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Research article

New classes of reverse super edge magic graphs

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Abstract: A reverse edge magic (REM) labeling of a graph $G(V, E)$ with p vertices and q edges is a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \cdots, p+q\}$ such that $k = f(uv) - \{f(u) + f(v)\}$ is a constant k for any edge $uv \in E(G)$. A REM labeling f is called reverse super edge magic (RSEM) labeling if $f(V(G)) = \{1, 2, 3, 4, 5, \ldots, v\}$ and $f(E(G)) = \{v+1, v+2, v+3, v+4, v+5, \ldots, v+e\}$. In this paper, we find some new classes of RSEM labeling and the investigation of the connection between the RSEM labeling and different classes of labeling.

Keywords: trees; lobster; Banana graph; cartesian product; cycle
Mathematics Subject Classification: 37E25, 05C38

1. Introduction

The edge magic labelings of graphs were introduced by Kotzig and A. Rosa [1] and they called also magic valuations of graphs. In [2], The super edge magic labelings of a graph then the idea of edge magic labelings is proved by H. Enomoto et al. In [8], R. M. Figueroa Centeno et al. proved all caterpillars are super edge magic also verified that $mK_{1,n}, m$ and m, n are positive integers with the super edge magic is odd. In [4], M. Figueroa Centeno et al. defined that the forest $P_m \cup K_{1,n}, m \geq 4$ each positive integer $n \geq 1$. All trees are edge magic is verified by G. Ringel and A. Llado [8]. H. Enomoto et al. proposed in [2] a more difficult hypothesis: that every tree is super edge magic. All the lobsters are gracefully demonstrated by J. C. Bermond [7].

If G be the (super) 2-regular edge magic graph with n positive integers, then $G \odot \overline{K_n}$ is (super) edge magic and therefore for every two integers $m \geq 3$ and $m \geq 1$, then $n-$ crown $C_m \odot \overline{K_n}$ is super edge magic these results proved by R. Figueroa Centeno et al. [6]. V. Yegnanarayanan [3] demonstrated that the graph is obtained through edge magic for $t \geq 2$ also introduced new pendant edges of the outermost C_3 in $P_t \times C_3$ at each vertex. In [10], The total graph $T(P_n)$ is harmonious is obtained by R. Balakrishnan and R. Sampath kumar. In [2], H. Enomoto et al. is obtained that the complete bipartite graph $K_{m,n}$ is super edge-magic iff $m = 1$ or $n = 1$. R. Balakrishnan et al. [11] obtained that the harmonious iff n is even and the graph $K_2 + 2K_2$ is magic iff $n = 3$. In [9], K. Kathiresan proved that the subdivision graph $S(L_n)$ obtained by subdividing every edge of G exactly one is graceful. In [5], V. Yegnanarayanan introduced several other variations of magic labelings and discuss what are called vertex-magic and vertex-antimagic of $(1,1),(1,0)$ and $(0,1)$ graphs. Also, discussed edge-magic and edge-antimagic of $(1,0)$ and $(0,1)$ graphs. Finally, exhibited such magic, anti-magic labelings for a number of classes of graphs and derived several general results governing these graphs.

A reverse edge magic (REM) labeling of a graph $G(V,E)$ with p vertices and q edges is a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, \cdots, p+q\}$ such that $k = f(uv) - \{f(u) + f(v)\}$ is a constant k for any edge $uv \in E(G)$. A REM labeling f is called reverse super edge magic (RSEM) labeling if $f(V(G)) = \{1, 2, 3, 4, 5, \ldots, v\}$ and $f(E(G)) = \{v+1, v+2, v+3, v+4, v+5, \ldots, v+e\}$. In this paper, we find some new classes of RSEM labeling and the investigation of the connection between the RSEM labeling and different classes of labeling.

Definition 1. Let a be the path P_n, $1 \le i \le n$ and T_1 be a caterpillar obtained by position one end vertex at each vertex. Let T be the lobster created by linking a copy of P_2 at each end vertex b_i of $1 \le i \le n$.

2. New structures of revese super edge magic graphs

The accompanying outcomes on trees give support to the conjecture that all trees are RSEM.

Lemma 1. A graph G with p vertices and q edges is RSEM iff $\exists$ a bijective function $f : V(G) \rightarrow \{1, 2, ..., p\}$ so that the set $S = \{f(x) + f(y) : xy \in E(G)\}$ contains q number of successive numbers. In such a case, f spreads to a RSEM labeling of the graph G with reverse magic constant $k = p + q - s$, where $s = max(S)$ and $S = \{(p+1) - k, (p+2) - k, (p+3) - k, ..., (p+q) - k\}$.

Theorem 1. If m is odd. Then 3−stat $S_{m,3}$ is RSEM.

