Research article

Stabilization of wind farm integrated transmission system with input delay

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Abstract: In the process of large-scale wind farms integration, the time delay is usually caused by the introduction of wide area control signal, which leads to the unstable operation of integrated transmission system. In order to solve this problem, using the control principle of interconnection and damping assignment passivity-based (IDA-PB), this paper puts forward a control method which applies voltage source converter high voltage direct current (VSC-HVDC) technology to the integrated system of time-delay wind farm and keeps the system running stably. In this method, the framework of time-delay port controlled Hamiltonian (PCH) system is constructed, and the energy shaping of the system is carried out by extending the IDA-PB control principle, thus the feedback controller of the system is designed. Around the problem of time-delay stabilization, the stability criterion is obtained by constructing Lyapunov-Krasovskii functional and introducing free weighting matrices. Finally, the simulation results show that the proposed method can effectively solve the time delay problem of the integrated transmission system and avoid the performance deterioration of the system.

Keywords: input delay; wind farm integrated transmission; Hamiltonian system model with time delay; energy shaping; feedback control; Lyapunov-Krasovskii functional; time-delay stabilization

Mathematics Subject Classification: 35Q93

1. Introduction

At present, the magnitude of wind farms in China has reached tens of millions, and large-scale wind farms are usually located at the far end of the load center. Considering the economy and reliability of grid connection, VSC-HVDC technology is a good technical choice. As a commutation-modulated direct current transmission technology with many advantages [1, 2], VSC-HVDC technology has been used in the grid-connection process of many wind farm systems [3–5]. However, the power input of remote large-scale wind farm will cause power oscillation between power grids. Wide area damping control can suppress the oscillation, but it will also lead to the existence of time-delay factors in the wind farm side [6–9], making the wind generation system outputs voltage with time-delay, affecting
the performance of the wind farm integrated VSC-HVDC system, and even leading to the system out of control. In order to avoid this kind of situation, it is necessary to choose an appropriate control method to ensure the stable operation of the integrated system.

In recent years, many scholars have studied the control problems of time-delay systems [10–13], and have even obtained some results on practical systems, including robotic systems [14–16], aircraft control systems [17, 18], vehicle control systems [19–21], and power systems [22–27]. In the research field of power systems, when considering the time delay factor, [22] has proposed a control method which can ensure the stable operation of the electrical system when using wide area control. In [24], the $H_{\infty}$ control problem of electrical systems is investigated in the case of considering more than one time delay. The problem of time-delay stabilization for multi-region electrical systems is considered in [25] when the frequency of the controller is constrained. References [26, 27] make different studies on the stability of power load frequency control system with time delay, and the corresponding stability conclusions are obtained. Most of the existing research results are processed by approximate linearization of the actual power system, which inevitably deviates from the attributes of the actual system itself, and the designed controller is not completely suitable for the original actual power system. Therefore, it is indispensable to adopt a system model which can fully express and maintain the structural characteristics of the original system in order to put forward a more suitable controller design scheme.

As a special passive nonlinear processing method, the control method based on PCH model has certain control flexibility and brilliant application characteristics, and has been widely used in the research of practical systems [28–36]. In [32], the rotation rate control problem of permanent magnet synchronous motor is solved based on Hamiltonian theory. A full-order observer design scheme for practical physical applications such as mechanical power systems is provided in [33] with the Hamiltonian model as the framework. The control method based on PCH model is adopted in [34] to ensure the stability of the converter system. However, most of the existing literatures on power grid integrated transmission only analyze the grid-connected part independently and do not take into account the time-delay input voltage transmitted by the front-end wide-area wind power system. In fact, there have been some research results on the stability and control of time-delay Hamiltonian systems [37–39]. For example, in [37], based on the Hamiltonian method, the authors propose an adaptive controller design strategy for nonlinear systems with input time-delay. As far as the author knows, there are no Hamiltonian control results for the integrated VSC-HVDC system of time-delay wind farms.

Based on the above analysis, the stabilization problem of wind farm integrated VSC-HVDC system with input delay is studied in this paper. To start with, the system model is transformed into PCH model by selecting energy function and state variables. Then, the controller is designed by extending IDA-PB control principle, and the time-delay stability criterion is obtained by using free weight matrix and Lyapunov-Krasovskii functional method. Finally, the effectiveness of the proposed method in solving the control problem of wind farm integrated VSC-HVDC system with input time delay is verified by experimental simulation.

The contributions of this paper are mainly reflected in the following aspects: (1) for the actual object of wind farm integrated transmission system, the stability control problem under the influence of time-varying delay is studied for the first time; (2) in the process of controller design, the traditional IDA-PB control principle is extended and used in the case of time delay. Compared with the controller
given in the previous research on time-delay PCH system, the controller designed in this paper doesn’t adopt a single output feedback control, in addition, it also takes into account the non-zero operating point characteristics under the actual working conditions, which can effectively restrain the impact of time delay on wind farm integrated transmission system; (3) the relationship between time-varying delay, time-delay upper bound and their difference is fully taken into account when constructing the Lyapunov-Krasovskii functional, and the conservatism of the conclusion is reduced to some extent by introducing the free weight matrix method.

