Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 6(7): 6882–6897.
DOI: 10.3934/math.2021403
Received: 04 March 2021
Accepted: 13 April 2021
Published: 22 April 2021

Research article

Study on the exit strategy selection mechanism of venture capital based on quantum game

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Abstract: Venture capital exit strategy is a key condition of realizing venture capital appreciation and circular operation. Based on the equity sale method of venture capital exit, this paper explores strategic choices of venture capitalists and venture entrepreneurs for external investment, and constructs a venture capital exit strategy model with the paradigm of classical game theory and quantum game theory, respectively. A series of experiments demonstrated the proposed method can achieve the unification of Nash equilibrium and Pareto equilibrium. Therefore, this paper expands the basic theoretical support and provide practical support for the choice of venture capital exit strategies.

Keywords: game theory; quantum games; venture capital; exit strategies
Mathematics Subject Classification: 62C05, 92Axx

1. Introduction

With the development of economy, finance, and high-tech technology, venture capital exit has been paid more and more attention. Many researchers have focused on the choice of exit method and exit timing, and the impact of information asymmetry on venture capital exit. Among them, in terms of the choice of exit methods and exit timing, the main method of venture capital exit include mergers, buybacks, and liquidations. Murray analyzed the statistics results from the National Venture Capital Association of the United States and concluded that venture capitalists prefer the sales method of venture capital exit in the early stage of growth [1]. Studying on factors affecting the sources of venture capital in 21 countries, Jeng and Wells noted that the IPO (Initial Public Offering) exit mode has the greatest influence on venture capital [2]. A combination of modeling and empirical evidence, Carsten Bienz concluded that while highly profitable companies choose IPOs as an exit method, companies with low profitability usually choose sales as an exit method, and unprofitable companies or companies that are even losing their capital choose liquidation as an exit method in order to avoid greater losses [3]. Kaplan et al. conducted an empirical example of combining several venture capital projects in several countries and found that differences in the financial system of each country, which could led to differences in their choice of venture capital exit [4]. Giot and Schwienbacher studied venture enterprises' IPO shares, sales, and liquidation for a comparative analysis, and then applied exclusion analysis to build a competitive risk model to determine the exit method [5]. Dong and Yu proposed the argument of crowding-out effect, and noted that the way in which venture capital is eventually exited actually results from the interaction between the venture capitalist and the venture entrepreneur's willingness [6]. Through the combination of theory and practice, Hou comprehensively studied the main channels of foreign venture capital exit and put forward corresponding countermeasure suggestions for the current situation and problems of China's venture capital exit channels [7]. Li analyzed the advantages of adopting MBOs (Management Buy-Outs), and suggested that China should adopt more such MBOs for venture capital exit in the short term, and proposed specific application countermeasures [8]. Liang pointed out that acquisition and merger are the best choices in the current situation of venture capital exit in China [9]. Chen ascertained that the choice of the exit method can maximize socio-economic utility by establishing a model [10]. The above literature analyzes the impact of strategic alliances on venture capital exits and presents that alliances increase the possibility of a successful exit from an initial public offering (IPO), but not the ones of a merger [11]. By investigating 3,416 investment and exit deals in India between 2000 and 2017 to model the investment duration of the Indian venture capital (VC) market, the result show that the dropout probability is low in most departments, and there is no positive correlation between investment duration and valuation, but it is impossible to exit most investments due to the illiquidity of the venture capital market [12]. A empirical example of the IPO of Shenzhen Growth Enterprise Market from 2015 to 2016 is studied by correlation and regression analysis. The paper first gives factors that affect the venture capital exit return through, and further puts forward some suggestions and countermeasures [13] based the empirical findings with the development status of China's venture capital industry.

Information economics holds that information asymmetry has a very strong impact on venture capital, and its problems can be divided into two main categories: moral hazard and adverse selection. The existing theoretical and empirical research results show that the moral hazard and adverse selection effects are mutual; in other words, there are double-sided moral hazards and double-sided adverse selections for venture capitalists and external investors, which makes the impact of information asymmetry on venture capital exit complex. In such circumstances, the impact of convertible bonds on the exit outcome of venture capital has been studied in conjunction with the financing ability of financial markets, and it is held that convertible bonds have advantages in the case of long exit cycles and high uncertainty. A large number of researchers in China and other countries on venture capital exit mainly focus on the analysis of the gaming of venture capitalists and venture entrepreneurs, including the choice of venture capital exit mode and timing for proposing exit, and the impact of information asymmetry on venture capital exit, but often ignores the participation of external investors. However, external investors as buyers of equity in venture enterprises in reality, are the direct participants at the exit stage of venture capital and have a pivotal role in the smooth exit of venture capital. At the same time, due to the inconsistent measure of payoff, venture capitalists and venture entrepreneurs often reflect the relationship of non-cooperative game theory, and classical game theory holds that both the Pareto optimal strategy and Nash equilibrium point of the insiders are often not consistent in the non-cooperative game.