Proof. Let m is odd. Assume m be the degrees of vertex x in $S_{m,3}$ and 3 is the length of i^{th} path of $xu_iv_iw_i$ for $1 \le i \le m$.

The paths are RSEM and since $S_{1,3} \cong P_4$, when $m = 1$ the outcome is true. Assume that m is an odd number and $m > 3$. Assume $n = 3m + 1$.

Define the vertex labeling, $f : V(S_{m,3}) \rightarrow \{1, 2, 3, 4, 5, ..., n\}$ such that

$$f(x) = \frac{n+2}{3}$$
$$f(u_1) = n$$

$$f(u_{2i}) = 2i \ \ for \ \ 1 \le i \le \frac{n-4}{6}$$
$$f(u_{2i+1}) = \frac{n+5}{3} + 2i - 1 \ \ for \ \ 1 \le i \le \frac{n-4}{6}$$
$$f(v_{2i}) = \frac{n+5}{3} + 2i - 1 \ \ for \ \ 1 \le i \le \frac{n-4}{6}$$
$$f(v_{2i+1}) = 2i + 1 \ \ for \ \ 0 \le i \le \frac{n-4}{6}$$

$$f(w_1) = \frac{2n+1}{3}$$

$$f(w_{2i}) = \frac{5n-2}{6} - i + 1 \;\; for \;\; 1 \le i \le \frac{n-4}{6}$$
$$f(w_{2i+1}) = n - i \;\; for \;\; 1 \le i \le \frac{n-4}{6}.$$

Note that

$$S = \{f(x) + f(y) : xy \in E(S_{m,3}), m \le 3 \;\; is \;\; odd\}$$
$$= \{\frac{4n+2}{3}, \frac{4n-1}{3}, ..., \frac{n+8}{3}, \frac{n+11}{3}\},$$

is one set of $n-1$ successive integers. Accordingly, by using the Lemma 1, f extend to a RSEM labeling of $S_{m,3}$ with valence. $k = p + q - s = n + n - 1 + \frac{n+8}{3} = \frac{2n-5}{3}$, when $m \le 3$ is odd.

Example 1. Figure 1 shows the RSEM labeling of the lobster T with $n = 13$.

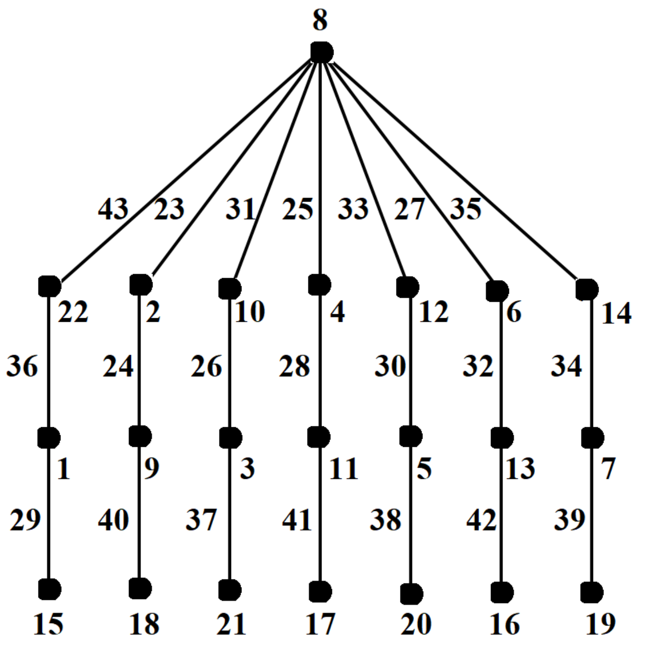

Figure 1. The RSEM labeling of the lobster T with $n = 13$.

Theorem 2. The lobster T characterized above is RSEM for total positive numbers $n > 3$.

Proof. We consider two cases.

Case 1: If n is even.

Let C_i denotes that the termination vertex of T at b_i, $1 \le i \le n$.

Characterize a vertex with labeling $f : V(T) \rightarrow \{1, 2, 3, ..., 3n\}$ such that

$$f(a_i) = \begin{cases} i, & \text{if } i \text{ is even}, 1 \le i \le n \\ 2n+i, & \text{if } i \text{ is odd}, 1 \le i \le n \end{cases}$$

$$f(b_i) = \begin{cases} i, & \text{if } i \text{ is even}, 1 \le i \le n \\ 2n+i, & \text{if } i \text{ is odd}, 1 \le i \le n \end{cases}$$

$$f(c_1) = 2n$$

$$f(c_{2i+3}) = \frac{3n}{2}(1+i) \ \ for \ \ 0 \le i \le \frac{n-4}{2}$$

$$f(c_{n \cong 2i}) = \frac{3n}{2} + i \ \ for \ \ 0 \le i \le \frac{n-2}{2}.$$

Therefore,

$$\begin{aligned} S &= \{f(x) + f(y) : xy \in E(T), n \ \ is \ \ even, n \ge 3\} \\ &= \{2n-2, 2n-1, 2n, 2n+1, ..., 5n-4\}. \end{aligned}$$

Accordingly, by using the Lemma 1, f extend to a RSEM labeling of T with valence $k = p+q-s = n+3$.

