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Research article

Evaluation of regularized long-wave equation via Caputo and Caputo-Fabrizio fractional derivatives

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Abstract: The analytical solution of fractional-order regularized long waves in the context of various operators is presented in this study as a framework for the homotopy perturbation transform technique. To investigate regularized long wave equations, we first establish the Yang transform of the fractional Caputo and Caputo-Fabrizio operators. The fractional order regularized long wave equation is solved using the Yang transform as well. The accuracy of the proposed operators are verified using numerical problems, and the resulting solutions are shown in the figures. The solutions demonstrate how the suggested approach is accurate and suitable for analyzing nonlinear physical and engineering challenges.

Keywords: Caputo and Caputo-Fabrizio derivatives; Yang transformation; approximate-analytical solution; regularized long wave equations
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1. Introduction

Integration and fractional-order derivatives are the sole topics covered in the mathematical area known as fractional calculus (FC). Different phenomena in science and engineering may be described using non-integer order integrations and derivatives, such as fractional integration and fractional derivatives. L'Hospital asked Leibniz in 1695, "What is the physical meaning of fractional derivatives?" This subject inspired many well-known scientists of the 18th and 19th centuries to focus on fractional calculus, which has several applications in applied technology and science [1–4]. Numerous research, including [5–12], have shown that fractional extensions of integer order models successfully represent real occurrences. The classic derivatives are those that are local. On the other hand, the Caputo fractional derivative is nonlocal, allowing us to examine changes near a location using classical derivatives. We can still analyze changes in an interval using Caputo fractional derivatives. By virtue of this property, the fractional Caputo operator can be used to model a wider range of physical phenomena, including steady dynamics [13, 14] , diffusion processes [15], continuum and numerical kinematics [16], electromagnetism [17], thermoelastic metals [18], plasma physics [19], spherical flame propagation [20], visco-elastic materials [21], and so on [22–25].

Fractional-order differential problems have been studied for years due to their widespread usage in applications. Fractional-order partial differential problems are used to explain several phenomena in the fields of electro-magnetics, viscoelasticity, acoustics, plasma physics, material science, and electrochemistry. Fractional differential equations have intriguing numerical solutions. For fractional differential equations, there is no method that provides an exact solution. The only procedures that can yield approximations are linearization and series solutions [26–29]. Nonlinear phenomena are present in many engineering and scientific fields, including nonlinear spectroscopy, chemical kinetics, fluid physics, solid-state physics, quantum mechanics, computational biology, thermodynamics, and others. Several higher-order nonlinear partial differential equations define the idea of nonlinearity (PDEs). There are nonlinear models for any physical system that can describe actual occurrences. The literature has documented the use of integral transform methods for solving fractional differential equations. Numerous academics have examined specific important methods for solving practical issues and numerical simulations arising from the novel integral transformation [30–37].

In this article, homogeneous fractional regularized long wave equations are examined. According to some researchers, these equations are superior than the traditional Korteweg-de Vries (KdV) equation [38]. To investigate three particular regularized long wave equations, we use the Yang transformation combinations with the Caputo and Caputo-Fabrizio operators [39]. Following the production of the analytical findings and analysis of the numerical computations of the results, the nonlinear regularized long wave equations [40, 41] are obtained.

$$D^{\delta}_{\mathfrak{I}}\varphi(\psi,\mathfrak{I}) - \varphi_{\psi\psi\mathfrak{I}}(\psi,\mathfrak{I}) + \varphi_{\psi}(\psi,\mathfrak{I}) + \varphi(\psi,\mathfrak{I})\varphi_{\psi}(\psi,\mathfrak{I}) = 0, \tag{1.1}$$

with the initial condition

$$\varphi(\psi,0) = 3\upsilon \sec h^2(\delta\zeta), \tag{1.2}$$

and

$$D^{\delta}_{\mathfrak{I}}\varphi(\psi,\mathfrak{I}) - 2\varphi_{\psi\psi\mathfrak{I}}(\psi,\mathfrak{I}) + \varphi_{\psi}(\psi,\mathfrak{I}) = 0, \tag{1.3}$$

with the initial condition

$$\varphi(\psi,0) = e^{-\psi}, \tag{1.4}$$

and

$$D^{\delta}_{\mathfrak{I}}\varphi(\psi,\mathfrak{I}) + \varphi_{\psi\psi\psi\psi}(\psi,\mathfrak{I}) = 0, \tag{1.5}$$

having initial source

$$\varphi(\psi,0) = \sin\psi. \tag{1.6}$$

Equation (1.1) is called as a general fractional regularized long-wave equation, the Eqs (1.3) and (1.5) show the regularized fractional long wave equations.

The magnetohydrodynamic waves in plasma, rotating tube flow, stress waves in compressed gas bubble mixtures, longitudinal dispersive waves in elastic rods, and ion-acoustic waves in plasma are only a few applications for the regularized long wave equations. In engineering and applied sciences, the regularized long wave equation is a great model for a number of important physical structures. Researchers study several liquid flow phenomena that call for diffusions, such shock or viscous conditions. It may be used to solve nonlinear wave diffusion issues and simulate dissipation. This dissipation may occur via a variety of methods, depending on the problem modeled, including viscosity, heat conduction, chemical reaction, mass diffusion, thermal radiation, or others [42]. Numerous important engineering phenomena, including minor frequency long and shallow waves, are described by fractional regularized long wave equations. Many experts in ocean shallow liquid waves are interested in the nonlinear waves modeled using fractional-order regularized long wave equations. In the representation of ocean nonlinear waves, fractional regularized long wave equations were used. In fact, the enormous surface waves of the tsunami are described by fractional regularized long wave equations. Massive internal waves in the ocean's core caused by temperature changes that might sink marine ships could be described as fractional regularized long wave equations in the current, exceedingly complicated framework.

Using the homotopy perturbation transform approach to resolve fractional-order regularized long wave equations is the main goal of this study. The fractional partial differential equations solution methodology provided by the homotopy perturbation transform method makes use of the Yang transformation approach. The rapid converging series output from the suggested homotopy perturbation transform technique, which might lead to a closed form solution, is required. As opposed to the variational iteration technique or the Adomian decomposition method, the proposed approach solves fractional nonlinear problems without the need of a Lagrange multiplier. By not needing linearization, predetermined assumptions, perturbation, or discretization, these strategies prevent round-off errors.

The following is the article: Some fundamental definitions are required to formulate the issue in Section 2. In Section 3, a unique integral transformation is used to explain the procedure. The primary findings, graphical representations, and numerical simulations are presented in Section 4. Finally, Section 5 summaries the significant results of the research investigation.

2. Preliminaries concepts

In this section, we discuss several fundamental ideas, concepts, and terms associated with fractional derivative operators involving index and exponential decay as a kernel, as well as the specific effects of the Yang transform.

Definition 2.1. The fractional Caputo derivative is defined as follow [43, 44]:

$$ {}_0^cD_{\Im}^{\delta}\mathbb{F}(\Im) = \begin{cases} \frac{1}{\Gamma(r-\delta)}\int_0^{\Im}\frac{\mathbb{F}^{(r)}(\mathbf{y}_1)}{(\Im-\varphi)^{\delta+1-r}}d\varphi, & r-1<\delta<r \\ \frac{d^r}{d\Im^r}\mathbb{F}(\Im), & \delta = r \end{cases} $$

where Γ show that the gamma function.

