AIMS Mathematics

AIMS Mathematics, 7(10): 18377–18392.
DOI: 10.3934/math.20221012
Received: 13 March 2022
Revised: 27 July 2022
Accepted: 01 August 2022
Published: 16 August 2022

http://www.aimspress.com/journal/Math

Research article

Investigation of chaos behavior and integral sliding mode control on financial risk model

Sukono[1,*], Siti Hadiaty Yuningsih[2], Endang Rusyaman[1], Sundarapandian Vaidyanathan[3] and Aceng Sambas[4,5]

[1] Department of Mathematics, Universitas Padjadjaran, Jatinangor, Kabupaten Sumedang 45363, Indonesia
[2] Magister Program of Mathematics, Universitas Padjadjaran, Jatinangor, Kabupaten Sumedang 45363, Indonesia
[3] Centre for Control Systems, Vel Tech University, Avadi, Chennai-600062, Tamil Nadu, India
[4] Department of Mechanical Engineering, Universitas Muhammadiyah Tasikmalaya, 46196, Indonesia
[5] Institute of Strategic Industrial Decision Modelling (ISIDM), Universiti Utara Malaysia, Sintok, Kedah 06010, Malaysia

* **Correspondence:** Email: sukono@unpad.ac.id.

Abstract: This paper reports the finding of a new financial chaotic system. A new control law for completely synchronizing the new financial chaotic system with itself has been established using adaptive integral sliding mode control. We also find that the new financial chaotic system has fascinating traits including symmetry, equilibrium points, multistability, Lyapunov exponents and bifurcation diagrams. We illustrate all the main results of this research work using MATLAB phase plots. The Lyapunov characteristic exponents and analysis using bifurcation diagrams have resulted in a new financial chaos system showing chaos phenomena in the intervals of parameters $0 < a < 15$, and parameters $0 < b < 0.25$. The results of this study can be used to predict if there is chaos in financial risk. Chaotic systems have many applications in engineering like cryptosystems and secure communication systems.

Keywords: chaos; financial chaotic system; integral sliding mode control; synchronization
Mathematics Subject Classification: 34H10, 34K18, 34K20, 37G15

1. Introduction

Chaos is widely applied to various fields of natural and social sciences, as well as forming new subjects or new research fields [1]. There are many applications of chaotic systems in finance and management such as business cycles, firm growth, investment, and chaotic behavior in foreign capital investment [2]. Until now, econophysics has been appointed as an alternative scientific methodology to understand the very complex dynamics of the economic and financial system [3]. In the economic field, financial risk is one of the most important issues that concerns the State and all economic entities [4]. In recent years, the chaotic state of the economy has received intensive attention and engineering applications have been made to understand the complex behavior of financial markets [5].

Control methods for nonlinear dynamical systems are important topics in chaos theory [6–8]. Much research attention has been given to system modelling and control in engineering and science [9–11]. Chaos phenomenon developed by creating irregular phenomena can be desirable for many types of control applications [12]. Chaotic systems have been studied using various techniques like passive control [13] adaptive control [14], fuzzy control [15,16], active control [17–19], sliding mode control [20–22], etc.

Sliding mode control (SMC) is attractive for nonlinear systems because of its insensitivity to parametric and nonparametric uncertainties [23–25]. SMC is a well-known method for chaotic system synchronization [26–28]. However, SMC invariance is not guaranteed in the attainment phase. Integral sliding mode control (ISMC) eliminates the gain phase in such a way that invariance is achieved in the entire system response. Hence, ISMC is a popular method for chaotic system control and synchronization [29,30].

There are several properties that identify the behavior of chaotic systems. Sensitivity to initial conditions is the most common trait of chaotic systems [31]. Any nonlinear dynamical system can have the ability to exhibit chaotic behavior if it is at least a 3-dimensional system for an autonomous system, and a 2-dimensional system for a non-autonomous system [32]. One way to identify a chaotic system is to use the method of Lyapunov exponents. A 3-D dynamical system is chaotic if it has a positive Lyapunov exponent. Also, a system is said to be dissipative if the sum of all Lyapunov exponents of the system is negative.

In the chaos literature, there is a significant interest in the modelling of financial chaotic systems. Gao and Ma [33] investigated the behavior of chaos and the existence of Hopf bifurcation in a financial chaotic system. Vaidyanathan et al. [34] proposed a new financial chaotic system and presented its circuit design. Tacha et al. [35] introduced a new financial chaotic system with dissaving and detailed its circuit simulation.

In this work, we propose a new finance chaotic system and study its dynamic behavior with help of bifurcation diagrams and Lyapunov exponents. We compare the new finance chaotic system with the three finance chaotic systems [33–35] and show that the proposed chaotic system has a great value of Maximum Lyapunov Exponent (MLE) and Kaplan-Yorke Dimension than the three finance chaotic systems [33–35].

