Mathematics

AIMS Mathematics, 7(2): 2820–2839.
DOI: 10.3934/math.2022156
Received: 15 September 2021
Accepted: 04 November 2021
Published: 19 November 2021

http://www.aimspress.com/journal/Math

Research article

Jaya algorithm in estimation of P[X>Y] for two parameter Weibull distribution

Saurabh L. Raikar[1,*], Dr. Rajesh S. Prabhu Gaonkar[2]

[1] Mechanical Engineering Department, Goa College of Engineering (affiliated to Goa University), Farmagudi, Ponda, Goa 403401, India
[2] Indian Institute of Technology Goa (IIT Goa), Farmagudi, Ponda, Goa 403401, India

* **Correspondence:** Email: saurabhphd@gec.ac.in.

Abstract: Jaya algorithm is a highly effective recent metaheuristic technique. This article presents a simple, precise, and faster method to estimate stress strength reliability for a two-parameter, Weibull distribution with common scale parameters but different shape parameters. The three most widely used estimation methods, namely the maximum likelihood estimation, least squares, and weighted least squares have been used, and their comparative analysis in estimating reliability has been presented. The simulation studies are carried out with different parameters and sample sizes to validate the proposed methodology. The technique is also applied to real-life data to demonstrate its implementation. The results show that the proposed methodology's reliability estimates are close to the actual values and proceeds closer as the sample size increases for all estimation methods. Jaya algorithm with maximum likelihood estimation outperforms the other methods regarding the bias and mean squared error.

Keywords: reliability; Weibull distribution; parameter estimation; optimization; Jaya algorithm
Mathematics Subject Classification: 62F10, 37N40

1. Introduction

Reliability of the form P[X>Y] is used in cases of stress strength interference [1]. Stress and strength are important properties of a material. These properties do not have a fixed single value because of the uncertainties present in the environment like temperature, humidity, etc. So, they can be considered to follow a certain distribution. According to the interference theory, if stress and strength follow a certain distribution, then their interference area gives the probability of failure. The concept of stress strength interference in evaluating reliability has been used by many researchers in their studies. Liu et al. [2] evaluated the reliability of automotive seat adjuster by using the stress strength interference model. The finite element model of the seat-adjuster was constructed and the analysis was verified with the bench test. The theory has also been used in medical applications by Miller and Freivalds [3] to obtain the probability of failure of tendons in carpel tunnel syndrome.

Weibull distribution has been widely used by researchers in their study as it is capable of fitting large data types [4–6]. If x and y are the random variables following Weibull distribution $W(\sigma, p_1)$ and $W(\sigma, p_2)$ respectively then their pdf can be given as:

$$f(x;\sigma,p_1) = \frac{p_1}{\sigma^{p_1}}(x)^{p_1-1}\exp\left\{-\left(\frac{x}{\sigma}\right)^{p_1}\right\}, x > 0, \sigma > 0, p_1 > 0 \tag{1}$$

And

$$f(y;\sigma,p_2) = \frac{p_2}{\sigma^{p_2}}(y)^{p_2-1}\exp\left\{-\left(\frac{y}{\sigma}\right)^{p_2}\right\}, y > 0, \sigma > 0, p_2 > 0 \tag{2}$$

respectively. The corresponding cumulative distribution function (cdf) for strength and stress is given by

$$F(x;\sigma,p_1) = 1 - \exp\left\{-\left(\frac{x}{\sigma}\right)^{p_1}\right\} \tag{3}$$

And

$$F(y;\sigma,p_2) = 1 - \exp\left\{-\left(\frac{y}{\sigma}\right)^{p_2}\right\} \tag{4}$$

where p_1 & p_2 are shape parameters for strength and stress respectively and σ is the common scale parameter. It is difficult to evaluate the stress strength reliability model when both parameters of Weibull distribution are different. The purpose of the present study is to show the effectiveness of Jaya algorithm in the estimation of reliability for Weibull distribution. Hence, it has been assumed that the scale parameter for stress and strength distribution remains the same. Stress strength Weibull distribution with common scale parameter has been used in estimating the reliability for strength of carbon fibers [7].

2. Estimation methods

In this study, some of the most widely used estimation methods are implemented namely maximum likelihood estimation, least squares estimation, and weighted least squares estimation. Louzada et al. [8] used these methods in estimating the parameters of extended exponential geometric distribution for medical data. Datsiou and Overend [9] presented a comparison of various methods including MLE and LSE in the estimation of parameters for Weibull distribution applied to a data of strength of glass fibers by evaluating the fitness of the parameters using Anderson Darling goodness of fit test. The above estimation methods are simple and easy to evaluate.

2.1. Maximum likelihood estimation (MLE)

Maximum likelihood estimation is one of the common and effective methods in the estimation of parameters [10–12]. Chacko and Mohan [13] used the MLE method in estimating the parameters of two-parameter Kumaraswamy-exponential distribution for progressive type-II censored samples. Tzavelas [14] proposed estimation of parameters of three-parameter gamma distribution using MLE via reparameterization of function and predictor-corrector method. MLE method has also been used by Aggarwala and Balakrishnan [15] in the estimation of scale and location parameters of Laplace distribution. Ng et al. [16] discussed estimating the parameters of three-parameter Weibull distribution for type II progressively censored samples using MLE and weighted MLE. Abushal [17] applied MLE technique to estimate the unknown parameters and reliability characteristics for Akash distribution.
Let x_1, x_2, x_3 …x_n be a random sample of size n drawn from $W(\sigma, p_1)$ and y_1, y_2, y_3, …., y_n be the random sample of size m from $W(\sigma, p_2)$. Then the likelihood function can be given as:

$$L = \prod_{i=1}^{n} f(x_i) \prod_{j=1}^{m} f(y_j) \tag{5}$$

$$L = \prod_{i=1}^{n} \frac{p_1}{\sigma^{p_1}} (x_i)^{p_1-1} \exp\left\{-\left(\frac{x_i}{\sigma}\right)^{p_1}\right\} . \prod_{j=1}^{m} \frac{p_2}{\sigma^{p_2}} (y_j)^{p_2-1} \exp\left\{-\left(\frac{y_j}{\sigma}\right)^{p_2}\right\} \tag{6}$$

$$\begin{gathered}\ln L = n \ln p_1 + m \ln p_2 - np_1 \ln \sigma - mp_2 \ln \sigma \\ +(p_1 - 1)\sum_{i=1}^{n} \ln(x_i) + (p_2 - 1)\sum_{j=1}^{m} \ln(y_j) \\ -\frac{1}{\sigma^{p_1}}\sum_{i=1}^{n}(x_i)^{p_1} - \frac{1}{\sigma^{p_2}}\sum_{\mathrm{j}=1}^{\mathrm{m}}(y_j)^{p_2}\end{gathered} \tag{7}$$

The log-likelihood function (7) is to be maximized in order to obtain the best estimates of parameters.

2.2. Least squares estimation (LSE) and weighted least squares estimation (WLSE)

The least squares estimation technique was used by Swain et al. [18] in Johnson's translation system for modeling glucose levels in diabetes, in the analysis of statistical models, and structural reliability. Ashour and Eltehiwy [19] proposed the application of the technique in estimation of parameters of exponentiated power Lindley distribution. Weighted least squares which is a modification of least squares estimation method has been used in many applications [20]. The method was used by Wu et al. [21] in moving identification and found the suitability of the application in time-varying systems. The technique has also been used in estimation of parameters of multiplicative generalization of binomial distribution [22]. Benchiha et al. [23] used LSE and WLSE techniques in estimating the parameters of weighted generalized Quasi Lindley distribution. The method of least square and weighted least square have a property of unbiased estimation for large number of observations and have been used in estimating stress-strength reliability for various distributions including Weibull distribution [24–27].

Consider $x_1, x_2, x_3, \ldots, x_n$ is the random sample in ascending order of size n following Weibull distribution W(σ, p_1) and $y_1, y_2, y_3, \ldots, y_m$ is the random sample in ascending order of size m following Weibull distribution W(σ, p_2). Then the least squares criterion can be obtained as [28]

$$Q = \sum_{i=1}^{n}\left(\ln\left(\ln\left(\frac{1}{1-\widehat{F(x_i)}}\right)\right) - p_1\ln(x_i) + p_1\ln(\sigma)\right)^2 + \sum_{j=1}^{m}\left(\ln\left(\ln\left(\frac{1}{1-\widehat{F(y_j)}}\right)\right) - p_2\ln(y_j) + p_2\ln(\sigma)\right)^2 \tag{8}$$

The estimates of parameters can be obtained by minimizing function (8). The estimate values of F(x) and F(y) can be obtained by mean rank as

$$\widehat{F(x_i)} = \frac{i}{n+1} \text{ and } \widehat{F(y_j)} = \frac{j}{m+1}.$$

Similarly, the criterion to be minimized for weighted least squares can be given as

$$Q = \sum_{i=1}^{n} w_i\left(\ln\left(\ln\left(\frac{1}{1-\widehat{F(x_i)}}\right)\right) - p_1\ln(x_i) + p_1\ln(\sigma)\right)^2 + \sum_{j=1}^{m} w_j\left(\ln\left(\ln\left(\frac{1}{1-\widehat{F(y_j)}}\right)\right) - p_2\ln(y_j) + p_2\ln(\sigma)\right)^2 \tag{9}$$

where $w_i = \left(1-\widehat{F(x_i)}\right) ln\left(1-\widehat{F(x_i)}\right)^2$ and $w_j = \left(1-\widehat{F(y_j)}\right) ln\left(1-\widehat{F(y_j)}\right)^2$.

