Research article

Prominent interior GE-filters of GE-algebras

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Abstract: The concept of a prominent interior GE-filter (of type 1 and type 2) is introduced, and their properties are investigated. The relationship between a prominent GE-filter and a prominent interior GE-filter and the relationship between an interior GE-filter and a prominent interior GE-filter are discussed. Examples to show that any interior GE-filter is not a prominent interior GE-filter and any prominent GE-filter is not a prominent interior GE-filter are provided. Conditions for an interior GE-filter to be a prominent interior GE-filter are given. Also, conditions under which an internal GE-filter larger than a given internal GE-filter can become a prominent internal GE-filter are considered, and an example describing it is given. The relationship between a prominent interior GE-filter and a prominent interior GE-filter of type 1 is discussed.

Keywords: (transitive) GE-algebra; GE-filter; interior GE-filter; prominent interior GE-filter (of type 1 and type 2)

Mathematics Subject Classification: 03G25, 06F35

1. Introduction

Henkin and Skolem introduced Hilbert algebras in the fifties for investigations in intuitionistic and other non-classical logics. Diego [4] proved that Hilbert algebras form a variety which is locally finite. Bandaru et al. introduced the notion of GE-algebras which is a generalization of Hilbert algebras, and investigated several properties (see [1, 2, 7–9]). The notion of interior operator is introduced by Vorster [12] in an arbitrary category, and it is used in [3] to study the notions of connectedness and disconnectedness in topology. Interior algebras are a certain type of algebraic structure that encodes the idea of the topological interior of a set, and are a generalization of topological spaces defined by means of topological interior operators. Rachůnek and Svoboda [6]
studied interior operators on bounded residuated lattices, and Svrcek [11] studied multiplicative interior operators on GMV-algebras. Lee et al. [5] applied the interior operator theory to GE-algebras, and they introduced the concepts of (commutative, transitive, left exchangeable, belligerent, antisymmetric) interior GE-algebras and bordered interior GE-algebras, and investigated their relations and properties. Later, Song et al. [10] introduced the notions of an interior GE-filter, a weak interior GE-filter and a belligerent interior GE-filter, and investigate their relations and properties. They provided relations between a belligerent interior GE-filter and an interior GE-filter and conditions for an interior GE-filter to be a belligerent interior GE-filter is considered. Given a subset and an element, they established an interior GE-filter, and they considered conditions for a subset to be a belligerent interior GE-filter. They studied the extensibility of the belligerent interior GE-filter and established relationships between weak interior GE-filter and belligerent interior GE-filter of type 1, type 2 and type 3. Rezaei et al. [7] studied prominent GE-filters in GE-algebras. The purpose of this paper is to study by applying interior operator theory to prominent GE-filters in GE-algebras. We introduce the concept of a prominent interior GE-filter, and investigate their properties. We discuss the relationship between a prominent GE-filter and a prominent interior GE-filter and the relationship between an interior GE-filter and a prominent interior GE-filter. We find and provide examples where any interior GE-filter is not a prominent interior GE-filter and any prominent GE-filter is not a prominent interior GE-filter. We provide conditions for an interior GE-filter to be a prominent interior GE-filter. We provide conditions under which an internal GE-filter larger than a given internal GE filter can become a prominent internal GE-filter, and give an example describing it. We also introduce the concept of a prominent interior GE-filter of type 1 and type 2, and investigate their properties. We discuss the relationship between a prominent interior GE-filter of type 1 and type 2, and investigate their properties. We find and provide examples where any interior GE-filter is not a prominent interior GE-filter and any prominent GE-filter is not a prominent interior GE-filter. We provide conditions for an interior GE-filter to be a prominent interior GE-filter. We provide conditions under which an internal GE-filter larger than a given internal GE filter can become a prominent internal GE-filter, and give an example describing it. We also introduce the concept of a prominent interior GE-filter of type 1 and type 2, and investigate their properties. We discuss the relationship between a prominent interior GE-filter and a prominent interior GE-filter of type 1. We give examples to show that A and B are independent of each other, where A and B are:

(1) \{ A: prominent interior GE-filter of type 1. \\
B: prominent interior GE-filter of type 2. \\
\}

(2) \{ A: prominent interior GE-filter. \\
B: prominent interior GE-filter of type 2. \\
\}

(3) \{ A: interior GE-filter. \\
B: prominent interior GE-filter of type 1. \\
\}

(4) \{ A: interior GE-filter. \\
B: prominent interior GE-filter of type 2. \\
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2. Preliminaries

Definition 2.1. [1] By a GE-algebra we mean a non-empty set $X$ with a constant $1$ and a binary operation $*$ satisfying the following axioms:

(GE1) $u * u = 1$,

(GE2) $1 * u = u$,

(GE3) $u * (v * w) = u * (v * (u * w))$

for all $u, v, w \in X$. 

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In a GE-algebra $X$, a binary relation “$\leq$” is defined by

$$(\forall x, y \in X) \ (x \leq y \iff x \ast y = 1).$$

(2.1)

**Definition 2.2.** [1, 2, 8] A GE-algebra $X$ is said to be transitive if it satisfies:

$$(\forall x, y, z \in X) \ (x \ast y \leq (z \ast x) \ast (z \ast y)).$$

(2.2)

**Proposition 2.3.** [1] Every GE-algebra $X$ satisfies the following items:

$$\begin{align*}
(\forall u \in X) \ (u \ast 1 = 1). \\
(\forall u, v \in X) \ (u \ast (u \ast v) = u \ast v). \\
(\forall u, v \in X) \ (u \ast v \leq v \ast u). \\
(\forall u, v, w \in X) \ (u \ast (v \ast w) \leq v \ast (u \ast w)). \\
(\forall u \in X) \ (1 \leq u \ \Rightarrow \ u = 1). \\
(\forall u, v \in X) \ (u \leq (v \ast u) \ast u). \\
(\forall u, v \in X) \ (u \leq (u \ast v) \ast v). \\
(\forall u, v, w \in X) \ (u \leq v \ast w \iff v \leq u \ast w).
\end{align*}$$

(2.3)-(2.10)

If $X$ is transitive, then

$$\begin{align*}
(\forall u, v, w \in X) \ (u \leq v \ \Rightarrow \ w \ast u \leq w \ast v, \ v \ast w \leq u \ast w). \\
(\forall u, v, w \in X) \ (u \ast v \leq (v \ast w) \ast (u \ast w)).
\end{align*}$$

(2.11)-(2.12)

**Lemma 2.4.** [1] In a GE-algebra $X$, the following facts are equivalent each other.

$$\begin{align*}
(\forall x, y, z \in X) \ (x \ast y \leq (z \ast x) \ast (z \ast y)). \\
(\forall x, y, z \in X) \ (x \ast y \leq (y \ast z) \ast (x \ast z)).
\end{align*}$$

(2.13)-(2.14)

**Definition 2.5.** [1] A subset $F$ of a GE-algebra $X$ is called a GE-filter of $X$ if it satisfies:

$$\begin{align*}
1 \in F, \\
(\forall x, y \in X) \ (x \ast y \in F, \ x \in F \ \Rightarrow \ y \in F).
\end{align*}$$

(2.15)-(2.16)

**Lemma 2.6.** [1] In a GE-algebra $X$, every filter $F$ of $X$ satisfies:

$$(\forall x, y \in X) \ (x \leq y, \ x \in F \ \Rightarrow \ y \in F).$$

(2.17)

**Definition 2.7.** [7] A subset $F$ of a GE-algebra $X$ is called a prominent GE-filter of $X$ if it satisfies (2.15) and

$$\begin{align*}
(\forall x, y, z \in X) & \ (x \ast (y \ast z) \in F, \ x \in F \ \Rightarrow \ ((z \ast y) \ast y) \ast z \in F).
\end{align*}$$

(2.18)