2. Problem formulation and preliminaries

A wind farm integrated VSC-HVDC system with input delay is studied in this paper. In this part, the research object is elicited and the research ideas are expounded.

2.1. Mathematical model of integrated VSC-HVDC system for single-ended wind farm with input time delay

During the integrated transmission process, the complete integrated VSC-HVDC system of input delay wind farm is shown in Figure 1.

![Figure 1. Topology of wind farm integrated VSC-HVDC system with input delay.](image)

In Figure 1, $R_1$ and $R_2$ are the alternating current side resistance of rectifier and inverter respectively, and $i_{zd1}$ and $i_{zd2}$ are the alternating current side current of rectifier and inverter respectively. $U_{s1}(t-b(t))$ is the time-delay voltage at the wind farm side, $U_{s2}(t-b(t))$ is the time-delay voltage at the grid side, $L_1$ and $L_2$ are the alternating current side inductance of the rectifier converter and the alternating current side inductance of the inverter converter, $U_z$ and $U_n$ are the alternating current side voltage of the rectifier converter and the alternating current side voltage of the inverter converter respectively, $U_{zd1}$ and $U_{zd2}$ are the direct current voltage of the rectifier converter and the direct current voltage of the inverter converter respectively, $C_1$ and $C_2$ are direct current side capacitors of rectifier and inverter respectively. $R_\omega$ is the line transmission resistance on the direct current side of the converter.

Because the power input of long-distance large-scale wind farm will cause power oscillation between power grids, wide-area damping control is usually used to suppress the oscillation, but at the same time, there is a time delay on the side of the wind farm ( [6–9]), As a result, the wind power generation system outputs voltage with time delay. This time-delay voltage is a single-ended (wind farm side) input time-delay factor for the wind farm integrated system with VSC-HVDC. In order to prevent the time-delay factor from continuing to pass downwards, we need to deal with it in time on the wind farm side and design an appropriate controller to ensure the stable operation of the system.
Therefore, this paper investigates stabilization of the system in the case of single-ended (wind farm side) input voltage $U_{sd}$ with time delay. Firstly, the mathematical model of the integrated VSC-HVDC system of single-ended wind farm with input time delay in $dq$ synchronous rotating coordinate system is established as follows

$$
\begin{align*}
L_1 \frac{di_{sd1}}{dt} &= U_{sd1}(t-b(t)) - S_d U_{zd1} - R_1 i_{sd1} - \omega L_1 i_{sq1}, \\
L_1 \frac{di_{sq1}}{dt} &= U_{sq1}(t-b(t)) - S_q U_{zd1} - R_1 i_{sq1} + \omega L_1 i_{sd1}, \\
2 \frac{2}{3} C_1 \frac{dU_{zd1}}{dt} &= S_d i_{sd1} + S_q i_{sq1} - \frac{2}{3} i_{zd1},
\end{align*}
$$

(2.1)

where $U_{sd1}(t-b(t))$ and $U_{sq1}(t-b(t))$ are the $d$-axis and $q$-axis components of the time-delay voltages on the wind farm side, $S_d$ and $S_q$ are respectively the $d$-axis and $q$-axis components of the switching function, $\omega$ is the angular frequency, and the time-delay $b(t)$ is a continuous time-varying function and satisfies

$$
0 \leq b(t) \leq \tau, \quad \dot{b}(t) \leq \mu < 1,
$$

(2.2)

where $\tau$ and $\mu$ are constants.

Because of $i_{zd1} = U_{zd1}/R_{zd1}$, the Eq (2.1) is further written as follows

$$
\begin{align*}
L_1 \frac{di_{sd1}}{dt} &= U_{sd1}(t-b(t)) - S_d U_{zd1} - R_1 i_{sd1} - \omega L_1 i_{sq1}, \\
L_1 \frac{di_{sq1}}{dt} &= U_{sq1}(t-b(t)) - S_q U_{zd1} - R_1 i_{sq1} + \omega L_1 i_{sd1}, \\
2 \frac{2}{3} C_1 \frac{dU_{zd1}}{dt} &= S_d i_{sd1} + S_q i_{sq1} - \frac{2}{3} U_{zd1},
\end{align*}
$$

(2.3)

where $R_{zd1}$ is the direct current side equivalent resistance of the rectifier converter.

The time delay voltage on the side of the wind farm is regarded as the control input. In order to research the stability control problem of the system with input time delay, the next work is to design the feedback controller

$$
u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} = \begin{bmatrix} U_{sd1}(t-b(t)) \\ U_{sq1}(t-b(t)) \end{bmatrix}
$$

to reduce the impact of time delay on the system, and finally ensure the stable operation of the system.

