AIMS Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(10): 18603–18615.
DOI: 10.3934/math.20221023
Received: 18 March 2022
Revised: 05 July 2022
Accepted: 18 July 2022
Published: 19 August 2022

Research article

Study of power law non-linearity in solitonic solutions using extended hyperbolic function method

Muhammad Imran Asjad[1], Naeem Ullah[1], Asma Taskeen[2] and Fahd Jarad[3,4,5,*]

[1] Department of Mathematics, University of Management and Technology, Lahore, 54770, Pakistan
[2] Department of Mathematics and Statistics, The University of Lahore, Lahore, Pakistan
[3] Department of Mathematics, Cankaya University, 06790 Etimesgut, Ankara, Turkey
[4] Department of Mathematics, King Abdulaziz University, Saudi Arabia
[5] Department of Medical Research, China Medical University, Taichung 40402, Taiwan

* **Correspondence:** Email: fahd@cankaya.edu.tr; Tel: +903122844500239; Fax: +903122868962.

Abstract: This paper retrieves the optical solitons to the Biswas-Arshed equation (BAE), which is examined with the lack of self-phase modulation by applying the extended hyperbolic function (EHF) method. Novel constructed solutions have the shape of bright, singular, periodic singular, and dark solitons. The achieved solutions have key applications in engineering and physics. These solutions define the wave performance of the governing models. The outcomes show that our scheme is very active and reliable. The acquired results are illustrated by 3-D and 2-D graphs to understand the real phenomena for such sort of non-linear models.

Keywords: EHF method; power law non-linearity; optical solitons
Mathematics Subject Classification: 35Q51, 35Q53

1. Introduction

Solitons have much importance in various areas of science and nature, such as waves, geology, wave propagation, population ecology, fluid dynamics, computer science, biology, plasmas, heat, birefringent fibers, mechanics and optics. There are many mathematical models that efficiently demonstrate the properties of soliton transmission such as Radhakrishnan-Kundu-Lakshmanan equation [1], the resonant nonlinear Schrodinger's equation [2] and so on [3–10]. The key object for the presence of solitons in the optical fibers is to keep balance between group velocity dispersion (GVD) and non-linearity. In special conditions, there may be circumstances which produce small non-linearity and low GVD. Recently, Biswas and Arshed [11–16], estimated a precise creative clue to grip the circumstances where non-linearity and GVD are small, which is presented in the model called BAE. This governing model can act as a feasible model to study the properties of solitons in crystals, Photonic crystal fiber, and optical fibers. Waves and solitons describe many real life phenomena. Several meaningful techniques have been recognized for solving wave structures of non-linear partial differential equations, such as the improved sub-equation scheme [17], expansion approach [18], extended Jacobi elliptic function expansion method [19], singular manifold method [20], modified fractional reduced differential transform method [21], modified (G'/G)-expansion approach [22], Sine-Gordon expansion method [23], extended modified mapping method [24], Hirota's bilinear method [25], the power series method [26], Lie symmetry method [27], Backlund transformation method [28], the generalized Darboux transformation [29], the Darboux transformation method [30], the extended Darboux transformation technique [31], the Hirota bilinear method [32], and the generalized Darboux transformation scheme [33]. This article studies the BAE with power law via the EHF method [34–36], which leads to the construction of singular, bright, dark-bright and periodic-singular optical soliton solutions. In the future, we will use fractional calculus to construct analytical solutions of nonlinear models under the Atangana-Baleanu derivative in Caputo sense (ABC) with the help of the Sumudu transform [37], group preserving scheme [38] and time-fractional Zakharov-Kuznetsov equation via the natural transform decomposition method with nonsingular kernel derivatives [39]. Studies on the physical behaviors of these non-linear models using different analytical methods have gained a lot of attention recently. In physics and engineering, a large number of non-linear dispersive models appear that have a large area of applications. These general solutions can also provide a useful help for researchers to study and understand the physical interpretations of systems.

The layouts of this paper are as follows. The governing model is described in Section 2. Analysis of the proposed EHF method is presented in Section 3. In Section 4 the EHF method is applied, and Section 5 consists of the findings and discussions. Conclusions of this paper are posted in Section 6.

2. Biswas-Arshed equation

The BAE with full non-linearity [11–16] is given by

$$iu_t + a_1 u_{xx} + a_2 u_{xt} + i(b_1 u_{xxx} + b_2 u_{xxt}) = i[\phi(|u|^{2n}u)_x + \sigma(|u|^{2n})_x u + \theta|u|^{2n}u_x], \tag{2.1}$$

where $u(x,t)$ and n are a complex-valued function and full non-linearity parameter, respectively. On the left of the (2.1), a_1 and a_2 indicate the temporal evolution, coefficient of GVD and coefficient of STD, respectively. Next, b_1 and b_2 are coefficients of 3OD and STD. θ and σ represent the effect of self-steepening and non-linear dispersion in the absence of SPM. This effect of dispersion and non-linearity provides the required balance for solitons, existence.