We show that the proposed chaotic financial system has three unstable equilibrium points. We discuss its properties such as rotational symmetry, multi-stability, etc. A nonlinear chaotic system is called multistable if it has coexistent chaotic attractors for the same parameter values but different initial states [36]. The existence of this new financial chaos system is expected to predict chaos in the financial sector with variables of interest rates, investment demand, price exponents, household savings, investment costs, and the elasticity of demand for the commercial market. Chaotic systems have applications in many engineering fields such as cryptosystems [37,38] and secure communication devices [39,40].

This work is organized as follows. In Section 2, we present a dynamic analysis of the new chaotic system. In Section 3, we obtain new results for complete synchronization of the new chaotic system via adaptive integral sliding mode control. In Section 4, we draw the main conclusions.

2. A new finance chaotic system

Gao & Ma [33] reported 3-D dynamics with a financial risk model provided by

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 \\ \dot{x}_2 = 1 - bx_2 - x_1^2 \\ \dot{x}_3 = -x_1 - x_3 \end{cases} . \tag{1}$$

In the 3-D financial system (1), x_1 represents the interest rate, x_2 represents the level of investment demand and x_3 represents the price exponential. Also, the constant a stands for the household savings rate and b represents the cost of investment.

We assume a and b are positive. Gao and Ma [33] showed that the finance system (1) is chaotic when $a = 6$ and $b = 0.1$.

Using MATLAB, the Lyapunov exponent (LE) values of the Gao-Ma finance system (1) are obtained for $(a,b) = (6, 0.1)$ and $X(0) = (0.5, 3, -0.4)$ as follows:

$$LE_1 = 0.0909, \;\; LE_2 = 0, \;\; LE_3 = -0.3944. \tag{2}$$

This calculation shows that the Gao-Ma financial system (1) is chaotic with the maximal Lyapunov exponent (MLE) value as $LE_1 = 0.0909$.

Also, the Kaplan-Yorke dimension of the Gao-Ma financial chaotic system (1) is calculated as follows:

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.2305. \tag{3}$$

Vaidyanathan et al. [34] proposed a new financial system by replacing the quadratic nonlinearity x_1^2 with the quartic nonlinearity x_1^4 in the second differential equation of the Gao-Ma financial chaotic system (1).

Vaidyanathan et al. [34] reported 3-D dynamics with a financial risk model provided by

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 \\ \dot{x}_2 = 1 - bx_2 - x_1^4 \\ \dot{x}_3 = -x_1 - x_3 \end{cases} . \tag{4}$$

Vaidyanathan et al. [34] showed that the finance system (4) is chaotic when $a = 7.5$ and $b = 0.1$.

Using MATLAB, the Lyapunov exponent (LE) values of the Vaidyanathan finance system (4) are obtained for $(a,b)=(7.5,0.1)$ and $X(0)=(0.5,3,-0.4)$ as follows:

$$LE_1 = 0.1285, \ \ LE_2 = 0, \ \ LE_3 = -0.3997. \tag{5}$$

This calculation shows that the Vaidyanathan financial system (3) is chaotic with the maximal Lyapunov exponent (MLE) value as $LE_1 = 0.1285$.

Also, the Kaplan-Yorke dimension of the Vaidyanathan financial chaotic system (3) is calculated as follows:

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.3215. \tag{6}$$

Tacha et al. [35] proposed a new financial system with dissaving by replacing the term $-ax_1$ with ax_1 in the first differential equation of the Gao-Ma financial chaotic system (1) and also by replacing the quadratic nonlinearity x_1^2 with the absolute function nonlinearity $|x_1|$ in the second differential equation of the Gao-Ma financial chaotic system (1).

Tacha et al. [35] reported 3-D dynamics with a financial risk model with dissaving provided by

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 + a)x_1 \\ \dot{x}_2 = 1 - bx_2 - |x_1| \\ \dot{x}_3 = -x_1 - x_3 \end{cases}. \tag{7}$$

Tacha et al. [34] showed that the finance system (7) is chaotic when $a = 1$ and $b = 0.2$.

Using MATLAB, the Lyapunov exponent (LE) values of the Tacha finance system (7) are obtained for $(a,b)=(1,0.2)$ and $X(0)=(0.5,3,-0.4)$ as follows:

$$LE_1 = 0.0673, \ \ LE_2 = 0, \ \ LE_3 = -0.5354. \tag{8}$$

This calculation shows that the Tacha financial system (7) is chaotic with the maximal Lyapunov exponent (MLE) value as $LE_1 = 0.0673$.

Also, the Kaplan-Yorke dimension of the Tacha financial chaotic system (7) is calculated as follows:

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.1257. \tag{9}$$

In this work, we propose a new financial chaos system by retaining the quadratic nonlinearity x_1^2 but adding the absolute function nonlinearity $|x_1|$ in the second differential equation of the Gao-Ma chaotic system (1).

We propose a new 3-D dynamics with a financial risk model provided by

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 \\ \dot{x}_2 = 1 - bx_2 - |x_1| - x_1^2 \\ \dot{x}_3 = -x_1 - x_3 \end{cases} . \tag{10}$$

We shall show that the new finance system (10) is chaotic when $a = 1$ and $b = 0.04$.