2.2. PCH modeling of integrated VSC-HVDC system for single-ended wind farm with input time delay

A general PCH system can be described as follows

$$
\begin{align*}
\dot{x} &= (J(x) - R(x)) \frac{dH(x)}{dx} + G(x)u, \\
y &= G^T(x) \frac{dH(x)}{dx},
\end{align*}
$$

(2.4)

where $x \in \mathbb{R}^n$ is a state variable, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$ are input and output vectors, respectively, $R(x) \in \mathbb{R}^{n \times n}$ is a non-negative symmetric matrix, $G(x) \in \mathbb{R}^{m \times m}$ represents a gain matrix, $J(x) \in \mathbb{R}^{n \times n}$ is an antisymmetric matrix, and $H(x)$ is the energy function of the system, which satisfies $H(x) \geq 0$ and $H(0) = 0$.
Let
\[
x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T = A \begin{bmatrix} i_{sd1} & i_{sq1} & U_{zd1} \end{bmatrix}^T
\]
be the state variable of the system and select the energy function as
\[
H(x) = \frac{1}{2} x^T A^{-1} x = \frac{1}{2} \left( \frac{1}{L_1} x_1^2 + \frac{1}{L_1} x_2^2 + \frac{3}{2C_1} x_3^2 \right)
\]
which satisfies Assumption 1, where \( A = \text{diag}(L_1, L_1, \frac{2}{3} C_1) \).

**Assumption 1.** The function \( H(x) \), the gradient \( \frac{\partial H(x)}{\partial x} \) and the Hessian matrix \( \text{Hess}(H(x)) \) of \( H(x) \) satisfy
\[
\epsilon_1 \| x_0(0) \|^2 := \epsilon_1 \| x(t - b(t)) \|^2_{t=0} \leq H(x) \leq \epsilon_2 \| x \|^2,
\]
\[
\sigma_1 \| x \| \leq \| \frac{\partial H(x)}{\partial x} \| \leq \sigma_2 \| x \|,
\]
\[
\| \text{Hess}(H(x)) \cdot \text{Hess}^T(H(x)) \| \leq \psi^2,
\]
where \( \epsilon_1, \epsilon_2, \sigma_1, \sigma_2, \psi \) are nonnegative constants.

By combining Eq (2.3) with Eq (2.4), the PCH model of the integrated VSC-HCDC system of single-ended wind farm with input time delay in \( dq \) synchronous rotating coordinate system can be obtained as shown below
\[
\begin{aligned}
\dot{x} &= (J - R) \frac{\partial H(x)}{\partial x} + Gu(t), \\
y &= G^T \frac{\partial H(x)}{\partial x},
\end{aligned}
\]
where
\[
J = \begin{bmatrix} 0 & -\omega L_1 & -S_d \\ \omega L_1 & 0 & -S_q \\ S_d & S_q & 0 \end{bmatrix}, \quad R = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_1 & 0 \\ 0 & 0 & \frac{2}{3R_{d1}} \end{bmatrix}, \quad G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\]

The expected balance point is
\[
x^* = \begin{bmatrix} x_1^* & x_2^* & x_3^* \end{bmatrix}^T = \begin{bmatrix} L_1 i_{sd1}^* & L_1 i_{sq1}^* & \frac{2}{3} C_1 U_{zd1}^* \end{bmatrix}^T.
\]

The goal of this paper is to design a stabilization controller for wind farm systems with input time delay (2.1). The specific steps are as follows: firstly, based on the IDA-PB control energy shaping principle, a controller \( u(t) \) is designed for the system with input time delay (2.6). Then, the stability criterion of the closed-loop system is given by dealing with the time delay of the closed-loop system. Finally, for the wind power grid-connected transmission system (2.1), the effectiveness of the proposed method is verified by combining with the simulation platform.

In order to achieve the above purpose, it is necessary to give the following lemmas.

**Lemma 1.** *(Schur complement)* The following linear matrix inequality holds
\[
\begin{bmatrix} \Lambda_1 & \Lambda_2 \\ * & \Lambda_3 \end{bmatrix} > 0
\]
if and only if
\[
\Lambda_1 - \Lambda_2 \Lambda_3^{-1} \Lambda_2^T > 0, \quad \Lambda_3 > 0,
\]
where \( \Lambda_1 = \Lambda_1^T, \Lambda_3 = \Lambda_3^T, \) and \( \Lambda_i \ (i = 1, 2, 3) \) are constant matrices.
Lemma 2. For given $\tau > 0$, a time varying function $b(t)$ that satisfies $0 \leq b(t) \leq \tau$, any positive definite matrix $Z$ and the vector function $\omega : [-\tau, \infty) \rightarrow \mathbb{R}^n$, the following relation is satisfied.

$$\tau \int_{t-b(t)}^t \omega^T(s)Z\omega(s)ds \geq \int_{t-b(t)}^t \omega^T(s)ds \cdot \int_{t-b(t)}^t \omega(s)ds, \quad t \geq 0.$$ 

3. Main results

This section puts forward the main results of this paper. Specifically, a feedback controller for single-ended VSC-HVDC systems with input time-delay is designed in subsection A, and stability criterion of time-delay are given in subsection B.

