Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 7(1): 1211–1223.
DOI: 10.3934/math.2022071
Received: 18 August 2021
Accepted: 11 October 2021
Published: 21 October 2021

Research article

Single valued neutrosophic (m, n)-ideals of ordered semirings

Saba Al-Kaseasbeh[1,*], **Madeline Al Tahan**[2], **Bijan Davvaz**[3] **and Mariam Hariri**[4]

[1] Department of Mathematics,Tafila Technical University, Tafila, Jordan
[2] Department of Mathematics, Lebanese International University, Lebanon
[3] Department of Mathematics, Yazd University, Yazd, Iran
[4] Department of Mathematics and Statistics, Zayed University, United Arab Emirates

* **Correspondence:** Email: saba.alkaseasbeh@gmail.com.

Abstract: The aim of this paper is to combine the innovative concept of single valued neutrosophic sets and ordered semirings. It studies ordered semirings by the properties of their single valued neutrosphic subsets. In this regard, we define single valued neutrosophic (m, n)-ideals (SVN-(m, n)-ideals) of ordered semirings. First, we illustrate our new definition by non-trivial examples. Second, we study these SVN-(m, n)-ideals under different operations of SVNS. Finally, we find a relationship between the (m, n)-ideals of ordered semirings and level sets by finding a necessary and sufficient condition for an SVNS of an ordered semiring R to be an SVN-(m, n)-ideal of R.

Keywords: semiring; ordered semiring; (m, n)-ideal; single valued neutrosophic set; single valued neutrosophic (m, n)-ideal
Mathematics Subject Classification: 16Y99

1. Introduction

A semiring is a natural generalization of ring and it has many applications to various fields such as optimization theory, max/min algebra, algebra of formal process, combinatorial optimization, automata theory, etc. Semirings were introduced explicitly by Vandiverin [21] in 1934. By considering the ring of rational numbers $(\mathbb{Q}, +, \cdot)$ that plays an important role in ring theory, we get the subset $(\mathbb{Q}^+ \cup \{0\}, +)$ of all non-negative rational numbers is an abelian additive semigroup which is closed under the usual multiplication of rational numbers, i.e., $(\mathbb{Q}^+ \cup \{0\}, +, \cdot)$ forms a semiring.

Fuzzy set, a generalization of the crisp set, was introduced in 1965 by Zadeh [23] where he assigned to each set's element a degree of belongingness (truth value: "t") with $0 \leq t \leq 1$. Intuitionistic fuzzy set, as an extension of the fuzzy set, was introduced in 1986 by Atanassov [8] where he assigned to each set's element a degree of belongingness (truth value: "t") and a degree of non-belongingness (falsity value: "f") with $0 \leq t, f \leq 1, 0 \leq t+f \leq 1$. Neutrosophic set, as an extension of intuitionistic fuzzy set, was introduced in 1998 by Smarandache [18] where he assigned to each set's element a truth value "T", indeterminacy value "I", and falsity value "F" with $0 \leq T, I, F \leq 1$. For detailed information about neutrosophic sets, we refer to [19, 20].

The connection between algebraic structures (hyperstructures) and each of fuzzy, intuitionistic fuzzy, and neutrosophic sets have been of great interest to many algebraists. Different concepts were introduced and studied. For more details, we refer to [3–7] for neutrosophic algebraic structures (hyperstructures) and to [1, 9, 10, 12, 15, 16] for fuzzy algebraic structures (hyperstructures).

In [3], Al-Tahan et al. discussed single valued neutrosophic (SVN) subsets of ordered groupoids and in [7], Akara et al. studied SVN subsets of ordered semigroups. In 2019, Mahboub et al. [15] studied fuzzy (m,n)-ideals in semigroups. Our paper is concerned about SVN-(m,n)-ideals in ordered semirings and it is structured as follows: In Section 2, we present basic definitions and examples on single valued neutrosophic sets and their operations. In Section 3, we discuss some results on (m,n)-ideals in ordered semirings. Finally in Section 4, we define SVN-(m,n)-ideals in ordered semirings, study their properties, present non-trivial examples on them, and find a relationship between (m,n)-ideals of ordered semirings and level sets.

2. Definitions and examples on SVNSs

In this section, we present some preliminaries related to single valued neutrosophic sets (SVNSs) and some of their operations.

Definition 2.1. [22] Let U be a non-empty space of elements. A single valued neutrosophic set (SVNS) A on U is characterized by the functions: T_A (truth-membership), I_A (indeterminacy-membership), and F_A (falsity-membership). Here, the range of each of these functions is a subset of the unit interval and A is denoted as follows:

$$A = \Big\{\frac{x}{(T_A(x), I_A(x), F_A(x))} : x \in U\Big\}.$$

Definition 2.2. [6] Let U be a non-empty set, A an SVNS over U, and $0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 1$. Then the $(\alpha_1, \alpha_2, \alpha_3)$-level set of A is defined as follows:

$$L_{(\alpha_1,\alpha_2,\alpha_3)} = \{x \in U : T_A(x) \geq \alpha_1, I_A(x) \leq \alpha_2, F_A(x) \leq \alpha_3\}.$$

Definition 2.3. [22] Let U be a non-empty set and A, B be SVNSs over U defined as follows.

$$A = \Big\{\frac{x}{(T_A(x), I_A(x), F_A(x))} : x \in U\Big\}, B = \{\frac{x}{(T_B(x), I_B(x), F_B(x))} : x \in U\Big\}.$$

Then

(1) A is called a single valued neutrosophic subset of B ($A \subseteq B$) if for every $x \in U$, we have $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, and $F_A(x) \geq F_B(x)$.
Moreover, A and B are said to be equal single valued neutrosophic sets ($A = B$) if A is a single valued neutrosophic subset of B and B is a single valued neutrosophic subset of A.

(2) The intersection of A and B is defined to be the SVNS over U:

$$A \cap B = \Big\{\frac{x}{(T_{A\cap B}(x), I_{A\cap B}(x), F_{A\cap B}(x))} : x \in U\Big\}.$$

Here, $T_{A\cap B}(x) = T_A(x) \wedge T_B(x)$, $I_{A\cap B}(x) = I_A(x) \vee I_B(x)$, and $F_{A\cap B}(x) = F_A(x) \vee F_B(x)$ for all $x \in U$.

