Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(3): 3653–3679.
DOI:10.3934/math.2022202
Received: 18 October 2021
Accepted: 28 November 2021
Published: 06 December 2021

Research article

Global exponential stability and existence of almost periodic solutions in distribution for Clifford-valued stochastic high-order Hopfield neural networks with time-varying delays

Nina Huo[1], **Bing Li**[2] **and Yongkun Li**[3,*]

[1] Key Laboratory of Applied Mathematics and Mechanism of Artificial Intelligence, Hefei University, Hefei, Anhui 230601, China

[2] School of Mathematics and Computer Science, Yunnan Minzu University, Kunming, Yunnan 650500, China

[3] School of Mathematics and Statistics, Yunnan University, Kunming, Yunnan 650091, China

* **Correspondence:** Email: yklie@ynu.edu.cn.

Abstract: In this paper, we consider a class of Clifford-valued stochastic high-order Hopfield neural networks with time-varying delays whose coefficients are Clifford numbers except the time delays. Based on the Banach fixed point theorem and inequality techniques, we obtain the existence and global exponential stability of almost periodic solutions in distribution of this class of neural networks. Even if the considered neural networks degenerate into real-valued, complex-valued and quaternion-valued ones, our results are new. Finally, we use a numerical example and its computer simulation to illustrate the validity and feasibility of our theoretical results.

Keywords: Clifford-valued neural network; stochastic neural network; high-order Hopfield neural network; almost periodic solution in distribution; global exponential stability
Mathematics Subject Classification: 34K14, 34K20, 34K50, 92B20

1. Introduction

Clifford-valued neural networks (NNs) are the NNs whose state variables, connection weights and external inputs are Clifford numbers. They are generalizations of real-valued, complex-valued and quaternion-valued neural networks. In recent years, due to their advantages over real-valued networks and their potential application values in many fields, they have attracted the attention of many researchers [1–12]. However, because the multiplication of Clifford numbers does not satisfy the commutative law, it is difficult to study the dynamics of Clifford-valued NNs. At this stage, there are few results on the dynamics of Clifford-valued NNs [8–14]. In addition, it is worth noting that in most of the existing results [5, 7–9, 14], the coefficients of the leakage terms in neural networks are assumed to be real numbers.

On the one hand, it is well known that high-order Hopfield NNs have more advantages than low-order Hopfield NNs. Therefore, in the past few decades, many scholars have done a lot of research on the dynamics of high-order Hopfield NN [13, 15–18]. This is because the application of neural networks in various fields largely depends on their dynamic performance. Moreover, the use of neural networks with complex or even chaotic dynamic behaviors in information processing is expected to improve the efficiency and flexibility of information processing.

In addition, noise interference is the main source of neural network instability, which can lead to poor neural network performance. In the real nervous system, synaptic transmission is a noisy process, caused by random fluctuations in neurotransmitter release and other probabilistic reasons. As we all know, neural networks can be stable or unstable through some random inputs [19]. For this reason, stochastic neural networks are widely studied [20–26].

Besides, we know that the existence and stability of equilibrium points are important dynamics of autonomous neural networks. For nonautonomous neural networks, there are generally no equilibrium points. Therefore, the existence and stability of periodic or almost periodic solutions are important dynamics. Since almost periodicity is more common than periodicity, in the past few decades, many scholars have studied the almost periodic solutions of deterministic neural networks [7, 27–29]. However, the existing results on the existence of almost periodic solutions of stochastic neural networks are almost all about mean-square almost periodic solutions. Unfortunately, in [30], some counterexamples show that the nontrivial solutions of some stochastic differential equations with almost periodic coefficients cannot be mean-square almost periodic. Therefore, it is more reasonable to study the almost periodic solutions in distribution of stochastic differential equations. Random almost periodic oscillation is a complex oscillation phenomenon. However, so far, no papers have been published on almost periodic solutions in distribution of Clifford-valued stochastic high-order Hopfield NNs. Therefore, it is necessary to study this issue.

Inspired by the above discussion, and considering the fact that time delay is inevitable, in this work, we consider the following Clifford-valued stochastic high-order Hopfield NN with time varying-delays:

$$\begin{aligned} dx_p(t) =& \Big[-c_p(t)x_p(t) + \sum_{q=1}^{n} a_{pq}(t)f_q(x_q(t-\tau_{pq}(t))) \\ &+ \sum_{q=1}^{n}\sum_{l=1}^{n} b_{pql}(t)g_q(x_q(t-\sigma_{pql}(t)))g_l(x_l(t-\nu_{pql}(t))) \\ &+ I_p(t)\Big]dt + \sum_{q=1}^{n} \delta_{pq}(x_q(t-\gamma_{pq}(t)))d\omega_q(t), \end{aligned} \tag{1.1}$$

where $p \in \{1, 2, \ldots, n\} := \mathcal{D}$, n is the number of neurons in layers; $x_p(t) \in \mathcal{A}$ is the state variable of the pth unit at time t and $\mathcal{A}$ is a Clifford algebra; $c_p(t) \in \mathcal{A}$ is the coefficient of the leakage term, which represents the rate with which the pth unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs; $a_{pq}(t), b_{pql}(t) \in \mathcal{A}$ are the first-order and second-order connection weights of the neural network; $\tau_{pq}(t) \geq 0$, $\sigma_{pql}(t) \geq 0$, $\nu_{pql}(t) \geq 0$ and $\gamma_{pq}(t) \geq 0$ correspond to the transmission delays; $I_p(t) \in \mathcal{A}$ denotes the external inputs at time t; $f_q, g_q : \mathcal{A} \to \mathcal{A}$

are the activation functions of signal transmission; $\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t))^T$ is an n-dimensional Brownian motion defined on a complete probability space; $\delta_{pq} : \mathcal{A} \to \mathcal{A}$ is a Borel measurable function.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. Denote by $CB_{\mathcal{F}_0}([-\varrho, 0], \mathcal{A}^n)$ the family of all bounded, $\mathcal{F}_0$-measurable, $C([-\varrho, 0], \mathcal{A}^n)$-valued random variables ϕ.

The initial values of system (1.1) are given by

$$x_p(s) = \phi_p(s), \ s \in [-\varrho, 0], \ p \in \mathcal{D}, \tag{1.2}$$

where $\phi_p \in CB_{\mathcal{F}_0}([-\varrho, 0], \mathcal{A}^n)$.

The main purpose of this paper is to study the existence and global exponential stability of almost periodic solutions in distribution of system (1.1). The innovations of this paper are as follows: (1) This is the first paper that uses a non-decomposition method to study stochastic NNs whose coefficients are all Clifford numbers except for time delays. (2) This is the first time to study almost periodic solutions in distribution of Clifford-valued stochastic high-order Hopfield NNs. (3) The method of dealing with time-varying delays in this paper can be used to study the corresponding problems of other types of stochastic NNs with time-varying delays. (4) When the system we consider degenerates into real-valued system, complex-valued system or quaternion-valued system, the results of this paper are also new.

The rest of this paper is organized as follows. In Sect. 2, we recollect some basic definitions and lemmas. In Sect. 3, based on the principle of contractive mapping, we establish the existence of almost periodic solutions in distribution for system (1.1). In Sect. 4, we study the global exponential stability of the almost periodic solution in distribution of system (1.1) by inequality techniques. In Sect. 5, we give an example to illustrate the feasibility of the theoretical results obtained in this paper. In Sect. 6, we give a concise conclusion to end this paper.

2. Preliminaries

The real Clifford algebra over $\mathbb{R}^m$ is defined as

$$\mathcal{A} = \Big\{ \sum_{A \in \Pi} x^A e_A, x^A \in \mathbb{R} \Big\},$$

where $\Pi = \{\emptyset, 1, 2, \ldots, A, \ldots, 12\cdots m\}$, $e_\emptyset = e_0 = 1$ and $e_p, p = 1, 2, \ldots, m$ are called the Clifford generators and satisfy there exits an $\iota\,(0 \leq \iota < m)$ such that

$$\begin{cases} e_p^2 = 1, & p = 1, 2, \ldots, \iota, \\ e_p^2 = -1, & p = \iota + 1, \iota + 2, \ldots, m, \\ e_{pq} + e_{qp} = 0, & 1 \leq p, q \leq m, p \neq q. \end{cases}$$

For $x = \sum_A x^A e_A \in \mathcal{A}$, let $\|x\|_{\mathcal{A}} = \max_A\{| \ x^A \ |\}$, where $\sum_A$ and $\max_A$ are short for $\sum_{A\in\Pi}$ and $\max_{A\in\Pi}$, respectively. For $y = (y_1, y_2, \ldots, y_n)^T \in \mathcal{A}^n$, we define $\|y\|_n = \max\{\|y_p\|_{\mathcal{A}}\}$. Then $(\mathcal{A}, \|\cdot\|_n)$ is a Banach space. For more information on Clifford analysis, see [31].

Throughout this paper, for $x = \sum_A x^A e_A \in \mathcal{A}$, we denote $x^{\eth} = \sum_{A\neq\emptyset} x^A e_A$ and $x^{\emptyset} = x - x^{\eth}$.

Definition 2.1. *[6] Let $BC(\mathbb{R},\mathcal{A}^n)$ denote the set of all bounded continuous functions from $\mathbb{R}$ to $\mathcal{A}^n$. A function $f \in BC(\mathbb{R},\mathcal{A}^n)$ is said to be almost periodic, if for every $\varepsilon > 0$ there exists a positive number ℓ such that every interval of length ℓ contains a number τ such that*

$$\|f(t+\tau) - f(t)\|_n < \varepsilon,\ \ t \in \mathbb{R}.$$

The τ is called the ε-translation number of f. Denote by $AP(\mathbb{R},\mathcal{A}^n)$ the set of all such functions.

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions.

A stochastic process $X = \{X(t) : t \geq 0\}$ (or, simply X_t) is called adapted to the filtration $\{\mathcal{F}_t\}_{t\geq 0}$ if X_t is $\mathcal{F}_t$-measurable for all $t \geq 0$.

Let $(\mathbb{E}, d)$ be a separable, complete metric space and $\mathcal{B}(\mathbb{E})$ be the σ-algebra of Borel sets of $\mathbb{E}$. We denote by $\mathcal{P}(\mathbb{E})$ the set of all probability measures defined on $\mathcal{B}(\mathbb{E})$ and by $CB(\mathbb{E})$ the set of all bounded continuous functions $f : \mathbb{E} \to \mathbb{R}$ with $\|f\|_\infty := \sup_{x\in\mathbb{E}} | f(x) |< \infty$.

For $f \in CB(\mathbb{E})$, $\mu, \nu \in \mathcal{P}(\mathbb{E})$, we define

$$\|f\|_L = \sup_{a\neq b} \frac{| f(a) - f(b) |}{d(a, b)},\ \ \|f\|_{BL} = \max\{\|f\|_\infty, \|f\|_L\},$$

$$d_{BL}(\mu, \nu) := \sup_{\|f\|_{BL}\leq 1} \left| \int_{\mathbb{E}} f d(\mu - \nu) \right| .$$

It is well known that the metric space $(\mathcal{P}(\mathbb{E}), d_{BL})$ is a Polish space [32]. For a random variable $X : (\Omega, \mathcal{F}, P) \to \mathbb{E}$, we will denote by $\mu(X) := P \circ X^{-1}$ its law and by $E(X)$ its expectation.

Let $\mathcal{L}^2(\Omega,\mathcal{A}^n)$ be the space of all $\mathcal{A}^n$-valued random variables such that

$$E(\|X\|_n^2) = \int_\Omega \|X\|_n^2 dP < \infty.$$

For $X \in \mathcal{L}^2(\Omega,\mathcal{A}^n)$, we denote

$$\|X\|_n^2 = \Big(\int_\Omega \|X\|_n^2 dP\Big)^{\frac{1}{2}} \quad \text{and} \quad E\|X\|_n^2 = \int_\Omega \|X\|_n^2 dP.$$

Definition 2.2. *[33] A stochastic process $X : \mathbb{R} \to \mathcal{L}^2(\Omega,\mathcal{A}^n)$ is said to be $\mathcal{L}^2$-continuous if for any $t_0 \in \mathbb{R}$,*

$$\lim_{t\to t_0} E\|X(t) - X(t_0)\|_n^2 = 0.$$

It is said to be $\mathcal{L}^2$-bounded if $\sup_{t\in\mathbb{R}} E\|X(t)\|_n^2 < \infty$.

Definition 2.3. *[34] An $\mathcal{L}^2$-continuous stochastic process $X : \mathbb{R} \to \mathcal{L}^2(\Omega,\mathcal{A}^n)$ is said to be square-mean almost periodic if for every $\varepsilon > 0$, there exist a positive number ℓ such that every interval of length ℓ contains a number τ such that*

$$E\|X(t+\tau) - X(t)\|_n^2 < \varepsilon,\ \ t \in \mathbb{R}.$$

Definition 2.4. *[35] A stochastic process $X:\mathbb{R}\to\mathcal{A}^n$ is said to be almost periodic in distribution if the mapping*

$$t\to\mu_t:=\mu(X(t))$$

is almost periodic, where $\mu(X(t))=P\circ[X(t)]^{-1}$ is the law of $X(t)$ under P, that is to say, if for every $\varepsilon>0$, there exists a positive number ℓ such that every interval of length ℓ contains a number τ such that

$$d_{BL}(P\circ[X(t+\tau)]^{-1},P\circ[X(t)]^{-1})<\varepsilon.$$

From Remark 2.12 in [33], one can deduce that

Lemma 2.1. *If an $\mathcal{L}^2$-continuous stochastic process $X(t)$ is square-mean almost periodic, then $X(t)$ is almost periodic in distribution; but the converse is not true.*

Lemma 2.2. *[36] Let $g:\mathbb{R}\to\mathbb{R}$ be a continuous function such that, for every $t\in\mathbb{R}$,*

$$0\le g(t)\le\alpha+\beta\int_{-\infty}^{t}e^{-\delta(t-s)}g(s)ds,$$

where $\alpha,\beta,\delta\ge0$ are constants and $\delta>\beta$. Then we have $g(t)\le\alpha\frac{\delta}{\delta-\beta}$.