Assume that

$$u(x,t) = P(\eta)e^{i\Psi(x,t)}, \tag{2.2}$$

where $P(\eta)$ indicates the amplitude, and

$$\eta = x - ct, \quad \Psi(x,t) = -kx + wt + \epsilon, \tag{2.3}$$

where c and $\Psi(x,t)$ are the velocity and the phase component. $k,\ w$ are frequency and wave number, accordingly, while ϵ is the phase constant of solitons. Inserting (2.2) and (2.3) in (2.1) and separating into real and imaginary parts, the real part gives

$$-(w+a_1k^2+b_1k^3-a_2wk-b_2wk^2)P+(a_1+3b_1k-a_2c-2b_2ck-b_2w)P''=k(\theta+\phi)P^{2n+1}, \quad (2.4)$$

while the imaginary part gives

$$\begin{aligned}(-3b_1k^2+b_2ck^2+2b_2wk-2a_1k-c+a_2ck+a_2w)P'+(-b_2c+b_1)P'''\\ =[\theta+2n\sigma+(2n+1)\phi]P^{2n}P'.\end{aligned} \quad (2.5)$$

Integrating (2.5) and taking the integration constant to be zero, we obtain

$$\begin{aligned}(-3b_1k^2+b_2ck^2+2b_2wk-2a_1k-c+a_2ck+a_2w)P+(-b_2c+b_1)P''\\ =[\theta+2n\sigma+(2n+1)\phi]P^{2n}.\end{aligned} \quad (2.6)$$

To get the solutions, we use following substitution:

$$P=W^{1/2n}. \quad (2.7)$$

Equations (2.4) and (2.6), yield

$$\begin{aligned}(a_1+3b_1k-a_2c-b_2w-2b_2ck)[(1-2n)(W')^2+2nWW'']\\ -4n^2(w+a_1k^2+b_1k^3-a_2wk-b_2wk^2)W^2-4n^2k(\theta+\phi)W^3=0,\end{aligned} \quad (2.8)$$

$$\begin{aligned}(2n+1)(b_1-b_2c)[(1-2n)(W')^2+2nWW'']-4n^2[2n\sigma+\theta+(2n+1)\phi]W^3\\ +(-2a_1k-3b_1k^2+b_2ck^2+2b_2wk+a_2ck-c+a_2w)4n^2(2n+1)[(1-2n)(W')^2\\ +2nWW'']W^2=0.\end{aligned} \quad (2.9)$$

As $W(\xi)$ satisfies (2.8) and (2.9), the constraint conditions are as follows:

$$\begin{aligned}\frac{(a_1-a_2c+3b_1k-2b_2ck-b_2w)}{(2n+1)(b_1-b_2c)}&=\frac{(\theta+\phi)k}{2n\sigma+\theta+(2n+1)\phi}\\ &=\frac{-(w+a_1k^2+b_1k^3-a_2wk-b_2wk^2)}{(2n+1)(b_2ck^2-3b_1k^2+2b_2wk-2a_1k+a_2ck-c+a_2w)}.\end{aligned} \quad (2.10)$$

Now, (2.8) is analyzed using the extended hyperbolic function method.

Setting $A=b_2ck^2-3b_1k^2+2b_2wk-2a_1k+a_2ck-c+a_2w$ and $B=w+a_1k^2-a_2wk+b_1k^3-b_2wk^2$, gives

$$A[(1-2n)(W')^2+2nWW'']-4n^2BW^2-4n^2k(\theta+\phi)W^3=0. \quad (2.11)$$

3. The EHF method

Two phases of the EHF method are as follows:
Form 1: Let (2.11) have the solution

$$W(\eta) = \sum_{j=0}^{N} F_j \Phi^j(\eta), \tag{3.1}$$

where F_j are constants and $\Phi(\eta)$ satisfies the auxiliary ODE as

$$\frac{d\Phi}{d\eta} = \Phi\sqrt{\tau + \mu\,\Phi^2},\ \ \tau, \mu \in R. \tag{3.2}$$

To find the number N, we use the balancing rule on (2.11). Substituting (3.1) in (2.11) along with (3.2) produces a system of algebraic equations for F_j $(0 \leq j \leq N)$. A set of solutions is acquired by solving this system, that accepts (3.2) as
Set 1: If $\tau > 0$ and $\mu > 0$,

$$\Phi(\eta) = -\sqrt{\frac{\tau}{\mu}}\operatorname{csch}(\sqrt{\tau}(\eta + \eta_0)). \tag{3.3}$$

Set 2: If $\tau < 0$ and $\mu > 0$,

$$\Phi(\eta) = \sqrt{\frac{-\tau}{\mu}}\sec(\sqrt{-\tau}(\eta + \eta_0)). \tag{3.4}$$

Set 3: If $\tau > 0$ and $\mu < 0$,

$$\Phi(\eta) = \sqrt{\frac{\tau}{-\mu}}\operatorname{sech}(\sqrt{\tau}(\eta + \eta_0)). \tag{3.5}$$

