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Research article

M-polynomial and topological indices of some transformed networks

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Abstract: In the chemical industry, topological indices play an important role in defining the properties of chemical compounds. They are numerical parameters and structure invariant. It is a proven fact by scientists that topological properties are influential tools for interconnection networks. In this paper, we will use stellation, medial and bounded dual operations to build transformed networks from zigzag and triangular benzenoid structures. Using M-polynomial, we compute the first and second Zagreb indices, second modified Zagreb indices, symmetric division index, general Randic index, reciprocal general Randic index. We also calculate atomic bond connectivity index, geometric arithmetic index, harmonic index, first and second Gourava indices, first and second hyper Gourava indices.

Keywords: M-polynomial; topological indices; zigzag benzenoid; triangular benzenoid
Mathematics Subject Classification: 05C92

1. Introduction

In today's world mathematics is necessary in all fields. It is an essential instrument for comprehension around us. It covers all the facts of life. Mathematics is the branch of science that deals with the reasoning of figures, numbers and order. In our daily life, we use mathematics in our routine work in various forms. There are many branches of mathematics like algebra, geometry, arithmetic, trigonometry, analysis and many other theories. Graph theory is the study of mathematical objects known as graph, which consist of vertices connected by edges. It is the mathematical theory which deals with the properties and applications of graphs. When we apply graph theory on chemistry then it is called chemical graph theory. In mathematical models graph theory is used to get a deep understanding of the physical properties of these chemical compounds. Some physical properties such as boiling point, melting point, density are associated to geometrical structure of the compound. Now a days, several ways are used in mathematical chemistry to understand chemical structure which are existing behind the chemical concepts, to create and inquire novel mathematical representation. In complete history of chemistry certain scientist, usually contemplates connections between mathematics with chemistry and the possibility of using mathematics to analyze and predict new chemical concepts. Mufti defined sanskruti and harmonic indices of certain graph structure [25]. Babujee calculated topological indices and new graph structures in 2012 [5]. Farahani worked on a new version of zagreb index of circumcoronene series of benzenoid in 2013 [7]. Hayat defined some degree based topological indices of certain nanotubes [8]. Imran worked on topological indices of certain interconnection networks in 2014 [14]. In 2016, Siddiqui computed zagreb indices and zagreb polynomials of some nanostars dendrimers [27]. Saleem computed retractions and homomorphisms on some operations of graphs [28]. Iqbal calculated eccentricity based topological indices of some benzenoid structures [12]. Yang examined two-point resistances and random walks on stellated regular graphs [29]. Islam defined M-polynomial and entropy of paraline graph of Napthalene in 2019 [11]. Iqbal worked on topological indices of subdivided and line graph of subdivided friendship graph [13]. In 2020, Afzal examined M-polynomial and topological indices of zigzag edge coronoid fused by starphene [1]. Archdeacon defined the medial graph and voltage-current duality [2]. Munir computed M-polynomial and degree-based topological indices of polyhex nanotubes [22]. Iqbal calculated ve topological indices of tickysim spinnaker model [15]. Azhar examined a note on valency dependence invariants of L(G(K)) Graph [4]. Jamil defined the first general zagreb eccentricity index [18]. Iqbal defined ABC4 and GA5 index of subdivided and line graph of subdivided dutch windmill graph [16]. Maji worked on the first entire zagreb index of various corona products and their bounds [23]. Imran describe computation of topological indices of NEPS of graphs [17]. Hayat computed topological indices for networks derived by applying graph operations from honeycomb structures [9], based on this idea we have computed the topological indices for transformed structures. Next we have few definitions [3, 9]:

Definition 1:

Let K be a simple connected graph its M-polynomial is defined as;

$$M(K;x,y) = \sum_{S\leq a\leq b\leq T} m_{ab}(K)x^a y^b, \qquad \text{(i)}$$

where: S=$Min\{d_\beta | \beta \in V(K)\}$, T=$Max\{d_\beta | \beta \in V(K)\}$, and $m_{ab}(K)$ is the number of edges $\gamma\beta \in E(K)$ such that $\{d_\gamma,\ d_\beta\}=\{a,b\}$.

Definition 2:

When we place a new vertex in each face of a planar graph G, and attach that vertex with all the vertices of the respective face of G, we get the *stellation* of G and denote it as St(G).

Definition 3:

We introduce a node in each edge of the graph and join the nodes if their corresponding edges are adjacent and is denoted by Md(G).

Definition 4:

We introduce a node in each bounded faces of the graph and join the nodes by an edge if the faces share an edge in graph it is called *bounded dual*. It is represented as Bdu(G).

T. Réti introduced First and Second Zagreb indices are [26],

$$M_1(K) = \sum_{\gamma\beta\in E(K)} (d_\gamma + d_\beta), \tag{1.1}$$

$$M_2(K) = \sum_{\gamma\beta\in E(K)} (d_\gamma d_\beta). \tag{1.2}$$

Second modified Zagreb index is defined as [10],

$$^{m}M_2(k) = \sum_{\gamma\beta\in E(K)} \left(\frac{1}{d_\gamma d_\beta}\right). \tag{1.3}$$

Symmetric division and reciprocal general Randic index is [19],

$$SDD(K) = \sum_{\gamma\beta\in E(K)} \left\{\frac{min(d_\gamma, d_\beta)}{max(d_\gamma, d_\beta)} + \frac{max(d_\gamma, d_\beta)}{min(d_\gamma, d_\beta)}\right\}, \tag{1.4}$$

$$RR_\alpha(K) = \sum_{\gamma\beta\in E(K)} \frac{1}{(d_\gamma d_\beta)^\alpha}. \tag{1.5}$$

Whereas, the General Randic index defined as [24],

$$R_\alpha(K) = \sum_{\gamma\beta\in E(K)} (d_\gamma d_\beta)^\alpha, \tag{1.6}$$

where α is an arbitrary real number.
In 1998, the Atomic Bond Connectivity Index is defined as [6],

$$ABC(K) = \sum_{\gamma\beta\in E(K)} \sqrt{\frac{d_\gamma + d_\beta - 2}{d_\gamma d_\beta}}. \tag{1.7}$$

Geometric-Arithmetic index is [30],

$$GA(K) = \sum_{\gamma\beta\in E(K)} \frac{2\sqrt{d_\gamma d_\beta}}{d_\gamma + d_\beta}. \tag{1.8}$$

In 2015, the General Version of Harmonic Index is defined [31],

$$H_k(K) = \sum_{\gamma\beta\in E(K)} \left(\frac{2}{d_\gamma + d_\beta}\right)^k. \tag{1.9}$$

First and Second Gourava Indices were introduced by Kalli which is defined as, respectively [20],

$$GO_1(K) = \sum_{\gamma\beta\in E(K)} [(d_\gamma + d_\beta) + (d_\gamma d_\beta)], \tag{1.10}$$

$$GO_2(K) = \sum_{\gamma\beta\in E(K)} [(d_\gamma + d_\beta)(d_\gamma d_\beta)]. \quad (1.11)$$

Kalli introduced the First and Second Hyper-Gourava Indices which is defined as, respectively [21],

$$HGO_1(K) = \sum_{\gamma\beta\in E(K)} [(d_\gamma + d_\beta) + (d_\gamma d_\beta)]^2, \quad (1.12)$$

$$HGO_2(K) = \sum_{\gamma\beta\in E(K)} [(d_\gamma + d_\beta)(d_\gamma d_\beta)]^2. \quad (1.13)$$

Table 1. Derivation of some degree-based topological indices from M-polynomial [3].

