_2 S_{p_n} \mathrm{I}_{q_n} - \kappa_1 I_{p_n}\right\}\right],$$

$$M[S_{q_{n+1}}(t)] = S_{q_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\Lambda_q - \mu S_{q_n}I_{p_n} - \kappa_1 S_{q_n}\right\}\right],$$

$$M[I_{q_{(n+1)}}(t)] = I_{q_n}(0) + ST^{-1}\left[\frac{1-\sigma}{q(\sigma)\sigma\Gamma(\sigma+1)N_\sigma\left(-\frac{1}{1-\sigma}V^\sigma\right)}ST\left\{\mu S_{q_n}I_{p_n} - \kappa_1 I_{q_n}\right\}\right],$$

Proof. In the first step we will show that M is a fixed point

$$\forall(m,n) \in N \times N,$$

$$M\left(S_{p_n}(t)\right) - M\left(S_{p_m}(t)\right) = S_{p_n}(t) - S_{p_m}(t) + ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\alpha+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\left[\Lambda_p - \beta_1 S_{p_n}I_{p_n} - \beta_2 S_{p_n}I_{q_n} - \kappa_1 S_{p_n}\right]\right] -$$

$$ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\alpha+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\left[\Lambda_p - \beta_1 S_{p_m}I_{p_m} - \beta_2 S_{p_m}I_{q_m} - \kappa_1 S_{p_m}\right]\right],$$

$$\mathrm{M}\left(I_{p_n}(t)\right) - M\left(I_{p_m}(t)\right) = I_{p_n}(t) - I_{p_m}(t) + ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\vartheta+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\left[\beta_1 S_{p_n}I_{p_n} + \beta_2 S_{p_n}\mathrm{I}_{q_n} - \kappa_1 I_{p_n}\right]\right] - ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\vartheta+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\left[\beta_1 S_{p_m}I_{p_m} + \beta_2 S_{p_m}\mathrm{I}_{q_m} - \kappa_1 I_{p_m}\right]\right], \tag{12}$$

$$M\left(S_{q_n}(t)\right) - M\left(S_{q_m}(t)\right) = S_{q_n}(t) - S_{q_m}(t) + ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\alpha+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\left[\Lambda_q - \mu S_{q_n}I_{p_n} - \kappa_1 S_{q_n}\right]\right] - ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\alpha+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\left[\Lambda_q - \mu S_{q_m}I_{p_n} - \kappa_1 S_{q_m}\right]\right],$$

$$M\left(I_{q_n}(t)\right) - M\left(I_{q_m}(t)\right) = I_{q_n}(t) - I_{q_m}(t) + ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\alpha+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\left[\mu S_{q_n}I_{p_n} - \kappa_1 I_{q_n}\right]\right] - ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\alpha+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\left[\mu S_{q_m}I_{p_m} - \kappa_1 I_{q_m}\right]\right].$$

Applying the properties of the norm and also taking into account the triangular inequality, we obtain

$$\left\|\mathrm{M}\left(S_{p_n}(t)\right)-M\left(S_{p_m}(t)\right)\right\| \leq \left\|S_{p_n}(t)-S_{p_m}(t)\right\| ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\vartheta+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\{\left\|\Lambda_p-\beta_1 S_{p_n} I_{p_n}-\beta_2 S_{p_n} I_{q_n}-\kappa_1 S_{p_n}\right\|+\left\|\Lambda_p-\beta_1 S_{p_m} I_{p_m}-\beta_2 S_{p_m} I_{q_m}-\kappa_1 S_{p_m}\right\|\right],$$

$$\left\|\mathrm{M}\left(I_{p_n}(t)\right)-M\left(I_{p_m}(t)\right)\right\| \leq \left\|I_{p_n}(t)-I_{p_m}(t)\right\|+ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\vartheta+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}\times ST\left\{\left\|\beta_1 S_{p_n} I_{p_n}+\beta_2 S_{p_n}\mathrm{I}_{q_n}-\kappa_1 I_{p_n}\right\|+\left\|\beta_1 S_{p_m} I_{p_m}+\beta_2 S_{p_m}\mathrm{I}_{q_m}-\kappa_1 I_{p_m}\right\|\right\}\right], \tag{13}$$

$$\left\|\mathrm{M}\left(S_{q_n}(t)\right)-M\left(S_{q_m}(t)\right)\right\| \leq \left\|q_n(t)-q_m(t)\right\| ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\vartheta+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\{\left\|\Lambda_q-\mu S_{q_n} I_{p_n}-\kappa_1 S_{q_n}\right\|+\left\|\Lambda_q-\mu S_{q_m} I_{p_n}-\kappa_1 S_{q_m}\right\|\right],$$

$$\left\|\mathrm{M}\left(I_{q_n}(t)\right)-M\left(I_{q_m}(t)\right)\right\| \leq \left\|I_{q_n}(t)-I_{q_m}(t)\right\| ST^{-1}\left[\frac{1-\vartheta}{B(\vartheta)\vartheta\Gamma(\vartheta+1)N_\vartheta\left(-\frac{1}{1-\vartheta}w^\vartheta\right)}ST\{\left\|\mu S_{q_n} I_{p_n}-\kappa_1 I_{q_n}\right\|+\left\|\mu S_{q_m} I_{p_m}-\kappa_1 I_{q_m}\right\|\right].$$

K fulfills the conditions associated with Theorem (4.1), when

θ=(0,0,0,0,0)=

$$\begin{cases} \left\|S_{p_n}(t)-S_{p_m}(t)\right\| \times \left\|-\left(S_{p_n}(t)+S_{p_m}(t)\right)\right\|+\Lambda_p-\beta_1\left\|S_{p_n}I_{p_n}-S_{p_m}I_{p_m}\right\|-\beta_2\left\|S_{p_n}I_{q_n}-S_{p_m}I_{q_m}\right\| \\ \quad -\kappa_1\left\|S_{p_n}-S_{p_m}\right\| \\ \times\left\|I_{p_n}(t)-I_{p_m}(t)\right\| \times \left\|-\left(I_{p_n}(t)+I_{p_m}(t)\right)\right\|+\beta_1\left\|S_{p_n}I_{p_n}-S_{p_m}I_{p_m}\right\|+\beta_2\left\|S_{p_n}I_{q_n}-S_{p_m}I_{q_m}\right\| \\ \quad -\kappa_1\left\|I_{p_n}-I_{p_m}\right\| \\ \times\left\|S_{q_n}(t)-S_{q_m}(t)\right\| \times \left\|-\left(S_{q_n}(t)+S_{q_m}(t)\right)\right\|+\Lambda_q-\mu\left\|S_{q_n}I_{p_n}-S_{q_m}I_{p_m}\right\|-\kappa_1\left\|S_{q_n}-S_{q_m}\right\| \\ \times\left\|I_{q_n}(t)-I_{q_m}(t)\right\| \times \left\|-\left(I_{q_n}(t)+I_{q_m}(t)\right)\right\|+\mu\left\|S_{q_n}I_{p_n}-S_{q_m}I_{p_m}\right\|-\kappa_1\left\|I_{q_n}-I_{q_m}\right\|. \end{cases}$$

Hence system is Picard P-Stable.

Theorem 4.3. Prove that system (11) has a special solution is unique.

Proof. Let Hilbert space $H=L^2\big((p,q)\times(0,T)\big)$ which is given as

$$h:(p,q)\times(0,T)\to\mathbb{R}, \iint ghdgdh<\infty.$$

In this regard, the following operators are considered

$$\theta(0,0,0,0,0), \theta=\begin{cases}\Lambda_p-\beta_1 S_p I_p-\beta_2 S_p I_q-\kappa_1 S_p, \\ \beta_1 S_p I_p+\beta_2 S_p I_q-\kappa_1 I_p, \\ \Lambda_q-\mu S_q I_p-\kappa_2 S_q, \\ \mu S_q I_p-\kappa_2 I_q.\end{cases} \tag{14}$$

We establish that the inner product of

$$P\left(\left(S_{p_{11}}-S_{p_{12}}, I_{p_{21}}-I_{p_{22}}, S_{q_{31}}-S_{q_{32}}, I_{q_{41}}-I_{q_{42}}\right),(V_1,V_2,V_3,V_4)\right).$$