Set 4: If $\tau < 0$ and $\mu > 0$,

$$\Phi(\eta) = \sqrt{\frac{-\tau}{\mu}}\csc(\sqrt{-\tau}(\eta + \eta_0)). \tag{3.6}$$

Set 5: If $\tau > 0$ and $\mu = 0$,

$$\Phi(\eta) = \exp(\sqrt{\tau}(\eta + \eta_0)). \tag{3.7}$$

Set 6: If $\tau < 0$ and $\mu = 0$,

$$\Phi(\eta) = \cos(\sqrt{-\tau}(\eta + \eta_0)) + i\,\sin(\sqrt{-\tau}(\eta + \eta_0)). \tag{3.8}$$

Set 7: If $\tau = 0$ and $\mu > 0$,

$$\Phi(\eta) = \pm\frac{1}{(\sqrt{\mu}(\eta + \eta_0))}. \tag{3.9}$$

Set 8: If $\tau = 0$ and $\mu < 0$,

$$\Phi(\eta) = \pm \frac{i}{(\sqrt{-\mu}(\eta + \eta_0))}. \tag{3.10}$$

Form 2: Adopting the same pattern as above, assume (3.1) satisfies the auxiliary ODE as follows:

$$\frac{d\Phi}{d\eta} = \tau + \mu\Phi^2,\ \tau, \mu \in R. \tag{3.11}$$

Substituting (3.1) into (2.11) with (3.11) yields the value of N, and it provides a set equations. Assume (3.11) has the solutions as follows:
Set 1: If $\tau\mu > 0$,

$$\Phi(\eta) = sn(\tau)\sqrt{\frac{\tau}{\mu}}\tan(\sqrt{\tau\mu}(\eta + \eta_0)). \tag{3.12}$$

Set 2: If $\tau\mu > 0$,

$$\Phi(\eta) = -sn(\tau)\sqrt{\frac{\tau}{\mu}}\cot(\sqrt{\tau\mu}(\eta + \eta_0)). \tag{3.13}$$

Set 3: If $\tau\mu < 0$,

$$\Phi(\eta) = sn(\tau)\sqrt{\frac{\tau}{-\mu}}\tanh(\sqrt{-\tau\mu}(\eta + \eta_0)). \tag{3.14}$$

Set 4: If $\tau\mu < 0$,

$$\Phi(\eta) = sn(\tau)\sqrt{\frac{\tau}{-\mu}}\coth(\sqrt{-\tau\mu}(\eta + \eta_0)). \tag{3.15}$$

Set 5: If $\tau = 0$ and $\mu > 0$,

$$\Phi(\eta) = -\frac{1}{\mu(\eta + \eta_0)}. \tag{3.16}$$

Set 6: If $\tau \in R$ and $\mu = 0$,

$$\Phi(\eta) = \tau(\eta + \eta_0). \tag{3.17}$$

Note: *sn* is the well-known sign function.

4. Application of the EHF method

Form 1: In this section, we utilize the above said method to solve the BAE with power law non-linearity. Using the balance principle in (2.11), yields $N = 2$, so (3.1) converts to

$$W(\eta) = F_0 + F_1\Phi(\eta) + F_2(\Phi(\eta))^2, \tag{4.1}$$

where $F_0,\ F_1$ and F_2 are constants. Inserting (4.1) in (2.11), one attains a set of equations in $F_0,\ F_1,\ F_2,\ \tau$ *and* μ is obtained. On working the set of equations, we achieve

$$F_0 = 0, \quad F_1 = 0,$$

$$F_2 = F_2, \ \ \tau = -\frac{Bn^2}{2An - 2n - A},$$

$$\mu = -\frac{k\, n^2(\theta + \phi)F_2}{2An - 3n - A}. \tag{4.2}$$

Set 1: If $\tau > 0$ and $\mu > 0$,

$$u_1(x,t) = \left[F_2\Big(- \sqrt{\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}}\operatorname{csch}(\sqrt{-\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0))\Big)^2\right]^{1/2n} \times (e^{\iota\Psi(x,\ t)}). \tag{4.3}$$

Set 2: If $\tau < 0$ and $\mu > 0$,

$$u_2(x,t) = \left[F_2\Big(\sqrt{-\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}}\sec(\sqrt{\frac{B\, n^2}{2An - 2n - A}}(\eta + \eta_0))\Big)^2\right]^{1/2n} \times (e^{\iota\Psi(x,\ t)}). \tag{4.4}$$

Set 3: If $\tau > 0$ and $\mu < 0$,

$$u_3(x,t) = \left[F_2\Big(\sqrt{-\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}}\operatorname{sech}(\sqrt{-\frac{B\, n^2}{2An - 2n - A}}(\eta + \eta_0))\Big)^2\right]^{1/2n} \times (e^{\iota\Psi(x,\ t)}). \tag{4.5}$$