Definition 2.2. The fractional Caputo-Fabrizio derivative is defined as follow [43, 44]:

$$ {}^{CF}D_{\Im}^{\delta}(\mathbb{F}(\Im)) = \frac{(2-\delta)\mathbb{B}(\delta)}{2(1-\delta)}\int_0^{\Im}\exp\left(-\frac{\delta(\Im-\varphi)}{1-\delta}\right)\mathbb{F}'(\Im)d\Im $$

where $\mathbb{F} \in \mathbf{H}^1(\mathbf{a},\mathbf{b})$ (Sobolev space), $\mathbf{a} < \mathbf{b}, \delta \in [0,1]$ and $\mathbb{B}(\delta)$ represents a normalization term as $\mathbb{B}(\delta) = \mathbb{B}(0) = \mathbb{B}(1) = 1$.

Definition 2.3. The fractional Caputo-Fabrizio integral operator is expressed as [43, 44]:

$$^{CF}\mathcal{I}_{\mathfrak{I}}^{\delta}(\mathbb{F}(\mathfrak{I})) = \frac{2(1-\delta)}{(2-\delta)\mathbb{B}(\delta)}\mathbb{F}(\mathfrak{I}) + \frac{2\delta}{(2-\delta)\mathbb{B}(\delta)}\int_0^{\mathfrak{I}} \mathbb{F}(\varphi)\, d\varphi$$

Definition 2.4. The Yang transformation is expressed as follow [43, 44]:

$$\mathbb{Y}[\mathbb{F}(\varphi)] = \mathbb{Y}(\omega) = \int_0^{\infty} \mathbb{F}(\varphi)\exp\left(-\frac{\varphi}{\omega}\right) d\varphi, \quad \varphi > 0.$$

Following is the Yang transformation of a variety of vital expressions:

$$\begin{aligned} &\mathbb{Y}[1] = \omega \\ &\mathbb{Y}[\varphi] = \omega^2 \\ &\vdots \\ &\mathbb{Y}\left[\frac{\varphi^{\delta}}{\Gamma(\delta+1)}\right] = \omega^{\delta+1} \end{aligned}$$

Definition 2.5. The inverse Yang transformation $\mathbb{Y}^{-1}$ is defined as

$$\mathbb{Y}^{-1}[\mathbb{Y}(\omega)] = h(\mathfrak{I}) = \frac{1}{2\pi\iota}\int_{\upsilon-\iota\infty}^{\upsilon+\iota\infty} h\left(\frac{1}{\omega}\right)e^{\omega\mathfrak{I}}\omega d\omega = \Sigma \ residues\ of\ h\left(\frac{1}{\omega}\right)e^{\omega\mathfrak{I}}\omega.$$

Definition 2.6. The Yang transformation Caputo-Fabrizio operator is given as [43, 44]:

$$\mathbb{Y}\left\{{}_0^cD_{\mathfrak{I}}^{\delta}(\mathbb{F}(\mathfrak{I})), \mathfrak{s}\right\} = \varphi^{-\delta}Q(\mathfrak{s}) - \sum_{\kappa=0}^{\delta-1}\varphi^{1-\delta-\kappa}(\mathfrak{s})\mathbb{F}^{(\kappa)}(0), \quad r-1 < \delta < r, \varphi > 0.$$

Definition 2.7. The fractional Caputo-Fabrizio Yang transformation derivative is given as [43, 44]:

$$\mathbb{Y}\left\{{}_0^{CF}D_e^{\delta}(\mathbb{F}(\varphi)), \omega\right\} = \frac{\mathbb{Y}[\mathbb{F}(\varphi) - \omega\mathbb{F}(0)]}{1+\delta(\omega-1)}$$

3. Road map of the proposed method

Consider the fractional order partial differential equation

$$\begin{cases} ^{CF}D_{\mathfrak{I}}^{\delta}\varphi(\psi,\mathfrak{I}) + L(\varphi(\psi,\mathfrak{I})) + N(\varphi(\psi,\mathfrak{I})) = g(\psi,\mathfrak{I}), \\ G(\psi,0) = h(\psi), \end{cases} \tag{3.1}$$

where the term $\varphi(\psi,\mathfrak{I})$ represents the source term. Implement Yang transform to Equation (3.2), and one can achieve

$$\frac{\mathbb{Y}[\varphi(\psi,\mathfrak{I}) - vG(\psi,0)]}{1+\delta(v-1)} = -\mathbb{Y}[L(\varphi(\psi,\mathfrak{I})) + N(\varphi(\psi,\mathfrak{I}))] + \mathbb{Y}[g(\psi,\mathfrak{I})],$$

$$\mathbb{Y}[\varphi(\psi,\mathfrak{I})] = vh(\psi) - (1+\delta(v-1))[\mathbb{Y}[L(\varphi(\psi,\mathfrak{I})) + N(\varphi(\psi,\mathfrak{I}))] + \mathbb{Y}[g(\psi,\mathfrak{I})]. \tag{3.2}$$

Applying inverse of Yang transform, we achieve

$$\varphi(\psi,\mathfrak{I}) = \mathcal{G}(\psi,\mathfrak{I}) - \mathbb{Y}^{-1}[(1+\delta(v-1))[\mathbb{Y}[L(\varphi(\psi,\mathfrak{I})) + N(\varphi(\psi,\mathfrak{I}))] + \mathbb{Y}[g(\psi,\mathfrak{I})]], \tag{3.3}$$

where the term $\varphi(\psi,\mathfrak{I})$ represents the source term and the given I.C (initial condition). Now, we utilize HPM:

$$\varphi(\psi,\mathfrak{I}) = \sum_{q=0}^{\infty} \rho^q \varphi_q(\psi,\mathfrak{I}). \tag{3.4}$$

We decompose the nonlinear term $N(\varphi(\psi,\mathfrak{I}))$ as

$$N(\varphi(\psi,\mathfrak{I})) = \sum_{q=0}^{\infty} \rho^q H_q(\varphi), \tag{3.5}$$

where $H_q(\varphi)$ represents the He's polynomial and is calculated through the formula:

$$H_q(\varphi_1,\varphi_2,\varphi_3,\cdots,\varphi_q) = \frac{1}{\Gamma(q+1)}\frac{\partial^q}{\partial\rho^q}\left[N\left(\sum_{i=0}^{\infty}\rho^i\varphi_i\right)\right]_{\rho=0}, \quad q=1,2,3,\cdots. \tag{3.6}$$

Putting Eqs (3.4) and (3.5) in Eq (3.3), we achieve

$$\sum_{q=0}^{\infty}\rho^q\varphi_q(\psi,\mathfrak{I}) = \mathcal{G}(\psi,\mathfrak{I}) - \rho\left(\mathbb{Y}^{-1}\left[(1+\delta(v-1))Y\left[L\sum_{q=0}^{\infty}\rho^q\varphi_q(\psi,\mathfrak{I}) + N\sum_{q=0}^{\infty}\rho^q H_q(\varphi)\right]\right]\right), \tag{3.7}$$