Where$\left(S_{p_{11}}-S_{p_{12}}, I_{p_{21}}-I_{p_{22}}, S_{q_{31}}-S_{q_{32}}, I_{q_{41}}-I_{q_{42}},\right)$, are the special solutions of the system. Taking into account the inner function and the norm, we have

$$\{\Lambda_p-\beta_1\left(S_{p_{11}}-S_{p_{12}}\right)\left(I_{p_{21}}-I_{p_{22}}\right)-\beta_2\left(S_{p_{11}}-S_{p_{12}}\right)\left(I_{q_{41}}-I_{q_{42}}\right)-\kappa_1\left(S_{p_{11}}-S_{p_{12}}\right)\}V_1$$
$$\leq \Lambda_p\|V_1\|+\beta_1\left\|S_{p_{11}}-S_{p_{12}}\right\|\left\|I_{p_{21}}-I_{p_{22}}\right\|\|V_1\|$$
$$+\beta_2\left\|S_{p_{11}}-S_{p_{12}}\right\|\left\|I_{q_{41}}-I_{q_{42}}\right\|\|V_1\|.$$

$$\left\{\beta_1\left(S_{p_{11}}-S_{p_{12}}\right)\left(I_{p_{21}}-I_{p_{22}}\right)+\beta_2\left(S_{p_{11}}-S_{p_{12}}\right)\left(I_{q_{41}}-I_{q_{42}}\right)-\kappa_1\left(I_{p_{21}}-I_{p_{22}}\right)\right\}V_2\leq\beta_1\left\|S_{p_{11}}-S_{p_{12}}\right\|\left\|I_{p_{21}}-I_{p_{22}}\right\|\|V_2\|+\beta_2\left\|S_{p_{11}}-S_{p_{12}}\right\|\left\|I_{q_{41}}-I_{q_{42}}\right\|\|V_2\|+\kappa_1\left\|I_{p_{21}}-I_{p_{22}}\right\|\|V_2\|. \tag{15}$$

$$\left\{\Lambda_q-\mu\left(S_{q_{31}}-S_{q_{32}}\right)\left(I_{p_{21}}-I_{p_{22}}\right)-\kappa_2\left(S_{q_{31}}-S_{q_{12}}\right)\right\}V_3$$
$$\leq \Lambda_p\|V_3\|-\mu\left\|S_{q_{31}}-S_{q_{32}}\right\|\left\|I_{p_{21}}-I_{p_{22}}\right\|\|V_3\|-\kappa_2\left\|S_{q_{31}}-S_{q_{12}}\right\|\|V_3\|.$$

$$\left\{\mu\left(S_{q_{31}}-S_{q_{32}}\right)\left(I_{p_{21}}-I_{p_{22}}\right)-\kappa_2\left(I_{q_{41}}-I_{q_{42}}\right)\right\}V_4\leq\mu\left\|S_{q_{31}}-S_{q_{32}}\right\|\left\|I_{p_{21}}-I_{p_{22}}\right\|\|V_4\|-\kappa_2\left\|I_{q_{41}}-I_{q_{42}}\right\|\|V_4\|.$$

Due to the large number of e_1, e_2, e_3, e_4 and e_5, both solutions converge to the exact solution. Applying the topological idea, we have the very small positive five parameters $\left(\chi_{e_1},\chi_{e_2},\chi_{e_3},\chi_{e_4} and \chi_{e_5}\right)$.

$$\left\|S_p-S_{p_{11}}\right\|,\left\|S_p-S_{p_{12}}\right\|\leq\frac{\chi_{e_1}}{\varpi},$$

$$\left\|I_p-I_{p_{21}}\right\|,\left\|I_p-I_{p_{22}}\right\|\leq\frac{\chi_{e_2}}{\varsigma},$$

$$\left\|S_q-S_{q_{31}}\right\|,\left\|S_q-S_{q_{32}}\right\|\leq\frac{\chi_{e_3}}{\upsilon}, \tag{16}$$

$$\left\|I_p-I_{q_{41}}\right\|,\left\|\mathrm{I}_p-I_{q_{42}}\right\|\leq\frac{\chi_{e_4}}{\kappa},$$

$$\varpi = 5\left(\Lambda_p + \beta_1 \left\|S_{p_{11}} - S_{p_{12}}\right\| \left\|I_{p_{21}} - I_{p_{22}}\right\| + \beta_2 \left\|S_{p_{11}} - S_{p_{12}}\right\| \left\|I_{q_{41}} - I_{q_{42}}\right\|\right) \|V_1\|,$$

$$\varsigma = 5\left(\beta_1 \left\|S_{p_{11}} - S_{p_{12}}\right\| \left\|I_{p_{21}} - I_{p_{22}}\right\| + \beta_2 \left\|S_{p_{11}} - S_{p_{12}}\right\| \left\|I_{q_{41}} - I_{q_{42}}\right\| + \kappa_1 \left\|I_{p_{21}} - I_{p_{22}}\right\|\right) \|V_2\|,$$

$$\upsilon = 5\left(\Lambda_p - \mu \left\|S_{q_{31}} - S_{q_{32}}\right\| \left\|I_{p_{21}} - I_{p_{22}}\right\| - \kappa_2 \left\|S_{q_{31}} - S_{q_{12}}\right\|\right) \|V_3\|, \tag{17}$$

$$\kappa = 5\left(\mu \left\|S_{q_{31}} - S_{q_{32}}\right\| \left\|I_{p_{21}} - I_{p_{22}}\right\| - \kappa_2 \left\|I_{q_{41}} - I_{q_{42}}\right\|\right) \|V_4\|.$$

But, it is obvious that

$$\left(\Lambda_p + \beta_1 \left\|S_{p_{11}} - S_{p_{12}}\right\| \left\|I_{p_{21}} - I_{p_{22}}\right\| + \beta_2 \left\|S_{p_{11}} - S_{p_{12}}\right\| \left\|I_{q_{41}} - I_{q_{42}}\right\|\right) \neq 0,$$

$$\left(\beta_1 \left\|S_{p_{11}} - S_{p_{12}}\right\| \left\|I_{p_{21}} - I_{p_{22}}\right\| + \beta_2 \left\|S_{p_{11}} - S_{p_{12}}\right\| \left\|I_{q_{41}} - I_{q_{42}}\right\| + \kappa_1 \left\|I_{p_{21}} - I_{p_{22}}\right\|\right) \neq 0,$$

$$\left(\Lambda_p - \mu \left\|S_{q_{31}} - S_{q_{32}}\right\| \left\|I_{p_{21}} - I_{p_{22}}\right\| - \kappa_2 \left\|S_{q_{31}} - S_{q_{12}}\right\|\right) \neq 0,$$

$$\left(\mu \left\|S_{q_{31}} - S_{q_{32}}\right\| \left\|I_{p_{21}} - I_{p_{22}}\right\| - \kappa_2 \left\|I_{q_{41}} - I_{q_{42}}\right\|\right) \neq 0,$$

where $\|V_1\|, \|V_2\|, \|V_3\|, \|V_4\|, \|V_5\| \neq 0$.
Therefore, we have

$$\left\|S_{p_{11}} - S_{p_{12}}\right\| = 0, \left\|I_{p_{21}} - I_{p_{22}}\right\| = 0,$$

$$\left\|S_{q_{31}} - S_{q_{12}}\right\| = 0, \left\|I_{q_{41}} - I_{q_{42}}\right\| = 0,$$

which yields that

$$S_{p_{11}} = S_{p_{12}}, I_{p_{21}} = I_{p_{22}}, S_{q_{31}} = S_{q_{12}}, I_{q_{41}} = I_{q_{42}}.$$

Hence proved.

5. Numerical scheme with Atangana-Toufik

In this section we consider the Atangana-Toufik technique given in [27] for fractional derivative model (7). For this purpose, we suppose that

$$\begin{cases} {}^{ABC}_{0}DA(t) = f\big(t, A(t)\big), \\ \qquad A(0) = A_0. \end{cases} \tag{18}$$

We express the Eq (8) in the form of a fractional integral equation by applying the fundamental theorem of fractional calculus.

$$A(t)-A(0)=\frac{(1-\sigma)}{ABC(\sigma)}f\big(t,A(t)\big)+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\int_0^t f\big(\tau,A(\tau)\big)(t-\tau)^{\sigma-1}d\tau. \tag{19}$$

At a given point $t_{n+1}, n=0,1,2,3,...,$ the above equation is reformulated as

$$A(t_{n+1})-A(0)=\frac{(1-\sigma)}{ABC(\sigma)}f\big(t_n,A(t_n)\big)+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\int_0^{t_{n+1}} f\big(\tau,A(\tau)\big)(t_{n+1}-\tau)^{\sigma-1}d\tau$$

$$=\frac{(1-\sigma)}{ABC(\sigma)}f\big(t_n,A(t_n)\big)+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} f\big(\tau,A(\tau)\big)(t_{n+1}-\tau)^{\sigma-1}d\tau. \tag{20}$$

Within the interval $[t_k,t_{k+1}]$, the function $f\big(\tau,A(\tau)\big)$, using the two-steps Lagrange polynomial interpolation can be approximate as follows:

$$P_k(\tau)=\frac{\tau-t_{j-1}}{t_j-t_{j-1}}f\left(t_j,A(t_j)\right)-\frac{\tau-t_j}{t_j-t_{j-1}}f\left(t_{j-1},A(t_{j-1})\right),$$

$$=\frac{f\left(t_j,A(t_j)\right)}{h}\big(\tau-t_{j-1}\big)-\frac{f\left(t_{j-1},A(t_{j-1})\right)}{h}\big(\tau-t_j\big),$$

$$\cong\frac{f(t_j,A_j)}{h}\big(\tau-t_{j-1}\big)-\frac{f(t_{j-1},A_{j-1})}{h}\big(\tau-t_j\big). \tag{21}$$