Set 4: If $\tau < 0$ and $\mu > 0$,

$$u_4(x,t) = \left[F_2\Big(\sqrt{-\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}}\csc(\sqrt{\frac{B\, n^2}{2An - 2n - A}}(\eta + \eta_0))\Big)^2\right]^{1/2n} \times (e^{\iota\Psi(x,\ t)}). \tag{4.6}$$

Set 5: If $\tau > 0$ and $\mu = 0$,

$$u_5(x,t) = \left[F_2\Big(\exp(\sqrt{-\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0))\Big)^2\right]^{1/2n} \times (e^{\iota\Psi(x,\ t)}). \tag{4.7}$$

Set 6: If $\tau < 0$ and $\mu = 0$,

$$u_6(x,t) = \left[F_2\Big(\cos(\sqrt{\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0)) + i\sin(\sqrt{\frac{B\, n^2}{2An - 2n - A}}(\eta + \eta_0))\Big)^2\right]^{1/2n}$$

$$\times(e^{\iota\Psi(x,\,t)}). \tag{4.8}$$

Where $\Psi(x,t) = -kx + wt + \epsilon, \quad \eta = x - ct$.
Form 2: Using the balance principle on (2.11), yields $N = 2$, so (3.1) gives

$$W(\eta) = F_0 + F_1\Phi(\eta) + F_2(\Phi(\eta))^2, \tag{4.9}$$

where F_0, F_1 and F_2 are constants. Inserting (4.9) in (2.11) and comparing the coefficients of each polynomial of $\Phi(\eta)$ to zero, we retrieve a set of equations in $F_0,\ F_1,\ F_2,\ \tau$ *and* μ.

Working on the set of equations, we acquire

$$F_0 = \frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta+\phi A(1-2n)+2n\phi)},$$

$$F_1 = 0, \qquad F_2 = \frac{\mu^2(A(1-2n)+3n)}{kn^2(\phi+\theta)},$$

$$\tau = -\frac{Bn^2}{(2n+A(1-2n))\mu}, \quad \mu = \mu. \tag{4.10}$$

Set 1: If $\tau\mu > 0$,

$$u_9(x,t) = \Bigg[\frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta+\phi A(1-2n)+2n\phi)}$$
$$+F_2\Big(\chi\sqrt{-\frac{Bn^2}{(2n+A(1-2n))\mu^2}}\tan(\sqrt{-\frac{Bn^2}{A+2n-2An}}(\eta+\eta_0))\Big)^2\Bigg]^{1/2n} \times (e^{\iota\Psi(x,\,t)}). \tag{4.11}$$

Set 2: If $\tau\mu > 0$,

$$u_{10}(x,t) = \Bigg[\frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta+\phi A(1-2n)+2n\phi)}$$
$$+F_2\Big(\chi\sqrt{-\frac{Bn^2}{(2n+A(1-2n))\mu^2}}\cot(\sqrt{-\frac{Bn^2}{A+2n-2An}}(\eta+\eta_0))\Big)^2\Bigg]^{1/2n} \times (e^{\iota\Psi(x,\,t)}). \tag{4.12}$$

Set 3: If $\tau\mu < 0$,

$$u_{11}(x,t) = \Bigg[\frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta+\phi A(1-2n)+2n\phi)}$$
$$+F_2\Big(\chi\sqrt{\frac{Bn^2}{(2n+A(1-2n))\mu^2}}\tanh(\sqrt{\frac{Bn^2}{A+2n-2An}}(\eta+\eta_0))\Big)^2\Bigg]^{1/2n} \times (e^{\iota\Psi(x,\,t)}). \tag{4.13}$$

Set 4: If $\tau\mu < 0$,

$$u_{12}(x,t) = \Bigg[\frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta+\phi A(1-2n)+2n\phi)}$$
$$+F_2\Big(\chi\sqrt{\frac{Bn^2}{(2n+A(1-2n))\mu^2}}\coth(\sqrt{\frac{Bn^2}{A+2n-2An}}(\eta+\eta_0))\Big)^2\Bigg]^{1/2n} \times (e^{\iota\Psi(x,\,t)}), \tag{4.14}$$

where $\chi = sgn(-\frac{Bn^2}{(2n+A(1-2n))\mu}), \quad \Psi(x,t) = -kx + wt + \epsilon, \quad \eta = x - ct$.

5. Results and discussion

We illustrated the solutions of the BAE in the form of optical solitons using the EHF method. These solutions have applications in telecommunication to transfer information, as solitons have the proficiency to travel long spaces without distortion and without altering their shapes. In this work, we added selected graphical representations of some solutions of the BAE to dodge overcrowded the manuscript. Figures 1–6 represent 2-D and 3-D plots of some optical solitons of (2.1), with the help of parameters involved. Figures 1 and 6 demonstrate the solutions obtained given in (4.3) and (4.14), accordingly, which are singular soliton waves. Figures 2 and 4 demonstrate the solutions (4.4) and (4.11), which are periodic singular soliton waves. Solution (4.5) given by Figure 3, represents a bright soliton, and solution (4.13) provides a dark soliton and is shown by Figure 5. These results reveal that the bright solitons are favorable in watching the order of the solitons. It is helpful to the eliminate internet jam that is a big problem nowadays in industry. With the recent disease of COVID-19, everywhere commercial dealings are being conducted virtually, and it is necessary to preserve a continuous flow of Internet communications. Similarly, dark solitons are too encouraging for soliton communication in the presence of background wave. Though, singular solitons are just involved the figure of solitons and display a whole band of soliton solutions formed using the model.