Using MATLAB, the Lyapunov exponent (LE) values of the new finance system (10) are obtained for $(a, b) = (1, 0.04)$ and $X(0) = (0.5, 3, -0.4)$ as follows:

$$LE_1 = 0.1862, \quad LE_2 = 0, \quad LE_3 = -0.5238. \tag{11}$$

This calculation shows that the new financial system (10) is chaotic with the maximal Lyapunov exponent (MLE) value as $LE_1 = 0.1862$.

Also, the Kaplan-Yorke dimension of the new financial chaotic system (10) is calculated as follows:

$$D_{KY} = 2 + \frac{LE_1 + LE_2}{|LE_3|} = 2.3555. \tag{12}$$

Table 1 shows a comparison of the Gao-Ma finance system [33], Vaidyanathan finance system [34], Tacha finance system [35] and the new finance chaotic system (10) proposed in this research work. From Table 1, it is very clear that the new finance chaotic system (10) is more chaotic than the Gao-Ma finance system [33], Vaidyanathan finance system [34] and Tacha finance system [35].

Table 1. A comparison of the four finance chaotic systems.

Chaotic System	Maximal Lyapunov Exponent	Kaplan-Yorke Dimension
Gao-Ma Finance System [33]	$MLE = 0.0909$	$D_{KY} = 2.2305$
Vaidyanathan Finance System [34]	$MLE = 0.1285$	$D_{KY} = 2.3215$
Tacha Finance System [35]	$MLE = 0.0673$	$D_{KY} = 2.1257$
New finance chaotic system	$MLE = 0.1862$	$D_{KY} = 2.3555$

We obtain the equilibrium point of the new finance chaotic system (10) by solving the following equation for the chaotic case of the parameters $(a, b) = (1, 0.04)$:

$$x_3 + (x_2 - 1)x_1 = 0 \tag{13a}$$

$$1 - 0.04x_2 - |x_1| - x_1^2 = 0 \tag{13b}$$

$$-x_1 - x_3 = 0 \tag{13c}$$

From Eq (13c), it is clear that

$$x_3 = -x_1 \tag{14}$$

Using (14), we can simplify the Eqs (13a) and (13b) as follows:

$$x_1(x_2 - 2) = 0 \tag{15a}$$

$$1 - 0.04x_2 - |x_1| - x_1^2 = 0 \tag{15b}$$

We have two cases to consider: (A) $x_1 = 0$ and (B) $x_1 \neq 0$.

In Case (A), we suppose that $x_1 = 0$. Since $x_3 = -x_1$, it is immediate that $x_3 = 0$. From Eq (15b), we get $1 - 0.04x_2 = 0$ or $x_2 = 25$. Thus, $E_0 = (0, 25, 0)$ is a balance point of the new finance system (10).

In Case (B), we suppose that $x_1 \neq 0$. From Eq (15a), we get $x_2 = 2$.

Thus, Eq (15b) can be simplified as follows:

$$x_1^2 + |x_1| = 0.92. \tag{16}$$

Solving Eq (16), we get two real roots $x_1 = \pm 0.5816653820$.

Since $x_3 = -x_1$, we deduce that $x_3 = \mp 0.5816653820$.

Thus, in case (B), we have two balance points given as follows: $E_1 = (0.5816, 2, -0.5816)$ and $E_2 = (-0.5816, 2, 0.5816)$ of the new finanace chaotic system (10).

The Jacobian matrix of the new financial chaos system (10) at $E_0 = (0,25,0)$ is written as follows:

$$W_0 = \begin{bmatrix} 1 & -0.5816 & 1 \\ 1 & 1.1233 & 0 \\ -1 & 0 & -1 \end{bmatrix}. \tag{17}$$

The spectral value of W_0 is determined using MATLAB as follows

$$\lambda_1 = -0.4505\,, \lambda_2 = 0.7869 + 0.8194i\,, \lambda_3 = \ 0.7869 - 0.8194i. \tag{18}$$

This establishes that E_0 is a saddle point and an unstable balance point for the new finance chaotic system (10).

The Jacobian matrix of the new financial chaos system (10) at $E_1 = (0.5816, 2, -0.5816)$ is shown as follows:

$$W_1 = \begin{bmatrix} 1 & -0.5816 & 1 \\ 1 & 1.1233 & 0 \\ -1 & 0 & -1 \end{bmatrix}. \tag{19}$$

The spectral value of W_1 was determined using MATLAB as follows

$$\lambda_1 = -0.4505\,, \lambda_2 = 0.7869 + 0.8194i\,, \lambda_3 = \ 0.7869 - 0.8194i. \tag{20}$$

This indicates that E_1 is a saddle focus and an unstable equilibrium point for a chaotic financial system (10).

The Jacobian matrix of the new financial chaos system (10) at $E_2 = (-0.5816,2,0.5816)$ is determined

$$W_2 = \begin{bmatrix} 1 & 0.5816 & 1 \\ 1 & -1.1233 & 0 \\ -1 & 0 & -1 \end{bmatrix}. \tag{21}$$

The spectral value of W_2 was determined using MATLAB as follows

$$\lambda_1 = -0.4505\,, \lambda_2 = 0.7869 + 0.8194i\,, \lambda_3 = 0.7869 - 0.8194i. \tag{22}$$

This establishes that E_1 is a saddle-focus and an unstable balance point for the new finance chaotic system (10).