Topological Index	Derivation from M (K; x,y)	
First Zagreb	$(D_x + D_y)(M(K; x, y))\vert_{x=1=y}$	(ii)
Second Zagreb	$(D_x D_y)(M(K; x, y))\vert_{x=1=y}$	(iii)
Second Modified Zagreb	$(S_x S_y)(M(K; x, y))\vert_{x=1=y}$	(iv)
general Randic	$(D_x^\alpha D_y^\alpha)(M(K; x, y))\vert_{x=1=y}$	(v)
Reciprocal general Randic	$(S_x^\alpha S_y^\alpha)(M(K; x, y))\vert_{x=1=y}$	(vi)
Symmetric Division Index	$(S_y D_x) + (S_x D_y)(M(K; x, y))\vert_{x=1=y}$	(vii)

Where: $D_x = x\frac{\partial f(x,y)}{\partial x}$, $D_y = y\frac{\partial f(x,y)}{\partial y}$, $S_x = \int_0^x \frac{f(t,y)}{t}dt$, $S_y = \int_0^y \frac{f(x,t)}{t}dt$.

2. Methodology

At first, we obtain transformed pattern of molecular structures zigzag and triangular benzenoid named as T_1 and T_2. Next we define M-polynomial and topological indices of these networks by applying the stellation, medial and bounded dual. Further we define edge partition depending upon degree based vertices of T_1 and T_2 and then calculate the indices.

Table 2. Planar and non-planar graph.

Graph	*Planar/ Non − Planar*
$St(G_1, G_2)$	*Planar*
$Md(G_1, G_2)$	*Planar*
$Bdu(G_1, G_2)$	*Non − Planar*
T_1	*Non − Planar*
T_2	*Non − Planar*

3. Results and discussion

3.1. M-polynomial and degree based topological indices of T_1 structure

Now we will construct the stellation, medial and bounded dual operations on simple and undirected zigzag benzenoid structure. We get a new transformed network (say) T_1, shown in Figure 1 for n = 2. After this we will compute M-polynomial and degree based topological indices of our newly obtained network. Where, blue is stellation, red is medial, green is bounded dual operations applied on T_1.

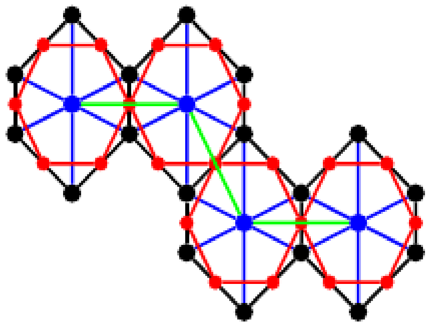

Figure 1. Transformed structure T_1.

Following Table 3 shows the types of edges and their count for network T_1; $n \geq 2$. Now, we will calculate the M-polynomial and vertex degree based topological indices namely first Zagreb index, second Zagreb index, second modified index, symmetric division index, general Randic index, reciprocal general Randic index, atomic bond connectivity index, geometric arithmetic index, general version of harmonic index, first and second gourava indices, first and second hyper gourava indices.

Table 3. Types and count of edges for T_1.

$Types\ of\ edges$	$Count\ of\ edges$
$(3,4)$	$8n+8$
$(3,7)$	8
$(3,8)$	$4n-4$
$(4,4)$	$4n+4$
$(4,5)$	$8n-4$
$(4,6)$	$8n-4$
$(5,6)$	$4n-2$
$(5,7)$	4
$(5,8)$	$8n-8$
$(7,8)$	2
$(8,8)$	$2n-3$

3.1.1. Theorem

Let T_1, be the transformed network then its M-polynomial is

$$\begin{aligned} M(T_1;x,y) &= (8n+8)x^3y^4 + 8x^3y^7 + (4n-4)x^3y^8 + (4n+4)x^4y^4 + (8n-4)x^4y^5 + (8n-4)x^4y^6 \\ &\quad +(4n-2)x^5y^6 + 4x^5y^7 + (8n-8)x^5y^8 + 2x^7y^8 + (2n-3)x^8y^8. \end{aligned}$$

Proof: Using definition of M-polynomial and information from Table 3, we have

$$\begin{aligned} M(T_1;x,y) &= \sum_{a\leq b} m_{ab}x^ay^b = \sum_{3\leq 4} m_{34}x^3y^4 + \sum_{3\leq 7} m_{37}x^3y^7 + \sum_{3\leq 8} m_{38}x^3y^8 + \sum_{4\leq 4} m_{44}x^4y^4 + \sum_{4\leq 5} m_{45}x^4y^5 \\ &\quad + \sum_{4\leq 6} m_{46}x^4y^6 + \sum_{5\leq 6} m_{56}x^5y^6 + \sum_{5\leq 7} m_{57}x^5y^7 + \sum_{5\leq 8} m_{58}x^5y^8 + \sum_{7\leq 8} m_{78}x^7y^8 + \sum_{8\leq 8} m_{88}x^8y^8 \\ &= |E_{3,4}|x^3y^4 + |E_{3,7}|x^3y^7 + |E_{3,8}|x^3y^8 + |E_{4,4}|x^4y^4 + |E_{4,5}|x^4y^5 + |E_{4,6}|x^4y^6 + |E_{5,6}|x^5y^6 \\ &\quad +|E_{5,7}|x^5y^7 + |E_{5,8}|x^5y^8 + |E_{7,8}|x^7y^8 + |E_{8,8}|x^8y^8 \\ &= (8n+8)x^3y^4 + 8x^3y^7 + (4n-4)x^3y^8 + (4n+4)x^4y^4 + (8n-4)x^4y^5 + (8n-4)x^4y^6 \\ &\quad +(4n-2)x^5y^6 + 4x^5y^7 + (8n-8)x^5y^8 + 2x^7y^8 + (2n-3)x^8y^8. \end{aligned}$$

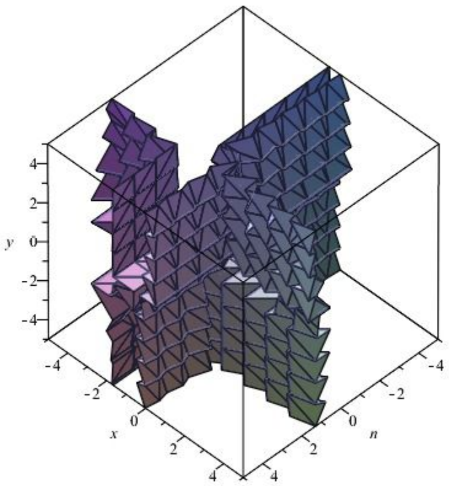

Figure 2. M-polynomial of T_1.

3.1.2. Theorem

Let T_1, be the transformed network and

$$\begin{aligned} M(T_1;x,y) &= (8n+8)x^3y^4 + 8x^3y^7 + (4n-4)x^3y^8 + (4n+4)x^4y^4 + (8n-4)x^4y^5 + (8n-4)x^4y^6 \\ &\quad +(4n-2)x^5y^6 + 4x^5y^7 + (8n-8)x^5y^8 + 2x^7y^8 + (2n-3)x^8y^8, \end{aligned}$$

be its M-polynomial. Then, the first Zagreb index $M_1(T_1)$, the second Zagreb index $M_2(T_1)$, the second modified Zagreb index ${}^mM_2(T_1)$, the general Randic index $R_\alpha(T_1)$, where $\alpha \in N$, reciprocal general

Randic index $RR_\alpha(T_1)$, where $\alpha \in N$, and the symmetric division degree index $SDD(T_1)$ obtained from M-polynomial are as follows:

$$\begin{aligned} M_1(T_1) &= 464n-48, \\ M_2(T_1) &= 1176n-264, \\ {}^mM_2(T_1) &= \frac{11}{12}(n+1)+\frac{11}{30}(n-1)+\frac{13}{30}(2n-1)+\frac{1}{64}(2n-3)+\frac{223}{420}, \\ R_\alpha(T_1) &= 12^\alpha(8n+8)+21^\alpha(8)+24^\alpha(4n-4)+16^\alpha(4n+4)+20^\alpha(8n-4)+24^\alpha(8n-4) \\ &\quad +30^\alpha(4n-2)+35^\alpha(4)+40^\alpha(8n-8)+56^\alpha(2)+64^\alpha(2n-3), \\ RR_\alpha(T_1) &= \frac{8}{12^\alpha}(n+1)+\frac{8}{21^\alpha}+\frac{4}{24^\alpha}(n-1)+\frac{4}{4^{2\alpha}}(n+1)+\frac{4}{20^\alpha}(2n-1)+\frac{4}{24^\alpha}(2n-1) \\ &\quad +\frac{2}{30^\alpha}(2n-1)+\frac{4}{35^\alpha}+\frac{8}{40^\alpha}(n-1)+\frac{2}{56^\alpha}+\frac{1}{64^\alpha}(2n-3), \\ SDD(T_1) &= \frac{74}{3}(n+1)+\frac{899}{30}(n-1)+\frac{314}{15}(2n-1)+2(2n-3)+\frac{14527}{420}. \end{aligned}$$