3.1. Controller design

Through energy shaping, a feedback controller $u(t) = c(t)$ will be designed and the related closed-loop system will be the following form

$$\dot{x} = (J_d - R_d) \left( \frac{\partial H_d(x)}{\partial x} + \frac{\partial H_d(x(t - b(t)))}{\partial x} \right), \quad (3.1)$$

where, $J_d = J + J_a$ and $R_d = R + R_a$ are skew symmetric and nonnegative symmetric matrices separately, $H_d(x) = H(x) + H_a(x)$ is a new energy function, $H_d(x(t - b(t))) = H(x(t - b(t))) + H_a(x(t - b(t)))$.

The closed loop system (3.1) is expanded to obtain the following equation

$$\dot{x} = (J - R) \frac{\partial H(x)}{\partial x} + (J_a - R_a) \frac{\partial H(x)}{\partial x} + (J_d - R_d) \frac{\partial H_a(x)}{\partial x} + (J_d - R_d) \frac{\partial H_d(x(t - b(t)))}{\partial x}. \quad (3.2)$$

To make the Eq (2.6) equal to the Eq (3.1), the following equation is necessary:

$$Gu(t) = (J_a - R_a) \frac{\partial H(x)}{\partial x} + (J_d - R_d) \frac{\partial H_a(x)}{\partial x} + (J_d - R_d) \frac{\partial H_d(x(t - b(t)))}{\partial x}. \quad (3.3)$$

Note $K(x) := \frac{\partial H_a(x)}{\partial x}$ and take $J_a = 0$. From Eq (2.6), it is known that the damping matrix $R$ contains the direct current side equivalent resistance $R_{zd1}$ of the converter. Consider offsetting $R_{zd1}$ in the process of damping matrix configuration, we select

$$R_d = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & -\frac{2}{2R_{zd1}}
\end{bmatrix}. \quad (3.4)$$

Take

$$K(x) = \begin{bmatrix}
k_x(x_1) \\
k_y(x_2) \\
k_y(x_3)
\end{bmatrix}.$$
By substituting $J, R, K(x), J_a, R_a, G, u(t)$ and $H(x)$ into (3.3), we get the following results

\[
\begin{aligned}
    u_1(t) &= -R_1 \cdot k_a(x_1) - \omega L_1 \cdot k_\beta(x_2) - S_d \cdot k_\gamma(x_3) - R_1\left[\frac{1}{L_1} x_1(t - b(t)) + k_a(x_1(t - b(t)))\right] \\
    &\quad - \omega L_1\left[\frac{1}{L_1} x_2(t - b(t)) + k_\beta(x_2(t - b(t)))\right] - S_d\left[\frac{3}{2C_1} x_3(t - b(t)) + k_\gamma(x_3(t - b(t)))\right], \\
    u_2(t) &= \omega L_1 \cdot k_a(x_1) - R_1 \cdot k_\beta(x_2) - S_q \cdot k_\gamma(x_3) + \omega L_1\left[\frac{1}{L_1} x_1(t - b(t)) + k_a(x_1(t - b(t)))\right] \\
    &\quad - R_1\left[\frac{1}{L_1} x_2(t - b(t)) + k_\beta(x_2(t - b(t)))\right] - S_q\left[\frac{3}{2C_1} x_3(t - b(t)) + k_\gamma(x_3(t - b(t)))\right].
\end{aligned}
\] (3.5)

Then the IDA-PB principle is extended to design the controller $u(t)$ for the system. In order to achieve this goal and ensure the asymptotic stability of the closed-loop system, so as to ensure the stable operation of the original integrated transmission system (2.1) in the presence of time delay, the following assumptions are made for $K(x)$.

At $x^*$, it satisfies

(a1) \[
\frac{\partial K(x)}{\partial x} \bigg|_{x=x^*} > -\frac{\partial^2 H(x)}{\partial x^2} \bigg|_{x=x^*},
\]

(a2) \[K(x^*) = -\frac{\partial H(x)}{\partial x} \bigg|_{x=x^*}.
\]

According to (a1) and (a2), we get

\[
\begin{aligned}
    k_a(x_1^*) &= -\frac{x_1^*}{L_1} = -i_{sd1}^*, \\
    k_\beta(x_2^*) &= -\frac{x_2^*}{L_{\beta}} = -i_{sq1}^*, \\
    k_\gamma(x_3^*) &= -\frac{x_3^*}{2C_1} = -U_{sd1}^*.
\end{aligned}
\] (3.6)

Then $K(x)$ is as follows

\[
\begin{aligned}
    k_a(x_1) &= -\frac{x_1}{L_1} + m \cdot (x_1 - x_1^*), \\
    k_\beta(x_2) &= -\frac{x_2}{L_{\beta}} + n \cdot (x_2 - x_2^*), \\
    k_\gamma(x_3) &= -\frac{x_3}{2C_1} + p \cdot (x_3 - x_3^*),
\end{aligned}
\] (3.7)

where $m, n, p > 0$ are the error parameters to be evaluated.

