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Research article

Multiplicity of positive radial solutions for systems with mean curvature operator in Minkowski space

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Abstract: In this paper, we are considered with the Dirichlet problem of quasilinear differential system with mean curvature operator in Minkowski space

$$\mathcal{M}(w) := \mathrm{div}\Big(\frac{\nabla w}{\sqrt{1-|\nabla w|^2}}\Big),$$

in a ball in $\mathbb{R}^N$. In particular, we deal with this system with Lane-Emden type nonlinearities in a superlinear case, by using the Leggett-Williams' fixed point theorem, we obtain the existence of three positive radial solutions.

Keywords: Minkowski curvature operator; system; positive radial solution; existence; Leggett-Williams' fixed point theorem
Mathematics Subject Classification: 34B15, 35J66

1. Introduction

In this paper, we consider the existence and multiplicity of positive radial solutions for non-variational radial system of type

$$\begin{cases} \mathcal{M}(u) + f_1(|x|, u, v) = 0 & \text{in } B, \\ \mathcal{M}(v) + f_2(|x|, u, v) = 0 & \text{in } B, \\ u|_{\partial B} = v|_{\partial B} = 0, \end{cases} \tag{1.1}$$

where $\mathcal{M}$ stands for the mean curvature operator in Minkowski space

$$\mathcal{M}(w) := \mathrm{div}\Big(\frac{\nabla w}{\sqrt{1-|\nabla w|^2}}\Big),$$

$B = \{x \in \mathbb{R}^N : |x| < 1\}$, $N \geq 2$ is an integer and the functions $f_1, f_2 : [0,1] \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ are continuous, where $\mathbb{R}^+ := [0, \infty)$.

Problems with the operator $\mathcal{M}$ are originated in differential geometry and theory of relativity. Geometrically, these are related to maximal and constant mean curvature spacelike hypersurfaces (space-like submanifolds of codimension one in the flat Minkowski space $\mathbb{L}^{N+1} := \{(x,t) : x \in \mathbb{R}^N, t \in \mathbb{R}\}$ endowed with the Lorentzian metric $\sum_{j=1}^{N}(dx_j)^2 - (dt)^2$, where (x,t) are the canonical coordinates in $\mathbb{R}^{N+1}$ having the property that the trace of the extrinsic curvature is zero, respectively, constant (see [17]).

It is known (see [1, 4]) that the study of spacelike submanifolds of codimension one in the flat Minkowski space $\mathbb{L}^{N+1}$ with prescribed mean extrinsic curvature can lead to the type

$$\mathcal{M}v = H(x, v) \text{ in } \Omega, \qquad v = 0 \text{ on } \partial\Omega, \tag{1.2}$$

where Ω is a bounded domain in $\mathbb{R}^N$ and the nonlinearity $H : \Omega \times \mathbb{R} \to \mathbb{R}$ is continuous.

If H is bounded, then it has been shown by Bartnik and Simon [1] that (1.2) has at least one solution $u \in C^1(\Omega) \cap W^{2,2}(\Omega)$. Also, when Ω is a ball or an annulus in $\mathbb{R}^N$ and the nonlinearity H has a radial structure, then it has been proved in [3] that (1.2) has at least one classical radial solution. In [5], by using Leray-Schauder degree argument and critical point theory, Bereanu and his coauthors gave a sharper result: there exists $\Lambda > 0$ such that it has zero, at least one or at least two positive radial solutions according to $\lambda \in (0, \Lambda)$, $\lambda = \Lambda$ or $\lambda > \Lambda$. Ma et al. [23] and Dai et al. [12–14] generalized the results in [5] to more general cases via bifurcation technique. For other recent various existence results concerning such problems with the operator $\mathcal{M}$, we refer the reader to [2–4, 6–11, 15, 18, 19, 22, 24–28] and the references therein.

On the other hand, inspired by [5], Gurban et al. [16] proved that the result from [5] for a single equation remains valid for the system (1.1) in the case f_1 and f_2 has the particular form

$$f_1(|x|, u, v) = \lambda\mu(|x|)(p+1)u^p v^{q+1}, \quad f_2(|x|, u, v) = \lambda\mu(|x|)(q+1)u^{p+1}v^q,$$

with the positive exponents p, q satisfying $\max\{p, q\} > 1$ (this guaranties a super-linear behavior of both f_1 and f_2 near the origin, with respect to (u, v)) which, in particular, include Hénon-Lane-Emden nonlinearities for $\mu(|x|) = |x|^\sigma$ ($\sigma > 0$).

