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Research article

Evaluation of time-fractional Fisher's equations with the help of analytical methods

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Abstract: This article shows how to solve the time-fractional Fisher's equation through the use of two well-known analytical methods. The techniques we propose are a modified form of the Adomian decomposition method and homotopy perturbation method with a Yang transform. To show the accuracy of the suggested techniques, illustrative examples are considered. It is confirmed that the solution we get by implementing the suggested techniques has the desired rate of convergence towards the accurate solution. The main benefit of the proposed techniques is the small number of calculations. To show the reliability of the suggested techniques, we present some graphical behaviors of the accurate and analytical results, absolute error graphs and tables that strongly agree with each other. Furthermore, it can be used for solving fractional-order physical problems in various fields of applied sciences.

Keywords: Yang transform decomposition method; homotopy perturbation Yang transform method; time-fractional Fisher's equation; Caputo operator
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1. Introduction

The history of fractional calculus (FC) is as old as classical calculus which as at least 300 years old and arose from Leibniz's letter to L'Hospital, wherein, for the first time-fractional derivatives were discussed [1]. The researchers did not pay much attention to FC initially, but it attracted researchers' attention due to its large number of science and engineering applications. Many phenomena in engineering and other fields can be well represented by models based on FC or the theory of fractional non-integer order derivatives and integrals. In recent years, there has been a growing interest in FC, as evidenced by various studies, such as regular variation in thermodynamics, blood flow phenomena, biophysics, aerodynamics, electrical circuits, electro-analytical chemistry, control theory and biology [2–7].

In recent years, fractional differential equations have attracted lots of interest due to their numerous applications in physics and engineering. Many important phenomena in acoustics, electromagnetics, electrochemistry, viscoelasticity and material science are well explained by fractional differential equations [8–13]. The accurate and explicit solutions of nonlinear partial differential equations are extremely significant in mathematical sciences, and it is one of the most stimulating and dynamic study topics. Nonlinear differential equations are used to describe the majority of natural occurrences. As a result, scientists from various fields try to solve them. Finding an accurate solution is difficult due to the nonlinear nature of these equations. Although significant progress has been made in developing methods for getting exact solutions to nonlinear equations in the last few decades, the progress made is insufficient. This is because, in our opinion, there is no one ideal way to obtain exact solutions to nonlinear differential equations of both types, and each method has its own set of advantages and disadvantages that are dependent on the researchers' experience. To find a solution, a variety of analytical techniques have been used, such as the natural transform decomposition method [14], optimal homotopy asymptotic method [15], homotopy perturbation method [16], elzaki transform decomposition method [17], reduced differential transform method [18], adomian decomposition method [19], iterative Laplace transform method [20] and laplace variational iteration method [21].

Here we solve the time-fractional Fisher's equation by implementing two methods, namely, the homotopy perturbation Yang transform method (HPYTM) and the Yang transform decomposition method (YTDM). Xiao-Jun Yang was the first to introduce the Yang transform and utilize it for different differential equations solutions with constant coefficients. In contrast, the Adomian decomposition method [22, 23], is a powerful method for solving both linear and nonlinear and homogeneous and nonhomogeneous partial and differential equations as well as integrodifferential equations of integer and non-integer order which gives us convergent series form exact solutions. Also, In 1998 he was the first to introduce the [24, 25]. Later on, the solutions of some nonlinear nonhomogeneous partial differential equations were obtained through this semi-analytical method [26–29]. The solution they obtained is in the form of an infinite sequence that converges rapidly to the exact solutions. Due to its quick results, the method has been further used for solving linear and nonlinear equations. In the present work, we used an approximate analytical technique that combines the Yang transform and HPM, known as the HPYTM. It is confirmed that the proposed methods are very easy to implement to find the analytical solution of the time-fractional Fisher's equation. The proposed techniques and solutions are in good agreement with the exact solution of the targeted problems. The fractional view analysis of the problems is also shown using the suggested techniques. It is demonstrated that the proposed methods can be modified for solving other fractional PDEs and their systems.

In 1937, Fisher [30] and Kolmogorov [31] were first to present Fisher- Kolmogorov-Petrovsky Piscounov (Fisher-KPP) equation; afterward, it was recalled as the Fisher equation. The Fisher equation has various applications in the domain of science and engineering [32–35]. Researchers examined a certain significant generalization of this equation [36–38]. Many reaction-diffusion equations have wavefronts that show a dynamic part in the description of physical, chemical and biological phenomena [39–41]. Mathematical models of reaction-diffusion systems show how the concentration of one or more compounds changes over time. Local chemical reactions turn compounds into one another, while diffusion allows substances to diffuse through the air with the simplest equation in one dimension, as follows:

$$\varphi_\kappa = R\varphi_{\mu\mu} + S(\varphi) \tag{1.1}$$

Here $\varphi(\mu)$ indicates the single material concentration, R denotes the diffusion coefficients and S characterizes all local reactions. In order to get the Fisher equation we will replace $S(\varphi)$ with $\varphi(1-\varphi)$ which is used to define the dispersion of biological populations. The Fisher KPP advection equation is used to model population dynamics in advective environments [42]. Fisher presented the following nonlinear partial differential equation as:

$$\varphi_\kappa = R\varphi_{\mu\mu} + s\varphi(1-\varphi) \tag{1.2}$$

As a model for propagating a mutant gene with an advantageous selection intensity s, the same equation can also be found in nuclear reactor theory, autocatalytic chemical reactions, neurophysiology, Brownian motion and flame propagation. Because of the above applications, the Fisher equation is considered to be the most important equation in engineering.

The rest of the paper is organized as follows. Section 2 presents some basic ideas of FC related to our present study. The implementation of the HPYTM to solve the fractional partial differential equations is given in Section 3, while the methodology of the YTDM is shown in Section 4. In section 5, we derive the solution of fractional order Fisher's equations with the help of the proposed methods. The discussion part of the obtained solutions is presented in Section 6. In the end, a brief conclusion is given.

2. Preliminaries

Here we discuss some basic definitions used in our present work.