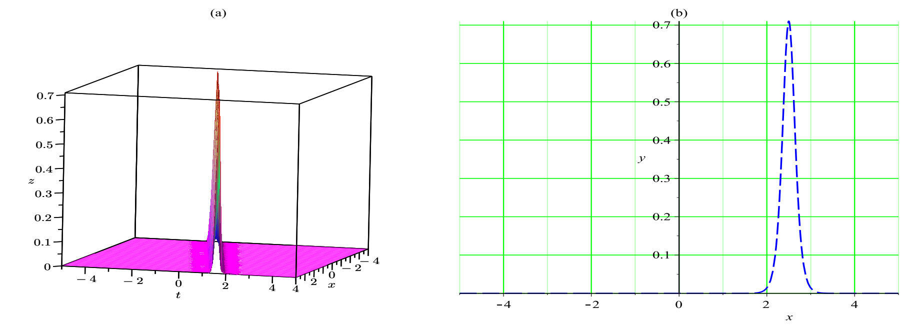

Figure 1. (a) 3D plot of singular soliton solution (4.3) with $F_2 = 1.7,\ A = -2.7,\ B = 11.4,\ n = 1\ k = -2,\ \theta = -4,\ \phi = 0.8,\ \omega = 3, c = 2.5,\ \epsilon = -4$. (b) 2D representation of (4.3) using $t = 1$.

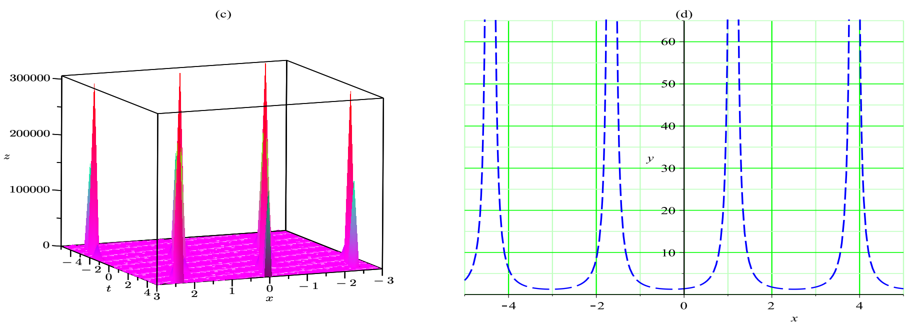

Figure 2. (c) 3D plot of periodic-singular soliton solution (4.4) with $F_2 = 1.7,\ A = -2.7,\ B = 11.4,\ n = 1\ k = -2,\ \theta = -4,\ \phi = 0.8,\ \omega = 3,\ \epsilon = 4, c = 2.5$. (d) 2D representation of (4.4) with $t = 1$.

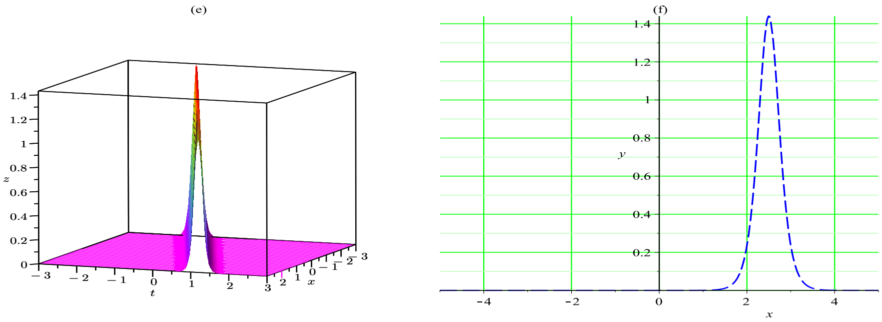

Figure 3. (e) 3D plot of bright soliton solution (4.5) for $F_2 = 1.7,\ A = -2.7,\ B = 11.4,\ n = 1\ \ k = -2,\ \theta = -4,\ \phi = 0.8,\ \ \omega = 3,\ \epsilon = -4, c = 2.5$. (f) 2D representation of (4.5) for $t = 1$.