In the rest part of this paper, we will adopt the following notation:

$$\overline{c}_p^{\eth}=\sup_{t\in\mathbb{R}}\|c_p^{\eth}(t)\|_{\mathcal{A}},\ \overline{c}_p^{\emptyset}=\sup_{t\in\mathbb{R}}\mid c_p^{\emptyset}(t)\mid,\ \underline{c}_p^{\emptyset}=\inf_{t\in\mathbb{R}}\mid c_p^{\emptyset}(t)\mid,\ a_{pq}^{+}=\sup_{t\in\mathbb{R}}\|a_{pq}(t)\|_{\mathcal{A}},$$

$$b_{pql}^{+}=\sup_{t\in\mathbb{R}}\|b_{pql}(t)\|_{\mathcal{A}},\ I_p^{+}=\sup_{t\in\mathbb{R}}\|I_p(t)\|_{\mathcal{A}},\ \tau_{pq}^{+}=\sup_{t\in\mathbb{R}}\tau_{pq}(t),$$

$$\sigma_{pql}^{+}=\sup_{t\in\mathbb{R}}\sigma_{pql}(t),\ \nu_{pql}^{+}=\sup_{t\in\mathbb{R}}\nu_{pql}(t),\ \gamma_{pq}^{+}=\sup_{t\in\mathbb{R}}\gamma_{pq}(t).$$

Throughout this paper, we assume that

(H_1) For $p,q,l\in\mathcal{D}$, $c_p^{\emptyset}\in AP(\mathbb{R},\mathbb{R}^+)$ with $\underline{c}_p^{\emptyset}>0$, $c_p^{\eth},a_{pq},b_{pql},I_p\in AP(\mathbb{R},\mathcal{A})$, $\tau_{pq},\sigma_{pql},\nu_{pql},\gamma_{pq}\in AP(\mathbb{R},\mathbb{R}^+)\cap C^1(\mathbb{R},\mathbb{R})$, there exist positive constants $\dot{\tau}_{pq}^{+},\dot{\sigma}_{pql}^{+},\dot{\nu}_{pql}^{+},\dot{\gamma}_{pq}^{+}$ such that $\dot{\tau}_{pq}(t)\le\dot{\tau}_{pq}^{+}<1$, $\dot{\sigma}_{pql}(t)\le\dot{\sigma}_{pql}^{+}<1$, $\dot{\nu}_{pql}(t)\le\dot{\nu}_{pql}^{+}<1$, $\dot{\gamma}_{pq}(t)\le\dot{\gamma}_{pq}^{+}<1$.

(H_2) For $q\in\mathcal{D}$, $f_q,h_q\in C(\mathcal{A},\mathcal{A})$, there exist constants L_q^f,L_q^g,M_q^g such that

$$\|f_q(x)-f_q(y)\|_{\mathcal{A}}\le L_q^f\|x-y\|_{\mathcal{A}},\ \|g_q(x)-g_q(y)\|_{\mathcal{A}}\le L_q^g\|x-y\|_{\mathcal{A}},$$

$$\|\delta_{pq}(x)-\delta_{pq}(y)\|_{\mathcal{A}}\le L_{pq}^{+}\|x-y\|_{\mathcal{A}},\ \|g_q(x)\|_{\mathcal{A}}\le M_q^g,$$

for all $x,y\in\mathcal{A}$, and $f_q(0)=g_q(0)=\delta_{pq}(0)=0$.

(H_3)

$$K:=\max_{p\in\mathcal{D}}\left\{\frac{4}{(\underline{c}_p^{\emptyset})^2}\Big[(\overline{c}_p^{\eth})^2+\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^f)^2+\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^{+})^2\sum_{l=1}^{n}(M_l^g)^2\Big)\right.$$
$$\left.\times\sum_{q=1}^{n}(L_q^g)^2+\frac{n\underline{c}_p^{\emptyset}}{2}\sum_{q=1}^{n}(L_{pq}^{+})^2\Big]\right\}<\frac{1}{4},$$

$$
\begin{aligned}
P := \max_{p\in D}\Big\{ & \frac{13}{\underline{c}_p^{\emptyset}}(\overline{c}_p^{\eth})^2 + \frac{26}{\underline{c}_p^{\emptyset}}\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^f)^2\frac{e^{\underline{c}_p^{\emptyset}\tau_{pq}^{+}}}{1-\dot{\tau}_{pq}^{+}} + \frac{52n}{\underline{c}_p^{\emptyset}}\sum_{q=1}^{n}\Big[\sum_{l=1}^{n}(b_{pql}^{+})^2 \\
& \times\sum_{l=1}^{n}\Big(M_q^g L_l^g\frac{e^{\underline{c}_p^{\emptyset}\nu_{pql}^{+}}}{1-\dot{\nu}_{pql}^{+}} + M_l^g L_q^g\frac{e^{\underline{c}_p^{\emptyset}\sigma_{pql}^{+}}}{1-\dot{\sigma}_{pql}^{+}}\Big)\Big] + 26n\sum_{q=1}^{n}(L_{pq}^{+})^2\frac{e^{2\underline{c}_p^{\emptyset}\gamma_{pq}^{+}}}{1-\dot{\gamma}_{pq}^{+}}\Big\} \\
& < \min_{p\in\mathcal{D}}\{\underline{c}_p^{\emptyset}\}.
\end{aligned}
$$

(H_4)

$$
\begin{aligned}
C := \max_{p\in\mathcal{D}}\Big\{ & \frac{5}{(\underline{c}_p^{\emptyset})^2}\Big[(\overline{c}_p^{\eth})^2 + \sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^f)^2 + 2n\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^{+})^2 \\
& \times\sum_{l=1}^{n}\Big((M_q^g L_l^g)^2 + (M_l^g L_q^g)^2\Big)\Big) + \frac{n\underline{c}_p^{\emptyset}}{2}\sum_{q=1}^{n}(L_{pq}^{+})^2\Big]\Big\} < 1.
\end{aligned}
$$

3. The existence of almost periodic solutions in distribution

We denote by $UCB(\mathbb{R}, \mathcal{L}^2(\Omega, \mathcal{A}^n))$ the space of all $\mathcal{L}^2$-bounded and uniformly $\mathcal{L}^2$-continuous functions from $\mathbb{R}$ to $\mathcal{L}^2(\Omega, \mathcal{A}^n)$. Let $\mathbb{B} = UCB(\mathbb{R}, \mathcal{L}^2(\Omega, \mathcal{A}^n))$ with the norm $\|x\|_{\mathbb{B}} = \Big(\sup_{t\in\mathbb{R}} E(\|x(t)\|_n^2)\Big)^{\frac{1}{2}}$, then $\mathbb{B}$ is a Banach space.

Set $x^0 = (x_1^0, x_2^0, \ldots, x_n^0)^T$, where $x_p^0(t) = \int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du} I_p(s)ds$, $t \in \mathbb{R}$, $p \in \mathcal{D}$ and take a constant κ such that $\|x^0\|_{\mathbb{B}} \leq \kappa$.

Definition 3.1. *An $\mathcal{F}_t$-progressively measurable stochastic process $x(t)$ is called a mild solution of system* (1.1)*, if $x(t)$ satisfies the following stochastic integral equation*

$$
\begin{aligned}
x_p(t) =& x_p(t_0)e^{-\int_{t_0}^{t} c_p^{\emptyset}(u)du} + \int_{t_0}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\Theta_p(s, x)ds \\
& + \int_{t_0}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\Gamma_p(s, x)d\omega_q(s),
\end{aligned}
$$

where $t \geq t_0$, $p \in \mathcal{D}$,

$$
\begin{aligned}
\Theta_p(s, x) =& -c_p^{\eth}(s)x_p(s) + \sum_{q=1}^{n} a_{pq}(s)f_q(x_q(s-\tau_{pq}(s))) + \sum_{q=1}^{n}\sum_{l=1}^{n} b_{pql}(s) \\
& \times g_q(x_q(s-\sigma_{pql}(s)))g_l(x_l(s-\nu_{pql}(s))) + I_p(s), \\
\Gamma_p(s, x) =& \sum_{q=1}^{n}\delta_{pq}(x_q(s-\gamma_{pq}(s))).
\end{aligned}
$$

Theorem 3.1. *Assume that* (H_1)*-*(H_4) *hold, then system* (1.1) *has a unique almost periodic solution in distribution in the closed ball $\mathbb{B}_\kappa = \{x \mid x \in \mathbb{B}, \|x - x^0\|_{\mathbb{B}} \leq \kappa\}$.*

Proof. According to Definition 3.1, taking the limit as $t_0 \longrightarrow -\infty$, we obtain

$$x_p(t) = \int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\Theta_p(s,x)ds + \int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\Gamma_p(s,x)d\omega_q(s), \ \ p \in \mathcal{D}, \tag{3.1}$$

which is a mild solution of system (1.1).

Define an operator $\Phi : \mathbb{B}_\kappa \to \mathbb{B}_\kappa$ by

$$\Phi x = ((\Phi x)_1, (\Phi x)_2, \cdots, (\Phi x)_n)^T,$$

where for $t \in \mathbb{R}$, $p \in \mathcal{D}$,

$$(\Phi x)_p(t) = \int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\Theta_p(s,x)ds + \int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\Gamma_p(s,x)d\omega_q(s).$$

Firstly, we will prove that the operator Φ is well defined.

In fact, for $x = (x_1, x_2, \dots, x_n)^T \in \mathbb{B}_\kappa$, one has

$$\|x\|_{\mathbb{B}} \le \|x^0\|_{\mathbb{B}} + \|x - x^0\|_{\mathbb{B}} \le 2\kappa \tag{3.2}$$

and

$$\begin{aligned}
&\|\Phi x - x^0\|_{\mathbb{B}}^2 \\
\le& 4\sup_{t\in\mathbb{R}}\max_{p\in\mathcal{D}}\Big\{E\Big\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du} c_p^{\eth}(s)x_p(s)ds\Big\|_{\mathcal{A}}^2\Big\} \\
&+ 4\sup_{t\in\mathbb{R}}\max_{p\in\mathcal{D}}\Big\{E\Big\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\sum_{q=1}^{n} a_{pq}(s)f_q(x_q(s-\tau_{pq}(s)))ds\Big\|_{\mathcal{A}}^2\Big\} \\
&+ 4\sup_{t\in\mathbb{R}}\max_{p\in\mathcal{D}}\Big\{E\Big\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\sum_{q=1}^{n}\sum_{l=1}^{n} b_{pql}(s)g_q(x_q(s-\sigma_{pql}(s))) \\
&\times g_l(x_l(s-\nu_{pql}(s)))ds\Big\|_{\mathcal{A}}^2\Big\} \\
&+ 4\sup_{t\in\mathbb{R}}\max_{p\in\mathcal{D}}\Big\{E\Big\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\sum_{q=1}^{n} \delta_{pq}(x_q(s-\gamma_{pq}(s)))d\omega_q(s)\Big\|_{\mathcal{A}}^2\Big\} \\
:=& A_1 + A_2 + A_3 + A_4.
\end{aligned} \tag{3.3}$$

By the Cauchy-Schwarz inequality, we have

$$\begin{aligned}
A_1 \le& 4\sup_{t\in\mathbb{R}}\max_{p\in\mathcal{D}}\Big\{E\Big(\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\overline{c}_p^{\eth}\|x_p\|_{\mathbb{B}}ds\Big)^2\Big\} \\
\le& \max_{p\in\mathcal{D}}\Big\{\frac{4(\overline{c}_p^{\eth})^2}{(\underline{c}_p^{\emptyset})^2}\Big\}\|x\|_{\mathbb{B}}^2
\end{aligned} \tag{3.4}$$

and

$$A_2 \le 4\sup_{t\in\mathbb{R}}\max_{p\in\mathcal{D}}\Big\{E\Big(\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du}\sum_{q=1}^{n} | a_{pq}(s) | \|f_q(x_q(s-\tau_{pq}(s)))\|_{\mathcal{A}}ds\Big)^2\Big\}$$

$$\leq 4\sup_{t\in\mathbb{R}}\max_{p\in\mathcal{D}}\Big\{E\Big(\int_{-\infty}^{t}e^{-\int_s^t c_p^{\emptyset}(u)du}\sum_{q=1}^{n}a_{pq}^{+}L_q^{f}\|x\|_{\mathbb{B}}ds\Big)^2\Big\}$$

$$\leq \max_{p\in\mathcal{D}}\Big\{\frac{4}{(\underline{c}_p^{\emptyset})^2}\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^{f})^2\Big\}\|x\|_{\mathbb{B}}^2. \tag{3.5}$$

Similarly, we have

$$A_3\leq \max_{p\in\mathcal{D}}\Big\{\frac{4}{(\underline{c}_p^{\emptyset})^2}\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^{+})^2\sum_{l=1}^{n}(M_l^{g})^2\Big)\sum_{q=1}^{n}(L_q^{g})^2\Big\}\|x\|_{\mathbb{B}}^2. \tag{3.6}$$