The above approximation can therefore be included in equation (18) to produce

$$A_{n+1}=A_0+\frac{(1-\sigma)}{ABC(\sigma)}f\big(t_n,A(t_n)\big)+\frac{\sigma}{\Gamma(\sigma)\times ABC(\sigma)}\sum_{j=0}^{n}\Big(\frac{f(t_j,A_j)}{h}\int_{t_j}^{t_{j+1}}\big(\tau-t_{j-1}\big)(t_{n+1}-\tau)^{\sigma-1}d\tau-\frac{f(t_{j-1},A_{j-1})}{h}\int_{t_j}^{t_{j+1}}\big(\tau-t_j\big)(t_{n+1}-\tau)^{\sigma-1}d\tau\Big). \tag{22}$$

For simplicity, we let

$$B_{a,j,1}=\int_{t_j}^{t_{j+1}}\big(\tau-t_{j-1}\big)(t_{n+1}-\tau)^{\sigma-1}d\tau,$$

and also

$$B_{a,j,2}=\int_{t_j}^{t_{j+1}}\big(\tau-t_j\big)(t_{n+1}-\tau)^{\sigma-1}d\tau,$$

$$B_{a,j,1}=h^{\sigma+1}\frac{(m+1-j)^{\sigma}(m-j+2+\sigma)-(m-j)^{\sigma}(m-j+2+2\sigma)}{\sigma(\sigma+1)}, \tag{23}$$

$$B_{a,j,2}=h^{\sigma+1}\frac{(m+1-j)^{\sigma+1}-(m-j)^{\sigma}(m-j+1+\sigma)}{\sigma(\sigma+1)}. \tag{24}$$

By using Eqs (23) and (24) we obtain

$$A_{n+1} = A_0 + \frac{(1-\sigma)}{ABC(\sigma)} f\big(t_n, A(t_n)\big) + \frac{\sigma}{ABC(\sigma)} \sum_{j=0}^{n} \left(\frac{h^\sigma f(t_j, A_j)}{\Gamma(\sigma+2)} (p_1 p_2 - p_3 p_4) - \right.$$

$$\left. \frac{h^\sigma f(t_{j-1}, A_{j-1})}{\Gamma(\sigma+2)} (p_5 - p_3 p_6) \right), \tag{25}$$

where

$p_1 = (m+1-j)^\sigma,\ p_2 = (m-j+2+\sigma),\ p_3 = (m-j)^\sigma,$

$p_4 = (m-j+2+2\sigma),\ p_5 = (m+1-j)^{\sigma+1},\ p_6 = (m-j+1+\sigma).$

We obtain the following for model (7)

$$S_{p_{(n+1)}} = S_{p_0} + \frac{(1-\sigma)}{ABC(\sigma)} f\left(t_n, S_p(t_n)\right) + \frac{\sigma}{ABC(\sigma)} \sum_{j=0}^{n} \left(\frac{h^\sigma f\left(t_j, S_{p_j}\right)}{\Gamma(\sigma+2)} \left(p_1 p_2 - p_3 p_4\right) - \frac{h^\sigma f\left(t_{j-1}, S_{p_{j-1}}\right)}{\Gamma(\sigma+2)} \left(p_5 - p_3 p_6\right) \right),$$

$$I_{p_{(n+1)}} = I_{p_0} + \frac{(1-\sigma)}{ABC(\sigma)} f\left(t_n, I_p(t_n)\right) + \frac{\sigma}{ABC(\sigma)} \sum_{j=0}^{n} \left(\frac{h^\sigma f\left(t_j, I_{p_j}\right)}{\Gamma(\sigma+2)} \left(p_1 p_2 - p_3 p_4\right) - \frac{h^\sigma f\left(t_{j-1}, I_{p_{j-1}}\right)}{\Gamma(\sigma+2)} \left(p_5 - p_3 p_6\right) \right),$$

$$S_{q_{n+1}} = S_{q_0} + \frac{(1-\sigma)}{ABC(\sigma)} f\left(t_n, S_q(t_n)\right) + \frac{\sigma}{ABC(\sigma)} \sum_{j=0}^{n} \left(\frac{h^\sigma f\left(t_j, S_{q_j}\right)}{\Gamma(\sigma+2)} \left(p_1 p_2 - p_3 p_4\right) - \frac{h^\sigma f\left(t_{j-1}, S_{q_{j-1}}\right)}{\Gamma(\sigma+2)} \left(p_5 - p_3 p_6\right) \right),$$

$$I_{q_{(n+1)}} = I_{q_0} + \frac{(1-\sigma)}{ABC(\sigma)} f\left(t_n, I_q(t_n)\right) + \frac{\sigma}{ABC(\sigma)} \sum_{j=0}^{n} \left(\frac{h^\sigma f\left(t_j, I_{q_j}\right)}{\Gamma(\sigma+2)} \left(p_1 p_2 - p_3 p_4\right) - \frac{h^\sigma f\left(t_{j-1}, I_{q_{j-1}}\right)}{\Gamma(\sigma+2)} \left(p_5 - p_3 p_6\right) \right).$$

6. Zika model with fractal fractional operator

In this section, we consider the Zika virus model (7) with fractal-fractional in ABC sense. We have

$$\begin{aligned}
{}^{FF}D_{0,\tau}^{\alpha_1,\alpha_2} S_p &= \Lambda_p - \beta_1 S_p I_p - \beta_2 S_p I_q - \kappa_1 S_p, \\
{}^{FF}D_{0,\tau}^{\alpha_1,\alpha_2} I_p &= \beta_1 S_p I_p + \beta_2 S_p I_q - \kappa_1 I_p, \\
{}^{FF}D_{0,\tau}^{\alpha_1,\alpha_2} S_q &= \Lambda_q - \mu S_q I_p - \kappa_2 S_q, \\
{}^{FF}D_{0,\tau}^{\alpha_1,\alpha_2} I_q &= \mu S_q I_p - \kappa_2 I_q.
\end{aligned} \tag{26}$$

The fractal-fractional Zika virus model algorithm for (19), we need to generalize the system and present steps by considering the Cauchy problem as below:

$${}^{FFM}_{\ \ 0}D_t^{\alpha_1,\alpha_2} y(t) = f\big(t, y(t)\big), \tag{27}$$

after integrating the above equation, we get:

$$y(t)-y(0)=\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2 t^{\alpha_2-1}f\big(t,y(t)\big)+\frac{\alpha_1\alpha_2}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^t \tau^{\alpha_2-1}f\big(\tau,y(\tau)\big)(t-\tau)^{\alpha_1-1}d\tau, \tag{28}$$

Let $k\big(t,y(t)\big)=\alpha_2 t^{\alpha_2-1}f\big(t,y(t)\big)$, then system (21) becomes

$$y(t)-y(0)=\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(t,y(t)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^t k\big(\tau,y(\tau)\big)(t-\tau)^{\alpha_1-1}d\tau, \tag{29}$$

At $t_{n+1}=(n+1)\Delta t$, we have

$$y(t_{n+1})-y(0)=\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(t_n,y(t_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\int_0^{t_{n+1}} k\big(\tau,y(\tau)\big)(t_{n+1}-\tau)^{\alpha_1-1}d\tau, \tag{30}$$

Also, we have

$$y(t_{n+1})=y(0)+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(t_n,y(t_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\int_{t_j}^{t_{j+1}} k\big(\tau,y(\tau)\big)(t_{n+1}-\tau)^{\alpha_1-1}d\tau, \tag{31}$$

Approximating the function $k\big(t,y(t)\big)$, using the Newton polynomial, we have

$$P_n(\tau)= k\big(t_{n-2},y(t_{n-2})\big)+\frac{k\big(t_{n-1},y(t_{n-1})\big)-k\big(t_{n-2},y(t_{n-2})\big)}{\Delta t}(\tau-t_{n-2})+ \frac{k\big(t_n,y(t_n)\big)-2k\big(t_{n-1},y(t_{n-1})\big)+k\big(t_{n-2},y(t_{n-2})\big)}{2(\Delta t)^2}(\tau-t_{n-2})(\tau-t_{n-1}). \tag{32}$$

Using Eq (32) into system (31), we have

$$y^{n+1}= y^0+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(t_n,y(t_n)\big)+ \frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\int_{t_j}^{t_{j+1}}\Big\{k\big(t_{n-2},y(t_{n-2})\big)+\frac{k\big(t_{n-1},y(t_{n-1})\big)-k\big(t_{n-2},y(t_{n-2})\big)}{\Delta t}(\tau-t_{n-2})+ \frac{k\big(t_n,y(t_n)\big)-2k\big(t_{n-1},y(t_{n-1})\big)+k\big(t_{n-2},y(t_{n-2})\big)}{2(\Delta t)^2}(\tau-t_{n-2})(\tau-t_{n-1})\Big\}(t_{n+1}-\tau)^{\alpha_1-1}d\tau, \tag{33}$$