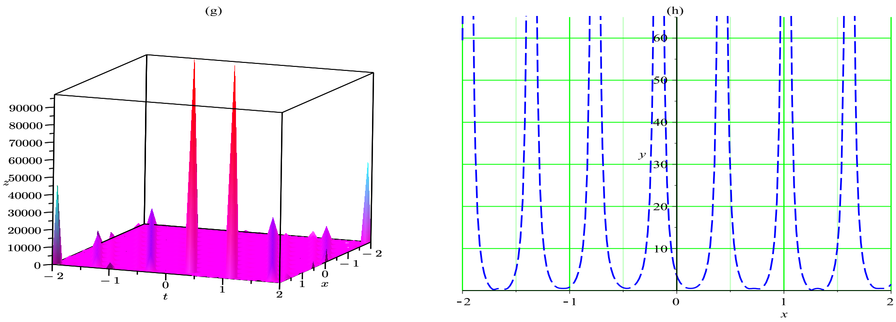

Figure 4. (g) 3D graph of periodic-singular soliton solution (4.11) for $F_2 = 1.7,\ A = -2.7,\ B = 11.4,\ n = 1\ \ k = -2,\ \theta = -4,\ \phi = 0.8,\ \ \omega = 3,\ \epsilon = -4, c = 2.5$. (h) 2D representation of (4.11) for $t = 1$.

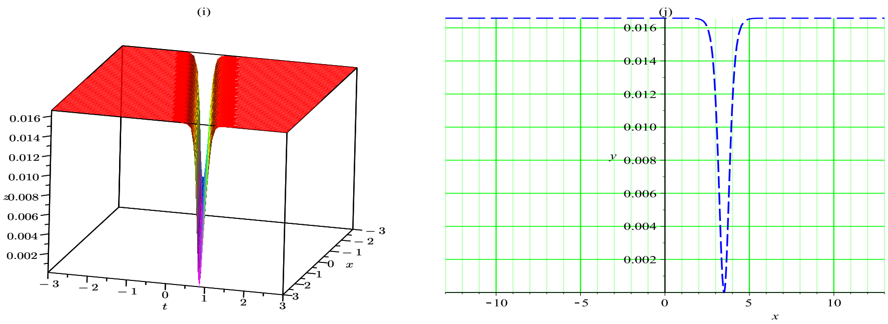

Figure 5. (i) 3D graph of singular soliton solution (4.13) for $F_2 = 1.7,\ A = -2.7,\ B = 11.4,\ n = 1\ \ k = -2,\ \theta = -4,\ \phi = 0.8,\ \ \omega = 3,\ \epsilon = -4, c = 2.5$. (j) 2D representation of (4.13) for $t = 1$.

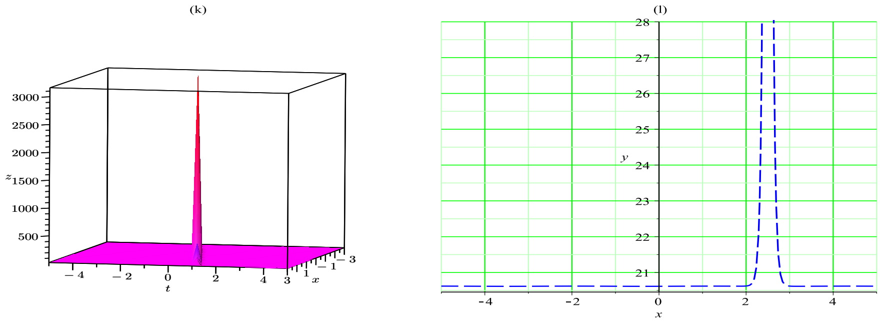

Figure 6. (k) 3D graph of periodic-singular soliton solution (4.14) with $F_2 = 1.7,\ A = -2.7,\ B = 11.4,\ n = 1\ \ k = -2,\ \theta = -4,\ \phi = 0.8,\ \ \omega = 3,\ \epsilon = -4, c = 2.5$. (l) 2D representation of (4.14) for $t = 1$.

6. Conclusions

In this paper, we have successfully employed the EHF method on the BAE and constructed the bright, periodic, dark, singular and combo solitons. These constructed results have much importance in many areas of non-linear sciences, such as physics, birefringent fibers, applied mathematics, optical fibers, engineering, pulse propagation and many more. The outcomes of this study are motivating and improve our understanding of the optical soliton solutions. By these outcomes, we can recognize that the present method is appropriate, useful and skilled for attaining the exact solutions of such kind of problems. The acquired results are fresh, correct and not reported earlier in literature.

Acknowledgments

The authors would like to express their sincere thanks to the Department of Mathematics, University of Management and Technology Lahore, Pakistan.

Conflict of interest

The authors declare that they have no conflict of interest.