Quantum game theory is a fusion of quantum information theory and game theory, which was first proposed by Meyer in 1999 and further applied by Eisert et al. in the prisoner's dilemma situation [14,15]. Subsequent researchers in the fields of physics and economics have also proposed related theorems to further enrich quantum game theory [16−19]. With the development of quantum game theory, it has expanded in various of application fields, including management, computer science, system science and information and communication engineering. Hen Yao et al. improved the cooperation model using quantum game theory [20]. Quantum information processing technology is rapidly developed and several quantum models of social science and economics have emerged [21,22]. The literature proposes a quantum game to study the reduction of food loss and waste (FLW) in a two-tier food supply chain (FSC) consisting of a single supplier and a single retailer. The results show that both parties will adopt all-out strategies to achieve a win-win situation and significantly increase the FLOW level of FSC [23]. Sujatha Babu applied Quantum Game to the Exploration of Cooperation Mechanism [24]. This paper introduces a new solution concept for normal forms of non-cooperative games with no connections and pure strategy: completely transparent balancing [25]. Soumik Mahanti et al. proposed that quantum robots are an excellent future application, and it can be implemented with the help of quantum computers [26]. The unprecedented growth of data volume has become a severe challenge to the traditional data mining and learning system model responsible for dealing with big data. Based on this, the Embedded Quantum Balance Game Paraegm (QEGP) is proposed to ensure that noisy attributes do not reduce the results of big data reduction [27]. The study proposes that equity of profit distribution should play an important role in promoting overall cooperation. It is hoped that when quantum networks and entanglement are accepted by the public, quantum entanglement and equity will promote full cooperation among people from afar from various interest groups [28]. Some researchers have introduced the concept of forming harmonious alliance into classical and quantum symmetric cooperative games. In both cases, the participants have an incentive to form alliances. At the same time, the main characteristic of this cooperative game is conservatism [29].

According to relevant research results, quantum games may be generated by the different properties of classical games [30,31]. As a new method of game research, quantum game has many unique characteristics that are different from classical games, and in many cases it has advantages over traditional games. Since exit in reality is not a binary "black or white" strategy setting game, participants can only choose "exit or not exit", so the degree of exit should be regarded as a continuous variable. Since there is an intermediate state between "complete exit" and "no exit at all", which is very similar to the concept of quantum superposition states in quantum mechanics, we quantify the classical exit strategy game and explore the unique properties of its exit mechanism. In this paper, the quantum game paradigm is applied to venture capital withdrawal to explore the related factors affecting venture capital withdrawal. In view of this, this paper will study venture capitalists and venture entrepreneurs' non-cooperative game selection of outside investors in the process of venture capital withdrawal, from the perspective of classical game theory and quantum game theory respectively in order to enrich the study of the role of game theory in venture capital withdrawal.

2. Relevant theories

2.1. Venture capital exit

Venture investment or venture capital was developed in the United States in the 1940s. It is a type of equity capital investment by professional financiers into new, rapidly growing enterprises with great competitive potential. With the three remarkable characteristics of high risk, high growth, and high payoff, it has become a popular investment and financing method for some financial institutions and entrepreneurial enterprises. The development of venture capital is back to 1940s, U.S. government began to allow pension funds and retirement funds to enter innovative enterprise sectors. In addition, after World War II, the rapid development of the economy, finance, and high technology was also an important condition for the rapid development of venture capital. However, venture capital started late in China, and only in the 1980s and 1990s, there were a small number of venture capital institutions appear. It was not until the 1990s when the market was liberalized, and China government began to restrict enterprises from taking loans directly from banks, that venture capital gradually emerged in response to the need to finance the development of innovative enterprises. Venture capital operates in the following ways:

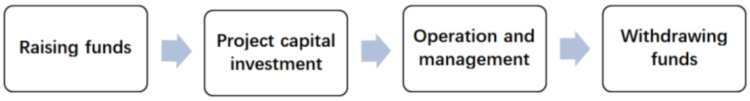

Figure 1. Schematic diagram of venture capital operations.

As shown in Figure 1, venture capital can also be viewed as a dynamic cycle. With their professional knowledge and practical experience in the relevant industry or sector, combined with their efficient business management skills and financial expertise, venture investors proactively participate in the management of the venture enterprise or venture project until the venture enterprise or venture project is publicly traded or achieves capital appreciation and liquidity through mergers and acquisitions. After the exit of a round of venture capital investment, the capital will be invested in the next venture enterprise or venture project selected, and so on, continuously obtaining the appreciation of venture capital.

Venture capital exit means that after a venture has reached a certain stage of development, the venture capitalist considers it necessary to withdraw the venture capital from the venture enterprise and therefore chooses a certain way (public listing, sale or repurchase, or liquidation) to withdraw the venture capital through the capital market in order to achieve capital appreciation or reduce losses to prepare for investing in the next project. High payoffs are achieved through a successful exit from the venture, and a viable exit mechanism is the key to successful venture capital investment.

There are three main types of venture capital exit.

1. Competitive transfer - initial public offering (IPO): Initial public offering (IPO) refers to the conversion of private equity owned by venture capitalists into public equity through the public listing of the shares of the venture enterprise, which, after gaining market recognition, is transferred in order to achieve capital appreciation. Public listing of shares is universally considered as the most desirable exit channel for venture capital, this mainly because public listing on the stock market allows venture capitalists to achieve high payoffs.

2. Contractual transfer-sale or repurchase: There are two forms of venture capital exit by contractual

means in the United States − share sale and repurchase. Share sale refers to an exit channel for an ordinary company or another venture capital company to acquire or merge shares held by a venture capital enterprise or venture capitalist at the negotiated price, also known as "acquisition". There are two types of share sales: statutory mergers and two-step mergers. Statutory mergers are primarily intercompany acquisitions and mergers, and two-step mergers are acquisitions by another venture capital firm that receives the second-step investment. Share repurchase is the purchase of shares from the hands of a venture capital entrepreneur by the venture enterprise or the venture entrepreneur himself. With the beginning of the fifth wave of mergers, venture capital has often exited by repurchase or sale.