Solving the following two equations

\[\frac{\partial H_d(x)}{\partial x} = K(x)\] (3.8)

and

\[H_d(x) = H(x) + H_d(x)\] (3.9)

yield the values of $H_d(x)$ and $H_d(x)$ as

\[H_d(x) = \frac{m}{2} x_1^2 - (m + \frac{1}{L_1})x_1^* x_1 + \frac{n}{2} x_2^2 - (n + \frac{1}{L_1})x_2^* x_2 + \frac{p}{2} x_3^2 - (p + \frac{3}{2C_1})x_3^* x_3,\] (3.10)
where
\[ \Delta = \text{diag}(m, n, p), \]
and
\[
E = \begin{bmatrix}
R_1 L_1 & \omega L_1^2 & \frac{3}{2} C_1 S_d \\
-\omega L_1^2 & R_1 L_1 & \frac{3}{2} C_1 S_q \\
\end{bmatrix},
\]
\[
\Phi = \begin{bmatrix}
2R_1 (m + \frac{1}{L_1}) & 2\omega (1 + nL_1) & 2S_d (p + \frac{3}{2C_1}) \\
-2\omega (1 + mL_1) & 2R_1 (n + \frac{1}{L_1}) & 2S_q (p + \frac{3}{2C_1}) \\
\end{bmatrix},
\]
\[
B = \begin{bmatrix}
\frac{1}{2} L_1 & 0 & 0 \\
0 & \frac{1}{2} L_1 & 0 \\
0 & 0 & \frac{1}{3} C_1 \\
\end{bmatrix}.
\]

3.2. Stability analysis

A time-delay stabilization results of the wind farm integrated transmission system (2.1) will be given in this section. Following Theorem 1 provides conditions to guarantee the time-delay stabilization of the system (2.6).

**Theorem 1.** Consider the system (2.6). For given constants \( \tau > 0, \mu < 1 \), if there exist positive definite matrices \( Q, Z \) and matrices \( Y, \Omega \) with appropriate dimensions, such that the following inequality (3.14) holds, then the system (2.6) is asymptotically stable under the feedback controller (3.13).

\[
\Lambda = \begin{bmatrix}
\Upsilon_{11} & Y^T & \Upsilon_{13} & G\Phi \cdot B & \Upsilon_{15} \\
* & \Upsilon_{22} & -\Omega^T & -Y & \Upsilon_{25} \\
* & * & 0 & -\Omega & \Upsilon_{35} \\
* & * & * & -Z & 0 \\
* & * & * & 0 & -Z^{-1} \\
\end{bmatrix} < 0,
\]

where \( \Upsilon_{ij}, \ U \subseteq \{1, 2, 3, 5\} \).
Proof. For the system (2.6), the following closed-loop system can be obtained under the feedback controller (3.13):

\[ \dot{x} = (J - R - GE \cdot \Delta) \frac{\partial H(x)}{\partial x} - G\Phi \cdot B \frac{\partial H(x_b)}{\partial x} + G\Phi x^*, \tag{3.15} \]

where \( x_b := x(t - b(t)) \).

Select a Lyapunov-Krasovskii functional as

\[ V(x_b) = V_a + V_b + V_c, \tag{3.16} \]

where,

\[ V_a = 2H(x), \]

\[ V_b = \int_{t-b(t)}^t \frac{\partial H^T(x(\theta))}{\partial x} Q \frac{\partial H(x(\theta))}{\partial x} d\theta, \]

\[ V_c = \tau \int_0^\tau \int_{t+b(t)}^{t+\beta} \left[ \frac{\partial H(x(\alpha))}{\partial x} \right]^T \left[ \frac{\partial H(x(\alpha))}{\partial x} \right] d\alpha d\beta, \]

\( Q \) and \( Z \) are nonnegative definite matrices to be determined.

For given

\[ H(x) = \frac{1}{2} x^T D^{-1} x = \frac{1}{2} \left( \frac{1}{L_1} x_1^2 + \frac{1}{L_1} x_2^2 + \frac{3}{2C_1} x_3^2 \right) \geq 0, \]

there must be

\[ \sigma_1 \|x\| \leq \|\frac{\partial H(x)}{\partial x}\| \leq \sigma_2 \|x\|, \]

and according to Assumption 1, we can get

\[ \epsilon_1 \|x_b(0)\|^2 \leq V(x_b) \leq \epsilon_2 \|x_b\|^2, \tag{3.17} \]

where \( \epsilon = 2\epsilon_2 + \tau \lambda_{\max}(Q) \sigma_2^2 + \tau \lambda_{\max}(Z). \)

From Newton-Leibniz formula, we have

\[ \int_{t-b(t)}^t \left[ \frac{\partial H(x(s))}{\partial x} \right]^T ds = \left[ \frac{\partial H(x(t))}{\partial x} \right] - \left[ \frac{\partial H(x_b)}{\partial x} \right]. \tag{3.18} \]

