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Research article

Soft α-separation axioms and α-fixed soft points

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Abstract: Soft set theory is a theme of interest for many authors working in various areas because of its rich potential for applications in many directions. it received the attention of the topologists who always seeking to generalize and apply the topological notions on different structures. To contribute to this research area, in this paper, we formulate new soft separation axioms, namely *tt*-soft αT_i $(i = 0, 1, 2, 3, 4)$ and *tt*-soft α-regular spaces. They are defined using total belong and total non-belong relations with respect to ordinary points. This way of definition helps us to generalize existing comparable properties via general topology and to remove a strict condition of the shape of soft open and closed subsets of soft α-regular spaces. With the help of examples, we show the relationships between them as well as with soft αT_i $(i = 0, 1, 2, 3, 4)$ and soft α-regular spaces. Also, we explore under what conditions they are kept between soft topological space and its parametric topological spaces. We characterize *tt*-soft αT_1 and *tt*-soft α-regular spaces and give some conditions that guarantee the equivalence of *tt*-soft αT_i $(i = 0, 1, 2)$ and the equivalence of *tt*-soft αT_i $(i = 1, 2, 3)$. Further, we investigate some interrelations of them and some soft topological notions such as soft compactness, product soft spaces and sum of soft topological spaces. In the end, we study the main properties of of α-fixed soft point theorem.

Keywords: soft α-open set; *tt*-soft αT_i-space $(i = 0, 1, 2, 3, 4)$; extended soft topology; additive property; topological property; soft α-compact space; α-fixed soft point
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1. Introduction

In 1999, Molodtsov [30] founded a novel mathematical tool for dealing with uncertainties, namely soft set. One of the merits of this tool is its free from the difficulties that the other existing methods such as fuzzy set theory and probability theory. This matter makes soft set theory very popular research area all over the globe. Immediately afterwards, Maji et al. [26] in 2003, established the basis of soft operations between soft sets. Although some of these operations were considered ill-defined, they formed the starting point of constructing soft set theory. In this regard, Ali et al. [5] redefined some soft operators to make them more functional for improving several new results and they explored new soft operators such as restricted union and restricted intersection of two soft sets.

In 2011, Shabir and Naz [32] exploited soft sets to introduce soft topological spaces. The fundamental soft topological notions such as the operators of soft closure and interior, soft subspace and soft separation axioms were investigated by them. Min [29] completed study of soft separation axioms and revised some results obtained in [32]. Soft compactness was introduced and discussed by Aygünoğlu and Aygün [14] in 2012. Hida [23] defined another type of soft compactness depending on the belong relations. Al-shami [10] did some amendments concern some types of soft union and intersection. Then, he [11] studied new types of soft compactness. The authors of [25] presented soft maps by using two crisp maps, one of them between the sets of parameters and the second one between the universal sets. However, the authors of [36] introduced soft maps by using the concept of soft points. Some applications of different types of soft maps were the goal of some articles, see [25, 27, 36].

Until 2018, the belong and non-belong relations that utilized in these studies are those given by [32]. In 2018, the authors of [20] came up new relations of belong and non-belong between an element and soft set, namely partial belong and total non-belong relations. In fact, these relations widely open the door to study and redefine many soft topological notions. This leads to obtain many fruitful properties and changes which can be seen significantly on the study of soft separation axioms as it was showed in [9, 20, 21]. As another path of study soft separation axioms, the authors of [16, 35] studied them with respect to the distinct soft points. Recently, some applications of compactness and soft separation axioms have been investigated in [6, 7, 33, 34].

Das and Samanta [19] studied the concept of a soft metric based on the soft real set and soft real numbers given in [18]. Wardowski [36] tackled the fixed point in the setup of soft topological spaces. Abbas et al. [1] presented soft contraction mappings and established a soft Banach fixed point theorem in the framework of soft metric spaces. Recently, many researchers explored fixed point findings in soft metric type spaces, see, for example, [2, 37]. Some interesting works regarding deferential equations were given in [17, 24, 28].

One of the significant ideas that helps to prove some properties and remove some problems on soft topology is the concept of a soft point. It was first defined by Zorlutuna et al. [38] in order to study interior points of a soft set and soft neighborhood systems. Then [19] and [31] simultaneously redefined soft points to discuss soft metric spaces. In fact, the recent definition of a soft point makes similarity between many set-theoretic properties and their counterparts on soft setting. Two types of soft topologies, namely enriched soft topology and extended soft topology were studied in [14] and [31], respectively. The equivalence between these two topologies have been recently proved by Al-shami and Kočinac [12].

We organized this paper as follows: After this introduction, we allocate Section (2) to recall some definitions and results of soft sets and soft topologies that will help us to understand this work. Section (3) introduces tt-soft αT_i $(i = 0, 1, 2, 3, 4)$ and tt-soft α-regular spaces with respect to ordinary points by using total belong and total non-belong relations. The relationships between them and their main properties are discussed with the help of interesting examples. In Section (4), we explore an α-fixed soft point theorem and study some main properties. In particular, we conclude under what conditions α-fixed soft points are preserved between a soft topological space and its parametric topological spaces. Section (6) concludes the paper.

2. Preliminaries

To well understand the results obtained in this study, we shall recall some basic concepts, definitions and properties from the literature.

2.1. Soft sets

Definition 2.1. *[30] For a nonempty set X and a set of parameters E, a pair (G, E) is said to be a soft set over X provided that G is a map of E into the power set $P(X)$.*

In this study, we use a symbol G_E to refer a soft set instead of (G, E) and we identify it as ordered pairs $G_E = \{(e, G(e)) : e \in E$ and $G(e) \in P(X)\}$.

A family of all soft sets defined over X with E is denoted by $S(X_E)$.

Definition 2.2. *[22] A soft set G_E is said to be a subset of a soft set H_E, denoted by $G_E \widetilde{\subseteq} H_E$, if $G(e) \subseteq H(e)$ for each $e \in E$.*

The soft sets G_E and H_E are said to be soft equal if each one of them is a subset of the other.

Definition 2.3. *[20, 32] Let G_E be a soft set over X and $x \in X$. We say that:*

(i) *$x \in G_E$, it is read: x totally belongs to G_E, if $x \in G(e)$ for each $e \in E$.*

(ii) *$x \notin G_E$, it is read: x does not partially belong to G_E, if $x \notin G(e)$ for some $e \in E$.*

(iii) *$x \Subset G_E$, it is read: x partially belongs to G_E, if $x \in G(e)$ for some $e \in E$.*

(iv) *$x \not\Subset G_E$, it is read: x does not totally belong to G_E, if $x \notin G(e)$ for each $e \in E$.*

Definition 2.4. *Let G_E be a soft set over X and $x \in X$. We say that:*

(i) *G_E totally contains x if $x \in G_E$.*

(ii) *G_E does not partially contain x if $x \notin G_E$.*

(iii) *G_E partially contains x if $x \Subset G_E$.*

(iv) *G_E does not totally contain x if $x \not\Subset G_E$.*

Definition 2.5. *[5] The relative complement of a soft set G_E is a soft set G^c_E, where $G^c : E \to 2^X$ is a mapping defined by $G^c(e) = X \setminus G(e)$ for all $e \in E$.*

Definition 2.6. *[19, 20, 26, 31] A soft set (G, E) over X is said to be:*

(i) *a null soft set, denoted by $\widetilde{\Phi}$, if $G(e) = \emptyset$ for each $e \in E$.*

(ii) *an absolute soft set, denoted by $\widetilde{X}$, if $G(e) = X$ for each $e \in E$.*

(iii) *a soft point P_e^x if there are $e \in E$ and $x \in X$ such that $G(e) = \{x\}$ and $G(e') = \emptyset$ for each $e' \in E \setminus \{e\}$. We write that $P_e^x \in G_E$ if $x \in G(e)$.*

(iv) *a stable soft set, denoted by $\widetilde{S}$, if there is a subset S of X such that $G(e) = S$ for each $e \in E$. In particular, we denoted by x_E if $S = \{x\}$.*

(v) *a countable (resp. finite) soft set if $G(e)$ is countable (resp. finite) for each $e \in E$. Otherwise, it is said to be uncountable (resp. infinite).*

Definition 2.7. *[5, 26] Let G_E and H_E be two soft sets over X.*

(i) *Their intersection, denoted by $G_E \widetilde{\bigcap} H_E$, is a soft set U_E, where a mapping $U : E \to 2^X$ is given by $U(e) = G(e) \bigcap H(e)$.*

(ii) *Their union, denoted by $G_E \widetilde{\bigcup} H_E$, is a soft set U_E, where a mapping $U : E \to 2^X$ is given by $U(e) = G(e) \bigcup H(e)$.*

Definition 2.8. *[15] Let G_E and H_F be two soft sets over X and Y, respectively. Then the cartesian product of G_E and H_F, denoted by $G \times H_{E\times F}$, is defined as $(G \times H)(e, f) = G(e) \times H(f)$ for each $(e, f) \in E \times F$.*

The soft union and intersection operators were generalized for any number of soft sets in a similar way.