The estimates of parameters using weighted least squares method can be obtained by minimizing Eq (9).

Equations (7)–(9) discussed above are optimization problems. Solutions to these equations using numerical computation do not yield precise results. It also has problems of slow convergence and non-convergence to real roots. So, these methods have to be assisted with a suitable optimization technique in order to improve their effectiveness. In this case, Jaya algorithm is used to optimize these functions.

3. Jaya algorithm

The application of metaheuristic techniques in optimization problems has seen increasing importance in modern times [29,30]. Jaya algorithm is a recent metaheuristic technique capable of solving a vast number of optimization problems with high effectiveness [31]. The researchers have used the technique in a number of applications and found satisfactory results. Meshram et al. [32] carried out electrical discharge machining with around eight control variables and two responses.

Taguchi's L12 orthogonal array was used in designing the experiment. The regression equations were taken as objective functions and Jaya algorithm was used in optimization observing improvement in response variables. Caldeira and Gnanavelbabu [33] presented the implementation of Jaya algorithm for effectively solving the flexible job-shop scheduling problem. Gupta et al. [34] discussed the superiority of Jaya algorithm over other similar metaheuristic techniques in optimizing standard functions for the application of workflow scheduling in cloud computing. Jin et al. [35] identified the parameters of wind turbine power models using Jaya algorithm and monitoring with multivariate control charts. Du et al. [36] proposed a hybrid objective function for identifying the sites and extent of damage in a damage identification problem wherein Jaya algorithm was used to optimize the function in obtaining the parameters with good accuracy. Similarly, the algorithm has been used by many other researchers in such optimization problems [37–40]. In Jaya algorithm, the initial population with sets of parameters is randomly generated using upper and lower bounds. Then, each candidate in the population is updated based on the equation:

$$Z'_{j,k,i}=Z_{j,k,i}+r_{1,j,i}(Z_{j,best,i}-\left|Z_{j,k,i}\right|)-r_{2,j,i}(Z_{j,worst,i}-\left|Z_{j,k,i}\right|) \tag{10}$$

where $Z'_{j,k,i}$ is the updated value of variable k for candidate solution j, $Z_{j,k,i}$ is the previous value of variable k for candidate solution j and i is the iteration number. $r_{1,j,i}$ and $r_{2,j,i}$ are the random variables between 0 and 1. The significance of the algorithm is that it continuously takes the candidate solution towards the best solution by the term $(Z_{j,best,i}-\left|Z_{j,k,i}\right|)$ and away from the worst solution by the term $(Z_{j,worst,i}-\left|Z_{j,k,i}\right|)$. Depending on the best and worst function value, the candidate solutions are updated. Figure 1 shows the flowchart of Jaya algorithm:

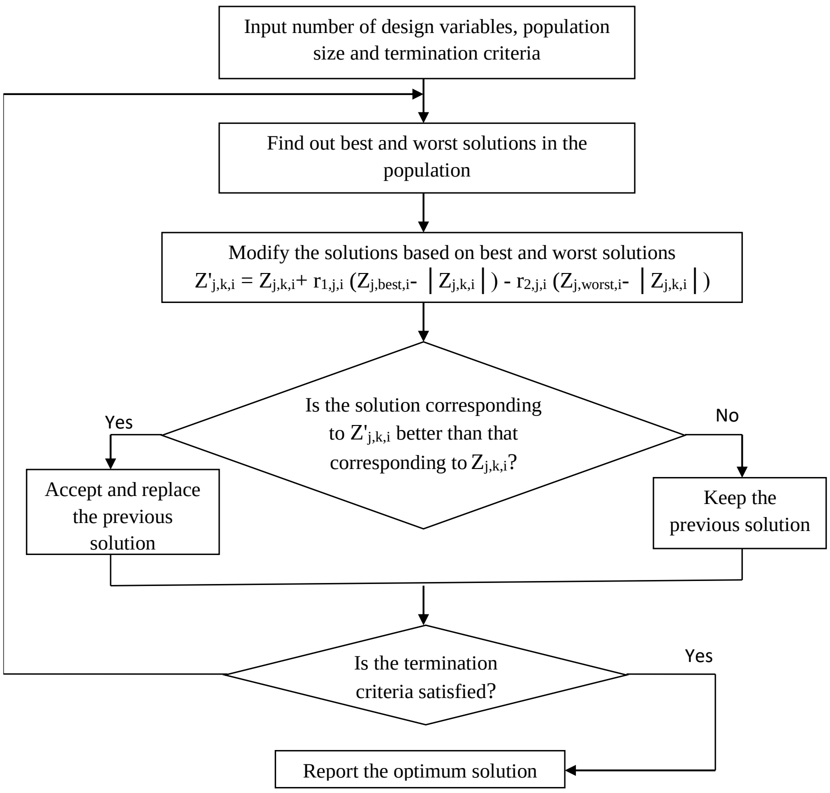

Figure 1. Flowchart of Jaya algorithm.

4. Reliability estimation

If X and Y denote the strength and stress distribution with common scale parameter but different shape parameter then according to interference theory the reliability can be given as

$$R = P(X > Y) = \int_0^\infty \left(f(x;\sigma,p_1) \int_0^x f(y;\sigma,p_2)dy \right) dx,$$

$$R = P(X > Y) = \int_0^\infty \begin{pmatrix} \frac{p_1}{\sigma^{p_1}} (x)^{p_1-1} \exp\left\{-\left(\frac{x}{\sigma}\right)^{p_1}\right\}. \\ \int_0^x \frac{p_2}{\sigma^{p_2}} (y)^{p_2-1} \exp\left\{-\left(\frac{y}{\sigma}\right)^{p_2}\right\} dy \end{pmatrix} dx,$$

$$R = P(X > Y) = \int_0^\infty \begin{pmatrix} \frac{p_1}{\sigma^{p_1}} (x)^{p_1-1} \exp\left\{-\left(\frac{x}{\sigma}\right)^{p_1}\right\}. \\ \left\{1 - \exp\left\{-\left(\frac{x}{\sigma}\right)^{p_2}\right\}\right\} \end{pmatrix} dx,$$

$$R = P(X > Y) = 1 - \int_0^\infty \frac{p_1}{\sigma^{p_1}} (x)^{p_1-1} \exp\left\{-\left(\left(\frac{x}{\sigma}\right)^{p_1} + \left(\frac{x}{\sigma}\right)^{p_2}\right)\right\} dx.$$

If $\widehat{p_1}$, $\widehat{p_2}$ and $\widehat{\sigma}$ are the estimated parameters of Weibull distribution then the estimated reliability $\hat{R}$ can be given as

$$\hat{R} = 1 - \int_0^\infty \frac{\widehat{p_1}}{\widehat{\sigma}^{\widehat{p_1}}} (x)^{\widehat{p_1}-1} \exp\left\{-\left(\left(\frac{x}{\widehat{\sigma}}\right)^{\widehat{p_1}} + \left(\frac{x}{\widehat{\sigma}}\right)^{\widehat{p_2}}\right)\right\} dx.$$

The detailed steps in using Jaya algorithm in estimation of reliability are as follows:

(1) Specify the population size and number of design variables.
(2) Set the boundary conditions.
(3) Generate a random set of parameters with the number of sets equal to population size and the number of parameters equal to the number of design variables.
(4) Trim the generated set as per boundary conditions.
(5) Calculate function value for each set based on the objective function (7)–(9) for MLE, LSE and WLSE respectively.
(6) Identify the best and the worst function value.
(7) Update the parameter set based on Eq (10) within the boundary conditions. Calculate the updated function value and identify the best & worst function values for the parameter sets.
(8) If the updated function value of a set is better than the earlier function value of the respective set, replace the earlier set of the design parameters with the updated parameter set. This completes the first iteration.
(9) The iteration number can be considered as the termination criteria.

5. Simulation studies

Random numbers were generated with shape parameters for strength, shape parameter for stress, and common scale parameter (p_1, p_2, σ) taken as (1.5, 2, 1), (2, 2, 1), (2.5, 2, 1) and (2.5, 2, 2). The sample sizes taken were (25, 25), (50, 50), (100, 100) and (500, 500). Total 500 experiments were conducted to check the repeatability of the estimation method. The parameters were estimated using the proposed methodology for MLE, LSE, and WLSE methods. The reliability was evaluated along with bias and mean squared error. The results of simulation studies are presented in Tables 1–3. The estimation using proposed methodology gives very good results with reliability estimates close to the actual reliability. It can be noted that the accuracy of estimation increases with increase in sample size. The trend is strongly followed by MLE method compared to the other two. But as the sample size increases, the time taken for compilation also increases. Another fact that can be observed is that if the shape parameter for strength increases in comparison to that of stress, the reliability increases. Also, the reliability decreases with an increase in common scale parameter. Figures 2–4 shows the box plots in estimation of reliability for 500 experiments across the sample sizes for considered different sets of parameters. It can be seen that the accuracy of the estimation increases as the sample size increases. For example, all the estimates of 500 experiments are very close to the actual reliability values in case of sample size (500, 500) whereas the spread increases with a decrease in sample size. Though the spread is observed to be more in case of a smaller sample size, the mean of reliability estimate is close to the actual reliability values. Also, the values for bias and MSE are lesser as compared to the other estimation methods in the literature. A notable observation can be made of many outliers wide away from the actual reliability in case of estimation with LSE and WLSE. Figures 5–7 shows the convergence behavior of Jaya algorithm for different sample sizes using MLE, LSE, and WLSE respectively. It can be noted that the algorithm converges to real roots after around 40 to 60 iterations for MLE, 80 to 100 iterations for LSE, and around 80–120 for WLSE. Figures 8–11 shows comparative graphs of bias and mean squared error (MSE) for the three estimation methods. It can be seen that the algorithm with MLE gives lesser bias and MSE in almost all the cases. This shows that Jaya algorithm with MLE is superior as compared to the other two estimation methods.