Moreover, by the Itô isometry, we obtain

$$A_4 = 4\sup_{t\in\mathbb{R}}\max_{p\in\mathcal{D}}\Big\{E\Big[\int_{-\infty}^{t}e^{-2\int_s^t c_p^{\emptyset}(u)du}\Big\|\sum_{q=1}^{n}\delta_{pq}(x_q(s-\gamma_{pq}(s)))\Big\|_{\mathcal{A}}^2ds\Big]\Big\}$$

$$\leq \max_{p\in\mathcal{D}}\Big\{\frac{2n}{\underline{c}_p^{\emptyset}}\sum_{q=1}^{n}(L_{pq}^{+})^2\Big\}\|x\|_{\mathbb{B}}^2. \tag{3.7}$$

Substituting (3.4)–(3.7) into (3.3), by (3.2) and (H_3), we have

$$\begin{aligned}
&\|\Phi x - x^0\|_{\mathbb{B}}^2\\
\leq &\max_{p\in\mathcal{D}}\Big\{\frac{4}{(\underline{c}_p^{\emptyset})^2}\Big[(\overline{c}_p^{\eth})^2+\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^{f})^2+\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^{+})^2\sum_{l=1}^{n}(M_l^{g})^2\Big)\\
&\times\sum_{q=1}^{n}(L_q^{g})^2+\frac{n\underline{c}_p^{\emptyset}}{2}\sum_{q=1}^{n}(L_{pq}^{+})^2\Big]\Big\}\|x\|_{\mathbb{B}}^2 = K\|x\|_{\mathbb{B}}^2\leq\frac{1}{4}(2\kappa)^2=\kappa^2.
\end{aligned}$$

For any $x\in\mathbb{B}_\kappa$ and $t_1,t_2\in\mathbb{R}$ with $t_1>t_2$, we derive that

$$\begin{aligned}
&E\|(\Phi x)(t_1)-(\Phi x)(t_2)\|_n^2\\
=&\max_{p\in\mathcal{D}}\Big\{E\Big\|\int_{-\infty}^{t_2}\Big[e^{-\int_s^{t_1}c_p^{\emptyset}(u)du}-e^{-\int_s^{t_2}c_p^{\emptyset}(u)du}\Big]\\
&\times\Big[-c_p^{\eth}(s)x_p(s)+\sum_{q=1}^{n}a_{pq}(s)f_q(x_q(s-\tau_{pq}(s)))\\
&+\sum_{q=1}^{n}\sum_{l=1}^{n}b_{pql}(s)g_q(x_q(s-\sigma_{pql}(s)))g_l(x_l(s-\nu_{pql}(s)))+I_p(s)\Big]ds\\
&+\int_{t_2}^{t_1}e^{-\int_s^{t_1}c_p^{\emptyset}(u)du}\Big[-c_p^{\eth}(s)x_p(s)+\sum_{q=1}^{n}a_{pq}(s)f_q(x_q(s-\tau_{pq}(s)))\\
&+\sum_{q=1}^{n}\sum_{l=1}^{n}b_{pql}(s)g_q(x_q(s-\sigma_{pql}(s)))g_l(x_l(s-\nu_{pql}(s)))+I_p(s)\Big]ds\\
&+\int_{-\infty}^{t_2}\Big[e^{-\int_s^{t_1}c_p^{\emptyset}(u)du}-e^{-\int_s^{t_2}c_p^{\emptyset}(u)du}\Big]\sum_{q=1}^{n}\delta_{pq}(x_q(s-\gamma_{pq}(s)))d\omega_q(s)
\end{aligned}$$

$$
\begin{aligned}
&+\int_{t_2}^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u)du} ds \sum_{q=1}^{n} \delta_{pq}(x_q(s-\gamma_{pq}(s)))d\omega_q(s)\Big\|_{\mathcal{A}}^2\Big\} \\
\le& \max_{p\in\mathcal{D}} \Big\{ E\Big[\int_{-\infty}^{t_2} \mid e^{-\int_s^{t_1} c_p^{\emptyset}(u)du} - e^{-\int_s^{t_2} c_p^{\emptyset}(u)du} \mid \Big(\overline{c}_p^{\eth}\|x\|_{\mathbb{B}} + \sum_{q=1}^{n} a_{pq}^{+} L_q^f \|x\|_{\mathbb{B}} \\
&+ \sum_{q=1}^{n}\sum_{l=1}^{n} b_{pql}^{+} L_q^g M_l^g \|x\|_{\mathbb{B}} + I_p^{+}\Big) ds + \int_{t_2}^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u)du}\Big(\overline{c}_p^{\eth}\|x\|_{\mathbb{B}} \\
&+ \sum_{q=1}^{n} a_{pq}^{+} L_q^f \|x\|_{\mathbb{B}} + \sum_{q=1}^{n}\sum_{l=1}^{n} b_{pql}^{+} L_q^g M_l^g \|x\|_{\mathbb{B}} + I_p^{+}\Big) ds \\
&+ \int_{-\infty}^{t_2} \mid e^{-\int_s^{t_1} c_p^{\emptyset}(u)du} - e^{-\int_s^{t_2} c_p^{\emptyset}(u)du} \mid \Big\| \sum_{q=1}^{n} \delta_{pq}(x_q(s-\gamma_{pq}(s)))d\omega_q(s) \\
&+ \int_{t_2}^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u)du} ds \sum_{q=1}^{n} \delta_{pq}(x_q(s-\gamma_{pq}(s)))d\omega_q(s)\Big\|_{\mathcal{A}}\Big]^2\Big\} \\
\le& \max_{p\in\mathcal{D}} 10\Big[\sum_{q=1}^{n}(a_{pq}^{+}L_q^f 2\kappa)^2 + \sum_{q=1}^{n}\sum_{l=1}^{n}(b_{pql}^{+}L_q^g 2\kappa)^2 + I_p^{+}\Big]\Big[\Big(\int_{-\infty}^{t_2} e^{-(t_2-s)\underline{c}_p^{\emptyset}} \\
&\times \mid \int_s^{t_2} c_p^{\emptyset}(u)du - \int_s^{t_1} c_p^{\emptyset}(u)du \mid ds\Big)^2 + \Big(\int_{t_2}^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u)du} ds\Big)^2\Big] \\
&+ 10n \max_{p\in\mathcal{D}} \sum_{q=1}^{n}(L_{pq}^{\delta}2\kappa)^2\Big[\int_{-\infty}^{t_2} e^{-2(t_2-s)\underline{c}_p^{\emptyset}} \\
&\times \mid \int_s^{t_2} c_p^{\emptyset}(u)du - \int_s^{t_1} c_p^{\emptyset}(u)du|^2 ds + \int_{t_2}^{t_1} e^{-2\int_s^{t_1} c_p^{\emptyset}(u)du} ds\Big] \\
\le& \max_{p\in\mathcal{D}} \Big\{10\Big[(\overline{c}_p^{\eth}2\kappa)^2 + \sum_{q=1}^{n}(a_{pq}^{+}L_q^f 2\kappa)^2 + \sum_{q=1}^{n}\sum_{l=1}^{n}(b_{pql}^{+}L_q^g 2\kappa)^2 + I_p^{+}\Big] \\
&\times \Big[(\frac{\overline{c}_p^{\emptyset}}{\underline{c}_p^{\emptyset}})^2 + 1 + \sum_{q=1}^{n}(L_{pq}^{\delta}2\kappa)^2\Big(\frac{5n(\overline{c}_p^{\emptyset})^2}{\underline{c}_p^{\emptyset}}\Big)\Big]\Big\} \mid t_1 - t_2|^2 \\
&+ \max_{p\in\mathcal{D}}\Big\{10n\sum_{q=1}^{n}(L_{pq}^{+}\kappa)^2\Big\} \mid t_1 - t_2 \mid \\
\le& \Lambda_1 \mid t_1 - t_2|^2 + \Lambda_2 \mid t_1 - t_2 \mid,
\end{aligned}
$$

where

$$
\begin{aligned}
\Lambda_1 =& \max_{p\in\mathcal{D}} \Big\{10\Big[(\overline{c}_p^{\eth}2\kappa)^2 + \sum_{q=1}^{n}(a_{pq}^{+}L_q^f 2\kappa)^2 + \sum_{q=1}^{n}\sum_{l=1}^{n}(b_{pql}^{+}L_q^g 2\kappa)^2 + I_p^{+}\Big] \\
&\times \Big[(\frac{\overline{c}_p^{\emptyset}}{\underline{c}_p^{\emptyset}})^2 + 1 + \sum_{q=1}^{n}(L_{pq}^{\delta}2\kappa)^2\Big(\frac{5n(\overline{c}_p^{\emptyset})^2}{\underline{c}_p^{\emptyset}}\Big)\Big]\Big\},
\end{aligned}
$$

$$\Lambda_2 = \max_{p\in\mathcal{D}} \Big\{10n \sum_{q=1}^{n} (L_{pq}^{+}\kappa)^2\Big\}.$$

Consequently, we deduce that $E\|(\Phi x)(t_1) - (\Phi x)(t_2)\|_0^2 \to 0$ as $t_1 \to t_2$, which implies Φx is uniformly $\mathcal{L}^2$-continuous. Therefore, we gain $\Phi(\mathbb{B}_\kappa) \subset \mathbb{B}_\kappa$, that is, Φ is well defined.

Next, we will show that Φ is a contraction operator. Actually, for every $x, y \in \mathbb{B}_\kappa$, one has

$$\begin{aligned}
&\|\Phi x - \Phi y\|_{\mathbb{B}}^2 \\
\leq& 4 \sup_{t\in\mathbb{R}} \max_{p\in\mathcal{D}} \Big\{E\Big\| \int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du} c_p^{\eth}(s)\Big(x_p(s) - y_p(s)\Big)ds\Big\|_{\mathcal{A}}^2\Big\} \\
&+ 4 \sup_{t\in\mathbb{R}} \max_{p\in\mathcal{D}} \Big\{E\Big\| \int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du} \sum_{q=1}^{n} a_{pq}(s)\Big(f_q(x_q(s-\tau_{pq}(s))) \\
&- f_q(y_q(s-\tau_{pq}(s)))\Big)ds\Big\|_{\mathcal{A}}^2\Big\} \\
&+ 4 \sup_{t\in\mathbb{R}} \max_{p\in\mathcal{D}} \Big\{E\Big\| \int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du} \sum_{q=1}^{n}\sum_{l=1}^{n} b_{pql}(s)\Big(g_q(x_q(s-\sigma_{pql}(s))) \\
&\times g_l(x_l(s-\nu_{pql}(s))) - g_q(y_q(s-\sigma_{pql}(s)))g_l(y_l(s-\nu_{pql}(s)))\Big)ds\Big\|_{\mathcal{A}}^2\Big\} \\
&+ 4 \sup_{t\in\mathbb{R}} \max_{p\in\mathcal{D}} \Big\{E\Big\| \int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u)du} \sum_{q=1}^{n} \Big(\delta_{pq}(x_q(s-\gamma_{pq}(s))) \\
&- \delta_{pq}(y_q(s-\gamma_{pq}(s)))\Big)d\omega_q(s)\Big\|_{\mathcal{A}}^2\Big\} \\
\leq& \max_{p\in\mathcal{D}} \Big\{\frac{4}{(\underline{c}_p^{\emptyset})^2}\Big[(\overline{c}_p^{\eth})^2 + \sum_{q=1}^{n}(a_{pq}^{+})^2 \sum_{q=1}^{n}(L_q^f)^2 + \sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^{+})^2 \sum_{l=1}^{n}(M_l^g)^2\Big) \\
&\times \sum_{q=1}^{n}(L_q^g)^2 + \frac{n\underline{c}_p^{\emptyset}}{2}\sum_{q=1}^{n}(L_{pq}^{+})^2\Big]\Big\}\|x-y\|_{\mathbb{B}}^2 = K\|x-y\|_{\mathbb{B}}^2.
\end{aligned}$$

Hence,

$$\|\Phi x - \Phi y\|_{\mathbb{B}} \leq \sqrt{K}\|x-y\|_{\mathbb{B}} < \frac{1}{2}\|x-y\|_{\mathbb{B}},$$

which implies that Φ is a contraction mapping. Therefore, Φ has a unique fixed point x in $\mathbb{B}_\kappa$, that is, system (1.1) possesses a unique solution x in $\mathbb{B}_\kappa$.

Finally, we will show that the unique solution x of system (1.1) in $\mathbb{B}_\kappa$ is almost periodic in distribution.