2.1. Definition

The fractional Caputo derivative is stated as [43,44]

$$D_\kappa^\wp \varphi(\mu,\kappa) = \frac{1}{\Gamma(k-\wp)}\int_0^\kappa (\kappa-\rho)^{k-\wp-1}\varphi^{(k)}(\mu,\rho)d\rho, \quad k-1<\wp\leq k, \quad k\in N. \tag{2.1}$$

2.2. Definition

Xiao Jun Yang was the first to present the Yang Laplace transform. The Yang transform of a function $\varphi(\kappa)$ is given by $Y\{\varphi(\kappa)\}$ or $M(u)$ as [45]

$$Y\{\varphi(\kappa)\} = M(u) = \int_0^\infty e^{\frac{-\kappa}{u}}\varphi(\kappa)d\kappa, \;\; \kappa>0, \;\; u\in(-\kappa_1,\kappa_2). \tag{2.2}$$

The inverse Yang transform is given as

$$Y^{-1}\{M(u)\} = \varphi(\kappa). \tag{2.3}$$

2.3. Definition

The nth derivatives of the Yang transform [46, 47]

$$Y\{\varphi^n(\kappa)\} = \frac{M(u)}{u^n} - \sum_{k=0}^{n-1} \frac{\varphi^k(0)}{u^{n-k-1}}, \ \ \forall \ n = 1, 2, 3, \cdots \tag{2.4}$$

2.4. Definition

The Yang transform of derivatives with a fractional order is as follows [46, 47]

$$Y\{\varphi^{\wp}(\kappa)\} = \frac{M(u)}{u^{\wp}} - \sum_{k=0}^{n-1} \frac{\varphi^k(0)}{u^{\wp-(k+1)}}, \ \ 0 < \wp \leq n. \tag{2.5}$$

3. Homotopy perturbation Yang transform method

The following equation is considered in order to understand the basic idea of this approach as:

$$D_{\kappa}^{\wp}\varphi(\mu,\kappa) = \mathcal{P}_1[\mu]\varphi(\mu,\kappa) + \mathcal{Q}_1[\mu]\varphi(\mu,\kappa), \ \ 1 < \wp \leq 2, \tag{3.1}$$

where the initial conditions are represented by

$$\varphi(\mu, 0) = \xi(\mu), \ \ \frac{\partial}{\partial \kappa}\varphi(\mu, 0) = \zeta(\mu);$$

additionally $D_{\kappa}^{\wp} = \frac{\partial^{\wp}}{\partial \kappa^{\wp}}$ denotes Caputo‘s derivative and $\mathcal{P}_1[\mu]$ and $\mathcal{Q}_1[\mu]$ represent the linear and nonlinear operators respectively.
Applying the Yang transform, we get

$$Y[D_{\kappa}^{\wp}\varphi(\mu,\kappa)] = Y[\mathcal{P}_1[\mu]\varphi(\mu,\kappa) + \mathcal{Q}_1[\mu]\varphi(\mu,\kappa)], \tag{3.2}$$

$$\frac{1}{u^{\wp}}\{M(u) - u\varphi(0) - u^2\varphi^{'}(0)\} = Y[\mathcal{P}_1[\mu]\varphi(\mu,\kappa) + \mathcal{Q}_1[\mu]\varphi(\mu,\kappa)]. \tag{3.3}$$

From the above equation, we have that

$$M(\varphi) = u\varphi(0) + u^2\varphi^{'}(0) + u^{\wp}Y[\mathcal{P}_1[\mu]\varphi(\mu,\kappa) + \mathcal{Q}_1[\mu]\varphi(\mu,\kappa)]. \tag{3.4}$$

On taking the inverse Yang transform, we get

$$\varphi(\mu,\kappa) = \varphi(0) + \varphi^{'}(0) + Y^{-1}[u^{\wp}Y[\mathcal{P}_1[\mu]\varphi(\mu,\kappa) + \mathcal{Q}_1[\mu]\varphi(\mu,\kappa)]]. \tag{3.5}$$

By applying the perturbation method with the parameter ϵ, it can be defined as

$$\varphi(\mu,\kappa) = \sum_{k=0}^{\infty} \epsilon^k \varphi_k(\mu,\kappa), \tag{3.6}$$

where $\epsilon \in [0, 1]$ which is the perturbation parameter.
The nonlinear terms decomposition is determined as

$$\mathcal{Q}_1[\mu]\varphi(\mu,\kappa) = \sum_{k=0}^{\infty} \epsilon^k H_n(\varphi), \tag{3.7}$$

where He's polynomials are represented by H_n of the form $\varphi_0, \varphi_1, \varphi_2, ..., \varphi_n$, which is calculated as

$$H_n(\varphi_0, \varphi_1, ..., \varphi_n) = \frac{1}{\Gamma(n+1)} D_\epsilon^k \left[Q_1 \left(\sum_{k=0}^{\infty} \epsilon^i \varphi_i \right) \right]_{\epsilon=0}, \tag{3.8}$$

where $D_\epsilon^k = \frac{\partial^k}{\partial \epsilon^k}$.
Substituting (3.7) and (3.8) in (3.5) and establishing the homotopy, we get

$$\sum_{k=0}^{\infty} \epsilon^k \varphi_k(\mu, \kappa) = \varphi(0) + \varphi^{'}(0) + \epsilon \times \left(Y^{-1} \left[u^\wp Y\{\mathcal{P}_1 \sum_{k=0}^{\infty} \epsilon^k \varphi_k(\mu, \kappa) + \sum_{k=0}^{\infty} \epsilon^k H_k(\varphi)\} \right] \right). \tag{3.9}$$

Thus, upon comparing ϵ coefficient of both sides, we have that

$$\begin{aligned}
&\epsilon^0 : \varphi_0(\mu, \kappa) = \varphi(0) + \varphi^{'}(0), \\
&\epsilon^1 : \varphi_1(\mu, \kappa) = Y^{-1} \left[u^\wp Y(\mathcal{P}_1[\mu]\varphi_0(\mu, \kappa) + H_0(\varphi)) \right], \\
&\epsilon^2 : \varphi_2(\mu, \kappa) = Y^{-1} \left[u^\wp Y(\mathcal{P}_1[\mu]\varphi_1(\mu, \kappa) + H_1(\varphi)) \right], \\
&. \\
&. \\
&. \\
&\epsilon^k : \varphi_k(\mu, \kappa) = Y^{-1} \left[u^\wp Y(\mathcal{P}_1[\mu]\varphi_{k-1}(\mu, \kappa) + H_{k-1}(\varphi)) \right], \\
&k > 0, k \in N.
\end{aligned} \tag{3.10}$$

The components $\varphi_k(\mu, \kappa)$ are easily computable. On taking $\epsilon \to 1$, we have that

$$\varphi(\mu, \kappa) = \lim_{M \to \infty} \sum_{k=1}^{M} \varphi_k(\mu, \kappa). \tag{3.11}$$

Thus the series form solution we obtain converges rapidly to the accurate solution.