Definition 2.9. *[25] A soft mapping between $S(X_A)$ and $S(Y_B)$ is a pair (f, ϕ), denoted also by f_ϕ, of mappings such that $f : X \to Y$, $\phi : A \to B$. Let G_A and H_B be subsets of $S(X_A)$ and $S(Y_B)$, respectively. Then the image of G_A and pre-image of H_B are defined as follows.*

(i) *$f_\phi(G_A) = (f_\phi(G))_B$ is a subset of $S(Y_B)$ such $f_\phi(G)(b) = \bigcup_{a\in\phi^{-1}(b)} f(G(a))$ for each $b \in B$.*

(ii) *$f_\phi^{-1}(H_B) = (f_\phi^{-1}(H))_A$ is a subset of $S(X_A)$ such that $f_\phi^{-1}(H)(a) = f^{-1}(H(\phi(a)))$ for each $a \in A$.*

Definition 2.10. *[38] A soft map $f_\phi : S(X_A) \to S(Y_B)$ is said to be injective (resp. surjective, bijective) if ϕ and f are injective (resp. surjective, bijective).*

2.2. Soft topology

Definition 2.11. *[32] A family τ of soft sets over X under a fixed set of parameters E is said to be a soft topology on X if it satisfies the following.*

(i) *$\widetilde{X}$ and $\widetilde{\Phi}$ are members of τ.*

(ii) *The intersection of a finite number of soft sets in τ is a member of τ.*

(iii) *The union of an arbitrary number of soft sets in τ is a member of τ.*

The triple (X, τ, E) is called a soft topological space. A member in τ is called soft open and its relative complement is called soft closed.

Proposition 2.12. *[32] In (X, τ, E), a family $\tau_e = \{G(e) : G_E \in \tau\}$ is a classical topology on X for each $e \in E$.*

τ_e is called a parametric topology and (X, τ_e) is called a parametric topological space.

Definition 2.13. *[32] Let (X, τ, E) be a soft topological space and $\emptyset \neq Y \subseteq X$. A family $\tau_Y = \{\widetilde{Y}\widetilde{\bigcap}G_E : G_E \in \tau\}$ is called a soft relative topology on Y and the triple (Y, τ_Y, E) is called a soft subspace of (X, τ, E).*

Definition 2.14. *[3] A subset G_E of (X, τ, E) is called soft α-open if $G_E \widetilde{\subseteq} int(cl(int(G_E)))$.*

The following result will help us to establish some properties of soft α-separation axioms and soft α-compact spaces, see, for example, Theorem (3.16) and Proposition (3.24). It implies that the family of soft α-open subsets of (X, τ, E) forms a new soft topology τ^α over X that is finer than τ. In fact, this characteristic of soft α-open sets that does not exist for the families of soft semi-open, soft pre-open, soft b-open and soft β-open sets.

Theorem 2.15. *[3, 8]*

(i) *Every soft open set is soft α-open.*

(ii) *The arbitrary union (finite intersection) of soft α-open sets is soft α-open.*

Definition 2.16. *[3] Let G_E be a subset of (X, τ, E). Then $\overline{G_E}^{\alpha}$ is the intersection of all soft α-closed sets containing G_E.*

It is clear that: $x \in \overline{G_E}^{\alpha}$ if and only if $G_E \widetilde{\bigcap} U_E \neq \widetilde{\Phi}$ for each soft α-open set U_E totally containing x; and $P^x_e \in \overline{G_E}^{\alpha}$ if and only if $G_E \widetilde{\bigcap} U_E \neq \widetilde{\Phi}$ for each soft α-open set U_E totally containing P^x_e.

Proposition 2.17. *[8] Let $\widetilde{Y}$ be soft open subset of (X, τ, E). Then:*

1. *If (H, E) is soft α-open and $\widetilde{Y}$ is soft open in (X, τ, E), then $(H, E)\widetilde{\bigcap}(Y, E)$ is a soft α-open subset of (Y, τ_Y, E).*
2. *If $\widetilde{Y}$ is soft open in (X, τ, E) and (H, E) is a soft α-open in (Y, τ_Y, E), then (H, E) is a soft α-open subset of (X, τ, E).*

Definition 2.18. *[4] (X, τ, E) is said to be:*

(i) *soft αT_0 if for every $x \neq y \in X$, there is a soft α-open set U_E such that $x \in U_E$ and $y \notin U_E$; or $y \in U_E$ and $x \notin U_E$.*

(ii) *soft αT_1 if for every $x \neq y \in X$, there are two soft α-open sets U_E and V_E such that $x \in U_E$ and $y \notin U_E$; and $y \in V_E$ and $x \notin V_E$.*

(iii) *soft αT_2 if for every $x \neq y \in X$, there are two disjoint soft α-open sets U_E and V_E such that $x \in G_E$ and $y \in F_E$.*

(iv) *soft α-regular if for every soft α-closed set H_E and $x \in X$ such that $x \notin H_E$, there are two disjoint soft α-open sets U_E and V_E such that $H_E \widetilde{\subseteq} U_E$ and $x \in V_E$.*

(v) *soft α-normal if for every two disjoint soft α-closed sets H_E and F_E, there are two disjoint soft α-open sets U_E and V_E such that $H_E \widetilde{\subseteq} U_E$ and $F_E \widetilde{\subseteq} V_E$.*

(vi) *soft αT_3 (resp. soft αT_4) if it is both soft α-regular (resp. soft α-normal) and soft αT_1-space.*

Definition 2.19. *[8] A family $\{G_{i_E} : i \in I\}$ of soft α-open subsets of (X, τ, E) is said to be a soft α-open cover of $\widetilde{X}$ if $\widetilde{X} = \widetilde{\bigcup}_{i \in I} G_{i_E}$.*

Definition 2.20. *[8]* (X, τ, E) *is said to be:*

(i) *soft $\alpha T_2'$ if for every $P_e^x \neq P_{e'}^y \in \widetilde{X}$, there are two disjoint soft α-open sets U_E and V_E containing P_e^x and $P_{e'}^y$, respectively.*

(ii) *soft α-compact if every soft α-open cover of $\widetilde{X}$ has a finite subcover.*

Proposition 2.21. *[8]*

(i) *A soft α-compact subset of a soft $\alpha T_2'$-space is soft α-closed.*

(ii) *A stable soft α-compact subset of a soft αT_2-space is soft α-closed.*

To study the properties that preserved under soft $\alpha^\star$-homeomorphism maps, the concept of a soft α-irresolute map will be presented in this work under the name of a soft $\alpha^\star$-continuous map.

Definition 2.22. *[8] $g_\varphi : (X, \tau, E) \to (Y, \theta, E)$ is called soft $\alpha^\star$-continuous if the inverse image of each soft α-open set is soft α-open.*

Proposition 2.23. *[8] The soft $\alpha^\star$-continuous image of a soft α-compact set is soft α-compact.*

Definition 2.24. *[3] A soft map $f_\varphi : (X, \tau, A) \to (Y, \theta, B)$ is said to be:*

(i) *soft α-continuous if the inverse image of each soft open set is soft α-open.*

(ii) *soft α-open (resp. soft α-closed) if the image of each soft open (resp. soft closed) set is soft α-open (resp. soft α-closed).*

(iii) *a soft α-homeomorphism if it is bijective, soft α-continuous and soft α-open.*

Definition 2.25. *A soft topology τ on X is said to be:*

(i) *an enriched soft topology [14] if all soft sets G_E such that $G(e) = \emptyset$ or X are members of τ.*

(ii) *an extended soft topology [31] if $\tau = \{G_E : G(e) \in \tau_e$ for each $e \in E\}$, where τ_e is a parametric topology on X.*

Al-shami and Kočinac [12] proved the equivalence of enriched and extended soft topologies and obtained many useful results that help to study the relationships between soft topological spaces and their parametric topological spaces.

Theorem 2.26. *[12] A subset (F, E) of an extended soft topological space (X, τ, E) is soft α-open if and only if each e-approximate element of (F, E) is α-open.*

Proposition 2.27. *[13] Let $\{(X_i, \tau_i, E) : i \in I\}$ be a family of pairwise disjoint soft topological spaces and $X = \bigcup_{i \in I} X_i$. Then the collection*

$$\tau = \{(G, E)\widetilde{\subseteq}\widetilde{X} : (G, E)\widetilde{\bigcap}\widetilde{X_i} \text{ is a soft open set in } (X_i, \tau_i, E) \text{ for every } i \in I\}$$

defines a soft topology on X with a fixed set of parameters E.

Definition 2.28. *[13] The soft topological space (X, τ, E) given in the above proposition is said to be the* sum of soft topological spaces *and is denoted by $(\oplus_{i \in I} X_i, \tau, E)$.*

Theorem 2.29. *[13] A soft set* $(G,E)\widetilde{\subseteq}\widetilde{\oplus_{i\in I}X_i}$ *is soft* α*-open (resp. soft* α*-closed) in* $(\oplus_{i\in I}X_i,\tau,E)$ *if and only if all* $(G,E)\widetilde{\bigcap}\widetilde{X_i}$ *are soft* α*-open (resp. soft* α*-closed) in* (X_i,τ_i,E)*.*

Proposition 2.30. *[36] Let* $g_\varphi:(X,\tau,E)\to(X,\tau,E)$ *be a soft map such that* $\widetilde{\bigcap}_{n\in\mathbb{N}}g^n_\varphi(\widetilde{X})$ *is a soft point* P^x_e*. Then* P^x_e *is a unique fixed point of* g_φ*.*

Theorem 2.31. *[38] Let* (X,τ,A) *and* (Y,θ,B) *be two soft topological spaces and* $\Omega=\{G_A\times F_B : G_A\in\tau$ *and* $F_B\in\theta\}$*. Then the family of all arbitrary union of elements of* Ω *is a soft topology over* $X\times Y$ *under a fixed set of parameters* $A\times B$*.*

Lemma 2.32. *[7] Let* (G,A) *and* (H,B) *be two subsets of* (X_1,τ_1,A) *and* (X_2,τ_2,B)*, respectively. Then:*

(i) $cl(G,A)\times cl(H,B)=cl((G,A)\times(H,B))$.