Example 2. Figure 2 shows the RSEM labeling of the lobster T with $n = 11$.

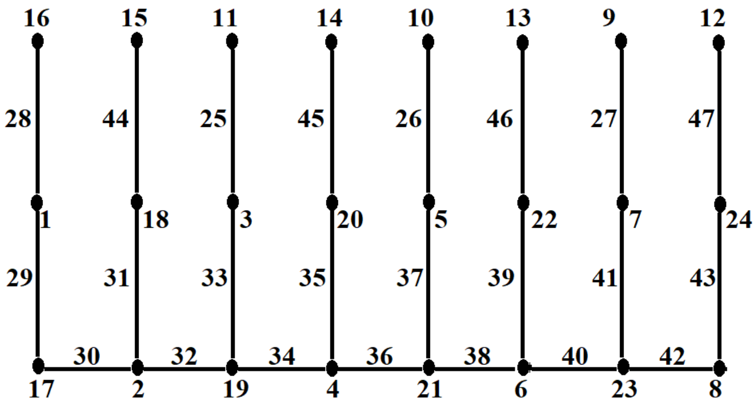

Figure 2. The RSEM labeling of the lobster T with $n = 11$.

Case 2: If n is odd.

Define the vertex labeling $f : V(T) \rightarrow \{1, 2, 3, ..., 3n\}$ here

$$f(a_i) = \begin{cases} i, & \text{if } i \text{ is even}, 1 \le i \le n \\ n+i, & \text{if } i \text{ is odd}, 1 \le i \le n \end{cases}$$

$$f(b_i) = \begin{cases} i, & \text{if } i \text{ is even}, 1 \le i \le n \\ n+i, & \text{if } i \text{ is odd}, 1 \le i \le n \end{cases}$$

$$f(c_1) = 3n$$
$$f(c_{2i+1}) = 3n - i \ for \ 1 \leq i \leq \frac{n-1}{2}$$
$$f(c_{n-2i+1}) = 2n + i \ for \ 1 \leq i \leq \frac{n-1}{2}.$$

Since,

$$\begin{aligned} S &= \{f(x) + f(y) : xy \in E(T), n \text{ is } odd, n \geq 3\} \\ &= \{n+2, n+3, 2n, ..., 4n, \}. \end{aligned}$$

Accordingly, by using the Lemma 1, f extend to a RSEM labeling of T with valence $k = p + q - s = 2n - 1$.

Example 3. Figure 3 shows the RSEM labeling of the lobster T with $n = 9$.

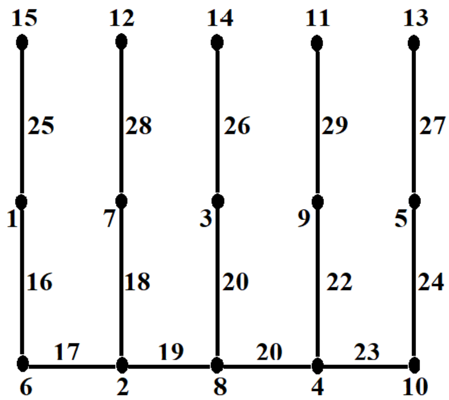

Figure 3. The RSEM labeling of the lobster T with $n = 9$.

Definition 2. Let $\{a_1k_{1,n_1}, a_2k_{1,n_2}, ..., a_pk_{1,n_p}\}$ be a family of stars where a_ik_{1,n_i}, a_i denotes the isomorphic disjoint copies of k_{1,n_i} for $1 \leq i \leq p$ and $a_i \geq 1$. Let k_{1,n_1} and v_{ijk} be the end vertices of H_{ij} be the j^{th} the isomorphic, $k = 1, 2, ..., n_i$ if one end vertex of each star which is adjacent to a vertex w adjoin. Thus the trees subsequently defined by $H_w^{a_1+a_2+--+^{-}a_p}$. These kinds of trees are prefers to as the banana tree.

Theorem 3. The banana tree $H_w^{a_1+a_2+--\mp a_p}$ corresponding to the family of stars $\{a_1k_{1,n_1}, a_2k_{1,n_2}, ..., a_pk_{1,n_p}\}$, $1 < n_1 < n_2 < ... < n_p, p \geq 2$ and $a_1 + a_2 + ... + a_i \geq n_i, i = 1, 2, ..., p$ is RSEM.