3. Forced transfer − bankruptcy liquidation: It is well known that a considerable majority of venture capital investments will not be very successful. When a venture enterprise is declared bankrupt in accordance with the law because it cannot settle its debts as they fall due, personnel from relevant authorities and agencies, as well as personnel with relevant expertise and who are qualified to practice from intermediaries such as law firms, and social intermediaries will be organized to set up a bankruptcy administrator to conduct bankruptcy liquidation. For venture capitalists, once it is confirmed that the venture enterprise has lost the possibility of development or is growing too slowly to provide the expected high payoffs, it is necessary to withdraw decisively and use the funds that can be recovered for the next investment cycle.

2.2. Quantum game theory

Game theory is the study of the decisions made by decision-making subjects, and they interact with each other and the equilibrium problem of those decisions, in other words, the decision-making and equilibrium problem when one subject (which may be a person, a school, or a firm) is influenced by the choices of other subjects and, in turn, influences other subjects (other people, other schools, or other firms).

Quantum game theory is a combination of classical game theory and quantum theory, which can also be said to extend classical games to the quantum system. The research process of quantum games is essentially same as that of classical games.

2.2.1. Eisert scheme

The basic symbolic representation of quantum games and the concept of quantum entanglement are as follows.

（1） Dirac representation

A quantum system can be described by the state in its entirety, using the Dirac representation to represent the state. $|\varphi\rangle$ denotes the right vector, and $\langle\varphi|$ denotes the left vector. The right vector is represented by a two-dimensional column vector, and the left vector by a two-dimensional row vector. An example follows:

$$|\varphi\rangle = \begin{pmatrix} m \\ n \end{pmatrix}, \langle\varphi| = (m^* \quad n^*).$$

The inner product operation of the left vector and right vector is defined as

$$\langle\varphi|\varphi\rangle=(m^* \quad n^*)\begin{pmatrix} m \\ n \end{pmatrix} = m^*m + n^*n.$$

（2） Quantum bit

A bit is a fundamental concept in classical information theory and quantum information theory, the basic unit is a quantum bit. In a two-dimensional Hilbert space, the orthogonal basis is $|0\rangle, |1\rangle$. Then a quantum bit can be expressed as $|\varphi\rangle = a|0\rangle + b|1\rangle$, where a and b are both complex numbers and satisfy $|a|^2 + |b|^2 = 1$, and a quantum bit is a superposition of pure states.

（3） Tensor product

The operator for tensor product is denoted by $\otimes$. Tensor product operation is defined as follows:

$$|\varphi_1\rangle = \begin{pmatrix} m_1 \\ n_1 \end{pmatrix}, |\varphi_2\rangle = \begin{pmatrix} m_2 \\ n_2 \end{pmatrix}$$

$$|\varphi_1\varphi_2\rangle = \begin{pmatrix} m_1 m_2 \\ m_1 n_2 \\ n_1 m_2 \\ n_1 n_2 \end{pmatrix}$$

The state space of a composite quantum system can be represented by the tensor product of the subsystems, where one quantum bit represents one quantum system, and the definition of entanglement is related to the tensor product.

（4） Entanglement

The entangled state of a quantum represents the mechanical property of association between the degrees of freedom of at least two quantum systems. When the superposition state of quantum bits cannot be decomposed into the form of a product of the tensor products of the respective quantum bits in any way, this superposition state is called a “quantum entangled state”.

Quantum games present the exchange of information and the realization of gains for the players of the game through the quantum scheme. The two main quantization schemes are the Marinatto-Weber scheme and the Eisert scheme [24,25]. The Eisert quantization scheme explores quantum games analytically from the viewpoint of entanglement degree. The quantization scheme proposed by Marinatto and Weber is called the “MW quantization scheme”, which is an analytical investigation of quantum games from the perspective of superposition states. The essence of two quantization schemes is the same, this is to say, the process of strategy selection is a local transformation of the state vector, and then the final state is transformed into a payoff result by some means. In game theory, Pareto optimality is often mentioned; it represents a state of resource allocation under which it is impossible to make some people’s situations better without making anyone’s situation worse. In game theory, this can be seen as the state of optimal cooperation, so the payoff that reaches the Pareto optimality state represents the best cooperative payoff.

In 1999, Eisert et al. studied the prisoners' dilemma, and the dilemma that existed in the classical game model disappeared [14]. The quantization scheme is known as the “Eisert scheme”. The basic knowledge used in the Eisert quantum scheme is as follows: create a Hilbert space of a two-state quantum system, in other words, a quantum bit, where the basis vector represents the same meaning as the basic strategy of the classical game. The game state at any moment can then be represented as a tensor product of two quantum bits.

The main steps of the Eisert scheme are as follows:

1. Define two quantum bits in the Hilbert space, each representing the state of a classical strategy;
2. The two quantum bits defined are entangled together by the quantum gate J to form the initial state;

3. Each of the two game players does a local unitary transformation on its own quantum bits;
4. Disentangle the two quantum bits through the quantum gate J† to obtain the final state of the game;
5. The final gain for both players is determined by the measurement device.

The scheme is represented schematically in Figure 2.