Table 1. Simulation results of 500 experiments for MLE using Jaya algorithm.

(p_1, p_2, σ)	R	(n,m)	$\hat{R}$	Bias	MSE	T(s)
(1.5, 2, 1)	0.48063	(25,25)	0.481730	0.001100	0.000214321	12.79
		(50,50)	0.481418	0.000788	0.000103129	21.26
		(100,100)	0.480968	0.000338	0.000042815	39.67
		(500,500)	0.480770	0.000140	0.000009908	191.6
(2, 2, 1)	0.5	(25,25)	0.498637	- 0.00136	0.000204486	12.38
		(50,50)	0.500416	0.000416	0.000114916	21.64
		(100,100)	0.499798	- 0.00020	0.000050960	40.26
		(500,500)	0.500074	0.000074	0.000008967	198.1
(2.5, 2, 1)	0.5151	(25,25)	0.516159	0.001059	0.000206911	12.48
		(50,50)	0.515342	0.000242	0.000109094	23.14
		(100,100)	0.515160	0.000060	0.000048278	39.50
		(500,500)	0.515126	0.000026	0.000010192	188.5
(2.5, 2, 2)	0.515049	(25,25)	0.515432	0.000383	0.000217130	12.64
		(50,50)	0.514981	- 0.00007	0.000103620	21.85
		(100,100)	0.515108	0.000059	0.000051235	38.13
		(500,500)	0.515019	- 0.00003	0.000009259	186.9

Table 2. Simulation results of 500 experiments for LSE using Jaya algorithm.

(p_1, p_2, σ)	R	(n,m)	$\hat{R}$	Bias	MSE	T(s)
(1.5, 2, 1)	0.48063	(25,25)	0.483988	0.003358	0.000374892	8.208
		(50,50)	0.482671	0.002041	0.000296409	13.72
		(100,100)	0.480902	0.000272	0.000120576	24.11
		(500,500)	0.481391	0.000761	0.000033845	116.5
(2, 2, 1)	0.5	(25,25)	0.501092	0.001092	0.000709956	8.773
		(50,50)	0.500238	0.000238	0.000227603	13.44
		(100,100)	0.500839	0.000839	0.000243236	23.76
		(500,500)	0.500048	0.000048	0.000022361	107.9
(2.5, 2, 1)	0.5151	(25,25)	0.515538	0.000438	0.000568429	8.163
		(50,50)	0.517007	0.001907	0.000734081	13.48
		(100,100)	0.515227	0.000127	0.000108498	23.93
		(500,500)	0.514984	- 0.00012	0.000023476	118.8
(2.5, 2, 2)	0.515049	(25,25)	0.517451	0.002402	0.001412757	8.760
		(50,50)	0.517467	0.002418	0.000708432	13.92
		(100,100)	0.515638	0.000589	0.000117623	23.84
		(500,500)	0.515736	0.000687	0.000178473	119.2

Table 3. Simulation results of 500 experiments for WLSE using Jaya algorithm.

(p_1, p_2, σ)	R	(n,m)	$\hat{R}$	Bias	MSE	T(s)
(1.5, 2, 1)	0.48063	(25,25)	0.484014	0.003384	0.001492711	10.32
		(50,50)	0.481896	0.001266	0.000637294	18.11
		(100,100)	0.481325	0.000695	0.000666559	30.80
		(500,500)	0.482667	0.002037	0.000730472	143.5
(2, 2, 1)	0.5	(25,25)	0.502422	0.002422	0.001433974	10.61
		(50,50)	0.503042	0.003042	0.001274580	18.11
		(100,100)	0.502669	0.002669	0.001481735	31.15
		(500,500)	0.503678	0.003678	0.001585831	143.5
(2.5, 2, 1)	0.5151	(25,25)	0.516737	0.001637	0.001899504	11.19
		(50,50)	0.518774	0.003674	0.001496135	17.10
		(100,100)	0.517434	0.002334	0.001370110	31.35
		(500,500)	0.517283	0.002183	0.000599416	143.0
(2.5, 2, 2)	0.515049	(25,25)	0.517838	0.002789	0.002217398	10.46
		(50,50)	0.516697	0.001648	0.000536541	17.38
		(100,100)	0.519082	0.004033	0.001620499	32.25
		(500,500)	0.516500	0.001451	0.000401925	143.3

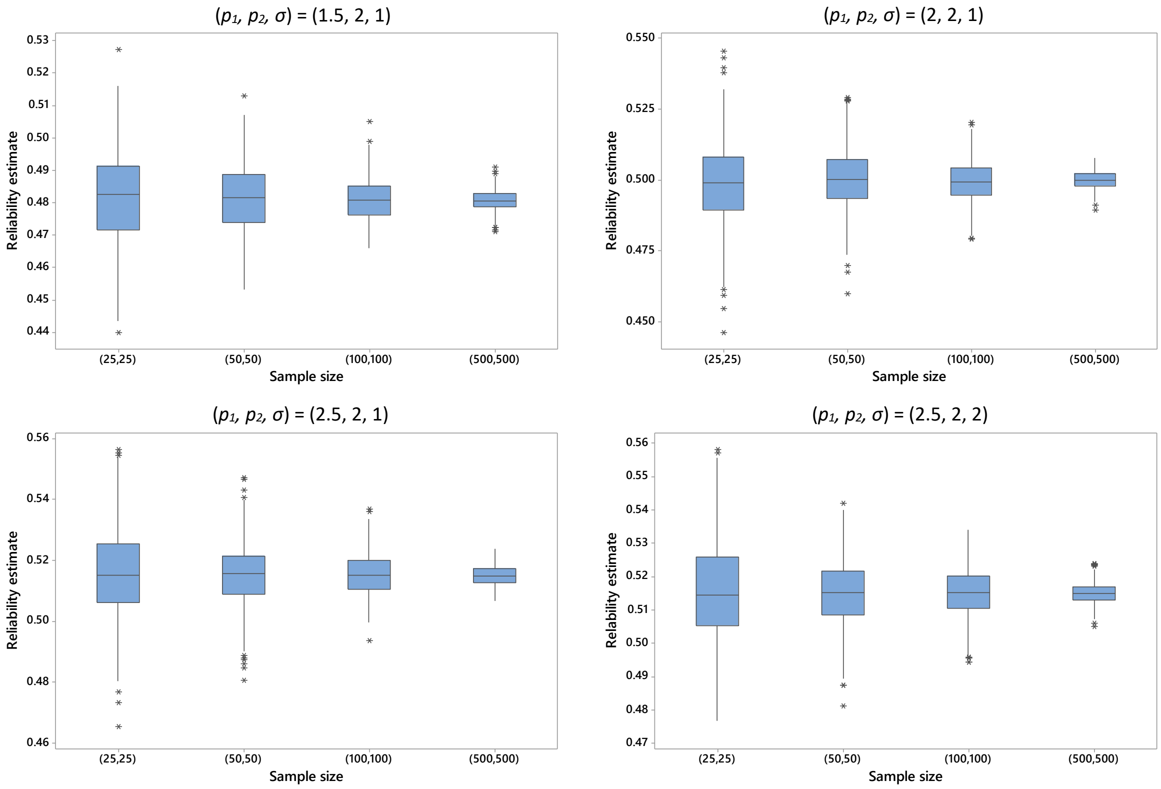

Figure 2. Box plots for reliability estimates across different sample sizes with MLE.