From the above discussion, we know that $x \in UCB(\mathbb{R}, \mathcal{L}^2(\Omega, \mathcal{A}^n))$. Hence, for given $\epsilon > 0$, there exists $\delta \in (0, \epsilon)$ such that, when $| t_1 - t_2 |< \delta$, we have $E\|x(t_1) - x(t_2)\|_0^2 < \epsilon$. According to (H_1), we see that, for the δ above, there exists $l > 0$ such that in every interval of length l of $\mathbb{R}$, we can find a number ς such that for $t \in \mathbb{R},\ p, q \in \mathcal{D}$,

$$| \tau_{pq}(t+\varsigma) - \tau_{pq}(t) |< \delta, \quad | \sigma_{pql}(t+\varsigma) - \sigma_{pql}(t) |< \delta, \quad | \nu_{pql}(t+\varsigma) - \nu_{pql}(t) |< \delta,$$

$$| \gamma_{pq}(t+\varsigma)-\gamma_{pq}(t) |<\delta, \quad | c_p^{\emptyset}(t+\varsigma)-c_p^{\emptyset}(t) |<\delta, \quad \|c_p^{\eth}(t+\varsigma)-c_p^{\eth}(t)\|_{\mathcal{A}}^2<\delta,$$

$$\|a_{pq}(t+\varsigma)-a_{pq}(t)\|_{\mathcal{A}}^2<\delta, \quad \|b_{pql}(t+\varsigma)-b_{pql}(t)\|_{\mathcal{A}}^2<\delta, \quad \|I_p(t+\varsigma)-I_p(t)\|_{\mathcal{A}}^2<\delta,$$

and hence, we have

$$E\|x(t-\tau_{pq}(t+\varsigma))-x(t-\tau_{pq}(t))\|_n^2<\epsilon,\ E\|x(t-\nu_{pql}(t+\varsigma))-x(t-\nu_{pql}(t))\|_n^2<\epsilon,$$

$$E\|x(t-\sigma_{pql}(t+\varsigma))-x(t-\sigma_{pql}(t))\|_n^2<\epsilon,\ E\|x(t-\gamma_{pq}(t+\varsigma))-x(t-\gamma_{pq}(t))\|_n^2<\epsilon.$$

By (3.1), we get

$$\begin{aligned}
&x_p(t+\varsigma)\\
=&\int_{-\infty}^{t+\varsigma} e^{-\int_s^{t+\varsigma} c_p^{\emptyset}(u)du}\Big[-c_p^{\eth}(s)x_p(s)+\sum_{q=1}^{n}a_{pq}(s)f_q(x_q(s-\tau_{pq}(s)))\\
&+\sum_{q=1}^{n}\sum_{l=1}^{n}b_{pql}(s)g_q(x_q(s-\sigma_{pql}(s)))g_l(x_l(s-\nu_{pql}(s)))+I_p(s)\Big]ds\\
&+\int_{-\infty}^{t+\varsigma} e^{-\int_s^{t+\varsigma} c_p^{\emptyset}(u)du}\sum_{q=1}^{n}\delta_{pq}(x_q(s-\gamma_{pq}(s)))d\omega_q(s)\\
=&\int_{-\infty}^{t} e^{-\int_s^{t} c_p^{\emptyset}(u+\varsigma)du}\Big[-c_p^{\eth}(s+\varsigma)x_p(s+\varsigma)+\sum_{q=1}^{n}a_{pq}(s+\varsigma)\\
&\times f_q(x_q(s+\varsigma-\tau_{pq}(s+\varsigma)))+\sum_{q=1}^{n}\sum_{l=1}^{n}b_{pql}(s+\varsigma)g_q(x_q(s+\varsigma-\sigma_{pql}(s+\varsigma)))\\
&\times g_l(x_l(s+\varsigma-\nu_{pql}(s+\varsigma)))+I_p(s+\varsigma)\Big]ds\\
&+\int_{-\infty}^{t} e^{-\int_s^{t} c_p^{\emptyset}(u+\varsigma)du}\sum_{q=1}^{n}\delta_{pq}(x_q(s+\varsigma-\gamma_{pq}(s+\varsigma)))d[\omega_q(s+\varsigma)-\omega_q(\varsigma)],
\end{aligned}$$

where $p\in\mathcal{D}$, $\omega_q(s+\varsigma)-\omega_q(\varsigma)$ is a Brownian motion with the same distribution as $\omega_q(s)$.

Let us consider the process

$$\begin{aligned}
&x_p(t+\varsigma)\\
=&\int_{-\infty}^{t} e^{-\int_s^{t} c_p^{\emptyset}(u+\varsigma)du}\Big[-c_p^{\eth}(s+\varsigma)x_p(s+\varsigma)+\sum_{q=1}^{n}a_{pq}(s+\varsigma)\\
&\times f_q(x_q(s+\varsigma-\tau_{pq}(s+\varsigma)))+\sum_{q=1}^{n}\sum_{l=1}^{n}b_{pql}(s+\varsigma)g_q(x_q(s+\varsigma-\sigma_{pql}(s+\varsigma)))\\
&\times g_l(x_l(s+\varsigma-\nu_{pql}(s+\varsigma)))+I_p(s+\varsigma)\Big]ds\\
&+\int_{-\infty}^{t} e^{-\int_s^{t} c_p^{\emptyset}(u+\varsigma)du}\sum_{q=1}^{n}\delta_{pq}(x_q(s+\varsigma-\gamma_{pq}(s+\varsigma)))d\omega_q(s).
\end{aligned}$$

Then

$$
\begin{aligned}
&E\|x(t+\varsigma)-x(t)\|_n^2 \\
\leq & 13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du} c_p^{\eth}(s+\varsigma)\Big(x_p(s+\varsigma)-x_p(s)\Big)ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\Big(c_p^{\eth}(s+\varsigma)-c_p^{\eth}(s)\Big)x_p(s)ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t} \Big(e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}-e^{-\int_s^t c_p^{\emptyset}(u)du}\Big)c_p^{\eth}(s)x_p(s)ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\sum_{q=1}^{n} a_{pq}(s+\varsigma)\Big(f_q(x_q(s+\varsigma-\tau_{pq}(s+\varsigma))) \\
&-f_q(x_q(s-\tau_{pq}(s)))\Big)ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\sum_{q=1}^{n}\Big(a_{pq}(s+\varsigma) \\
&-a_{pq}(s)\Big)f_q(x_q(s-\tau_{pq}(s)))ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t}\Big(e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}-e^{-\int_s^t c_p^{\emptyset}(u)du}\Big) \\
&\times\sum_{q=1}^{n} a_{pq}(s)f_q(x_q(s-\tau_{pq}(s)))ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\sum_{q=1}^{n}\sum_{l=1}^{n} b_{pql}(s+\varsigma)\Big(g_q(x_q(s+\varsigma-\sigma_{pql}(s+\varsigma))) \\
&\times g_l(x_l(s+\varsigma-\nu_{pql}(s+\varsigma)))-g_q(x_q(s-\sigma_{pql}(s)))g_l(x_l(s-\nu_{pql}(s)))\Big)ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\sum_{l=1}^{n}\Big(b_{pql}(s+\varsigma)-b_{pql}(s)\Big)g_q(x_q(s-\sigma_{pql}(s))) \\
&\times g_l(x_l(s-\nu_{pql}(s)))ds\bigg\|_{\mathcal{A}}^2+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t}\Big(e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}-e^{-\int_s^t c_p^{\emptyset}(u)du}\Big) \\
&\times\sum_{l=1}^{n} b_{pql}(s)g_q(x_q(s-\sigma_{pql}(s))\times g_l(x_l(s-\nu_{pql}(s)))ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\Big(I_p(s+\varsigma)-I_p(s)\Big)ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t}\Big(e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}-e^{-\int_s^t c_p^{\emptyset}(u)du}\Big)I_p(s)ds\bigg\|_{\mathcal{A}}^2 \\
&+13\max_{p\in\mathcal{D}} E\bigg\|\int_{-\infty}^{t} e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\sum_{q=1}^{n}\Big(\delta_{pq}(x_q(s+\varsigma-\gamma_{pq}(s+\varsigma)))
\end{aligned}
$$

$$
\begin{aligned}
&-\delta_{pq}(x_q(s-\gamma_{pq}(s)))\Big)d\omega_q(s)\Big\|_{\mathcal{A}}^2\\
&+13\max_{p\in\mathcal{D}}E\Big\|\int_{-\infty}^{t}\Big(e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}-e^{-\int_s^t c_p^{\emptyset}(u)du}\Big)\\
&\times\sum_{q=1}^{n}\delta_{pq}(x_q(s-\gamma_{pq}(s))d\omega_q(s)\Big\|_{\mathcal{A}}^2\\
&:=\sum_{i=1}^{13}S_i(t).
\end{aligned}
\tag{3.8}
$$

According to the Cauchy-Schwarz inequality, we have that

$$
\begin{aligned}
S_1(t)=&13\max_{p\in\mathcal{D}}E\Big\|\int_{-\infty}^{t}e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}c_p^{\eth}(s+\varsigma)\Big(x_p(s+\varsigma)-x_p(s)\Big)ds\Big\|_{\mathcal{A}}^2\\
\le&\max_{p\in\mathcal{D}}\Big\{\frac{13}{\underline{c}_p^{\emptyset}}(\overline{c}_p^{\eth})^2\int_{-\infty}^{t}e^{-\underline{c}_p^{\emptyset}(t-s)}E\|x(s+\varsigma)-x(s)\|_0^2ds\Big\},
\end{aligned}
\tag{3.9}
$$

$$
\begin{aligned}
S_2(t)\le&\max_{p\in\mathcal{D}}\Big\{\frac{13}{\underline{c}_p^{\emptyset}}\int_{-\infty}^{t}e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}E\|c_p^{\eth}(s+\varsigma)-c_p^{\eth}(s)\|_{\mathcal{A}}^2E\|x(s)\|_0^2ds\Big\}\\
\le&\max_{p\in\mathcal{D}}\Big\{\frac{13}{(\underline{c}_p^{\emptyset})^2}(2\kappa)^2\epsilon\Big\},
\end{aligned}
\tag{3.10}
$$

$$
\begin{aligned}
S_3(t)\le&13\max_{p\in\mathcal{D}}\Big\{\Big(\int_{-\infty}^{t}\mid e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}-e^{-\int_s^t c_p^{\emptyset}(u)du}\mid ds\Big)^2(\overline{c}_p^{\eth})^2E\|x(s)\|_0^2\Big\}\\
\le&13\max_{p\in\mathcal{D}}\Big\{\Big(\int_{-\infty}^{t}e^{-\underline{c}_p^{\emptyset}(t-s)}\int_s^t\mid c_p^{\emptyset}(u+\varsigma)du-c_p^{\emptyset}(u)\mid duds\Big)^2(\overline{c}_p^{\eth})^2(2\kappa)^2\Big\}\\
\le&\max_{p\in\mathcal{D}}\Big\{\frac{13}{(\underline{c}_p^{\emptyset})^4}(\overline{c}_p^{\eth})^2(2\kappa)^2\epsilon^2\Big\}.
\end{aligned}
\tag{3.11}
$$

Similarly, we can get

$$S_5(t)\le\max_{p\in\mathcal{D}}\Big\{\frac{13}{(\underline{c}_p^{\emptyset})^2}\sum_{q=1}^{n}(L_q^f)^2(2\kappa)^2n\epsilon\Big\},\tag{3.12}$$

$$S_6(t)\le\max_{p\in\mathcal{D}}\Big\{\frac{13}{(\underline{c}_p^{\emptyset})^4}\sum_{q=1}^{n}(a_{pq}^+)^2\sum_{q=1}^{n}(L_q^f)^2(2\kappa)^2\epsilon^2\Big\},\tag{3.13}$$

$$S_8(t)\le\max_{p\in\mathcal{D}}\Big\{\frac{13}{(\underline{c}_p^{\emptyset})^2}\sum_{q=1}^{n}(L_q^g)^2(2\kappa)^2\sum_{l=1}^{n}(M_l^g)^2n^2\epsilon\Big\},\tag{3.14}$$

$$S_9(t)\le\max_{p\in\mathcal{D}}\Big\{\frac{13}{(\underline{c}_p^{\emptyset})^4}\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^+)^2\sum_{l=1}^{n}(M_l^g)^2\Big)\sum_{q=1}^{n}(L_q^g)^2(2\kappa)^2\epsilon^2\Big\},\tag{3.15}$$

$$S_{10}(t)\le\max_{p\in\mathcal{D}}\Big\{\frac{13}{(\underline{c}_p^{\emptyset})^2}\epsilon\Big\},\tag{3.16}$$

$$S_{11}(t)\le\max_{p\in\mathcal{D}}\Big\{\frac{13}{(\underline{c}_p^{\emptyset})^4}M_I\epsilon^2\Big\}.\tag{3.17}$$

Note that

$$
\begin{aligned}
S_4(t) \leq & \max_{p\in\mathcal{D}}\Big\{\frac{26}{\underline{c}_p^{\emptyset}}\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^f)^2\Big[\int_{-\infty}^{t}e^{-\underline{c}_p^{\emptyset}(t-s)}E\|x(s+\varsigma-\tau_{pq}(s+\varsigma)) \\
& - x(s-\tau_{pq}(s+\varsigma))\|_0^2 ds \\
& + \int_{-\infty}^{t}e^{-\underline{c}_p^{\emptyset}(t-s)}E\|x(s-\tau_{pq}(s+\varsigma))-x(s-\tau_{pq}(s))\|_0^2 ds\Big]\Big\},
\end{aligned}
$$

let $u = s - \tau_{pq}(s+\varsigma)$, we obtain

$$
\begin{aligned}
S_4(t) \leq & \max_{p\in\mathcal{D}}\Big\{\frac{26}{\underline{c}_p^{\emptyset}}\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^f)^2\Big[\int_{-\infty}^{t-\tau_{pq}(t+\varsigma)}\frac{e^{-\underline{c}_p^{\emptyset}(t-u-\tau_{pq}(s+\varsigma))}}{1-\dot{\tau}_{pq}^{+}} \\
& \times E\|x(u+\varsigma)-x(u)\|_0^2 du + \frac{\epsilon}{\underline{c}_p^{\emptyset}}\Big]\Big\} \\
\leq & \max_{p\in\mathcal{D}}\Big\{\frac{26}{\underline{c}_p^{\emptyset}}\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^f)^2\frac{e^{\underline{c}_p^{\emptyset}\tau_{pq}^{+}}}{1-\dot{\tau}_{pq}^{+}}\int_{-\infty}^{t}e^{-\underline{c}_p^{\emptyset}(t-s)}E\|x(s+\varsigma)-x(s)\|_0^2 ds \\
& + \frac{26}{(\underline{c}_p^{\emptyset})^2}\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^f)^2\epsilon\Big\}.
\end{aligned}
\tag{3.18}
$$