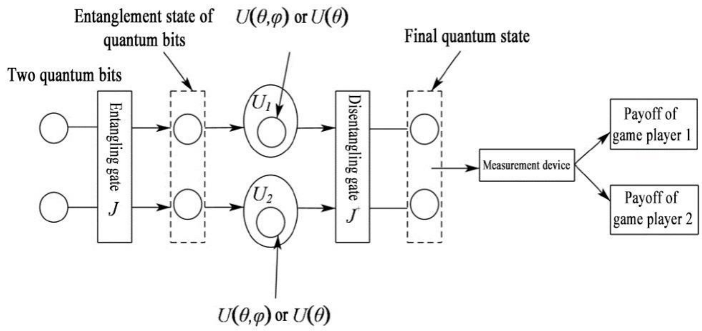

Figure 2. Eisert quantization steps.

2.2.2. MW scheme

In 2000, Marinatto and Weber conducted a quantitative analysis of gender warfare, and the quantization scheme they used has come to be known as the "Marinatto-Weber scheme" [15]. The main steps of the MW scheme are as follows:

1. Pick any initial quantum state in the Hilbert space.
2. Each of the two game players applies a local transformation to the initial quantum state (there are two types of transformation operators: the unitary operator I and the inverse operator C), and the final-state density matrix is obtained after the transformation.
3. The payoff results of the game are obtained based on the defined payoff operator and the end-state density matrix.

The scheme is represented schematically, as shown in Figure 3.

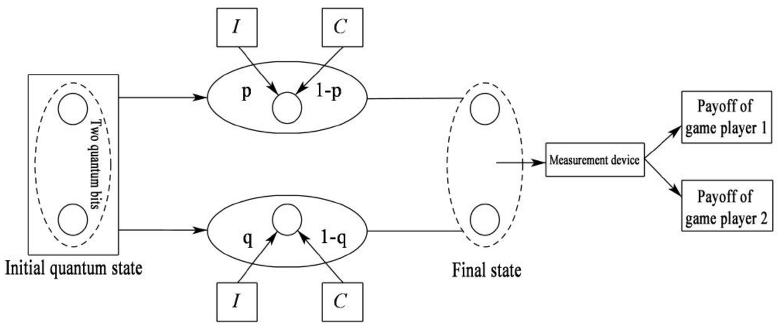

Figure 3. MW quantization steps.

2.3. Game problem in the venture capital exit process

Due to the finite duration of venture capital funds, venture capital will inevitably choose to exit the venture enterprise eventually, and the venture capitalist will only be able to obtain the desired payoff and build a reputation in the market through a successful exit from the investment project. When ownership and control are separated, a principal–agent relationship arises. The inconsistency of goals and preferences of venture capitalists and venture entrepreneurs in the exit decision process, as well as the complex nature of venture capital projects and their development, lead to a conflict of interest in the choice of exit method between the two decision makers. The choice of external investors in the equity sale between venture capitalists and venture entrepreneurs is the game process between the two players. On this basis, we will establish a classical game model and a quantum game model for two players. In the classical game model, we analyze the strategic choices and payoff of both players in the basic state, and in the quantum game model, we explore the influence of degree of entanglement and initial state on the strategies and payoffs of both players, in order to better solve the problem of venture capital exit for both players of the game.

3. Classical game models

3.1. Model hypotheses

The game mechanism imparts mutual rationality on the participants, and the relevant information is fully disclosed to all players of the game. Under the conditions of a perfectly rational game, the following hypotheses are about the participants.

Hypothesis 1: There are only three types of market participants in venture capital exits: venture entrepreneur A, venture capitalist B, and external investors C and D, where C has a higher ability and D has a higher bid price.

Hypothesis 2: A smooth exit of venture capital can only be achieved when the choices of the venture capitalist and venture entrepreneur for external investors are aligned, and when the choices of the two are not aligned, the venture capitalist cannot exit the venture enterprise.

3.2. Model construction

Three important concepts, namely, participants, strategies, and payoff functions, are involved in this paper.

(1) Participants: venture entrepreneurs and venture capitalists.

(2) Strategies: the formula for the construction of strategic alternatives for both players is U = (C, D). As to the appropriate choice of strategy, that is to be judged by the specific payoffs of both players.

(3) Payoff functions:

When both the venture entrepreneur and the capitalist choose to sell the venture enterprise's equity to C, the payoffs obtained are R1 and R2, respectively, at which time the venture entrepreneur obtains the maximum payoff through the game; when both players choose D, the payoffs obtained are R2 and R1, respectively, at which time the venture entrepreneur obtains the maximum payoff through the game; when the two players make different choices, it is not possible to achieve the venture exit, and both players suffer certain losses and obtain the payoff R3, where R1 > R2 > R3.

Under various choice conditions, this paper studies the game between A and B based on the above game hypotheses, and the payoff matrix is presented in Table 1.

Table 1. Payoff matrix.

	C	D
C	(R1, R2)	(R3, R3)
D	(R3, R3)	(R2, R1)

4. Quantum game model

Based on the previous discussion and analysis, in order to explore the properties of quantum games that may arise differently from classical games, therefore, this section will employ the Eisert quantization scheme for the rumor propagation game with quantum information processing to investigate the impact of quantization on gaming of the two parties, focusing on the impact of quantum entanglement on user payoff and the difference between quantum and classical strategies.