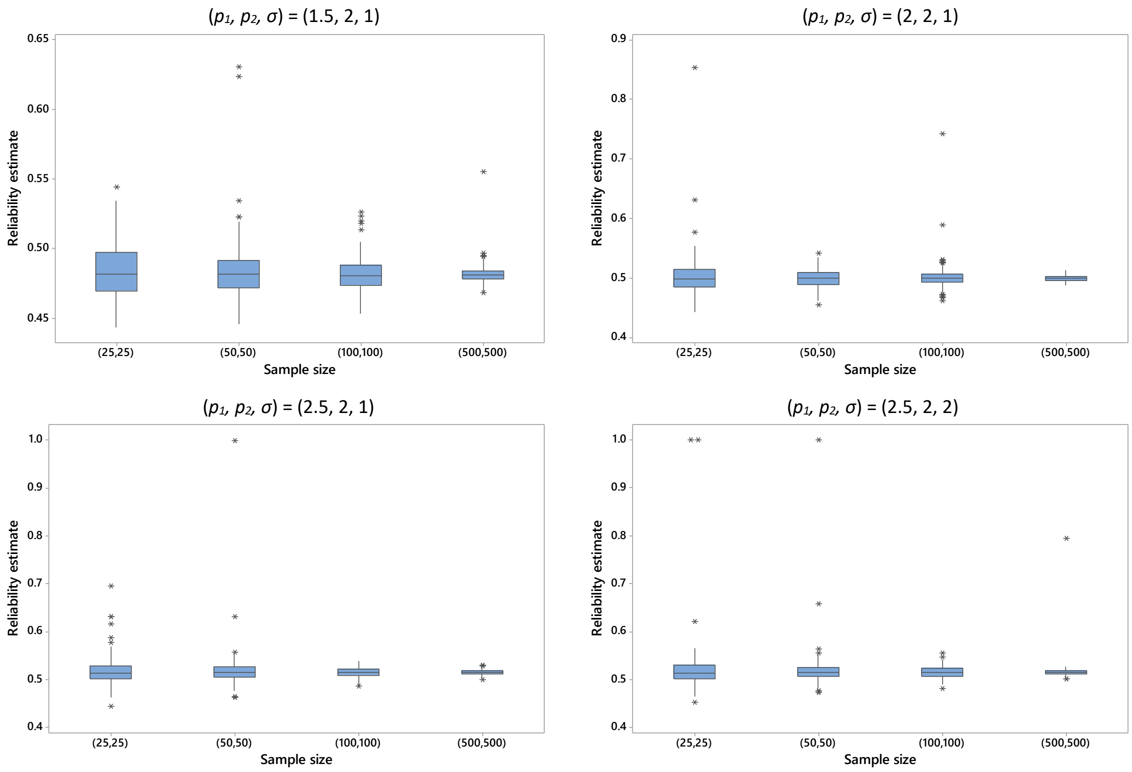

Figure 3. Box plots for reliability estimates across different sample sizes with LSE.

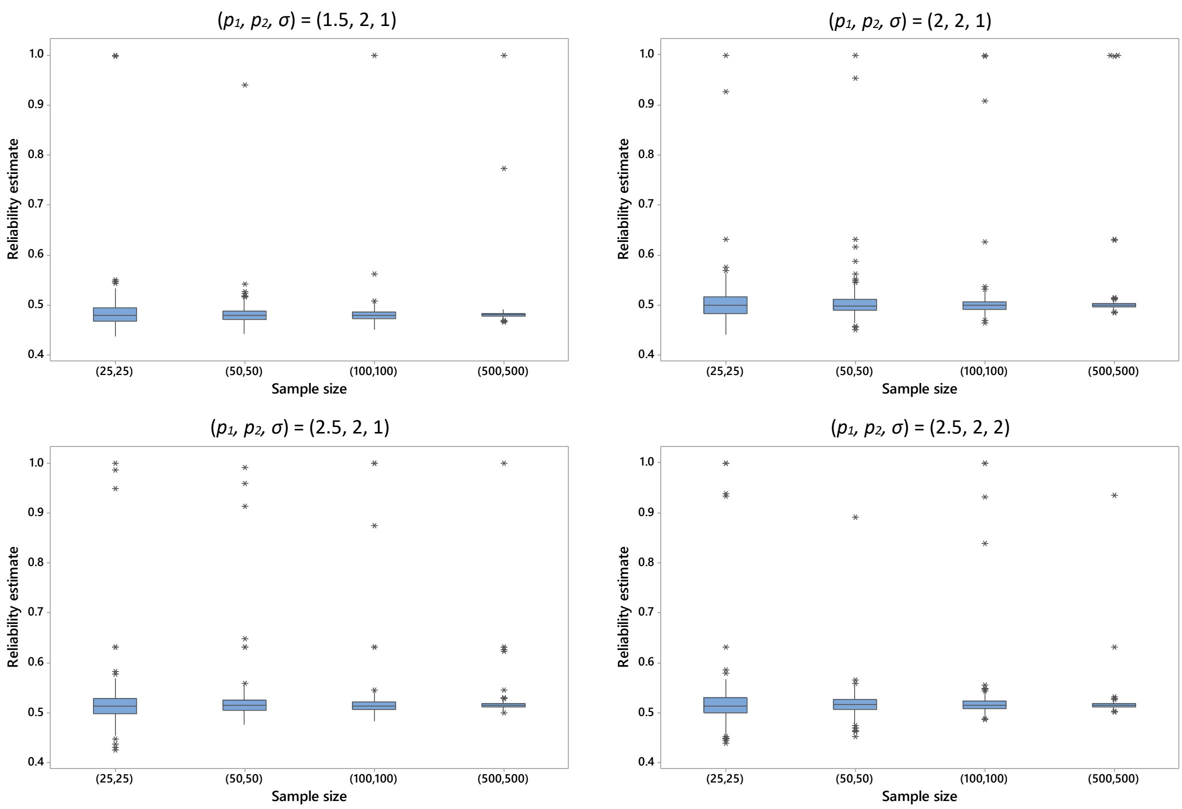

Figure 4. Box plots for reliability estimates across different sample sizes with WLSE.

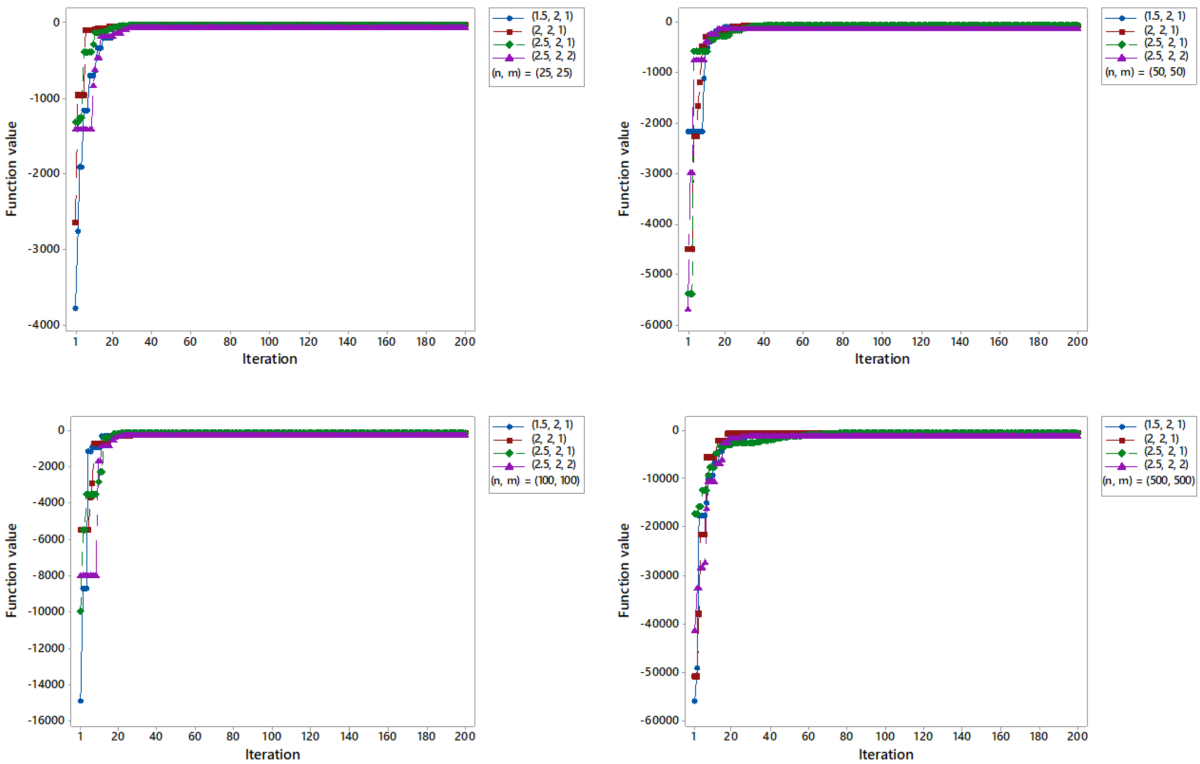

Figure 5. Convergence behavior of Jaya algorithm for different sample sizes with MLE.