The phase plot of the new financial chaotic system (10) for $X(0) = (0.5,3,-0.4)$ and $(a,b) = (1,0.04)$ in the (x_1,x_2) −plane is shown in Figure 1. Plane phase orbit in Figure 1 depicts the high complexity of the new financial chaotic system (10) in the (x_1,x_2) −plane.

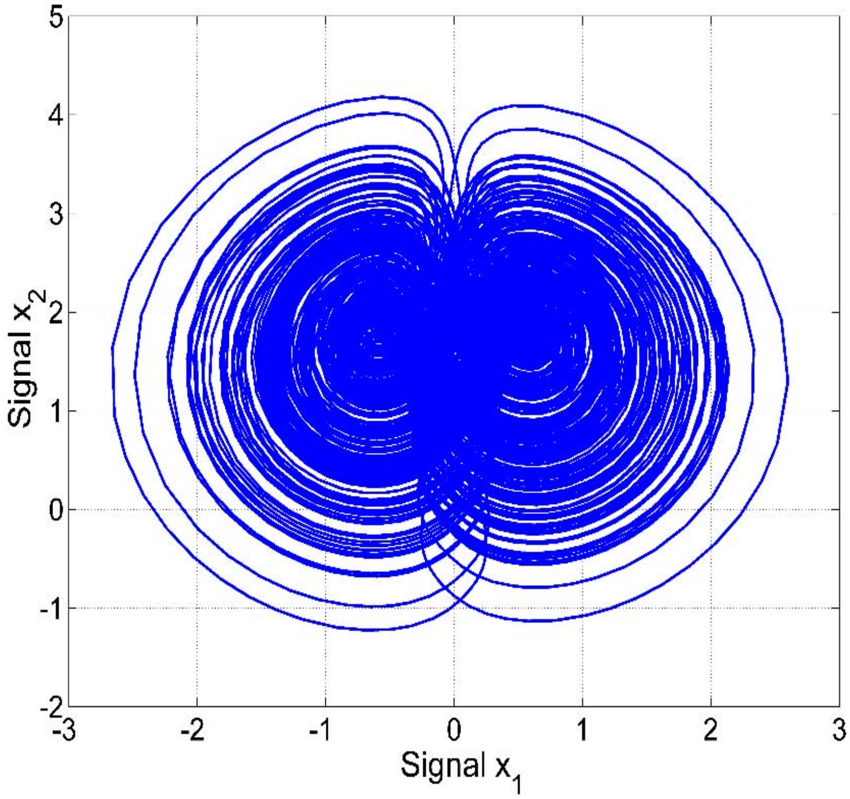

Figure 1. 2-D phase plot of the new financial chaotic system (10) in the (x_1,x_2) – plane for $X(0) = (0.5,3,-0.4)$ and $(a,b) = (1,0.04)$.

The phase plot of the new financial chaotic system (10) for $X(0) = (0.5,3,-0.4)$ and $(a,b) = (1,0.04)$ in the (x_2,x_3) −plane is shown in Figure 2. Plane phase orbit in Figure 2 depicts the high complexity of the new financial chaotic system (10) in the (x_2,x_3) −plane.

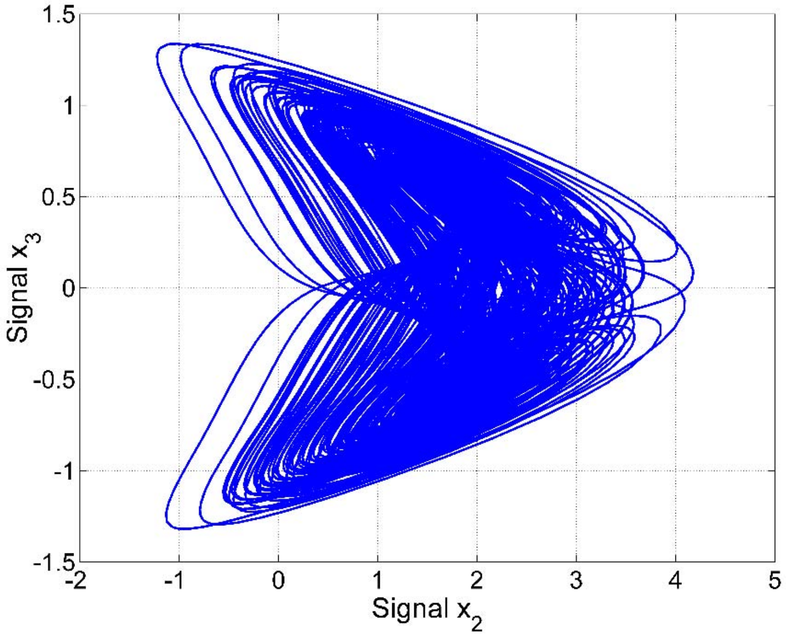

Figure 2. 2-D phase plot of the new financial chaotic system (10) in the (x_2, x_3) – plane for $X(0) = (0.5,3,-0.4)$ and $(a, b) = (1,0.04)$.