Proof. Conider the family of stars $\{a_1k_{1,n_1}, a_2k_{1,n_2}, ..., a_pk_{1,n_p}\}$. Let $k_{1,n_i}, i = 1, 2, ..., p$, is H_{ij} be the j^{th} the isomorphic copy. Assume H_{ij} is the end-vertices of $v_{ijk}, k = 1, 2, 3, 4, ..., n_i$ and u_{ij} be the H_{ij} is center. Let the new vertex be w which is adjacent to one end vertex $v_{ij\beta_{ij}}$ from every star H_{ij} of the family where $\beta_{ij} = a_0 + a_1 + ... + a_{i-1} + j$ and $a_0 = 0$. The new tree obtained is denoted by $H_w^{a_1+a_2+--\mp a_p}$. and has $a_1(n_1 + 1) + a_2(n_2 + 1) + ... + a_p(n_p + 1)$ vertices and $a_1n_1 + a_2n_2 + ... + a_pn_p + (a_1 + a_2 + ... + a_p)$ edges.

Let $p = a_1(n_1 + 1) + a_2(n_2 + 1) + ... + a_p(n_p + 1)$.

Define a vertex labeling $f : V\left(H_w^{a_1+a_2+--\mp a_p}\right) \to \{1, 2, ..., p_1\}$, such that

$$f(v_{1jk}) = (j-1)n_1 + k \ \ for \ \ 1 \le j \le a_1, 1 \le k \le n_1.$$
$$f(v_{ijk}) = f\left(v_{i-1a_{i-1}n_{i-1}}\right) + (j-1)n_i + k, \ for$$
$$2 \le i \le p, 1 \le j \le a_1, 1 \le k \le n_1.$$
$$f(w) = f(v_{pa_pn_p}) + 1.$$
$$f(u_{ij}) = f(w) + (a_0 + a_1 + ... + a_{i-1} + j),$$
$$2 \le i \le p, 1 \le j \le a_1.$$

Note that, $S = \{a_1n_1+a_2n_2+...+a_pn_p+2, a_2n_2+...+a_pn_p+3, ..., 2(a_1n_1+a_2n_2+...+a_pn_p)+(a_1+...+a_p)+1\}$. Accordingly, by using the Lemma 1, f extend to a RSEM labeling of $H_w^{a_1+a_2+--\mp a_p}$ with valence $k = p + q - s = a_0 + a_1 + ... + a_p$.

Definition 3. Consider the graph $G(t, m) = P_1 \times C_{2m+1}$ where x have t vertices ($t \ge 2$) is an odd cycle when the cartesian product of the path. Consider a new graph $G(t, m, n)$ by defining the new pendant edges n at each vertex of the furthest odd numbered cycle in $G(t, m)$.

Theorem 4. The graph $G(t, m, n)$ is RSEM, for $t \ge 2$ and $m \ge 2$.

Proof. Let C_{2m+1} be the fixed vertex of innermost of v_{11} and we will collect the different types of vertices $v_{12}, v_{13}, ..., v_{1(2m+1)}$ in clock-wise. For $2 \le i \le t$, let v_{i1} be the i^{th} copy of C_{2m+1} vertex was end to end to the vertex $v_{(i-1)(2m+1)}$ in the $(i-1)^{th}$ copy of C_{2m+1} and take the other is adjacent to the vertex v_{ijk} is the outermost C_{2m+1} for $1 \le k \le n$ and $1 \le j \le (2m + 1)$.

Define the vertex marking $f : V(G(t, m, n)) \to \{1, 2, ..., (2m + 1)(t + n)\}$ such that

$$f(v_{ij}) = \begin{cases} (i-1)(2m+1) + \frac{j+1}{2}, & if\, j \ is \ odd, \\ (i-1)(2m+1) + m + \frac{j+2}{2}, & if\, j \ is \ even, \end{cases}$$

for $1 \le i \le t$ and $1 \le j \le (2m + 1)$, $f(v_{ijk}) = (2m + 1)(t + k - 1) + (2m + 2 - j) \ for \ 1 \le j \le (2m + 1)$ *and* $1 \le k \le n$.

Note that

$$S = \{f(x) + f(y) : xy \in E(G(t, m, n)), t \ge 2, m \ge 2\}$$
$$= \{m + 2, m + 3, ..., (m + 1) + (2m + 1)(2t + n - 1)\}$$

is a set of all consecutive integers.

Accordingly, by using the Lemma 1, f extend to a RSEM labeling of $G(t, m, n)$ with valence $k = p + q - s = (2m + 3)n + (2t + m), \ for \ t \ge 2 \ and \ m \ge 2$.

Example 4. Figure 4 shows the RSEM labeling of the graph $G(3, 2, 2)$ with $n = 22$.