References

1. A. Biswas, Optical soliton perturbation with Radhakrishnan-Kundu-Lakshmanan equation by traveling wave hypothesis, *Optik*, **171** (2018), 217–220. https://doi.org/10.1016/j.ijleo.2018.06.043

2. A. Biswas, A. J. M. Jawad, Q. Zhou, Resonant optical solitons with anti-cubic nonlinearity, *Optik*, **157** (2018), 525–531. https://doi.org/10.1016/j.ijleo.2017.11.125

3. S. Arshed, Two reliable techniques for the soliton solutions of perturbed Gerdjikov-Ivanov equation, *Optik*, **164** (2018), 93–99. https://doi.org/10.1016/j.ijleo.2018.02.119

4. M. S. Osman, Nonlinear interaction of solitary waves described by multi-rational wave solutions of the (2+1)-dimensional Kadomtsev-Petviashvili equation with variable coefficients, *Nonlinear Dyn.*, **87** (2017), 1209–1216. https://doi.org/10.1007/s11071-016-3110-9
5. F. Tahir, M. Younis, H. U. Rehman, Optical Gaussons and dark solitons in directional couplers with spatiotemporal dispersion, *Opt. Quant. Electron.*, **50** (2018), 422. https://doi.org/10.1007/s11082-017-1259-1
6. N. Ullah, H. Rehman, M. A. Imran, T. Abdeljawad, Highly dispersive optical solitons with cubic law and cubic-quintic-septic law nonlinearities, *Results Phys.*, **17** (2020), 103021. https://doi.org/10.1016/j.rinp.2020.103021
7. Y. Ren, H. Zhang, New generalized hyperbolic functions and auto-Backlund transformations to find new exact solutions of the (2+1)-dimensional NNV equation, *Phys. Lett. A*, **357** (2006), 438–448. https://doi.org/10.1016/j.physleta.2006.04.082
8. H. Rezazadeh, S. M. Mirhosseini-Alizamini, M. Eslami, M. Rezazadeh, M. Mirzazadeh, S. Abbagari, New optical solitons of nonlinear conformable fractional Schrodinger-Hirota equation, *Optik*, **172** (2018), 545–553. https://doi.org/10.1016/j.ijleo.2018.06.111
9. M. Quiroga-Teixeiro, H. Michinel, Stable azimuthal stationary state in quintic nonlinear optical media, *J. Opt. Soc. Am. B*, **14** (1997), 2004–2009. https://doi.org/10.1364/JOSAB.14.002004
10. H. U. Rehman, N. Ullah, M. A. Imran, Highly dispersive optical solitons using Kudryashov's method, *Optik*, **199** (2019), 163349. https://doi.org/10.1016/j.ijleo.2019.163349
11. H. U. Rehman, S. Jafar, A. Javed, S. Hussain, M. Tahir, New optical soliotons of Biswas-Arshed equation using different technique, *Optik*, **206** (2020), 163670. https://doi.org/10.1016/j.ijleo.2019.163670
12. M. Tahir, A. U. Awan, H. U. Rehman, Dark and singular optical solitons to the Biswas-Arshed model with Kerr and power law nonlinearity, *Optik*, **185** (2019), 777–783. https://doi.org/10.1016/j.ijleo.2019.03.108
13. P. K. Das, Chirped and chirp-free optical exact solutions of the Biswas-Arshed equation with full nonlinearity by the rapidly convergent approximation method, *Optik*, **223** (2020), 165293. https://doi.org/10.1016/j.ijleo.2020.165293
14. Z. Korpinar, M. Inc, M. Bayram, M. S. Hashemi, New optical solitons for Biswas-Arshed equation with higher order dispersions and full nonlinearity, *Optik*, **206** (2020), 163332. https://doi.org/10.1016/j.ijleo.2019.163332
15. M. Tahir, A. U. Awan, Optical singular and dark solitons with Biswas-Arshed model by modified simple equation method, *Optik*, **202** (2020), 163523. https://doi.org/10.1016/j.ijleo.2019.163523
16. M. Ekici, A. Sonmezoglu, Optical solitons with Biswas-Arshed equation by extended trial function method, *Optik*, **177** (2019), 13–20. https://doi.org/10.1016/j.ijleo.2018.09.134
17. B. Karaagac, New exact solutions for some fractional order differential equations via improved sub-equation method, *Discret Cont. Dyn. S*, **12** (2019), 447–54. https://doi.org/10.3934/dcdss.2019029
18. M. Alam, F. Belgacem, Microtubules nonlinear models dynamics investigations through the $\exp(G'(G))$-expansion method implementation, *Mathematics*, **4** (2016), 6. https://doi.org/10.3390/math4010006

19. A. Sonmezoglu, Exact solutions for some fractional differential equations, *Adv. Math. Phys.*, **2015** (2015), 567842. https://doi.org/10.1155/2015/567842

20. R. Saleh, M. Kassem, S. M. Mabrouk, Exact solutions of nonlinear fractional order partial differential equations via singular manifold method, *Chinese J. Phys.*, **61** (2019), 290–300. https://doi.org/10.1016/j.cjph.2019.09.005

21. M. Iqbal, A. R. Seadawy, D. Lu, Construction of solitary wave solutions to the nonlinear modified Kortewegede Vries dynamical equation in unmagnetized plasma via mathematical methods, *Mod. Phys. Lett. A*, **33** (2018), 1850183. https://doi.org/10.1142/S0217732318501833