We achieve the following terms by comparing coefficients of ρ in (3.7):

$$\begin{aligned}
\rho^0 &: \varphi_0(\psi,\mathfrak{I}) = \varphi(\psi,\mathfrak{I}),\\
\rho^1 &: \varphi_1(\psi,\mathfrak{I}) = \mathbb{Y}^{-1}\left[(1+\delta(v-1))Y\left[L(\varphi_0(\psi,\mathfrak{I})) + H_0(\varphi)\right]\right],\\
\rho^2 &: \varphi_2(\psi,\mathfrak{I}) = \mathbb{Y}^{-1}\left[(1+\delta(v-1))Y\left[L(\varphi_1(\psi,\mathfrak{I})) + H_1(\varphi)\right]\right],\\
\rho^3 &: \varphi_3(\psi,\mathfrak{I}) = \mathbb{Y}^{-1}\left[(1+\delta(v-1))Y\left[L(\varphi_2(\psi,\mathfrak{I})) + H_2(\varphi)\right]\right],\\
&\vdots\\
\rho^q &: \varphi_q(\psi,\mathfrak{I}) = \mathbb{Y}^{-1}\left[(1+\delta(v-1))Y\left[L(\varphi_q(\psi,\mathfrak{I})) + H_q(\varphi)\right]\right].
\end{aligned} \tag{3.8}$$

Thus, we may write the acquired solution of Equation (3.1) as follows:

$$\varphi(\psi,\mathfrak{I}) = \varphi_0(\psi,\mathfrak{I}) + \varphi_1(\psi,\mathfrak{I}) + \cdots. \tag{3.9}$$

Convergence and Error Analysis

The following theorems are based on the method's mechanism and address the original problem's (3.1) convergence and error analysis.

Theorem

Let $\varphi(\psi,\mathfrak{I})$ be the exact solution of (3.1) and let $\varphi(\psi,\mathfrak{I})$, $\varphi_n(\psi,\mathfrak{I}) \in H$ and $\sigma \in (0,1)$, where H denotes the Hilbert space. Then, the obtained solution $\sum_{q=0}^{\infty}\varphi_q(\psi,\mathfrak{I})$ will converge $\varphi(\psi,\mathfrak{I})$ if $\varphi_q(\psi,\mathfrak{I}) \leq \varphi_{q-1}(\psi,\mathfrak{I})$ $\forall q > A$, i.e., for any $\omega > 0 \exists A > 0$, such that $\|\varphi_{q+n}(\psi,\mathfrak{I})\| \leq \beta, \forall m,n \in N$.

Proof

We make a sequence of $\sum_{q=0}^{\infty}\varphi_q(\psi,\Im)$.

$$\begin{aligned}
\varphi_0(\psi,\Im) =&\varphi_0(\psi,\Im),\\
\varphi_1(\psi,\Im) =&\varphi_0(\psi,\Im)+\varphi_1(\psi,\Im),\\
\varphi_2(\psi,\Im) =&\varphi_0(\psi,\Im)+\varphi_1(\psi,\Im)+\varphi_2(\psi,\Im),\\
\varphi_3(\psi,\Im) =&\varphi_0(\psi,\Im)+\varphi_1(\psi,\Im)+\varphi_2(\psi,\Im)+\varphi_3(\psi,\Im),\\
&\vdots\\
\varphi_q(\psi,\Im) =&\varphi_0(\psi,\Im)+\varphi_1(\psi,\Im)+\varphi_2(\psi,\Im)+\cdots+\varphi_q(\psi,\Im),
\end{aligned} \tag{3.10}$$

To get the desired result, we have to prove that $\varphi_q(\psi,\Im)$ forms a "Cauchy sequence." Further, let us take

$$\begin{aligned}
\|\varphi_{q+1}(\psi,\Im)-\varphi_q(\psi,\Im)\| = \|\varphi_{q+1}(\psi,\Im)\| \leq \sigma\|\varphi_q(\psi,\Im)\| \leq \sigma^2\|\varphi_{q-1}(\psi,\Im)\| \leq \sigma^3\|\varphi_{q-2}(\psi,\Im)\|\cdots\\
\leq \sigma_{q+1}\|\varphi_0(\psi,\Im)\|.
\end{aligned} \tag{3.11}$$

For $q, n \in N$, we acquire

$$\begin{aligned}
\|\varphi_q(\psi,\Im)-\varphi_n(\psi,\Im)\| =&\|\varphi_{q+n}(\psi,\Im)\| = \|\varphi_q(\psi,\Im)-\varphi_{q-1}(\psi,\Im)+(\varphi_{q-1}(\psi,\Im)-\varphi_{q-2}(\psi,\Im))\\
&+(\varphi_{q-2}(\psi,\Im)-\varphi_{q-3}(\psi,\Im))+\cdots+(\varphi_{n+1}(\psi,\Im)-\varphi_n(\psi,\Im))\|\\
\leq&\|\varphi_q(\psi,\Im)-\varphi_{q-1}(\psi,\Im)\|+\|(\varphi_{q-1}(\psi,\Im)-\varphi_{q-2}(\psi,\Im))\|\\
&+\|(\varphi_{q-2}(\psi,\Im)-\varphi_{q-3}(\psi,\Im))\|+\cdots+\|(\varphi_{n+1}(\psi,\Im)-\varphi_n(\psi,\Im))\|\\
\leq&\sigma^q\|\varphi_0(\psi,\Im)\|+\sigma^{q-1}\|\varphi_0(\psi,\Im)\|+\cdots+\sigma^{q+1}\|\varphi_0(\psi,\Im)\|\\
=&\|\varphi_0(\psi,\Im)\|(\sigma^q+\sigma^{q-1}+\sigma^{q+1})\\
=&\|\varphi_0(\psi,\Im)\|\frac{1-\sigma^{q-n}}{1-\sigma^{q+1}}\sigma^{n+1}.
\end{aligned} \tag{3.12}$$

Since $0<\sigma<1$, and $\varphi_0(\psi,\Im)$ is bounded, let us take $\beta = 1-\sigma/(1-\sigma_{q-n})\sigma^{n+1}\|\varphi_0(\psi,\Im)\|$, and we obtain Thus, $\{\varphi_q(\psi,\Im)\}_{q=0}^{\infty}$ forms a "Cauchy sequence" in H. It follows that the sequence $\{\varphi_q(\psi,\Im)\}_{q=0}^{\infty}$ is a convergent sequence with the limit $\lim_{q\to\infty}\varphi_q(\psi,\Im)=\varphi(\psi,\Im)$ for $\exists\varphi(\psi,\Im)\in\mathcal{H}$. Hence, this ends the proof.

Theorem

Let $\sum_{h=0}^{k}\varphi_h(\psi,\Im)$ is finite and $\varphi(\psi,\Im)$ represents the obtained series solution. Let $\sigma>0$ such that $\|\varphi_{h+1}(\psi,\Im)\| \leq \|\varphi_h(\psi,\Im)\|$, then the following relation gives the maximum absolute error.

$$\|\varphi(\psi,\Im)-\sum_{h=0}^{k}\varphi_h(\psi,\Im)\| < \frac{\sigma^{k+1}}{1-\sigma}\|\varphi_0(\psi,\Im)\|. \tag{3.13}$$

Proof

Since $\sum_{h=0}^{k}\varphi_h(\psi,\mathfrak{I})$ is finite, this implies that $\sum_{h=0}^{k}\varphi_h(\psi,\mathfrak{I})<\infty$.
Consider

$$\begin{aligned}\|\varphi(\psi,\mathfrak{I})-\sum_{h=0}^{k}\varphi_h(\psi,\mathfrak{I})\| =&\|\sum_{h=k+1}^{\infty}\varphi_h(\psi,\mathfrak{I})\|\\ \leq&\sum_{h=k+1}^{\infty}\|\varphi_h(\psi,\mathfrak{I})\|\\ \leq&\sum_{h=k+1}^{\infty}\sigma^h\|\varphi_0(\psi,\mathfrak{I})\|\\ \leq&\sigma^{k+1}(1+\sigma+\sigma^2+\cdots)\|\varphi_0(\psi,\mathfrak{I})\|\\ \leq&\frac{\sigma^{k+1}}{1-\sigma}\|\varphi_0(\psi,\mathfrak{I})\|.\end{aligned} \tag{3.14}$$

This ends the theorem's proof.

