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Research article

Some new results of difference perfect functions in topological spaces

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Abstract: Everyday problems are characterized by voluminous data and varying levels of ambiguity. Thereupon, it is critical to develop new mathematical approaches to dealing with them. In this context, the perfect functions are anticipated to be the best instrument for this purpose. Therefore, we investigate in this paper how to generate perfect functions using a variety of set operators. Symmetry is related to the interactions among specific types of perfect functions and their classical topologies. We can explore the properties and behaviors of classical topological concepts through the study of sets, thanks to symmetry. In this paper, we introduce a novel class of perfect functions in topological spaces that we term D-perfect functions and analyze them. Additionally, we establish the links between this new class of perfect functions and classes of generalized functions. Furthermore, while introducing the herein proposed D-perfect functions and analyzing them, we illustrate this new idea, explicate the associated relationships, determine the conditions necessary for their successful application, and give examples and counter-examples. Alternative proofs for the Hausdorff topological spaces and the D-compact topological spaces are also provided. For each of these functions, we examine the images and inverse images of specific topological features. Lastly, product theorems relating to these concepts have been discovered.

Keywords: topological spaces; *D*-set; *D*-compact space; locally indiscrete space; *D*-perfect functions
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1. Introduction

In recent years, some generalized topological structures have been proposed. Because of importance of the topological space in analysis and in a variety of applications, see [20–23]. The perfect functions stand as one of the most important generalizations of the topological space. We know from general topology that the open sets play a critical role in creation of new forms of sets and key topological properties of the new sets. Tong in [19] developed the concept of difference sets (or D-sets) using open sets in 1982, used these new sets to formulate and define a new set of separation axioms, termed D_k $(k = 0, 1, 2)$ spaces. Subsequently, many scholars developed these concepts, including (in 1997) Caldas [5], Jafari [11], Caldas et al. [6], Ekici and Jafari [7], Keskin and Noiri [13], Balasubramanian [3], Balasubramanian and Lakshim [4], Sreeja and Janaki [18], Gnanachandra and Thangavelu [10], Padma et al. [16]. In the field of locally-compact spaces, the perfect functions were first introduced in 1952 by Vainstein [24]. However, in the field of metric spaces, the class of perfect functions was developed and introduced for the first time in 1952 by Leray [19]. Unless otherwise stated, $(O, \varkappa)$ and (P, ϱ) or (O and P) refer to topological spaces with no separation axioms are assumed throughout the work. The letters $\varkappa$-closure and $\varkappa$-interior of a set R are replaced in this study with $CL(R)$ and $Int(R)$, respectively. The product of $\varkappa_1$ and $\varkappa_2$ will be replaced with $\varkappa_1 \times \varkappa_2$. The rest of this paper is organized in five sections. Section 2 presents the basic definitions that are used in this study. Section 3 discusses the topological properties of the D-perfect functions and the link between the D-perfect functions and the perfect functions. Then, Section 4 discusses some of the more advanced properties of the D-perfect functions. Lastly, Section 5 highlights some of the characteristics of the operation of cartesian multiplication of D-perfect functions under special circumstances.

2. Preliminaries

In this section, we state basic definitions and theorems that we will use in this paper to prove our main results. We start with the definition of the D-set.

Definition 2.1. [19] A subset R of a topological space $(O, \varkappa)$ is called a D-set if there are two open sets F and G such that $F \neq O$ and $R = F - G$. In this case, we say that R is a D-set generated by F and G.

Every open set F is a D-set.

Definition 2.2. [17] A cover $\tilde{D} = \{D_\kappa : \kappa \in \Upsilon\}$ of a topological space $(O, \varkappa)$ is said to be D-cover if each D_κ is a D-set for all $\kappa \in \Upsilon$.

Definition 2.3. [17] A topological space $(O, \varkappa)$ is called D-compact if every D-cover of the space $(O, \varkappa)$ has a finite subcover.

Definition 2.4. [9] A space $(O, \varkappa)$ is said to be locally indiscrete if every open set is clopen.

Definition 2.5. [9] Let $(O, \varkappa)$ be a topological space, and let $F \subseteq \varkappa$ be a collection of open subsets of O. We say F is an open cover of O if $O = \bigcup F$.