Motivated by above mentioned results, in this paper, we obtain the existence of three and arbitrarily many positive radial solutions of system (1.1). In particular, we deal with the case when f_1 (resp. f_2) has a superlinear growth near the origin with respect to $\phi(u)$ (resp. $\phi(v)$), where $\phi(s) := s/\sqrt{1-s^2}$. In this respect, we obtain (Theorem 3.2), the existence of at least three positive radial solutions. Here we have in view extensions of some results obtained in [22, 26] for a single equation to systems of type (1.1). Moreover, our results are a complement to the result of Gurban et al. [15, 16], where they obtained the existence and multiplicity (at least two) of positive radial solutions of system (1.1) under the suitable conditions on the nonlinearities.

This paper is organized as follows: In Section 2, some preliminaries are given; in Section 3, we obtain the main results.

2. Preliminaries

In order to present existence results of positive radial solutions for system (1.1), setting $r = |x|$ and $u(|x|) = u(r), v(|x|) = v(r)$, the system (1.1) reduces to the homogeneous mixed boundary value problem:

$$\begin{cases} \left(r^{N-1}\phi(u')\right)' + r^{N-1}f_1(r,u,v) = 0, \\ \left(r^{N-1}\phi(v')\right)' + r^{N-1}f_2(r,u,v) = 0, \\ u'(0) = u(1) = 0 = v(1) = v'(0). \end{cases} \tag{2.1}$$

By a solution of (2.1) we mean a couple of nonnegative functions $(u,v) \in C^1[0,1] \times C^1[0,1]$ with $\|u'\| < 1, \|v'\| < 1$ and $r \mapsto r^{N-1}\phi(u'(r))$, $r \mapsto r^{N-1}\phi(v'(r))$ of class C^1 on $[0,1]$, which satisfies problem (2.1). Here and below, $\|\cdot\|$ stands for the usual sup-norm on $C := C[0,1]$, while the product space $X := C \times C$ will be endowed with the norm $\|(u,v)\| = \|u\| + \|v\|$. Let $X = C \times C$. Also, we denote $B_\rho = \{(u,v) \in X : \|(u,v)\| < \rho\}$.

Throughout the paper, we make the following hypotheses on the nonlinearity

(H1) $f_i : [0,1] \times \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ is continuous, and for any $(r,u,v) \in [0,1] \times \mathbb{R}^+ \times \mathbb{R}^+$, $f_i(r,u,v) > 0$, if $(u,v) \neq (0,0), i = 1,2$.

The following lemma is a direct consequence of [20, Lemma 2.2].

Lemma 2.1. For any $u \in C([0,1],[0,\infty))$ for which $u'(r)$ is decreasing in $[0,1]$ we have

$$\min_{r\in[\frac{1}{4},\frac{3}{4}]} u(r) \geq \frac{1}{4}\|u\|.$$

Let P be a cone in X defined as

$$P = \left\{\tilde{\mathbf{u}} = (u_1,u_2) \in X : u_i(t) \geq 0, t \in [0,1],\ i = 1,2, \text{and} \min_{t\in[\frac{1}{4},\frac{3}{4}]} \sum_{i=1}^{2} u_i(t) \geq \frac{1}{4}\|(u_1,u_2)\|\right\}.$$

Let $T : P \to X$ be a map with components (T_1, T_2). We define $T_i, i = 1,2$ by

$$(T_i\tilde{\mathbf{u}})(r) = \int_r^1 \phi^{-1}\left(\frac{1}{t^{N-1}} \int_0^t s^{N-1} f_i(s,u_1(s),u_2(s))ds\right)dt, \quad \tilde{\mathbf{u}} \in P \cap B_1. \tag{2.2}$$

From a standard procedure(see [15,22]), we have

Lemma 2.2. Assume (H1) holds. Then $T(P) \subset P$ and $T : P \to P$ is compact and continuous. We now introduce the following well-known Leggett-Williams' fixed point theorem. Let K be a cone in the real Banach space X. A map α is a nonnegative continuous concave functional on the cone K if it satisfies the following conditions:

(i) $\alpha : K \to [0,\infty)$ is continuous;

(ii) $\alpha(tx + (1-t)y) \geq t\alpha(x) + (1-t)\alpha(y)$ for all $x,y \in K$ and $0 \leq t \leq 1$.