Rearranging the above system, we have

$$y^{n+1}=y^0+\frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}k\big(t_n,y(t_n)\big)+\frac{\alpha_1}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1)}\sum_{j=2}^{n}\Big\{\int_{t_j}^{t_{j+1}}k\big(t_{j-2},y^{j-2}\big)(t_{n+1}-\tau)^{\alpha_1-1}d\tau+ \int_{t_j}^{t_{j+1}}\frac{k\big(t_{j-1},y^{j-1}\big)-k\big(t_{j-2},y^{j-2}\big)}{\Delta t}\big(\tau-t_{j-2}\big)(t_{n+1}-\tau)^{\alpha_1-1}d\tau+\int_{t_j}^{t_{j+1}}\frac{k\big(t_j,y^j\big)-2k\big(t_{j-1},y^{j-1}\big)+k\big(t_{j-2},y^{j-2}\big)}{2(\Delta t)^2}\big(\tau- t_{j-2}\big)\big(\tau-t_{j-1}\big)(t_{n+1}-\tau)^{\alpha_1-1}d\tau\Big\}. \tag{34}$$

Writing further system (34), we have

$$y^{n+1} = y^0 + \frac{(1-\alpha_1)}{C(\alpha_1)} k\big(t_n, y(t_n)\big) + \frac{\alpha_1}{C(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} k\big(t_{j-2}, y^{j-2}\big) \int_{t_j}^{t_{j+1}} (t_{n+1}-\tau)^{\alpha_1-1} d\tau + \frac{\alpha_1}{C(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} \frac{k\big(t_{j-1},y^{j-1}\big)-k\big(t_{j-2},y^{j-2}\big)}{\Delta t} \int_{t_j}^{t_{j+1}} \big(\tau - t_{j-2}\big)(t_{n+1}-\tau)^{\alpha_1-1} d\tau + \frac{\alpha_1}{C(\alpha_1)\Gamma(\alpha_1)} \sum_{j=2}^{n} \frac{k\big(t_j,y^j\big)-2k\big(t_{j-1},y^{j-1}\big)+k\big(t_{j-2},y^{j-2}\big)}{2(\Delta t)^2} \int_{t_j}^{t_{j+1}} \big(\tau - t_{j-2}\big)\big(\tau - t_{j-1}\big)\,(t_{n+1} - \tau)^{\alpha_1-1} d\tau. \tag{35}$$

Now, calculating the integrals in the system (35), we get

$$\int_{t_j}^{t_{j+1}} (t_{n+1}-\tau)^{\alpha_1-1} d\tau = \frac{(\Delta t)^{\alpha_1}}{\alpha_1}\left[(m-j+1)^{\alpha_1} - (m-j)^{\alpha_1}\right],$$

$$\int_{t_j}^{t_{j+1}} \big(\tau - t_{j-2}\big)(t_{n+1}-\tau)^{\alpha_1-1} d\tau = \frac{(\Delta t)^{\alpha_1+1}}{\alpha_1(\alpha_1+1)}\left[(m-j+1)^{\alpha_1}(m-j+3+2\alpha_1) - (m-j+1)^{\alpha_1}(m-j+3+3\alpha_1)\right],$$

$$\int_{t_j}^{t_{j+1}} \big(\tau - t_{j-2}\big)\big(\tau - t_{j-1}\big)\,(t_{n+1}-\tau)^{\alpha_1-1} d\tau$$
$$= \frac{(\Delta t)^{\alpha_1+2}}{\alpha_1(\alpha_1+1)(\alpha_1+2)}\Big[(m-j+1)^{\alpha_1}\{2(m-j)^2 + (3\alpha_1+10)(m-j) + 2{\alpha_1}^2 + 9\alpha_1 + 12\} - (m-j)^{\alpha_1}\{2(m-j)^2 + (5\alpha_1+10)(m-j) + 6{\alpha_1}^2 + 18\alpha_1 + 12\}\Big].$$

Inserting them into system (35), we get

$$y^{n+1} = y^0 + \frac{(1-\alpha_1)}{C(\alpha_1)} k\big(t_n, y(t_n)\big) + \frac{\alpha_1(\Delta t)^{\alpha_1}}{C(\alpha_1)\Gamma(\alpha_1+1)} \sum_{j=2}^{n} k\big(t_{j-2}, y^{j-2}\big)\left[(m-j+1)^{\alpha_1} - (m-j)^{\alpha_1}\right] + \frac{\alpha_1(\Delta t)^{\alpha_1}}{C(\alpha_1)\Gamma(\alpha_1+2)} \sum_{j=2}^{n} \left[k\big(t_{j-1}, y^{j-1}\big) - k\big(t_{j-2}, y^{j-2}\big)\right]\left[(m-j+1)^{\alpha_1}(m-j+3+2\alpha_1) - (m-j+1)^{\alpha_1}(m-j+3+3\alpha_1)\right] + \frac{\alpha_1(\Delta t)^{\alpha_1}}{C(\alpha_1)\Gamma(\alpha_1+3)} \sum_{j=2}^{n} \left[k\big(t_j, y^j\big) - 2k\big(t_{j-1}, y^{j-1}\big) + k\big(t_{j-2}, y^{j-2}\big)\right]\left[(m-j+1)^{\alpha_1}\{2(m-j)^2 + (3\alpha_1+10)(m-j) + 2{\alpha_1}^2 + 9\alpha_1 + 12\} - (m-j)^{\alpha_1}\{2(m-j)^2 + (5\alpha_1+10)(m-j) + 6{\alpha_1}^2 + 18\alpha_1 + 12\}\right]. \tag{36}$$

Finally, we have the following approximation:

$$y^{n+1} = y^0 + \frac{(1-\alpha_1)}{C(\alpha_1)} \alpha_2 t_n^{\alpha_2-1} f\big(t_n, y(t_n)\big) + \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{C(\alpha_1)\Gamma(\alpha_1+1)} \sum_{j=2}^{n} t_{j-2}^{\alpha_2-1} f\big(t_{j-2}, y^{j-2}\big)\left[(m-j+1)^{\alpha_1} - (m-j)^{\alpha_1}\right] + \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{C(\alpha_1)\Gamma(\alpha_1+2)} \sum_{j=2}^{n} \left[t_{j-1}^{\alpha_2-1} f\big(t_{j-1}, y^{j-1}\big) - t_{j-2}^{\alpha_2-1} f\big(t_{j-2}, y^{j-2}\big)\right]\left[(m-j+1)^{\alpha_1}(m-j+3+2\alpha_1) - (m-j+1)^{\alpha_1}(m-j+3+3\alpha_1)\right] + \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{C(\alpha_1)\Gamma(\alpha_1+3)} \sum_{j=2}^{n} \left[t_j^{\alpha_2-1} f\big(t_j, y^j\big) - 2t_{j-1}^{\alpha_2-1} f\big(t_{j-1}, y^{j-1}\big) + t_{j-2}^{\alpha_2-1} f\big(t_{j-2}, y^{j-2}\big)\right]\left[(m-j+1)^{\alpha_1}\{2(m-j)^2 + (3\alpha_1+10)(m-j) + 2{\alpha_1}^2 + 9\alpha_1 + 12\} - (m-j)^{\alpha_1}\{2(m-j)^2 + (5\alpha_1+10)(m-j) + 6{\alpha_1}^2 + 18\alpha_1 + 12\}\right]. \tag{37}$$

We obtain the following for system (26)

$$S_p^{\,n+1} = S_p^{\,0} + \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2 t_n^{\alpha_2-1} f\left(t_n, S_p(t_n)\right) + \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+1)}\sum_{j=2}^{n} t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, S_p^{\,j-2}\right)\left[(m-j+1)^{\alpha_1} - (m-j)^{\alpha_1}\right] + \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+2)}\sum_{j=2}^{n}\left[t_{j-1}^{\alpha_2-1} f\left(t_{j-1}, S_p^{\,j-1}\right) - t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, S_p^{\,j-2}\right)\right]\left[(m-j+1)^{\alpha_1}(m-j+3+2\alpha_1) - (m-j+1)^{\alpha_1}(m-j+3+3\alpha_1)\right] + \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+3)}\sum_{j=2}^{n}\left[t_j^{\alpha_2-1} f\left(t_j, S_p^{\,j}\right) - 2t_{j-1}^{\alpha_2-1} f\left(t_{j-1}, S_p^{\,j-1}\right) + t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, S_p^{\,j-2}\right)\right]\left[(m-j+1)^{\alpha_1}\{2(m-j)^2 + (3\alpha_1+10)(m-j) + 2\alpha_1^{\,2} + 9\alpha_1 + 12\} - (m-j)^{\alpha_1}\{2(m-j)^2 + (5\alpha_1+10)(m-j) + 6\alpha_1^{\,2} + 18\alpha_1 + 12\}\right].$$