Proof: Let $f(x,y)$=$M(T_1;x,y)$ be the M-polynomial of the transformed network T_1. Then

$$\begin{aligned} M(T_1;x,y) &= (8n+8)x^3y^4+8x^3y^7+(4n-4)x^3y^8+(4n+4)x^4y^4+(8n-4)x^4y^5+(8n-4)x^4y^6 \\ &\quad +(4n-2)x^5y^6+4x^5y^7+(8n-8)x^5y^8+2x^7y^8+(2n-3)x^8y^8. \end{aligned}$$

Now, the required partial derivatives and integrals are obtained as:
By using the information given in Table 3 and formulas for Table 1;

$$\begin{aligned} D_xf(x,y) &= 3(8n+8)x^3y^4+24x^3y^7+3(4n-4)x^3y^8+4(4n+4)x^4y^4+4(8n-4)x^4y^5 \\ &\quad +4(8n-4)x^4y^6+5(4n-2)x^5y^6+20x^5y^7+5(8n-8)x^5y^8+14x^7y^8+8(2n-3)x^8y^8, \\ D_yf(x,y) &= 4(8n+8)x^3y^4+56x^3y^7+8(4n-4)x^3y^8+4(4n+4)x^4y^4+5(8n-4)x^4y^5 \\ &\quad +6(8n-4)x^4y^6+6(4n-2)x^5y^6+28x^5y^7+8(8n-8)x^5y^8+16x^7y^8+8(2n-3)x^8y^8, \\ D_xD_yf(x,y) &= 12(8n+8)x^3y^4+168x^3y^7+24(4n-4)x^3y^8+16(4n+4)x^4y^4+20(8n-4)x^4y^5 \\ &\quad +24(8n-4)x^4y^6+30(4n-2)x^5y^6+140x^5y^7+40(8n-8)x^5y^8+112x^7y^8 \\ &\quad +64(2n-3)x^8y^8, \\ S_xS_yf(x,y) &= \frac{2}{3}(n+1)x^3y^4+\frac{8}{21}x^3y^7+\frac{1}{6}(n-1)x^3y^8+\frac{1}{4}(n+1)x^4y^4+\frac{1}{5}(2n-1)x^4y^5 \\ &\quad +\frac{1}{6}(2n-1)x^4y^6+\frac{1}{15}(2n-1)x^5y^6+\frac{4}{35}x^5y^7+\frac{1}{5}(n-1)x^5y^8 \\ &\quad +\frac{1}{28}x^7y^8+\frac{1}{64}(2n-3)x^8y^8, \\ D_x^\alpha D_y^\alpha f(x,y) &= 12^\alpha(8n+8)x^3y^4+21^\alpha(8)x^3y^7+24^\alpha(4n-4)x^3y^8+4^{2\alpha}(4n+4)x^4y^4+20^\alpha(8n-4)x^4y^5 \\ &\quad +24^\alpha(8n-4)x^4y^6+30^\alpha(4n-2)x^5y^6+35^\alpha(4)x^5y^7+40^\alpha(8n-8)x^5y^8+56^\alpha(2)x^7y^8 \\ &\quad +46^\alpha(2n-3)x^8y^8, \\ S_x^\alpha S_y^\alpha f(x,y) &= \frac{8}{12^\alpha}(n+1)x^3y^4+\frac{8}{21^\alpha}x^3y^7+\frac{4}{24^\alpha}(n-1)x^3y^8+\frac{4}{4^{2\alpha}}(n+1)x^4y^4 \\ &\quad +\frac{4}{20^\alpha}(2n-1)x^4y^5+\frac{4}{24^\alpha}(2n-1)x^4y^6+\frac{2}{30^\alpha}(2n-1)x^5y^6+\frac{4}{35^\alpha}x^5y^7 \end{aligned}$$

$$
\begin{aligned}
&+\frac{8}{40^{\alpha}}(n-1)x^5y^8+\frac{2}{56^{\alpha}}x^7y^8+\frac{1}{64^{\alpha}}(2n-3)x^8y^8,\\
S_yD_xf(x,y) &= 6(n+1)x^3y^4+\frac{24}{7}x^3y^7+\frac{3}{2}(n-1)x^3y^8+4(n+1)x^4y^4+\frac{16}{5}(2n-1)x^4y^5\\
&+\frac{8}{3}(2n-1)x^4y^6+\frac{5}{3}(2n-1)x^5y^6+\frac{20}{7}x^5y^7+5(n-1)x^5y^8\\
&+\frac{7}{4}x^7y^8+(2n-3)x^8y^8,\\
S_xD_yf(x,y) &= \frac{32}{3}(n+1)x^3y^4+\frac{56}{3}x^3y^7+\frac{32}{3}(n-1)x^3y^8+4(n+1)x^4y^4+5(2n-1)x^4y^5\\
&+6(2n-1)x^4y^6+\frac{12}{5}(2n-1)x^5y^6+\frac{28}{5}x^5y^7+\frac{64}{5}(n-1)x^5y^8\\
&+\frac{16}{7}x^7y^8+(2n-3)x^8y^8.
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
M_1(T_1) &= (D_x+D_y)f(x,y)|_{x=1=y}=464n-48,\\
M_2(T_1) &= D_xD_yf(x,y)|_{x=1=y}=1176n-264,\\
{}^mM_2(T_1) &= S_xS_yf(x,y)|_{x=1=y}=\frac{11}{12}(n+1)+\frac{11}{30}(n-1)+\frac{13}{30}(2n-1)+\frac{1}{64}(2n-3)+\frac{223}{420},\\
R_\alpha(T_1) &= D_x^\alpha D_y^\alpha f(x,y)|_{x=1=y}=12^\alpha(8n+8)+(21)^\alpha 8+24^\alpha(4n-4)+16^\alpha(4n+4)+20^\alpha(8n-4)\\
&+24^\alpha(8n-4)+30^\alpha(4n-2)+(35)^\alpha 4+40^\alpha(8n-8)+(56)^\alpha 2+64^\alpha(2n-3),\\
RR_\alpha(T_1) &= S_x^\alpha S_y^\alpha f(x,y)|_{x=1=y}=\frac{8}{12^\alpha}(n+1)+\frac{8}{21^\alpha}+\frac{4}{24^\alpha}(n-1)+\frac{4}{4^{2\alpha}}(n+1)+\frac{4}{20^\alpha}(2n-1)\\
&+\frac{4}{24^\alpha}(2n-1)+\frac{2}{30^\alpha}(2n-1)+\frac{4}{35^\alpha}+\frac{8}{40^\alpha}(n-1)+\frac{2}{56^\alpha}+\frac{1}{64^\alpha}(2n-3),\\
SDD(T_1) &= (S_yD_x+S_xD_y)(x,y)|_{x=1=y}=\frac{74}{3}(n+1)+\frac{899}{30}(n-1)+\frac{314}{15}(2n-1)+2(2n-3)+\frac{14527}{420}.
\end{aligned}
$$

3.1.3. Theorem

For T_1, the Atomic Bond Connectivity, Geometric Arithematic Index and General Harmonic Index are as follows, respectively.