4. Applications

4.1. Example

Initially, we implement the Yang transform method with the help of the Caputo derivative to solve the initial condition of problem (1.1). By applying the Yang transformation, we achieved

$$\varphi(\psi,\varpi)=\varpi^{\delta}\mathbb{Y}\left[\varphi_{\psi\psi\mathfrak{I}}(\psi,\mathfrak{I})-\varphi_{\psi}(\psi,\mathfrak{I})-\varphi(\psi,\mathfrak{I})\varphi_{\psi}(\psi,\mathfrak{I})\right]+\varpi^2\varphi(\psi,0). \tag{4.1}$$

We apply the Yang perturbation transformation technique to analysis Eq. (4.1), we get

$$\begin{aligned}\sum_{\jmath=0}^{\infty}\rho^{\jmath}\tilde{\varepsilon}_{\jmath}(\psi,\varpi) =&\rho\varpi^{\delta}\mathbb{Y}\left[\left(\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi,\mathfrak{I})\right)_{\psi\psi\mathfrak{I}}\right.\\ &\left.-\left(\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi,\mathfrak{I})\right)_{\psi}\right]-\rho\varpi^{\delta}\mathbb{Y}\left[\left(\sum_{\jmath=0}^{\infty}\rho^{\jmath}\Psi_{\jmath}(\psi,\mathfrak{I})\right)\right]+\varpi^2\varphi(\psi,0).\end{aligned} \tag{4.2}$$

Now we apply inverse Yang transformation to Eq (4.2), we get

$$\begin{aligned}\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi,\varpi) =&\rho\mathbb{Y}^{-1}\left[\varpi^{\delta}\mathbb{Y}\left[\left(\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi,\mathfrak{I})\right)_{\psi\psi\mathfrak{I}}-\left(\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi,\mathfrak{I})\right)_{\psi}\right]\right]\\ &-\rho\mathbb{Y}^{-1}\left[\varpi^{\delta}\mathbb{Y}\left[\left(\sum_{\jmath=0}^{\infty}\rho^{\jmath}\Psi_{\jmath}(\psi,\mathfrak{I})\right)\right]\right]+\mathbb{Y}^{-1}[\varpi^2\varphi(\psi,0)].\end{aligned} \tag{4.3}$$

The $\Psi_J(\psi,\varpi)$ values in Eq (4.3) are terms show that the non-linear functions in Eq (3.6), and are investigated as follows:

$$\begin{aligned}\Psi_0(\varphi) =&\varphi_0(\varphi_0)_\psi,\\ \Psi_1(\varphi) =&\varphi_0(\varphi_1)_\psi + \varphi_1(\varphi_0)_\psi,\\ \Psi_2(\varphi) =&\varphi_0(\varphi_2)_\psi + \varphi_1(\varphi_1)_\psi + \varphi_2(\varphi_0)_\psi,\\ &\vdots\end{aligned} \tag{4.4}$$

We then achieve the terms of the Caputo operator solution by investigating the associated powers of ρ:

$$\begin{aligned}\rho^0 :\ &\varphi_0(\psi,\Im) =\mathbb{Y}^{-1}[\varpi^2 3\upsilon \sec h^2(\delta\psi)] = 3\upsilon \sec h^2(\delta\psi),\\ \rho^1 :\ &\varphi_1(\psi,\Im) =\mathbb{Y}^{-1}[\varpi^\delta \mathbb{Y}[L(\varphi_0(\psi,\Im))]] - \mathbb{Y}^{-1}[\varpi^\delta \mathbb{Y}[\Psi_0(\psi,\Im)]] = 3\upsilon\delta\{1+6\upsilon\delta+\cosh(2\delta\psi)\}\\ &\quad \operatorname{sech}^4(\delta\psi)\tanh(\delta\psi)\frac{\Im^\delta}{\Gamma(\delta+1)},\\ \rho^2 :\ &\varphi_2(\psi,\Im) =\mathbb{Y}^{-1}[\varpi^\delta \mathbb{Y}[L(\varphi_1(\psi,\Im))]] - \mathbb{Y}^{-1}[\varpi^\delta \mathbb{Y}[\Psi_1(\psi,\Im)]] = -\frac{3}{32}\upsilon\delta^2\{-8-96\upsilon-576\upsilon^2+\\ &\quad 3(-3-16\upsilon+144\upsilon^2)\cosh(2\delta\psi)+48\upsilon\cosh(4\delta\psi)+\cosh(6\delta\psi)\}\operatorname{sech}^8(\delta\psi)\frac{\Im^{2\delta}}{\Gamma(2\delta+1)},\\ &\quad\vdots\end{aligned} \tag{4.5}$$

The analytical solutions of the given problem is

$$\begin{aligned}\varphi(\psi,\Im) = \Bigg(&3\upsilon\operatorname{sech}^2(\delta\psi) + 3\upsilon\delta\{1+6\upsilon\delta+\cosh(2\delta\psi)\}\operatorname{sech}^4(\delta\psi)\tanh(\delta\psi)\frac{\Im^\delta}{\Gamma(\delta+1)} - \frac{3}{32}\upsilon\delta^2\\ &\{-8-96\upsilon-576\upsilon^2+3(-3-16\upsilon+144\upsilon^2)\cosh(2\delta\psi)+48\upsilon\cosh(4\delta\psi)+\cosh(6\delta\psi)\}\operatorname{sech}^8(\delta\psi)\\ &\frac{\Im^{2\delta}}{\Gamma(2\delta+1)},+\cdots\Bigg),\end{aligned} \tag{4.6}$$

The exact solution is, $\varphi(\psi,\Im) = 3\upsilon\operatorname{sech}^2(\delta(\psi-(1+\upsilon)\Im))$.

In contrast, we solve the problem by combining the Yang transformation with the Caputo-Fabrizio operator. Next, we solve the problem using the Yang transformation:

$$\varphi(\psi,\varpi) = (1+\delta(\varpi-1))\mathbb{Y}\left[\varphi_{\psi\psi\Im}(\psi,\Im) - \varphi_\psi(\psi,\Im) - \varphi(\psi,\Im)\varphi_\psi(\psi,\Im)\right] + \varpi^2\varphi(\psi,0). \tag{4.7}$$

To Eq (4.7), we apply the Yang perturbation transform method and obtain as

$$\begin{aligned}\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\varpi) =&\rho(1+\delta(\varpi-1))\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\Im)\right)_{\psi\psi\Im} - \left(\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\Im)\right)_{\psi}\right]\\ &-\rho(1+\delta(\varpi-1))\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^J\Psi_J(\psi,\Im)\right)\right] + \varpi^2\varphi(\psi,0).\end{aligned} \tag{4.8}$$