Note that every prominent GE-filter is a GE-filter in a GE-algebra (see [7]).
Definition 2.8. [5] By an interior GE-algebra we mean a pair \((X, f)\) in which \(X\) is a GE-algebra and \(f : X \rightarrow X\) is a mapping such that
\[
(\forall x \in X)(x \leq f(x)), \quad (2.19)
\]
\[
(\forall x \in X)((f \circ f)(x) = f(x)), \quad (2.20)
\]
\[
(\forall x, y \in X)(x \leq y \Rightarrow f(x) \leq f(y)). \quad (2.21)
\]

Definition 2.9. [10] Let \((X, f)\) be an interior GE-algebra. A GE-filter \(F\) of \(X\) is said to be interior if it satisfies:
\[
(\forall x \in X)(f(x) \in F \Rightarrow x \in F). \quad (2.22)
\]

3. Prominent interior GE-filters

Definition 3.1. Let \((X, f)\) be an interior GE-algebra. Then a subset \(F\) of \(X\) is called a prominent interior GE-filter in \((X, f)\) if \(F\) is a prominent GE-filter of \(X\) which satisfies the condition (2.22).

Example 3.2. Let \(X = \{1, 2, 3, 4, 5\}\) be a set with the Cayley table which is given in Table 1.

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Then \(X\) is a GE-algebra. If we define a mapping \(f\) on \(X\) as follows:
\[
f : X \rightarrow X, \quad x \mapsto \begin{cases} 
1 & \text{if } x \in \{1, 4, 5\}, \\
2 & \text{if } x \in \{2, 3\},
\end{cases}
\]
then \((X, f)\) is an interior GE-algebra and \(F = \{1, 4, 5\}\) is a prominent interior GE-filter in \((X, f)\).

It is clear that every prominent interior GE-filter is a prominent GE-filter. But any prominent GE-filter may not be a prominent interior GE-filter in an interior GE-algebra as seen in the following example.

Example 3.3. Let \(X = \{1, 2, 3, 4, 5\}\) be a set with the Cayley table which is given in Table 2.

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and define a mapping $f$ on $X$ as follows:

$$f : X \rightarrow X, \ x \mapsto \begin{cases} 1 & \text{if } x \in \{1, 2, 3, 5\}, \\ 4 & \text{if } x = 4. \end{cases}$$

Then $(X, f)$ is an interior GE-algebra and $F := \{1\}$ is a prominent GE-filter of $X$. But it is not a prominent interior GE-filter in $(X, f)$ since $f(2) = 1 \in F$ but $2 \not\in F$.

We discuss relationship between interior GE-filter and prominent interior GE-filter.

**Theorem 3.4.** *In an interior GE-algebra, every prominent interior GE-filter is an interior GE-filter.*

**Proof.** It is straightforward because every prominent GE-filter is a GE-filter in a GE-algebra. □

In the next example, we can see that any interior GE-filter is not a prominent interior GE-filter in general.

**Example 3.5.** Let $X = \{1, 2, 3, 4, 5\}$ be a set with the Cayley table which is given in Table 3.

**Table 3.** Cayley table for the binary operation “$\ast$”.

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Then $X$ is a GE-algebra. If we define a mapping $f$ on $X$ as follows:

$$f : X \rightarrow X, \ x \mapsto \begin{cases} 1 & \text{if } x = 1, \\ 2 & \text{if } x \in \{2, 4, 5\}, \\ 3 & \text{if } x = 3, \end{cases}$$

then $(X, f)$ is an interior GE-algebra and $F = \{1\}$ is an interior GE-filter in $(X, f)$. But it is not a prominent interior GE-filter in $(X, f)$ since $1 \ast (2 \ast 3) = 1 \in F$ but $((3 \ast 2) \ast 3) = 3 \not\in F$.