$$I_p^{\,n+1} = I_p^{\,0} + \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2 t_n^{\alpha_2-1} f\left(t_n, I_p(t_n)\right) + \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+1)}\sum_{j=2}^{n} t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, I_p^{\,j-2}\right)\left[(m-j+1)^{\alpha_1} - (m-j)^{\alpha_1}\right] + \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+2)}\sum_{j=2}^{n}\left[t_{j-1}^{\alpha_2-1} f\left(t_{j-1}, I_p^{\,j-1}\right) - t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, I_p^{\,j-2}\right)\right]\left[(m-j+1)^{\alpha_1}(m-j+3+2\alpha_1) - (m-j+1)^{\alpha_1}(m-j+3+3\alpha_1)\right] + \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+3)}\sum_{j=2}^{n}\left[t_j^{\alpha_2-1} f\left(t_j, I_p^{\,j}\right) - 2t_{j-1}^{\alpha_2-1} f\left(t_{j-1}, I_p^{\,j-1}\right) + t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, I_p^{\,j-2}\right)\right]\left[(m-j+1)^{\alpha_1}\{2(m-j)^2 + (3\alpha_1+10)(m-j) + 2\alpha_1^{\,2} + 9\alpha_1 + 12\} - (m-j)^{\alpha_1}\{2(m-j)^2 + (5\alpha_1+10)(m-j) + 6\alpha_1^{\,2} + 18\alpha_1 + 12\}\right]. \tag{38}$$

$$\begin{aligned}
S_q^{\,n+1} = S_q^{\,0} &+ \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2 t_n^{\alpha_2-1} f\left(t_n, S_q(t_n)\right) \\
&+ \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+1)}\sum_{j=2}^{n} t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, S_q^{\,j-2}\right)\left[(m-j+1)^{\alpha_1} - (m-j)^{\alpha_1}\right] \\
&+ \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+2)}\sum_{j=2}^{n}\left[t_{j-1}^{\alpha_2-1} f\left(t_{j-1}, S_q^{\,j-1}\right)\right. \\
&\left. - t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, S_q^{\,j-2}\right)\right]\left[(m-j+1)^{\alpha_1}(m-j+3+2\alpha_1)\right. \\
&\left. - (m-j+1)^{\alpha_1}(m-j+3+3\alpha_1)\right] \\
&+ \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+3)}\sum_{j=2}^{n}\left[t_j^{\alpha_2-1} f\left(t_j, S_q^{\,j}\right) - 2t_{j-1}^{\alpha_2-1} f\left(t_{j-1}, S_q^{\,j-1}\right)\right. \\
&\left. + t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, S_q^{\,j-2}\right)\right]\left[(m-j+1)^{\alpha_1}\{2(m-j)^2 + (3\alpha_1+10)(m-j) + 2\alpha_1^{\,2}\right. \\
&\left. + 9\alpha_1 + 12\} - (m-j)^{\alpha_1}\{2(m-j)^2 + (5\alpha_1+10)(m-j) + 6\alpha_1^{\,2} + 18\alpha_1 + 12\}\right].
\end{aligned}$$

$$
\begin{aligned}
{I_q}^{n+1} = {I_q}^{0} &+ \frac{(1-\alpha_1)}{\mathrm{C}(\alpha_1)}\alpha_2 t_n^{\alpha_2-1} f\left(t_n, I_q(t_n)\right) \\
&+ \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+1)} \sum_{j=2}^{n} t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, {I_q}^{j-2}\right)\left[(m-j+1)^{\alpha_1} - (m-j)^{\alpha_1}\right] \\
&+ \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+2)} \sum_{j=2}^{n} \left[t_{j-1}^{\alpha_2-1} f\left(t_{j-1}, {I_q}^{j-1}\right)\right. \\
&- \left. t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, {I_q}^{j-2}\right)\right]\left[(m-j+1)^{\alpha_1}(m-j+3+2\alpha_1)\right. \\
&- \left.(m-j+1)^{\alpha_1}(m-j+3+3\alpha_1)\right] \\
&+ \frac{\alpha_1\alpha_2(\Delta t)^{\alpha_1}}{\mathrm{C}(\alpha_1)\Gamma(\alpha_1+3)} \sum_{j=2}^{n} \left[t_{j}^{\alpha_2-1} f\left(t_{j}, {I_q}^{j}\right) - 2t_{j-1}^{\alpha_2-1} f\left(t_{j-1}, {I_q}^{j-1}\right)\right. \\
&+ \left. t_{j-2}^{\alpha_2-1} f\left(t_{j-2}, {I_q}^{j-2}\right)\right]\left[(m-j+1)^{\alpha_1}\left\{2(m-j)^2 + (3\alpha_1+10)(m-j) + 2{\alpha_1}^2\right.\right. \\
&+ \left.\left. 9\alpha_1 + 12\right\} - (m-j)^{\alpha_1}\left\{2(m-j)^2 + (5\alpha_1+10)(m-j) + 6{\alpha_1}^2 + 18\alpha_1 + 12\right\}\right].
\end{aligned}
$$

7. Numerical results and discussions

To identify the potential effectiveness of Zika virus transmission in the Community, We consider the following parameters values and initial conditions [18] for our simulations:

$$\Lambda_p = 1.2, \Lambda_q = 0.3,\ \kappa_1 = 0.004,\ \kappa_2 = 0.0014,\ \beta_1 = 0.125 \times 10^{-4},\ \beta_2 = 0.4 \times 10^{-4},\ \mu = 0.475 \times 10^{-5}.$$

The mechanical features of the fractional-order model are identified by the various numerical methods with the time-fractional parameters. We demonstrate our results using different techniques in Figures 1–12 to check the efficiency of obtained solutions. The results of the nonlinear system memory were also detected with the help of fractional value. It provides a better way of understanding to control the disease without defining other parameters. Figures 1–4 represents the dynamical behavior of the Zika virus by using ABC derivative, $S_h(t)$, $S_m(t)$ and $I_h(t)$ start increase steadily by decreasing the fractional values while $I_m(t)$start decreasing by decreasing the fractional values. Figures 5–8 represents the dynamical behavior of the Zika virus by using fractal fractional derivative with dimensions 0.9, $S_h(t)$, $S_m(t)$ and $I_h(t)$ start increase strictly by decreasing the fractional values while $I_m(t)$ start decreasing strictly by decreasing the fractional values. Figures 9–12 represents the dynamical behavior of the Zika virus by using fractal fractional derivative with dimensions 0.8, $S_h(t)$, $S_m(t)$ and $I_h(t)$ start increase strictly by decreasing the fractional values while $I_m(t)$start decreasing strictly by decreasing the fractional values. Similar behavior can be seen with both techniques, but fractal fractional gives results fastly with minor effects of dimensions according to steady state. Moreover, it provides better results by decreasing the fractional value.

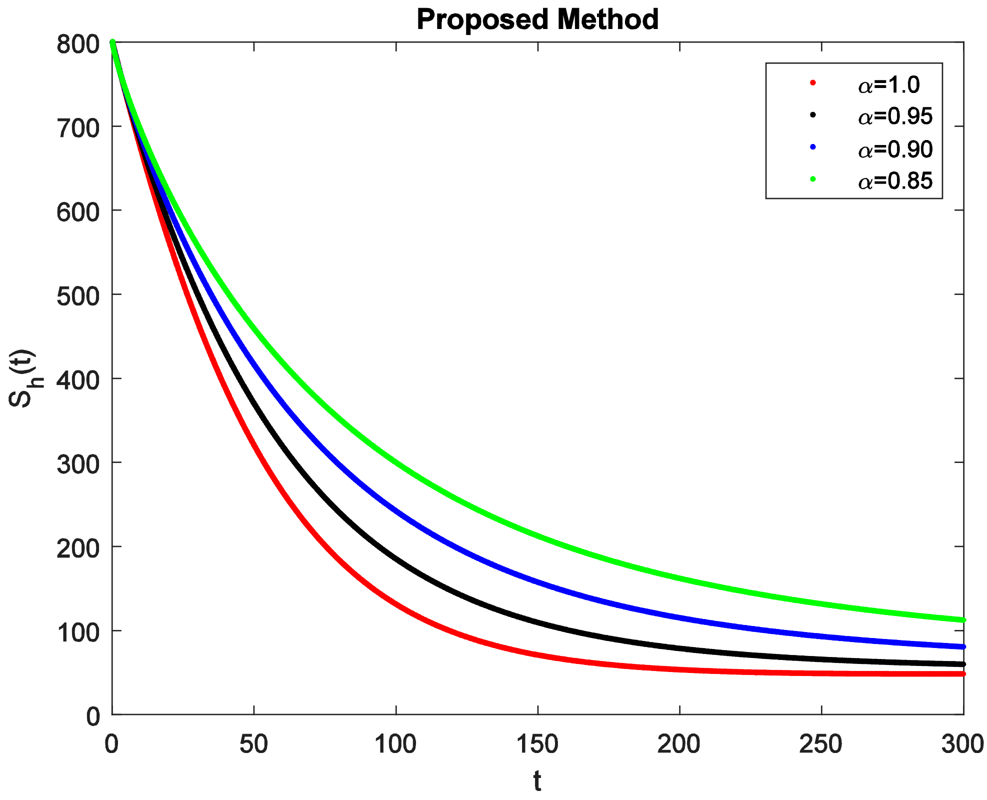

Figure 1. Simulation of $S_h(t)$ at different fractal orders with ABC operator.