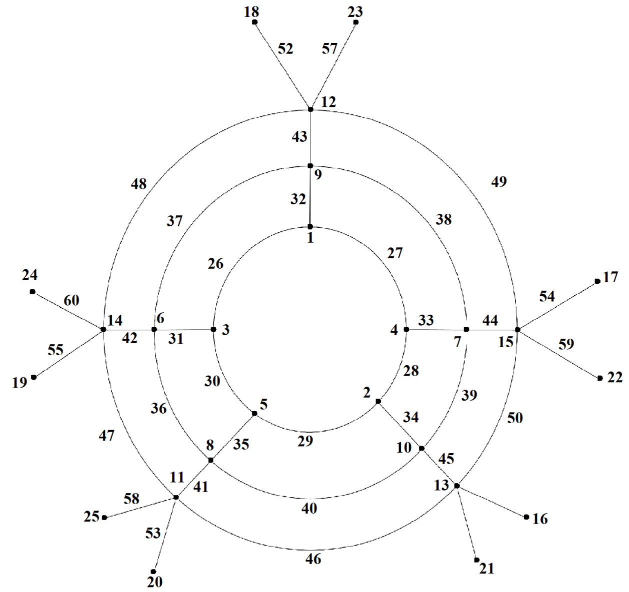

Figure 4. The RSEM labeling of the graph $G(3,2,2)$ with $n = 22$.

Theorem 5. The graph $C_n \odot P_2$ is RSEM for all odd $n \geq 3$.

Proof. Let $n = 2m + 1 \geq 3$, here n is an odd integer. Let $v_1, v_2, ..., v_n$ be the vertices of the cycle C_n. Now $c_n \odot P_2$ is the graph defined by the attaching P_2 to every vertex of C_n. Let the rim vertices v_i of C_n in $C_n \odot P_2$ is adjacent to the vertices $a_i, b_i, 1 \leq i \leq n$. The graph $C_n \odot P_2$ has $3n$ vertices and $4n$ edges. Consider a labeling of vertex $f : V(C_n \odot P_n \to \{1, 2, ..., 3n\})$ such that

$$f(a_i) = \begin{cases} \frac{i+1}{2}, & \textit{if i is odd,} \\ m + \frac{i+2}{2}, & \textit{if i is even,} \end{cases}$$

$$\begin{aligned} f(a_i) &= 2n + 1 - i \;\; \textit{for} \;\; 1 \leq i \leq n \\ f(b_{2i}) &= 2n + i \;\; \textit{for} \;\; 1 \leq i \leq m \\ f(b_1) &= 2n + m + 1 \\ f(b_{2i+1}) &= 2n + m + 1 + i \;\; \textit{for} \;\; 1 \leq i \leq m. \end{aligned}$$

Define

$$\begin{aligned} S &= \{I(x) + f(y) : xy \in E(c_n \odot p_2)\} \\ &= \{m + 2, m + 3, m + 4, ..., m + 4n + 1\} \end{aligned}$$

is a set of $4n$ successive integers.

Accordingly, by using the Lemma 1, f extend to a RSEM labeling of $C_n \odot P_2$ with valence $k = p + q - s = \frac{15n-1}{2}$, when $n \geq 3$ is odd number.

Example 5. Figure 5 shows the RSEM labeling of the graph $C_5 \odot P_2$ with $n = 12$.

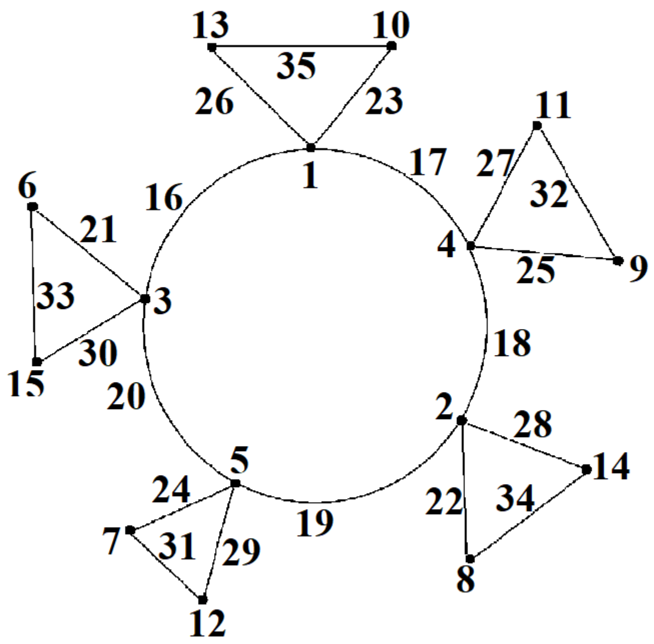

Figure 5. The RSEM labeling of the graph $C_5 \odot P_2$ with $n = 12$.