Definition 2.6. [8] A topological space $(O, \varkappa)$ is said to be compact if every open cover O has a finite subcover.

Definition 2.7. [5] A topological space $(O, \varkappa)$ is Hausdorff if for any $o,\ p \in O$ with $o \neq p$, there exist open sets F and G containing o and p, respectively such that $F \cap G = \phi$.

Definition 2.8. [8] A topological space $(O, \varkappa)$ is said to be locally compact if every point has an open nieghbourhood with compact closure.

Definition 2.9. [9] Let $(O, \varkappa)$ and (P, ϱ) be topological spaces. A function $\Gamma : O \to P$ is said to be continuous, if the inverse image of every open subset of P is open in O.

In other words, if $G \in \varrho$, then its inverse image $\Gamma^{-1}(G) \in \varkappa$.

Definition 2.10. [5] Let $(O, \varkappa)$ and (P, ϱ) be topological spaces. A function $\Gamma : O \to P$ is said to be open if, for any open set F in O, the image $\Gamma(F)$ is open in P.

Definition 2.11. [8] Let $(O, \varkappa)$ and (P, ϱ) be topological spaces. A function $\Gamma : O \to P$ is said to be closed if, for any closed set F in O, the image $\Gamma(F)$ is closed in P.

Definition 2.12. [5] A topological space $(O, \varkappa)$ is said to be countably compact if every countable open cover of O has a finite subcover.

Definition 2.13. [5] A topological space $(O, \varkappa)$ is said to be a paracompact space if every open cover of O has a locally finite open refinement.

Definition 2.14. [19] A continuous function $\Gamma : (O, \varkappa) \longrightarrow (P, \varrho)$ is said to be perfect if Γ is closed and the fibers $\Gamma^{-1}(p)$ are compact subsets of O.

Definition 2.15. [17] A topological space $(O, \varkappa)$ is called D-countably compact if every countable D-cover of the space $(O, \varkappa)$ has a finite subcover.

Theorem 2.16. *[17] The continuous image of a D-compact space is D-compact.*

Theorem 2.17. *[17] Every D-compact space is compact.*

Theorem 2.18. *[17] Let $(O, \varkappa)$ be a topological space and $M \subseteq O$, then $(M, \varkappa_M)$ is D-compact if and only if every cover of M by D-sets in O has a finite subcover.*

Theorem 2.19. *[17] Every closed subspace of a D-compact space is D-compact.*

Theorem 2.20. *[17] Any D-compact subset of a T_2-space is closed.*

3. A new classification of perfect functions

In this section, we introduce the D-perfect functions in topological spaces and we show their links with other spaces.

Definition 3.1. A function $\Gamma : (O, \varkappa) \to (P, \varrho)$ is called D-perfect, if Γ is continuous, closed, and for each $p \in P$, $\Gamma^{-1}(p)$ is D-compact.

The ideas of definition and the relationship between perfect and D-perfect functions are explained in the following examples.

Example 3.1. Let $O = \mathbb{R}$ and $\varkappa = \{\phi, \mathbb{R}, \{0\}, \mathbb{R} - \{0\}\}$, if $\Gamma : (O, \varkappa) \to (O, \varkappa)$ is the identity function, then Γ is D-perfect function.

Since, Γ is continuous, closed, and for each $p \in P$ any D-cover $\hat{F}$ of $\Gamma^{-1}(p)$ has a finite subcover, so $(\mathbb{R}, \varkappa)$ a D-compact space. Hence Γ is D-perfect function.

Example 3.2. Let $X = \{a, b, c\}$, $\iota_X = \{\phi, \{a, b, c\}, \{a, b\}, \{c\}\}$. If the function $\Gamma : (X, \iota_X) \to (\mathbb{R}, \iota_u)$ is defined by $\Gamma(a) = \Gamma(b) = 1$ and $\Gamma(c) = 2$, then Γ is continuous, closed and D-perfect function, because for every $p \in \mathbb{R}$, we have $\Gamma^{-1}(p)$ is a finite set, hence D-compact.