(ii) $int(G,A)\times int(H,B)=int((G,A)\times(H,B))$.

3. α-soft separation axioms

This section introduces the concepts of *tt*-soft αT_i ($i=0,1,2,3,4$) and *tt*-soft α-regular spaces, where *tt* denote the total belong and total non-belong relations that are utilized in the definitions of these concepts. The relationships between them are showed and their main features are studied. In addition, their behaviours with the concepts of hereditary, topological and additive properties are investigated. Some examples are provided to elucidate the obtained results.

Definition 3.1. (X,τ,E) *is said to be:*

(i) *tt-soft* αT_0 *if for every* $x\neq y\in X$*, there exists a soft* α*-open set* U_E *such that* $x\in U_E$ *and* $y\notin U_E$ *or* $y\in U_E$ *and* $x\notin U_E$*.*

(ii) *tt-soft* αT_1 *if for every* $x\neq y\in X$*, there exist soft* α*-open sets* U_E *and* V_E *such that* $x\in U_E$ *and* $y\notin U_E$*; and* $y\in V_E$ *and* $x\notin V_E$*.*

(iii) *tt-soft* αT_2 *if for every* $x\neq y\in X$*, there exist two disjoint soft* α*-open sets* U_E *and* V_E *such that* $x\in U_E$ *and* $y\notin U_E$*; and* $y\in V_E$ *and* $x\notin V_E$*.*

(iv) *tt-soft* α*-regular if for every soft* α*-closed set* H_E *and* $x\in X$ *such that* $x\notin H_E$*, there exist disjoint soft* α*-open sets* U_E *and* V_E *such that* $H_E\widetilde{\subseteq}U_E$ *and* $x\in V_E$*.*

(v) *tt-soft* αT_3 *(resp. tt-soft* αT_4*) if it is both tt-soft* α*-regular (resp. soft* α*-normal) and tt-soft* αT_1*.*

Remark 3.2. *It can be noted that: If* F_E *and* G_E *are disjoint soft set, then* $x\in F_E$ *if and only if* $x\notin G_E$*. This implies that* (X,τ,E) *is a tt-soft* αT_2*-space if and only if is a soft* αT_2*-space. That is, the concepts of a tt-soft* αT_2*-space and a soft* αT_2*-space are equivalent.*

We can say that: (X,τ,E) *is tt-soft* αT_2 *if for every* $x\neq y\in X$*, there exist two disjoint soft* α*-open sets* U_E *and* V_E *totally contain x and y, respectively.*

Remark 3.3. *The soft* α*-regular spaces imply a strict condition on the shape of soft* α*-open and soft* α*-closed subsets. To explain this matter, let* F_E *be a soft* α*-closed set such that* $x\notin H_E$*. Then we have two cases:*

(i) *There are $e, e' \in E$ such that $x \notin H(e)$ and $x \in H(e')$. This case is impossible because there do not exist two disjoint soft sets U_E and V_E containing x and H_E, respectively.*

(ii) *For each $e \in E$, $x \notin H(e)$. This implies that H_E must be stable.*

As a direct consequence, we infer that every soft α-closed and soft α-open subsets of a soft α-regular space must be stable. However, this matter does not hold on the tt-soft α-regular spaces because we replace a partial non-belong relation by a total non-belong relation. Therefore a tt-soft α-regular space need not be stable.

Proposition 3.4. **(i)** *Every tt-soft αT_i-space is soft αT_i for $i = 0, 1, 4$.*

(ii) *Every soft α-regular space is tt-soft α-regular.*

(iii) *Every soft αT_3-space is tt-soft αT_3.*

Proof. The proofs of (i) and (ii) follow from the fact that a total non-belong relation $\not\Subset$ implies a partial non-belong relation $\not\in$.

To prove (iii), it suffices to prove that a soft αT_i-space is *tt*-soft αT_i when (X, τ, E) is soft α-regular. Suppose $x \neq y \in X$. Then there exist two soft α-open sets U_E and V_E such that $x \in U_E$ and $y \notin U_E$; and $y \in V_E$ and $x \notin V_E$. Since U_E and V_E are soft α-open subsets of a soft α-regular space, then they are stable. So $y \not\Subset U_E$ and $x \not\Subset V_E$. Thus (X, τ, E) is *tt*-soft αT_1. Hence, we obtain the desired result. □

To clarify that the converse of the above proposition does not hold in general, we give the following examples.

Example 3.5. *Let $E = \{e_1, e_2\}$ and $\tau = \{\widetilde{\Phi}, \widetilde{X}, G_{i_E} : i = 1, 2, 3\}$ be a soft topology on $X = \{x, y\}$, where*

$G_{1_E} = \{(e_1, \{x\}), (e_2, X)\}$;
$G_{2_E} = \{(e_1, X), (e_2, \{y\})\}$ and
$G_{3_E} = \{(e_1, \{x\}), (e_2, \{y\})\}$.

One can examine that $\tau = \tau^\alpha$. Then (X, τ, E) is a soft αT_1-space. On the other hand, it is not tt-soft αT_0 because there does not exist a soft α-open set containing one of the points x or y such that the other point does not totally belong to it.

Example 3.6. *Let $E = \{e_1, e_2\}$ and $\tau = \{\widetilde{\Phi}, \widetilde{X}, G_{i_E} : i = 1, 2, ..., 8\}$ be a soft topology on $X = \{x, y\}$, where*

$G_{1_E} = \{(e_1, X), (e_2, \{x\})\}$;
$G_{2_E} = \{(e_1, \emptyset), (e_2, \{y\})\}$;
$G_{3_E} = \{(e_1, \{y\}), (e_2, \emptyset)\}$;
$G_{4_E} = \{(e_1, \{y\}), (e_2, \{y\})\}$;
$G_{5_E} = \{(e_1, \{x\}), (e_2, \{y\})\}$;
$G_{6_E} = \{(e_1, X), (e_2, \{y\})\}$;
$G_{7_E} = \{(e_1, \{x\}), (e_2, \emptyset)\}$ and
$G_{8_E} = \{(e_1, X), (e_2, \emptyset)\}$.

By calculating, we find that $\tau^\alpha = \tau$.

Then (X, τ, E) is a soft αT_4-space. On the other hand, there does not exist a soft α-open set totally containing x such that y does not totally belong to it. So (X, τ, E) is not a tt-soft αT_1-space, hence it is not tt-soft αT_4.

Example 3.7. *Let X be any universal set X and E be any set of parameters such that $|X| \geq 2$ and $|E| \geq 2$. The discrete soft topology (X, τ, E) is a tt-soft α-regular space, but it is not soft α-regular. Hence, it is a tt-soft αT_3-space, but it is not soft αT_3.*

Before we show the relationship between *tt*-soft αT_i-spaces, we need to prove the following useful lemma.

Lemma 3.8. *(X, τ, E) is a tt-soft αT_1-space if and only if x_E is soft α-closed for every $x \in X$.*

Proof. Necessity: For each $y_i \in X \backslash \{x\}$, there is a soft α-open set G_{i_E} such that $y_i \in G_{i_E}$ and $x \notin G_{i_E}$. Therefore $X \backslash \{x\} = \bigcup_{i \in I} G_i(e)$ and $x \notin \bigcup_{i \in I} G_i(e)$ for each $e \in E$. Thus $\widetilde{\bigcup}_{i \in I} G_{i_E} = \widetilde{X \backslash \{x\}}$ is soft α-open. Hence, x_E is soft α-closed.

Sufficiency: Let $x \neq y$. By hypothesis, x_E and y_E are soft α-closed sets. Then x_E^c and y_E^c are soft α-open sets such that $x \in (y_E)^c$ and $y \in (x_E)^c$. Obviously, $y \notin (y_E)^c$ and $x \notin (x_E)^c$. Hence, (X, τ, E) is *tt*-soft αT_1. □

Proposition 3.9. *Every tt-soft αT_i-space is tt-soft αT_{i-1} for $i = 1, 2, 3, 4$.*

Proof. We prove the proposition in the cases of $i = 3, 4$. The other cases follow similar lines.

For $i = 3$, let $x \neq y$ in a *tt*-soft αT_3-space (X, τ, E). Then x_E is soft α-closed. Since $y \notin x_E$ and (X, τ, E) is *tt*-soft α-regular, then there are disjoint soft α-open sets G_E and F_E such that $x_E \widetilde{\subseteq} G_E$ and $y \in F_E$. Therefore (X, τ, E) is *tt*-soft αT_2.

For $i = 4$, let $x \in X$ and H_E be a soft α-closed set such that $x \notin H_E$. Since (X, τ, E) is *tt*-soft αT_1, then x_E is soft α-closed. Since $x_E \widetilde{\bigcap} H_E = \widetilde{\Phi}$ and (X, τ, E) is soft α-normal, then there are disjoint soft α-open sets G_E and F_E such that $H_E \widetilde{\subseteq} G_E$ and $x_E \widetilde{\subseteq} F_E$. Hence, (X, τ, E) is *tt*-soft αT_3. □

The following examples show that the converse of the above proposition is not always true.