(3) The union of A and B is defined to be the SVNS over U:

$$A \cup B = \Big\{\frac{x}{(T_{A\cup B}(x), I_{A\cup B}(x), F_{A\cup B}(x))} : x \in U\Big\}.$$

Here, $T_{A\cup B}(x) = T_A(x) \vee T_B(x)$, $I_{A\cup B}(x) = I_A(x) \wedge I_B(x)$, and $F_{A\cup B}(x) = F_A(x) \wedge F_B(x)$ for all $x \in U$.

Example 1. Let $U = \{m_0, m_1, m_2\}$ and A_1, A_2 be SVNS over U defined as follows.

$$A_1 = \Big\{\frac{m_0}{(0.67, 0.64, 0.45)}, \frac{m_1}{(0.83, 0.54, 0.32)}, \frac{m_2}{(0.12, 0.75, 1)}\Big\},$$

$$A_2 = \Big\{\frac{m_0}{(0.89, 0.01, 0.7)}, \frac{m_1}{(1, 0, 0.76)}, \frac{m_2}{(0.99, 0.53, 0.763)}\Big\}.$$

Then the SVNSs $A_1 \cup A_2$ and $A_1 \cap A_2$ over U are as follows.

$$A_1 \cup A_2 = \Big\{\frac{m_0}{(0.89, 0.01, 0.45)}, \frac{m_1}{(1, 0, 0.32)}, \frac{m_2}{(0.99, 0.53, 0.763)}\Big\},$$

$$A_1 \cap A_2 = \Big\{\frac{m_0}{(0.67, 0.64, 0.7)}, \frac{m_1}{(0.83, 0.54, 0.76)}, \frac{m_2}{(0.12, 0.75, 1)}\Big\}.$$

Definition 2.4. Let X, Y be non-empty sets, $f : X \to Y$ a function, and A be an SVNS over Y defined as follows.

$$A = \Big\{\frac{y}{(T_A(y), I_A(y), F_A(y))} : y \in Y\Big\}.$$

Then $f^{-1}(A)$ is a single valued neutrosophic set over X defined as follows.

$$f^{-1}(A) = \Big\{\frac{x}{(T_{f^{-1}(A)}(x), I_{f^{-1}(A)}(x), F_{f^{-1}(A)}(x))} : x \in X\Big\}.$$

Here, $T_{f^{-1}(A)}(x) = T_A(f(x))$, $I_{f^{-1}(A)}(x) = I_A(f(x))$, and $F_{f^{-1}(A)}(x) = F_A(f(x))$.

Definition 2.5. Let X, Y be non-empty sets, A, B be SVNSs over X, Y respectively defined as follows.

$$A = \Big\{\frac{x}{(T_A(x), I_A(x), F_A(x)} : x \in X\Big\}, B = \Big\{\frac{y}{(T_A(y), I_A(y), F_A(y))} : y \in Y\Big\}.$$

Then $A \times B$ is an SVNS over $X \times Y$ defined as follows.

$$A \times B = \Big\{\frac{(x, y)}{(T_{A\times B}((x, y)), I_{A\times B}((x, y)), F_{A\times B}((x, y)))} : x \in X, y \in Y\Big\}.$$

Here, $T_{A\times B}((x, y)) = T_A(x) \wedge T_B(y)$, $I_{A\times B}((x, y)) = I_A(x) \vee I_B(y)$, and $F_{A\times B}((x, y)) = F_A(x) \vee F_B(y)$.

3. (m, n)-ideals of ordered semirings

In this section, we deal with ordered semirings by presenting some results related to their (m, n)-ideals. For more details about (ordered) semirings, we refer to the books [13, 14].

Definition 3.1. [14] Let R be a non-empty set with binary operations "+" and "·". Then $(R, +, \cdot)$ is called a *semiring* if the following conditions hold for all $a, b, c \in R$:

(1) $(R, +)$ is an abelian semigroup with identity "0";
(2) $(R, \cdot)$ is a semigroup with "0" as bilaterally absorbing element (i.e. $a \cdot 0 = 0 \cdot a = 0$);
(3) $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$.

A semiring $(R, +, \cdot)$ is called *semifield* if $(R, \cdot)$ is a commutative semigroup with unity "1", and for every non-zero element $r_1 \in R$, there exists $r_2 \in R$ with $r_1 \cdot r_2 = 1$.

Definition 3.2. [13] Let $(R, +, \cdot)$ be a semiring with $0 \in R$ and "$\leq$" be a partial order on R. Then $(R, +, \cdot, \leq)$ is called an *ordered semiring* if the following conditions are satisfied for all $x, y, a \in R$.

(1) If $x \leq y$ then $a + x \leq a + y$;
(2) If $x \leq y$ then $a \cdot x \leq a \cdot y$ and $x \cdot a \leq y \cdot a$ for all $0 \leq a$.

Remark 1. Every semiring R is an ordered semiring under the trivial order (i.e., $u \leq v$ if and only if $u = v$ for all $u, v \in R$).

Example 2. [11] Let $Q^+ \cup \{0\}$ be the set of non-negative rational numbers, $\mathbb{R}$ be the set of real numbers, and "$\leq$" be the usual order of real numbers. Then $(Q^+ \cup \{0\}, +, \cdot, \leq)$ and $(\mathbb{R} \cup \{\pm\infty\}, \vee, \wedge, \leq)$ are examples of infinite ordered semirings. Here "$\wedge$" and "$\vee$" denote the minimum and maximum respectively.

Example 3. Let $\mathbb{R}$ be the set of real numbers. Then $(\mathbb{R} \cup \{-\infty\}, \vee, +, \leq)$ is an ordered semifield.

Example 4. [16] Let "$\oplus$" and "$\odot$" be the operations defined on the unit interval $[0, 1]$ as follows: For all $x, y \in [0, 1]$,

$$x \oplus y = x \vee y \qquad \text{and} \qquad x \odot y = (x + y - 1) \vee 0.$$

Then $([0, 1], \oplus, \odot, \leq)$ is an ordered semiring (with "0" as a zero element) under the usual order of real numbers. Moreover, it is not an ordered semifield.

We can present finite semirings by means of Cayley's tables.

Example 5. Let $M = \{0, t_1, t_2\}$ and define $(M, \oplus, \odot)$ by the following tables.