The phase plot of the new financial chaotic system (10) for $X(0) = (0.5,3,-0.4)$ and $(a, b) = (1,0.04)$ in the (x_1, x_3) −plane is shown in Figure 3. Plane phase orbit in Figure 3 depicts the high complexity of the new financial chaotic system (10) in the (x_1, x_3) −plane.

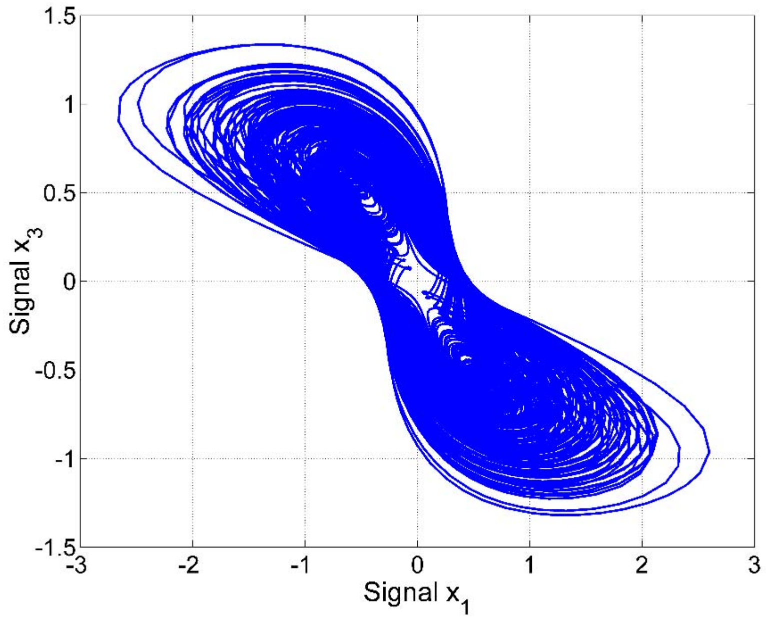

Figure 3. 2-D phase plot of the new financial chaotic system (10) in the (x_1, x_3) – plane for $X(0) = (0.5,3,-0.4)$ and $(a, b) = (1,0.04)$.

Figure 4 shows a Lyapunov exponents diagram for the new financial chaotic system (10) when a takes values in the interval $[0,18]$ and $b=0.04$. Figure 5 shows a bifurcation diagram of the new financial system (10) showing a graph of X3max versus a for $b=0.04$. These results show that the new finance system (10) has a chaotic attractor for $a\in[0,15]$ and a periodic orbit for $a>15$.

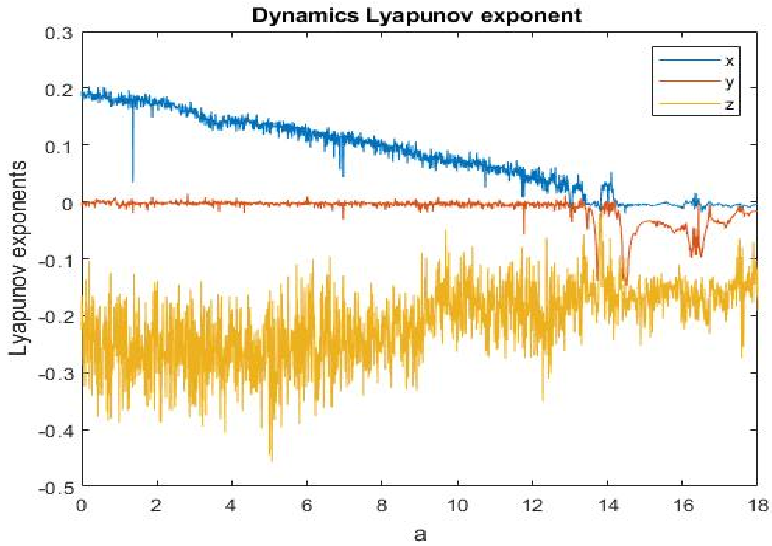

Figure 4. Lyapunov exponents of the new financial system (10) versus a for $b = 0.04$.

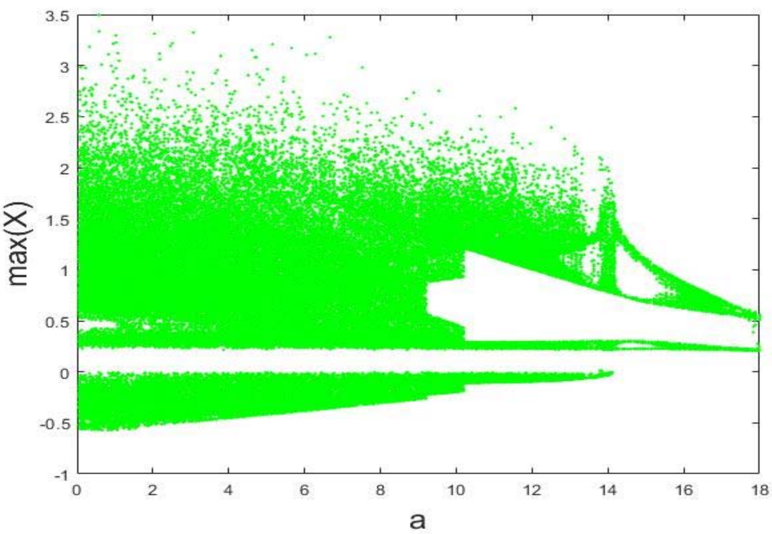

Figure 5. Bifurcation diagram of the new financial system (10) showing a graph of X3max versus a for $b = 0.04$.