Example 3.10. *Let (X, τ, E) be a soft topological space given in Example (3.6). For $x \neq y$, we have G_{4_E} is a soft α-open set such that $y \in G_{4_E}$ and $x \notin G_{4_E}$. Then (X, τ, E) is tt-soft αT_0. However, it is not tt-soft αT_1 because there does not exist a soft α-open set totally containing x and does not totally contain y.*

Example 3.11. *Let E be any set of parameters and $\tau = \{\widetilde{\Phi}, G_E \widetilde{\subseteq} \mathbb{N} : G_E^c$ is finite$\}$ be a soft topology on the set of natural numbers $\mathbb{N}$. It is clear that a soft subset of $(\mathbb{N}, \tau, E)$ is soft α-open if and only if it is soft open. For each $x \neq y \in \mathbb{N}$, we have $\widetilde{\mathbb{N} \setminus \{y\}}$ and $\widetilde{\mathbb{N} \setminus \{x\}}$ are soft α-open sets such that $x \in \widetilde{\mathbb{N} \setminus \{y\}}$ and $y \notin \widetilde{\mathbb{N} \setminus \{y\}}$; and $y \in \widetilde{\mathbb{N} \setminus \{x\}}$ and $x \notin \widetilde{\mathbb{N} \setminus \{x\}}$. Therefore $(\mathbb{N}, \tau, E)$ is tt-soft αT_1. On the other hand, there do not exist two disjoint soft α-open sets except for the null and absolute soft sets. Hence, $(\mathbb{N}, \tau, E)$ is not tt-soft αT_2.*

Example 3.12. *It is well known that a soft topological space is a classical topological space if E is a singleton. Then it suffices to consider examples that satisfy an αT_2-space but not αT_3; satisfy an αT_3-space but not αT_4.*

In what follows, we establish some properties of *tt*-soft αT_i and *tt*-soft α-regular.

Lemma 3.13. *Let U_E be a subset of (X, τ, E) and $x \in X$. Then $x \notin \overline{U_E}^{\alpha}$ iff there exists a soft α-open set V_E totally containing x such that $U_E \widetilde{\bigcap} V_E = \widetilde{\Phi}$.*

Proof. Let $x \notin \overline{U_E}^{\alpha}$. Then $x \in (\overline{U_E}^{\alpha})^c = V_E$. So $U_E \widetilde{\bigcap} V_E = \widetilde{\Phi}$. Conversely, if there exists a soft α-open set V_E totally containing x such that $U_E \widetilde{\bigcap} V_E = \widetilde{\Phi}$, then $U_E \subseteq V_E^c$. Therefore $\overline{U_E}^{\alpha} \subseteq V_E^c$. Since $x \notin V_E^c$, then $x \notin \overline{U_E}^{\alpha}$. □

Proposition 3.14. *If (X, τ, E) is a tt-soft αT_0-space, then $\overline{x_E}^{\alpha} \neq \overline{y_E}^{\alpha}$ for every $x \neq y \in X$.*

Proof. Let $x \neq y$ in a *tt*-soft αT_0-space. Then there is a soft α-open set U_E such that $x \in U_E$ and $y \notin U_E$ or $y \in U_E$ and $x \notin U_E$. Say, $x \in U_E$ and $y \notin U_E$. Now, $y_E \widetilde{\bigcap} U_E = \widetilde{\Phi}$. So, by the above lemma, $x \notin \overline{y_E}^{\alpha}$. But $x \in \overline{x_E}^{\alpha}$. Hence, we obtain the desired result. □

Corollary 3.15. *If (X, τ, E) is a tt-soft αT_0-space, then $\overline{P_e^x}^{\alpha} \neq \overline{P_{e'}^y}^{\alpha}$ for all $x \neq y$ and $e, e' \in E$.*

Theorem 3.16. *Let E be a finite set. Then (X, τ, E) is a tt-soft αT_1-space if and only if $x_E = \widetilde{\bigcap}\{U_E : x \in U_E \in \tau^{\alpha}\}$ for each $x \in X$.*

Proof. To prove the "*if*" part, let $y \in X$. Then for each $x \in X \setminus \{y\}$, we have a soft α-open set U_E such that $x \in U_E$ and $y \notin U_E$. Therefore $y \notin \widetilde{\bigcap}\{U_E : x_E \widetilde{\subseteq} U_E \in \tau^{\alpha}\}$. Since y is chosen arbitrary, then the desired result is proved.

To prove the "*only if*" part, let the given conditions be satisfied and let $x \neq y$. Let $|E| = m$. Since $y \notin x_E$, then for each $j = 1, 2, ..., m$ there is a soft α-open set U_{i_E} such that $y \notin U_i(e_j)$ and $x \in U_{i_E}$. Therefore $\widetilde{\bigcap}_{i=1}^m U_{i_E}$ is a soft α-open set such that $y \notin \widetilde{\bigcap}_{i=1}^m U_{i_E}$ and $x \in \widetilde{\bigcap}_{i=1}^m U_{i_E}$. Similarly, we can get a soft α-open set V_E such that $y \in V_E$ and $x \notin V_E$. Thus (X, τ, E) is a *tt*-soft αT_1-space. □

Theorem 3.17. *If (X, τ, E) is an extended tt-soft αT_1-space, then P_e^x is soft α-closed for all $P_e^x \in \widetilde{X}$.*

Proof. It follows from Lemma (3.8) that $\widetilde{X \setminus \{x\}}$ is a soft α-open set. Since (X, τ, E) is extended, then a soft set H_E, where $H(e) = \emptyset$ and $H(e') = X$ for each $e' \neq e$, is a soft α-open set. Therefore $\widetilde{X \setminus \{x\}} \widetilde{\bigcup} H_E$ is soft α-open. Thus $(\widetilde{X \setminus \{x\}} \widetilde{\bigcup} H_E)^c = P_e^x$ is soft α-closed. □

Corollary 3.18. *If (X, τ, E) is an extended tt-soft αT_1-space, then the intersection of all soft α-open sets containing U_E is exactly U_E for each $U_E \widetilde{\subseteq} \widetilde{X}$.*

Proof. Let U_E be a soft subset of $\widetilde{X}$. Since P_e^x is a soft α-closed set for every $P_e^x \in U_E^c$, then $\widetilde{X} \setminus P_e^x$ is a soft α-open set containing U_E. Therefore $U_E = \widetilde{\bigcap}\{\widetilde{X} \setminus P_e^x : P_e^x \in U_E^c\}$, as required. □

Theorem 3.19. *A finite (X, τ, E) is tt-soft αT_2 if and only if it is tt-soft αT_1.*

Proof. Necessity: It is obtained from Proposition (3.9).

Sufficiency: For each $x \neq y$, we have x_E and y_E are soft α-closed sets. Since X is finite, then $\widetilde{\bigcup}_{y \in X \setminus \{x\}} y_E$ and $\widetilde{\bigcup}_{x \in X \setminus \{y\}} x_E$ are soft α-closed sets. Therefore $(\widetilde{\bigcup}_{y \in X \setminus \{x\}} y_E)^c = x_E$ and $(\widetilde{\bigcup}_{x \in X \setminus \{y\}} x_E)^c = y_E$ are disjoint soft α-open sets. Thus (X, τ, E) is a *tt*-soft αT_2-space. □

Corollary 3.20. *A finite tt-soft αT_1-space is soft α-disconnected.*

Remark 3.21. *In Example* (3.11)*, note that x_E is not a soft α-open set for each $x \in \mathbb{N}$. This clarifies that a soft set x_E in a tt-soft αT_1-space need not be soft α-open if the universal set is infinite.*

Theorem 3.22. (X, τ, E) *is tt-soft α-regular iff for every soft α-open subset F_E of (X, τ, E) totally containing x, there is a soft α-open set V_E such that $x \in V_E \widetilde{\subseteq} \overline{V_E}^{\alpha} \widetilde{\subseteq} F_E$.*

Proof. Let $x \in X$ and F_E be a soft α-open set totally containing x. Then F_E^c is α-soft closed and $x_E \widetilde{\bigcap} F_E^c = \widetilde{\Phi}$. Therefore there are disjoint soft α-open sets U_E and V_E such that $F_E^c \widetilde{\subseteq} U_E$ and $x \in V_E$. Thus $V_E \widetilde{\subseteq} U_E^c \widetilde{\subseteq} F_E$. Hence, $\overline{V_E}^{\alpha} \widetilde{\subseteq} U_E^c \widetilde{\subseteq} F_E$. Conversely, let F_E^c be a soft α-closed set. Then for each $x \notin F_E^c$, we have $x \in F_E$. By hypothesis, there is a soft α-open set V_E totally containing x such that $\overline{V_E}^{\alpha} \widetilde{\subseteq} F_E$. Therefore $F_E^c \widetilde{\subseteq} (\overline{V_E}^{\alpha})^c$ and $V_E \widetilde{\bigcap} (\overline{V_E}^{\alpha})^c = \widetilde{\Phi}$. Thus (X, τ, E) is *tt*-soft α-regular, as required. □

Theorem 3.23. *The following properties are equivalent if (X, τ, E) is a tt-soft α-regular space.*

(i) *a tt-soft αT_2-space.*

(ii) *a tt-soft αT_1-space.*

(iii) *a tt-soft αT_0-space.*

Proof. The directions **(i)** → **(ii)** and **(ii)** → **(iii)** are obvious.