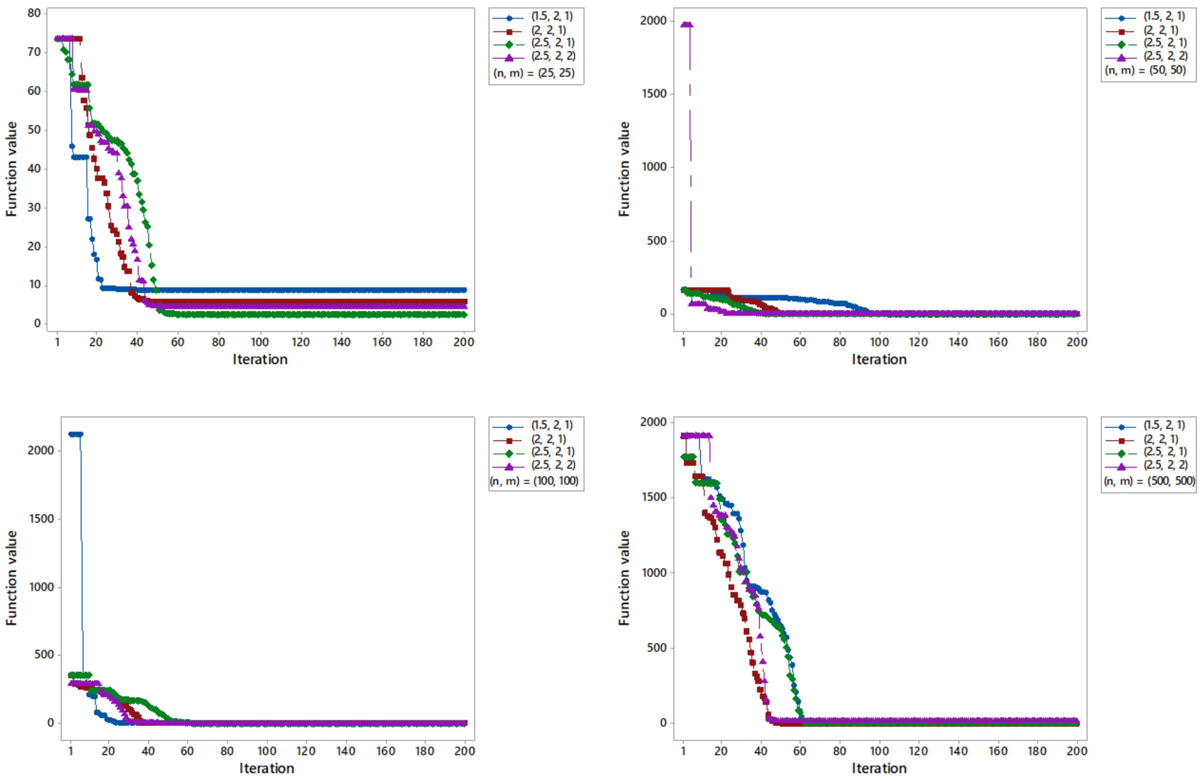

Figure 6. Convergence behavior of Jaya algorithm for different sample sizes with LSE.

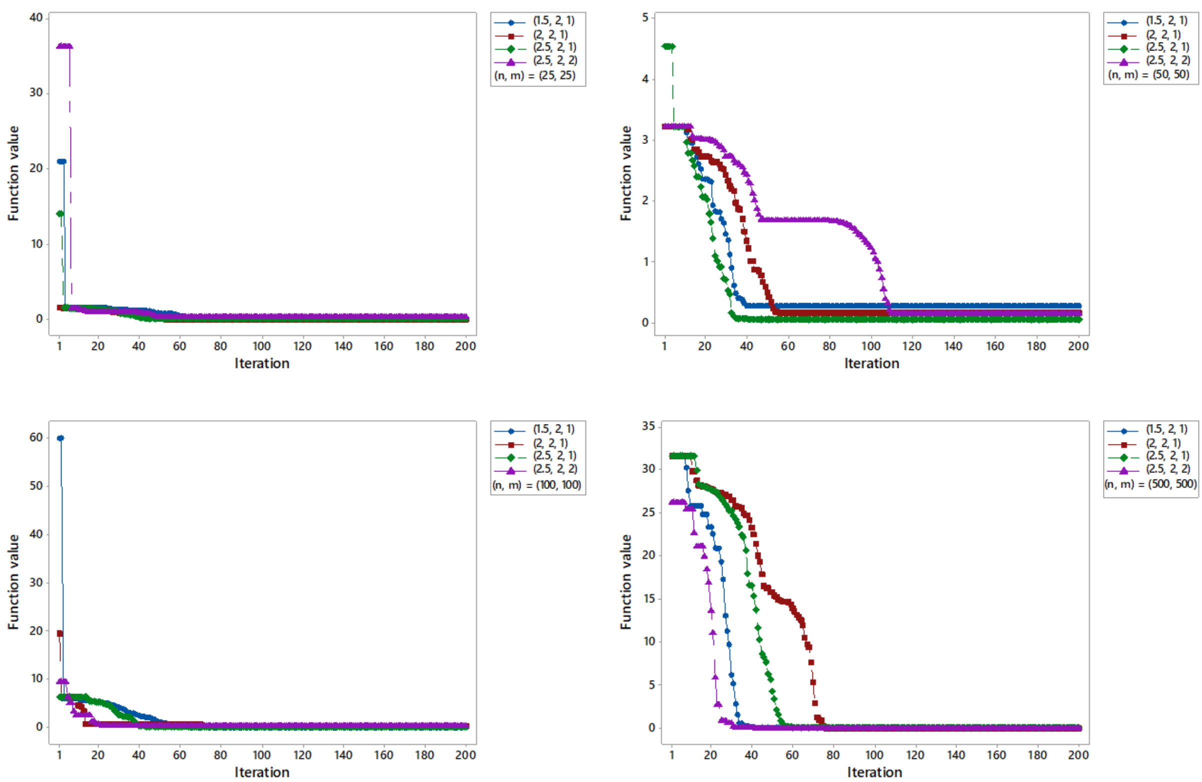

Figure 7. Convergence behavior of Jaya algorithm for different sample sizes with WLSE.

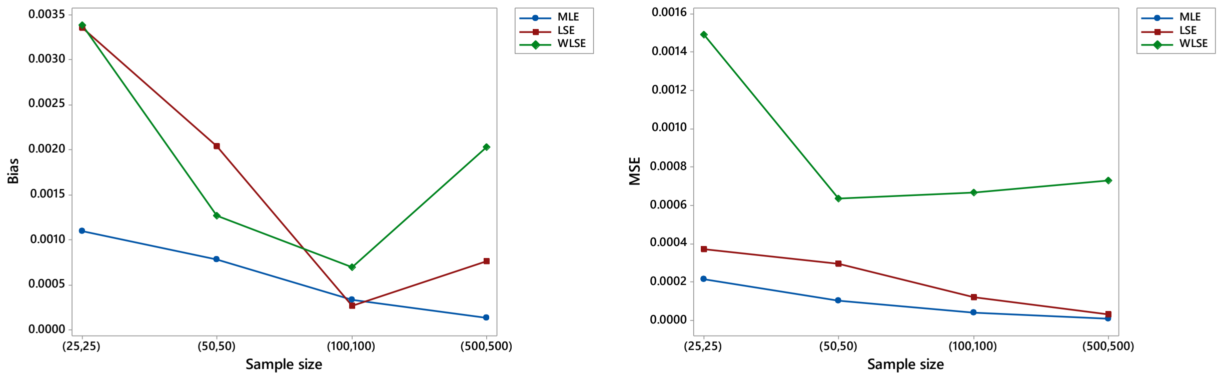

Figure 8. Comparison of bias and MSE for estimation methods using Jaya algorithm for (p_1, p_2, σ)=(1.5, 2, 1).

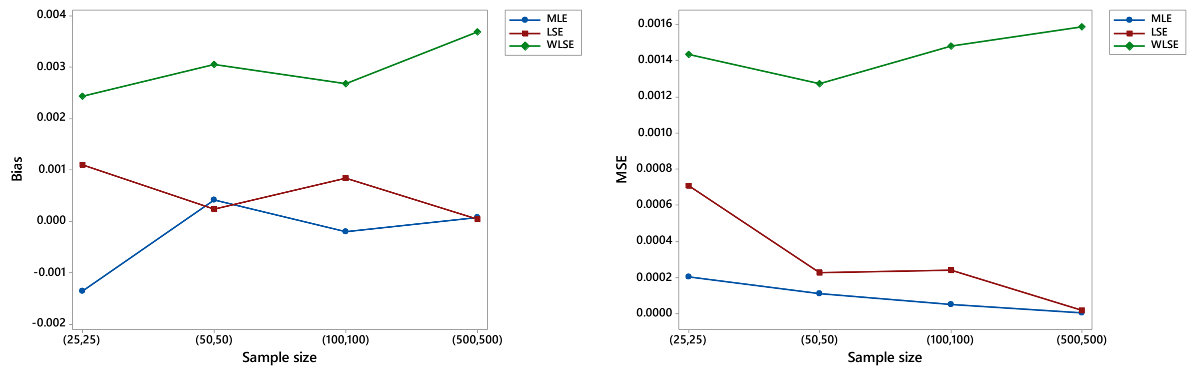

Figure 9. Comparison of bias and MSE for estimation methods using Jaya algorithm for (p_1, p_2, σ)=(2, 2, 1).

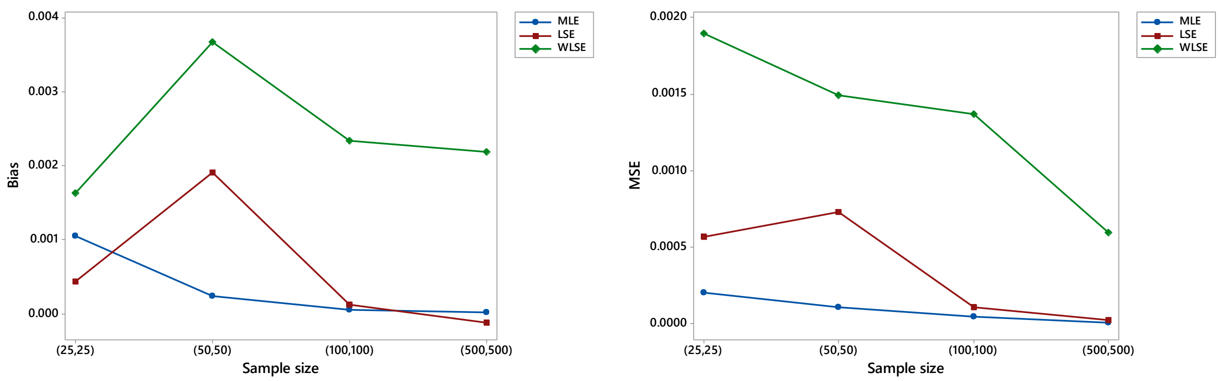

Figure 10. Comparison of bias and MSE for estimation methods using Jaya algorithm for (p_1, p_2, σ) =(2.5, 2, 1).