Figure 6 shows a Lyapunov exponents diagram for the new financial chaotic system (10) when b takes values in the interval $[0,0.8]$ and $a=1$. Figure 7 shows a bifurcation diagram of the new financial system (10) showing a graph of X3max versus b for $a=1$. These results show that the new finance system (10) has a chaotic attractor for $b\in[0,0.25]$ and a periodic orbit for $b>0.25$.

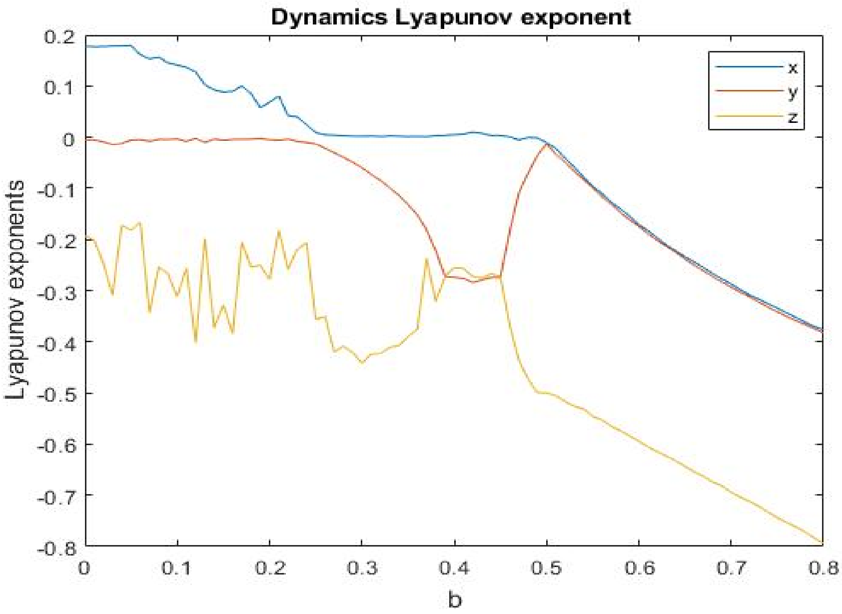

Figure 6. Lyapunov exponents of the new financial system (10) versus b for $a = 1$.

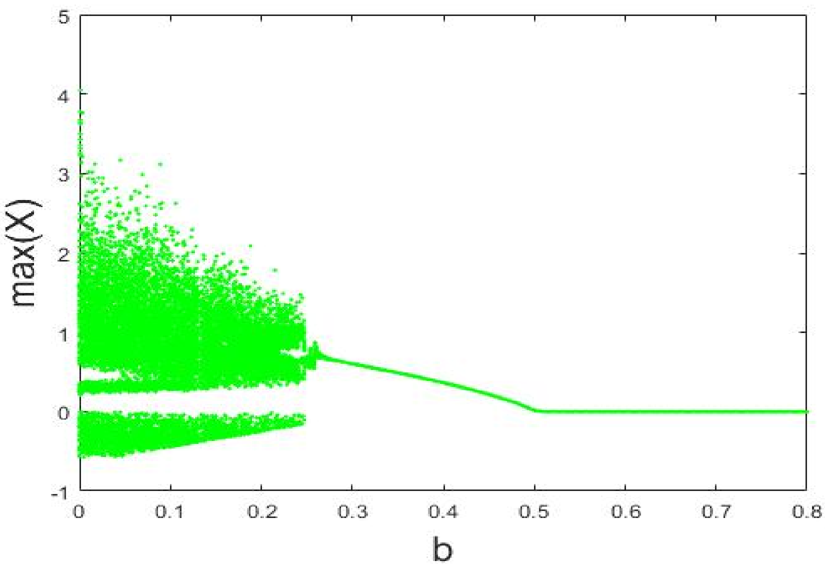

Figure 7. Bifurcation diagram of the new financial system (10) showing a graph of X3max versus b for $a = 1$.

Figure 8 shows the multistability of the financial chaotic system (10) where the blue color trajectory starts with the initial state $X_0 = (0.5,3,-0.4)$, and the red color trajectory starts with the initial state $Y_0 = (-1,3,1)$, and the parameter values are kept fixed as $(a, b) = (1, 0.04)$.