To prove **(iii)** → **(i)**, let $x \neq y$ in a *tt*-soft αT_0-space (X, τ, E). Then there exists a soft α-open set G_E such that $x \in G_E$ and $y \notin G_E$, or $y \in G_E$ and $x \notin G_E$. Say, $x \in G_E$ and $y \notin G_E$. Obviously, $x \notin G_E^c$ and $y \in G_E^c$. Since (X, τ, E) is *tt*-soft α-regular, then there exist two disjoint soft α-open sets U_E and V_E such that $x \in U_E$ and $y \in G_E^c \widetilde{\subseteq} V_E$. Hence, (X, τ, E) is *tt*-soft αT_2. □

Proposition 3.24. *A finite tt-soft αT_2-space (X, τ, E) is tt-soft α-regular.*

Proof. Let H_E be a soft α-closed set and $x \in X$ such that $x \notin H_E$. Then $x \neq y$ for each $y \Subset H_E$. By hypothesis, there are two disjoint soft α-open sets U_{i_E} and V_{i_E} such that $x \in U_{i_E}$ and $y \in V_{i_E}$. Since $\{y : y \in X\}$ is a finite set, then there is a finite number of soft α-open sets V_{i_E} such that $H_E \widetilde{\subseteq} \widetilde{\bigcup}_{i=1}^{m} V_{i_E}$. Now, $\widetilde{\bigcap}_{i=1}^{m} U_{i_E}$ is a soft α-open set containing x and $[\widetilde{\bigcup}_{i=1}^{m} V_{i_E}] \widetilde{\bigcap} [\widetilde{\bigcap}_{i=1}^{m} U_{i_E}] = \widetilde{\Phi}$. Hence, (X, τ, E) is *tt*-soft α-regular. □

Corollary 3.25. *The following properties are equivalent if (X, τ, E) is finite.*

(i) *a tt-soft αT_3-space.*

(ii) *a tt-soft αT_2-space.*

(iii) *a tt-soft αT_1-space.*

Proof. The directions **(i)** → **(ii)** and **(ii)** → **(iii)** follow from Proposition (3.9).

The direction **(iii)** → **(ii)** follows from Theorem (3.19).

The direction **(ii)** → **(i)** follows from Proposition (3.24). □

Theorem 3.26. *The property of being a tt-soft αT_i-space $(i = 0, 1, 2, 3)$ is a soft open hereditary.*

Proof. We prove the theorem in the case of $i = 3$ and the other cases follow similar lines.

Let (Y, τ_Y, E) be a soft open subspace of a *tt*-soft αT_3-space (X, τ, E). To prove that (Y, τ_Y, E) is *tt*-soft αT_1, let $x \neq y \in Y$. Since (X, τ, E) is a *tt*-soft αT_1-space, then there exist two soft α-open sets G_E and F_E such that $x \in G_E$ and $y \notin G_E$; and $y \in F_E$ and $x \notin F_E$. Therefore $x \in U_E = \widetilde{Y} \widetilde{\bigcap} G_E$ and $y \in V_E = \widetilde{Y} \widetilde{\bigcap} F_E$ such that $y \notin U_E$ and $x \notin V_E$. It follows from Proposition (2.17), that U_E and V_E are soft α-open subsets of (Y, τ_Y, E), so that (Y, τ_Y, E) is *tt*-soft αT_1.

To prove that (Y, τ_Y, E) is *tt*-soft α-regular, let $y \in Y$ and F_E be a soft α-closed subset of (Y, τ_Y, E) such that $y \notin F_E$. Then $F_E \widetilde{\bigcup} \widetilde{Y}^c$ is a soft α-closed subset of (X, τ, E) such thaty $\notin F_E \widetilde{\bigcup} \widetilde{Y}^c$. Therefore there exist disjoint soft α-open subsets U_E and V_E of (X, τ, E) such that $F_E \widetilde{\bigcup} \widetilde{Y^c} \widetilde{\subseteq} U_E$ and $y \in V_E$. Now, $U_E \widetilde{\bigcap} \widetilde{Y}$ and $V_E \widetilde{\bigcap} \widetilde{Y}$ are disjoint soft α-open subsets of (Y, τ_Y, E) such that $F_E \widetilde{\subseteq} U_E \widetilde{\bigcap} \widetilde{Y}$ and $y \in V_E \widetilde{\bigcap} \widetilde{Y}$. Thus (Y, τ_Y, E) is *tt*-soft α-regular.

Hence, (Y, τ_Y, E) is *tt*-soft αT_3, as required. □

Theorem 3.27. *Let (X, τ, E) be extended and $i = 0, 1, 2, 3, 4$. Then (X, τ, E) is tt-soft αT_i iff (X, τ_e) is αT_i for each $e \in E$.*

Proof. We prove the theorem in the case of $i = 4$ and one can similarly prove the other cases.

Necessity: Let $x \neq y$ in X. Then there exist two soft α-open sets U_E and V_E such that $x \in U_E$ and $y \notin U_E$; and $y \in V_E$ and $x \notin V_E$. Obviously, $x \in U(e)$ and $y \notin U(e)$; and $y \in V(e)$ and $x \notin V(e)$. Since (X, τ, E) is extended, then it follows from Theorem (2.26) that $U(e)$ and $V(e)$ are α-open subsets of (X, τ_e) for each $e \in E$. Thus, (X, τ_e) is an αT_1-space. To prove that (X, τ_e) is α-normal, let F_e and H_e be two disjoint α-closed subsets of (X, τ_e). Let F_E and H_E be two soft sets given by $F(e) = F_e$, $H(e) = H_e$ and $F(e') = H(e') = \emptyset$ for each $e' \neq e$. It follows, from Theorem (2.26) that F_E and H_E are two disjoint soft α-closed subsets of (X, τ, E). By hypothesis, there exist two disjoint soft α-open sets G_E and W_E such that $F_E \widetilde{\subseteq} G_E$ and $H_E \widetilde{\subseteq} W_E$. This implies that $F(e) = F_e \subseteq G(e)$ and $H(e) = H_e \subseteq W(e)$. Since (X, τ, E) is extended, then it follows from Theorem (2.26) that $G(e)$ and $W(e)$ are α-open subsets of (X, τ_e). Thus, (X, τ_e) is an α-normal space. Hence, it is an αT_4-space.

Sufficiency: Let $x \neq y$ in X. Then there exists two α-open subsets U_e and V_e of (X, τ_e) such that $x \in U_e$ and $y \notin U_e$; and $y \in V_e$ and $x \notin V_e$. Let U_E and V_E be two soft sets given by $U(e) = U_e$, $V(e) = V_e$ for each $e \in E$. Since (X, τ, E) is extended, then it follows from Theorem (2.26) that U_E and V_E are soft α-open subsets of (X, τ, E) such that $x \in U_E$ and $y \notin U_E$; and $y \in V_E$ and $x \notin V_E$. Thus, (X, τ, E) is a *tt*-soft αT_1-space. To prove that (X, τ, E) is soft α-normal, let F_E and H_E be two disjoint soft α-closed subsets of (X, τ, E). Since (X, τ, E) is extended, then it follows from Theorem (2.26) that $F(e)$ and $H(e)$ are two disjoint α-closed subsets of (X, τ_e). By hypothesis, there exist two disjoint α-open subsets G_e and W_e of (X, τ_e) such that $F(e) \subseteq G_e$ and $H(e) \subseteq W_e$. Let G_E and W_E be two soft sets given by $G(e) = G_e$ and $W(e) = W_e$ for each $e \in E$. Since (X, τ, E) is extended, then it follows from Theorem (2.26) that G_E and W_E are two disjoint soft α-open subsets of (X, τ, E) such that $F_E \widetilde{\subseteq} G_E$ and $H_E \widetilde{\subseteq} W_E$. Thus (X, τ, E) is soft α-normal. Hence, it is a *tt*-soft αT_4-space. □

In the following examples, we show that a condition of an extended soft topology given in the above theorem is not superfluous.

Example 3.28. *Let $E = \{e_1, e_2\}$ and $\tau = \{\widetilde{\Phi}, \widetilde{X}, G_{1_E}, G_{2_E}\}$ be a soft topology on $X = \{x, y\}$, where*

$G_{1_E} = \{(e_1, \{x\}), (e_2, \{y\})\}$ *and*

$G_{2_E} = \{(e_1, \{y\}), (e_2, \{x\})\}$.

One can examine that $\tau = \tau^\alpha$. It is clear that (X, τ, E) is not a tt-soft αT_0-space. On the other hand, τ_{e_1} and τ_{e_2} are the discrete topology on X. Hence, the two parametric topological spaces (X, τ_{e_1}) and (X, τ_{e_2}) are αT_4.

Theorem 3.29. *The property of being a tt-soft αT_i-space $(i = 0, 1, 2)$ is preserved under a finite product soft spaces.*

Proof. We prove the theorem in case of $i = 2$. The other cases follow similar lines.