$$
\begin{aligned}
1)\ ABC(T_1) &= 2n\sqrt{6}+(n+1)\frac{4\sqrt{15}}{3}+(14n+11)\frac{\sqrt{14}}{56}+(n-1)\frac{2\sqrt{110}}{5}\\
&+(2n-1)\Big(2\sqrt{\frac{7}{5}}+\frac{4}{\sqrt{3}}+\sqrt{\frac{6}{5}}\Big)+\frac{16\sqrt{42}}{21}+\frac{\sqrt{182}}{14},\\
2)\ GA(T_1) &= (n+1)\Big(\frac{32\sqrt{3}}{7}+4\Big)+(n-1)\frac{32\sqrt{10}}{13}+(32n-21)\frac{8\sqrt{6}}{55}+(2n-1)\Big(\frac{16\sqrt{5}}{9}+\frac{4\sqrt{30}}{11}\Big)\\
&+(2n-3)+\frac{8\sqrt{21}}{5}+\frac{2\sqrt{35}}{3}+\frac{8\sqrt{14}}{15},\\
3)\ H_k(T_1) &= (8n+8)(\frac{2}{7})^k+(8n+4)(\frac{1}{5})^k+(4n-4)(\frac{2}{11})^k+(4n+4)(\frac{1}{4})^k+(8n-4)(\frac{2}{9})^k
\end{aligned}
$$

$$+(4n-2)(\frac{2}{11})^k + 4(\frac{1}{6})^k + (8n-8)(\frac{2}{13})^k + 2(\frac{2}{15})^k + (2n-3)(\frac{1}{8})^k.$$

Proof:

1). According to Eq (1.7)

$$ABC(T_1) \;=\; \sum_{\gamma\beta\in E(K)} \sqrt{\frac{d_\gamma + d_\beta - 2}{d_\gamma d_\beta}}.$$

By using the information given in Table 3.

$$\begin{aligned}
&= (8n+8)\Big(\sqrt{\frac{3+4-2}{3\times 4}}\Big) + 8\Big(\sqrt{\frac{3+7-2}{3\times 7}}\Big) + (4n-4)\Big(\sqrt{\frac{3+8-2}{3\times 8}}\Big)\\
&\quad +(4n+4)\Big(\sqrt{\frac{4+4-2}{4\times 4}}\Big) + (8n-4)\Big(\sqrt{\frac{4+5-2}{4\times 5}}\Big) + (8n-4)\Big(\sqrt{\frac{4+6-2}{4\times 6}}\Big)\\
&\quad +(4n-2)\Big(\sqrt{\frac{5+6-2}{5\times 6}}\Big) + 4\Big(\sqrt{\frac{5+7-2}{5\times 7}}\Big) + (8n-8)\Big(\sqrt{\frac{5+8-2}{5\times 8}}\Big)\\
&\quad +2\Big(\sqrt{\frac{7+8-2}{7\times 8}}\Big) + (2n-3)\Big(\sqrt{\frac{8+8-2}{8\times 8}}\Big)\\
&= 2n\sqrt{6} + (n+1)\frac{4\sqrt{15}}{3} + (14n+11)\frac{\sqrt{14}}{56} + (n-1)\frac{2\sqrt{110}}{5}\\
&\quad +(2n-1)\Big(2\sqrt{\frac{7}{5}} + \frac{4}{\sqrt{3}} + \sqrt{\frac{6}{5}}\Big) + \frac{16\sqrt{42}}{21} + \frac{\sqrt{182}}{14}.
\end{aligned}$$

2). According to Eq (1.8)

$$GA(T_1) \;=\; \sum_{\gamma\beta\in E(K)} \frac{2\sqrt{d_\gamma d_\beta}}{d_\gamma + d_\beta}.$$

By using the information given in Table 3.

$$\begin{aligned}
&= (8n+8)\Big(\frac{2\sqrt{3\times 4}}{3+4}\Big) + 8\Big(\frac{2\sqrt{3\times 7}}{3+7}\Big) + (4n-4)\Big(\frac{2\sqrt{3\times 8}}{3+8}\Big) + (4n+4)\Big(\frac{2\sqrt{4\times 4}}{4+4}\Big)\\
&\quad +(8n-4)\Big(\frac{2\sqrt{4\times 5}}{4+5}\Big) + (8n-4)\Big(\frac{2\sqrt{4\times 6}}{4+6}\Big) + (4n-2)\Big(\frac{2\sqrt{5\times 6}}{5+6}\Big) + 4\Big(\frac{2\sqrt{5\times 7}}{5+7}\Big)\\
&\quad +(8n-8)\Big(\frac{2\sqrt{5\times 8}}{5+8}\Big) + 2\Big(\frac{2\sqrt{7\times 8}}{7+8}\Big) + (2n-3)\Big(\frac{2\sqrt{8\times 8}}{8+8}\Big)\\
&= (n+1)\Big(\frac{32\sqrt{3}}{7} + 4\Big) + (n-1)\frac{32\sqrt{10}}{13} + (32n-21)\frac{8\sqrt{6}}{55} + (2n-1)\Big(\frac{16\sqrt{5}}{9} + \frac{4\sqrt{30}}{11}\Big)\\
&\quad +(2n-3) + \frac{8\sqrt{21}}{5} + \frac{2\sqrt{35}}{3} + \frac{8\sqrt{14}}{15}.
\end{aligned}$$

3). According to Eq (1.9)

$$H_k(T_1) = \sum_{\gamma\beta\in E(K)} \Big(\frac{2}{d_\gamma + d_\beta}\Big)^k.$$

By using the information given in Table 3.

$$\begin{aligned}
&= (8n+8)\Big(\frac{2}{3+4}\Big)^k + 8\Big(\frac{2}{3+7}\Big)^k + (4n-4)\Big(\frac{2}{3+8}\Big)^k + (4n+4)\Big(\frac{2}{4+4}\Big)^k + (8n-4)\Big(\frac{2}{4+5}\Big)^k \\
&\quad +(8n-4)\Big(\frac{2}{4+6}\Big)^k + (4n-2)\Big(\frac{2}{5+6}\Big)^k + 4\Big(\frac{2}{5+7}\Big)^k + (8n-8)\Big(\frac{2}{5+8}\Big)^k + 2\Big(\frac{2}{7+8}\Big)^k + (2n-3)\Big(\frac{2}{8+8}\Big)^k \\
&= (8n+8)(\frac{2}{7})^k + (8n+4)(\frac{1}{5})^k + (4n-4)(\frac{2}{11})^k + (4n+4)(\frac{1}{4})^k + (8n-4)(\frac{2}{9})^k \\
&\quad +(4n-2)(\frac{2}{11})^k + 4(\frac{1}{6})^k + (8n-8)(\frac{2}{13})^k + 2(\frac{2}{15})^k + (2n-3)(\frac{1}{8})^k.
\end{aligned}$$

3.1.4. Theorem

For T_1, the First, Second Gourava Indices and the First, Second Hyper Gourava Indices are as follows, respectively.

$$\begin{aligned}
1)\ GO_1(T_1) &= 1640n - 312, \\
2)\ GO_2(T_1) &= 13128n - 4404, \\
3)\ HGO_1(T_1) &= 68064n - 26124, \\
4)\ HGO_2(T_1) &= 5816720n - 3573928.
\end{aligned}$$

Proof:

1). According to Eq (1.10)

$$GO_1(T_1) = \sum_{\gamma\beta\in E(K)} [(d_\gamma + d_\beta) + (d_\gamma d_\beta)].$$

By using the information given in Table 3.

$$\begin{aligned}
&= (8n+8)[(3+4)+(3\times4)] + 8[(3+7)+(3\times7)] + (4n-4)[(3+8)+(3\times8)] + (4n+4)[(4+4) \\
&\quad +(4\times4)] + (8n-4)[(4+5)+(4\times5)] + (8n-4)[(4+6)+(4\times6)] + (4n-2)[(5+6)+(5\times6)] \\
&\quad +4[(5+7)+(5\times7)] + (8n-8)[(5+8)+(5\times8)] + 2[(7+8)+(7\times8)] + (2n-3)[(8+8)+(8\times8)] \\
&= 1640n - 312.
\end{aligned}$$