Theorem 6. The graph $C_n \odot P_3$ is RSEM for every odd $n \geq 3$.

Proof. Let C_n be an odd cycle with $n = 2m + 1 \geq 3$ vertices. The cycle C_n with the vertices $v_1, v_2, ..., v_n$. Let the path of three vertices is P_3. Now $4n$ vertices and $6n$ edges is a graph $C_n \odot P_3$ is obtained by attaching P_3.

Consider a vertex labeling $f : V(C_n \odot P_3) \rightarrow \{1, 2, ..., 4n\}$ such that

$$f(v_i) = \begin{cases} \frac{i+1}{2}, & \textit{if } i \textit{ is odd,} \\ m + \frac{i+2}{2}, & \textit{if } i \textit{ is even.} \end{cases}$$

If n is odd, Then m even for $f-$values and the rim vertices of $f-$values is $m + 1$ odd. Let us the label $3n$ vertices outside the rim of C_n in $C_n \odot P_3$ as follows. Let $u_1, u_2, ...u_m$, be the vertex degree two outside the rim, $f-$values are $2m, 2m - 2, ..., 4, 2$ is adjacent to the rim vertices respectively. Again let $u_{n+1}, u_{n+2}, ...u_{n+m}$ be the remaining vertices of degree two, whose $f-$values are $2m, 2m - 2, ..., 4, 2$ is adjacent to the rim vertices respectively. Let $u_{m+1}, u_{m+2}, ..., u_n$ be the vertex degree two outside the rim, whose $f-$values are $n, n - 2, ..., 3, 1$ is adjacent to the rim vertices respectively. Also, let $u_{n+m+1}, u_{n+m+2}, ..., u_{2n}$ be the continuing vertex degree two outside the rim, whose $f-$values are $n, n - 2, ..., 3, 1$ is adjacent to the rim vertices respectively. Let $u_{2n+1}.u_{2n+2}, ..., u_{2n+m+1}$ be the vertices of degree three outside the rim whose $f-$values are $n, n-2, ..., 3, 1$ is adjacent to the rim vertices respectively. Finally, let $u_{2n+m+2}, u_{2n+m+3}, ..., u_{3n}$ be the vertices of degree three whose $f-$values are $2m, 2m-2, ..., 4, 2$ is adjacent to the rim vertices respectively.

Consider $f(u_i) = n + i \; for \; 1 \le i \le 3n$.

Note that

$$S = \{f(x) + f(y) : xy \in E(C_n \odot P_3)\}$$
$$= \{m + 2, m + 3, .., m + 6n + 1\}$$

is a set of all consecutive integers.

Accordingly, by using the Lemma 1, f extend to a RSEM labeling of $C_n \odot P_3$ with valence $k = p + q - s = \frac{7n-1}{2}$.

Example 6. Figure 6 shows the RSEM labeling of the graph $C_7 \odot P_3$ with $n = 24$.

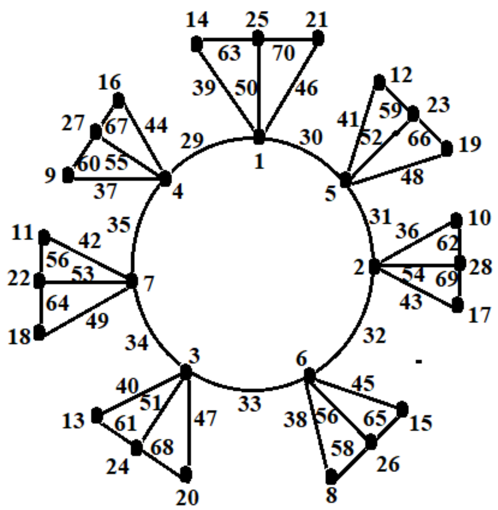

Figure 6. The RSEM labeling of the graph $C_7 \odot P_3$ with $n = 24$.

Definition 4. Let L_n denote the ladder graph $P_n \times P_2$ and $L_n \odot K_1$ be the graph containing the connecting an edge at every vertex of L_n.

Theorem 7. The graph $L_n \odot K_1$ is RSEM for odd n.

Proof. Let $V((L_n) = \{u_1, u_2, ..., u_n; v_1, v_2, ..., v_n\}$ and $E((L_n) = \{u_i u_{i+1}, v_i v_{i+1}, u_j v_j, 1 \le i \le (n-1), 1 \le j \le n\}$.

Let u_i^1 and v_i^1 be the vertices is adjacent to the u_i and v_i respectively in $L_n \odot K_1$. Then $V((L_n \odot K_1) = \{u_i, v_i, u_i^1, v_i^1 : 1 \; leqi \le n, \}$. and $V((L_n \odot K_1) = \{u_i u_{i+1}, v_i v_{i+1}, u_j v_j, u_j u_j^1, v_j v_j^1, 1 \le i \le (n-1), 1 \le j \le n\}$. The graph $L_n \odot K_1$ has $4n$ vertices and $5n - 2$ edges.