Let $K_c := \{x \in K : \|x\| < c\}$ and $K(\alpha,b,d) := \{x \in K : b \leq \alpha(x), \|x\| \leq d\}$.

Lemma 2.3 ([21])**.** Let K be a cone in the real Banach space X, $A : \bar{K}_c \to \bar{K}_c$ be completely continuous and α be a nonnegative continuous concave functional on K with $\alpha(x) \leq \|x\|$ for all $x \in \bar{K}_c$. Suppose there exist $0 < a < b < d \leq c$ such that the following conditions hold:

(i) $\{x \in K(\alpha,b,d) : \alpha(x) > b\} \neq \emptyset$ and $\alpha(Ax) > b$ for all $x \in K(\alpha,b,d)$;

(ii) $\|Ax\| < a$ for $x \in \bar{K}_a$;

(iii) $\alpha(Ax) > b$ for $x \in K(\alpha,b,c)$ with $\|Ax\| > d$.

Then, A has at least three fixed points $x_1,x_2,x_3 \in \bar{K}_c$ satisfying

$$\|x_1\| < a, \quad \alpha(x_2) > b, \quad a < \|x_3\| \text{ with } \alpha(x_3) < b. \tag{2.3}$$

3. Main result

Theorem 3.1. Assume that there exist positive constants a, c and d with $0 < d < a < \frac{1}{4}c < c < 1$ such that

(H2) $f(r,u,v) < N\phi(d/2)$, for all $r \in [0,1], u, v \geq 0$ and $0 \leq u+v < d$;

(H3) there exists $i_0 \in \{1,2\}$, for all $r \in [\frac{1}{4}, \frac{3}{4}], u, v \geq 0$, and $u+v \in [a, 4a]$, such that

$$f_{i_0}(r,u,v) \geq \phi(\frac{4a}{\Gamma}), \text{ with } \Gamma = \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{1}{N}(t^N - (\frac{1}{4})^N)\right) dt.$$

(H4) $f(r,u,v) < N\phi(c/2)$, for all $r \in [0,1], u, v \geq 0$ and $0 \leq u+v < c$.

Then system (1.1) has at least three positive radial solutions $\mathbf{u}^1 = (u^1, v^1), \mathbf{u}^2 = (u^2, v^2), \mathbf{u}^3 = (u^3, v^3)$ satisfying

$$\|\mathbf{u}^1\| < d, a < \min_{\frac{1}{4} \leq t \leq \frac{3}{4}} \left(u^2(t) + v^2(t)\right), \text{ and } \|\mathbf{u}^3\| > d \text{ with } \min_{\frac{1}{4} \leq t \leq \frac{3}{4}} \left(u^3(t) + v^3(t)\right) < a. \tag{3.1}$$

Proof. For $\mathbf{u} := (u,v) \in P$, define

$$\alpha(\mathbf{u}) = \min_{\frac{1}{4} \leq t \leq \frac{3}{4}} (u(t) + v(t)),$$

then it is easy to see that α is a nonnegative continuous concave functional on P with $\alpha(\mathbf{u}) \leq \|\mathbf{u}\|$ for $\mathbf{u} \in P$.

Set $b = 4a$. First, we show that $T : \bar{P}_c \to \bar{P}_c$, where $c > b$. In fact, for any $\mathbf{u} \in \bar{P}_c$, then $\|\mathbf{u}\| \leq c$. Since ϕ^{-1} is increasing in $\mathbb{R}$, by (2.2) and (H4), we have

$$\begin{aligned}
\|T_i\mathbf{u}\| &= \sup_{r\in[0,1]} \int_r^1 \phi^{-1}\left(\frac{1}{t^{N-1}} \int_0^t s^{N-1} f_i(s, u(s), v(s)) ds\right) dt \\
&= \int_0^1 \phi^{-1}\left(\frac{1}{t^{N-1}} \int_0^t s^{N-1} N\phi(\frac{c}{2}) ds\right) dt \\
&= \int_0^1 \phi^{-1}(\phi(\frac{c}{2})t) dt \\
&\leq \int_0^1 \phi^{-1}(\phi(\frac{c}{2})) dt = \frac{c}{2}.
\end{aligned}$$

So $\|T\mathbf{u}\| = \|T_1\mathbf{u}(t)\| + \|T_2\mathbf{u}(t)\| \leq c$. Therefore, $T\bar{P}_c \subset \bar{P}_c$. Similarly, $\|T\mathbf{u}\| < d$, thus condition (ii) of Lemma 2.3 holds.