2). According to Eq (1.11)

$$GO_2(T_1) = \sum_{\gamma\beta\in E(K)} [(d_\gamma + d_\beta)(d_\gamma d_\beta)].$$

By using the information given in Table 3.

$$= (8n+8)[(3+4)(3\times4)] + 8[(3+7)(3\times7)] + (4n-4)[(3+8)(3\times8)] + (4n+4)[(4+4)(4\times4)]$$

$$+(8n-4)[(4+5)(4\times5)]+(8n-4)[(4+6)(4\times6)]+(4n-2)[(5+6)(5\times6)]+4[(5+7)(5\times7)]$$
$$+(8n-8)[(5+8)(5\times8)]+2[(7+8)(7\times8)]+(2n-3)[(8+8)(8\times8)]$$
$$=\ 13128n-4404.$$

3). According to Eq (1.12)

$$HGO_1(T_1) = \sum_{\gamma\beta\in E(K)}[(d_\gamma+d_\beta)+(d_\gamma d_\beta)]^2.$$

By using the information given in Table 3.

$$\begin{aligned}
&= (8n+8)[(3+4)+(3\times4)]^2+8[(3+7)+(3\times7)]^2+(4n-4)[(3+8)+(3\times8)]^2\\
&\quad +(4n+4)[(4+4)+(4\times4)]^2+(8n-4)[(4+5)+(4\times5)]^2+(8n-4)[(4+6)+(4\times6)]^2\\
&\quad +(4n-2)[(5+6)+(5\times6)]^2+4[(5+7)+(5\times7)]^2+(8n-8)[(5+8)+(5\times8)]^2\\
&\quad +2[(7+8)+(7\times8)]^2+(2n-3)[(8+8)+(8\times8)]^2\\
&= 68064n-26124.
\end{aligned}$$

4). According to Eq (1.13)

$$HGO_2(T_1) = \sum_{\gamma\beta\in E(K)}[(d_\gamma+d_\beta)(d_\gamma d_\beta)]^2.$$

By using the information given in Table 3.

$$\begin{aligned}
&= (8n+8)[(3+4)(3\times4)]^2+8[(3+7)(3\times7)]^2+(4n-4)[(3+8)(3\times8)]^2+(4n+4)[(4+4)(4\times4)]^2\\
&\quad +(8n-4)[(4+5)(4\times5)]^2+(8n-4)[(4+6)(4\times6)]^2+(4n-2)[(5+6)(5\times6)]^2\\
&\quad +4[(5+7)(5\times7)]^2+(8n-8)[(5+8)(5\times8)]^2+2[(7+8)(7\times8)]^2+(2n-3)[(8+8)(8\times8)]^2\\
&= 5816720n-3573928.
\end{aligned}$$

3.2. M-polynomial and degree based topological indices of T_2 structure

Now we will construct the stellation, medial and bounded dual operations on simple and undirected triangular benzenoid structure. We get a new transformed network (say) T_2, shown in Figure 3 for n=5. After this we will compute M-polynomial and degree based topological indices of our newly obtained network. Where, blue is stellation, red is medial, green is bounded dual operations applied on T_2. Following Table 4 shows the types of edges and their count for network T_2; $n \geq 5$. Now, we will calculate the different M-polynomial and vertex degree based topological indices namely first Zagreb index, second Zagreb index, second modified index, symmetric division index, general Randic index, reciprocal general Randic index, atomic bond connectivity index, geometric arithmetic index, general version of harmonic index, first and second gourava indices, first and second hyper gourava indices.

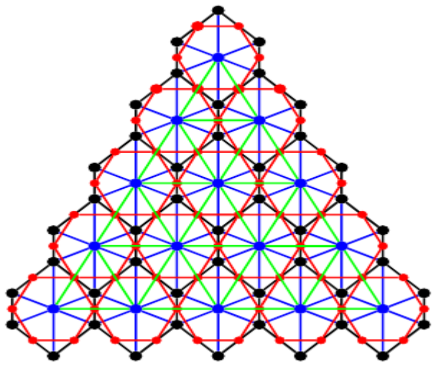

Figure 3. Transformed structure T_2.

3.2.1. Theorem

Let T_2, be the transformed network then its M-polynomial is

$$\begin{aligned} M(T_2;x,y) &= (6n+6)x^3y^4 + 9x^3y^8 + (3n-6)x^3y^{10} + (3n+3)x^4y^4 + (6n-6)x^4y^5 + (6n-6)x^4y^6 \\ &+(3n-3)x^5y^6 + 6x^5y^8 + (6n-12)x^5y^{10} + 6(n-1)^2x^6y^6 + 3x^6y^8 + (9n-18)x^6y^{10} \\ &+6[(n-(n-1)) + (n-(n-2)) + ... + (n-4) + (n-3)]x^6y^{12} + 6x^8y^{10} + (3n-6)x^{10}y^{10} \\ &+(6n-18)x^{10}y^{12} + 3[(n-4) + (n-5) + ... + (n-(n-1))]x^{12}y^{12}. \end{aligned}$$

Proof: Using definition of M-polynomial and information from Table 4, we have

$$\begin{aligned} M(T_2;x,y) &= \sum_{a\le b} m_{ab}x^ay^b = \sum_{3\le 4} m_{34}x^3y^4 + \sum_{3\le 8} m_{38}x^3y^8 + \sum_{3\le 10} m_{3\ 10}x^3y^{10} + \sum_{4\le 4} m_{44}x^4y^4 \\ &+ \sum_{4\le 5} m_{45}x^4y^5 + \sum_{4\le 6} m_{46}x^4y^6 + \sum_{5\le 6} m_{56}x^5y^6 + \sum_{5\le 8} m_{58}x^5y^8 + \sum_{5\le 10} m_{5\ 10}x^5y^{10} \\ &+ \sum_{6\le 6} m_{66}x^6y^6 + \sum_{6\le 8} m_{68}x^6y^8 + \sum_{6\le 10} m_{6\ 10}x^6y^{10} + \sum_{6\le 12} m_{6\ 12}x^6y^{12} + \sum_{8\le 10} m_{8\ 10}x^8y^{10} \\ &+ \sum_{10\le 10} m_{10\ 10}x^{10}y^{10} + \sum_{10\le 12} m_{10\ 12}x^{10}y^{12} + \sum_{12\le 12} m_{12\ 12}x^{12}y^{12} \\ &= |E_{3,4}|x^3y^4 + |E_{3,8}|x^3y^8 + |E_{3,10}|x^3y^{10} + |E_{4,4}|x^4y^4 + |E_{4,5}|x^4y^5 + |E_{4,6}|x^4y^6 + |E_{5,6}|x^5y^6 \\ &+|E_{5,8}|x^5y^8 + |E_{5,10}|x^5y^{10} + |E_{6,6}|x^6y^6 + |E_{6,8}|x^6y^8 + |E_{6,10}|x^6y^{10} + |E_{6,12}|x^6y^{12} \\ &+|E_{8,10}|x^8y^{10} + |E_{10,10}|x^{10}y^{10} + |E_{10,12}|x^{10}y^{12} + |E_{12,12}|x^{12}y^{12} \\ &= (6n+6)x^3y^4 + 9x^3y^8 + (3n-6)x^3y^{10} + (3n+3)x^4y^4 + (6n-6)x^4y^5 + (6n-6)x^4y^6 \\ &+(3n-3)x^5y^6 + 6x^5y^8 + (6n-12)x^5y^{10} + 6(n-1)^2x^6y^6 + 3x^6y^8 + (9n-18)x^6y^{10} \\ &+6[(n-(n-1)) + (n-(n-2)) + ... + (n-4) + (n-3)]x^6y^{12} + 6x^8y^{10} + (3n-6)x^{10}y^{10} \\ &+(6n-18)x^{10}y^{12} + 3[(n-4) + (n-5) + ... + (n-(n-1))]x^{12}y^{12}. \end{aligned}$$

Table 4. Types and count of edges for T_2.