Theorem 3.2. *If the function $\Gamma : (O, \varkappa) \to (P, \varrho)$ is a D-perfect functions, then Γ is a perfect function, but the converse need not be true.*

Proof. It is obvious, that Γ is continuous, closed, (and by using Theorem 2.17) and for each $p \in P$, $\Gamma^{-1}(p)$ is D-compact space, then $\Gamma^{-1}(p)$ is compact. Hence Γ is a perfect function. □

The following example shows that the converse of Theorem 3.2 is not true in general.

Example 3.3. Let $\Gamma : \left(\mathbb{R}, \varkappa_{cof}\right) \to (\{a, b\}, \varkappa_{discrete})$, be defined by $\Gamma(p) = a$, for all $p \in \mathbb{R}$. Then Γ is continuous, closed and perfect function, but not D-perfect function, because $\Gamma^{-1}(a) = \mathbb{R}$ which is compact but not D-compact (see Example 3.14 in [17]).

4. Some *D*-perfect function theorems

In this section, we present more results topological features of the D-perfect functions and show the main link between these functions.

Theorem 4.1. *If $\Gamma : (O, \varkappa) \to (P, \varrho)$ is a D-perfect function and $(O, \varkappa)$ is locally indiscrete, then O is D-compact, if P is a D-compact.*

Proof. Let $\hat{Q} = \{Q_\kappa : \kappa \in \Upsilon\}$ be a D-cover of O. Since Γ is a D-perfect function, then for any $p \in P$, $\Gamma^{-1}(p)$ is a D-compact subset of O. So there exists a finite subset ϱ_p of Υ, such that $\Gamma^{-1}(p) \subseteq \bigcup\limits_{\kappa \in \varrho_p} G_\kappa$. Also, $\hat{Q}$ is an open cover of O. Now, let $L_p = P - \Gamma(O - \bigcup\limits_{\kappa \in \varrho_p} G_\kappa)$ is a D-open subset of P containing p. Since $\Gamma^{-1}(L_p) \subseteq \bigcup\limits_{\kappa \in \varrho_p} G_\kappa$, then $\tilde{L} = \{L_p : p \in P\}$ is a D-open cover of P. Since P is D-compact, $\tilde{L}$ has a finite subcover $\hat{G} = \{L_{p_1}, L_{p_2}, ...L_{p_n}\}$, such that $P \subseteq \bigcup\limits_{i=1}^{n} L_{p_i}$. Thus, $O = \Gamma^{-1}(P) \subseteq \Gamma^{-1}(\bigcup\limits_{i=1}^{n} L_{p_i}) \subseteq \bigcup\limits_{i=1}^{n} \Gamma^{-1}(L_{p_i})$. Hence O is covered by a finite sets which are subsets of the union of a finite numbers of members of $\hat{Q}$. Hence, O is D-compact. □

The same argument in the proof of Theorem 4.1, leads to the following corollary.

Corollary 4.2. *The composition of two D-perfect functions is a D-perfect function.*

Proposition 4.3. *If the composition $\Theta \circ \Gamma$ of continuous functions, $\Gamma : (O, \varkappa) \overset{onto}{\to}$*

(P, ϱ) and $\Theta : (P, \varrho) \overset{onto}{\to} (J, \vartheta)$ is closed, then the function $\Theta : (P, \varrho) \overset{onto}{\to} (J, \vartheta)$ is closed.

Proof. Let R be a closed subset of P, then $\Gamma^{-1}(R)$ is a closed subset of O. Since $\Theta \circ \Gamma$ is closed, then $\Theta(\Gamma\left(\Gamma^{-1}(R)\right)) = \Theta(R)$ is a closed subset of J. Thus Θ is closed. □

Theorem 4.4. *If the composition function $\Theta \circ \Gamma$ of continuous funcions $\Gamma : (O, \varkappa) \overset{onto}{\to} (P, \varrho)$ and $\Theta : (P, \varrho) \overset{onto}{\to} (J, \vartheta)$ is a D-perfect, then the function $\Theta : (P, \varrho) \overset{onto}{\to} (J, \vartheta)$ is D-perfect.*