By taking the inverse YT of Eq (4.8), we get

$$\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\mathfrak{I}) =\rho\mathbb{Y}^{-1}\left[(1+\delta(\varpi-1))\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\mathfrak{I})\right)_{\psi\psi\mathfrak{I}}-\left(\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\mathfrak{I})\right)_{\psi}\right]\right] - \rho\mathbb{Y}^{-1}\left[(1+\delta(\varpi-1))\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^J\Psi_J(\psi,\mathfrak{I})\right)\right]\right]+\mathbb{Y}^{-1}[\varpi^2\varphi(\psi,0)]. \tag{4.9}$$

The $\Psi_J(.)$ terms in Eq (4.9) are the nonlinear polynomials described in Eq (3.5). By repeating the methods for nonlinear polynomials, we arrive at the following results:

$$\begin{aligned}
\rho^0 &: \varphi_0(\psi,\mathfrak{I}) =\mathbb{Y}^{-1}[\varpi^2 3\upsilon\sec h^2(\delta\psi)] = 3\upsilon\sec h^2(\delta\psi),\\
\rho^1 &: \varphi_1(\psi,\mathfrak{I}) =\mathbb{Y}^{-1}[(1+\delta(\varpi-1))\mathbb{Y}[\varphi_0(\psi,\mathfrak{I})]]-\mathbb{Y}^{-1}[(1+\delta(\varpi-1))\mathbb{Y}[\Psi_0(\psi,\mathfrak{I})]]\\
&= -3\upsilon\delta\{1+6\upsilon\delta+\cosh(2\delta\psi)\}\,\text{sech}^4(\delta\psi)\tanh(\delta\psi)\Big\{1+\delta\mathfrak{I}-\delta\Big\},
\end{aligned} \tag{4.10}$$

$$\begin{aligned}
\rho^2 : \varphi_2(\psi,\mathfrak{I}) = \mathbb{Y}^{-1}[(1+\delta(\varpi-1))\mathbb{Y}[\varphi_1(\psi,\mathfrak{I})]]-\mathbb{Y}^{-1}[(1+\delta(\varpi-1))\mathbb{Y}[\Psi_1(\psi,\mathfrak{I})]] =\\
-\frac{3}{32}\upsilon\delta^2\{-8-96\upsilon-576\upsilon^2+3(-3-16\upsilon+144\upsilon^2)\cosh(2\delta\psi)+48\upsilon\cosh(4\delta\psi)+\cosh(6\delta\psi)\}\,\text{sech}^8(\delta\psi)\\
\Big\{(1-\delta)2\delta\mathfrak{I}+(1-\delta)^2+\frac{\delta^2\mathfrak{I}^2}{2}\Big\},\\
\vdots
\end{aligned} \tag{4.11}$$

Now find the solution, based on the Caputo-Fabrizio operator, the analytical result is given as:

$$\begin{aligned}
\varphi(\psi,\mathfrak{I}) &= \sum_{\sigma=0}^{n}\varphi_\sigma(\psi,\mathfrak{I})\\
&=3\upsilon\,\text{sech}^2(\delta\psi)-3\upsilon\delta\{1+6\upsilon\delta+\cosh(2\delta\psi)\}\,\text{sech}^4(\delta\psi)\tanh(\delta\psi)\Big\{1+\delta\mathfrak{I}-\delta\Big\}\\
&-\frac{3}{32}\upsilon\delta^2\{-8-96\upsilon-576\upsilon^2+3(-3-16\upsilon+144\upsilon^2)\cosh(2\delta\psi)+48\upsilon\cosh(4\delta\psi)+\cosh(6\delta\psi)\}\,\text{sech}^8(\delta\psi)\\
&\Big\{(1-\delta)2\delta\mathfrak{I}+(1-\delta)^2+\frac{\delta^2\mathfrak{I}^2}{2}\Big\}+\cdots
\end{aligned} \tag{4.12}$$

The exact solution is ($\delta = 1$) solution, $\varphi(\psi,\mathfrak{I}) = 3\upsilon\,\text{sech}^2(\delta(\psi-(1+\upsilon)\mathfrak{I}))$.

The analytical solution the fractional-order regularized long wave equations with the help of homotopy perturbation transform method. Example 4.1 is graphical simulation is shown in Figure 1 (a) the exact and (b) the analytical solution at $\delta = 1$. Figure 1 is a graphical depiction of the solution acquired by the offered methods and the exact outcome of Example 4.1. Figure 1, (c) and (d) show that depicts the results of proposed techniques with different fractional order δ, respectively.

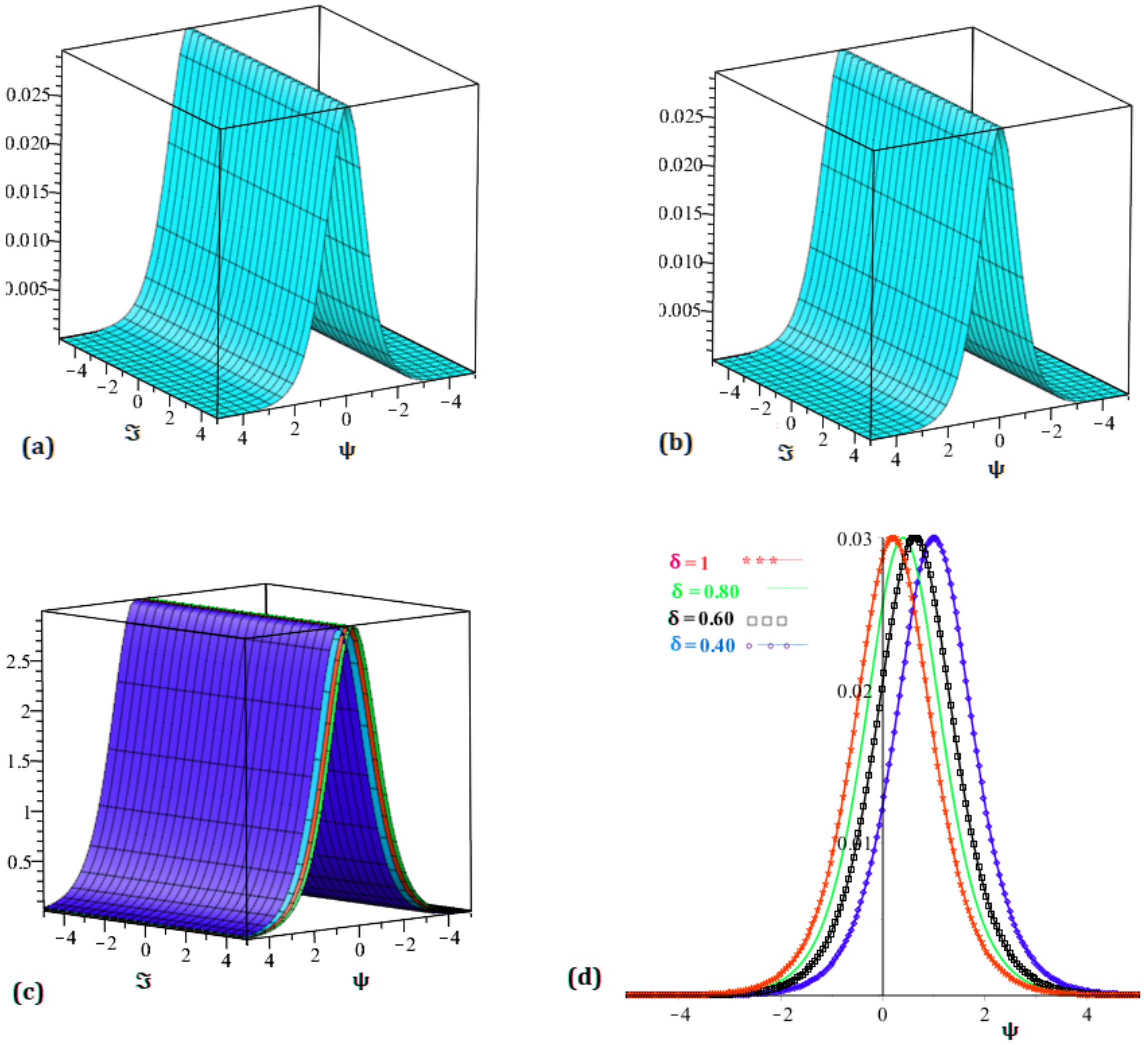

Figure 1. Example 4.1 solutions figure (a) Actual result (b) Analytic result at $\delta = 1$, (c) Analytic result at different fractional-order of δ (d) $\Im = 0.5$.