4. Yang transform decomposition method

The following equation is considered in order to understand the basic idea of this approach as:

$$D_\kappa^\wp \varphi(\mu, \kappa) = \mathcal{P}_1(\mu, \kappa) + Q_1(\mu, \kappa), 1 < \wp \leq 2, \tag{4.1}$$

which has some initial conditions:

$$\varphi(\mu, 0) = \xi(\mu), \quad \frac{\partial}{\partial \kappa}\varphi(\mu, 0) = \zeta(\mu),$$

where $D_\kappa^\wp = \frac{\partial^\wp}{\partial \kappa^\wp}$ is the fractional Caputo derivative of order $\wp$ and $\mathcal{P}_1$ and Q_1 are linear and nonlinear functions, respectively.
Applying the Yang transform, we get:

$$Y[D_\kappa^\wp \varphi(\mu, \kappa)] = Y[\mathcal{P}_1(\mu, \kappa) + Q_1(\mu, \kappa)]. \tag{4.2}$$

By the transformation property of Yang differentiation, we get

$$\frac{1}{u^{\wp}}\{M(u) - u\varphi(0) - u^2\varphi^{'}(0)\} = Y[\mathcal{P}_1(\mu,\kappa) + Q_1(\mu,\kappa)]. \tag{4.3}$$

Equation (4.3) implies that

$$M(\varphi) = u\varphi(0) + u^2\varphi^{'}(0) + u^{\wp}Y[\mathcal{P}_1(\mu,\kappa) + Q_1(\mu,\kappa)]. \tag{4.4}$$

On employing the inverse Yang transform to (4.4), we have that

$$\varphi(\mu,\kappa) = \varphi(0) + \varphi^{'}(0) + Y^{-1}[u^{\wp}Y[\mathcal{P}_1(\mu,\kappa) + Q_1(\mu,\kappa)]. \tag{4.5}$$

The YTDM determines the infinite sequence $\varphi(\mu,\kappa)$ solution as

$$\varphi(\mu,\kappa) = \sum_{m=0}^{\infty}\varphi_m(\mu,\kappa). \tag{4.6}$$

Now, by the Adomian polynomials Q_1 the nonlinear terms can be decomposed as

$$Q_1(\mu,\kappa) = \sum_{m=0}^{\infty}\mathcal{A}_m. \tag{4.7}$$

All forms of nonlinearity of the Adomian polynomials are represented by

$$\mathcal{A}_m = \frac{1}{m!}\left[\frac{\partial^m}{\partial\ell^m}\left\{Q_1\left(\sum_{k=0}^{\infty}\ell^k\mu_k, \sum_{k=0}^{\infty}\ell^k\kappa_k\right)\right\}\right]_{\ell=0}. \tag{4.8}$$

Substituting (4.7) and (4.8) into (4.5), we have that

$$\sum_{m=0}^{\infty}\varphi_m(\mu,\kappa) = \varphi(0) + \varphi^{'}(0) + Y^{-1}u^{\wp}\left[Y\left\{\mathcal{P}_1(\sum_{m=0}^{\infty}\mu_m, \sum_{m=0}^{\infty}\kappa_m) + \sum_{m=0}^{\infty}\mathcal{A}_m\right\}\right]. \tag{4.9}$$

The following terms are described:

$$\varphi_0(\mu,\kappa) = \varphi(0) + \kappa\varphi^{'}(0), \tag{4.10}$$

$$\varphi_1(\mu,\kappa) = Y^{-1}\left[u^{\wp}Y\{\mathcal{P}_1(\mu_0,\kappa_0) + \mathcal{A}_0\}\right],$$

where the generalization for $m \geq 1$ is defined as

$$\varphi_{m+1}(\mu,\kappa) = Y^{-1}\left[u^{\wp}Y\{\mathcal{P}_1(\mu_m,\kappa_m) + \mathcal{A}_m\}\right].$$

5. Applications

In this section, the solutions of some fractional order Fisher's equations are determined by using the YTDM and HPYTM.

5.1. Example

Consider the Fisher equation in the time-fractional domain, as given by

$$\frac{\partial^{\wp}\varphi}{\partial\kappa^{\wp}} = \frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi), \qquad 0 < \wp \leq 1, \tag{5.1}$$

with the initial condition

$$\varphi(\mu, 0) = \gamma.$$

Applying the Yang transform, we get

$$Y\left(\frac{\partial^{\wp}\varphi}{\partial\kappa^{\wp}}\right) = Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right). \tag{5.2}$$

By applying the Yang transform differential property, we get

$$\frac{1}{u^{\wp}}\{M(u) - u\varphi(0)\} = Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right), \tag{5.3}$$

$$M(u) = u\varphi(0) + u^{\wp}Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right). \tag{5.4}$$

On taking the inverse Yang transform, we have that

$$\begin{aligned} \varphi(\mu,\kappa) &= \varphi(0) + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right)\right\}\right], \\ \varphi(\mu,\kappa) &= \gamma + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right)\right\}\right]. \end{aligned} \tag{5.5}$$

By applying the homotopy perturbation technique given in (3.9), we have that

$$\sum_{k=0}^{\infty}\epsilon^k\varphi_k(\mu,\kappa) = \gamma + \epsilon\left(Y^{-1}\left[u^{\wp}Y\left[\left(\sum_{k=0}^{\infty}\epsilon^k\varphi_k(\mu,\kappa)\right)_{\mu\mu} + \sum_{k=0}^{\infty}\epsilon^k\varphi_k(\mu,\kappa) - \sum_{k=0}^{\infty}\epsilon^k H_k(\varphi)\right]\right]\right), \tag{5.6}$$

where He's polynomials are given as

$$H_0(\varphi) = \varphi_0^2, \quad H_1(\varphi) = 2\varphi_0\varphi_1, \quad H_2(\varphi) = 2\varphi_0\varphi_2 + (\varphi_2)^2$$

When we compare the same ϵ power coefficient, we get

$$\begin{aligned}
&\epsilon^0 : \varphi_0(\mu,\kappa) = \gamma, \\
&\epsilon^1 : \varphi_1(\mu,\kappa) = Y^{-1}\left(u^{\wp}Y[\frac{\partial^2\varphi_0}{\partial\mu^2} + \varphi_0 - \varphi_0^2]\right) = \gamma(1-\gamma)\frac{\kappa^{\wp}}{\Gamma(\wp+1)}, \\
&\epsilon^2 : \varphi_2(\mu,\kappa) = Y^{-1}\left(u^{\wp}Y[\frac{\partial^2\varphi_1}{\partial\mu^2} + \varphi_1 - 2\varphi_0\varphi_1]\right) = \gamma(1-\gamma)(1-2\gamma)\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)}, \\
&\epsilon^3 : \varphi_3(\mu,\kappa) = Y^{-1}\left(u^{\wp}Y[\frac{\partial^2\varphi_2}{\partial\mu^2}\varphi_2 - 2\varphi_0\varphi_2 - (\varphi_2)^2 + \varphi_2]\right) = (\gamma - 5\gamma^2 + 8\gamma^3 - 4\gamma^4) \\
&\frac{\kappa^{3\wp}}{\Gamma(3\wp+1)} - (\gamma^2 - 2\gamma^3 + \gamma^4)(\frac{\Gamma(2\wp+1)}{\Gamma(\wp+1)^2})\frac{\kappa^{3\wp}}{\Gamma(3\wp+1)}, \\
&. \\
&. \\
&.
\end{aligned}$$

Thus on taking $\epsilon \to 1$, we can calculate the solution with a convergent series form as

$$\begin{aligned}\varphi(\mu,\kappa) &= \varphi_0(\mu,\kappa) + \varphi_1(\mu,\kappa) + \varphi_2(\mu,\kappa) + \varphi_3(\mu,\kappa) + \cdots \\ &= \gamma + \gamma(1-\gamma)\frac{\kappa^{\wp}}{\Gamma(\wp+1)} + \gamma(1-\gamma)(1-2\gamma)\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)} + (\gamma - 5\gamma^2 + 8\gamma^3 - 4\gamma^4)\frac{\kappa^{3\wp}}{\Gamma(3\wp+1)} \\ &- (\gamma^2 - 2\gamma^3 + \gamma^4)(\frac{\Gamma(2\wp+1)}{\Gamma(\wp+1)^2})\frac{\kappa^{3\wp}}{\Gamma(3\wp+1)} + \cdots\end{aligned}$$