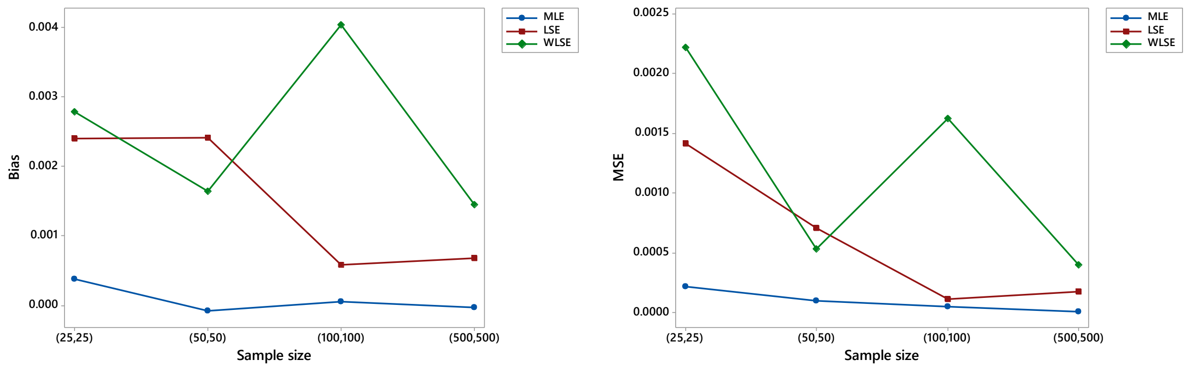

Figure 11. Comparison of bias and MSE for estimation methods using Jaya algorithm for (p_1, p_2, σ) =(2.5, 2, 2).

6. Application to real life data

The methodology has been applied to real-life data of strength of carbon fibers of gauge length 10 mm and 20 mm first studied by Badar and Priest [41] and then transformed by Valiollahi et al. [7] to fit for a common scale parameter. The transformed data sets are shown in Tables 4 and 5. Using the proposed methodology, the estimated parameters $(\widehat{p_1}, \widehat{p_2}, \widehat{\sigma})$ are obtained as (5.5061, 5.0514, 0.9999) using MLE, (5.5647, 5.7374, 0.9982) using LSE and (5.6584, 4.9353, 0.9880) using WLSE method. The Kolmogorov-Smirnov test was used to check the fit of the estimated Weibull model to the data sets. The K-S statistic, p-value, and estimated reliability using the three methods are given in Table 6. Figures 12–17 shows the fitted pdf and probability plot with estimated parameters using various methods for data sets I & II. The proposed methodology gives rapid results in a very short time compared to other common estimation methods using metaheuristic techniques [42–45]. The Akaike information criterion was used to find the best fit model for the given data among MLE, LSE and WLSE. The results are displayed in Table 7. It can be seen that the minimum value for AIC is obtained with MLE and the maximum value is obtained for LSE. Thus, it can be inferred that the proposed methodology using MLE gives the best fit model and LSE gives the worst fit model for the given data sets.

Table 4. Data of gauge length 20 mm (Data set I).

0.495	0.496	0.558	0.585	0.641	0.68	0.702	0.704	0.733	0.739
0.742	0.753	0.757	0.762	0.765	0.775	0.778	0.791	0.807	0.822
0.839	0.845	0.85	0.856	0.857	0.858	0.868	0.868	0.89	0.899
0.899	0.915	0.918	0.919	0.935	0.939	0.947	0.948	0.956	0.963
0.968	0.97	0.976	0.992	0.993	0.997	0.999	1.013	1.017	1.028
1.045	1.046	1.056	1.06	1.063	1.064	1.074	1.086	1.114	1.136
1.157	1.163	1.166	1.168	1.18	1.22	1.295	1.352	1.352	

Table 5. Data of gauge length 10 mm (Data set II).

0.573	0.643	0.665	0.672	0.681	0.709	0.712	0.723	0.723	0.738
0.74	0.746	0.76	0.761	0.762	0.764	0.777	0.789	0.789	0.79
0.792	0.802	0.807	0.826	0.827	0.862	0.88	0.883	0.886	0.886
0.898	0.904	0.914	0.943	0.947	0.949	0.971	0.972	0.976	0.978
0.985	0.987	0.994	1.005	1.009	1.019	1.028	1.036	1.054	1.056
1.067	1.072	1.074	1.094	1.162	1.168	1.172	1.198	1.214	1.215
1.274	1.326	1.514							

Table 6. Comparison of MLE, LSE and WLSE in data fit and estimation of reliability.

Estimation method	$\widehat{p_1}$	$\widehat{p_2}$	$\hat{\sigma}$	K-S	p-value	Reliability	Compilation time(s)
MLE	5.5061	5.0514	0.9999	0.0564 (DS I) 0.0881 (DS II)	0.9773 (DS I) 0.6929 (DS II)	0.505824	0.079715
LSE	5.5647	5.7374	0.9982	0.0513 (DS I) 0.1096 (DS II)	0.9920 (DS I) 0.4145 (DS II)	0.497935	0.888459
WLSE	5.6584	4.9353	0.9880	0.0523 (DS I) 0.1002 (DS II)	0.9900 (DS I) 0.5295 (DS II)	0.509234	1.359188

Table 7. Model comparison based on AIC.

	No. of estimated parameters	Log Likelihood	AIC	Delta AIC
MLE	3	-40861.12	81728.24	0
LSE	3	-81588.05	163182.1	81453.86
WLSE	3	-42321.46	84648.92	2920.68

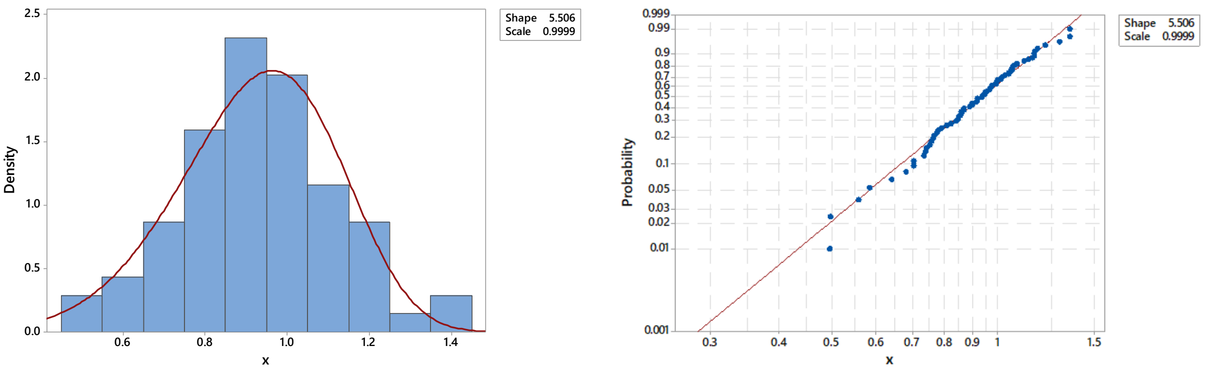

Figure 12. The fitted pdf and probability plot for Data set I with MLE.

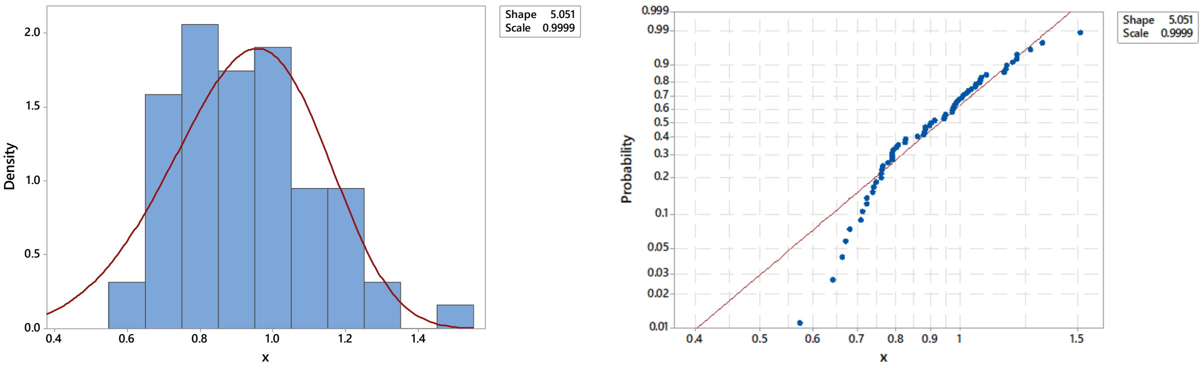

Figure 13. The fitted pdf and probability plot for Data set II with MLE.