4.1. Quantum game model based on ES scheme

4.1.1. Model construction

If we assume that the venture entrepreneur and venture capitalist know the game payoff matrix of both players and continuously adjust their initial quantum strategies accordingly, the quantum game payoff matrix will be a function of the quantum strategies of both players and changes as the strategies of both players in the game change.

First, we set the basis vectors of the two-dimensional Hilbert space to $|C\rangle$, $|D\rangle$, and $|C\rangle = (1, 0)^T$ *and* $|D\rangle = (0, 1)^T$, the state of the game at any moment in time can be represented by the quantities in the tensor product space of these two basis vectors. This tensor product space takes $|CC\rangle, |CD\rangle, |DC\rangle, |DD\rangle$ as the basis vectors, where the former term denotes the quantum bits of the game player A, and the latter term denotes the quantum bits of the game player B.

We denote the initial state of the game process as $|\psi_i\rangle = J|CC\rangle$, where J is a unitary operator known by both players. The set of strategies of game player A is denoted by U_1, and the set of strategies of game player B is denoted by U_2. After both players of the game implement their strategies separately, the state of the game becomes $(U_1 \otimes U_2)J|CC\rangle$, and then after the conjugate transposition by operator $J^\dagger$, the state of both players is reduced, and the final state of the game is obtained: $|\psi_f\rangle = J^\dagger(U_1 \otimes U_2)J|CC\rangle$, where the final payoffs of both players of the game are given by the measurement device.

According to Table 1, the final payoffs for both players are expressed as follows:

$$\$_A = (R1)P_{CC} + (R3)P_{CD} + (R3)P_{DC} + (R2)P_{DD} \tag{4.1}$$

$$\$_B = (R2)P_{CC} + (R3)P_{CD} + (R3)P_{DC} + (R1)P_{DD} \tag{4.2}$$

$$P_{\sigma\sigma'} = |\langle\sigma\sigma'|\psi_f\rangle|^2, \sigma\sigma' = \mathrm{C}, \mathrm{D} \tag{4.3}$$

We combine the strategy matrices U_1 and U_2, denoted as follows:

$$U_i = (\theta_i, \varphi_i) = \begin{bmatrix} e^{i\varphi_i}\cos\frac{\theta_i}{2} & \sin\frac{\theta_i}{2} \\ -\sin\frac{\theta_i}{2} & e^{-i\varphi_i}\cos\frac{\theta_i}{2} \end{bmatrix}, \tag{4.4}$$

where $(i,j=1,2), 0 \le \theta_i \le \pi, 0 \le \varphi_i \le \frac{\pi}{2}$. From equation 4.3, it can be noted that $U_1=(0,0), U_1=(\pi,0)$.

We denote the unitary operator J as follows:

$$J=\exp\left\{\frac{i\gamma}{2}\sigma_x \otimes \sigma_x\right\}=\cos\frac{\gamma}{2}\cdot I+i\sin\frac{\gamma}{2}\cdot(\sigma_x\otimes\sigma_x), \tag{4.5}$$

where $\gamma \in \left[0, \frac{\pi}{2}\right]$, which represents the entanglement degree of the two players of the game. When $\gamma=\frac{\pi}{2}$, the degree of entanglement is maximum. σ_x is the variant of Pauli-x matrix $\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$; I is the 4×4 unit matrix.

We denote operator $J^{\dagger}$ as follows:

$$J^{\dagger}=\cos\frac{\gamma}{2}\cdot I-i\sin\frac{\gamma}{2}\cdot(\sigma_x\otimes\sigma_x). \tag{4.6}$$

According to Eqs 4.4–4.6, we will get the final results as follows:

$$\begin{aligned}
|\psi_f\rangle &= J^{\dagger}(U_1\otimes U_2)J|SS\rangle \\
&= [\cos(\varphi_1+\varphi_2)-i\cdot\cos\gamma\cdot\sin(\varphi_1+\varphi_2)]\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\left|SS\right\rangle \\
&+ [\cos\varphi_1-i\cdot\cos\gamma\cdot\sin\varphi_1]\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\left|SN\right\rangle+[\sin\gamma\cdot\sin\varphi_2]\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}|SN\rangle \\
&+ [\cos\varphi_2-i\cdot\cos\gamma\cdot\sin\varphi_2]\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}\left|NS\right\rangle+[\sin\gamma\cdot\sin\varphi_1]\cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}\left|NS\right\rangle \\
&+ [\sin\gamma\cdot\sin(\varphi_1+\varphi_2)]\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}|NN\rangle+\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}|NN\rangle
\end{aligned} \tag{4.7}$$

According to Eq 4.3, the probability of each quantum state is calculated as

$$P_{CC}=[cos^2(\varphi_1+\varphi_2)+sin^2(\varphi_1+\varphi_2)\cdot cos^2\gamma]cos^2\frac{\theta_1}{2}cos^2\frac{\theta_2}{2} \tag{4.8}$$

$$P_{CD}=[cos^2\varphi_1+sin^2\varphi_1\cdot cos^2\gamma]cos^2\frac{\theta_1}{2}sin^2\frac{\theta_2}{2}+[sin^2\varphi_2\cdot sin^2\gamma]sin^2\frac{\theta_1}{2}cos^2\frac{\theta_2}{2} \tag{4.9}$$

$$P_{DC}=[cos^2\varphi_2+sin^2\varphi_2\cdot cos^2\gamma]cos^2\frac{\theta_2}{2}sin^2\frac{\theta_1}{2}+[sin^2\varphi_1\cdot sin^2\gamma]sin^2\frac{\theta_2}{2}cos^2\frac{\theta_1}{2} \tag{4.10}$$

$$P_{DD}=[sin^2(\varphi_1+\varphi_2)+sin^2\gamma]cos^2\frac{\theta_1}{2}cos^2\frac{\theta_2}{2}+sin^2\frac{\theta_1}{2}sin^2\frac{\theta_2}{2} \tag{4.11}$$