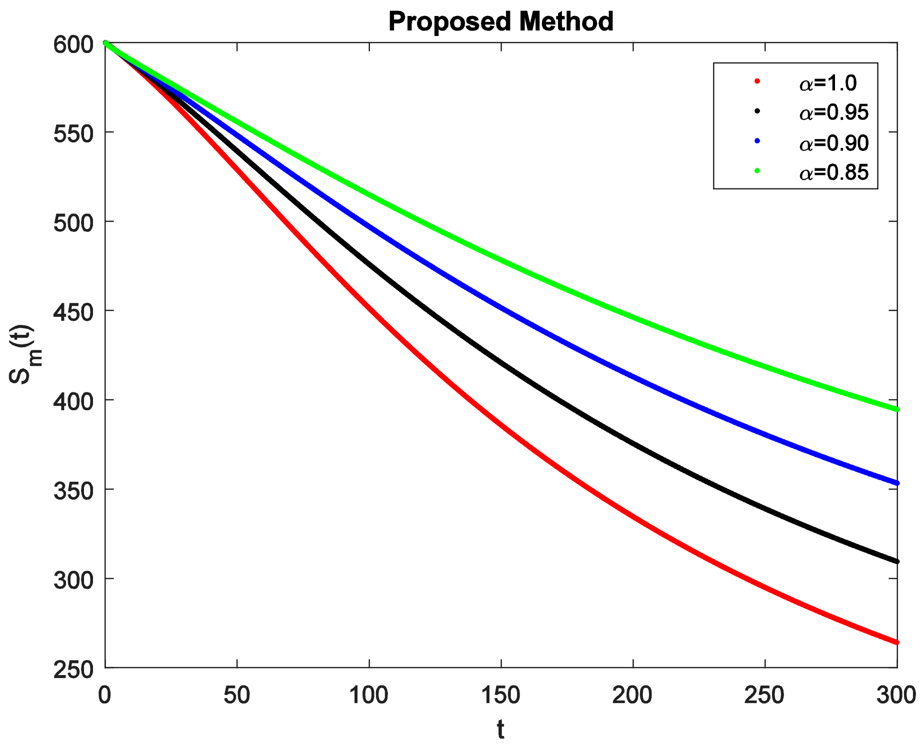

Figure 2. Simulation of $S_m(t)$ at different fractal orders with ABC operator.

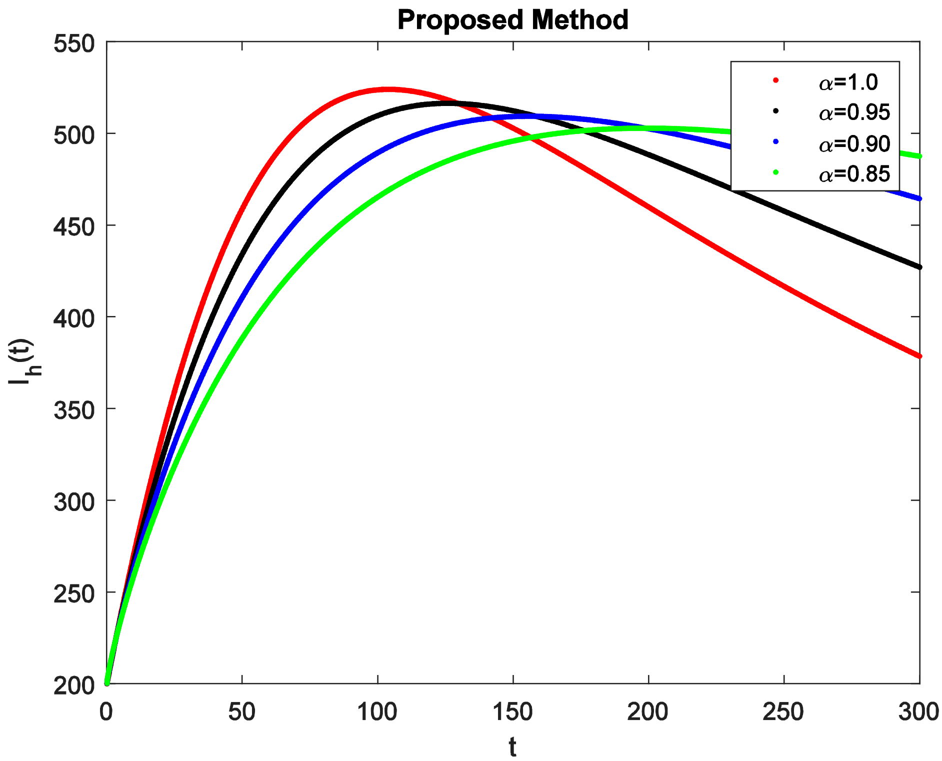

Figure 3. Simulation of $I_h(t)$ at different fractal orders with ABC operator.

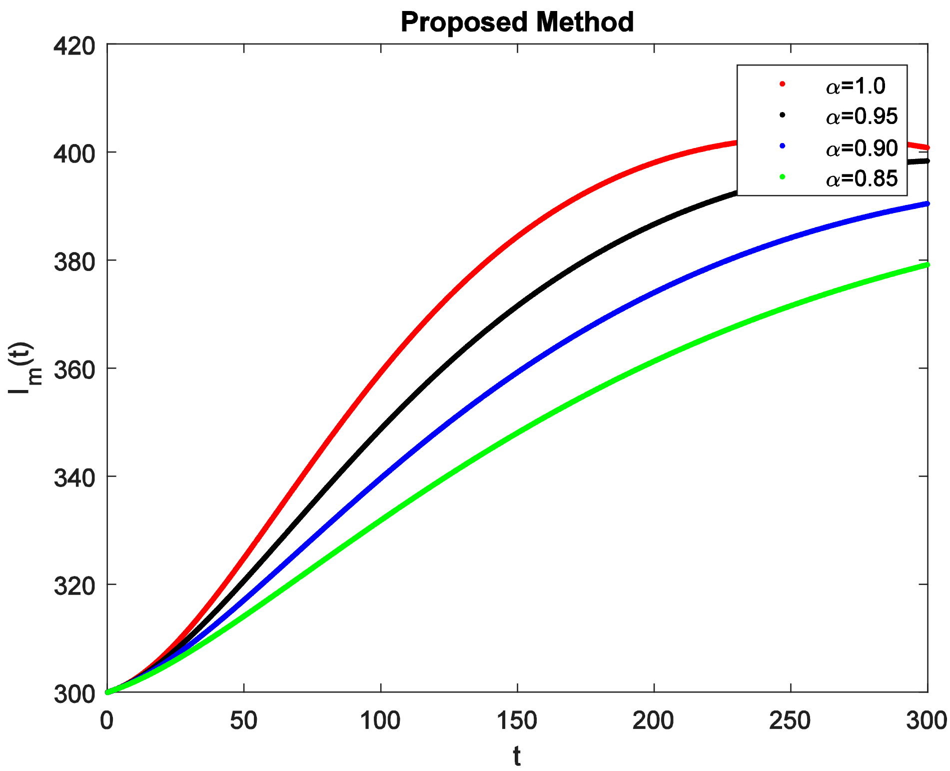

Figure 4. Simulation of $I_m(t)$ at different fractal orders with ABC operator.

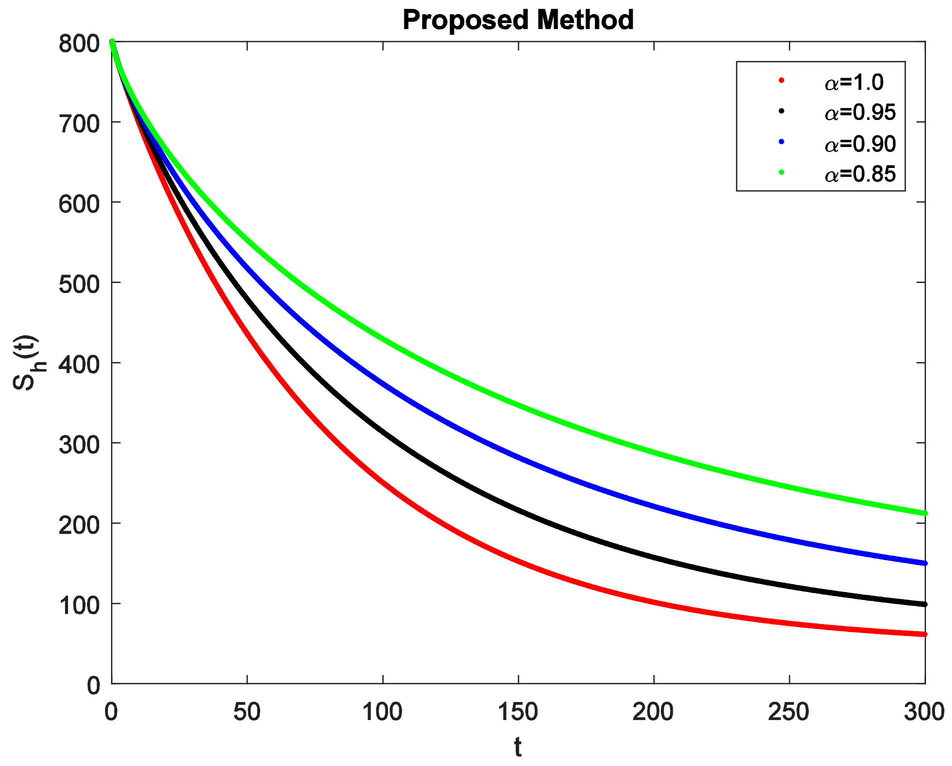

Figure 5. Simulation of $S_h(t)$ at different fractional values with dimension 0.9.