$\oplus$	0	t_1	t_2
0	0	t_1	t_2
t_1	t_1	t_1	t_2
t_2	t_2	t_2	t_2

$\odot$	0	t_1	t_2
0	0	0	0
t_1	0	0	0
t_2	0	0	0

Then $(M, \oplus, \odot)$ is a semiring. By defining the partial order "$\leq$" on M as follows

$$\leq = \{(0, 0), (0, t_1), (0, t_2), (t_1, t_1), (t_1, t_2), (t_2, t_2)\},$$

we get that $(M, \oplus, \odot)$ is a commutative ordered semiring.

Example 6. Let $N = \{0, 1, 2, 3\}$ and define $(N, \boxplus, \boxdot)$ by the following tables.

$\boxplus$	0	1	2	3
0	0	1	2	3
1	1	1	3	3
2	2	3	2	2
3	3	3	2	2

$\boxdot$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	1	2	3
3	0	1	2	3

Then $(N, \boxplus, \boxdot)$ is a semiring. By defining the partial order "$\leq$" on N as follows

$$\leq = \{(0, 1), (0, 1), (1, 1), (2, 2), (2, 3), (3, 3)\},$$

we get that $(N, \boxplus, \boxdot, \leq)$ is an ordered semiring.

Definition 3.3. Let $(R, +, \cdot, \leq)$ be an ordered semiring and $S \neq \emptyset \subseteq R$. Then S is a subsemiring of R if the following conditions hold.

(1) $0 \in S$;
(2) $S + S \subseteq S$ and $S^2 \subseteq S$;
(3) $(S] = \{x \in R : x \leq s \text{ for some } s \in S\} \subseteq S$.

Example 7. Let $(N, \boxplus, \boxdot, \leq)$ be the ordered semiring of Example 6. Then $\{0, 1\}$ is a subsemiring of N.

Ideals play an important role in semiring theory. Some generalizations of ideals were established so that one can study a semiring by the properties of its (generalized) ideals.

Definition 3.4. Let $(R, +, \cdot, \leq)$ be an ordered semiring, I a subsemiring of R, and m, n be non-negative integers with $(m, n) \neq (0, 0)$. Then I is:

(1) a left ideal of R if $RI \subseteq I$.
(2) a right ideal of R if $IR \subseteq I$.
(3) m a bi-ideal of R if $IRI \subseteq I$.
(4) an (m, n)- ideal of R if $I^mRI^n \subseteq I$.

Remark 2. Let $(R, +, \cdot, \leq)$ be an ordered semiring and $I \subseteq R$. Then the following statements hold.

(1) If I is an ideal of R then I is a left (right) ideal, bi-ideal, and an (m, n)-ideal of R for all $m, n \geq 1$.
(2) If I is a left ideal of R and $n \geq 1$ then I is a $(0, n)$-ideal of R.
(3) m If I is a right ideal of R and $m \geq 1$ then I is an $(m, 0)$-ideal of R.
(4) m If I is an (m, n)-ideal of R and $s \geq m, t \geq n$ then I is an (s, t)-ideal of R.

Proposition 3.5. *Let $(R, +, \cdot, \leq)$ be an ordered semifield, $I \neq \emptyset \subseteq R$, and m, n be non-negative integers with $(m, n) \neq (0, 0)$. If I is an (m, n)-ideal of R then $I = \{0\}$ or $I = R$. Moreover, $I = \{0\}$ is an (m, n)-ideal of R if and only if $(\{0\}] = \{0\}$.*

Proof. Let $I \neq \{0\}$ be an (m, n)-ideal of R. Since R is commutative, it follows that I is an $(m+n, 0)$-ideal of R. Let $x \neq 0 \in I$. Then $1 = \underbrace{x^{-1} \dots x^{-1}}_{m+n \text{ times}} x^{m+n} \in I$. For any $r \in R$, we have $r = \underbrace{1 \dots 1}_{m+n \text{ times}} r \in I$. Therefore, $I = R$.

It is clear that $I = \{0\}$ is an (m, n)-ideal of R if and only if $(\{0\}] = \{0\}$. $\square$

Example 8. Let $(\mathbb{R} \cup \{-\infty\}, \vee, +, \leq)$ be the ordered semifield in Example 3 and m, n be non-negative integers with $(m, n) \neq (0, 0)$. Proposition 3.5 asserts that $\{-\infty\}$ and $\mathbb{R}$ are the only (m, n)-ideals of $\mathbb{R} \cup \{-\infty\}$.

Definition 3.6. Let $(R_1, +_1, \cdot_1, \leq_1)$ and $(R_2, +_2, \cdot_2, \leq_2)$ be ordered semirings and $\phi : R_1 \to R_2$ be a function. Then ϕ is an ordered semiring homomorphism if the following conditions hold for all $r, s \in R_1$.

(1) $\phi(r +_1 s) = \phi(r) +_2 \phi(s)$;
(2) $\phi(r \cdot_1 s) = \phi(r) \cdot_2 \phi(s)$;
(3) If $r \leq_1 s$ then $\phi(r) \leq_2 \phi(s)$.

If ϕ is a bijective ordered semiring homomorphism then ϕ is an *ordered semiring isomorphism* and R_1 and R_2 are said to be isomorphic ordered semirings.

Theorem 3.7. *Let $(R_1, +_1, \cdot_1, \leq_1)$ and $(R_2, +_2, \cdot_2, \leq_2)$ be ordered semirings. Then $(R_1 \times R_2, +, \cdot, \leq)$ is an ordered semiring, where "+", "·", and "≤" is defined as follows for all $(r, s), (r', s') \in R_1 \times R_2$.*

$$((r, s) + (r', s') = (r +_1 r', s + s'), (r, s) \cdot (r', s') = (r \cdot_1 r', s \cdot_2 s'),$$

$$(r, s) \leq (r', s') \textit{ if and only if } r \leq_1 r' \textit{ and } s \leq_2 s'.$$

4. SVN-(m, n)-ideals in ordered semirings

In this section, we define SVN-(m, n)-ideals of ordered semirings and present some related non-trivial examples. Furthermore, we study various properties of them and find a relationship between their level sets and the (m, n)-ideals.