Let (X_1, τ_1, E_1) and (X_2, τ_2, E_2) be two *tt*-soft αT_2-spaces and let $(x_1, y_1) \neq (x_2, y_2)$ in $X_1 \times X_2$. Then $x_1 \neq x_2$ or $y_1 \neq y_2$. Without loss of generality, let $x_1 \neq x_2$. Then there exist two disjoint soft α-open subsets G_{E_1} and H_{E_1} of (X_1, τ_1, E_1) such that $x_1 \in G_{E_1}$ and $x_2 \notin G_{E_1}$; and $x_2 \in H_{E_1}$ and $x_1 \notin H_{E_1}$. Obviously, $G_{E_1} \times \widetilde{X_2}$ and $H_{E_1} \times \widetilde{X_2}$ are two disjoint soft α-open subsets $X_1 \times X_2$ such that $(x_1, y_1) \in G_{E_1} \times \widetilde{X_2}$ and $(x_2, y_2) \notin G_{E_1} \times \widetilde{X_2}$; and $(x_2, y_2) \in H_{E_1} \times \widetilde{X_2}$ and $(x_1, y_1) \notin H_{E_1} \times \widetilde{X_2}$. Hence, $X_1 \times X_2$ is a *tt*-soft αT_2-space. □

Theorem 3.30. *The property of being a tt-soft αT_i-space is an additive property for $i = 0, 1, 2, 3, 4$.*

Proof. To prove the theorem in the cases of $i = 2$. Let $x \neq y \in \oplus_{i \in I} X_i$. Then we have the following two cases:

1. There exists $i_0 \in I$ such that $x, y \in X_{i_0}$. Since (X_{i_0}, τ_{i_0}, E) is *tt*-soft αT_2, then there exist two disjoint soft α-open subsets G_E and H_E of (X_{i_0}, τ_{i_0}, E) such that $x \in G_E$ and $y \in H_E$. It follows from Theorem (2.29), that G_E and H_E are disjoint soft α-open subsets of $(\oplus_{i \in I} X_i, \tau, E)$.
2. There exist $i_0 \neq j_0 \in I$ such that $x \in X_{i_0}$ and $y \in X_{j_0}$. Now, $\widetilde{X_{i_0}}$ and $\widetilde{X_{j_0}}$ are soft α-open subsets of (X_{i_0}, τ_{i_0}, E) and (X_{j_0}, τ_{j_0}, E), respectively. It follows from Theorem (2.29), that $\widetilde{X_{i_0}}$ and $\widetilde{X_{j_0}}$ are disjoint soft α-open subsets of $(\oplus_{i \in I} X_i, \tau, E)$.

It follows from the two cases above that $(\oplus_{i \in I} X_i, \tau, E)$ is a *tt*-soft αT_2-space.

The theorem can be proved similarly in the cases of $i = 0, 1$.

To prove the theorem in the cases of $i = 3$ and $i = 4$, it suffices to prove the *tt*-soft α-regularity and soft α-normality, respectively.

First, we prove the *tt*-soft α-regularity property. Let F_E be a soft α-closed subset of $(\oplus_{i \in I} X_i, \tau, E)$ such that $x \notin F_E$. It follows from Theorem (2.29) that $F_E \widetilde{\bigcap} \widetilde{X_i}$ is soft α-closed in (X_i, τ_i, E) for each $i \in I$. Since $x \in \oplus_{i \in I} X_i$, there is only $i_0 \in I$ such that $x \in X_{i_0}$. This implies that there are disjoint soft α-open subsets G_E and H_E of (X_{i_0}, τ_{i_0}, E) such that $F_E \widetilde{\bigcap} \widetilde{X_{i_0}} \widetilde{\subseteq} G_E$ and $y \in H_E$. Now, $G_E \widetilde{\bigcup_{i \neq i_0}} \widetilde{X_i}$ is a soft α-open subset of $(\oplus_{i \in I} X_i, \tau, E)$ containing F_E. The disjointness of $G_E \bigcup_{i \neq i_0} X_i$ and H_E ends the proof that $(\oplus_{i \in I} X_i, \tau, E)$ is a *tt*-soft α-regular space.

Second, we prove the soft α-normality property. Let F_E and H_E be two disjoint soft α-closed subsets of $(\oplus_{i \in I} X_i, \tau, E)$. It follows from Theorem (2.29) that $F_E \widetilde{\bigcap} \widetilde{X_i}$ and $H_E \widetilde{\bigcap} \widetilde{X_i}$ are soft α-closed in (X_i, τ_i, E) for each $i \in I$. Since (X_i, τ_i, E) is soft α-normal for each $i \in I$, then there there exist two disjoint soft α-open subsets U_{i_E} and V_{i_E} of (X_i, τ_i, E) such that $F_E \widetilde{\bigcap} \widetilde{X_i} \widetilde{\subseteq} U_{i_E}$ and $H_E \widetilde{\bigcap} \widetilde{X_i} \widetilde{\subseteq} V_{i_E}$. This implies that $F_E \widetilde{\subseteq} \widetilde{\bigcup_{i \in I}} U_{i_E}$, $H_E \widetilde{\subseteq} \widetilde{\bigcup_{i \in I}} V_{i_E}$ and $[\widetilde{\bigcup_{i \in I}} U_{i_E}] \widetilde{\bigcap} [\widetilde{\bigcup_{i \in I}} V_{i_E}] = \widetilde{\Phi}$. Hence, $(\oplus_{i \in I} X_i, \tau, E)$ is a soft α-normal space. □

In the following we probe the behaviours of *tt*-soft αT_i-spaces under some soft maps.

Definition 3.31. *A map $f_\varphi : (X, \tau, A) \to (Y, \theta, B)$ is said to be:*

1. *soft $\alpha^\star$-continuous if the inverse image of soft α-open set is soft α-open.*
2. *soft $\alpha^\star$-open (resp. soft $\alpha^\star$-closed) if the image of soft α-open (resp. soft α-closed) set is soft α-open (resp. soft α-closed).*
3. *soft $\alpha^\star$-homeomorphism if it is bijective, soft $\alpha^\star$-continuous and soft $\alpha^\star$-open.*

Proposition 3.32. *Let $f_\varphi : (X, \tau, A) \to (Y, \theta, B)$ be a soft α-continuous map such that f is injective. Then if (Y, θ, B) is a p-soft T_i-space, then (X, τ, A) is a tt-soft αT_i-space for $i = 0, 1, 2$.*

Proof. We only prove the proposition for $i = 2$.

Let $f_\varphi : (X, \tau, A) \to (Y, \theta, B)$ be a soft α-continuous map and $a \neq b \in X$. Since f is injective, then there are two distinct points x and y in Y such that $f(a) = x$ and $f(b) = y$. Since (Y, θ, B) is a p-soft T_2-space, then there are two disjoint soft open sets G_B and F_B such that $x \in G_B$ and $y \in F_B$. Now, $f_\varphi^{-1}(G_B)$ and $f_\varphi^{-1}(F_B)$ are two disjoint soft α-open subsets of (X, τ, A) such that $a \in f_\varphi^{-1}(G_B)$ and $b \in f_\varphi^{-1}(F_B)$. Thus (X, τ, A) is a *tt*-soft αT_2-space. □

In a similar way, one can prove the following result.

Proposition 3.33. *Let $f_\varphi : (X, \tau, A) \to (Y, \theta, B)$ be a soft $\alpha^\star$-continuous map such that f is injective. Then if (Y, θ, B) is a tt-soft αT_i-space, then (X, τ, A) is a tt-soft αT_i-space for $i = 0, 1, 2$.*

Proposition 3.34. *Let $f_\varphi : (X, \tau, A) \to (Y, \theta, B)$ be a bijective soft α-open map. Then if (X, τ, A) is a p-soft T_i-space, then (Y, θ, B) is a tt-soft αT_i-space for $i = 0, 1, 2$.*

Proof. We only prove the proposition for $i = 2$.

Let $f_\varphi : (X, \tau, A) \to (Y, \theta, B)$ be a soft α-open map and $x \neq y \in Y$. Since f is bijective, then there are two distinct points a and b in X such that $a = f^{-1}(x)$ and $b = f^{-1}(y)$. Since (X, τ, A) is a p-soft T_2-space, then there are two disjoint soft open sets U_A and V_A such that $x \in U_A$ and $y \in V_A$. Now, $f_\varphi(U_A)$ and $f_\varphi(V_A)$ are two disjoint soft α-open subsets of (Y, θ, B) such that $x \in f_\varphi(U_A)$ and $y \in f_\varphi(V_A)$. Thus (Y, θ, B) is a *tt*-soft αT_2-space. □

In a similar way, one can prove the following result.

Proposition 3.35. *Let $f_\varphi : (X, \tau, A) \to (Y, \theta, B)$ be a bijective soft $\alpha^\star$-open map. Then if (X, τ, A) is a tt-soft αT_i-space, then (Y, θ, B) is a tt-soft αT_i-space for $i = 0, 1, 2$.*

Proposition 3.36. *The property of being tt-soft αT_i $(i = 0, 1, 2, 3, 4)$ is preserved under a soft $\alpha^\star$-homeomorphism map.*

We complete this section by discussing some interrelations between *tt*-soft αT_i-spaces $(i = 2, 3, 4)$ and soft α-compact spaces.