4.2. Example

Second, we employ the Yang transform method with the help of the Caputo derivative to solve the initial condition of problem (1.5). The result of applying the Yang transformation is:

$$\varphi(\psi, \varpi) = \varpi^{\delta}\mathbb{Y}\left[2\varphi_{\psi\psi\Im}(\psi, \Im) - \varphi_{\psi}(\psi, \Im)\right] + \varpi^{2}\varphi(\psi, 0). \tag{4.13}$$

We apply the Yang perturbation transformation technique to investigate Eq (4.13), we get

$$\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi, \varpi) = \rho\varpi^{\delta}\mathbb{Y}\left[\left(\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi, \Im)\right)_{\psi\psi\Im} - \left(\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi, \Im)\right)_{\psi}\right] + \varpi^{2}\varphi(\psi, 0). \tag{4.14}$$

Now we use inverse Yang transformation to Eq (4.14), we get

$$\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi, \Im) = \rho\mathbb{Y}^{-1}\left[\varpi^{\delta}\mathbb{Y}\left[\left(\sum_{\jmath=0}^{\infty}\rho^{\jmath}\varphi_{\jmath}(\psi, \Im)\right)\right]\right] + \mathbb{Y}^{-1}\left[\varpi^{2}\varphi(\psi, 0)\right]. \tag{4.15}$$

We then achieve the terms of the Caputo operator solution by investigating the associated powers of ρ:

$$\begin{aligned}
\rho^0 : \varphi_0(\psi,\mathfrak{I}) =& \mathbb{Y}^{-1}[\varpi^2 e^{-\psi}] = e^{-\psi},\\
\rho^1 : \varphi_1(\psi,\mathfrak{I}) =& \mathbb{Y}^{-1}[\varpi^\delta \mathbb{Y}[L(\varphi_0(\psi,\mathfrak{I}))]],\\
=& e^{-\psi}\frac{\mathfrak{I}^\delta}{\Gamma(\delta+1)},\\
\rho^2 : \varphi_1(\psi,\mathfrak{I}) =& \mathbb{Y}^{-1}[\varpi^\delta \mathbb{Y}[L(\varphi_1(\psi,\mathfrak{I}))]],\\
=& e^{-\psi}\frac{\mathfrak{I}^{2\delta}}{\Gamma(2\delta+1)},\\
&\vdots
\end{aligned} \tag{4.16}$$

The analytical series form solution is given as

$$\varphi(\psi,\mathfrak{I}) = e^{-\psi} + e^{-\psi}\frac{\mathfrak{I}^\delta}{\Gamma(\delta+1)} + e^{-\psi}\frac{\mathfrak{I}^{2\delta}}{\Gamma(2\delta+1)} + \cdots \tag{4.17}$$

The exact solution is $\varphi(\psi,\mathfrak{I}) = e^{(\mathfrak{I}-\psi)}$.
In contrast, we solve the problem by combining the Yang transform with the Caputo-Fabrizio operator. First, we solve the problem using the Yang transform:

$$\varphi(\psi,\varpi) = (1+\delta(\varpi-1))\left(\mathbb{Y}\left[\varphi_{\psi\psi\mathfrak{I}}(\psi,\mathfrak{I}) - \varphi_\psi(\psi,\mathfrak{I})\right]\right) + \varpi^2\varphi(\psi,0). \tag{4.18}$$

To Eq (4.18), apply the Yang perturbation transformation method, we get

$$\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\varpi) = (1+\delta(\varpi-1))\Big(\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\mathfrak{I})\right)_{\psi\psi\mathfrak{I}} - \left(\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\mathfrak{I})\right)_\psi\right]\Big) + \varpi^2\varphi(\psi,0). \tag{4.19}$$

By inverse Yang transform of the above equation, we have

$$\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\mathfrak{I}) = \rho\mathbb{Y}^{-1}\left[(1+\delta(\varpi-1))\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^J\varphi_J(\psi,\mathfrak{I})\right)\right]\right] + \mathbb{Y}^{-1}[\varpi^2\varphi(\psi,0)]. \tag{4.20}$$

Both sides comparing

$$\begin{aligned}
\rho^0 : \varphi_0(\psi,\mathfrak{I}) =& \mathbb{Y}^{-1}[\varpi^2 e^{-\psi}] = e^{-\psi},\\
\rho^1 : \varphi_1(\psi,\mathfrak{I}) =& \mathbb{Y}^{-1}\left[(1+\delta(\varpi-1))\mathbb{Y}\left[(\varphi_0)_{\psi\psi\mathfrak{I}} - (\varphi_0)_\psi\right]\right]\\
=& e^{-\psi}\Big\{1+\delta\mathfrak{I}-\delta\Big\},\\
\rho^2 : \varphi_2(\psi,\mathfrak{I}) =& \mathbb{Y}^{-1}\left[(1+\delta(\varpi-1))\mathbb{Y}\left[(\varphi_1)_{\psi\psi\mathfrak{I}} - (\varphi_1)_\psi\right]\right]\\
=& e^{-\psi}\Big\{(1-\delta)2\delta\mathfrak{I} + (1-\delta)^2 + \frac{\delta^2\mathfrak{I}^2}{2}\Big\}
\end{aligned} \tag{4.21}$$

The series form solutions based on Caputo-Fabrizio operator is given as

$$\varphi(\psi,\mathfrak{I}) = e^{-\psi} + e^{-\psi}\Big\{1+\delta\mathfrak{I}-\delta\Big\} + e^{-\psi}\Big\{(1-\delta)2\delta\mathfrak{I} + (1-\delta)^2 + \frac{\delta^2\mathfrak{I}^2}{2}\Big\} + \cdots, \tag{4.22}$$

The exact solution is $\varphi(\psi, \Im) = e^{(\Im-\psi)}$.

The analytical solution the fractional-order regularized long wave equations with the help of homotopy perturbation transform method. Example 4.2 is graphical simulation is shown in Figure 2 (a) the exact and (b) the analytical solution at $\delta = 1$. Figure 2 is a graphical depiction of the solution acquired by the offered methods and the exact outcome of Example 4.2. Figure 2, (c) and (d) show that depicts the results of proposed techniques with different fractional order δ, respectively.