By a same method, one gets

$$
\begin{aligned}
S_7(t) \leq & 13\max_{p\in\mathcal{D}}\Big\{E\Big\|\Big[\int_{-\infty}^{t}e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}ds\Big]\Big[\int_{-\infty}^{t}e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du} \\
& \times\Big(\sum_{q=1}^{n}\sum_{l=1}^{n}b_{pql}(s+\varsigma)\Big(g_q(x_q(s+\varsigma-\sigma_{pql}(s+\varsigma)))g_l(x_l(s+\varsigma-\nu_{pql}(s+\varsigma))) \\
& - g_q(x_q(s-\sigma_{pql}(s)))g_l(x_l(s-\nu_{pql}(s)))\Big)\Big)^2 ds\Big]\Big\|_{\mathcal{A}}\Big\} \\
\leq & \max_{p\in\mathcal{D}} E\Big\{\frac{13n}{\underline{c}_p^{\emptyset}}\int_{-\infty}^{t}e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\sum_{q=1}^{n}\Big[\sum_{l=1}^{n}(b_{pql}^{+})^2 \\
& \times\sum_{l=1}^{n}\Big(M_q^g L_l^g(\|x_l(s+\varsigma-\nu_{pql}(s+\varsigma))-x_l(s-\nu_{pql}(s+\varsigma))\|_{\mathcal{A}} \\
& + \|x_l(s-\nu_{pql}(s+\varsigma))-x_l(s-\nu_{pql}(s))\|_{\mathcal{A}}) \\
& + M_l^g L_q^g(\|x_q(s+\varsigma-\sigma_{pql}(s+\varsigma))-x_q(s-\sigma_{pql}(s+\varsigma))\|_{\mathcal{A}} \\
& + \|x_q(s-\sigma_{pql}(s+\varsigma))-x_q(s-\sigma_{pql}(s))\|_{\mathcal{A}})\Big)^2\Big]ds\Big\} \\
\leq & \max_{p\in\mathcal{D}} E\Big\{\frac{52n}{\underline{c}_p^{\emptyset}}\int_{-\infty}^{t}e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\sum_{q=1}^{n}\Big[\sum_{l=1}^{n}(b_{pql}^{+})^2 \\
& \times\sum_{l=1}^{n}\Big(M_q^g L_l^g(\|x_l(s+\varsigma-\nu_{pql}(s+\varsigma))
\end{aligned}
$$

$$
\begin{aligned}
&- x_l(s-\nu_{pql}(s+\varsigma))\|_{\mathcal{A}}^2 + \|x_l(s-\nu_{pql}(s+\varsigma)) - x_l(s-\nu_{pql}(s))\|_{\mathcal{A}}^2))\\
&+ M_l^g L_q^g(\|x_q(s+\varsigma-\sigma_{pql}(s+\varsigma)) - x_q(s-\sigma_{pql}(s+\varsigma))\|_{\mathcal{A}}^2\\
&+ \|x_q(s-\sigma_{pql}(s+\varsigma)) - x_q(s-\sigma_{pql}(s))\|_{\mathcal{A}}^2))\Big)\Big]ds\Big\}\\
\le& \max_{p\in\mathcal{D}}\Big\{\frac{52n}{\underline{c}_p^{\emptyset}}\int_{-\infty}^t e^{-\int_s^t c_p^{\emptyset}(u+\varsigma)du}\sum_{q=1}^n\Big[\sum_{l=1}^n (b_{pql}^+)^2\\
&\times\sum_{l=1}^n\Big(M_q^g L_l^g E\|x(s+\varsigma-\nu_{pql}(s+\varsigma)) - x(s-\nu_{pql}(s+\varsigma))\|_0^2\\
&+ M_l^g L_q^g E\|x(s+\varsigma-\sigma_{pql}(s+\varsigma)) - x(s-\sigma_{pql}(s+\varsigma))\|_0^2\\
&+ M_q^g L_l^g\epsilon + M_l^g L_q^g\epsilon\Big)\Big]ds\Big\}\\
\le& \max_{p\in\mathcal{D}}\Big\{\frac{52n}{\underline{c}_p^{\emptyset}}\sum_{q=1}^n\Big[\sum_{l=1}^n (b_{pql}^+)^2\cdot\sum_{l=1}^n\Big(M_q^g L_l^g\int_{-\infty}^t e^{-\underline{c}_p^{\emptyset}(t-s)}E\|x(s+\varsigma-\nu_{pql}(s+\varsigma))\\
&- x(s-\nu_{pql}(s+\varsigma))\|_0^2 ds + M_l^g L_q^g\int_{-\infty}^t e^{-\underline{c}_p^{\emptyset}(t-s)}E\|x(s+\varsigma-\sigma_{pql}(s+\varsigma))\\
&- x(s-\sigma_{pql}(s+\varsigma))\|_0^2 ds + \frac{M_q^g L_l^g\epsilon + M_l^g L_q^g\epsilon}{\underline{c}_p^{\emptyset}}\Big)\Big]ds\Big\}\\
\le& \max_{p\in\mathcal{D}}\Big\{\frac{52n}{\underline{c}_p^{\emptyset}}\sum_{q=1}^n\Big[\sum_{l=1}^n (b_{pql}^+)^2\cdot\sum_{l=1}^n\Big(M_q^g L_l^g\frac{e^{\underline{c}_p^{\emptyset}\nu_{pql}^+}}{1-\dot{\nu}_{pql}^+} + M_l^g L_q^g\frac{e^{\underline{c}_p^{\emptyset}\sigma_{pql}^+}}{1-\dot{\sigma}_{pql}^+}\Big)\\
&\times\int_{-\infty}^t e^{-\underline{c}_p^{\emptyset}(t-s)}E\|x(s+\varsigma) - x(s)\|_0^2 ds + \frac{M_q^g L_l^g + M_l^g L_q^g}{\underline{c}_p^{\emptyset}}\epsilon\Big)\Big]\Big\}.
\end{aligned}
\tag{3.19}
$$

Similar to (3.18) and by the Itô isometry, one has

$$
\begin{aligned}
S_{12}(t) \le& \max_{p\in\mathcal{D}}\Big\{26n\sum_{q=1}^n (L_{pq}^+)^2\frac{e^{2\underline{c}_p^{\emptyset}\gamma_{pq}^+}}{1-\dot{\gamma}_{pq}^+}\int_{-\infty}^t e^{-2\underline{c}_p^{\emptyset}(t-s)}E\|x(s+\varsigma) - x(s)\|_0^2 ds\\
&+ \frac{13n}{\underline{c}_p^{\emptyset}}\sum_{q=1}^n (L_{pq}^+)^2\epsilon\Big\}
\end{aligned}
\tag{3.20}
$$

and

$$
S_{13}(t) \le \max_{p\in\mathcal{D}}\Big\{\frac{13n}{(\underline{c}_p^{\emptyset})^4}\sum_{q=1}^n (L_{pq}^+)^2(2\kappa)^2\epsilon^2\Big\}. \tag{3.21}
$$

Substituting (3.9)–(3.21) into (3.8), we get

$$
E\|x(t+\tau) - x(t)\|_n^2 \le N\epsilon + P\int_{-\infty}^t e^{-c^-(t-s)}E\|x(s+\tau) - x(s)\|_n^2 ds,
$$

where $c^- = \min_{p\in\mathcal{D}}\{\underline{c}_p^{\emptyset}\}$,

$$
N = \max_{p\in\mathcal{D}}\Big\{\frac{13}{(\underline{c}_p^{\emptyset})^2}\Big[(2\kappa)^2 + 2\sum_{q=1}^n (a_{pq}^+)^2\sum_{q=1}^n (L_q^f)^2 + \sum_{q=1}^n (L_q^f)^2(2\kappa)^2 n
$$

$$+4n\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^{+})^{2}\sum_{l=1}^{n}(M_q^gL_l^g+M_l^gL_q^g)\Big)+\sum_{q=1}^{n}(L_q^g)^2(2\kappa)^2\sum_{l=1}^{n}(M_l^g)^2n^2$$

$$+1+n\underline{c}_p^{\emptyset}\sum_{q=1}^{n}(L_{pq}^{+})^2\Big]+\frac{13}{(\underline{c}_p^{\emptyset})^4}\Big[(\overline{c}_p^{\eth})^2(2\kappa)^2\epsilon+\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^f)^2(2\kappa)^2\epsilon$$

$$+\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^{+})^2\sum_{l=1}^{n}(M_l^g)^2\Big)\sum_{q=1}^{n}(L_q^g)^2(2\kappa)^2\epsilon+I_p^{+}\epsilon+n\sum_{q=1}^{n}(L_{pq}^{+})^2(2\kappa)^2\epsilon\Big]\Big\}.$$

Thus, by Lemma 2.2, we conclude that

$$E\|x(t+\tau)-x(t)\|_n^2<N\epsilon\frac{c^-}{c^--P},$$

which implies that $x(t)$ is square-mean almost periodic. According to Lemma 2.1, we deduce that $x(t)$ is almost periodic in distribution. The proof is complete. □

4. Global exponential stability in mean square

Definition 4.1. *[37] Let x be an almost periodic solution in distribution of* (1.1) *with the initial value φ. If there exist positive constants $\lambda > 0$ and $M > 0$ such that for every solution y with initial value ψ satisfies*

$$E\|x(t)-y(t)\|_n^2\le ME\|\varphi-\phi\|_\theta^2e^{-\lambda t},\ \ t>0,$$

where $\|\varphi-\phi\|_\theta^2=\sup\limits_{s\in[-\theta,0]}\|\varphi(s)-\phi(s)\|_n^2$, then the almost periodic solution $x(t)$ in distribution of (1.1) *is said to be globally exponentially stable.*

Theorem 4.1. *Assume that* (H_1)–(H_4) *hold, then the unique almost periodic solution in distribution of system* (1.1) *is globally exponentially stable.*

Proof. From Theorem 3.1, we know that system (1.1) has a unique almost periodic solution x in distribution with the initial value φ. Suppose that y is an arbitrary solution of (1.1) with initial value ψ. Set $z = x - y$, then from (1.1), we get

$$\begin{aligned}dz_p(t)=&\Big[-c_p^{\emptyset}(t)z_p(t)-c_p^{\eth}(t)z_p(t)+\sum_{q=1}^{n}a_{pq}(t)\Big(f_q(x_q(t-\tau_{pq}(t)))\\&-f_q(y_q(t-\tau_{pq}(t)))\Big)+\sum_{q=1}^{n}\sum_{l=1}^{n}b_{pql}(t)\Big(g_q(x_q(t-\sigma_{pql}(t)))\\&\times g_l(x_l(t-\nu_{pql}(t)))-g_q(y_q(t-\sigma_{pql}(t)))g_l(y_l(t-\nu_{pql}(t)))\Big)\Big]dt\\&+\sum_{q=1}^{n}\Big(\delta_{pq}(x_q(t-\gamma_{pq}(t)))-\delta_{pq}(y_q(t-\gamma_{pq}(t)))\Big)d\omega_q(t),\ \ p\in\mathcal{D}.\end{aligned}\tag{4.1}$$

Let $\Upsilon_p:\mathbb{R}\to\mathbb{R}$ be defined as follows:

$$\Upsilon_p(\varpi)=\underline{c}_p^{\emptyset}(\underline{c}_p^{\emptyset}-\varpi)-\varpi-5\Big[(\overline{c}_p^{\eth})^2+\sum_{q=1}^{n}(a_{pq}^{+})^2\sum_{q=1}^{n}(L_q^f)^2e^{\varpi\tau_{pq}^{+}}+2n\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^{+})^2$$

$$\times \sum_{l=1}^{n}\Big((M_q^g L_l^g)^2\, e^{\varpi \nu_{pql}^+} + (M_l^g L_q^g)^2 e^{\varpi \sigma_{pql}^+}\Big)\Big) + \frac{n\underline{c}_p^{\emptyset}}{2}\sum_{q=1}^{n}(L_{pq}^+)^2 e^{\varpi \gamma_{pq}^+}\Big].$$

By (H_4), we have $\Upsilon_p(0) > 0$. Due to the continuity of Υ_p on $[0, +\infty)$ and the fact that $\Upsilon_p(\varpi) \to -\infty$, as $\varpi \to +\infty$, there exists $\varsigma_p > 0$ such that $\Upsilon_p(\varsigma_p) = 0$ and $\Upsilon_p(\varpi) > 0$ for $\varpi \in (0, \varsigma_p)$. Consequently, one can take a positive constant $0 < \lambda < \min_{1\le p\le n}\{\varsigma_p, \underline{c}_p^{\emptyset}\}$ such that $\Upsilon(\lambda) > 0,\ p \in \mathcal{D}$, which implies that

$$\frac{5}{\underline{c}_p^{\emptyset}(\underline{c}_p^{\emptyset} - \lambda)}\Big[(\overline{c}_p^{\eth})^2 + \sum_{q=1}^{n}(a_{pq}^+)^2\sum_{q=1}^{n}(L_q^f)^2 e^{\lambda\tau_{pq}^+} + 2n\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^+)^2$$
$$\times \sum_{l=1}^{n}\Big((M_q^g L_l^g)^2 e^{\lambda\nu_{pql}^+} + (M_l^g L_q^g)^2 e^{\lambda\sigma_{pql}^+}\Big)\Big) + \frac{n\underline{c}_p^{\emptyset}}{2}\sum_{q=1}^{n}(L_{pq}^+)^2 e^{\lambda\gamma_{pq}^+}\Big] < 1.$$