Define $f : V(L_n \odot K_1) \to \{1, 2, ..., 4n\}$ is the vertex labeling where

$$f(x)=\begin{cases}\frac{4n+i+1}{2}, & if\ x=u_i\ i\ is\ odd\ and\ 1\le i\le n\\ \frac{5n+i+1}{2}, & if\ x=u_i\ i\ is\ even\ and\ 1\le i\le n\\ \frac{3n+i}{2}, & if\ x=v_i\ i\ is\ odd\ and\ 1\le i\le n\\ \frac{2n+i}{2}, & if\ x=v_i\ i\ is\ even\ and\ 1\le i\le n\\ n, & if\ x=v_1^1\\ \frac{7n+1}{2}, & if\ x=v_2^1\\ i, & if\ x=v_{2i+1}^1,\ 1\le i\le(\frac{n-1}{2})\\ \frac{n+2i-1}{2}, & if\ x=v_{2i}^1,\ 2\le i\le(\frac{n-1}{2})\\ \frac{7n+2i+1}{2}, & if\ x=u_{2i-1}^1,\ 2\le i\le(\frac{n-1}{2})\\ 3n+1+i, & if\ x=u_{2i}^1,\ 1\le i\le(\frac{n-3}{2})\\ \frac{n+1}{2}, & if\ x=u_{n-1}^1\\ 3n+1, & if\ x=u_n^1.\end{cases}$$

Note that $S=\{f(x)+f(y):xy\in E(L_n\odot K_1)\}=\{\frac{3n+5}{2},\frac{3n+7}{2},...,\frac{13n-1}{2}\}$ is a set of alternative integers.

Accordingly, by using the Lemma 1, f extend to a RSEM labeling of $L_n\odot K_1$ with valence $k=p+q-s=\frac{5n-3}{2}$, for all odd n. Accordingly, Lemma 1, if G is a RSEM labeling of (p,q) graph then $q\le 2p-3$.

The next theorem gives a RSEM graph with $q=2p-3$.

Example 7. Figure 7 shows the RSEM labeling of the graph $L_5\odot K_1$ with $n=11$.

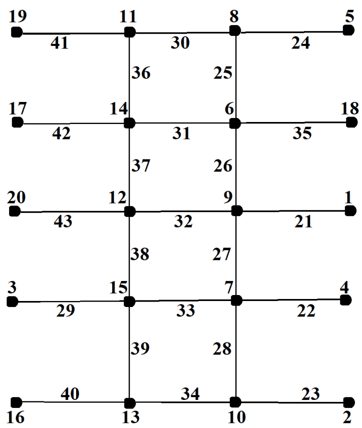

Figure 7. The RSEM labeling of the graph $L_5\odot K_1$ with $n=11$.

Theorem 8. The total graph $T(P_n)$ is RSEM for every integer n.

Proof. Let P_n be the path $u_1,u_2,...,u_n$ and e_j be the edge u_j,u_{j+1} for $1\le j\le(n-1)$. Then the vertex and edge set of $T(P_n)$ as denoted as $V(T(P_n))=\{u_j,e_j:1\le j\le n,1\le j\le(n-1).\}$

Note that $T(P_n)$ has $2n-1$ vertices and $4n-5$ edges, then $q=2p-3$.

Now define $f : V(T(P_n)) \rightarrow \{1, 2, ..., (2n-1)\}$ as the vertex labeling such that

$$f(u_i) = 2i - 1, for \ \ 1 \le i \le n$$
$$f(e_i) = 2i, for \ \ 1 \le i \le (n-1).$$

Since $S = \{f(x) + f(y) : xy \in E(T(P_n))\} = \{3, 4, ..., (4n-3)\}$ is a set of successive integers.

Accordingly, by using the Lemma 1, f extend to a RSEM labeling of $T(P_n)$ with valence $k = 2n-3$.

Example 8. Figure 8 shows the RSEM labeling of the graph $T(P_4)$ with $n = 5$.

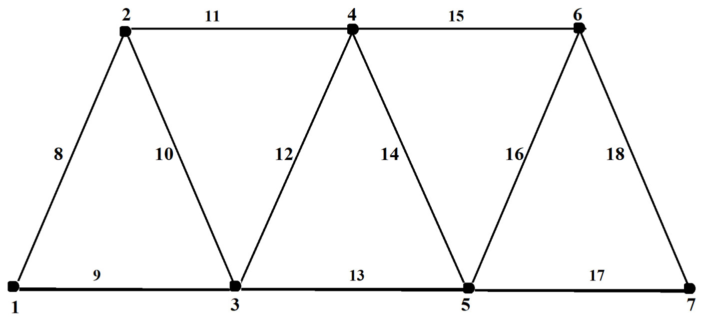

Figure 8. The RSEM labeling of the graph $T(P_4)$ with $n = 5$.