4.1.2. Equilibrium solutions and stability analysis of the quantum model

In the above quantization process, we have gotten the quantization model, and then we will analyze the equilibrium solution of the quantization model and its evolutionary stability. Since the equilibrium conditions are based on the payoffs of both players, the analysis process will start with the payoffs of both players, which can be gotten from Eq (4.1),

$$
\begin{aligned}
\$_A(U_{1,}U_2) &= (R1)P_{CC} + (R3)P_{CD} + (R3)P_{DC} + (R2)P_{DD} \\
&= R1[cos^2(\varphi_1+\varphi_2) + sin^2(\varphi_1+\varphi_2)\cdot cos^2\gamma]cos^2\frac{\theta_1}{2}cos^2\frac{\theta_2}{2} \\
&+ R3[cos^2\varphi_1 + sin^2\varphi_1\cdot cos^2\gamma]cos^2\frac{\theta_1}{2}sin^2\frac{\theta_2}{2} + [sin^2\varphi_2\cdot sin^2\gamma]sin^2\frac{\theta_1}{2}cos^2\frac{\theta_2}{2} \\
&+ R3[cos^2\varphi_2 + sin^2\varphi_2\cdot cos^2\gamma]cos^2\frac{\theta_2}{2}sin^2\frac{\theta_1}{2} + [sin^2\varphi_1\cdot sin^2\gamma]sin^2\frac{\theta_2}{2}cos^2\frac{\theta_1}{2} \\
&+ R2[sin^2(\varphi_1+\varphi_2) + sin^2\gamma]cos^2\frac{\theta_1}{2}cos^2\frac{\theta_2}{2} + sin^2\frac{\theta_1}{2}sin^2\frac{\theta_2}{2}
\end{aligned}
\tag{4.12}
$$

$$
\begin{aligned}
\$_B(U_{1,}U_2) &= (R2)P_{CC} + (R3)P_{CD} + (R3)P_{DC} + (R1)P_{DD} \\
&= R2[cos^2(\varphi_1+\varphi_2) + sin^2(\varphi_1+\varphi_2)\cdot cos^2\gamma]cos^2\frac{\theta_1}{2}cos^2\frac{\theta_2}{2} \\
&+ R3[cos^2\varphi_1 + sin^2\varphi_1\cdot cos^2\gamma]cos^2\frac{\theta_1}{2}sin^2\frac{\theta_2}{2} + [sin^2\varphi_2\cdot sin^2\gamma]sin^2\frac{\theta_1}{2}cos^2\frac{\theta_2}{2} \\
&+ R3[cos^2\varphi_2 + sin^2\varphi_2\cdot cos^2\gamma]cos^2\frac{\theta_2}{2}sin^2\frac{\theta_1}{2} + [sin^2\varphi_1\cdot sin^2\gamma]sin^2\frac{\theta_2}{2}cos^2\frac{\theta_1}{2} \\
&+ R1[sin^2(\varphi_1+\varphi_2) + sin^2\gamma]cos^2\frac{\theta_1}{2}cos^2\frac{\theta_2}{2} + sin^2\frac{\theta_1}{2}sin^2\frac{\theta_2}{2}
\end{aligned}
\tag{4.13}
$$

The formula involves many variables, which is very inconvenient to discuss. Actually, it is found that in other quantum game models under the Eisert quantization scheme, as the entanglement increases, the classical features of the game are transformed into quantum features, and when the degree of entanglement takes the maximum value $\frac{\pi}{2}$, there is a Nash equilibrium. For this reason, we only study the case in which the degree of entanglement is $\frac{\pi}{2}$.

If there is an equilibrium strategy $(U_1^*, U_2^*) = [(\theta_1^*, \varphi_1^*), (\theta_2^*, \varphi_2^*)]$, in formula (2.13), let the entanglement degree be $\frac{\pi}{2}$, and the payoffs of both players are derived. If the strategy (U_1^*, U_2^*) is a Nash equilibrium strategy, according to the conditions of the Nash equilibrium,

$$\$_A(U_{1,}, U_2^*) \le \$_A(U_1^*, U_2^*)$$

$$\$_B(U_1^*, U_2) \le \$_B(U_1^*, U_2^*)$$

So the strategy (U_1^*, U_2^*) is a quantum equilibrium strategy. On the basis of this process, we conduct further to analyze the evolutionary stability of the strategy.

For any $U_1 \ne U_1^*$, there is

$$\$_A(U_1^*, U_2) < \$_B(U_1^*, U_2^*).$$

For any $U_2 \ne U_2^*$, there is

$$\$_B(U_{1,}, U_2^*) < \$_A(U_1^*, U_2^*).$$

In summary, we obtain the following conclusion: when $\gamma = \frac{\pi}{2}$, the strategy (U_1^*, U_2^*) is the only Nash equilibrium strategy, and it is evolutionarily stable.