Notation 1. *Let U be a non-empty set, $x, y \in U$, and A be an SVNS over U. Then $N_A(x) \geq N_A(y)$ is equivalent to $T_A(x) \geq T_A(y)$, $I_A(x) \leq I_A(y)$, $F_A(x) \leq F_A(y)$.*

Definition 4.1. Let $(R, +, \cdot, \leq)$ be an ordered semiring and A an SVNS on R. Then A is an SVN-subsemiring of R if the following conditions hold for all $x, y \in R$.

(1) $N_A(x + y) \geq N_A(x) \wedge N_A(y)$;
(2) $N_A(x \cdot y) \geq N_A(x) \wedge N_A(y)$;
(3) If $x \leq y$ then $N_A(x) \geq N_A(y)$.

Definition 4.2. Let $(R, +, \cdot, \leq)$ be an ordered semiring and A an SVNS on R. Then A is an SVN-left (right) ideal of R if the following conditions hold for all $x, y \in R$.

(1) $N_A(x + y) \geq N_A(x) \wedge N_A(y)$;
(2) $N_A(x \cdot y) \geq N_A(y)$ (respectively $N_A(x \cdot y) \geq N_A(x)$);
(3) If $x \leq y$ then $N_A(x) \geq N_A(y)$.

Moreover, A is an SVN-ideal of R if it is both: an SVN-left ideal of R and an SVN-right ideal of R.

Example 9. Let $([0, 1], \oplus, \odot, \leq)$ be the ordered semiring of Example 4 and A be the SVNS on $[0, 1]$ defined as follows:

$$N_A(x) = \begin{cases} (0.6, 0.3, 0.2) & \text{if } 0 \leq x \leq 0.5; \\ (0.3, 0.4, 0.7) & \text{otherwise.} \end{cases}$$

Then A is an SVN-ideal of $[0, 1]$.

Next, we give an example on an SVN-right ideal of R that is not an SVN-ideal of R.

Example 10. Let $O = \{0, k_1, k_2, k_3\}$ and define $(O, +, \cdot)$ by the following tables.

$+$	0	k_1	k_2	k_3
0	0	k_1	b	k_3
k_1	k_1	k_1	k_1	k_1
k_2	k_2	k_1	k_2	k_3
k_3	k_3	k_1	k_3	k_3

$\cdot$	0	k_1	k_2	k_3
0	0	0	0	0
k_1	0	k_1	k_1	k_1
k_2	0	k_2	k_2	k_2
k_3	0	k_1	k_1	k_1

Then $(O, +, \cdot)$ is a semiring [17]. By considering the trivial order "$\leq$" on O, we get that $(O, +, \cdot, \leq)$ is an ordered semiring. Let A be the SVNS on R defined as follows:

$$A = \left\{\frac{0}{(0.7, 0.2, 0.35)}, \frac{k_1}{(0.65, 0.32, 0.4)}, \frac{k_2}{(0.1, 0.6, 0.9)}, \frac{k_3}{(0.1, 0.6, 0.9)}\right\}.$$

Then A is an SVN-right ideal of R. Moreover, it is not an SVN-left ideal of R as $N_A(b) = N_A(a \cdot b) \ngeq N_A(a)$. Thus, it is not an SVN-ideal of R.

Definition 4.3. Let $(R, +, \cdot, \leq)$ be an ordered semiring, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A an SVNS on R. Then A is an SVN-(m, n)-ideal of R if the following conditions hold for all $x, y, x_1, \ldots, x_m, y_1, \ldots, y_n, r \in R$.

(1) $N_A(x + y) \geq N_A(x) \wedge N_A(y)$;
(2) $N_A(x_1 \cdot \ldots \cdot x_m \cdot r \cdot y_1 \ldots \cdot y_n) \geq N_A(x_1) \wedge \ldots \wedge N_A(x_m) \wedge N_A(y_1) \wedge \ldots \wedge N_A(y_n)$;
(3) If $x \leq y$ then $N_A(x) \geq N_A(y)$.

Moreover, if $m = n = 1$ then A is an SVN-bi-ideal of R.

Next, we give an example on an SVN-bi-ideal of an ordered semiring that is not an SVN-left ideal nor an SVN-right ideal.

Example 11. Let M be the set of non-negative real numbers and $(M_2(M), +, \cdot)$ be the semiring of all 2×2 matrices with non-negative real entries under usual addition and multiplication of matrices. By defining "$\leq$" on $M_2(M)$ to be the trivial order, we get that $(M_2(M), +, \cdot, \leq)$ is an ordered semiring.

Let A be the SVNS on $M_2(M)$ defined as follows:

$$N_A(\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}) = \begin{cases} (0.9, 0.2, 0.204) & \text{if } a_{12} = a_{21} = a_{22} = 0; \\ (0.54, 0.367, 0.359) & \text{otherwise.} \end{cases}$$

Then A is an SVN-bi-ideal of $M_2(S)$.

Since $N_A(\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}) = (0.54, 0.367, 0.359) \ngeq (0.9, 0.2, 0.204) = N_A(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix})$, it follows that A is not an SVN-left ideal of $M_2(M)$. Furthermore, because $N_A(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}) = N_A(\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}) = (0.54, 0.367, 0.359) \ngeq (0.9, 0.2, 0.204) = N_A(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix})$, it follows that A is not an SVN-right ideal of $M_2(M)$.

Remark 3. Let $(R, +, \cdot, \leq)$ be an ordered semiring, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A an SVNS on R. Then the following are true.

(1) If A is an SVN-ideal of R then A is an SVN left (right)-SVN-ideal, SVN-bi-ideal, and an SVN-(m, n)-ideal of R for all $m, n \geq 1$.
(2) If A is an SVN-left ideal of R then A is an SVN-$(0, n)$-ideal of R for all $n \geq 1$.
(3) If A is an SVN-right ideal of R then A is an SVN-$(m, 0)$-ideal of R for all $m \geq 1$.
(4) If A is an SVN-(m, n)-ideal of R then A is an SVN-(s, t)-ideal of R for all $s \geq m, t \geq n$.

Next, we give an example on an SVN-(m, n)-ideal of an ordered semiring that is not an SVN-bi-ideal.

Example 12. Let $T = \mathbb{Q}^+ \cup \{0\}$ and $(M_3(T), +, \cdot)$ be the semiring of all 3×3 matrices with non-negative rational entries under usual addition and multiplication of matrices. By defining "$\leq$" on $M_3(T)$ to be the trivial order, we get that $(M_3(T), +, \cdot, \leq)$ is an ordered semiring.