Next, we shall show that condition (i) of Lemma 2.3 is satisfied. To do this, let $\bar{\mathbf{u}} = (\frac{a+b}{4}, \frac{a+b}{4}) \in \{\mathbf{u} = (u,v) \in P(\alpha, a, b) : \alpha(\mathbf{u}) > a\}$. Hence

$$\{\mathbf{u} \in P(\alpha, a, b) : \alpha(\mathbf{u}) > a\} \neq \emptyset.$$

Then for any $\mathbf{u} \in P(\alpha, a, b)$ and $t \in [\frac{1}{4}, \frac{3}{4}]$, it is easy to obtain that

$$b \geq \|u\| + \|v\| \geq u(t) + v(t) \geq \min_{\frac{1}{4} \leq t \leq \frac{3}{4}} (u(t) + v(t)) = \alpha(\mathbf{u}) > a.$$

Then from (H3) and the fact that $\phi^{-1}(s_1 s_2) \geq s_1\phi^{-1}(s_2)$ with $s_1 \in (0, 1)$, we have

$$\begin{aligned}
\alpha(T\mathbf{u}) &= \min_{t\in[\frac{1}{4},\frac{3}{4}]} \sum_{i=1}^{2} T_i\mathbf{u}(t) \\
&\geq \min_{t\in[\frac{1}{4},\frac{3}{4}]} T_{i_0}\mathbf{u}(t) \\
&\geq \frac{1}{4}\|T_{i_0}\mathbf{u}(t)\| \\
&\geq \frac{1}{4}\int_{\frac{1}{4}}^{\frac{3}{4}} \phi^{-1}\left(\frac{1}{t^{N-1}}\int_{\frac{1}{4}}^{t} s^{N-1} f_{i_0}(s, u(s), v(s))ds\right)dt \\
&\geq \frac{1}{4}\int_{\frac{1}{4}}^{\frac{3}{4}} \phi^{-1}\left(\int_{\frac{1}{4}}^{t} s^{N-1}\phi(\frac{4a}{\Gamma})ds\right)dt \\
&\geq \frac{a}{\Gamma}\int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{1}{N}(t^N - (\frac{1}{4})^N)\right)dt \\
&= a.
\end{aligned}$$

Finally, we check condition (iii) of Lemma 2.3. Suppose that $\mathbf{u} \in P(\alpha, a, c)$ with $\|T\mathbf{u}\| > b$, then from Lemma 2.2, we obtain

$$\alpha(T\mathbf{u}) = \min_{t\in[\frac{1}{4},\frac{3}{4}]} \sum_{i=1}^{2} T_i\mathbf{u}(t) \geq \frac{1}{4}\sum_{i=1}^{2}\|T_i\mathbf{u}\| > \frac{b}{4} = a.$$

In summary, all conditions of Lemma 2.3 are satisfied. Hence (2.1) has at least three positive solutions $\mathbf{u}^1 = (u^1, v^1)$, $\mathbf{u}^2 = (u^2, v^2)$ and $\mathbf{u}^3 = (u^3, v^3)$ such that $\|\mathbf{u}^1\| < d, a < \min_{\frac{1}{4}\leq t\leq\frac{3}{4}}\left(u^2(t) + v^2(t)\right)$, and $\|\mathbf{u}^3\| > d$ with

$$\min_{\frac{1}{4}\leq t\leq\frac{3}{4}}\left(u^3(t) + v^3(t)\right) < a.$$

Then the results of Theorem 3.1 hold. □

Consider problem (2.1), in addition to (H1), that f_1 and f_2 satisfy

(H_f) (i) $f_1(r, s, t) > 0,\ f_2(r, s, t) > 0,\ \forall s, t > 0,\ \forall r \in [0, 1]$;

(ii) $f_1(r, \xi, 0) = f_2(r, 0, \xi) = 0,\ \forall \xi > 0,\ \forall r \in [0, 1]$.