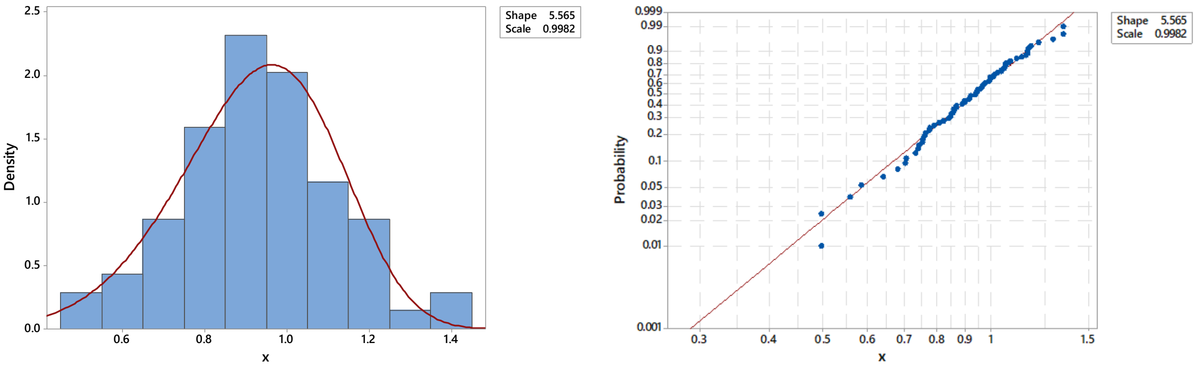

Figure 14. The fitted pdf and probability plot for Data set I with LSE.

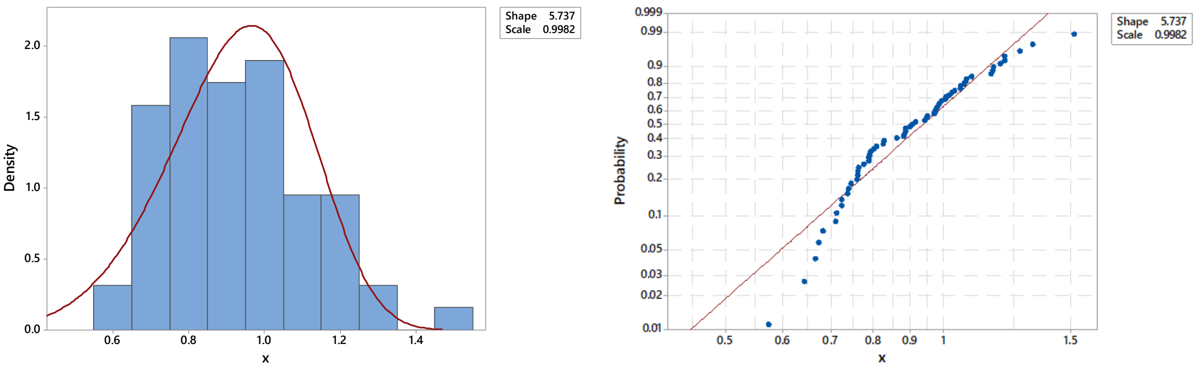

Figure 15. The fitted pdf and probability plot for Data set II with LSE.

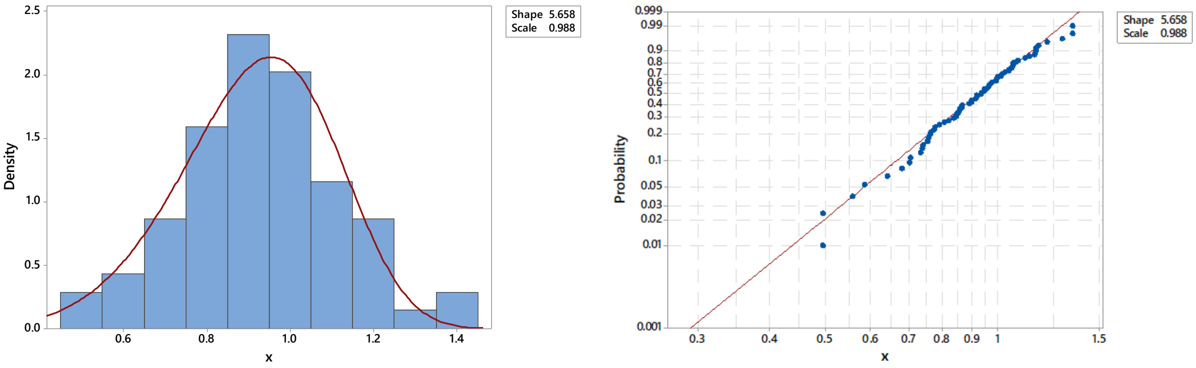

Figure 16. The fitted pdf and probability plot for Data set I with WLSE.

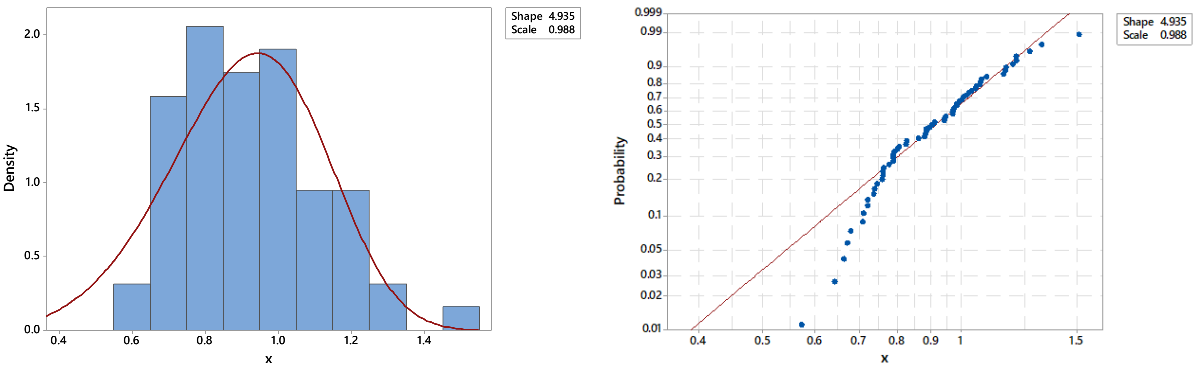

Figure 17. The fitted pdf and probability plot for Data set II with WLSE.

7. Conclusions

This study deals with estimation P(X>Y) for X and Y following Weibull distribution with different shape parameters and same scale parameters. The estimation methods used are maximum likelihood estimation, least squares estimation, and weighted least squares estimation. Jaya algorithm has been used in optimizing the estimation functions. The reliability estimate equation has been presented and simulation studies are carried out in order to validate the model and compare the performance of the algorithm with the above estimation methods. Box plots showed the increasing accuracy of estimation with an increase in sample size. Jaya algorithm shows a consistent convergence towards the real roots. It was observed that the algorithm with maximum likelihood estimation outperforms the other two techniques studied with respect to the bias and mean squared error. The technique was applied to real-life data and it was observed that the estimated models with the proposed methodology give a very good fit for all the three estimation methods which were confirmed by the Kolmogorov-Smirnov test. The proposed methodology using MLE gives the best fit followed by WLSE and then LSE for the real-life data of strength of carbon fibers. There are many methods for estimating the parameters via various optimization techniques. But the proposed methodology gives highly accurate results with faster compilation time compared to most of these methods. Further studies can be carried out using the proposed methodology considering the location parameter and investigating its effects on reliability calculation. Also, the methodology can be applied to X & Y values following other distributions like gamma, exponential, Laplace, etc.

Conflict of interest

The authors confirm that there are no known conflicts of interest associated with this publication.

References

1. M. A. H. Sabry, E. M. Almetwally, O. A. Alamri, M. Yusuf, H. M. Almongy, A. S. Eldeeb, Inference of fuzzy reliability model for inverse rayleigh distribution, *AIMS Math.*, **6** (2021), 9770–9785. doi: 10.3934/math.2021568.
2. X. Liu, L. Liu, Q. Wu, X. Yuan, H. Huang, Reliability analysis and evaluation of automotive seat angle-adjuster, *Aust. J. Mech. Eng.*, **18** (2020), 481–489. doi: 10.1080/14484846.2018.1548720.
3. S. A. Miller, A. Freivalds, A stress-strength interference model for predicting CTD probabilities, *Int. J. Ind. Ergon.*, **15** (1995), 447–457. doi: 10.1016/0169-8141(94)00063-9.
4. A. Kumar, M. Ram, System reliability analysis based on Weibull distribution and Hesitant fuzzy set, *Int. J. Math. Eng. Manag. Sci.*, **3** (2018), 513–521. doi: 10.33889/IJMEMS.2018.3.4-037.
5. Q. Ramzan, M. Amin, A. Elhassanein, M. Ikram, The extended generalized inverted kumaraswamy weibull distribution: Properties and applications, *AIMS Math.*, **6** (2021), 9955–9980. doi: 10.3934/math.2021579.
6. E. Ramos, P. L. Ramos, F. Louzada, Posterior properties of the Weibull distribution for censored data, *Stat. Probabil. Lett.*, **166** (2020), 108873. doi: 10.1016/j.spl.2020.108873.
7. R. Valiollahi, A. Asgharzadeh, M. Z. Raqab, Estimation of P (Y < X) for weibull distribution under progressive type-II censoring, *Commun. Stat.-Theory Methods.*, **42** (2013), 4476–4498. doi: 10.1080/03610926.2011.650265.
8. F. Louzada, P. L. Ramos, G. S. C. Perdoná, Different estimation procedures for the parameters of the extended exponential geometric distribution for medical data, *Comput. Math. Methods Med.*, **2016** (2016), 8727951. doi: 10.1155/2016/8727951.
9. K. C. Datsiou, M. Overend, Weibull parameter estimation and goodness-of-fit for glass strength data, *Struct. Saf.*, **73** (2018) 29–41. doi: 10.1016/j.strusafe.2018.02.002.
10. F. Louzada, L. F. A. Alegria, D. Colombo, D. E. A. Martins, H. F. L. Santos, J. A. Cuminato, et al., A repairable system subjected to hierarchical competing risks: Modeling and applications, *IEEE Access*, **7** (2019), 171707–171723. doi: 10.1109/ACCESS.2019.2954767.
11. F. Louzada, J. A. Cuminato, O. M. H. Rodriguez, V. L. D. Tomazella, P. H. Ferreira, P. L. Ramos, et al., Improved objective Bayesian estimator for a PLP model hierarchically represented subject to competing risks under minimal repair regime, *PLoS One*, **16** (2021), 1–25. doi: 10.1371/journal.pone.0255944.
12. M. P. Almeida, R. S. Paixão, P. L. Ramos, V. Tomazella, F. Louzada, R. S. Ehlers, Bayesian non-parametric frailty model for dependent competing risks in a repairable systems framework, *Reliab. Eng. Syst. Safe.*, **204** (2020), 107145. doi: 10.1016/j.ress.2020.107145.