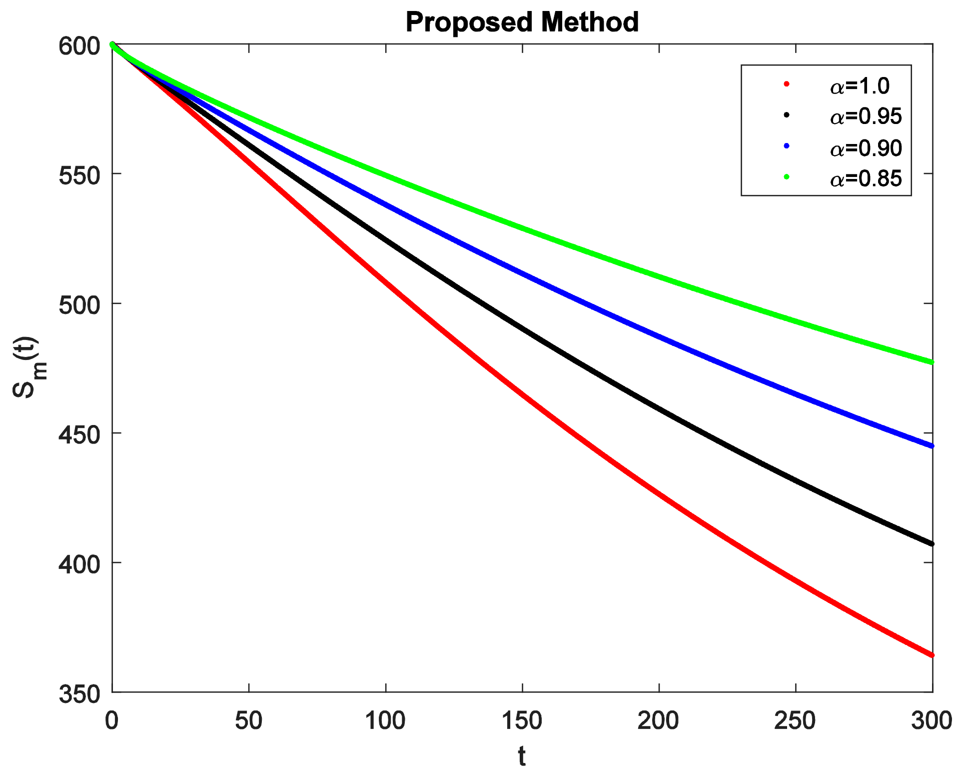

Figure 6. Simulation of $S_m(t)$ at different fractional values with dimension 0.9.

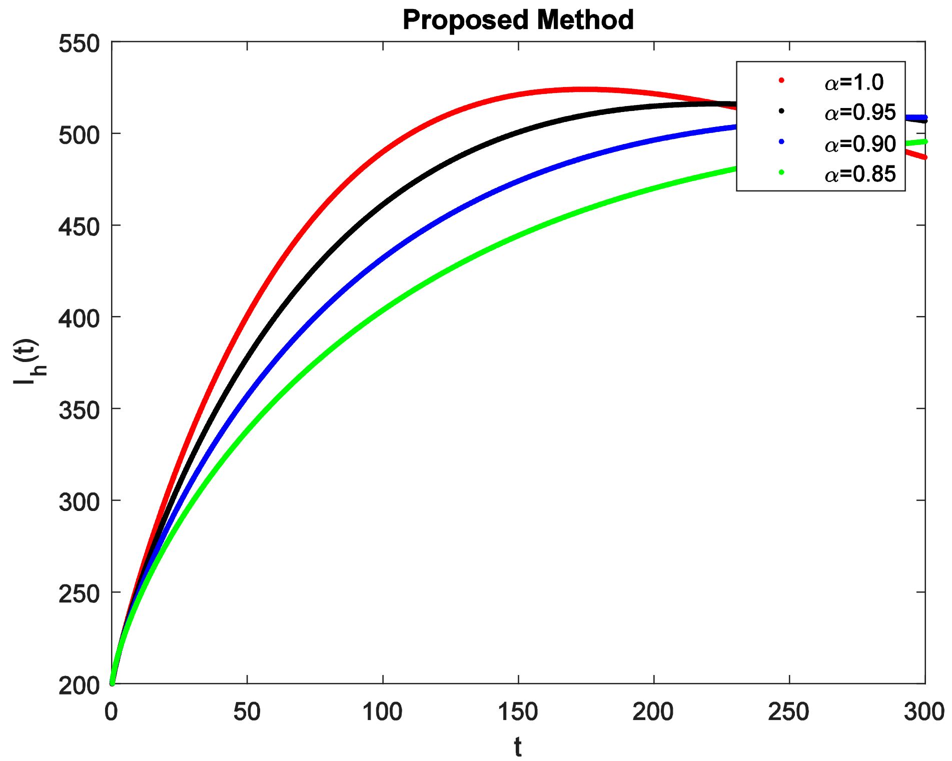

Figure 7. Simulation of $I_h(t)$ at different fractional values with dimension 0.9.

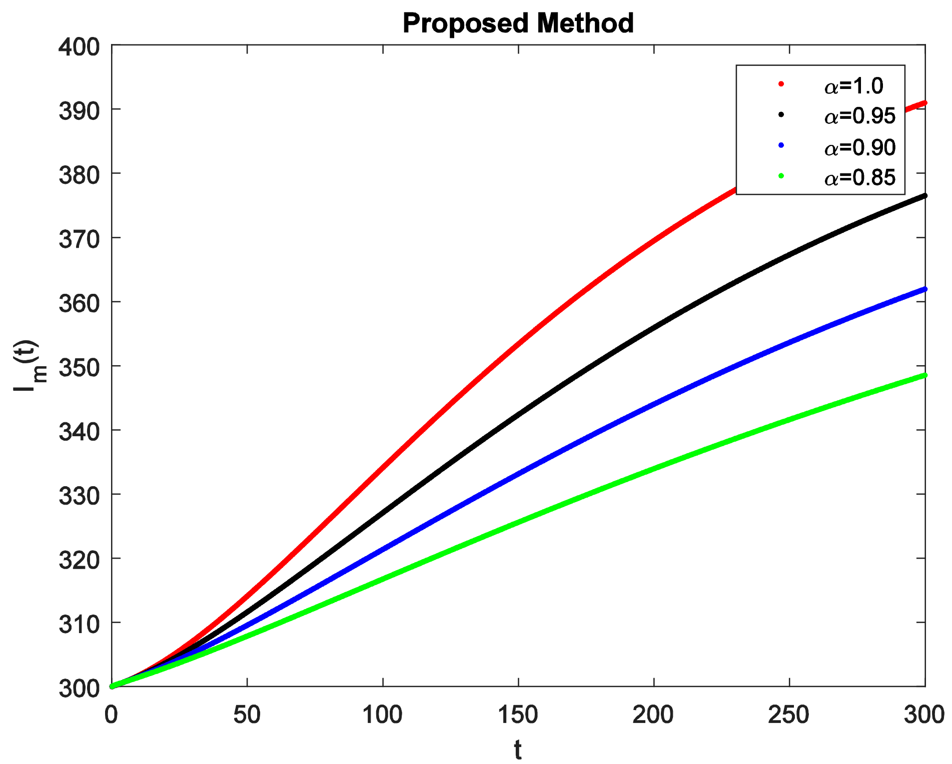

Figure 8. Simulation of $I_m(t)$at different fractional values with dimension 0.9.

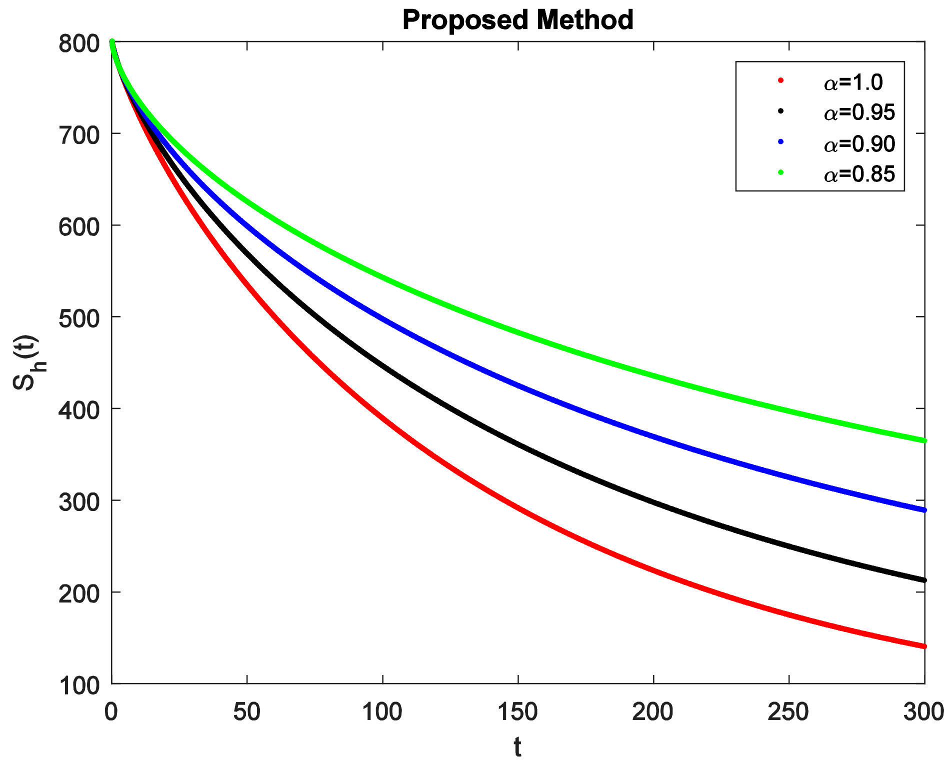

Figure 9. Simulation of $S_h(t)$ at different fractional values with dimension 0.8.