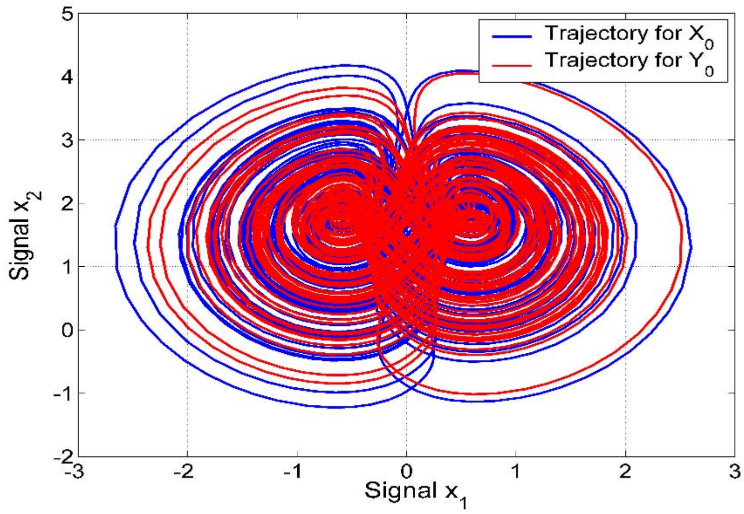

Figure 8. A 2-D orbit in (x_1, x_2) plane showing multistability of the new financial system (10).

3. Complete synchronization of the new finance chaotic system via integral sliding mode control

In this section, we achieve global chaos synchronization of the new finance chaotic system (10) with itself via feedback controllers designed with integral sliding mode control (ISMC).

The drive system is taken as the new finance chaotic system:

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 \\ \dot{x}_2 = 1 - bx_2 - |x_1| - x_1^2 \\ \dot{x}_3 = -x_1 - x_3 \end{cases}. \tag{23}$$

The response system is taken as the new finance chaotic system with sliding mode controls:

$$\begin{cases} \dot{y}_1 = y_3 + (y_2 - a)y_1 + v_1 \\ \dot{y}_2 = 1 - by_2 - |y_1| - x_1^2 + v_2 \\ \dot{y}_3 = -y_1 - y_3 + v_3 \end{cases}. \tag{24}$$

We denote the states of the systems (23) and (24) as $X = (x_1, x_2, x_3)$ and $Y = (y_1, y_2, y_3)$ respectively. In (24), v_1, v_2, v_3 are the integral sliding mode controls (ISMC) to be determined.

First, we define the synchronization error between the finance systems (23) and (24) as:

$$\varepsilon_i = y_i - x_i, \text{ for } i = 1,2,3. \tag{25}$$

It is easy to see that

$$\begin{cases} \dot{\varepsilon}_1 = -a\varepsilon_1 + \varepsilon_3 + y_1 y_2 - x_1 x_2 + v_1 \\ \dot{\varepsilon}_2 = -b\varepsilon_2 - |y_1| - |x_1| - y_1^2 + x_1^2 + v_2 \\ \dot{\varepsilon}_3 = -\varepsilon_1 - \varepsilon_3 + v_3 \end{cases}. \tag{26}$$

In the IMSC design, we associate an integral sliding surface with every synchronization error variable as follows:

$$\begin{cases} S_1 = \varepsilon_1 + \mu_1 \int_0^t \varepsilon_1(\tau) d\tau \\ S_2 = \varepsilon_2 + \mu_2 \int_0^t \varepsilon_2(\tau) d\tau \\ S_3 = \varepsilon_3 + \mu_3 \int_0^t \varepsilon_3(\tau) d\tau \end{cases}. \tag{27}$$

Differentiating the surface equations given in (27), we get:

$$\begin{cases} \dot{S}_1 = \dot{\varepsilon}_1 + \mu_1 \varepsilon_1 \\ \dot{S}_2 = \dot{\varepsilon}_2 + \mu_2 \varepsilon_2 \\ \dot{S}_3 = \dot{\varepsilon}_3 + \mu_3 \varepsilon_3 \end{cases}. \tag{28}$$

We suppose that μ_1, μ_2, μ_3 are positive constants, and consider the integral sliding mode controls as follows:

$$\begin{cases} v_1 = a\varepsilon_1 - \varepsilon_3 - y_1 y_2 + x_1 x_2 - \mu_1 \varepsilon_1 - \alpha_1 \operatorname{sgn}(S_1) - k_1 S_1 \\ v_2 = -b\varepsilon_2 + |y_1| - |x_1| + y_1^2 - x_1^2 - \mu_2 \varepsilon_2 - \alpha_2 \operatorname{sgn}(S_2) - k_2 S_2 \\ v_3 = \varepsilon_1 + \varepsilon_3 - \mu_3 \varepsilon_3 - \alpha_3 \operatorname{sgn}(S_3) - k_3 S_3 \end{cases}. \tag{29}$$

In Eq (29), $\alpha_i, k_i, (i = 1,2,3)$ are positive constants.

Substituting (29) into the error Eq (26), we obtain:

$$\begin{cases} \dot{\varepsilon}_1 = -\mu_1 \varepsilon_1 - \alpha_1 \operatorname{sgn}(S_1) - k_1 S_1 \\ \dot{\varepsilon}_2 = -\mu_2 \varepsilon_2 - \alpha_2 \operatorname{sgn}(S_2) - k_2 S_2 \\ \dot{\varepsilon}_3 = -\mu_3 \varepsilon_3 - \alpha_3 \operatorname{sgn}(S_3) - k_3 S_3 \end{cases}. \tag{30}$$

Next, we shall establish the main synchronization result of this section.