Thus, for matrices \( Y \) and \( \Omega \) of any appropriate dimensions, the following equation

\[ 2\left[ \frac{\partial H^T(x_b)}{\partial x} Y + (x')^T \Omega \right] \times \left[ \frac{\partial H(x)}{\partial x} - \int_{t-b(t)}^t \left[ \frac{\partial H(x(s))}{\partial x} \right]' ds - \frac{\partial H(x_b)}{\partial x} \right] = 0 \tag{3.19} \]
From Eq (3.18), the system (3.15) can be rewritten as
\[ \dot{x} = (J - R - G(E \cdot \Delta + \Phi \cdot B)) \frac{\partial H(x)}{\partial x} + G\Phi \cdot B \cdot \int_{t_{b(t)}}^{t} \left[ \frac{\partial H(x(s))}{\partial x} \right]' ds + G\Phi x'. \] (3.20)

Next, the derivatives of \( V_a, V_b \) and \( V_c \) along the trajectory of the system (3.20) are calculated respectively
\[ \dot{V}_a = 2 \frac{\partial H^T(x)}{\partial x} (J - R - G(E \cdot \Delta + \Phi \cdot B)) \frac{\partial H(x)}{\partial x} + 2 \frac{\partial H^T(x)}{\partial x} G\Phi \cdot B \cdot \int_{t_{b(t)}}^{t} \left[ \frac{\partial H(x(s))}{\partial x} \right]' ds + 2 \frac{\partial H^T(x)}{\partial x} G\Phi x'. \] (3.21)

\[ \dot{V}_b = \frac{\partial H^T(x)}{\partial x} Q \frac{\partial H(x)}{\partial x} - (1 - \dot{b}(t)) \frac{\partial H^T(x_b)}{\partial x} Q \frac{\partial H(x_b)}{\partial x} \leq \frac{\partial H^T(x)}{\partial x} Q \frac{\partial H(x)}{\partial x} - (1 - \mu) \frac{\partial H^T(x_b)}{\partial x} Q \frac{\partial H(x_b)}{\partial x}, \] (3.22)

\[ \dot{V}_c = \tau^2 \dot{x}^T \cdot \text{Hess}(H(x)) \cdot Z \cdot \text{Hess}^T(H(x)) \cdot \dot{x} - \tau \int_{t_{b(t)}}^{t} \left[ \frac{\partial H^T(x(\alpha))}{\partial x} \right]' \left[ \frac{\partial H(x(\alpha))}{\partial x} \right]' d\alpha \leq \tau^2 \dot{x}^T \cdot \text{Hess}(H(x)) \cdot Z \cdot \text{Hess}^T(H(x)) \cdot \dot{x} - \tau \int_{t_{b(t)}}^{t} \left[ \frac{\partial H^T(x(\alpha))}{\partial x} \right]' \left[ \frac{\partial H(x(\alpha))}{\partial x} \right]' d\alpha \leq \tau \psi^2 \left( (J - R - G(E \cdot \Delta)) \frac{\partial H(x)}{\partial x} - G\Phi \cdot B \cdot \frac{\partial H(x_b)}{\partial x} + G\Phi x^* \right) \cdot Z \cdot (J - R - G(E \cdot \Delta)) \cdot \frac{\partial H(x)}{\partial x} \cdot \dot{x} - \tau \int_{t_{b(t)}}^{t} \left[ \frac{\partial H^T(x(\alpha))}{\partial x} \right]' d\alpha \cdot Z \cdot \int_{t_{b(t)}}^{t} \left[ \frac{\partial H(x(\alpha))}{\partial x} \right]' d\alpha. \] (3.23)

By combining (3.19), (3.21)– (3.23), we can obtain
\[ \dot{V}(x_b) = \dot{V}_a + \dot{V}_b + \dot{V}_c \leq \xi^T(t) \Theta \xi(t), \] (3.24)

where
\[ \xi(t) = \left[ \begin{array}{c} \frac{\partial H^T(x)}{\partial x} \\ \frac{\partial H^T(x_b)}{\partial x} \\ x^T \int_{t_{b(t)}}^{t} \left[ \frac{\partial H^T(x(\alpha))}{\partial x} \right]' d\alpha \end{array} \right]^T, \]
\[ \Theta = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \Theta_{13} & G\Phi \cdot B \\ \ast & \Theta_{22} & \Theta_{23} & -Y \\ \ast & \ast & \Theta_{33} & -\Omega \\ \ast & \ast & \ast & -Z \end{bmatrix}, \]
\[
\Theta_{11} = -2R - G(E \cdot \Delta + \Phi \cdot B) - (E \cdot \Delta + \Phi \cdot B)^{T} \cdot G^{T} + Q
\]
\[
+ \tau^{2} \psi^{2}(J - R - G \cdot E \cdot \Delta)^{T} Z \cdot (J - R - G \cdot E \cdot \Delta),
\]
\[
\Theta_{12} = Y^{T} - \tau^{2} \psi^{2}(J - R - G \cdot E \cdot \Delta)^{T} ZG \Phi \cdot B,
\]
\[
\Theta_{13} = \Omega^{T} + G \Phi + \tau^{2} \psi^{2}(J - R - G \cdot E \cdot \Delta)^{T} ZG \Phi,
\]
\[
\Theta_{22} = -Y - Y^{T} - (1 - \mu)Q + \tau^{2} \psi^{2}(\Phi \cdot B)^{T} G^{T} ZG \Phi \cdot B,
\]
\[
\Theta_{23} = -\Omega^{T} - \tau^{2} \psi^{2}(\Phi \cdot B)^{T} G^{T} ZG \Phi,
\]
\[
\Theta_{33} = \tau^{2} \psi^{2} \Phi^{T} G^{T} ZG \Phi.
\]