Solution via the YTDM

Applying the Yang transform to (5.1), we get

$$Y\left\{\frac{\partial^{\wp}\varphi}{\partial\kappa^{\wp}}\right\} = Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right]. \tag{5.7}$$

By applying the Yang transform differential property, we get

$$\frac{1}{u^{\wp}}\{M(u) - u\varphi(0)\} = Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right], \tag{5.8}$$

$$M(u) = u\varphi(0) + u^{\wp}Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right]. \tag{5.9}$$

On taking the inverse Yang transform, we have that

$$\begin{aligned}\varphi(\mu,\kappa) &= \varphi(0) + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right)\right\}\right], \\ \varphi(\mu,\kappa) &= \gamma + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi)\right)\right\}\right].\end{aligned} \tag{5.10}$$

The infinite series form solution for the function $\varphi(\mu,\kappa)$ is stated as

$$\varphi(\mu,\kappa) = \sum_{m=0}^{\infty}\varphi_m(\mu,\kappa). \tag{5.11}$$

Thus the nonlinear terms can be defined by the Adomian polynomial $\varphi^2 = \sum_{m=0}^{\infty}\mathcal{A}_m$. Using specific concepts, (5.10) can be rewritten in the following form:

$$\begin{aligned}\sum_{m=0}^{\infty}\varphi_m(\mu,\kappa) &= \varphi(\mu,0) + Y^{-1}\left[u^{\wp}Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + \varphi - \sum_{m=0}^{\infty}\mathcal{A}_m\right]\right], \\ \sum_{m=0}^{\infty}\varphi_m(\mu,\kappa) &= \gamma + Y^{-1}\left[u^{\wp}Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + \varphi - \sum_{m=0}^{\infty}\mathcal{A}_m\right]\right].\end{aligned} \tag{5.12}$$

Now, by applying the Adomian polynomials Q_1 the nonlinear terms can be decomposed as given in (4.8):

$$\mathcal{A}_0 = \varphi_0^2, \quad \mathcal{A}_1 = 2\varphi_0\varphi_1, \quad \mathcal{A}_2 = 2\varphi_0\varphi_2 + (\varphi_2)^2.$$

Thus, upon comparing both sides of (5.12), we get

$$\varphi_0(\mu,\kappa) = \gamma.$$

$m = 0$

$$\varphi_1(\mu,\kappa) = \gamma(1-\gamma)\frac{\kappa^{\wp}}{\Gamma(\wp+1)}.$$

$m = 1$

$$\varphi_2(\mu,\kappa) = \gamma(1-\gamma)(1-2\gamma)\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)}.$$

$m = 2$

$$\varphi_3(\mu,\kappa) = (\gamma - 5\gamma^2 + 8\gamma^3 - 4\gamma^4)$$
$$\frac{\kappa^{3\wp}}{\Gamma(3\wp+1)} - (\gamma^2 - 2\gamma^3 + \gamma^4)(\frac{\Gamma(2\wp+1)}{\Gamma(\wp+1)^2})\frac{\kappa^{3\wp}}{\Gamma(3\wp+1)}.$$

In the same manner, the YTDM remaining elements, as denoted by ρ_m with $m \geq 3$ are easy to calculate. Thus, we define the series of possibilities as follows:

$$\varphi(\mu,\kappa) = \sum_{m=0}^{\infty}\varphi_m(\mu,\kappa) = \varphi_0(\mu,\kappa) + \varphi_1(\mu,\kappa) + \varphi_2(\mu,\kappa) + \varphi_3(\mu,\kappa) + \cdots$$

$$\varphi(\mu,\kappa) = \gamma + \gamma(1-\gamma)\frac{\kappa^{\wp}}{\Gamma(\wp+1)} + \gamma(1-\gamma)(1-2\gamma)\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)} + (\gamma - 5\gamma^2 + 8\gamma^3 - 4\gamma^4)\frac{\kappa^{3\wp}}{\Gamma(3\wp+1)}$$
$$- (\gamma^2 - 2\gamma^3 + \gamma^4)(\frac{\Gamma(2\wp+1)}{\Gamma(\wp+1)^2})\frac{\kappa^{3\wp}}{\Gamma(3\wp+1)} + \cdots$$

Hence the YTDM solution at $\wp = 1$ as follows:

$$\varphi(\mu,\kappa) = \frac{\gamma\exp^{\kappa}}{1-\gamma+\gamma\exp^{\kappa}}. \tag{5.13}$$

5.2. Example

Consider the Fisher equation in time-fractional domain, as given by

$$\frac{\partial^{\wp}\varphi}{\partial\kappa^{\wp}} = \frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi), \quad 0 < \wp \leq 1, \tag{5.14}$$

with the following initial condition:

$$\varphi(\mu,0) = \frac{1}{(1+\exp^{\mu})^2}.$$

Applying the Yang transform, we get

$$Y\left(\frac{\partial^{\wp}\varphi}{\partial\kappa^{\wp}}\right) = Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right). \tag{5.15}$$

By applying the Yang transform differential property, we have that

$$\frac{1}{u^{\wp}}\{M(u) - u\varphi(0)\} = Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right), \tag{5.16}$$

$$M(u) = u\varphi(0) + u^{\wp} Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right). \tag{5.17}$$

On taking the inverse Yang transform, we have that

$$\begin{aligned} \varphi(\mu,\kappa) &= \varphi(0) + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right)\right\}\right], \\ \varphi(\mu,\kappa) &= \frac{1}{(1+\exp^{\mu})^2} + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right)\right\}\right]. \end{aligned} \tag{5.18}$$

By employing the homotopy perturbation technique given in (3.9), we get

$$\sum_{k=0}^{\infty} \epsilon^k \varphi_k(\mu,\kappa) = \frac{1}{(1+\exp^{\mu})^2} + \epsilon\left(Y^{-1}\left[u^{\wp} Y\left[\left(\sum_{k=0}^{\infty} \epsilon^k \varphi_k(\mu,\kappa)\right)_{\mu\mu} + 6\sum_{k=0}^{\infty} \epsilon^k \varphi_k(\mu,\kappa) - 6\sum_{k=0}^{\infty} \epsilon^k H_k(\varphi)\right]\right]\right). \tag{5.19}$$

where He's polynomials are given as

$$H_0(\varphi) = \varphi_0^2, \quad H_1(\varphi) = 2\varphi_0\varphi_1, \quad H_2(\varphi) = 2\varphi_0\varphi_2 + (\varphi_2)^2$$