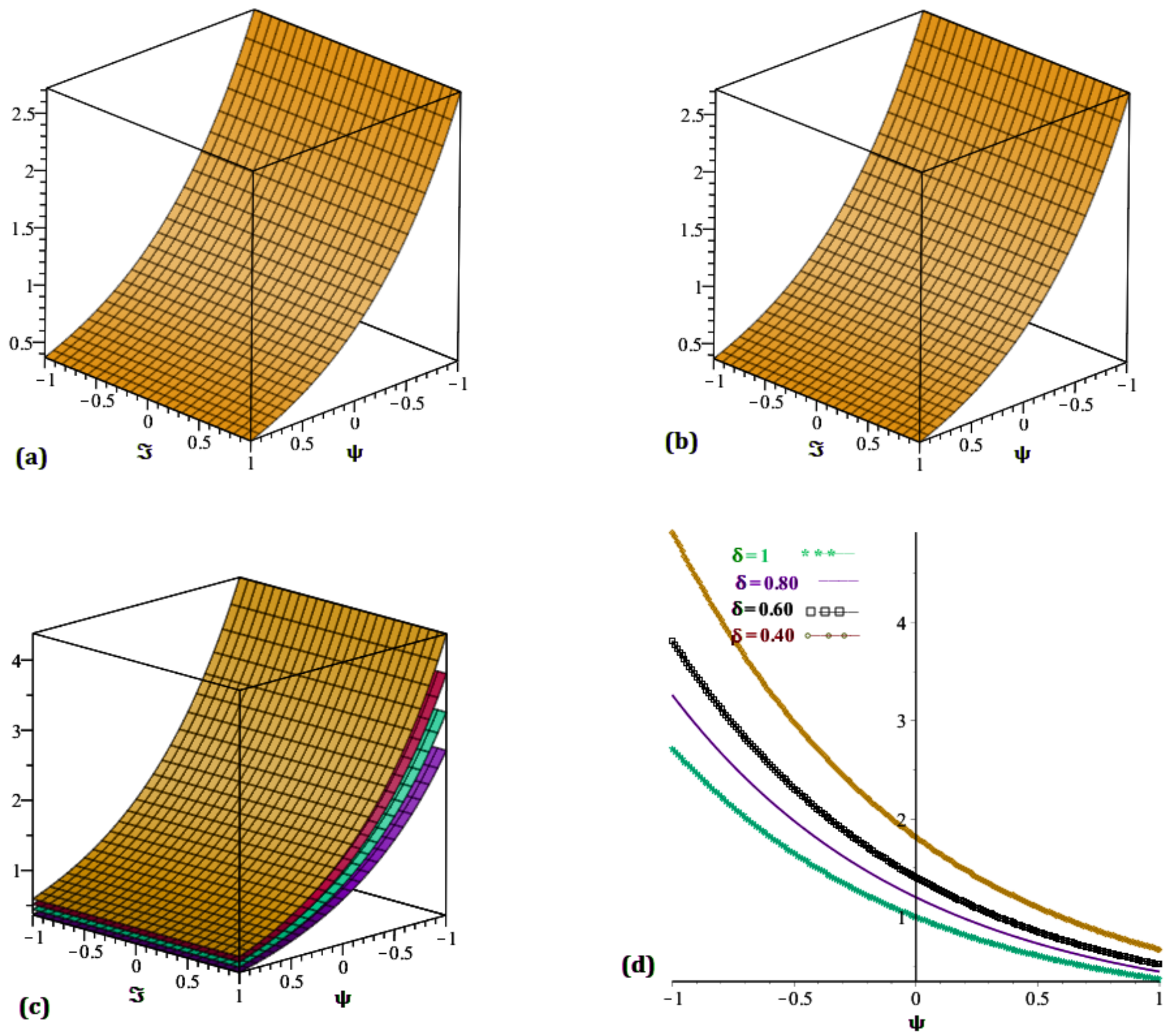

Figure 2. Example 4.2 result figure (a) Actual result, (b) Analytic result at $\lambda = 1$, (c) Analytic result at different fractional-order of δ (d)$\Im = 0.5$.

4.3. Example

Finally, we apply the Yang transformation in the sense of Caputo and Caputo-Fabrizio operators to analysis the problem in Eq (1.5). To Eq (1.5), now use the Yang transformation with the help of Caputo derivative:

$$\varphi(\psi, \varpi) = \varpi^{\delta}\mathbb{Y}\left[\varphi_{\psi\psi\psi\psi}(\psi, \Im)\right] + \varpi^{2}\varphi(\psi, 0). \tag{4.23}$$

We apply the Yang perturbation transformation method to analysis equation (4.23), we get

$$\sum_{J=0}^{\infty}\rho^{J}\varphi_{J}(\psi,\varpi)=\rho\varpi^{\delta}\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^{J}\varphi_{J}(\psi,\varpi)\right)_{\psi\psi\psi\psi}\right]+\varpi^{2}\varphi(\psi,0). \tag{4.24}$$

The inverse Yang transformation to Eq (4.24)

$$\sum_{J=0}^{\infty}\rho^{J}\varphi_{J}(\psi,\mathfrak{I})=\rho\mathbb{Y}^{-1}\left[\varpi^{\delta}\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^{J}\varphi_{J}(\psi,\mathfrak{I})\right)\right]\right]+\mathbb{Y}^{-1}\left[\varpi^{2}\varphi(\psi,0)\right]. \tag{4.25}$$

We then achieve the terms of the Caputo operator solution by investigating the associated powers of ρ:

$$\begin{aligned}
\rho^{0}:\varphi_{0}(\psi,\mathfrak{I})&=\mathbb{Y}^{-1}[\varpi^{2}\sin\psi]=\sin\psi,\\
\rho^{1}:\varphi_{1}(\psi,\mathfrak{I})&=\mathbb{Y}^{-1}[\varpi^{\delta}\mathbb{Y}[L(\varphi_{0}(\psi,\mathfrak{I}))]],\\
&=-\sin\psi\frac{\mathfrak{I}^{\delta}}{\Gamma(\delta+1)},\\
\rho^{2}:\varphi_{1}(\psi,\mathfrak{I})&=\mathbb{Y}^{-1}[\varpi^{\delta}\mathbb{Y}[L(\varphi_{1}(\psi,\mathfrak{I}))]],\\
&=\sin\psi\frac{\mathfrak{I}^{2\delta}}{\Gamma(2\delta+1)},\\
&\vdots
\end{aligned} \tag{4.26}$$

The analytical series form solution to the given problem is

$$\varphi(\psi,\mathfrak{I})=\sin\psi-\sin\psi\frac{\mathfrak{I}^{\delta}}{\Gamma(\delta+1)}+\sin\psi\frac{\mathfrak{I}^{2\delta}}{\Gamma(2\delta+1)}+\cdots \tag{4.27}$$

The exact solution is $\varphi(\psi,\mathfrak{I})=\sin\psi e^{(-\mathfrak{I})}$.
In contrast, we solve the problem by combining the Yang transformation with the Caputo-Fabrizio operator. Initially, we solve the problem using the Yang transformation:

$$\varphi(\psi,\varpi)=(1+\delta(\varpi-1))\left(\mathbb{Y}\left[\varphi_{\psi\psi\psi\psi}(\psi,\mathfrak{I})\right]\right)+\varpi^{2}\varphi(\psi,0). \tag{4.28}$$

To Eq (4.28), the Yang perturbation transformation method apply. we get

$$\sum_{J=0}^{\infty}\rho^{J}\varphi_{J}(\psi,\varpi)=(1+\delta(\varpi-1))\left(\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^{J}\varphi_{J}(\psi,\mathfrak{I})\right)_{\psi\psi\psi\psi}\right]\right)+\varpi^{2}\varphi(\psi,0). \tag{4.29}$$