From inequality (3.14) and Lemma 1, it is easy to get \( \Theta < 0 \) and \( \dot{V}(x_{b}) < 0 \). According to Lyapunov-Krasovskii stability theorem, the closed-loop system (3.15) is asymptotically stable, that is, the system (2.6) is asymptotically stable under the feedback controller (3.13). The proof is complete. \( \square \)

For the wind farm integrated transmission system (2.1), the following theorem provides sufficient conditions of stabilization.

**Theorem 2.** Considering the wind farm integrated transmission system (2.1). For given constants \( \tau > 0, \mu < 1 \), if there exist positive definite matrices \( Q, Z \) and matrices \( Y \) and \( \Omega \) with appropriate dimensions, such that inequality (3.14) holds, and the wind farm side delay voltages \( U_{sd1}(t - b(t)) \) and \( U_{sq1}(t - b(t)) \) of the integrated transmission system (2.1) are maintained in the form of (3.25), then the system (2.1) can overcome the influence of time delay and maintain stable operation.

\[
\begin{aligned}
U_{sd1}(t - b(t)) &= -mR_{1}L_{1}i_{sd1} - n\omega L_{1}^{2}i_{sq1} - \frac{2}{3} \cdot p \cdot C_{1} \cdot S_{d} \cdot U_{zd1} - R_{1}(mL_{1} + 1) \cdot i_{sd1}(t - b(t)) \\
& \quad - \omega L_{1}(nL_{1} + 1) \cdot i_{sq1}(t - b(t)) - S_{d} \left( \frac{2}{3} pC_{1} + 1 \right) \cdot U_{zd1}(t - b(t)) + 2R_{1}(mL_{1}) \\
& \quad + 1)i_{sd1}^{*} + 2\omega L_{1}(1 + nL_{1}) \cdot i_{sq1}^{*} + 2S_{d} \left( \frac{2}{3} p \cdot C_{1} + 1 \right) U_{zd1}^{*},
\end{aligned}
\]
\[
\begin{aligned}
U_{sq1}(t - b(t)) &= m\omega L_{1}^{2}i_{sd1} - nR_{1}L_{1}i_{sq1} - \frac{2}{3} \cdot p \cdot C_{1} \cdot S_{q} \cdot U_{zd1} + \omega L_{1}(mL_{1} + 1) \cdot i_{sd1}(t - b(t)) \\
& \quad - R_{1}(nL_{1} + 1) \cdot i_{sq1}(t - b(t)) - S_{q} \left( \frac{2}{3} pC_{1} + 1 \right) \cdot U_{zd1}(t - b(t)) - 2\omega L_{1}(mL_{1}) \\
& \quad + 1)i_{sd1}^{*} + 2R_{1}(nL_{1} + 1) \cdot i_{sq1}^{*} + 2S_{q} \left( \frac{2}{3} p \cdot C_{1} + 1 \right) U_{zd1}^{*}.
\end{aligned}
\]

(3.25)

4. Illustrative examples

In order to verify the effectiveness of the proposed method, a simulation model of the two-machine and two-region wind farm integrated transmission system with input delay is established on the simulation platform, as shown in Figure 2. There is a wind farm in area A (wide area signal is introduced into the wind power generation link). The main parameters of the integrated transmission system are set as follows: The inductance of the alternating current side of the rectifying converter \( L_{1} = 0.15 \text{pu} \), the resistance of the alternating current side of the rectifying converter \( R_{1} = 2.5 \text{pu} \), \( S_{d} = 0.15, S_{q} = 0.05 \), the capacitor of the direct current side of the rectifying converter \( C_{1} = 0.04 \text{pu} \), the equivalent resistance of the direct current side of the rectifying converter \( R_{zd1} = 5.1 \text{pu} \), the frequency \( \omega = 5 \text{pu} \).
Figure 2. Two-machine and two-region wind farm integrated transmission system with input delay.