Proof. For every $j \in J$, $\Theta^{-1}(j) = \Gamma((\Theta \circ \Gamma)^{-1}(j))$ is a D-compact subset of P, because $\Theta \circ \Gamma$ is a D-perfect. Since Θ is a closed by Proposition 4.3, we get that Θ is D-perfect. □

Theorem 4.5. *If $\Gamma : (O, \varkappa) \overset{onto}{\rightarrow} (P, \varrho)$ is a closed function, then for any $M \subset P$ the restriction $\Gamma_M : \Gamma^{-1}(M) \rightarrow M$ is closed.*

Proof. Let $M \subset P$. Consider the function $\Gamma : (O, \varkappa) \rightarrow (P, \varrho)$. Let R be a closed subset of O. Then $\Gamma_M (R \bigcap \Gamma^{-1}(M)) = \Gamma(R) \bigcap M$ is a closed subset of M. Thus $\Gamma_M : \Gamma^{-1}(M) \rightarrow M$ is closed. □

The proof of the following theorem follows directly from Theorems 4.5 and 2.18.

Theorem 4.6. *If $\Gamma : (O, \varkappa) \overset{onto}{\rightarrow} (P, \varrho)$ is a D-perfect function, then for any $M \subset P$ the restriction $\Gamma_M : \Gamma^{-1}(M) \rightarrow M$ is a D-perfect.*

Theorem 4.7. *Let $\Gamma : (O, \varkappa) \rightarrow (P, \varrho)$ be a continuous bijection function. If (P, ϱ) is a Hausdörff space, and $(O, \varkappa)$ is a D-compact, then Γ is a homeomorphism function.*

Theorem 4.8. *If $\Gamma : (O, \varkappa) \rightarrow (P, \varrho)$ is a D-perfect function, where $(O, \varkappa)$ is a D-compact, and (P, ϱ) is Hausdörff, then Γ is closed.*

Proof. If R is a closed subset of $(O, \varkappa)$, then it is D-compact because $(O, \varkappa)$ is D-compact. Since Γ is continuous, $\Gamma(R)$ is D-compact subset of (P, ϱ). Since (P, ϱ) is Hausdörff, then $\Gamma(R)$ is a closed subset of (P, ϱ). Hence we give the result. □

Theorem 4.9. *Let $\Gamma : (O, \varkappa) \rightarrow (P, \varrho)$ be a bijective continuous function. If (P, ϱ) is Hausdörff, D-locally compact space, and $(O, \varkappa)$ is locally indiscrete space, Then the following are equivalent:*

(A) Γ is a D-perfect function.
(B) For every D-compact subset $J \subset P$ the set $\Gamma^{-1}(J)$ is a D-compact subset of O.

Proof. $(A) \Rightarrow (B)$: The proof follows from Theorem 4.1. $(B) \Rightarrow (A)$: It suffices to show that $\Gamma : (O, \varkappa) \rightarrow (P, \varrho)$ is a closed function. Let R be a closed subset of O, and p be a cluster point of $\Gamma(R)$. Suppose $p \notin \Gamma(R)$. Since P is D-locally compact, there is a D-set G containing p such that $CL(G)$ is D-compact. Now, $\Gamma^{-1}(CL(G) \bigcap \Gamma(R)) = \Gamma^{-1}(CL(G)) \bigcap R$. By using (B) $\Gamma^{-1}(CL(G))$ is D-compact and $\Gamma^{-1}(CL(G)) \bigcap R$ is a closed, D-compact subset of O, we obtain $\Gamma(\Gamma^{-1}(CL(G)) \bigcap R) = CL(G) \bigcap \Gamma(R)$ is a D-compact subset that closed of P. Now, $P - (CL(G) \bigcap \Gamma(R)) = F$ is open set containing p and $F \bigcap \Gamma(R) = \phi$, which contradicts the fact that p is a cluster point of $\Gamma(R)$. Hence $p \in \Gamma(R)$ and $\Gamma(R)$ is closed. Thus $\Gamma : (O, \varkappa) \rightarrow (P, \varrho)$ is a closed function. □

Definition 4.10. A function $\Gamma : (O, \varkappa) \rightarrow (P, \varrho)$ is called a strongly function, if for every open cover $\hat{Q} = \{Q_\kappa : \kappa \in \Upsilon\}$ of O there exists an open cover $\hat{G} = \{G_\kappa : \kappa \in \Lambda\}$ of P,
such that $\Gamma^{-1}(G) \subseteq \bigcup \{Q_\kappa : \kappa \in \Omega, \ \Omega \text{ is a finite subset of } \Upsilon\}$, for all $G \in \hat{G}$.