Let A be the SVNS on $M_3(T)$ defined as follows:

$$N_A(\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}) = \begin{cases} (0.889, 0.12, 0.0322) & \text{if } a_{11} = a_{21} = a_{22} = a_{31} = a_{32} = a_{33} = 0; \\ (0.687, 0.597, 0.549) & \text{otherwise.} \end{cases}$$

Then A is an SVN-$(2, 1)$-ideal of $M_3(T)$ that is not an SVN-bi-ideal of $M_3(T)$. Moreover, A is an SVN-$(m, 0)$-ideal of $M_3(T)$ and an SVN-$(0, n)$-ideal of $M_3(T)$ for all $m, n \geq 3$.

Proposition 4.4. *Let $(R, +, \cdot, \leq)$ be an ordered semiring, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A an SVNS on R. If A is an SVN-(m, n)-ideal of R then $N_A(0) \geq N_A(x)$ for all $x \in R$.*

Proof. Having A an SVN-(m, n)-ideal of R implies that $N_A(0) = N_A(x^m \cdot 0 \cdot x^n) \geq N_A(x)$ for all $x \in R$. □

Proposition 4.5. *Let $(R, +, \cdot, \leq)$ be an ordered semifield, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A an SVNS on R. If A is an SVN-(m, n)-ideal of R then $N_A(r) = N_A(1)$ for all $r \in R - \{0\}$.*

Proof. Having A an SVN-(m, n)-ideal of R and R commutative implies that $N_A(r) = N_A(1^{m+n} \cdot r) \geq N_A(1)$ for all $r \in R$. Moreover, we have $N_A(1) = N_A(r^{-m-n} \cdot r^{m+n}) \geq N_A(r)$ for all $r \in R - \{0\}$. Therefore, $N_A(r) = N_A(1)$ for all $x \in R - \{0\}$. □

Example 13. Let $(\mathbb{R} \cup \{-\infty\}, \vee, +, \leq)$ be the ordered semifield in Example 3, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A be the SVNS on $\mathbb{R} \cup \{-\infty\}$ defined as follows.

$$N_A(x) = \begin{cases} (0.687, 0.34, 0.28) & \text{if } x \in [1, \infty[; \\ (0.956, 0.14, 0.08) & \text{otherwise.} \end{cases}$$

Proposition 4.5 asserts that A is not an SVN-(m, n)-ideal of $\mathbb{R} \cup \{-\infty\}$.

Next, we deal with some operations involving SVN-(m, n)-ideals of ordered semirings.

Lemma 4.6. *Let $(R_1, +_1, \cdot_1, \leq_1)$ and $(R_2, +_2, \cdot_2, \leq_2)$ be ordered semirings, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A, B be SVN-$(m, 0)$-ideal and SVN-$(0, n)$-ideal of R_1, R_2 respectively. Then $A \times B$ is an SVN-(m, n)-ideal of $R_1 \times R_2$.*

Proof. Let $(x, y), (z, w) \in R_1 \times R_2$. Then $N_{A\times B}(x +_1 z, y +_2 w) = N_A(x +_1 z) \wedge N_A(y +_2 w)$. Having A, B SVN-$(m, 0)$-ideal and SVN-$(0, n)$-ideal of R_1, R_2 respectively implies that $N_{A\times B}(x+_1 z, y+_2 w) \geq N_A(x)\wedge N_A(z)\wedge N_B(y)\wedge N_B(w) = N_{A\times B}(x, y)\wedge N_{A\times B}(z, w)$. Let $(x_1, y_1), \ldots, (x_m, y_m), (r_1, r_2), (z_1, w_1), \ldots, (z_n, w_n) \in R_1 \times R_2$,
$v = (x_1, y_1)\ldots(x_m, y_m)(r_1, r_2)(z_1, w_1)\ldots(z_n, w_n)$, $v_1 = x_1 \ldots x_m r_1 z_1 \ldots z_n$, and $v_2 = y_1 \ldots y_m r_2 w_1 \ldots w_n$. Then $N_{A\times B}(v) = N_{A\times B}((v_1, v_2)) = N_A(v_1) \wedge N_B(v_2)$. Having A, B SVN-$(m, 0)$-ideal and SVN-$(0, n)$-ideal of R_1, R_2 respectively implies that $N_A(v_1) = N_A(x_1 \ldots x_m(r_1 z_1 \ldots z_n)) \geq N_A(x_1) \wedge \ldots \wedge N_A(x_m)$ and $N_B(v_2) = N_B((y_1 \ldots y_m r_2)w_1 \ldots w_n) \geq N_B(w_1) \wedge \ldots \wedge N_B(w_n)$. The latter implies that $N_{A\times B}(v) \geq N_A(x_1) \wedge \ldots \wedge N_A(x_m) \wedge N_B(w_1) \wedge \ldots \wedge N_B(w_n) \geq N_A(x_1) \wedge \ldots \wedge N_A(x_m) \wedge N_A(z_1) \ldots \wedge N_A(z_n) \wedge N_B(y_1) \wedge \ldots \wedge N_B(y_m) \wedge N_B(w_1) \wedge \ldots \wedge N_B(w_n)$ and hence,

$$N_{A\times B}(v) \geq N_{A\times B}((x_1, y_1)) \wedge \ldots \wedge N_{A\times B}((x_m, y_m)) \wedge N_{A\times B}((z_1, w_1)) \wedge \ldots \wedge N_{A\times B}((z_n, w_n)).$$

Let $(x, y) \leq (z, w) \in R_1 \times R_2$. Having $x \leq_1 z, y \leq_2 w$ and A, B SVN-$(m, 0)$-ideal and SVN-$(0, n)$-ideal of R_1, R_2 respectively implies that $N_A(x) \geq N_A(z)$ and $N_B(y) \geq N_B(w)$. The latter implies that $N_{A\times B}((x, y)) \geq N_{A\times B}((z, w))$. Therefore, $A \times B$ is an SVN-(m, n)-ideal of $R_1 \times R_2$. □

Corollary 4.7. Let $(R_1, +_1, \cdot_1, \leq_1)$ and $(R_2, +_2, \cdot_2, \leq_2)$ be ordered semirings and A, B be SVN-right-ideal and SVN-left-ideal of R_1, R_2 respectively. Then $A \times B$ is an SVN-bi-ideal of $R_1 \times R_2$.