Theorem 9. The cycle graph C_n with a chord of distance 3 consisting two vertices is RSEM for each odd number n, where $n > 7$.

Proof: Let G be the graph and C_n is a chord with consisting two vertices of $C_n(n \ge 7)$ at a distance 3. Let $(G) = \{v_1, v_2, ..., v_n\}$, join the vertices v_1 and v_{n-2} as a chord for G so that $d(v_1, v_n) = 3$. Note that G has n vertices and $n + 1$ edges. Define $f : V(G) \rightarrow \{1, 2, ..., n\}$ the vertex labeling such that

$$f(v_i) = \begin{cases} \frac{i+1}{2}, & \text{i is odd} \\ \frac{n+i+1}{2}, & \text{i is even.} \end{cases}$$

Note that $S = \{f(x) + f(y) : xy \in E(G)\} = \{\frac{n+1}{2}, \frac{n+3}{2}, ..., \frac{3n+1}{2}\}$ is a set of successive integers.

Accordingly, by using the Lemma 1, f extend to a RSEM labeling of G with valence $k = p+q-s = \frac{n+1}{2}$.

Example 9. Figure 9 shows the RSEM labeling of the graph C_7 with $n = 4$.

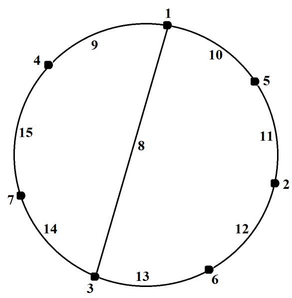

Figure 9. The RSEM labeling of the graph C_7 with $n = 4$.

Theorem 10. Let $G_1, G_2, ..., G_m$ be m disconnect and n cycles having vertex sets $v_i = \{v_1^i, v_2^i, ..., v_n^i\}$, $i = 1, 2, ..., m$ here n is odd and $n \geq 3$. Let G the graph attained by connecting v_n^1 to v_j^2, $1 \leq j \leq n$ and v_n^k to v_j^{k+1}, $1 \leq j \leq n, 2 \leq j \leq (m-1)$. Then G is a RSEM graph.

Proof. The graph G has containing mn vertices and $n(2m-1)$ edges.

Consider $f : V(G) \to \{1, 2, ..., mn\}$ a vertex labeling such that

$$f(v_i^1) = \begin{cases} \frac{i+1}{2}, & \textit{if i is odd}, 1 \leq i \leq n \\ \frac{n+1+i}{2}, & \textit{if i is even}, 1 \leq i \leq n \end{cases}$$

$f(v_i^r) = 5r - 4$, if $2 \leq r \leq m$

$$f(v_i^r) = \begin{cases} f(v_1^r) + \frac{i-1}{2}, & \textit{if i is odd}, 1 \leq i \leq n, 2 \leq r \leq m \\ f(v_n^r) + \frac{i}{2}, & \textit{if i is even}, 2 \leq i \leq n, 2 \leq r \leq m. \end{cases}$$

It is easy to see that $S = \{f(x) + f(y) : xy \in E(G)\} = \{\frac{n+3}{2}, \frac{n+5}{2}, ..., n + 10m - 7\}$ is a set of $n(2m-1)$ successive integers.

Accordingly, by using the Lemma 1, f extend to a RSEM labeling of G with valence $k = p + q - s = n(3m-2) - 10m + 7$.

Example 10. Figure 10 shows the RSEM labeling of the graph G_1 with $n = 4$.

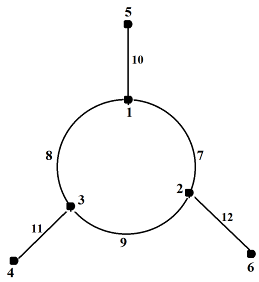

Figure 10. The RSEM labeling of the graph G_1 with $n = 4$.

3. Conclusions

Permitting to the outcome and argument we establish reverse edge magic valuation of the 3-star $S_{m,3}$ if m is odd, the lobster T characterized above is RSEM for all integers $n > 3$, the banana tree $H_w^{a_1+a_2+--+a_p}$. for $t \geq 2$ and $m \geq 2$ the graph $G(t, m, n)$ the graphs $C_n \odot P_2, C_n \odot P_3$ for all odd $m \geq 3$, the graph $L_n \odot K_1$ for odd n, the total graph $T(P_n)$ for any positive integer n and the graph C_n is a cycle with a chord conncection two vertices at the distance of 3 units for all odd n, $n > 7$.

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