Theorem 4.11. *Let $\Gamma : (O, \varkappa) \rightarrow (P, \varrho)$ be a strongly onto function. If $(O, \varkappa)$ is locally indiscrete, then $(O, \varkappa)$ is D-compact, if (P, ϱ) is so.*

Proof. Let $\hat{Q} = \{Q_\kappa : \kappa \in \Upsilon\}$ be a D-cover of $(O, \varkappa)$. Since Γ is a strongly function, there exists an open cover $\hat{G} = \{G_\alpha : \alpha \in \Lambda\}$ of (P, ϱ), such that $\Gamma^{-1}(G) \subseteq \bigcup\{Q_\kappa : \kappa \in \Omega, \Omega \text{ is a finite subset of } \Upsilon\}$, for all $G \in \hat{G}$, but (P, ϱ) is D-compact, so there exists a finite subset Λ_1 of Λ, such that $P = \bigcup_{\alpha\in\Lambda_1} G_\alpha$. Hence, $O = \bigcup_{\alpha\in\Lambda_1} \Gamma^{-1}(G_\alpha)$. So each $\Gamma^{-1}(G_\alpha)$ contains in the union of a finite number of members of $\hat{Q}$. Thus O is a D-compact . □

5. Some characterisations of *D*-perfect functions

This section highlights some sophisticated properties of the *D*-perfect functions and some of the peculiarities of the Cartesian process of multiplication of these functions in unusual situations.

Theorem 5.1. *Let $\Gamma : (O, \varkappa) \to (P, \varrho)$ be a D-perfect function such that P is countable set and $(O, \varkappa)$ is locally indiscrete. If (P, ϱ) is a D-countably compact, then $(O, \varkappa)$ is so.*

Proof. Let $\hat{Q} = \{Q_\kappa : \kappa \in \Upsilon\}$ be a countable D-cover of $(O, \varkappa)$ since Γ is a D-perfect function, then for each $p \in P$, $\Gamma^{-1}(p)$ is a *D*-compact subset of O. So there exists a finite subsets ϱ_p of Υ, such that $\Gamma^{-1}(p) \subseteq \bigcup_{\kappa\in\varrho_p} G_\kappa$. Note that $G_\kappa$is D-open subset of O for $\kappa \in \varrho_p$,because $(O, \varkappa)$ is locally indiscrete. Now, $L_p = P - \Gamma(O - \bigcup_{\kappa\in\varrho_p} G_\kappa)$ is a D-set containing p. Also, $\Gamma^{-1}(L_p) \subseteq \bigcup_{\kappa\in\varrho_p} G_\kappa$. Thus, $\tilde{L} = \{L_p : p \in P\}$, is a countable D-cover of P, since (P, ϱ) is D-countably compact, then $\tilde{L}$ has a finite subcover say: $\tilde{L}^* = \{L_{p_1}, L_{p_2}, ...L_{p_n}\}$. Thus, $O = \Gamma^{-1}(P) \subseteq \Gamma^{-1}(\bigcup_{i=1}^{n} L_{p_i}) \subseteq \bigcup_{i=1}^{n} \Gamma^{-1}(L_{p_i})$. Hence O is covered by a finite sets which are subsets of the union of a finite numbers of members of $\hat{Q}$. Hence $(O, \varkappa)$ is a D-countably compact. □

The following theorem shows that paracompactness is an inverse invariant under D-perfect function.