When we compare the same ϵ power coefficient, we get

$$\begin{aligned} &\epsilon^0 : \varphi_0(\mu,\kappa) = \frac{1}{(1+\exp^{\mu})^2}, \\ &\epsilon^1 : \varphi_1(\mu,\kappa) = Y^{-1}\left(u^{\wp} Y[\frac{\partial^2\varphi_0}{\partial\mu^2} + 6\varphi_0 - 6\varphi_0^2]\right) = 10\frac{\exp^{\mu}}{(1+\exp^{\mu})^3}\frac{\kappa^{\wp}}{\Gamma(\wp+1)}, \\ &\epsilon^2 : \varphi_2(\mu,\kappa) = Y^{-1}\left(u^{\wp} Y[\frac{\partial^2\varphi_1}{\partial\mu^2} + 6\varphi_1 - 12\varphi_0\varphi_1]\right) = 50\frac{\exp^{\mu}(-1+2\exp^{\mu})}{(1+\exp^{\mu})^4}\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)}, \\ &\epsilon^3 : \varphi_3(\mu,\kappa) = Y^{-1}\left(u^{\wp} Y[\frac{\partial^2\varphi_2}{\partial\mu^2}\varphi_2 - 12\varphi_0\varphi_2 - 6(\varphi_2)^2 + 6\varphi_2]\right) = 50\exp^{\mu}(5 - 6\exp^{\mu} - 15\exp^{2\mu} \\ &\quad + 20\exp^{3\mu} - 12\exp^{\mu}\frac{\Gamma(2\wp+1)}{(\Gamma(\wp+1))^2})\frac{\kappa^{3\wp}}{(1+\exp^{\mu})^6\Gamma(3\wp+1)}, \\ &. \\ &. \\ &. \end{aligned}$$

Thus on taking $\epsilon \to 1$, we can calculate the solution with the convergent series form as

$$\begin{aligned} \varphi(\mu,\kappa) &= \varphi_0(\mu,\kappa) + \varphi_1(\mu,\kappa) + \varphi_2(\mu,\kappa) + \varphi_3(\mu,\kappa) + \cdots \\ &= \frac{1}{(1+\exp^{\mu})^2} + 10\frac{\exp^{\mu}}{(1+\exp^{\mu})^3}\frac{\kappa^{\wp}}{\Gamma(\wp+1)} + 50\frac{\exp^{\mu}(-1+2\exp^{\mu})}{(1+\exp^{\mu})^4}\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)} \\ &+ 50\exp^{\mu}(5 - 6\exp^{\mu} - 15\exp^{2\mu} + 20\exp^{3\mu} - 12\exp^{\mu}\frac{\Gamma(2\wp+1)}{(\Gamma(\wp+1))^2})\frac{\kappa^{3\wp}}{(1+\exp^{\mu})^6\Gamma(3\wp+1)} + \cdots \end{aligned}$$

Solution via the YTDM

Applying the Yang transform to (5.14), we get

$$Y\left\{\frac{\partial^{\wp}\varphi}{\partial\kappa^{\wp}}\right\} = Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right]. \tag{5.20}$$

By applying the Yang transform differential property, we have that

$$\frac{1}{u^{\wp}}\{M(u) - u\varphi(0)\} = Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right], \tag{5.21}$$

$$M(u) = u\varphi(0) + u^{\wp} Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right]. \tag{5.22}$$

On taking the inverse Yang transform, we have that

$$\begin{aligned}\varphi(\mu,\kappa) &= \varphi(0) + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right)\right\}\right],\\ \varphi(\mu,\kappa) &= \frac{1}{(1+\exp^{\mu})^2} + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi(1-\varphi)\right)\right\}\right].\end{aligned} \tag{5.23}$$

The infinite series form solution for the function $\varphi(\mu,\kappa)$ is given as

$$\varphi(\mu,\kappa) = \sum_{m=0}^{\infty}\varphi_m(\mu,\kappa). \tag{5.24}$$

Thus the nonlinear terms can be defined by the Adomian polynomial $\varphi^2 = \sum_{m=0}^{\infty}\mathcal{A}_m$. Using specific concepts, Eq (5.23) can be rewritten in the following form:

$$\begin{aligned}\sum_{m=0}^{\infty}\varphi_m(\mu,\kappa) &= \varphi(\mu,0) + Y^{-1}\left[u^{\wp} Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi - 6\sum_{m=0}^{\infty}\mathcal{A}_m\right]\right],\\ \sum_{m=0}^{\infty}\varphi_m(\mu,\kappa) &= \frac{1}{(1+\exp^{\mu})^2} + Y^{-1}\left[u^{\wp} Y\left[\frac{\partial^2\varphi}{\partial\mu^2} + 6\varphi - 6\sum_{m=0}^{\infty}\mathcal{A}_m\right]\right].\end{aligned} \tag{5.25}$$

Now, by applying the Adomian polynomials Q_1 the nonlinear terms can be decomposed as given in (4.8):

$$\mathcal{A}_0 = \varphi_0^2, \quad \mathcal{A}_1 = 2\varphi_0\varphi_1, \quad \mathcal{A}_2 = 2\varphi_0\varphi_2 + (\varphi_2)^2.$$

Thus, upon comparing both sides of (5.25), we have that

$$\varphi_0(\mu,\kappa) = \frac{1}{(1+\exp^{\mu})^2}.$$

$m = 0$

$$\varphi_1(\mu,\kappa) = 10\frac{\exp^{\mu}}{(1+\exp^{\mu})^3}\frac{\kappa^{\wp}}{\Gamma(\wp+1)}.$$

$m = 1$

$$\varphi_2(\mu,\kappa) = 50\frac{\exp^{\mu}(-1+2\exp^{\mu})}{(1+\exp^{\mu})^4}\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)}.$$

$m = 2$

$$\begin{aligned}\varphi_3(\mu,\kappa) &= 50\exp^{\mu}(5 - 6\exp^{\mu} - 15\exp^{2\mu}\\ &+ 20\exp^{3\mu} - 12\exp^{\mu}\frac{\Gamma(2\wp+1)}{(\Gamma(\wp+1))^2})\frac{\kappa^{3\wp}}{(1+\exp^{\mu})^6\Gamma(3\wp+1)}.\end{aligned}$$

In the same manner, the YTDM remaining elements, as denoted by ρ_m with $m \geq 3$ are easy to calculate. Thus, we define the series of possibilities as follows:

$$\varphi(\mu,\kappa) = \sum_{m=0}^{\infty} \varphi_m(\mu,\kappa) = \varphi_0(\mu,\kappa) + \varphi_1(\mu,\kappa) + \varphi_2(\mu,\kappa) + \varphi_3(\mu,\kappa) + \cdots$$

$$\begin{aligned}\varphi(\mu,\kappa) = {} & \frac{1}{(1+\exp^{\mu})^2} + 10\frac{\exp^{\mu}}{(1+\exp^{\mu})^3}\frac{\kappa^{\wp}}{\Gamma(\wp+1)} + 50\frac{\exp^{\mu}(-1+2\exp^{\mu})}{(1+\exp^{\mu})^4}\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)} \\ & + 50\exp^{\mu}(5-6\exp^{\mu} - 15\exp^{2\mu} + 20\exp^{3\mu} - 12\exp^{\mu}\frac{\Gamma(2\wp+1)}{(\Gamma(\wp+1))^2})\frac{\kappa^{3\wp}}{(1+\exp^{\mu})^6\Gamma(3\wp+1)} + \cdots\end{aligned}$$