Theorem 3.2. Assume that f_1, f_2 are continuous and satisfy (H3) and (H_f). If there is some $M > 0$ such that either

$$\lim_{s\to 0^+}\frac{f_1(r, s, t)}{\phi(s)} = 0, \quad \lim_{s\to 1^-}\frac{f_1(r, s, t)}{\phi(s)} = 0 \quad \text{uniformly with } r \in [0, 1], t \in [0, M], \tag{3.2}$$

or

$$\lim_{t\to 0^+}\frac{f_2(r, s, t)}{\phi(t)} = 0, \quad \lim_{t\to 1^-}\frac{f_2(r, s, t)}{\phi(t)} = 0 \quad \text{uniformly with } r \in [0, 1], s \in [0, M]. \tag{3.3}$$

Then the system (1.1) has at least three positive radial solutions.

Proof. First, we shall show that there exists a positive number c with $c \geq b = 4a$ such that $T : \bar{P}_c \to \bar{P}_c$.

Let f_1 and f_2 be from Theorem 3.2. Without loss of generality, we assume that (3.2) is true. Then there exists $d \in (0, a)$ such that

$$f_i(r, u, v) \le \frac{1}{2} N\phi(\|u\|),\ \forall r \in [0, 1],\ u + v \in [0, d].$$

If $\mathbf{u} \in \bar{P}_d$, then we have

$$\begin{aligned}
\|T\mathbf{u}\| &= \sum_{i=1}^{2} \|T_i\mathbf{u}(t)\| \\
&\le \sum_{i=1}^{2} \sup_{t\in[0,1]} \int_t^1 \phi^{-1}\left(\int_0^s \tau^{N-1} f_i(\tau, u(\tau), v(\tau))d\tau\right) ds \\
&\le \sum_{i=1}^{2} \int_0^1 \phi^{-1}\left(\int_0^1 \tau^{N-1} f_i(\tau, u(\tau), v(\tau))d\tau\right) ds \\
&\le \sum_{i=1}^{2} \phi^{-1}\left(\int_0^1 \tau^{N-1}\left(\frac{1}{2}N\phi\|u\|\right) d\tau\right) \\
&\le \sum_{i=1}^{2} \phi^{-1}(\frac{d}{2}) \le d.
\end{aligned}$$

On the other hand, by the second condition of (3.2) and (3.3), there exists $\delta \in (0, 1)$, such that

$$f_i(r, u, v) \le \frac{1}{4} N\phi(\|u\|),\ \forall r \in [0, 1],\ \|u\| \ge \delta, v \in [0, M].$$

Set

$$\frac{1}{2} NM_i = \max\{f_i(r, u, v) : r \in [0, 1], u \in [0, \delta], v \in [0, M]\}.$$

First, we shall show that there exists a positive number c with

$$c > \max\{b, 2M_1, 2M_2\}.$$

If $\mathbf{u} \in \bar{P}_c$, then

$$\begin{aligned}
\|T\mathbf{u}\| &= \sum_{i=1}^{2} \|T_i\mathbf{u}(t)\| \\
&\le \sum_{i=1}^{2} \sup_{t\in[0,1]} \int_t^1 \phi^{-1}\left(\int_0^s \tau^{N-1} f_i(\tau, u(\tau), v(\tau))d\tau\right) ds \\
&\le \sum_{i=1}^{2} \phi^{-1}\left(\int_0^1 \tau^{N-1} f_i(\tau, u(\tau), v(\tau))d\tau\right) \\
&\le \sum_{i=1}^{2} \phi^{-1}\left(\int_0^1 \tau^{N-1}\left(\frac{1}{4}N\|u\| + \frac{1}{2}NM_i\right) d\tau\right) \\
&\le \sum_{i=1}^{2} \phi^{-1}(\frac{c}{2}) \le c.
\end{aligned}$$

Finally, the rest of the proof is similar to Theorem 3.1, we omit it. □

Remark 3.1. Suppose there exist constants d, a, c and $\eta \in (0, \frac{1}{2})$ with $0 < d < a < \eta c < c < 1$ such that (H1)–(H4) hold. Then the conclusion of Theorem 3.1 remains true.