The time-varying delay \( b(t) \) considered in this paper is selected as \( b(t) = 0.1 \sin(3t)+0.2 \). Obviously, \( 0 \leq 0.1 \leq b(t) \leq 0.3 \), \( b(t) \leq 0.3 < 1 \). According to the current research situation, the time delay of wide area signal is generally \( 0.1 \sim 0.3s \). Therefore, the input delay in this paper meets the engineering requirements. Here we set \( \tau = 0.3s, \mu = 0.3 \).

The Hessian matrix of \( H(x) \) satisfies \( \|\text{Hess}(H(x)) \cdot \text{Hess}^T(H(x))\| \leq \psi^2 \), so we can get \( \psi = 0.87 \).

Next, we intend to verify that the wind farm side delay voltages \( U_{sd1}(t - b(t)) \) and \( U_{sq1}(t - b(t)) \) in the form of (3.25) can ensure the stable operation of the system (2.1).

Select

\[
Q = \begin{bmatrix}
3 & 5 & 10 \\
5 & 15 & 0 \\
10 & 0 & 3 \\
\end{bmatrix}, \quad Z = \begin{bmatrix}
10 & 0 & 10 \\
0 & 20 & 1 \\
10 & 1 & 10 \\
\end{bmatrix}.
\]

\[
Y = \begin{bmatrix}
20 & 2 & 10 \\
2 & 20 & 1 \\
10 & 0 & 30 \\
\end{bmatrix}, \quad \Omega = \begin{bmatrix}
10 & 0 & 3 \\
1 & 12 & 0 \\
2 & 0 & 5 \\
\end{bmatrix}.
\]

Then, using MATLAB, we can get

\[
\Delta = \text{diag}(2.1201, 2.1201, 2.1201),
\]

\[
\Phi = \begin{bmatrix}
43.934 & 13.18015 & 11.88603 \\
-13.18015 & 43.934 & 3.96201 \\
\end{bmatrix}.
\]

Therefore, it can be known that the error parameters \( m = n = p = 2.1201 \).

When the system runs stably, the expected results are as follows: the \( d \)-axis component \( i_{sd1} \) of the alternating current side current of the rectifier converter is stable at 1pu, the \( q \)-axis component \( i_{sq1} \) is stable at 2pu, and the direct current side voltage \( U_{zd1} \) of the rectifier converter is stable at 2pu, that is, \( i_{sd1}^* = 1pu, i_{sq1}^* = 2pu, U_{zd1}^* = 2pu \).

Taking the above parameters, the wind farm side delay voltages \( U_{sd1}(t - b(t)) \) and \( U_{sq1}(t - b(t)) \) are
as follows

\[
\begin{align*}
U_{sd1}(t - b(t)) &= -0.7950375i_{sd1} - 0.23851125i_{sq1} - 0.0084804U_{zd1} \\
&\quad - 3.2950375 \cdot i_{sd1}(t - b(t)) - 0.98851125 \cdot i_{sq1}(t - b(t)) \\
&\quad - 0.1584804 \cdot U_{zd1}(t - b(t)) + 11.1780416, \\
U_{sq1}(t - b(t)) &= 0.23851125i_{sd1} - 0.7950375i_{sq1} - 0.0028268U_{zd1} \\
&\quad + 0.98851125 \cdot i_{sd1}(t - b(t)) - 3.2950375 \cdot i_{sq1}(t - b(t)) \\
&\quad - 0.0528268 \cdot U_{zd1}(t - b(t)) + 11.4144347.
\end{align*}
\] (4.1)

The corresponding waveforms of \(U_{sd1}(t - b(t))\), \(U_{sq1}(t - b(t))\), \(i_{sd1}\), \(i_{sq1}\) and \(U_{zd1}\) under time-varying delay are shown in Figures 3–7, respectively.

**Figure 3.** The waveform of \(U_{sd1}(t - b(t))\) under time-varying delay.

**Figure 4.** The waveform of \(U_{sq1}(t - b(t))\) under time-varying delay.
It can be seen from the figures that, in the case of time-varying delay, the proposed method can be used to reasonably regulate the time-delay voltages $U_{sd1}(t - b(t))$ and $U_{sq1}(t - b(t))$ of the wind farm integrated transmission system, so as to avoid the deterioration of the system performance and make the system run stably in the case of input time-delay.
5. Conclusions

In this paper, the stabilization problem of wind farm integrated VSC-HVDC system with input delay has been researched. Based on the PCH theory, we have transformed the model of wind farm integrated VSC-HVDC system with input delay, and have designed the feedback controller of the system by extending IDA-PB control principle. Meanwhile, under the feedback controller, we have proposed the criterion to ensure the time-delay stabilization of the closed-loop system. Simulation results have shown that the proposed method can effectively avoid the system performance degradation caused by time delay. The limitation of this paper is that the proposed conditions are still conservative to a certain extent, and there is room for further optimization and improvement. The next work of the authors is to investigate the control problem of wind farm integrated VSC-HVDC system with saturation constraints under the influence of time-varying input delay.

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Conflict of interest

The authors state that there is no conflict of interest.

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