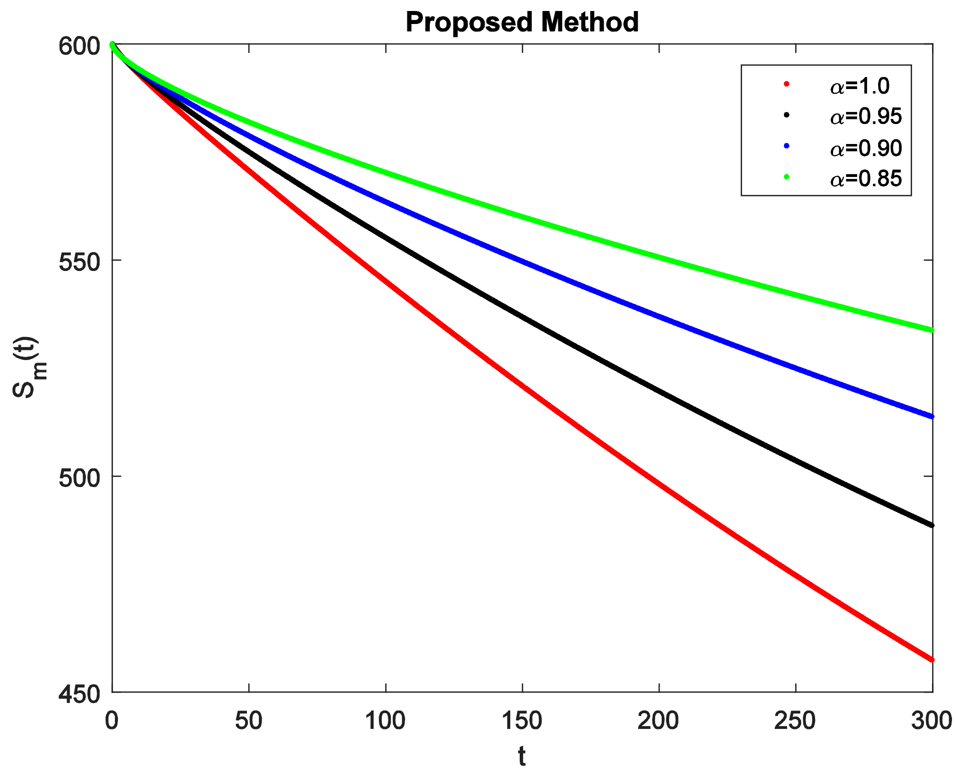

Figure 10. Simulation of $S_m(t)$ at different fractional values with dimension 0.8.

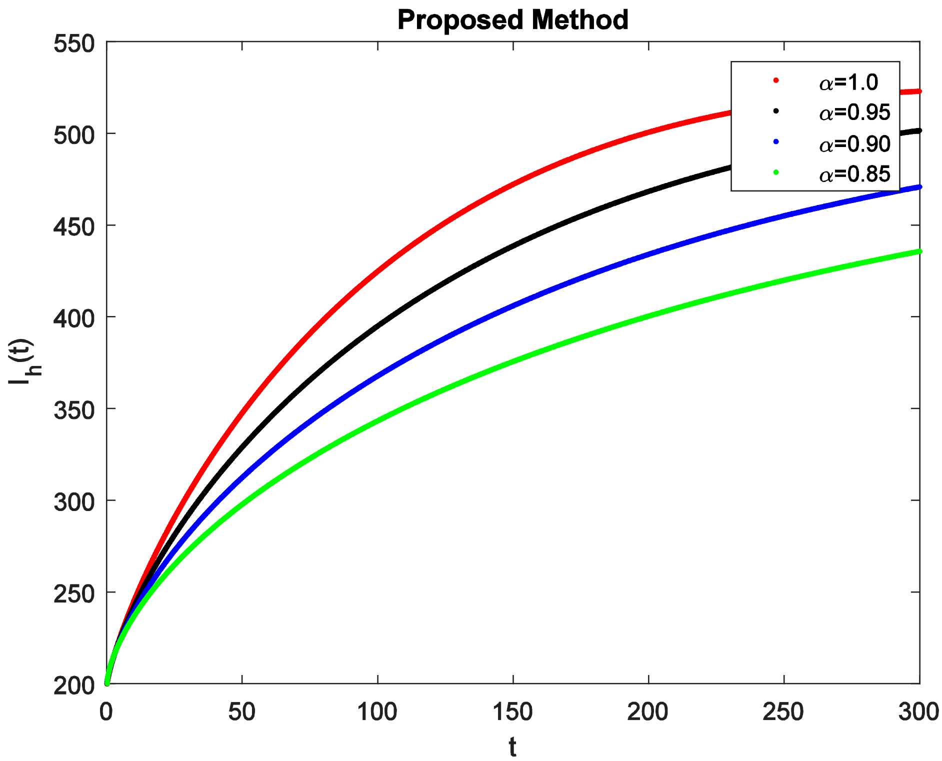

Figure 11. Simulation of $I_h(t)$ at different fractional values with dimension 0.8.

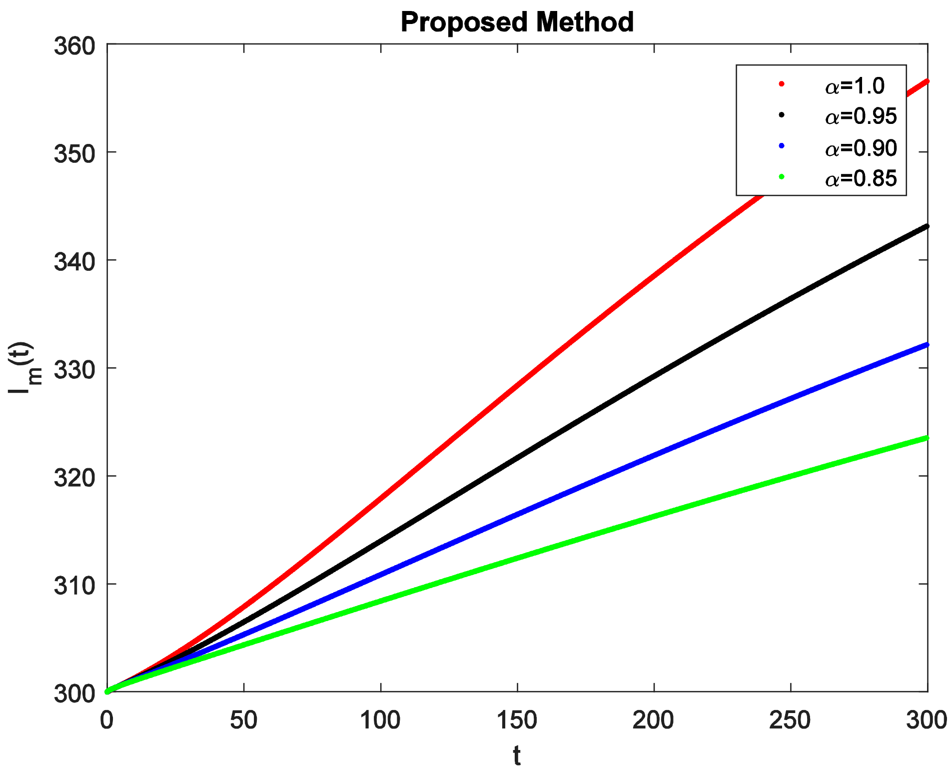

Figure 12. Simulation of $I_h(t)$ at different fractional values with dimension 0.8.

8. Conclusions

A fractional order differential equation model has been investigated in this article for the Zika virus. By using the fixed point theory, stability and uniqueness of the Zika virus model have been investigated. The arbitrary derivative of fractional order has been taken in the Attangana Baleeno in Caputo sense with no singular kernel and fractal fractional with Mittag-Leffler kernel respectively to analyses the Zika virus. Theoretical results are investigated for the fractional-order model, which proved the efficiency of the developed schemes. Numerical simulation has been made to check the actual behavior of the Zika virus outbreak. Such type of study will be helpful in future to understand the outbreak of this epidemic and to control the disease in a community.

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Conflict of interest

The authors have no conflict of interest.

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