In order to show the effect of entanglement on both players' payoffs more accurately and vividly, taking game player A as an example, and taking R1 = 3, R2 = 4, R3 = 1, a payoff change graph is plotted in MATLAB.

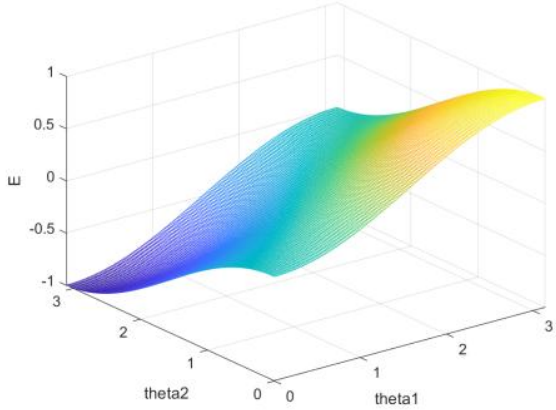

Figure 4. Three-dimensional function graph of user payoff.

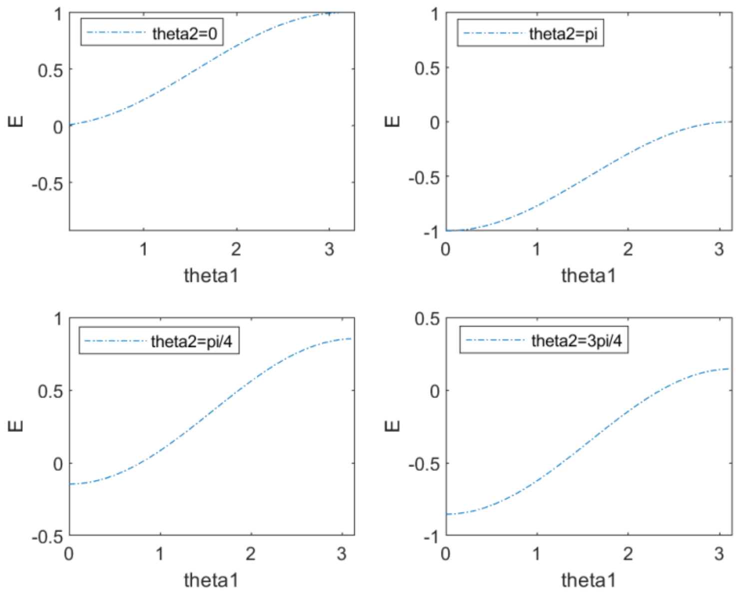

Figure 5. Two-dimensional function graph of user payoff under different θ_2 values.

It can be seen from Figure 4 that the closer the value of θ_2 is to 0, the more moderate the trend of the user's payoff increase as θ_1 increases; the closer the value of θ_2 is to π, the more significant the trend of the user's payoff decrease as θ_1 increases. In order to make a clearer judgment, taking $\theta_2 = \frac{\pi}{4}$ and $\theta_2 = \frac{3\pi}{4}$ respectively, the binary function diagram of changes in user payoff with θ_1 is obtained as shown in Figure 5. It is easy to see from Figure 5 that no matter what value is taken for θ_2, the user's payoff increases with the increase of θ_1.

4.2. Quantum game model based on MW scheme

4.2.1. Model building

First, let $|S\rangle = (1, 0)^T$, $|N\rangle = (0, 1)^T$. Let the initial state of users A and B be $|\psi_i\rangle$, we can get:

$$|\psi_i\rangle = u_{00}|00\rangle + u_{01}|01\rangle + u_{10}|10\rangle + u_{11}|11\rangle,$$

where $|u_{00}|^2 + |u_{01}|^2 + |u_{10}|^2 + |u_{11}|^2 = 1$. The first quantum bit represents user A, and the second quantum bit represents user B.

Let I represent the unit operator $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, and H represent the unitary operator $\mathrm{H} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, then they satisfy:

$$\mathrm{I}|C\rangle = |C\rangle, \mathrm{I}|D\rangle = |D\rangle, \mathrm{H}|C\rangle = |D\rangle, \mathrm{H}|D\rangle = |C\rangle.$$

Assuming that user A chooses strategy I with probability p, and chooses strategy C with probability 1-q; Individual B chooses strategy I with probability q, and chooses strategy C with probability 1-q. According to the quantum game mechanism, the final density matrix ρ_{fin} is

$$\rho_{fin} = pq\left[(I_A \otimes I_B)\rho_{in}\left(I_A^{\dagger} \otimes I_B^{\dagger}\right)\right] + p(1-q)\left[(I_A \otimes C_B)\rho_{in}\left(I_A^{\dagger} \otimes C_B^{\dagger}\right)\right]$$
$$+ (1-p)q\left[(C_A \otimes I_B)\rho_{in}\left(C_A^{\dagger} \otimes I_B^{\dagger}\right)\right] + (1-p)(1-q)\left[(C_A \otimes C_B)\rho_{in}\left(C_A^{\dagger} \otimes C_B^{\dagger}\right)\right],$$

where $\rho_{in} = |\psi_{in}\rangle\langle\psi_{in}|$.