Hence, the YTDM solution at $\wp = 1$ is given as

$$\varphi(\mu,\kappa) = \frac{1}{(1-\exp^{\mu-5\kappa})^2}. \tag{5.26}$$

5.3. Example

Consider the Fisher equation time-fractional domain, as given by

$$\frac{\partial^{\wp}\varphi}{\partial\kappa^{\wp}} = \frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi^6), \qquad 0 < \wp \leq 1, \tag{5.27}$$

with the following initial condition:

$$\varphi(\mu,0) = \frac{1}{(1+\exp^{\frac{3}{2}\mu})^{\frac{1}{3}}}.$$

Applying the Yang transform, we get

$$Y\left(\frac{\partial^{\wp}\varphi}{\partial\kappa^{\wp}}\right) = Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi^6)\right). \tag{5.28}$$

By applying the Yang transform differential property, we have that

$$\frac{1}{u^{\wp}}\{M(u) - u\varphi(0)\} = Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi^6)\right), \tag{5.29}$$

$$M(u) = u\varphi(0) + u^{\wp}Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi^6)\right). \tag{5.30}$$

On taking the inverse Yang transform, we have that

$$\begin{aligned}\varphi(\mu,\kappa) &= \varphi(0) + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi^6)\right)\right\}\right], \\ \varphi(\mu,\kappa) &= \frac{1}{(1+\exp^{\frac{3}{2}\mu})^{\frac{1}{3}}} + Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2} + \varphi(1-\varphi^6)\right)\right\}\right].\end{aligned} \tag{5.31}$$

By employing the homotopy perturbation technique given in (3.9), we get

$$\sum_{k=0}^{\infty}\epsilon^k\varphi_k(\mu,\kappa)=\frac{1}{(1+\exp^{\frac{3}{2}\mu})^{\frac{1}{3}}}+\epsilon\left(Y^{-1}\left[u^{\wp}Y\left[\left(\sum_{k=0}^{\infty}\epsilon^k\varphi_k(\mu,\kappa)\right)_{\mu\mu}+\sum_{k=0}^{\infty}\epsilon^k\varphi_k(\mu,\kappa)-\sum_{k=0}^{\infty}\epsilon^k H_k(\varphi)\right]\right]\right). \tag{5.32}$$

where He's polynomials are given as

$$H_0(\varphi)=\varphi_0^7,\quad H_1(\varphi)=7\varphi_0^6\varphi_1.$$

When we compare the same ϵ power coefficient, we get

$$\epsilon^0:\varphi_0(\mu,\kappa)=\frac{1}{(1+\exp^{\frac{3}{2}\mu})^{\frac{1}{3}}},$$

$$\epsilon^1:\varphi_1(\mu,\kappa)=Y^{-1}\left(u^{\wp}Y[\frac{\partial^2\varphi_0}{\partial\mu^2}+\varphi_0-\varphi_0^7]\right)=\frac{5\exp^{\frac{3}{2}\mu}}{4(1+\exp^{\frac{3}{2}\mu})^{\frac{4}{3}}}\frac{\kappa^{\wp}}{\Gamma(\wp+1)},$$

$$\epsilon^2:\varphi_2(\mu,\kappa)=Y^{-1}\left(u^{\wp}Y[\frac{\partial^2\varphi_1}{\partial\mu^2}+\varphi_1-7\varphi_0^6\varphi_1]\right)=\frac{25\exp^{\frac{3}{2}\mu}(\exp^{\frac{3}{2}\mu}-3)}{16(1+\exp^{\frac{3}{2}\mu})^{\frac{7}{3}}}\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)},$$

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Thus on taking $\epsilon\to 1$, we can calculate the solution with the convergent series form as

$$\begin{aligned}\varphi(\mu,\kappa)&=\varphi_0(\mu,\kappa)+\varphi_1(\mu,\kappa)+\varphi_2(\mu,\kappa)+\cdots\\&=\frac{1}{(1+\exp^{\frac{3}{2}\mu})^{\frac{1}{3}}}+\frac{5\exp^{\frac{3}{2}\mu}}{4(1+\exp^{\frac{3}{2}\mu})^{\frac{4}{3}}}\frac{\kappa^{\wp}}{\Gamma(\wp+1)}+\frac{25\exp^{\frac{3}{2}\mu}(\exp^{\frac{3}{2}\mu}-3)}{16(1+\exp^{\frac{3}{2}\mu})^{\frac{7}{3}}}\frac{\kappa^{2\wp}}{\Gamma(2\wp+1)}+\cdots\end{aligned}$$

Solution via the YTDM

Applying the Yang transform to (5.27), we get

$$Y\left\{\frac{\partial^{\wp}\varphi}{\partial\kappa^{\wp}}\right\}=Y\left[\frac{\partial^2\varphi}{\partial\mu^2}+\varphi(1-\varphi^6)\right]. \tag{5.33}$$

By applying the Yang transform differential property, we have that

$$\frac{1}{u^{\wp}}\{M(u)-u\varphi(0)\}=Y\left[\frac{\partial^2\varphi}{\partial\mu^2}+\varphi(1-\varphi^6)\right], \tag{5.34}$$

$$M(u)=u\varphi(0)+u^{\wp}Y\left[\frac{\partial^2\varphi}{\partial\mu^2}+\varphi(1-\varphi^6)\right]. \tag{5.35}$$

On taking the inverse Yang transform, we have that

$$\begin{aligned}\varphi(\mu,\kappa)&=\varphi(0)+Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2}+\varphi(1-\varphi^6)\right)\right\}\right],\\\varphi(\mu,\kappa)&=\frac{1}{(1+\exp^{\frac{3}{2}\mu})^{\frac{1}{3}}}+Y^{-1}\left[u^{\wp}\left\{Y\left(\frac{\partial^2\varphi}{\partial\mu^2}+\varphi(1-\varphi^6)\right)\right\}\right].\end{aligned} \tag{5.36}$$

The infinite series form solution for the function $\varphi(\mu, \kappa)$ is stated as

$$\varphi(\mu, \kappa) = \sum_{m=0}^{\infty} \varphi_m(\mu, \kappa). \tag{5.37}$$

Thus the nonlinear terms can be defined by the Adomian polynomial $\varphi^2 = \sum_{m=0}^{\infty} \mathcal{A}_m$. Using specific concepts, (5.36) can be rewritten in the following form:

$$\begin{aligned} \sum_{m=0}^{\infty} \varphi_m(\mu, \kappa) &= \varphi(\mu, 0) + Y^{-1}\left[u^{\wp} Y\left[\frac{\partial^2 \varphi}{\partial \mu^2} + \varphi - \sum_{m=0}^{\infty} \mathcal{A}_m\right]\right], \\ \sum_{m=0}^{\infty} \varphi_m(\mu, \kappa) &= \frac{1}{(1 + \exp^{\frac{3}{2}\mu})^{\frac{1}{3}}} + Y^{-1}\left[u^{\wp} Y\left[\frac{\partial^2 \varphi}{\partial \mu^2} + \varphi - \sum_{m=0}^{\infty} \mathcal{A}_m\right]\right]. \end{aligned} \tag{5.38}$$