From Theorem 3.1, we can obtain arbitrarily many positive radial solutions of system (1.1).

Corollary 3.1. Suppose there exist positive constants $0 < \eta < \frac{1}{2}, 0 < d_1 < a_1 < \eta c_1 < c_1 < d_2 < a_2 < \eta c_2 < c_2 < \cdots < d_{N-1} < a_{N-1} < \eta c_{N-1} < c_{N-1} < 1$, $N = 2, 3, \cdots$, such that
(H5) $f(r, u, v) < N\phi(d_i/2)$, for all $r \in [0, 1], u, v \geq 0$ and $0 \leq u + v < d_i$, $1 \leq i \leq N - 1$;
(H6) there exists $i_0 \in \{1, 2, \cdots\}$, for all $r \in [\eta, 1 - \eta], u, v \geq 0$, and $u + v \in [a_i, \frac{a_i}{\eta}]$,

$$f_{i_0}(r, u, v) \geq \phi(\frac{\eta a_i}{\Gamma}),\ 1 \leq i \leq N - 1;$$

(H7) $f(r, u, v) < N\phi(c_i/2)$, for all $r \in [0, 1], u, v \geq 0$ and $0 \leq u + v < c_i$, $1 \leq i \leq N - 1$.
Then the system (1.1) has at least $2N - 1$ positive radial solutions.

Proof. If $N = 2$, from condition (H5), we have $T\bar{P}_{d_1} \subset \bar{P}_{d_1}$. Then by the Schauder's fixed point theorem, problem (1.1) has at least one positive radial solution. If $N = 3$, it is clear that Theorem 3.1 holds with $c_1 = d_2$. Thus system (1.1) has at least three positive radial solutions. Following the same method, by the induction method we immediately complete our proof. □

Example 3.1. Consider the following Dirichlet problem of quasilinear differential system

$$\begin{cases} \left(r^{N-1}\phi(u')\right)' + r^{N-1}\mu_1(r)u^p v^q = 0, \\ \left(r^{N-1}\phi(v')\right)' + r^{N-1}\mu_2(r)u^p v^q = 0, \\ u'(0) = u(1) = 0 = v(1) = v'(0), \end{cases} \tag{3.4}$$

where the positive exponents p, q satisfy $\min\{p, q\} > 1$ and the function $\mu_i : [0, 1] \to [0, \infty)$ is continuous and $\mu_i(r) > 0, i = 1, 2$ for all $r \in (0, 1]$. Clearly, all of the conditions in Theorem 3.2 are fulfilled. By Theorem 3.2 and [15, Lemma 2.1], system (3.4) has at least three positive and radially strictly decreasing solutions.

Remark 3.2. In [15], Gurban et al. studied the more general Dirichlet problem of quasilinear differential system

$$\begin{cases} \left(r^{N-1}\phi(u')\right)' + \lambda_1 r^{N-1}\mu_1(r)u^p v^q = 0, \\ \left(r^{N-1}\phi(v')\right)' + \lambda_2 r^{N-1}\mu_2(r)u^p v^q = 0, \\ u'(0) = u(1) = 0 = v(1) = v'(0). \end{cases} \tag{3.5}$$

By using the fixed point index, they proved that there exist $\lambda_1^* = \frac{\phi(\frac{\alpha}{1-b})}{\alpha^{p+q}\int_0^b \tau^{N-1}\mu_1(\tau)d\tau}$, $\lambda_2^* = \frac{\phi(\frac{\alpha}{1-b})}{\alpha^{p+q}\int_0^b \tau^{N-1}\mu_2(\tau)d\tau}$, where $b \in (0, 1), \alpha \in (0, 1 - b)$ are constants. Then for all $\lambda_1 > \lambda_1^*$ and $\lambda_2 > \lambda_2^*$, system (3.5) has a solution (u, v) with both u and v positive in B and radially strictly decreasing. In particular, in the case $\mu_i \equiv 1, i = 1, 2$, it is easy to see that $\lambda_i^* > 1, i = 1, 2$. Our main result (Theorem 3.2) shows that in the case $\lambda_1 = \lambda_2 = 1$, system (3.5) has three positive radial solutions.

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Conflict of interest

The authors declare that they have no conflicts of interest.

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