Example 14. Let $O^{\circledast} = \{0, p_1, p_2, p_3\}$ and define $(O^{\circledast}, +^{\circledast}, \cdot^{\circledast})$ by the following tables.

$+^{\circledast}$	0	p_1	p_2	p_3
0	0	p_1	p_2	p_3
p_1	p_1	p_1	p_1	p_1
p_2	p_2	p_1	p_2	p_3
p_3	p_3	p_1	p_3	p_3

$\cdot^{\circledast}$	0	p_1	p_2	p_3
0	0	0	0	0
p_1	0	p_1	p_2	p_3
p_2	0	p_1	p_2	p_3
p_3	0	p_1	p_2	p_3

Then $(O^{\circledast}, +^{\circledast}, \cdot^{\circledast})$ is a semiring. By defining the trivial order "$\leq$" on $O^{\circledast}$, we get that $(O^{\circledast}, +^{\circledast}, \cdot^{\circledast}, \leq)$ is an ordered semiring [2]. Let $A^{\circledast}$ be the SVNS on $O^{\circledast}$ defined as follows:

$$A^{\circledast} = \left\{\frac{0}{(0.7, 0.2, 0.35)}, \frac{p_1}{(0.65, 0.32, 0.4)}, \frac{p_2}{(0.1, 0.6, 0.9)}, \frac{p_3}{(0.1, 0.6, 0.9)}\right\}.$$

Then $A^{\circledast}$ is an SVN-left ideal of $O^{\circledast}$. By taking the ordered semiring $(O, +, \cdot, \leq)$ in Example 10 and its SVN-right ideal A, we get by using Corollary 4.7 that $A \times B$ is an SVN-bi-ideal of $O \times O^{\circledast}$. Here, $A \times A^{\circledast}$ is defined as follows.

$$N_A((x, y)) = \begin{cases} (0.7, 0.2, 0.35) & \text{if } (x, y) = (0, 0); \\ (0.65, 0.32, 0.4) & \text{if } x \in \{0, k_1\}, y \in \{0, p_1\}, \text{ and } (x, y) \neq (0, 0); \\ (0.1, 0.6, 0.9) & \text{otherwise.} \end{cases}$$

Proposition 4.8. *Let $(R, +, \cdot, \leq)$ be an ordered semiring, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A_1, A_2 be SVN-(m, n)-ideals of R. Then $A_1 \cap A_2$ is an SVN-(m, n)-ideal of R.*

Proof. The proof is straightforward. □

Remark 4. Let $(R, +, \cdot, \leq)$ be an ordered semiring, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A, B be SVN-(m, n)-ideals of R. Then $A \cup B$ is not necessary an SVN-(m, n)-ideal of R.

Example 15. Let $(N, \boxplus, \boxdot, \leq)$ be the ordered semiring of Example 6 and A, B be the SVNS on R defined as follows.

$$A = \Big\{\frac{0}{(0.67, 0.02, 0.135)}, \frac{1}{(0.55, 0.25, 0.24)}, \frac{2}{(0.21, 0.46, 0.89)}, \frac{3}{(0.21, 0.46, 0.89)}\Big\},$$

$$B = \Big\{\frac{0}{(0.63, 0.01, 0.335)}, \frac{1}{(0.321, 0.56, 0.91)}, \frac{2}{(0.55, 0.15, 0.44)}, \frac{3}{(0.321, 0.56, 0.91)}\Big\}.$$

It is easy to see that A, B are SVN-$(0, 1)$-ideals of N. Having

$$A \cup B = \Big\{\frac{0}{(0.67, 0.01, 0.135)}, \frac{1}{(0.55, 0.25, 0.24)}, \frac{2}{(0.55, 0.15, 0.44)}, \frac{3}{(0.321, 0.46, 0.89)}\Big\},$$

$3 = 1 + 2$, and $N_{A\cup B}(3) \ngeq N_{A\cup B}(1) \wedge N_{A\cup B}(2)$ implies that $A \cup B$ is not an SVN-$(0, 1)$-ideal of N.

Lemma 4.9. *Let $(R_1, +_1, \cdot_1, \leq_1)$ and $(R_2, +_2, \cdot_2, \leq_2)$ be ordered semirings, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A, B be SVN-(m, n)-ideals of R_1, R_2 respectively. Then $A \times B$ is an SVN-(m, n)-ideal of $R_1 \times R_2$.*

Proof. The proof is similar to that of Lemma 4.6. □

Proposition 4.10. *Let $(R, +, \cdot, \leq)$ be an ordered semiring with $a^2 = a$ for all $a \in R$, m, n be positive integers, and A an SVNS on R. Then A is an SVN-(m, n)-ideal of R if and only if A is an SVN-bi-ideal of R.*

Proof. The proof is straightforward as $x^m r y^n = xry$ for all $x, y, r \in R$. □

Proposition 4.11. *Let $(R, +, \cdot, \leq)$ be a commutative ordered semiring with $a^2 = a$ for all $a \in R$, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and A an SVNS on R. Then A is an SVN-(m, n)-ideal of R if and only if A is an SVN-ideal of R.*

Proof. The proof is straightforward as $x^{m+n} = x$ for all $x \in R$. □

Next, we show that the pre-image of an SVN-(m, n)-ideal under ordered semiring homomorphism is an SVN-(m, n)-ideal.

Proposition 4.12. *Let $(R_1, +_1, \cdot_1, \leq_1)$ and $(R_2, +_2, \cdot_2, \leq_2)$ be ordered semirings, A an SVNS on R_2, m, n be non-negative integers with $(m, n) \neq (0, 0)$, and $f : R_1 \to R_2$ an ordered semiring homomorphism. If A is an SVN-(m, n)-ideal of R_2 then $f^{-1}(A)$ is an SVN-(m, n)-ideal of R_1.*