$Types\ of\ edges$	$Count\ of\ edges$
$(3,4)$	$6n+6$
$(3,8)$	9
$(3,10)$	$3n-6$
$(4,4)$	$3n+3$
$(4,5)$	$6n-6$
$(4,6)$	$6n-6$
$(5,6)$	$3n-3$
$(5,8)$	6
$(5,10)$	$6n-12$
$(6,6)$	$6(n-1)^2$
$(6,8)$	3
$(6,10)$	$9n-18$
$(6,12)$	$6[(n-(n-1))+(n-(n-2))+...$ $+(n-4)+(n-3)]$
$(8,10)$	6
$(10,10)$	$3n-6$
$(10,12)$	$6n-18$
$(12,12)$	$3[(n-4)+(n-5)+...$ $+(n-(n-1))]$

3.2.2. Theorem

Let T_2, be the transformed network and

$$\begin{aligned}M(T_2;x,y) &= (6n+6)x^3y^4+9x^3y^8+(3n-6)x^3y^{10}+(3n+3)x^4y^4+(6n-6)x^4y^5+(6n-6)x^4y^6\\&+(3n-3)x^5y^6+6x^5y^8+(6n-12)x^5y^{10}+6(n-1)^2x^6y^6+3x^6y^8+(9n-18)x^6y^{10}\\&+6[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]x^6y^{12}+6x^8y^{10}+(3n-6)x^{10}y^{10}\\&+(6n-18)x^{10}y^{12}+3[(n-4)+(n-5)+...+(n-(n-1))]x^{12}y^{12},\end{aligned}$$

be its M-polynomial. Then, the first Zagreb index $M_1(T_2)$, the second Zagreb index $M_2(T_2)$, the second modified Zagreb ${}^mM_2(T_2)$, the general Randic index $R_\alpha(T_2)$, where $\alpha \epsilon N$, reciprocal general Randic index $RR_\alpha(T_2)$, where $\alpha \epsilon N$, and the symmetric division degree index $SDD(T_2)$ obtained from M-polynomial are as follows:

$$\begin{aligned}M_1(T_2) &= 678n-816+72(n-1)^2+108[(n-(n-1))+(n-(n-2))\\&+...+(n-4)+(n-3)]+72[(n-4)+(n-5)+...+(n-(n-1))],\\M_2(T_2) &= 2424n-3774+216(n-1)^2+432[(n-(n-1))+(n-(n-2))\\&+...+(n-4)+(n-3)]+432[(n-4)+(n-5)+...+(n-(n-1))],\\{}^mM_2(T_2) &= \frac{11}{16}(n+1)+\frac{2}{5}(n-2)+\frac{13}{20}(n-1)+\frac{1}{20}(n-3)+\frac{53}{80}+\frac{1}{6}(n-1)^2\end{aligned}$$

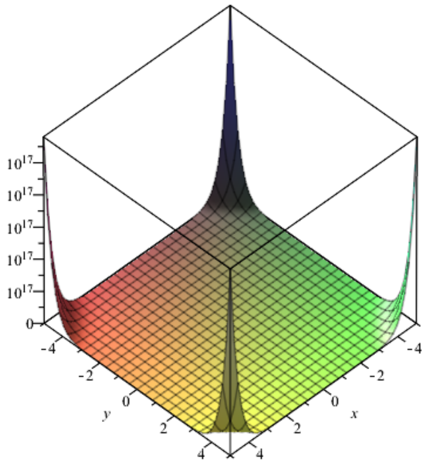

Figure 4. M-polynomial of T_2 for n = 6.

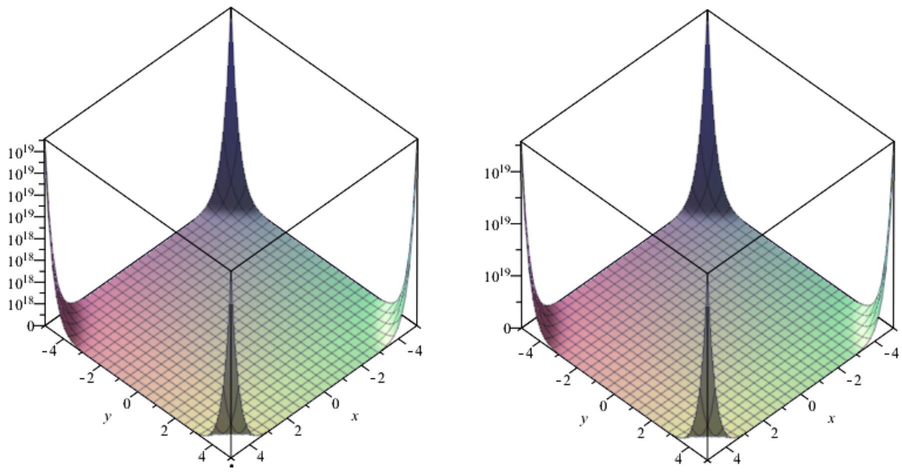

Figure 5. M-polynomial of T_2.

$$
\begin{aligned}
& +\frac{1}{12}[(n-(n-1))+(n-(n-2))+\ldots+(n-4)+(n-3)] \\
& +\frac{1}{48}[(n-4)+(n-5)+\ldots+(n-(n-1))], \\
R_\alpha(T_2) = \; & 12^\alpha(6n+6)+24^\alpha(9)+30^\alpha(3n-6)+4^{2\alpha}(3n+3)+20^\alpha(6n-6)+24^\alpha(6n-6) \\
& +30^\alpha(3n-3)+40^\alpha(6)+50^\alpha(6n-12)+6(n-1)^2(36)^\alpha+48^\alpha(3)+60^\alpha(9n-18) \\
& +72^\alpha[(n-(n-1))+(n-(n-2))+\ldots+(n-4)+(n-3)]6+80^\alpha(6)+100^\alpha(3n-6) \\
& +120^\alpha(6n-18)+144^\alpha[(n-4)+(n-5)+\ldots+(n-(n-1))]3, \\
RR_\alpha(T_2) = \; & \frac{6}{12^\alpha}(n+1)+\frac{9}{24^\alpha}+\frac{3}{30^\alpha}(n-2)+\frac{3}{4^{2\alpha}}(n+1)+\frac{6}{20^\alpha}(n-1)+\frac{6}{24^\alpha}(n-1) \\
& +\frac{3}{30^\alpha}(n-1)+\frac{6}{40^\alpha}+\frac{6}{50^\alpha}(n-2)+\frac{6}{36^\alpha}(n-1)^2+\frac{3}{48^\alpha}+\frac{9}{60^\alpha}(n-2)+\frac{6}{80^\alpha} \\
& +\frac{3}{100^\alpha}(n-2)+\frac{1}{72^\alpha}[(n-(n-1))+(n-(n-2))+\ldots+(n-4)+(n-3)]6
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{6}{120^{\alpha}}(n-3)+\frac{1}{144^{\alpha}}[(n-4)+(n-5)+...+(n-(n-1))]3, \\
SDD(T_2) \;=\; & \frac{37}{2}(n+1)+\frac{157}{5}(n-1)+\frac{523}{10}(n-2)+\frac{61}{5}(n-3)+\frac{2371}{40}+12(n-1)^2 \\
& +15[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)] \\
& +6[(n-4)+(n-5)+...+(n-(n-1))].
\end{aligned}
$$

Proof: Let $f(x,y)$=$M(T_2;x,y)$ be the M-polynomial of the transformed network T_2. Then

$$
\begin{aligned}
M(T_2;x,y) \;=\; & (6n+6)x^3y^4+9x^3y^8+(3n-6)x^3y^{10}+(3n+3)x^4y^4+(6n-6)x^4y^5+(6n-6)x^4y^6 \\
& +(3n-3)x^5y^6+6x^5y^8+(6n-12)x^5y^{10}+6(n-1)^2x^6y^6+3x^6y^8+(9n-18)x^6y^{10} \\
& +6[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]x^6y^{12}+6x^8y^{10}+(3n-6)x^{10}y^{10} \\
& +(6n-18)x^{10}y^{12}+3[(n-4)+(n-5)+...+(n-(n-1))]x^{12}y^{12}.
\end{aligned}
$$