Let $M = \max_{p\in\mathcal{D}}\Big\{\frac{(\underline{c}_p^{\emptyset})^2}{\gamma_p}\Big\}$, where

$$\gamma_p = (\overline{c}_p^{\eth})^2 + \sum_{q=1}^{n}(a_{pq}^+)^2\sum_{q=1}^{n}(L_q^f)^2 + 2n\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^+)^2\sum_{l=1}^{n}\Big((M_q^g L_l^g)^2 + (M_l^g L_q^g)^2\Big)\Big)$$
$$+ \frac{n\underline{c}_p^{\emptyset}}{2}\sum_{q=1}^{n}(L_{pq}^+)^2,$$

then by (H_4), we know $M > 1$, and further, we can deduce that

$$\frac{1}{M} - \frac{1}{\underline{c}_p^{\emptyset}(\underline{c}_p^{\emptyset} - \lambda)}\Big[(\overline{c}_p^{\eth})^2 + \sum_{q=1}^{n}(a_{pq}^+)^2\sum_{q=1}^{n}(L_q^f)^2 e^{\lambda\tau_{pq}^+} + 2n\sum_{q=1}^{n}\Big(\sum_{l=1}^{n}(b_{pql}^+)^2$$
$$\times \sum_{l=1}^{n}\Big((M_q^g L_l^g)^2 e^{\lambda\nu_{pql}^+} + (M_l^g L_q^g)^2 e^{\lambda\sigma_{pql}^+}\Big)\Big) + \frac{n\underline{c}_p^{\emptyset}}{2}\sum_{q=1}^{n}(L_{pq}^+)^2 e^{\lambda\gamma_{pq}^+}\Big] \le 0.$$

Hence, for any $\epsilon > 0$, it is easy to see that

$$E\|z(0)\|_n^2 \le E\|\varphi - \psi\|_\theta^2 + \epsilon$$

and for any $t \in [-\varrho, 0]$,

$$E\|z(t)\|_n^2 \le (E\|\varphi - \psi\|_\theta^2 + \epsilon)e^{-\lambda t} < M(E\|\varphi - \psi\|_\theta^2 + \epsilon)e^{-\lambda t}.$$

We assert that

$$E\|z(t)\|_n^2 < M(E\|\varphi - \psi\|_\theta^2 + \epsilon)e^{-\lambda t},\ \ t > 0. \tag{4.2}$$

On the contrary, there exists a certain $t_1 > 0$ such that

$$E\|z(t_1)\|_n^2 = M(E\|\varphi - \psi\|_\theta^2 + \epsilon)e^{-\lambda t_1}, \tag{4.3}$$

$$E\|z(t)\|_n^2 < M(E\|\varphi-\psi\|_\theta^2+\epsilon)e^{-\lambda t},\ \ t<t_1. \tag{4.4}$$

Multiplying (4.1) by $e^{\int_0^s c_p^{\emptyset}(u)du}$ and integrating it over the interval $[0,t]$, we deduce that, for $p\in\mathcal{D}$,

$$\begin{aligned} z_p(t) =& z_p(0)e^{-\int_0^t c_p^{\emptyset}(u)du} + \int_0^t e^{-\int_s^t c_p^{\emptyset}(u)du}\Big[-c_p^{\eth}(s)z_p(s) \\ &+ \sum_{q=1}^n a_{pq}(s)\Big(f_q(x_q(s-\tau_{pq}(s))) - f_q(y_q(s-\tau_{pq}(s)))\Big) \\ &+ \sum_{q=1}^n\sum_{l=1}^n b_{pql}(s)\Big(g_q(x_q(s-\sigma_{pql}(s)))g_l(x_l(s-\nu_{pql}(s))) \\ &- g_q(y_q(s-\sigma_{pql}(s)))g_l(y_l(s-\nu_{pql}(s)))\Big)\Big]ds \\ &+ \int_0^t e^{-\int_s^t c_p^{\emptyset}(u)du}\sum_{q=1}^n\Big(\delta_{pq}(x_q(s-\gamma_{pq}(s))) - \delta_{pq}(y_q(s-\gamma_{pq}(s)))\Big)d\omega_q(s). \end{aligned}$$

From the above equation, we have

$$\begin{aligned} E\|z_p(t_1)\|_{\mathcal{A}}^2 \le& 5E\Big\|z_p(0)e^{-\int_0^{t_1} c_p^{\emptyset}(u)du}\Big\|_{\mathcal{A}}^2 + 5E\Big\|\int_0^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u)du}c_p^{\eth}(s)z_p(s)ds\Big\|_{\mathcal{A}}^2 \\ &+ 5E\Big\|\int_0^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u)du}\sum_{q=1}^n a_{pq}(s)\Big(f_q(x_q(s-\tau_{pq}(s))) \\ &- f_q(y_q(s-\tau_{pq}(s)))\Big)ds\Big\|_{\mathcal{A}}^2 \\ &+ 5E\Big\|\int_0^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u)du}\sum_{q=1}^n\sum_{l=1}^n b_{pql}(s)\Big(g_q(x_q(s-\sigma_{pql}(s)))g_l(x_l(s-\nu_{pql}(s))) \\ &- g_q(y_q(s-\sigma_{pql}(s)))g_l(y_l(s-\nu_{pql}(s)))\Big)ds\Big\|_{\mathcal{A}}^2 \\ &+ 5E\Big\|\int_0^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u)du}\sum_{q=1}^n\Big(\delta_{pq}(x_q(s-\gamma_{pq}(s))) \\ &- \delta_{pq}(y_q(s-\gamma_{pq}(s)))\Big)d\omega_q(s)\Big\|_{\mathcal{A}}^2 \\ :=& \sum_{i=1}^5 \Xi_{ip}. \end{aligned} \tag{4.5}$$

By the Cauchy-Schwarz inequality, (4.3) and (4.4), one can get

$$\begin{aligned} \Xi_{2p} \le& \frac{5}{\underline{c}_p^{\emptyset}}(\overline{c}_p^{\eth})^2\int_0^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u)du}E\|z(s)\|_n^2ds \\ \le& \frac{5}{\underline{c}_p^{\emptyset}}(\overline{c}_p^{\eth})^2\int_0^{t_1} e^{-\int_s^{t_1}(c_p^{\emptyset}(u)-\lambda)du}dsM(E\|\psi-\varphi\|_\theta^2+\epsilon)e^{-\lambda t_1}, \end{aligned} \tag{4.6}$$

$$\Xi_{3p} \leq \frac{5}{\underline{c}_p^{\emptyset}} \sum_{q=1}^{n} (a_{pq}^+)^2 \sum_{q=1}^{n} (L_q^f)^2 e^{\lambda \tau_{pq}^+} \int_0^{t_1} e^{-\int_s^{t_1} c_p^{\emptyset}(u) du} E\|z(s)\|_n^2 ds$$

$$\leq \frac{5}{\underline{c}_p^{\emptyset}} \sum_{q=1}^{n} (a_{pq}^+)^2 \sum_{q=1}^{n} (L_q^f)^2 e^{\lambda \tau_{pq}^+} \int_0^{t_1} e^{-\int_s^{t_1} (c_p^{\emptyset}(u)-\lambda) du} ds M(E\|\psi-\varphi\|_\theta^2+\epsilon) e^{-\lambda t_1}, \tag{4.7}$$

$$\Xi_{4p} \leq \frac{10n}{\underline{c}_p^{\emptyset}} \sum_{q=1}^{n} \Big[\sum_{l=1}^{n} (b_{pql}^+)^2 \sum_{l=1}^{n} \Big((M_q^g L_l^g)^2 \, e^{\lambda \nu_{pql}^+} + (M_l^g L_q^g)^2 \, e^{\lambda \sigma_{pql}^+} \Big) \Big]$$

$$\times \int_0^{t_1} e^{-\int_s^{t_1} (c_p^{\emptyset}(u)-\lambda) du} ds M(E\|\psi-\varphi\|_\theta^2+\epsilon) e^{-\lambda t_1}. \tag{4.8}$$

By the Itô isometry, one gets

$$\Xi_{5p} \leq \frac{5n}{2} \sum_{q=1}^{n} (L_{pq}^+)^2 e^{\lambda \gamma_{pq}^+} \int_0^{t_1} e^{-\int_s^{t_1} (c_p^{\emptyset}(u)-\lambda) du} ds M(E\|\psi-\varphi\|_\theta^2+\epsilon) e^{-\lambda t_1}. \tag{4.9}$$

Substituting (4.6)–(4.9) into (4.5), one has

$$\begin{aligned}
E\|z_p(t_1)\|_{\mathcal{A}}^2 \leq & 5(E\|\varphi-\psi\|_\theta^2+\epsilon) e^{-\underline{c}_p^{\emptyset} t_1} + M(E\|\varphi-\psi\|_\theta^2+\epsilon) e^{-\lambda t_1} \int_0^{t_1} e^{-\int_s^{t_1} (c_p(u)-\lambda) du} ds \\
& \times \frac{5}{\underline{c}_p^{\emptyset}} \Big[(\overline{c}_p^{\eth})^2 + \sum_{q=1}^{n} (a_{pq}^+)^2 \sum_{q=1}^{n} (L_q^f)^2 e^{\lambda \tau_{pq}^+} + 2n \sum_{q=1}^{n} \Big(\sum_{l=1}^{n} (b_{pql}^+)^2 \\
& \times \sum_{l=1}^{n} \Big((M_q^g L_l^g)^2 \, e^{\lambda \nu_{pql}^+} + (M_l^g L_q^g)^2 e^{\lambda \sigma_{pql}^+} \Big) \Big) + \frac{n \underline{c}_p^{\emptyset}}{2} \sum_{q=1}^{n} (L_{pq}^+)^2 e^{\lambda \gamma_{pq}^+} \Big] \\
\leq & M(E\|\psi-\varphi\|_\theta^2+\epsilon) e^{-\lambda t_1} \Big\{ 5 \Big[\frac{1}{M} - \frac{1}{\underline{c}_p^{\emptyset}(\underline{c}_p^{\emptyset}-\lambda)} \Big((\overline{c}_p^{\eth})^2 + \sum_{q=1}^{n} (a_{pq}^+)^2 \sum_{q=1}^{n} (L_q^f)^2 e^{\lambda \tau_{pq}^+} \\
& + 2n \sum_{q=1}^{n} \Big(\sum_{l=1}^{n} (b_{pql}^+)^2 \sum_{l=1}^{n} \Big((M_q^g L_l^g)^2 \, e^{\lambda \nu_{pql}^+} + (M_l^g L_q^g)^2 e^{\lambda \sigma_{pql}^+} \Big) \Big) \\
& + \frac{n \underline{c}_p^{\emptyset}}{2} \sum_{q=1}^{n} (L_{pq}^+)^2 e^{\lambda \gamma_{pq}^+} \Big) \Big] e^{(\lambda - \underline{c}_p^{\emptyset}) t_1} + \frac{5}{\underline{c}_p^{\emptyset}(\underline{c}_p^{\emptyset}-\lambda)} \Big[(\overline{c}_p^{\eth})^2 + \sum_{q=1}^{n} (a_{pq}^+)^2 \\
& \times \sum_{q=1}^{n} (L_q^f)^2 e^{\lambda \tau_{pq}^+} + 2n \sum_{q=1}^{n} \Big(\sum_{l=1}^{n} (b_{pql}^+)^2 \sum_{l=1}^{n} \Big((M_q^g L_l^g)^2 \, e^{\lambda \nu_{pql}^+} + (M_l^g L_q^g)^2 e^{\lambda \sigma_{pql}^+} \Big) \Big) \\
& + \frac{n \underline{c}_p^{\emptyset}}{2} \sum_{q=1}^{n} (L_{pq}^+)^2 e^{\lambda \gamma_{pq}^+} \Big] \Big\} \\
< & M(E\|\psi-\varphi\|_\theta^2+\epsilon) e^{-\lambda t_1}.
\end{aligned}$$

Therefore,

$$E\|z(t_1)\|_n^2 < M(E\|\psi-\varphi\|_\theta^2+\epsilon) e^{-\lambda t_1},$$

which contradicts (4.3). Thus, (4.2) holds. Letting $\epsilon \to 0^+$, by (4.2), one has

$$E\|z(t)\|_n^2 \leq M E\|\psi-\varphi\|_\theta^2 e^{-\lambda t_1}, \ t > 0.$$

Consequently, the almost periodic solution in distribution of system (1.1) is globally exponentially stable. This completes the proof. □

Remark 4.1. *In [37], a class of Clifford-valued stochastic recurrent neural networks whose leakage term coefficients are real numbers are considered. In this paper, we consider a class of Clifford-valued stochastic high-order Hopfield neural networks whose leakage term coefficients are also Clifford numbers. Therefore, even if the* $c_p(t)$ *in* (1.1) *is a real-valued function, the conclusions of theorems 3.1 and 4.1 in this paper cannot be obtained from the corresponding results in [37].*

5. A numerical example

Our example is as follow.