Proposition 3.37. *A stable soft α-compact subset of a tt-soft αT_2-space is soft α-closed.*

Proof. It follows from Proposition (2.21) and Remark (3.2). □

Theorem 3.38. *Let H_E be a soft α-compact subset of a tt-soft αT_2-space. If $x \notin H_E$, then there are disjoint soft α-open sets U_E and V_E such that $x \in U_E$ and $H_E \subseteq V_E$.*

Proof. Let $x \notin H_E$. Then $x \neq y$ for each $y \Subset H_E$. Since (X, τ, E) is a *tt*-soft αT_2-space, then there exist disjoint soft α-open sets U_{i_E} and V_{i_E} such that $x \in U_{i_E}$ and $y \in V_{i_E}$. Therefore $\{V_{i_E}\}$ forms a soft α-open cover of H_E. Since H_E is soft α-compact, then $H_E \subseteq \widetilde{\bigcup}_{i=1}^{i=n} V_{i_E}$. By letting $\widetilde{\bigcup}_{i=1}^{i=n} V_{i_E} = V_E$ and $\widetilde{\bigcap}_{i=1}^{i=n} U_{i_E} = U_E$, we obtain the desired result. □

Theorem 3.39. *Every soft α-compact and tt-soft αT_2-space is tt-soft α-regular.*

Proof. Let H_E be a soft α-closed subset of soft α-compact and *tt*-soft αT_2-space (X, τ, E) such that $x \notin H_E$. Then H_E is soft α-compact. By Theorem (3.38), there exist disjoint soft α-open sets U_E and V_E such that $x \in U_E$ and $H_E \subseteq V_E$. Thus, (X, τ, E) is *tt*-soft α-regular. □

Corollary 3.40. *Every soft α-compact and tt-soft αT_2-space is tt-soft αT_3.*

Lemma 3.41. *Let F_E be a soft α-open subset of a soft α-regular space. Then for each $P_e^x \in F_E$, there exists a soft α-open set G_E such that $P_e^x \in \overline{G_E}^{\alpha} \widetilde{\subseteq} F_E$.*

Proof. Let F_E be a soft α-open set such that $P_e^x \in F_E$. Then $x \notin F_E^c$. Since (X, τ, E) is soft α-regular, then there exist two disjoint soft α-open sets G_E and W_E containing x and F_E^c, respectively. Thus $x \in G_E \widetilde{\subseteq} W_E^c \widetilde{\subseteq} F_E$. Hence, $P_e^x \in G_E \widetilde{\subseteq} \overline{G_E}^{\alpha} \widetilde{\subseteq} W_E^c \widetilde{\subseteq} F_E$. □

Theorem 3.42. *Let H_E be a soft α-compact subset of a soft α-regular space and F_E be a soft α-open set containing H_E. Then there exists a soft α-open set G_E such that $H_E \widetilde{\subseteq} G_E \widetilde{\subseteq} \overline{G_E}^{\alpha} \widetilde{\subseteq} F_E$.*

Proof. Let the given conditions be satisfied. Then for each $P_e^x \in H_E$, we have $P_e^x \in F_E$. Therefore there is a soft α-open set W_{xe_E} such that $P_e^x \in W_{xe_E} \widetilde{\subseteq} \overline{W_{xe_E}}^{\alpha} \widetilde{\subseteq} F_E$. Now, $\{W_{xe_E} : P_e^x \in F_E\}$ is a soft α-open cover of H_E. Since H_E is soft α-compact, then $H_E \widetilde{\subseteq} \widetilde{\bigcup}_{i=1}^{i=n} W_{xe_E}$. Putting $G_E = \widetilde{\bigcup}_{i=1}^{i=n} W_{xe_E}$. Thus $H_E \widetilde{\subseteq} G_E \widetilde{\subseteq} \overline{G_E}^{\alpha} \widetilde{\subseteq} F_E$. □

Corollary 3.43. *If (X, τ, E) is soft α-compact and soft αT_3, then it is tt-soft αT_4.*

Proof. Suppose that F_{1_E} and F_{2_E} are two disjoint soft α-closed sets. Then $F_{2_E} \widetilde{\subseteq} F_{1_E}^c$. Since (X, τ, E) is soft α-compact, then F_{2_E} is soft α-compact and since (X, τ, E) is soft α-regular, then there is a soft α-open set G_E such that $F_{2_E} \widetilde{\subseteq} G_E \widetilde{\subseteq} \overline{G_E}^{\alpha} \widetilde{\subseteq} F_{1_E}^c$. Obviously, $F_{2_E} \widetilde{\subseteq} G_E, F_{1_E} \widetilde{\subseteq} (\overline{G_E}^{\alpha})^c$ and $G_E \widetilde{\bigcap} (\overline{G_E}^{\alpha})^c = \widetilde{\Phi}$. Thus (X, τ, E) is soft α-normal. Since (X, τ, E) is soft αT_3, then it is *tt*-soft αT_1. Hence, it is *tt*-soft αT_4. □

4. α-fixed soft points of soft mappings

In this section, we investigate main features of an α-fixed soft point, in particular, those are related to parametric topological spaces.

Theorem 4.1. *Let $\{\mathcal{B}_n : n \in \mathbb{N}\}$ be a collection of soft subsets of a soft α-compact space (X, τ, E) satisfying:*

(i) *$\mathcal{B}_n \neq \widetilde{\Phi}$ for each $n \in \mathbb{N}$;*

(ii) *$\mathcal{B}_n$ is a soft α-closed set for each $n \in \mathbb{N}$;*

(iii) *$\mathcal{B}_{n+1} \widetilde{\subseteq} \mathcal{B}_n$ for each $n \in \mathbb{N}$.*

Then $\widetilde{\bigcap}_{n \in \mathbb{N}} \mathcal{B}_n \neq \widetilde{\Phi}$.

Proof. Suppose that $\widetilde{\bigcap}_{n \in \mathbb{N}} \mathcal{B}_n = \widetilde{\Phi}$. Then $\widetilde{\bigcup}_{n \in \mathbb{N}} \mathcal{B}_n^c = \widetilde{X}$. It follows from (ii) that $\{\mathcal{B}_n^c : n \in \mathbb{N}\}$ is a soft α-open cover of $\widetilde{X}$. By hypothesis of soft α-compactness, there exist $i_1, i_2, ..., i_k \in \mathbb{N}$, $i_1 < i_2 < ... < i_k$ such that $\widetilde{X} = \mathcal{B}_{i_1}^c \widetilde{\bigcup} \mathcal{B}_{i_2}^c \widetilde{\bigcup} ... \widetilde{\bigcup} \mathcal{B}_{i_k}^c$. It follows from (iii) that $\mathcal{B}_{i_k} \widetilde{\subseteq} \widetilde{X} = \mathcal{B}_{i_1}^c \widetilde{\bigcup} \mathcal{B}_{i_2}^c \widetilde{\bigcup} ... \widetilde{\bigcup} \mathcal{B}_{i_k}^c = [\mathcal{B}_{i_1} \widetilde{\bigcap} \mathcal{B}_{i_2} \widetilde{\bigcap} ... \widetilde{\bigcap} \mathcal{B}_{i_k}]^c = \mathcal{B}_{i_k}^c$. This yields a contradiction. Thus we obtain the proof that $\widetilde{\bigcap}_{n \in \mathbb{N}} \mathcal{B}_n \neq \widetilde{\Phi}$. □

To illustrate the above theorem, we give the following example

Example 4.2. *As we mentioned that a soft topological space is a classical topological space if $E = \{e\}$ is a singleton. Then we show the above theorem in the crisp setting. Let $\tau = \mathbb{R} \bigcup \{G \subseteq \mathbb{R} : 1 \notin \mathbb{R}\}$ be a (soft) topology on $\mathbb{R}$ (it is called an excluding point topology). One can examined that $(\mathbb{R}, \tau, E)$ is a soft α-compact space. Let $\{\mathcal{M}_n : n \in \mathbb{N}\}$ be a collection of soft subsets of $(\mathbb{R}, \tau, E)$ defined as follows: $\mathcal{M}_n = \mathbb{N} \setminus \{2, ..., n+1\}$; that is $\mathcal{M}_1 = \mathbb{N} \setminus \{2\}$, $\mathcal{M}_2 = \mathbb{N} \setminus \{2, 3\}$, and so on. It is clear that $\mathcal{M}_n$ satisfied the three conditions (i)-(iii) given in the above theorem. Now, $1 \in \bigcap_{n \in \mathbb{N}} \mathcal{M}_n$, as required.*

Proposition 4.3. *Let (X, τ, E) be a soft α-compact and soft $\alpha T_2'$-space and $g_\varphi : (X, \tau, E) \to (X, \tau, E)$ be a soft $\alpha^\star$-continuous map. Then there exists a unique soft point $P_e^x \in \widetilde{X}$ of g_φ.*