Using the inverse Yang transform of the above equation, we get

$$\sum_{J=0}^{\infty}\rho^{J}\varphi_{J}(\psi,\mathfrak{I})=\rho\mathbb{Y}^{-1}\left[(1+\delta(\varpi-1))\mathbb{Y}\left[\left(\sum_{J=0}^{\infty}\rho^{J}\varphi_{J}(\psi,\mathfrak{I})\right)\right]\right]+\mathbb{Y}^{-1}[\varpi^{2}\varphi(\psi,0)]. \tag{4.30}$$

On both sides comparing

$$
\begin{aligned}
\rho^0 : \varphi_0(\psi, \mathfrak{I}) =& \mathbb{Y}^{-1}[\varpi^2 \sin\psi] = \sin\psi, \\
\rho^1 : \varphi_1(\psi, \mathfrak{I}) =& \mathbb{Y}^{-1}\left[(1 + \delta(\varpi - 1))\mathbb{Y}\left[(\varphi_0)_{\psi\psi\psi\psi}\right]\right] \\
=& \sin\psi\Big\{1 + \delta\mathfrak{I} - \delta\Big\}, \\
\rho^2 : \varphi_2(\psi, \mathfrak{I}) =& \mathbb{Y}^{-1}\left[(1 + \delta(\varpi - 1))\mathbb{Y}\left[(\varphi_1)_{\psi\psi\psi\psi}\right]\right] \\
=& \sin\psi\Big\{(1 - \delta)2\delta\mathfrak{I} + (1 - \delta)^2 + \frac{\delta^2\mathfrak{I}^2}{2}\Big\}
\end{aligned}
\tag{4.31}
$$

The series form solution is given as

$$
\varphi(\psi, \mathfrak{I}) = \sin\psi + \sin\psi\Big\{1 + \delta\mathfrak{I} - \delta\Big\} + \sin\psi\Big\{(1 - \delta)2\delta\mathfrak{I} + (1 - \delta)^2 + \frac{\delta^2\mathfrak{I}^2}{2}\Big\} + \cdots, \tag{4.32}
$$

The exact solution is $\varphi(\psi, \mathfrak{I}) = \sin\psi \exp(-\mathfrak{I})$.

The analytical solution the fractional-order regularized long wave equations with the help of homotopy perturbation transform method. Example 4.3 is graphical simulation is shown in Figure 3 (a) the exact and (b) the analytical solution at $\alpha = 1$. Figure 2 is a graphical depiction of the solution acquired by the offered methods and the exact outcome of Example 4.3. Figure 3, (c) and (d) show that depicts the results of proposed techniques with different fractional order δ, respectively.

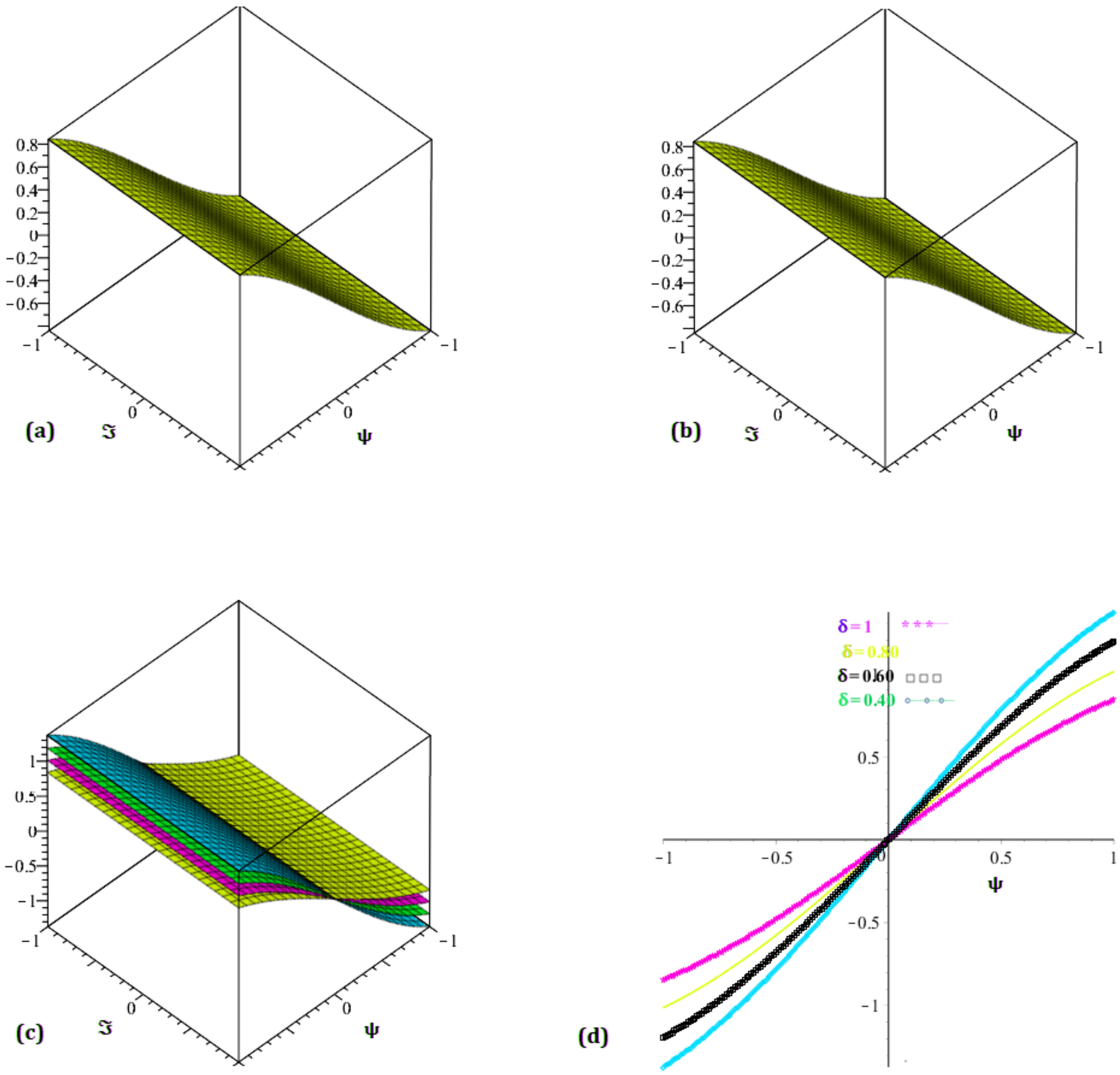

Figure 3. Example 4.3 solutions figure (a) Actual result, (b) Analytic result at $\delta = 1$, (c) Analytic result at different fractional-order of δ (d) $\Im = 0.5$.

5. Conclusions

In this study, the Yang transform, a modified integral transformation method, is used to determine the estimated solutions to a set of regularized fractional-order long-wave equations. We resolved the mentioned issues starting with the fractional Caputo-Fabrizio operator Yang transform. The capacity of the used system to provide a suitable convergence area for the outcome determines its dependability and efficacy. The suggested method's superiority to existing numerical approaches is shown by the findings' high accuracy and simplicity. Additionally, we have shown how the Caputo and Caputo-Fabrizio fractional operators vary when it comes to examining analytical solutions to the example problems. We presented the data in graphs to demonstrate the suggested method's accuracy.

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Conflict of interest

The authors declare that there are no conflicts of interest.

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