Now, the required partial derivatives and integrals are obtained.
By using the information given in Table 4 and formulas for Table 1:

$$
\begin{aligned}
D_xf(x,y) \;=\; & 3(6n+6)x^3y^4+27x^3y^8+3(3n-6)x^3y^{10}+4(3n+3)x^4y^4+4(6n-6)x^4y^5 \\
& +4(6n-6)x^4y^6+5(3n-3)x^5y^6+30x^5y^8+5(6n-12)x^5y^{10}+36(n-1)^2x^6y^6+18x^6y^8 \\
& +6(9n-18)x^6y^{10}+36[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]x^6y^{12} \\
& +48x^8y^{10}+10(3n-6)x^{10}y^{10}+10(6n-18)x^{10}y^{12}+36[(n-4)+(n-5)+... \\
& +(n-(n-1))]x^{12}y^{12}, \\
D_yf(x,y) \;=\; & 4(6n+6)x^3y^4+72x^3y^8+10(3n-6)x^3y^{10}+4(3n+3)x^4y^4+5(6n-6)x^4y^5 \\
& +6(6n-6)x^4y^6+6(3n-3)x^5y^6+48x^5y^8+10(6n-12)x^5y^{10}+36(n-1)^2x^6y^6+24x^6y^8 \\
& +10(9n-18)x^6y^{10}+72[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]x^6y^{12} \\
& +60x^8y^{10}+10(3n-6)x^{10}y^{10}+12(6n-18)x^{10}y^{12}+36[(n-4)+(n-5)+... \\
& +(n-(n-1))]x^{12}y^{12}, \\
D_xD_yf(x,y) \;=\; & 12(6n+6)x^3y^4+216x^3y^8+30(3n-6)x^3y^{10}+16(3n+3)x^4y^4+20(6n-6)x^4y^5 \\
& +24(6n-6)x^4y^6+30(3n-3)x^5y^6+240x^5y^8+50(6n-12)x^5y^{10}+216(n-1)^2x^6y^6 \\
& +144x^6y^8+60(9n-18)x^6y^{10}+432[(n-(n-1))+(n-(n-2))+... \\
& +(n-4)+(n-3)]x^6y^{12}+480x^8y^{10}+100(3n-6)x^{10}y^{10}+120(6n-18)x^{10}y^{12} \\
& +432[(n-4)+(n-5)+...+(n-(n-1))]x^{12}y^{12}, \\
S_xS_yf(x,y) \;=\; & \frac{1}{2}(n+1)x^3y^4+\frac{3}{8}x^3y^8+\frac{1}{10}(n-2)x^3y^{10}+\frac{3}{16}(n+1)x^4y^4+\frac{3}{10}(n-1)x^4y^5 \\
& +\frac{1}{4}(n-1)x^4y^6+\frac{1}{10}(n-1)x^5y^6+\frac{3}{20}x^5y^8+\frac{3}{25}(n-2)x^5y^{10}+\frac{1}{6}(n-1)^2x^6y^6+\frac{1}{16}x^6y^8 \\
& +\frac{3}{20}(n-2)x^6y^{10}+\frac{1}{12}[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]x^6y^{12} \\
& +\frac{3}{40}x^8y^{10}+\frac{3}{100}(n-2)x^{10}y^{10}+\frac{1}{20}(n-3)x^{10}y^{12} \\
& +\frac{1}{48}[(n-4)+(n-5)+...+(n-(n-1))]x^{12}y^{12},
\end{aligned}
$$

$$
\begin{aligned}
D_x^\alpha D_y^\alpha f(x,y) &= 12^\alpha(6n+6)x^3y^4+24^\alpha(9)x^3y^8+30^\alpha(3n-6)x^3y^{10}+4^{2\alpha}(3n+3)x^4y^4+20^\alpha(6n-6)x^4y^5\\
&+24^\alpha(6n-6)x^4y^6+30^\alpha(3n-3)x^5y^6+40^\alpha(6)x^5y^8+50^\alpha(6n-12)x^5y^{10}\\
&+(36)^\alpha(n-1)^2 6x^6y^6+48^\alpha(3)x^6y^8+60^\alpha(9n-18)x^6y^{10}+72^\alpha[(n-(n-1))+(n-(n-2))\\
&+...+(n-4)+(n-3)]6x^6y^{12}+80^\alpha(6)x^8y^{10}+100^\alpha(3n-6)x^{10}y^{10}+120^\alpha(6n-18)x^{10}y^{12}\\
&+144^\alpha[(n-4)+(n-5)+...+(n-(n-1))]3x^{12}y^{12},\\
S_x^\alpha S_y^\alpha f(x,y) &= \frac{6}{12^\alpha}(n+1)x^3y^4+\frac{9}{24^\alpha}x^3y^8+\frac{3}{30^\alpha}(n-2)x^3y^{10}+\frac{3}{4^{2\alpha}}(n+1)x^4y^4\\
&+\frac{6}{20^\alpha}(n-1)x^4y^5+\frac{6}{24^\alpha}(n-1)x^4y^6+\frac{3}{30^\alpha}(n-1(n-1)x^5y^6+\frac{6}{40^\alpha}x^5y^8\\
&+\frac{6}{50^\alpha}(n-2)x^5y^{10}+\frac{6}{36^\alpha}(n-1)^2x^6y^6+\frac{3}{48^\alpha}x^6y^8+\frac{9}{60^\alpha}(n-2)x^6y^{10}\\
&+\frac{1}{72^\alpha}\Big[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)\Big]6x^6y^{12}\\
&+\frac{6}{80^\alpha}x^8y^{10}+\frac{3}{100^\alpha}(n-2)x^{10}y^{10}+\frac{6}{120^\alpha}(n-3)x^{10}y^{12}\\
&+\frac{1}{144^\alpha}\Big[(n-4)+(n-5)+...+(n-(n-1))\Big]3x^{12}y^{12},\\
S_yD_xf(x,y) &= \frac{9}{2}(n+1)x^3y^4+\frac{27}{8}x^3y^8+\frac{9}{10}(n-2)x^3y^{10}+3(n+1)x^4y^4+\frac{24}{5}(n-1)x^4y^5\\
&+4(n-1)x^4y^6+\frac{5}{2}(n-1)x^5y^6+\frac{15}{4}x^5y^8+3(n-2)x^5y^{10}+6(n-1)^2x^6y^6+\frac{9}{4}x^6y^8\\
&+\frac{27}{5}(n-2)x^6y^{10}+3[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]x^6y^{12}\\
&+\frac{24}{5}x^8y^{10}+3(n-2)x^{10}y^{10}+5(n-3)x^{10}y^{12}+3[(n-4)+(n-5)+...+(n-(n-1))]x^{12}y^{12},\\
S_xD_yf(x,y) &= 8(n+1)x^3y^4+24x^3y^8+10(n-2)x^3y^{10}+3(n+1)x^4y^4+\frac{15}{2}(n-1)x^4y^5\\
&+9(n-1)x^4y^6+\frac{18}{5}(n-1)x^5y^6+\frac{48}{5}x^5y^8+12(n-2)x^5y^{10}+6(n-1)^2x^6y^6+4x^6y^8\\
&+15(n-2)x^6y^{10}+12[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]x^6y^{12}\\
&+\frac{15}{2}x^8y^{10}+3(n-2)x^{10}y^{10}+\frac{36}{5}(n-3)x^{10}y^{12}\\
&+3[(n-4)+(n-5)+...\\
&+(n-(n-1))]x^{12}y^{12}.
\end{aligned}
$$