Now, by applying the Adomian polynomials Q_1 the nonlinear terms can be decomposed as given in (4.8):

$$\mathcal{A}_0 = \varphi_0^7, \quad \mathcal{A}_1 = 7\varphi_0^6 \varphi_1.$$

Thus, upon comparing both sides of (5.38), we have that

$$\varphi_0(\mu, \kappa) = \frac{1}{(1 + \exp^{\frac{3}{2}\mu})^{\frac{1}{3}}}.$$

$m = 0$

$$\varphi_1(\mu, \kappa) = \frac{5\exp^{\frac{3}{2}\mu}}{4(1 + \exp^{\frac{3}{2}\mu})^{\frac{4}{3}}} \frac{\kappa^{\wp}}{\Gamma(\wp + 1)}.$$

$m = 1$

$$\varphi_2(\mu, \kappa) = \frac{25\exp^{\frac{3}{2}\mu}(\exp^{\frac{3}{2}\mu} - 3)}{16(1 + \exp^{\frac{3}{2}\mu})^{\frac{7}{3}}} \frac{\kappa^{2\wp}}{\Gamma(2\wp + 1)}.$$

In the same manner, the YTDM remaining elements, as denoted by ρ_m with $m \geq 2$ are easy to calculate. Thus, we define the series of possibilities as follows:

$$\varphi(\mu, \kappa) = \sum_{m=0}^{\infty} \varphi_m(\mu, \kappa) = \varphi_0(\mu, \kappa) + \varphi_1(\mu, \kappa) + \varphi_2(\mu, \kappa) + \cdots$$

$$\varphi(\mu, \kappa) = \frac{1}{(1 + \exp^{\frac{3}{2}\mu})^{\frac{1}{3}}} + \frac{5\exp^{\frac{3}{2}\mu}}{4(1 + \exp^{\frac{3}{2}\mu})^{\frac{4}{3}}} \frac{\kappa^{\wp}}{\Gamma(\wp + 1)} + \frac{25\exp^{\frac{3}{2}\mu}(\exp^{\frac{3}{2}\mu} - 3)}{16(1 + \exp^{\frac{3}{2}\mu})^{\frac{7}{3}}} \frac{\kappa^{2\wp}}{\Gamma(2\wp + 1)} + \cdots$$

Hence, the YTDM solution at $\wp = 1$ is given as

$$\varphi(\mu, \kappa) = \left\{\frac{1}{2}\tanh\left(\frac{15}{8}\kappa - \frac{3}{4}\mu\right) + \frac{1}{2}\right\}^{\frac{1}{3}}. \tag{5.39}$$

6. Results and discussion

Figure 1a shows the exact solution and Figure 1b shows the YTDM and HPYTM solutions at $\wp = 1$ for Example 1 given $0 \leq \kappa \leq 1$. The validity of the proposed methods is demonstrated in Table 1 via a comparison of the exact solution and our method solutions for Example 1. Figure 2a shows the exact solution graph while Figure 2b presents the solution obtained via the proposed methods. Also Figure 2c and 2d respectively show the 3D and 2D solution graphs at different values of $\wp = 1, 0.8, 0.7, 0.6$ for $0 \leq \mu \leq 1$ and $\kappa = 0.5$. Table 2 presents the (AE) comparison between the exact solution and the solutions of our approaches at various fractional orders for Example 2. From the figures and tables, it is clear that the fractional order solution converges to the exact solution as the value of $\wp$ shifts from fractional order to integer order. Graphical views of the exact solution and our method solutions for Example 3 are respectively shown in Figure 3a and 3b. The absolute error graph for the same problem is shown in Figure 4, whereas Table 3 presents the absolute error comparison between the exact solution and the solutions of our approaches at various fractional orders for Example 3. From the figures and tables it is clear that the HPYTM and YTDM solutions are in good Contact with the accurate solutions of the targeted problem.

Table 1. Example 1: Exact solution, proposed technique solutions and AE for $\varphi(\mu, \kappa)$.

$\kappa = 0.001$	Exact solution	Proposed techniques solution	AE of our methods	AE of our methods	AE of our methods
γ	$\wp = 1$	$\wp = 1$	$\wp = 1$	$\wp = 0.9$	$\wp = 0.8$
0	0.000000000000000	0.000000000000000	0.0000000000E+00	0.0000000000E+00	0.0000000000E+00
0.1	0.100000900000000	0.100001000000000	1.0000000000E-07	2.2624000000E-06	9.1005000000E-06
0.2	0.200001600000000	0.200002000000000	4.0000000000E-07	4.7247000000E-06	1.8401000000E-05
0.3	0.300002100000000	0.300003000000000	9.0000000000E-07	7.3870000000E-06	2.7901500000E-05
0.4	0.400002400000000	0.400004000000000	1.6000000000E-06	1.0249300000E-05	3.7602000000E-05
0.5	0.500002500000000	0.500005000000000	2.5000000000E-06	1.3311600000E-05	4.7502500000E-05
0.6	0.600002400000000	0.600006000000000	3.6000000000E-06	1.6574000000E-05	5.7603000000E-05
0.7	0.700002100000000	0.700007000000000	4.9000000000E-06	2.0036200000E-05	6.7903500000E-05
0.8	0.800001600000000	0.800008000000000	6.4000000000E-06	2.3698600000E-05	7.8404000000E-05
0.9	0.900000900000000	0.900009000000000	8.1000000000E-06	2.7561000000E-05	8.9104500000E-05
1.0	1.000000000000000	1.000010000000000	1.0000000000E-05	3.1624000000E-05	1.0000500000E-04

Table 2. Example 2: Exact solution, proposed technique solutions and AE for $\varphi(\mu, \kappa)$.