Proof. Let $x, y \in R_1$. Then $N_{f^{-1}(A)}(x +_1 y) = N_A(f(x +_1 y)) = N_A(f(x) +_2 f(y))$. Having A an SVN-(m, n)-ideal of R_2 implies that $N_A(f(x) +_2 f(y)) \geq N_A(f(x)) \wedge N_A(f(y)) = N_{f^{-1}(A)}(x) \wedge N_{f^{-1}(A)}(y)$ and hence, $N_{f^{-1}(A)}(x +_1 y) \geq N_{f^{-1}(A)}(x) \wedge N_{f^{-1}(A)}(y)$. Let $x_1, \ldots, x_m, r, y_1, \ldots, y_n \in R_1$. Then

$$N_{f^{-1}(A)}(x_1 \cdot_1 \ldots \cdot_1 x_m \cdot_1 r \cdot_1 y_1 \cdot_1 \ldots \cdot_1 y_m) = N_A(f(x_1 \cdot_1 \ldots \cdot_1 x_m \cdot_1 r \cdot_1 y_1 \cdot_1 \ldots \cdot_1 y_m)).$$

Since f is a homomorphism, it follows that

$$N_{f^{-1}(A)}(x_1 \cdot_1 \ldots \cdot_1 x_m \cdot_1 r \cdot_1 y_1 \cdot_1 \ldots \cdot_1 y_m) = N_A(f(x_1) \cdot_2 \ldots \cdot_2 f(x_m) \cdot_2 f(r) \cdot_2 f(y_1) \cdot_2 \ldots \cdot_2 f(y_m)).$$

The latter and having A an SVN-(m,n)-ideal of R_2 implies that for $z = x_1 \cdot_1 \ldots \cdot_1 x_m \cdot_1 r \cdot_1 y_1 \cdot_1 \ldots \cdot_1 y_m$,

$$N_{f^{-1}(A)}(z) \geq N_{f^{-1}(A)}(x_1) \wedge \ldots N_{f^{-1}(A)}(x_m) \wedge N_{f^{-1}(A)}(y_1) \wedge \ldots \wedge N_{f^{-1}(A)}(y_n).$$

Let $x \leq_1 y \in R_1$. Having $f(x) \leq_2 f(y) \in R_2$ and A an SVN-(m,n)-ideal of R_2 implies that $N_A(f(x)) \geq N_A(f(y))$. The latter implies that $N_{f^{-1}(A)}(x) \geq N_{f^{-1}(A)}(y)$. Therefore, $f^{-1}(A)$ is an SVN-(m,n)-ideal of R_1. □

Corollary 4.13. Let $(R,+,\cdot,\leq)$ be an ordered semiring, m,n be non-negative integers with $(m,n) \neq (0,0)$, and $A = \{\frac{x}{(T_A(x),I_A(x),F_A(x))} : x \in R\}$ an SVNS on R, and S a subsemiring of R. If A is an SVN-(m,n)-ideal of R then $A' = \{\frac{x}{(T_A(x),I_A(x),F_A(x))} : x \in S\}$ is an SVN-(m,n)-ideal of S.

Proof. Let $f : S \to R$ be the inclusion map (i.e. $f(x) = x$ for all $x \in S$). Proposition 4.12 asserts that $f^{-1}(A) = A'$ is an SVN-(m,n)-ideal of S. □

Next, we find a relationship between (m,n)-ideals of ordered semirings and level sets.

Lemma 4.14. *Let $(R,+,\cdot,\leq)$ be an ordered semiring, m,n be non-negative integers with $(m,n) \neq (0,0)$, and $I \subseteq R$ an (m,n)-ideal of R. Then I is a level set of an SVN-(m,n)-ideal of R.*

Proof. Let A be the SVNS on R defined as follows:

$$N(x) = \begin{cases} (1,0,0) & \text{if } x \in I; \\ (0,1,1) & \text{otherwise.} \end{cases}$$

One can easily see that A is an SVN-(m,n)-ideal of R and that $A_{(1,0,0)} = I$. □

Theorem 4.15. *Let $(R,+,\cdot,\leq)$ be an ordered semiring, m,n be non-negative integers with $(m,n) \neq (0,0)$, $0 \leq \alpha,\beta,\gamma \leq 1$, and A an SVNS on R. Then A is an SVN-(m,n)-ideal of R if and only if $A_{(\alpha,\beta,\gamma)}$ is either the empty set or an (m,n)-ideal of R.*

Proof. Let A be an SVN-(m,n)-ideal of R and $x,y \in A_{(\alpha,\beta,\gamma)} \neq \emptyset$. Then $N_A(x), N_A(y) \geq (\alpha,\beta,\gamma)$. Having A an SVN-(m,n)-ideal of R implies that $N_A(x+y) \geq N_A(x) \wedge N_A(y) \geq (\alpha,\beta,\gamma)$ and hence $x + y \in A_{(\alpha,\beta,\gamma)}$. Let $x_1,\ldots,x_m,y_1,\ldots,y_n \in A_{(\alpha,\beta,\gamma)}$ and $r \in R$. Having A an SVN-(m,n)-ideal of R implies that $N_A(x_1 \ldots x_m r y_1 \ldots y_n) \geq N_A(x_1) \wedge \ldots N_A(x_m) \wedge N_A(y_1) \wedge \ldots \wedge N_A(y_n) \geq (\alpha,\beta,\gamma)$ and hence, $x_1 \ldots x_m r y_1 \ldots y_n \in A_{(\alpha,\beta,\gamma)}$. Let $y \in A_{(\alpha,\beta,\gamma)}$ and $x \leq y$. Having A an SVN-(m,n)-ideal of R implies that $N_A(y) \geq N_A(x) \geq (\alpha,\beta,\gamma)$ and hence, $y \in A_{(\alpha,\beta,\gamma)}$. Thus, $A_{(\alpha,\beta,\gamma)}$ is an (m,n)-ideal of R.