Consequently,

$$
\begin{aligned}
M_1(T_2) &= (D_x+D_y)f(x,y)|_{x=1=y}=678n-816+72(n-1)^2+108[(n-(n-1))+(n-(n-2))\\
&+...+(n-4)+(n-3)]+72[(n-4)+(n-5)+...+(n-(n-1))],\\
M_2(T_2) &= D_xD_yf(x,y)|_{x=1=y}=2424n-3774+216(n-1)^2+432[(n-(n-1))+(n-(n-2))\\
&+...+(n-4)+(n-3)]+432[(n-4)+(n-5)+...+(n-(n-1))],\\
{}^mM_2(T_2) &= S_xS_yf(x,y)|_{x=1=y}=\frac{11}{16}(n+1)+\frac{2}{5}(n-2)+\frac{13}{20}(n-1)+\frac{1}{20}(n-3)+\frac{7}{10}+\frac{1}{6}(n-1)^2
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{1}{12}[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)] \\
& +\frac{1}{48}[(n-4)+(n-5)+...+(n-(n-1))], \\
R_\alpha(T_2) &= D_x^\alpha D_y^\alpha f(x,y)|_{x=1=y} = 12^\alpha(6n+6)+24^\alpha(9)+30^\alpha(3n-6)+4^{2\alpha}(3n+3)+20^\alpha(6n-6) \\
& +24^\alpha(6n-6)+30^\alpha(3n-3)+40^\alpha(6)+50^\alpha(6n-12)+6(n-1)^2(36)^\alpha+48^\alpha(3) \\
& +60^\alpha(9n-18)+72^\alpha[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]6 \\
& +80^\alpha(6)+100^\alpha(3n-6)+120^\alpha(6n-18)+144^\alpha[(n-4)+(n-5)+...+(n-(n-1))]3, \\
RR_\alpha(T_2) &= S_x^\alpha S_y^\alpha f(x,y)|_{x=1=y} = \frac{6}{12^\alpha}(n+1)+\frac{9}{24^\alpha}+\frac{3}{30^\alpha}(n-2)+\frac{3}{4^{2\alpha}}(n+1)+\frac{6}{20^\alpha}(n-1)+\frac{6}{24^\alpha}(n-1) \\
& +\frac{3}{30^\alpha}(n-1)+\frac{6}{40^\alpha}+\frac{6}{50^\alpha}(n-2)+\frac{6}{36^\alpha}(n-1)^2+\frac{3}{48^\alpha}+\frac{9}{60^\alpha}(n-2)+\frac{6}{80^\alpha} \\
& +\frac{3}{100^\alpha}(n-2)+\frac{1}{72^\alpha}[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]6 \\
& +\frac{6}{120^\alpha}(n-3)+\frac{1}{144^\alpha}[(n-4)+(n-5)+...+(n-(n-1))]3, \\
SDD(T_2) &= (S_yD_x+S_xD_y)(x,y)|_{x=1=y} = \frac{37}{2}(n+1)+\frac{157}{5}(n-1)+\frac{523}{10}(n-2)+\frac{61}{5}(n-3)+\frac{2371}{40} \\
& +12(n-1)^2+15[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)] \\
& +6[(n-4)+(n-5)+...+(n-(n-1))].
\end{aligned}
$$

3.2.3. Theorem

For T_2, the Atomic Bond Connectivity, Geometric Arithmetic Index and General Harmonic Index are as follows, respectively.

$$
\begin{aligned}
1)\quad ABC(T_2) &= (7n)\frac{\sqrt{6}}{4}+(n+1)\sqrt{15}+(n-1)\Big(3\sqrt{\frac{7}{5}}+2\sqrt{3}+3\sqrt{\frac{3}{10}}\Big) \\
& (n-2)\Big(\frac{\sqrt{330}}{10}+\frac{3\sqrt{26}}{5}+\frac{3\sqrt{210}}{10}+\frac{9\sqrt{2}}{10}\Big)+\frac{3\sqrt{110}}{10}+\frac{3}{2}+\frac{6}{\sqrt{5}} \\
& +(n-1)^2\sqrt{10}+[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]2\sqrt{2} \\
& +[(n-4)+(n-5)+...+(n-(n-1))]\frac{\sqrt{22}}{4}, \\
2)\quad GA(T_2) &= (24n+36)\frac{\sqrt{3}}{7}+(132n+48)\frac{\sqrt{6}}{55}+(50n-113)\frac{6\sqrt{30}}{143}+3(n+1) \\
& +(8n)\frac{\sqrt{5}}{3}+(n-2)\Big(4\sqrt{2}+\frac{9\sqrt{15}}{4}+3\Big)+\frac{24\sqrt{10}}{13}+6(n-1)^2 \\
& +[(n-(n-1))+(n-(n-2))+...+(n-4)+(n-3)]4\sqrt{2} \\
& +3[(n-4)+(n-5)+...+(n-(n-1))], \\
3)\quad H_k(T_2) &= (6n+6)(\frac{2}{7})^k+9(\frac{2}{11})^k+(3n-6)(\frac{2}{13})^k+(3n+3)(\frac{1}{4})^k+(6n-6) \\
& (\frac{2}{9})^k+(6n-6)(\frac{1}{5})^k+(3n-3)(\frac{2}{11})^k+6(\frac{2}{13})^k+(6n-12)(\frac{2}{15})^k
\end{aligned}
$$

$$+6(n-1)^2)(\frac{1}{6})^k + 3(\frac{1}{7})^k + (9n-18)(\frac{1}{8})^k + 6[(n-(n-1))$$
$$+(n-(n-2)) + ... + (n-4) + (n-3)](\frac{1}{9})^k + 6(\frac{1}{9})^k + (3n-6)(\frac{1}{10})^k$$
$$+(6n-18)(\frac{1}{11})^k + 3[(n-4) + (n-5) + ... + (n-(n-1))](\frac{1}{12})^k.$$

Proof: Using Eqs 1.7–1.9 and Table 4, we get the results 1–3.

3.2.4. Theorem

For T_2, the First, Second Gourava Indices and the First, Second Hyper Gourava Indices are as follows, respectively.

$$\begin{aligned}
1)\ GO_1(T_2) &= 3102n - 4590 + 288(n-1)^2 + 540[(n-(n-1)) + (n-(n-2)) \\
&\quad +... + (n-4) + (n-3)] + 504[(n-4) + (n-5) + ... + (n-(n-1))], \\
2)\ GO_2(T_2) &= 40548n - 74610 + 2592(n-1)^2 + 7776[(n-(n-1)) + (n-(n-2)) \\
&\quad +... + (n-4) + (n-3)] + 10368[(n-4) + (n-5) + ... + (n-(n-1))], \\
3)\ HGO_1(T_2) &= 267984n - 531210 + 13824(n-1)^2 \\
&\quad +48600[(n-(n-1)) + (n-(n-2)) + ... + (n-4) + (n-3)] \\
&\quad +84672[(n-4) + (n-5) + ... + (n-(n-1))], \\
4)\ HGO_2(T_2) &= 66901488n - 158433396 + 1119744(n-1)^2 \\
&\quad +10077696[(n-(n-1)) + (n-(n-2)) + ... + (n-4) + (n-3)] \\
&\quad +35831808[(n-4) + (n-5) + ... + (n-(n-1))].
\end{aligned}$$

Proof: Using Eqs 1.10–1.13 and Table 4 we get the above results 1–4.

4. Conclusions

In this paper, we defined M-polynomial of the transformed zigzag benzenoid and transformed triangular benzenoid structures by applying stellation, medial and bounded dual operations to get networks named as T_1 and T_2. With the help of M-polynomial, we computed certain degree-based topological indices such as first Zagreb index, second Zagreb index, second modified Zagreb index, general Randic index, reciprocal general Randic index, symmetric division degree index. We also computed atomic bound connectivity index, geometric arithmetic index, general harmonic index, first and second gourava indices, first and second hyper gourava indices. M-polynomial is used to calculate the certain degree based topological indices as a latest developed instrument in the chemical graph theory. In future, we can compute other indices on these structures and some additional transformed structures can be studied for a variety of topological indices to have an insight about their properties.

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Conflict of interest

The authors declare no conflict of interest.

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