$\kappa = 0.0001$	Exact solution	Proposed techniques solution	AE of our methods	AE of our methods	AE of our methods
μ	$\wp = 1$	$\wp = 1$	$\wp = 1$	$\wp = 0.9$	$\wp = 0.8$
0	0.250125015600000	0.250125015600000	0.0000000000E+00	1.8906890000E-04	6.6430520000E-04
0.1	0.225763248100000	0.225763245200000	2.9000000000E-09	1.7916820000E-04	6.2951110000E-04
0.2	0.202760871700000	0.202760866000000	5.7000000000E-09	1.6852090000E-04	5.9209510000E-04
0.3	0.181203221400000	0.181203213600000	7.8000000000E-09	1.5733420000E-04	5.5278500000E-04
0.4	0.161148032900000	0.161148023400000	9.5000000000E-09	1.4581750000E-04	5.1231600000E-04
0.5	0.142625699300000	0.142625688500000	1.0800000000E-08	1.3417360000E-04	4.7140140000E-04
0.6	0.125640540500000	0.125640528900000	1.1600000000E-08	1.2259260000E-04	4.3070870000E-04
0.7	0.110172940000000	0.110172927700000	1.2300000000E-08	1.1124550000E-04	3.9083840000E-04
0.8	0.096182157570000	0.096182145000000	1.2570000000E-08	1.0027947000E-04	3.5230901000E-04
0.9	0.083609606820000	0.083609594320000	1.2500000000E-08	8.9816590000E-05	3.1554770000E-04
1.0	0.072382381010000	0.072382368820000	1.2190000000E-08	7.9951180000E-05	2.8088627000E-04

Table 3. Example 3: Exact solution, proposed technique solutions and AE for $\varphi(\mu, \kappa)$.

$\kappa = 0.0001$	Exact solution	Proposed techniques solution	AE of our methods	AE of our methods	AE of our methods
μ	$\wp = 1$	$\wp = 1$	$\wp = 1$	$\wp = 0.9$	$\wp = 0.8$
0	0.793750129200000	0.793750133900000	4.7000000000E-09	7.5012100000E-05	2.6345310000E-04
0.1	0.773431238400000	0.773431242500000	4.1000000000E-09	7.8561700000E-05	2.7591940000E-04
0.2	0.752229901300000	0.752229905000000	3.7000000000E-09	8.1668800000E-05	2.8683080000E-04
0.3	0.730270700500000	0.730270703800000	3.3000000000E-09	18.4278900000E-05	2.9599690000E-04
0.4	0.707690367400000	0.707690370200000	2.8000000000E-09	8.6354800000E-05	3.0328690000E-04
0.5	0.684633165000000	0.684633167100000	2.1000000000E-09	8.7877000000E-05	3.0863230000E-04
0.6	0.661246165700000	0.661246167400000	1.7000000000E-09	8.8844000000E-05	3.1202730000E-04
0.7	0.637674763400000	0.637674764500000	1.1000000000E-09	8.9269700000E-05	3.1352190000E-04
0.8	0.614058694900000	0.614058695500000	6.0000000000E-10	8.9182600000E-05	3.1321500000E-04
0.9	0.590528767000000	0.590528767200000	2.0000000000E-10	8.8621500000E-05	3.1124330000E-04
1.0	0.567204388900000	0.567204388600000	3.0000000000E-10	8.7632900000E-05	3.0777110000E-04

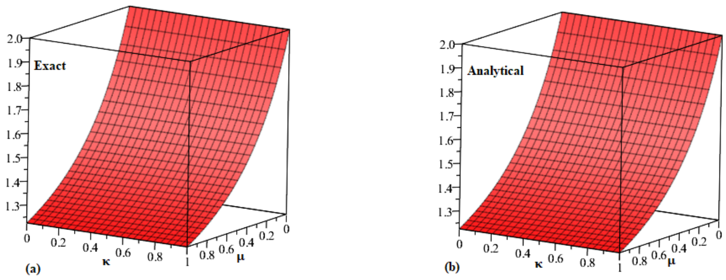

Figure 1. Behavior of Example **1**: (a) Exact and (b) Proposed techniques at $\wp = 1$.

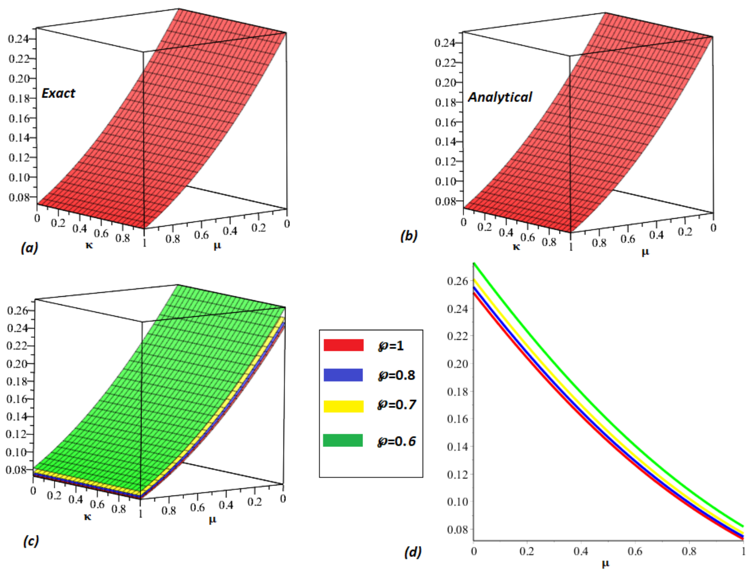

Figure 2. (a) Exact solution, (b) proposed techniques solution, (c)different fractional orders and (d) $\kappa = 0.5$ solution graph for Example **2** at $\wp = 1$.

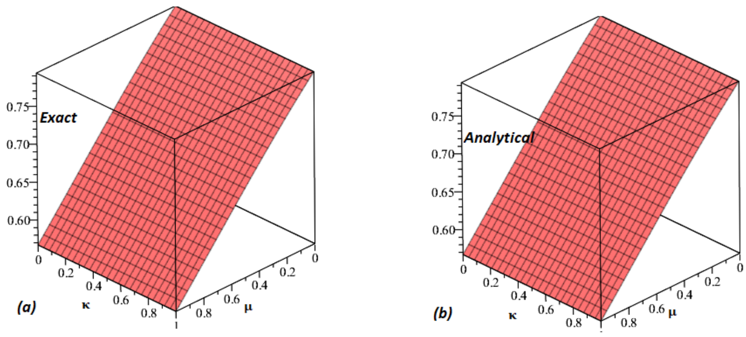

Figure 3. Behavior of Example **3**: (a) Exact and (b) proposed technique solution at $\wp = 1$.

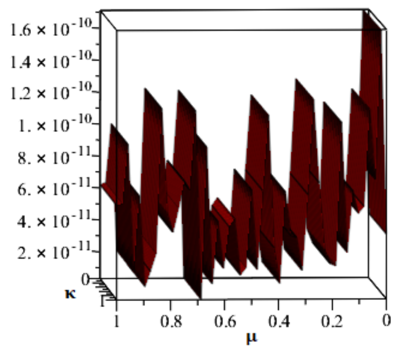

Figure 4. Error graph at $\wp = 1$ for Example **3**.

7. Conclusions

In the present article, a fractional view analysis of the time-fractional Fisher's equation is shown through the use of two different analytical methods. In the Caputo manner, the fractional derivative is taken. The following strategies are shown to be the most effective for solving fractional partial differential equations. The behavior of the graphs and tables confirm the strong agreement between the exact and proposed techniques results. The suggested techniques provide series form solutions with a higher rate of convergence to the exact results. Furthermore, fractional-order problem solutions have been proven to converge to integer-order problem solutions. The reliability of the proposed techniques have been confirmed by the convergence phenomena.

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