Proof. Let $\{\mathcal{B}_1 = g_\varphi(\widetilde{X})$ and $\mathcal{B}_n = g_\varphi(\mathcal{B}_{n-1}) = g_\varphi^n(\widetilde{X})$ for each $n \in \mathbb{N}\}$ be a family of soft subsets of (X, τ, E). It is clear that $\mathcal{B}_{n+1} \widetilde{\subseteq} \mathcal{B}_n$ for each $n \in \mathbb{N}$. Since g_φ is soft $\alpha^\star$-continuous, then $\mathcal{B}_n$ is a soft α-compact set for each $n \in \mathbb{N}$ and since (X, τ, E) is soft $\alpha T_2'$, then $\mathcal{B}_n$ is also a soft α-closed set for each $n \in \mathbb{N}$. It follows from Theorem (4.1) that $(H, E) = \widetilde{\bigcap}_{n \in \mathbb{N}} \mathcal{B}_n$ is a non null soft set. Note that $g_\varphi(H, E) = g_\varphi(\widetilde{\bigcap}_{n \in \mathbb{N}} g_\varphi^n(\widetilde{X})) \widetilde{\subseteq} \widetilde{\bigcap}_{n \in \mathbb{N}} g_\varphi^{n+1}(\widetilde{X}) \widetilde{\subseteq} \widetilde{\bigcap}_{n \in \mathbb{N}} g_\varphi^n(\widetilde{X}) = (H, E)$. To show that $(H, E) \widetilde{\subseteq} g_\varphi(H, E)$, suppose that there is a $P_e^x \in (H, E)$ such that $P_e^x \notin g_\varphi(H, E)$. Let $C_n = g_\varphi^{-1}(P_e^x) \widetilde{\bigcap} \mathcal{B}_n$. Obviously, $C_n \neq \widetilde{\Phi}$ and $C_n \widetilde{\subseteq} C_{n-1}$ for each $n \in \mathbb{N}$. By Theorem (2.15), C_n is a soft α-closed set for each $n \in \mathbb{N}$; and by Theorem (4.1), there exists a soft point P_m^y such that $P_m^y \in g_\varphi^{-1}(P_e^x) \widetilde{\bigcap} \mathcal{B}_n$. Therefore $P_e^x = g_\varphi(P_m^y) \in g_\varphi(H, E)$. This is a contradiction. Thus, $g_\varphi(H, E) = (H, E)$. Hence, the proof is complete. □

Definition 4.4. **(i)** *(X, τ, E) is said to have an α-fixed soft point property if every soft $\alpha^\star$-continuous map $g_\varphi : (X, \tau, E) \to (X, \tau, E)$ has a fixed soft point.*

(ii) *A property is said to be an $\alpha^\star$-soft topological property if the property is preserved by soft $\alpha^\star$-homeomorphism maps.*

Proposition 4.5. *The property of being an α-fixed soft point is an $\alpha^\star$-soft topological property.*

Proof. Let (X, τ, E) and (Y, θ, E) be a soft $\alpha^\star$-homeomorphic. Then there is a bijective soft map $f_\varphi : (X, \tau, E) \to (Y, \theta, E)$ such that f_φ and f_φ^{-1} are soft $\alpha^\star$-continuous. Since (X, τ, E) has an α-fixed soft point property, then every soft $\alpha^\star$-continuous map $g_\varphi : (X, \tau, E) \to (X, \tau, E)$ has an α-fixed soft point. Now, let $h_\varphi : (Y, \theta, E) \to (Y, \theta, E)$ be a soft $\alpha^\star$-continuous. Obviously, $h_\varphi \circ f_\varphi : (X, \tau, E) \to (Y, \theta, E)$ is a soft $\alpha^\star$-continuous. Also, $f_\varphi^{-1} \circ h_\varphi \circ f_\varphi : (X, \tau, E) \to (X, \tau, E)$ is a soft $\alpha^\star$-continuous. Since (X, τ, E) has an α-fixed soft point property, then $f_\varphi^{-1}(h_\varphi(f_\varphi(P_e^x))) = P_e^x$ for some $P_e^x \in \widetilde{X}$. consequently, $f_\varphi(f_\varphi^{-1}(h_\varphi(f_\varphi(P_e^x)))) = f_\varphi(P_e^x)$. This implies that $h_\varphi(f_\varphi(P_e^x)) = f_\varphi(P_e^x)$. Thus $f_\varphi(P_e^x)$ is an α-fixed soft point of h_φ. Hence, (Y, θ, E) has an α-fixed soft point property, as required. □

Before we investigate a relationship between soft topological space and their parametric topological spaces in terms of possessing a fixed (soft) point, we need to prove the following result.

Theorem 4.6. *Let τ be an extended soft topology on X. Then a soft map $g_\varphi : (X, \tau, E) \to (Y, \theta, E)$ is soft $\alpha^\star$-continuous if and only if a map $g : (X, \tau_e) \to (Y, \theta_{\phi(e)})$ is $\alpha^\star$-continuous.*

Proof. Necessity: Let U be an α-open subset of $(Y, \theta_{\phi(e)})$. Then there exists a soft α-open subset G_E of (Y, θ, E) such that $G(\phi(e)) = U$. Since g_φ is a soft $\alpha^\star$-continuous map, then $g_\phi^{-1}(G_E)$ is a soft α-open set. From Definition (2.9), it follows that a soft subset $g_\phi^{-1}(G_E) = (g_\phi^{-1}(G))_E$ of (X, τ, E) is given by $g_\phi^{-1}(G)(e) = g^{-1}(G(\phi(e)))$ for each $e \in E$. By hypothesis, τ is an extended soft topology on X, we obtain from Theorem (2.26) that a subset $g^{-1}(G(\phi(e))) = g^{-1}(U)$ of (X, τ_e) is α-open. Hence, a map g is $\alpha^\star$-continuous.

Sufficiency: Let G_E be a soft α-open subset of (Y, θ, E). Then from Definition (2.9), it follows that a soft subset $g_\phi^{-1}(G_E) = (g_\phi^{-1}(G))_E$ of (X, τ, E) is given by $g_\phi^{-1}(G)(e) = g^{-1}(G(\phi(e)))$ for each $e \in E$. Since a map g is $\alpha^\star$-continuous, then a subset $g^{-1}(G(\phi(e)))$ of (X, τ_e) is α-open. By hypothesis, τ is an extended soft topology on X, we obtain from Theorem (2.26) that $g_\phi^{-1}(G_E)$ is a soft α-open subset of (X, τ, E). Hence, a soft map g_φ is soft $\alpha^\star$-continuous. □

Definition 4.7. (X, τ) *is said to have an α-fixed point property if every $\alpha^\star$-continuous map $g : (X, \tau) \to (X, \tau)$ has a fixed point.*

Proposition 4.8. (X, τ, E) *has the property of an α-fixed soft point iff (X, τ_e) has the property of an α-fixed point for each $e \in E$.*

Proof. Necessity: Let (X, τ, E) has the property of an α-fixed soft point. Then every soft $\alpha^\star$-continuous map $g_\varphi : (X, \tau, E) \to (X, \tau, E)$ has a fixed soft point. Say, P_e^x. It follows from the above theorem that $g_e : (X, \tau_e) \to (X, \theta_{\phi(e)})$ is $\alpha^\star$-continuous. Since P_e^x is a fixed soft point of g_φ, then it must be that $g_e(x) = x$. Thus, g_e has a fixed point. Hence, we obtain the desired result.

Sufficiency: Let (X, τ_e) has the property of an α-fixed point for each $e \in E$. Then every $\alpha^\star$-continuous map $g_e : (X, \tau_e) \to (X, \theta_{\phi(e)})$ has a fixed point. Say, x. It follows from the above theorem that $g_\varphi : (X, \tau, E) \to (X, \theta, E)$ is soft $\alpha^\star$-continuous. Since x is a fixed point of g_e, then it must be that $g_\varphi(P_e^x) = P_e^x$. Thus, g_φ has a fixed soft point. Hence, we obtain the desired result. □

5. Summary

This work presents new types of soft separation axioms with respect to three factors:

(i) ordinary points.

(ii) total belong and total non-belong relations.

(iii) soft α-open sets.

We show the interrelationships between these soft separation axioms and investigate some properties. The main contributions of this work are the following:

(i) formulate new soft separation axioms, namely *tt*-soft $\alpha T_i (i = 0, 1, 2, 3, 4)$ and *tt*-soft α-regular spaces.

(ii) illustrate the relationships between them as well as with soft $\alpha T_i (i = 0, 1, 2, 3, 4)$ and soft α-regular spaces.

(iii) study the "transmission" of these soft separation axioms between soft topological space and its parametric topological spaces.

(iv) give some conditions that guarantee the equivalence of *tt*-soft $\alpha T_i (i = 0, 1, 2)$ and the equivalence of *tt*-soft $\alpha T_i (i = 1, 2, 3)$.

(v) characterize some of these soft separation axioms such as *tt*-soft αT_1 and *tt*-soft α-regular spaces

(vi) explore the interrelations of some of these soft separation axioms and soft compact spaces.

(vii) discuss the behaviours of these soft separation axioms with some notions such as product soft spaces and sum of soft topological spaces.

(viii) define α-fixed soft point and establish fundamental properties.

6. Conclusions

Soft separation axioms are among the most widespread and important concepts in soft topology because they are utilized to classify the objects of study and to construct different families of soft topological spaces. In this work, we have introduced new soft separation axioms with respect to ordinary points by using total belong and total non-belong relations. This way of definition helps us to generalize existing comparable properties via general topology and to remove a strict condition of the shape of soft open and closed subsets of soft α-regular spaces. In general, we study their main properties and illustrate the interrelations between them and some soft topological notions such as soft compactness, product soft spaces and sum of soft topological spaces. We complete this work by defining α-fixed soft point theorem and investigating its basic properties.

We plan in the upcoming works to study the concepts and results presented herein by using some celebrated types of generalizations of soft open sets such as soft preopen, soft b-open and soft β-open sets. In addition, we will explore these concepts on some contents such as supra soft topology and fuzzy soft topology. In the end, we hope that the concepts initiated herein will find their applications in many fields soon.

Conflict of interest

The authors declare that they have no competing interests.

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