13. M. Chacko, R. Mohan, Estimation of parameters of Kumaraswamy-Exponential distribution under progressive type-II censoring, *J. Stat. Comput. Simul.*, **87** (2017), 1951–1963. doi: 10.1080/00949655.2017.1300662.
14. G. Tzavelas, Maximum likelihood parameter estimation in the three-parameter gamma distribution with the use of Mathematica, *J. Stat. Comput. Sim.*, **79** (2009), 1457–1466. doi: 10.1080/00949650802403663.
15. R. Aggarwala, N. Balakrishnan, Maximum likelihood estimation of the Laplace parameters based on progressive type-II censored samples, In: *Advances on methodological and applied aspects of probability and statistics*, CRC Press, 2002.
16. H. K. T. Ng, L. Luo, Y. Hu, F. Duan, Parameter estimation of three-parameter Weibull distribution based on progressively Type-II censored samples, *J. Stat. Comput. Sim.*, **82** (2012), 1661–1678. doi: 10.1080/00949655.2011.591797.
17. T. A. Abushal, Parametric inference of akash distribution for type-ii censoring with analyzing of relief times of patients, *AIMS Math.*, **6** (2021), 12911–12912. doi: 10.3934/math.2021627.
18. J. J. Swain, S. Venkatraman, J. R. Wilson, Least-squares estimation of distribution functions in Johnson's translation system, *J. Stat. Comput. Sim.*, **29** (1988), 271–297. doi: 10.1080/00949658808811068.
19. S. K. Ashour, M. A. Eltehiwy, Exponentiated power Lindley distribution, *J. Adv. Res.*, **6** (2015) 895–905. doi: 10.1016/j.jare.2014.08.005.
20. C. B. Read, Weighted least squares, In: *Encyclopedia of statistical sciences*, Wiley, 2006, 179–196. doi: 10.1002/0471667196.ess2909.pub2.
21. W. T. Wu, Y. T. Chu, K. C. Chen, Moving identification via weighted least-squares estimation, *Int. J. Syst. Sci.*, **18** (1987), 477–486. doi: 10.1080/00207728708963981.
22. S. G. From, A weighted least-squares procedure for estimating the parameters of Altham's multiplicative generalization of the binomial distribution, *Stat. Probabil. Lett.*, **25** (1995), 193–199. doi: 10.1016/0167-7152(94)00222-t.
23. S. Benchiha, A. I. Al-Omari, N. Alotaibi, M. Shrahili, Weighted generalized quasi lindley distribution: Different methods of estimation, applications for covid-19 and engineering data, *AIMS Math.*, **6** (2021), 11850–11878. doi: 10.3934/math.2021688.
24. A. M. Almarashi, A. Algarni, M. Nassar, On estimation procedures of stress-strength reliability for Weibull distribution with application, *PLoS One*, **15** (2020), 1–23. doi: 10.1371/journal.pone.0237997.
25. W. S. Abu El Azm, E. M. Almetwally, A. S. Alghamdi, H. M. Aljohani, A. H. Muse, O. E. Abo-Kasem, Stress-strength reliability for exponentiated inverted Weibull distribution with application on breaking of Jute fiber and Carbon fibers, *Comput. Intel. Neurosc.*, **2021** (2021), 1–21. doi: 10.1155/2021/4227346.
26. A. M. Hamad, B. B. Salman, Different estimation methods of the stress-strength reliability restricted exponentiated Lomax distribution, *Math. Model. Eng. Probl.*, **8** (2021), 477–484. doi: 10.18280/mmep.080319.
27. R. M. Alotaibi, Y. M. Tripathi, S. Dey, H. R. Rezk, Bayesian and non-Bayesian reliability estimation of multicomponent stress–strength model for unit Weibull distribution, *J. Taibah Univ. Sci.*, **14** (2020), 1164–1181. doi: 10.1080/16583655.2020.1806525.

28. I. Pobočíková, Z. Sedliačková, Comparison of four methods for estimating the Weibull distribution parameters, Appl. Math. Sci., **8** (2014), 4137–4149. doi: 10.12988/ams.2014.45389.
29. S. Pant, A. Kumar, S. Bhan, M. Ram, A modified particle swarm optimization algorithm for nonlinear optimization, *Nonlinear Stud.*, **24** (2017), 127–138.
30. L. Sahoo, A. K. Bhunia, D. Roy, Reliability optimization in stochastic domain via genetic algorithm, *Int. J. Qual. Reliab. Manage.*, **31** (2014), 698–717. doi: 10.1108/IJQRM-06-2011-0090.
31. R. V. Rao, Jaya: An advanced optimization algorithm and its engineering applications, Springer International Publishing, 2019. doi: 10.1007/978-3-319-78922-4.
32. D. B. Meshram, Y. M. Puri, N. K. Sahu, Multi-objective optimization for improving performance characteristics of novel curved EDM process using Jaya algorithm, In: *Nature-inspired optimization in advanced manufacturing processes and systems*, CRC Press, 2020.
33. R. H. Caldeira, A. Gnanavelbabu, Solving the flexible job shop scheduling problem using an improved Jaya algorithm, *Comput. Ind. Eng.*, **137** (2019), 106064. doi: 10.1016/j.cie.2019.106064.
34. S. Gupta, I. Agarwal, R. S. Singh, Workflow scheduling using Jaya algorithm in cloud, *Concurr. Comput.*, **31** (2019), 1–13. doi: 10.1002/cpe.5251.
35. R. Jin, L. Wang, C. Huang, S. Jiang, Wind turbine generation performance monitoring with Jaya algorithm, *Int. J. Energy Res.*, **43** (2019), 1604–1611. doi: 10.1002/er.4382.
36. D. C. Du, H. H. Vinh, V. D. Trung, N. T. Hong Quyen, N. T. Trung, Efficiency of Jaya algorithm for solving the optimization-based structural damage identification problem based on a hybrid objective function, *Eng. Optim.*, **50** (2018), 1233–1251. doi: 10.1080/0305215X.2017.1367392.
37. D. Ezzat, S. Amin, H. A. Shedeed, M. F. Tolba, Directed jaya algorithm for delivering nano-robots to cancer area, *Comput. Methods Biomech. Biomed. Eng.*, **23** (2020), 1306–1316. doi: 10.1080/10255842.2020.1797698.
38. W. H. El-Ashmawi, A. F. Ali, A. Slowik, An improved Jaya algorithm with a modified swap operator for solving team formation problem, *Soft Comput.*, **24** (2020), 16627–16641. doi: 10.1007/s00500-020-04965-x.
39. R. V. Rao, D. P. Rai, Optimization of submerged arc welding process parameters using quasi-oppositional based Jaya algorithm, *J. Mech. Sci. Technol.*, **31** (2017), 2513–2522. doi: 10.1007/s12206-017-0449-x.
40. S. P. Singh, T. Prakash, V. P. Singh, M. G. Babu, Analytic hierarchy process based automatic generation control of multi-area interconnected power system using Jaya algorithm, *Eng. Appl. Artif. Intell.*, **60** (2017), 35–44. doi: 10.1016/j.engappai.2017.01.008.
41. M. G. Badar, A. M. Priest, Statistical aspects of fibre and bundle strength in hybrid composites, In: T. Hayashi, K. Kawata, S. Umekawa, *Progress in science and engineering composites*, Tokyo: ICCM-IV, 1982, 1129–1136.
42. H. H. Örkcü, E. Aksoy, M. I. Dogan, Estimating the parameters of 3-p Weibull distribution through differential evolution, *Appl. Math. Comput.*, **251** (2015), 211–224. doi: 10.1016/j.amc.2014.10.127.

43. H. H. Örkcü, V. S. Özsoy, E. Aksoy, M. I. Dogan, Estimating the parameters of 3-p Weibull distribution using particle swarm optimization: A comprehensive experimental comparison, *Appl. Math. Comput.*, **268** (2015), 201–226. doi: 10.1016/j.amc.2015.06.043.
44. B. Abbasi, A. H. Eshragh Jahromi, J. Arkat, M. Hosseinkouchack, Estimating the parameters of Weibull distribution using simulated annealing algorithm, *Appl. Math. Comput.*, **183** (2006), 85–93. doi: 10.1016/j.amc.2006.05.063.
45. S. Acitas, C. H. Aladag, B. Senoglu, A new approach for estimating the parameters of Weibull distribution via particle swarm optimization: An application to the strengths of glass fibre data, *Reliab. Eng. Syst. Saf.*, **183** (2019), 116–127. doi: 10.1016/j.ress.2018.07.024.