Conversely, let $x,y \in R$ with $N_A(x) \wedge N_A(y) = (\alpha,\beta,\gamma)$. Having $A_{(\alpha,\beta,\gamma)} \neq \emptyset$ an (m,n)-ideal of R and $x,y \in A_{(\alpha,\beta,\gamma)}$ implies that $x + y \in A_{(\alpha,\beta,\gamma)}$ and hence $N_A(x+y) \geq (\alpha,\beta,\gamma) = N_A(x) \wedge N_A(y)$. Let $x_1,\ldots,x_m,r,y_1,\ldots,y_n \in R$ with $N_A(x_1) \wedge \ldots \wedge N_A(x_m) \wedge N_A(y_1) \wedge \ldots \wedge N_A(y_n) = (\alpha',\beta',\gamma')$. Having $A_{(\alpha',\beta',\gamma')} \neq \emptyset$ an (m,n)-ideal of R implies that $x_1 \ldots x_m r y_1 \ldots y_n \in A_{(\alpha',\beta',\gamma')}$ and hence, $N_A(x_1 \ldots x_m r y_1 \ldots y_n) \geq (\alpha',\beta',\gamma') = N_A(x_1) \wedge \ldots \wedge N_A(x_m) \wedge N_A(y_1) \wedge \ldots \wedge N_A(y_n)$. Let $x \leq y$ and $N_A(y) = (\alpha_0,\beta_0,\gamma_0)$. Having $A_{(\alpha_0,\beta_0,\gamma_0)} \neq \emptyset$ an (m,n)-ideal of R and $y \in A_{(\alpha_0,\beta_0,\gamma_0)}$ implies that $x \in A_{(\alpha_0,\beta_0,\gamma_0)}$ and hence, $N_A(x) \geq (\alpha_0,\beta_0,\gamma_0) = N_A(y)$. Thus, A is an SVN-(m,n)-ideal of R. □

5. Conclusions

This paper added to the neutrosophic algebraic structures theory by considering SVN subsets of ordered semirings. The concept of SVN-(m, n)-ideals of ordered semirings was defined and studied. Having every SVN-ideal (bi-ideal) of ordered semiring an SVN-(m, n)-ideal makes our results a generalization of SVN-ideals (bi-ideals) of ordered semirings. Also, having SVNSs a generalization of both: fuzzy sets and intuitionistic fuzzy sets makes the study of fuzzy-(m, n)-ideals and intuitionistic fuzzy-(m, n)-ideals of ordered semirings special cases of our study.

It is well known that the concept of single valued neutrosophic sets and that of ordered semirings are well established in dealing with many real life problems. So, the new defined concepts in this paper would help to approach these problems with a different perspective. For future research, it would be interesting to consider other types of SVN substructures in ordered semirings.

Conflict of interest

The authors declare that they have no conflict of interest.

References

1. M. O. Alsarahead, A. G. Ahmad, Complex fuzzy subgroups, *Appl. Math. Sci.*, **11** (2017), 2011–2021. doi: 10.12988/ams.2017.64115.
2. M. Al-Tahan, B. Davvaz, On (m, n)-hyperideals in ordered semihyperrings: Applications to ordered semirings, *J. Algebra Appl.*, 2021. doi: 10.1142/S0219498822501018.
3. M. Al-Tahan, B. Davvaz, M. Parimala, A note on single valued neutrosophic sets in ordered groupoids, *IJNS*, **10** (2020), 73–83.
4. M. Al-Tahan, B. Davvaz, Some results on single valued neutrosophic (weak) polygroups, *IJNS*, **2** (2020), 38–46. doi: 10.5281/zenodo.3719350.
5. M. Al-Tahan, B. Davvaz, On single valued neutrosophic sets and neutrosophic $\aleph$-structures: Applications on algebraic structures (hyperstructures), *IJNS*, **3** (2020), 108–117. doi: 10.5281/zenodo.3750220.
6. M. Al-Tahan, F. Smarandache, B. Davvaz, NeutroOrderedAlgebra: Applications to semigroups, *Neutrosophic Sets Syst.*, **39** (2021), 133–147.
7. H. Akara, M. Al-Tahan, J. Vimala, Some results on single valued neutrosophic bi-ideals in ordered semigroups, *Neutrosophic Sets Syst.*, **45** (2021), 181–196.
8. K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Set. Syst.*, **20** (1986), 87–96. doi: 10.1016/S0165-0114(86)80034-3.
9. R. Biswas, Fuzzy subgroups and anti fuzzy subgroups, *Fuzzy Set. Syst.*, **35** (1990), 121–124. doi: 10.1016/0165-0114(90)90025-2.
10. J. J. Chen, S. G. Li, S. Q. Ma, X. P. Wang, m-Polar fuzzy sets: An extension of bipolar fuzzy sets, *Sci. World J.*, **2014** (2014), 416530. doi: 10.1155/2014/416530.

11. B. Davvaz, S. Subiono, M. Al-Tahan, Calculus of meet plus hyperalgebra (tropical semihyperrings), *Commun. Algebra*, **48** (2020), 2143–2159. doi: 10.1080/00927872.2019.1710178.

12. B. Davvaz, I. Cristea, *Fuzzy algebraic hyperstructures-An introduction*, Springer, 2015.

13. K. Głazek, *A guide to the literature on semirings and their applications in mathematics and information sciences*, Dordrecht: Springer, 2002. doi: 10.1007/978-94-015-9964-1.

14. J. S. Golan, *Semirings and their applications*, Dordrecht: Kluwer Academic publisher, 1999.

15. A. Mahboob, B. Davvaz, N. M. Khan, Fuzzy (m, n)-ideals in semigroups, *Comput. Appl. Math.*, **38** (2019), 189. doi: 10.1007/s40314-019-0930-5.

16. D. Mandal, Fuzzy ideals and fuzzy interior ideals in ordered semirings, *Fuzzy Inform. Eng.*, **6** (2014) 101–114. doi: 10.1016/j.fiae.2014.06.008.

17. S. Omidi, B. Davvaz, Basic notions and properties of ordered semihyperrings, *Categ. Gen. Algebraic*, **4** (2016), 43–62.

18. F. Smarandache, *Neutrosophy, neutrosophic probability, set, and logic*, USA: American Research Press, 1998. doi: 10.5281/zenodo.57726.

19. F. Smarandache, Neutrosophic set–A generalization of the intuitionistic fuzzy set, *Int. J. Pure Appl. Math.*, **24** (2005), 287–297.

20. F. Smarandache, *A unifying field in logics: Neutrosophic logic, neutrosophy, neutrosophic set, neutrosophic probability and statistics*, USA: InfoLearnQuest, 2007.

21. H. S. Vandiver, Note on a simple type of algebra in which the cancellation law of addition does not hold, *B. Am. Math. Soc.*, **40** (1934), 916–920.

22. H. B. Wang, F. Smarandache, Y. Q. Zhang, R. Sunderraman, Single valued neutrosophic sets, *Review of the Air Force Academy*, **1** (2010), 10–14.

23. L. A. Zadeh, Fuzzy Sets, *Inform. Contr.*, **8** (1965), 338–353. doi: 10.1016/S0019-9958(65)90241-X.