Example 5.1. *In system* (1.1)*, let* $m = n = 2$, $\iota = 1$, *and for* $p, q, l = 1, 2$, *take*

$$x_p = x_p^0 e_0 + x_p^1 e_1 + x_p^2 e_2 + x_p^{12} e_{12} \in \mathcal{A},$$

$$\begin{aligned} f_p(x_p) =& 0.021 e_0 \sin(x_p^2 + 2x^{12}) + 0.038 e_1 \arctan(3x_p^1 + x_p^2) \\ &+ 0.2 e_2 \tanh(x_p^0 - x_p^{12}) + 0.05 e_{12} \sin x_p^2, \\ g_p(x_p) =& 0.034 e_0 \sin(x_p^2 + x^{12}) + 0.1 e_1 \tanh(3x_p^{12} - 2x_p^0) \\ &+ 0.1 e_2 \sin 5x_p^1 + 0.02 e_{12} \sin 2x_p^1, \end{aligned}$$

$$\delta_{pq}(x_p) = 0.02 e_0 \tanh(x_p^2 + x^{12}) + 0.01 e_1 \tanh(6x_p^1 - 3x_p^0) + 0.03 e_{12} \sin 3x_p^{12},$$

$$a_{11}(t) = a_{12}(t) = 0.05 e_0 \sin \sqrt{2}t - 0.009 e_1 \cos \sqrt{7}t + 0.05 e_2 \cos \sqrt{3}t + 0.007 e_{12} \tanh t,$$

$$a_{21}(t) = a_{22}(t) = 0.07 e_0 \cos t - 0.09 e_1 \sin \sqrt{3}t + 0.05 e_2 \sin \sqrt{2}t - 0.009 e_{12} \cos \sqrt{7}t,$$

$$b_{1ql}(t) = 0.05 e_0 \sin \sqrt{7}t - 0.01 e_1 \cos \sqrt{3}t + 0.027 e_2 \arctan \sqrt{7}t + 0.034 e_{12} \tanh 2t,$$

$$b_{2ql}(t) = 0.056 e_0 \sin \sqrt{2}t - 0.06 e_1 \sin \sqrt{7}t + 0.012 e_2 \sin \sqrt{2}t - 0.009 e_{12} \cos \sqrt{7}t,$$

$$c_1(t) = (1.2 + 0.2 \sin t) e_0 + 0.01 e_1 \cos \sqrt{2}t + 0.0036 e_{12} \tanh \sqrt{2}t,$$

$$c_2(t) = (3 - 2\sin t) e_0 + 0.02 e_1 \sin \sqrt{3}t + 0.01 e_2 \arctan \sqrt{2}t, \ \tau_{pq}(t) = 0.04 + 0.01 \sin 0.1t,$$

$$\sigma_{pql}(t) = 0.9 + 0.1 \cos 0.4t, \ \nu_{pql}(t) = 0.5 + 0.1 \cos 0.1t, \ \gamma_{pq}(t) = 0.05 + 0.01 \sin 0.0001t,$$

$$I_1(t) = 2e_0 \sin t + e_1 \cos \sqrt{3}t + 0.1 e_2 \sin \sqrt{2}t + 2 e_{12} \cos \sqrt{2}t,$$

$$I_2(t) = 4 e_0 \sin t + 2 e_1 \sin \sqrt{11}t + 0.5 e_2 \sin \sqrt{2}t + e_{12} \cos t.$$

By a simple calculation, we have

$$\underline{c}_1^{\emptyset} = \underline{c}_2^{\emptyset} = 1, \ c^- = 1, \ \overline{c}_1^{\eth} = 0.01, \ \overline{c}_2^{\eth} = 0.02, \ L_q^f = 0.2, \ L_q^g = 0.1,$$

$$M_q^g = 0.1, \ L_{pq}^+ = 0.03, \ \tau_{pq}^+ = 0.05, \ \dot{\tau}_{pq}^+ = 0.001, \ \sigma_{pql}^+ = 1,$$

$$\dot{\sigma}_{pql}^+ = 0.04, \ \nu_{pql}^+ = 0.6, \ \dot{\nu}_{pql}^+ = 0.01, \ \gamma_{pq}^+ = 0.06, \ \dot{\gamma}_{pq}^+ = 0.000001,$$

$$a_{11}^+ = a_{12}^+ = 0.05, \ a_{21}^+ = a_{22}^+ = 0.09, \ b_{1ql}^+ = 0.05, \ b_{2ql}^+ = 0.06,$$

$$K \approx 0.014 < \frac{1}{4},\ P \approx 0.287 < c^-,\ C \approx 0.009 < 1.$$

Therefore, all of the conditions of Theorem 4.1 are satisfied. Hence, system (1.1) *has a unique almost periodic solution in distribution that is globally exponentially stable (see Figures 1–10).*

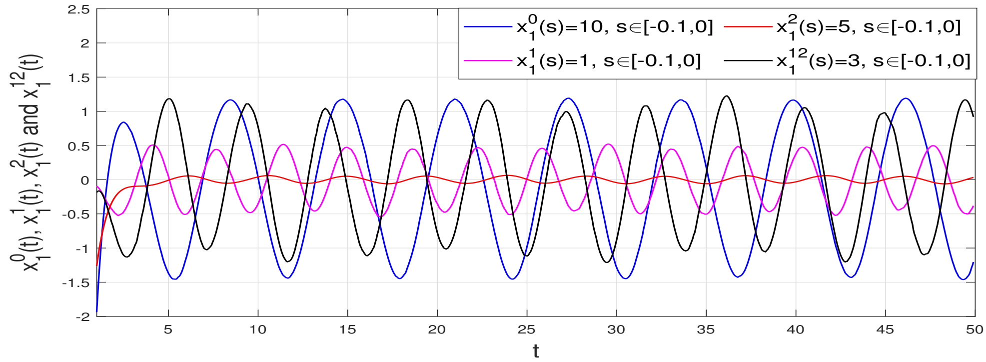

Figure 1. States $(x_1^0, x_1^1, x_1^2, x_1^{12})^T$ of (1.1) with initial values $x_1^0(s) = 10$, $x_1^1(s) = 1$, $x_1^2(s) = 5$ and $x_1^{12}(s) = 3$ for $s \in [-0.1, 0]$, respectively.

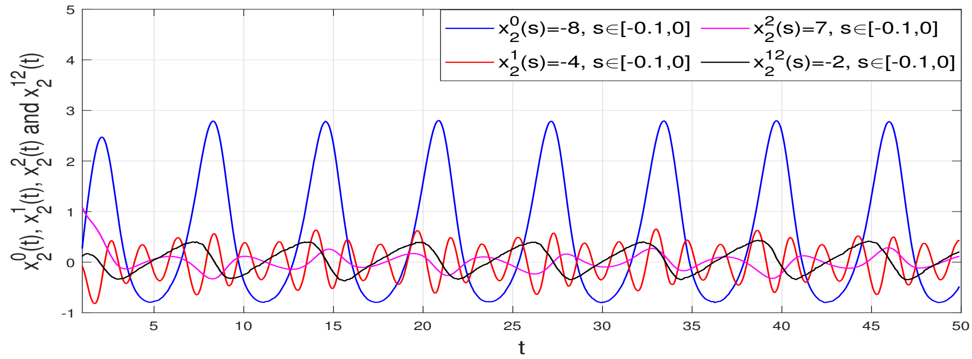

Figure 2. States $(x_2^0, x_2^1, x_2^2, x_2^{12})^T$ of (1.1) with initial values $x_2^0(s) = -8$, $x_2^1(s) = -4$, $x_2^2(s) = 7$ and $x_2^{12}(s) = -2$ for $s \in [-0.1, 0]$, respectively.

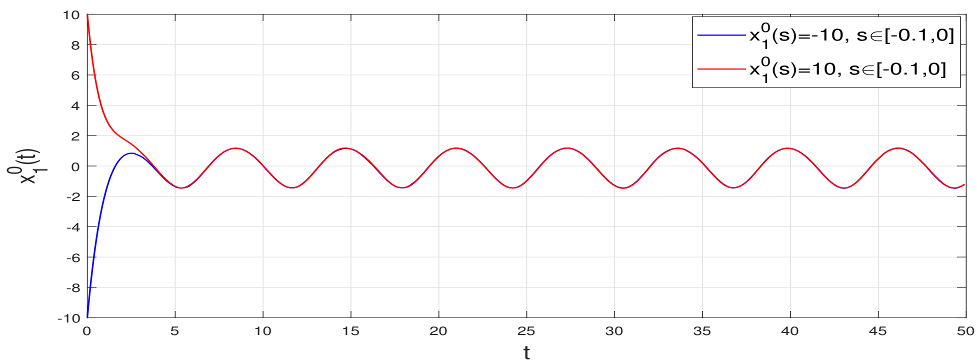

Figure 3. Global exponential stability of state x_1^0 of (1.1) with different initial values $x_1^0(s) = 10$ or $x_1^0(s) = -10$ for $s \in [-0.1, 0]$.

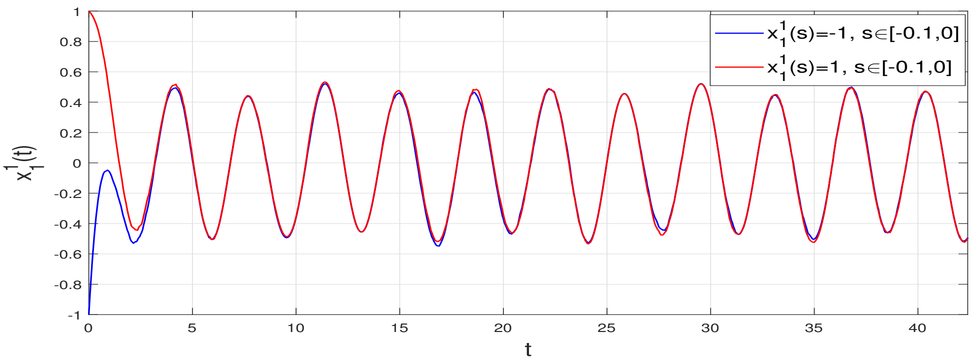

Figure 4. Global exponential stability of state x_1^1 of (1.1) with different initial values $x_1^1(s) = 1$ or $x_1^1(s) = -1$ for $s \in [-0.1, 0]$.

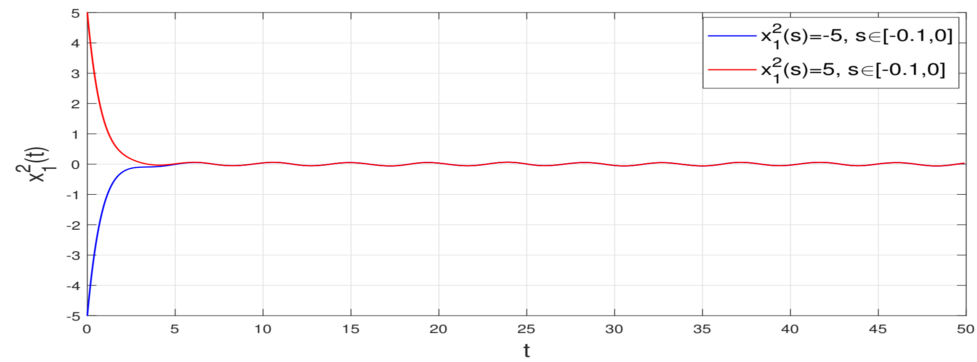

Figure 5. Global exponential stability of state x_1^2 of (1.1) with different initial values $x_1^2(s) = 5$ or $x_1^2(s) = -5$ for $s \in [-0.1, 0]$.

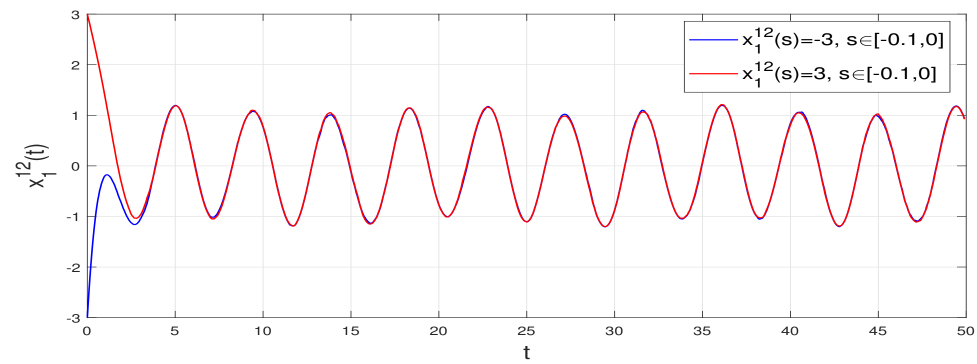

Figure 6. Global exponential stability of state x_1^{12} of (1.1) with different initial values $x_1^{12}(s) = 3$ or $x_1^{12}(s) = -3$ for $s \in [-0.1, 0]$.

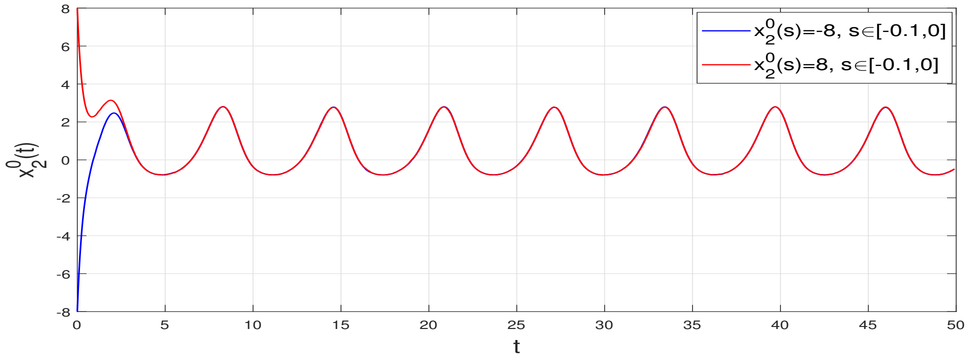

Figure 7. Global exponential stability of state x_2^0 of (1.1) with different initial values $x_2^0(s) = 8$ or $x_2^0(s) = -8$ for $s \in [-0.1, 0]$.