Theorem 1. *The integral sliding mode control law (29) renders the drive financial system (23) and response system (24) synchronized globally for all values of initial states* $X(0), Y(0) \in R^3$, *where it is assumed that* $\mu_i, \alpha_i, k_i, (i = 1,2,3)$ *are positive constants.*

Proof. We start the proof by taking the Lyapunov function defined by

$$V(S_1, S_2, S_3) = \frac{1}{2}\left(S_1^2 + S_2^2 + S_3^2\right). \tag{31}$$

It is easy to see that V is a strictly positive definite function defined on R^3.

Next, we find that

$$\dot{V} = S_1 \dot{S}_1 + S_2 \dot{S}_2 + S_3 \dot{S}_3 = \sum_{i=1}^{3} \left[S_i (-\alpha_i \operatorname{sgn}(S_i) - k_i S_i) \right]. \tag{32}$$

A simple calculation gives

$$\dot{V} = -\sum_{i=1}^{3}\left[\alpha_i \,|\, S_i \,|+k_i S_i^2\right]. \tag{33}$$

Clearly, $\dot{V}$ is a negative definite function since $\alpha_i, k_i > 0$ for $i = 1,2,3$.

Using Lyapunov stability theory, we deduce that $S_i \to 0, (i = 1,2,3)$ as $t \to \infty$.

Hence, it follows that $\varepsilon_i \to 0, (i = 1.2.3)$ as $t \to \infty$. This completes the proof.■

For MATLAB simulations, the constants are taken as in the chaos case, *viz.* $a = 1$ and $b = 0.04$.

We assume the sliding constants as $\alpha_i = 0.2, k_i = 8$ and $\mu_i = 8$ for $i = 1,2,3$.

The initial state of the drive system (23) is taken as $X(0) = (1.4, 2.9, 8.3)$ and the initial state of the response system (24) as $Y(0) = (2.7, 10.4, 1.5)$.

Figure 9 shows the complete synchronization between the drive system (23) and response system (24) as the synchronization errors converge to zero in 1.5 sec. The fast convergence of the synchronization error shows the efficiency of the integral sliding mode control (ISMC) effect. Chaos synchronization has been carried out by using integral sliding mode control to synchronize the new financial chaotic system, taken as the master-slave systems. ISMC is a very useful technique for nonlinear control systems for both parametric and nonparametric uncertainties.

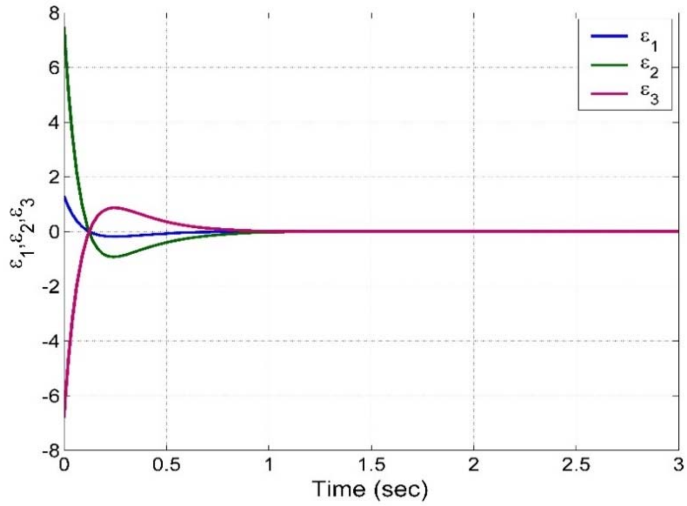

Figure 9. Time series showing the convergence of the synchronization errors between the drive system (23) and the response system (24).

4. Conclusions

In this paper, we proposed a new 3-D financial chaotic system and detailed about the novelty of the new financial system by comparing it with three financial chaotic systems in the literature, *viz.* Gao-Ma system [33], Vaidyanathan system [34] and Tacha system [35]. We showed that the proposed chaotic system is more chaotic than the three financial chaotic systems [33–35] by considering the maximal Lyapunov exponent (MLE) and the Kaplan-Yorke dimension of systems. We carried out a bifurcation analysis of the proposed financial system using bifurcation diagrams and Lyapunov exponents. Using integral sliding mode control and Lyapunov stability theory, we derived new results for global chaos synchronization for a pair of new financial chaotic systems taken as the drive-response systems. We illustrated all the main results of this study using MATLAB simulation plots.

Acknowledgments

We would like to thank Universitas Padjadjaran, which has provided the Universitas Padjadjaran Internal Research Grant, Fiscal Year 2022, the "ACADEMIC LEADERSHIP GRANT (ALG)" program under Prof. Dr. Sukono, MM., M.Si., with contract Number: 2203/UN6.3.1/PT.00/2022.

Conflict of interest

We declare that we have no conflict of interest.

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