22. M. N. Alam, C. Tunc, Constructions of the optical solitons and others soliton to the conformable fractional zakharov-kuznetsov equation with power law nonlinearity, *J. Taibah Univ. Sci.*, **14** (2020), 94–100. https://doi.org/10.1080/16583655.2019.1708542

23. N. Kadkhoda, H. Jafari, An analytical approach to obtain exact solutions of some space-time conformable fractional differential equations, *Adv. Differ. Equ.*, **2019** (2019), 428. https://doi.org/10.1186/s13662-019-2349-0

24. A. R. Seadawy, Three-dimensional weakly nonlinear shallow water waves regime and its travelling wave solutions, *Int. J. Comp. Meth.*, **15** (2018), 1850017. https://doi.org/10.1142/S0219876218500172

25. S. Tian, J. M. Tu, T. T. Zhang, Y. R. Chen, Integrable discretizations and soliton solutions of an Eckhaus-Kundu equation, *Appl. Math. Lett.*, **122** (2021), 107507. https://doi.org/10.1016/j.aml.2021.107507

26. S. Tian, M. J. Xu, T. T. Zhang, A symmetry-preserving difference scheme and analytical solutions of a generalized higher-order beam equation, *P. Roy. Soc. A*, **477** (2021), 20210455. https://doi.org/10.1098/rspa.2021.0455

27. S. Tian, Lie symmetry analysis, conservation laws and solitary wave solutions to a fourth-order nonlinear generalized Boussinesq water wave equation, *Appl. Math. Lett.*, **100** (2020), 106056. https://doi.org/10.1016/j.aml.2019.106056

28. S. Tian, D. Guo, X. Wang, T. Zhang, Traveling wave, lump Wave, rogue wave, multi-kink solitary wave and interaction solutions in a (3+1)-dimensional Kadomtsev-Petviashvili equation with Backlund transformation, *J. Appl. Anal. Comput.*, **11** (2021), 45–58. https://doi.org/10.11948/20190086

29. B. Q. Li, Optical rogue wave structures and phase transitions in a light guide fiber system doped with two-level resonant atoms, *Optik*, **253** (2022), 168541. https://doi.org/10.1016/j.ijleo.2021.168541

30. B. Q. Li, Y. L. Ma, Interaction properties between rogue wave and breathers to the manakov system arising from stationary self-focusing electromagnetic systems, *Chaos Soliton. Fract.*, **156** (2022), 111832. https://doi.org/10.1016/j.chaos.2022.111832

31. B. Q. Li, Interaction behaviors between breather and rogue wave in a Heisenberg ferromagnetic equation, *Optik*, **227** (2020), 166101. https://doi.org/10.1016/j.ijleo.2020.166101

32. B. Q. Li, Y. L. Ma, Interaction dynamics of hybrid solitons and breathers for extended generalization of Vakhnenko equation, *Nonlinear Dyn.*, **102** (2020), 1787–1799. https://doi.org/10.1007/s11071-020-06024-4
33. B. Q. Li, Y. L. Ma, N-order rogue waves and their novel colliding dynamics for a transient stimulated Raman scattering system arising from nonlinear optics, *Nonlinear Dyn.*, **101** (2020), 2449–2461. https://doi.org/10.1007/s11071-020-05906-x
34. Y. Shang, The extended hyperbolic function method and exact solutions of the long-short wave resonance equations, *Chaos Soliton. Fract.*, **36** (2008), 762–771. https://doi.org/10.1016/j.chaos.2006.07.007
35. Y. Shang, Y. Huang, W. Yuan, The extended hyperbolic functions method and new exact solutions to the Zakharov equations, *Appl. Math. Comput.*, **200** (2008), 110–122. https://doi.org/10.1016/j.amc.2007.10.059
36. S. Nestor, A. Houwe, G. Betchewe, M. Inc, S. Y. Doka, A series of abundant new optical solitons to the conformable space-time fractional perturbed nonlinear Schrödinger equation, *Phys. Scr.*, **95** (2020), 085108. https://doi.org/10.1088/1402-4896/ab9dad
37. K. S. Nisar, A. Ahmad, M. Inc, M. Farman, H. Rezazadeh, L. Akinyemi, et al., Analysis of dengue transmission using fractional order scheme, *AIMS Math.*, **7** (2022), 8408–8429. https://doi.org/10.3934/math.2022469
38. M. S. Hashemi, H. Rezazadeh, H. Almusawa, H. Ahmad, A Lie group integrator to solve the hydromagnetic stagnation point flow of a second grade fluid over a stretching sheet, *AIMS Math.*, **6** (2021), 13392–13406. https://doi.org/10.3934/math.2021775
39. M. X. Zhou, A. S. V. Ravi Kanth, K. Aruna, K. Raghavendar, H. Rezazadeh, M. Inc, et al., Numerical solutions of time fractional Zakharov-Kuznetsov equation via natural transform decomposition method with nonsingular kernel derivatives, *J. Funct. Space.*, **2021** (2021), 9884027. https://doi.org/10.1155/2021/9884027