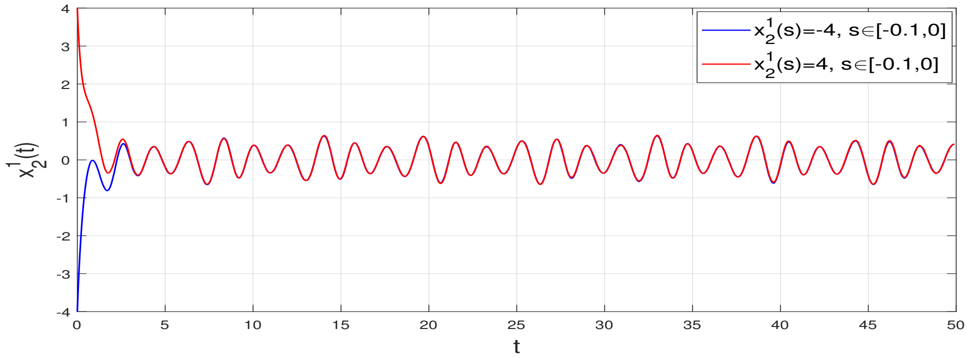

Figure 8. Global exponential stability of state x_2^1 of (1.1) with different initial values $x_2^1(s) = 4$ or $x_2^1(s) = -4$ for $s \in [-0.1, 0]$.

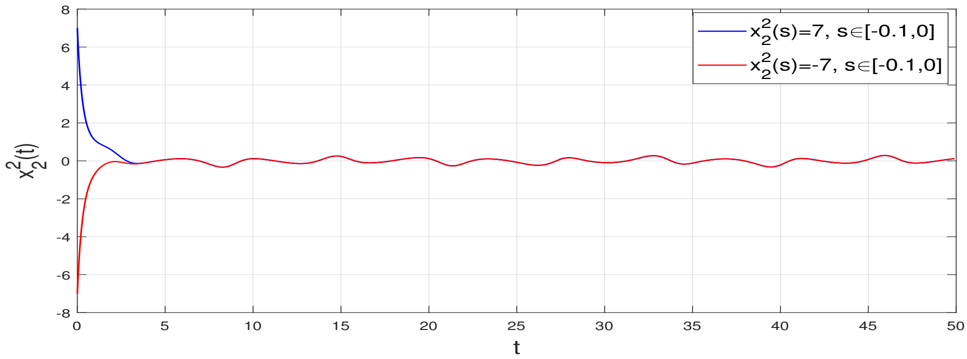

Figure 9. Global exponential stability of state x_2^2 of (1.1) with different initial values $x_2^2(s) = 7$ or $x_2^2(s) = -7$ for $s \in [-0.1, 0]$.

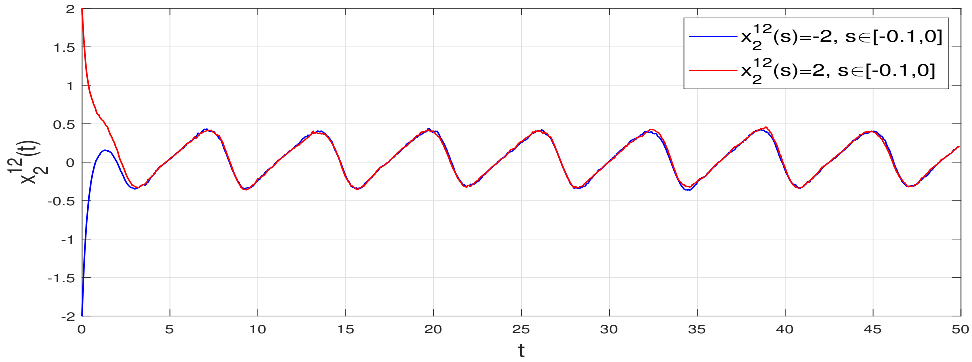

Figure 10. Global exponential stability of state x_2^{12} of (1.1) with different initial values $x_2^{12}(s) = 2$ or $x_2^{12}(s) = -2$ for $s \in [-0.1, 0]$.

Remark 5.1. *The results of Example 5.1 cannot be deduced from the existing results.*

6. Conclusions

In this work, we use a direct method to prove the existence and global exponential stability of almost periodic solutions in distribution for Clifford-valued stochastic higher-order Hopfield neural networks with all parameters being Clifford numbers except time delays. Even when the neural network considered in this paper degenerates into real-valued, complex-valued and quaternion-valued neural networks, our results are new. In addition, the method proposed in this paper can be used to study the almost periodic solutions in distribution for other types of Clifford-valued stochastic neural networks with time-varying delays.

Acknowledgments

This work is supported by the National Natural Science Foundation of China under Grant No. 11861072, the Natural Science Foundation of Anhui Province under Grant No. 2108085QA10 and the Applied Basic Research Foundation of Yunnan Province under Grant No. 2019FB003.

Conflict of interest

The authors declare that they have no competing interests.

References

1. J. Pearson, D. Bisset, Neural networks in the Clifford domain, *Proceedings of 1994 IEEE International Conference on Neural Networks (ICNN'94)*, 1994, 1465–1469. doi: 10.1109/ICNN.1994.374502.
2. E. Bayro-Corrochano, R. Vallejo, N. Arana-Daniel, Geometric preprocessing, geometric feedforward neural networks and Clifford support vector machines for visual learning, *Neurocomputing*, **67** (2005), 54–105. doi: 10.1016/j.neucom.2004.11.041.

3. S. Buchholz, G. Sommer, On Clifford neurons and Clifford multi-layer perceptrons, *Neural Networks*, **21** (2008), 925–935. doi: 10.1016/j.neunet.2008.03.004.

4. E. Hitzer, T. Nitta, Y. Kuroe, Applications of Clifford's geometric algebra, *Adv. Appl. Clifford Algebras*, **23** (2013), 377–404. doi: 10.1007/s00006-013-0378-4.

5. Y. Liu, P. Xu, J. Lu, J. Liang, Global stability of Clifford-valued recurrent neural networks with time delays, *Nonlinear Dyn.*, **84** (2016), 767–777. doi: 10.1007/s11071-015-2526-y.

6. Y. Li, Y. Wang, B. Li, The existence and global exponential stability of μ-pseudo almost periodic solutions of Clifford-valued semi-linear delay differential equations and an application, *Adv. Appl. Clifford Algebras*, **29** (2019), 105. doi: 10.1007/s00006-019-1025-5.

7. S. Shen, Y. Li, S^p-almost periodic solutions of Clifford-valued fuzzy cellular neural networks with time-varying delays, *Neural Process. Lett.*, **51** (2020), 1749–1769. doi: 10.1007/s11063-019-10176-9.

8. Y. Li, N. Huo, B. Li, On μ-pseudo almost periodic solutions for Clifford-valued neutral type neural networks with delays in the leakage term, *IEEE T. Neur. Net. Lear.*, **32** (2021), 1365–1374. doi: 10.1109/TNNLS.2020.2984655.

9. G. Rajchakit, R. Sriraman, P. Vignesh, C. P. Lim, Impulsive effects on Clifford-valued neural networks with time-varying delays: An asymptotic stability analysis, *Appl. Math. Comput.*, **407** (2021), 126309. doi: 10.1016/j.amc.2021.126309.

10. A. Chaouki, F. Touati, Global dissipativity of Clifford-valued multidirectional associative memory neural networks with mixed delays, *Comp. Appl. Math.*, **39** (2020), 310. doi: 10.1007/s40314-020-01367-5.

11. C. Aouiti, I. Gharbia, Dynamics behavior for second-order neutral Clifford differential equations: inertial neural networks with mixed delays, *Comp. Appl. Math.*, **39** (2020), 120. doi: 10.1007/s40314-020-01148-0.

12. C. Aouiti, F. Dridi, Weighted pseudo almost automorphic solutions for neutral type fuzzy cellular neural networks with mixed delays and D operator in Clifford algebra, *Int. J. Syst. Sci.*, **51** (2020), 1759–1781. doi: 10.1080/00207721.2020.1777345.

13. S. Shen, Y. Li, Weighted pseudo almost periodic solutions for Clifford-valued neutral-type neural networks with leakage delays on time scales, *Adv. Differ. Equ.*, **2020** (2020), 286. doi: 10.1186/s13662-020-02754-2.

14. N. Huo, B. Li, Y. Li, Anti-periodic solutions for Clifford-valued high-order Hopfield neural networks with state-dependent and leakage delays, *Int. J. Appl. Math. Comput. Sci.*, **30** (2020), 83–98. doi: 10.34768/amcs-2020-0007.

15. X. Liu, Q. Wang, Impulsive stabilization of high-order Hopfield-type neural networks with time-varying delays, *IEEE Trans. Neural Networ*, **19** (2008), 71–79. doi: 10.1109/TNN.2007.902725.

16. C. Ou, Anti-periodic solutions for high-order Hopfield neural networks, *Comput. Math. Appl.*, **56** (2008), 1838–1844. doi: 10.1016/j.camwa.2008.04.029.

17. Z. He, C. Li, H. Li, Q. Zhang, Global exponential stability of high-order Hopfield neural networks with state-dependent impulses, *Physica A*, **542** (2020), 123434. doi: 10.1016/j.physa.2019.123434.

18. X. Meng, Y. Li, Pseudo almost periodic solutions for quaternion-valued high-order Hopfield neural networks with time-varying delays and leakage delays on time scales, *AIMS Mathematics*, **6** (2021), 10070–10091. doi: 10.3934/math.2021585.

19. S. Blythe, X. Mao, X. Liao, Stability of stochastic delay neural networks, *J. Franklin I.*, **338** (2001), 481–495. doi: 10.1016/S0016-0032(01)00016-3.

20. Y. Ren, Q. He, Y. Gu, R. Sakthivel, Mean-square stability of delayed stochastic neural networks with impulsive effects driven by G-Brownian motion, *Stat. Probabil. Lett.*, **143** (2018), 56–66. doi: 10.1016/j.spl.2018.07.024.

21. L. Liu, A. Wu, Z. Zeng, T. Huang, Global mean square exponential stability of stochastic neural networks with retarded and advanced argument, *Neurocomputing*, **247** (2017), 156–164. doi: 10.1016/j.neucom.2017.03.057.

22. R. Suresh, A. Manivannan, Robust stability analysis of delayed stochastic neural networks via Wirtinger-based integral inequality, *Neural Comput.*, **33** (2021), 227–243. doi: 10.1162/neco_a_01344.

23. Y. Wang, J. Lou, H. Yan, J. Lu, Stability criteria for stochastic neural networks with unstable subnetworks under mixed switchings, *Neurocomputing*, **452** (2021), 827–833. doi: 10.1016/j.neucom.2019.10.119.

24. Q. Zhu, Stabilization of stochastic nonlinear delay systems with exogenous disturbances and the event-triggered feedback control, *IEEE Trans. Auto. Control*, **64** (2019), 3764–3771. doi: 10.1109/TAC.2018.2882067.

25. Q. Zhu, T. Huang, Stability analysis for a class of stochastic delay nonlinear systems driven by G-Brownian motion, *Syst. Control Lett.*, **140** (2020), 104699. doi: 10.1016/j.sysconle.2020.104699.

26. H. Wang, Q. Zhu, Global stabilization of a class of stochastic nonlinear time-delay systems with SISS inverse dynamics, *IEEE Trans. Auto. Control*, **65** (2020), 4448–4455. doi: 10.1109/TAC.2020.3005149.

27. R. Rajan, V. Gandhi, P. Soundharajan, Y. Joo, Almost periodic dynamics of memristive inertial neural networks with mixed delays, *Inform. Sciences*, **536** (2020), 332–350. doi: 10.1016/j.ins.2020.05.055.

28. P. Wan, D. Sun, M. Zhao, S. Huang, Multistability for almost-periodic solutions of takagi-sugeno fuzzy neural networks with nonmonotonic discontinuous activation functions and time-varying delays, *IEEE T. Fuzzy Syst.*, **29** (2021), 400–414. doi: 10.1109/TFUZZ.2019.2955886.

29. M. Bohner, G. Stamov, I. Stamova, Almost periodic solutions of Cohen-Grossberg neural networks with time-varying delay and variable impulsive perturbations, *Commun. Nonlinear Sci.*, **80** (2020), 104952. doi: 10.1016/j.cnsns.2019.104952.

30. O. Mellah, P. Fitte, Counterexamples to mean square almost periodicity of the solutions of some SDEs with almost periodic coefficients, *Electron. J. Differ. Eq.*, **2013** (2013), 1–7.

31. F. Brackx, R. Delanghe, F. Sommen, *Clifford analysis*, Boston: Pitman Books Limited, 1982.

32. A. Klenke, *Probability theory: a comprehensive course*, Berlin: Springer, 2008.

33. Z. Liu, K. Sun, Almost automorphic solutions for stochastic differential equations driven by Lévy noise, *J. Funct. Anal.*, **266** (2014), 1115–1149. doi: 10.1016/j.jfa.2013.11.011.

34. P. H. Bezandry, T. Diagana, *Almost periodic stochastic processes*, New York: Springer, 2011.

35. T. Morozan, C. Tudor, Almost periodic solutions of affine Itô equations, *Stoch. Anal. Appl.*, **7** (1989), 451–474. doi: 10.1080/07362998908809194.

36. M. Kamenskii, O. Mellah, P. Raynaud-de-Fitte, Weak averaging of semilinear stochastic differential equations with almost periodic coefficients, *J. Math. Anal. Appl.*, **427** (2015), 336–364. doi: 10.1016/j.jmaa.2015.02.036.

37. Y. Li, X. Wang, Almost periodic solutions in distribution of Clifford-valued stochastic recurrent neural networks with time-varying delays, *Chaos Soliton. Fract.*, **153** (2021), 111536. doi: 10.1016/j.chaos.2021.111536.

AIMS Press