Theorem 5.2. *Let $\Gamma : (O, \varkappa) \to (P, \varrho)$ be D-perfect function. If (P, ϱ) is paracompact space, then $(O, \varkappa)$ is so.*

Proof. Let $\hat{Q} = \{Q_\kappa : \kappa \in \Upsilon\}$ be an open cover of $(O, \varkappa)$, hence $\hat{Q}$ is D-cover of O. Since Γ is a D-perfect function, then for any $p \in P$, $\Gamma^{-1}(p)$ is a D-compact, there exist a finite subsets ϱ_p of Υ, such that $\Gamma^{-1}(p) \subseteq \bigcup_{\kappa\in\varrho_p} G_\kappa$. Let $L_p(\kappa, p) = P - \Gamma(O - \bigcup_{\kappa\in\varrho_p} G_\kappa)$ is open set containing p, where $\Gamma^{-1}(L_p) \subseteq \bigcup_{\kappa\in\varrho_p} G_\kappa$. Now, $\tilde{L} = \{L_p(\kappa, p) : p \in P\}$ is open cover of P. Since (P, ϱ) is a paracompact, $\tilde{L}$ has an open locally finite parallel refinement let us say $\grave{C} = \{C_M : M \in \Delta\}$. Let $T = \{\Gamma^{-1}(C_M) \bigcap G_\kappa : M \in \Delta, \kappa \in \varrho_p\}$ then T is an open locally finite parallel refinement of $\hat{Q}$. Hence, $(O, \varkappa)$ is paracompact space. □

Theorem 5.3. *The Hausdorffness is invariant under D-perfect onto function.*

Proof. Let $(O, \varkappa)$ be a Hausdorff space, $\Gamma : (O, \varkappa) \to (P, \varrho)$ be a *D*-perfect onto function, and $p_1, p_2 \in P$, such that $p_1 \neq p_2$, then $\Gamma^{-1}(p_1)$,$\Gamma^{-1}(p_2)$ are disjoint and D-compact ness subset of $(O, \varkappa)$. Since $(O, \varkappa)$ is a Hausdroff space, there exist neighborhoods F, G in O, and such that $\Gamma^{-1}(p_1) \subseteq F$, $\Gamma^{-1}(p_1) \subseteq G$, and $F \bigcap G = \phi$. Now, the sets $P - \Gamma(O - F)$ is an open subset in P containing p_1 and $P - \Gamma(O - F)$ is an open subset in P containing p_2, such that $[P - \Gamma(O - F) \bigcap P - \Gamma(O - G)] = P - [\Gamma(O - F) \bigcup \Gamma(O - G)] = P - \Gamma(O - F \bigcap G) = P - \Gamma(O) = \phi$. Hence (P, ϱ) is a Hausdorff space. □

Theorem 5.4. *Let $(O, \varkappa)$, (P, ϱ) be any two topological spaces. If $(O, \varkappa)$ is a compact, and (P, ϱ) is D-compact, then the projection function $F : (O \times P, \varkappa \times \varrho) \to (P, \varrho)$ is closed.*

Proof. Since $(O, \varkappa)$ is a compact and (P, ϱ) is D-compact, then $(O \times P, \varkappa \times \varrho)$ is D-compact, so the projection function $F : (O \times P, \varkappa \times \varrho) \to (P, \varrho)$ closed function. □

6. Conclusions

This study investigated the links between the perfect spaces and the D-perfect functions in the topological spaces and the topological spaces that functions generate. The study determined the necessary conditions for harmonizing the D-sets and the locally-indiscrete spaces according to the herein proposed concept of D-perfect functions. We examined the link between these two concepts and characterized them using different sorts of sets. One other objective of this study was to highlights some sophisticated properties of the D-perfect functions and some of the peculiarities of the Cartesian process of multiplication of these functions in unusual situations. In addition, dominant features of these ideas and some instructive cases were thoroughly examined. We pinpointed their primary qualities in general and clarified the necessary criteria for achieving equivalent relationships between them. We discussed their primary characteristics and showed how they interact. Furthermore, the paper underlined the properties of these functions and provided a variety of examples of them. These functions will be a starting point for investigations of the many futures of these functions. Future research may consider exploring further varieties of these functions [1, 2].

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Conflict of interest

We declare that we have no conflict of interest.

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