According to the payoff operator:

$$\mathrm{P}_{AB} = \Psi(C,C)|CC\rangle\langle CC| + \Psi(C,D)|CD\rangle D| + \Psi(C,D)|DC\rangle\langle DC| + \Psi(D,D)|DD\rangle\langle DD|$$

$$\$_{A,B}(p,q) = Tr(\mathrm{P}_{AB})\rho_{fin}$$

The final payoff function of users A and B can be gotten as the following formula,

$$\$_A(p,q) = p(R1 = R2 - 2R3)(|u_{00}|^2 - |u_{01}|^2 - |u_{10}|^2 + |u_{11}|^2) - (R2 - R3)(|u_{00}|^2 - |u_{10}|^2)$$
$$+ (R1 - R3)(|u_{01}|^2 - |u_{11}|^2)$$
$$+ q[(R1 - R3)(|u_{10}|^2 - |u_{11}|^2) + (R2 - R3)(|u_{01}|^2 - |u_{00}|^2)] + R1|u_{11}|^2$$
$$+ R2|u_{00}|^2 + R3(|u_{01}|^2 + |u_{10}|^2)$$

$$\$_B(p,q) = q(|u_{00}|^2 - |u_{01}|^2 - |u_{10}|^2 + |u_{11}|^2) - (R2 - R3)(|u_{11}|^2 - |u_{10}|^2)$$
$$+ (R1 - R3)(|u_{01}|^2 - |u_{00}|^2)$$
$$+ q\left[(R1 - R3)\left(|u_{10}|^2 - |u_{00}|^{2^2}\right) + (R2 - R3)(|u_{01}|^2 - |u_{00}|^2)\right] + R1|u_{00}|^2$$
$$+ R2|u_{11}|^2 + R3(|u_{01}|^2 + |u_{10}|^2)$$

4.2.2. Equilibrium solution and stability analysis of quantum model

For a quantum game, its Nash equilibrium $(p*, q*)$ must satisfy the following conditions, ∀p, q ϵ [0, 1]

$$\$_A(p^*, q^*) - \$_A(p, q^*) =\geq 0$$

$$\$_B(p^*, q^*) - \$_B(p, q^*) =\geq 0$$

If $p* = q* = 0$, the Nash equilibrium is (D,D);
If $p* = q* = 1$, the Nash equilibrium is (C,C);
If $p* = 0, q* = 1$, the Nash equilibrium is (D,C);
If $p* = 1$, $q* = 0$, the Nash equilibrium is (C,D).

From the above analysis, when the values of $|u_{00}|^2, |u_{01}|^2, |u_{10}|^2, |u_{11}|^2$ change, the quantum Nash equilibrium solution also changes. Under the above four conditions, points (0,0), (1,1), (0,1) and (1,0) are the Nash equilibrium points, $|u_{00}|^2 + |u_{01}|^2 + |u_{10}|^2 + |u_{11}|^2 = 1$, and the values of $|u_{00}|^2, |u_{01}|^2, |u_{10}|^2$ *and* $|u_{11}|^2$ are uncertain, so the equilibrium point of the quantum game system undergoes phase transitions with the parameters. Now, let us take the following case as the background and assign values to it to examine the phase transition process of quantum games.

Let us assume that venture capitalist B wants to sell the equity of the venture project he holds. External investor C and external investor D participate in the equity bidding. Due to the high bidding price of C, the venture capitalist prefers to sell the equity to C, but the external investor D has more social capital and higher management capabilities, so the venture entrepreneur wants to sell the equity to D. Only when the two players choose the same can the smooth exit of venture capital be realized. If the two players' choices are inconsistent, the venture capital exit cannot be realized.

We assume that the payoff matrix of the venture capitalist and venture entrepreneur is as shown in Table 2:

Table 2. Payoff matrix.

	C	D
C	(6,2)	(1,1)
D	(1,1)	(2,6)

From the classic game, it can be known that there are two equilibrium points, namely (C, C) and (D, D). At this time, the chief interests of the venture entrepreneur and venture capitalist are not the same, so it is difficult to achieve venture capital exit.

If a quantum game is used, when $|u_{01}|^2 = |u_{10}|^2 = 0$, $|u_{00}|^2 + |u_{11}|^2 = \frac{1}{\sqrt{2}}$, a new payoff matrix can be obtained (Table 3):

Table 3. Payoff matrix.

	C	D
C	(4,4)	(1,1)
D	(1,1)	(4,4)

Nash equilibrium strategies (C, C) and (D, D) can be obtained, and at this time, the biggest interests of venture entrepreneur and venture capitalist are the same, so the smooth exit of venture capital can be realized.

5. Conclusion

Based on the ES quantum game mechanism and the MW game scheme, this paper considers the game analysis of the choices of external investors by the venture entrepreneur and venture capitalist. In classic games, it is difficult to find an equilibrium solution for both players. In quantum games, the outcome of the game is closely related to the degree of entanglement. When the degree of entanglement is $\frac{\pi}{2}$, the game has a unique equilibrium solution. When the degree of entanglement gradually increases, various peculiar properties will appear. The final payoff matrix will change as the initial state changes, and the optimal result will appear until the entanglement degree reaches the maximum. This process reflects the mutual influence of the two players' preferences for strategies and choices of strategies.

In the game model established based on the MW quantization scheme, the results show critical conditions for phase transitions in the quantum game. When the value changes, the final Nash equilibrium solution to the quantum game can fundamentally change, indicating that the quantum initial state and the preferences for strategies have significant influences on the final game result. Under certain conditions, both players can realize the exit of venture capital.

Conflict of interest

The author declares no conflict of interest in this paper.

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