**Proposition 3.6.** *Every prominent interior GE-filter $F$ in an interior GE-algebra $(X, f)$ satisfies:*

$$(\forall x, y \in X) \left( f(x \ast y) \in F \Rightarrow ((y \ast x) \ast x) \ast y \in F \right). \quad (3.1)$$

**Proof.** Let $F$ be a prominent interior GE-filter in $(X, f)$. Let $x, y \in X$ be such that $f(x \ast y) \in F$. Then $x \ast y \in F$ by (2.22), and so $1 \ast (x \ast y) = x \ast y \in F$ by (GE2). Since $1 \in F$, it follows from (2.18) that $((y \ast x) \ast x) \ast y \in F$. □

**Corollary 3.7.** *Every prominent interior GE-filter $F$ in an interior GE-algebra $(X, f)$ satisfies:*

$$(\forall x, y \in X) \left( x \ast y \in F \Rightarrow ((y \ast x) \ast x) \ast y \in F \right). \quad (3.2)$$
**Proof.** Let $F$ be a prominent interior GE-filter in $(X, f)$. Then $F$ is an interior GE-filter in $(X, f)$ by Theorem 3.4. Let $x, y \in X$ be such that $x \ast y \in F$. Since $x \ast y \leq f(x \ast y)$ by (2.19), it follows from Lemma 2.6 that $f(x \ast y) \in F$. Hence $((y \ast x) \ast x) \ast y \in F$ by Proposition 3.6. 

**Corollary 3.8.** Every prominent interior GE-filter $F$ in an interior GE-algebra $(X, f)$ satisfies:

$$(\forall x, y \in X) (x \ast y \in F \Rightarrow f(((y \ast x) \ast x) \ast y) \in F).$$

**Proof.** Straightforward. 

**Corollary 3.9.** Every prominent interior GE-filter $F$ in an interior GE-algebra $(X, f)$ satisfies:

$$(\forall x, y \in X) (f(x \ast y) \in F \Rightarrow f(((y \ast x) \ast x) \ast y) \in F).$$

**Proof.** Straightforward. 

In the following example, we can see that any interior GE-filter $F$ in an interior GE-algebra $(X, f)$ does not satisfy the conditions (3.1) and (3.2).

**Example 3.10.** Consider the interior GE-algebra $(X, f)$ in Example 3.5. The interior GE-filter $F := \{1\}$ does not satisfy conditions (3.1) and (3.2) since $f(2 \ast 3) = f(1) = 1 \in F$ and $2 \ast 3 = 1 \in F$ but $((3 \ast 2) \ast 2) \ast 3 = 3 \notin F$.

We provide conditions for an interior GE-filter to be a prominent interior GE-filter.

**Theorem 3.11.** If an interior GE-filter $F$ in an interior GE-algebra $(X, f)$ satisfies the condition (3.1), then $F$ is a prominent interior GE-filter in $(X, f)$.

**Proof.** Let $F$ be an interior GE-filter in $(X, f)$ that satisfies the condition (3.1). Let $x, y, z \in X$ be such that $x \ast (y \ast z) \in F$ and $x \in F$. Then $y \ast z \in F$. Since $y \ast z \leq f(y \ast z)$ by (2.19), it follows from Lemma 2.6 that $f(y \ast z) \in F$. Hence $((z \ast y) \ast y) \ast z \in F$ by (3.1), and therefore $F$ is a prominent interior GE-filter in $(X, f)$. 

**Lemma 3.12.** [10] In an interior GE-algebra, the intersection of interior GE-filters is also an interior GE-filter.

**Theorem 3.13.** In an interior GE-algebra, the intersection of prominent interior GE-filters is also a prominent interior GE-filter.

**Proof.** Let $\{F_i \mid i \in \Lambda\}$ be a set of prominent interior GE-filters in an interior GE-algebra $(X, f)$ where $\Lambda$ is an index set. Then $\{F_i \mid i \in \Lambda\}$ is a set of interior GE-filters in $(X, f)$, and so $\bigcap \{F_i \mid i \in \Lambda\}$ is an interior GE-filter in $(X, f)$ by Lemma 3.12. Let $x, y \in X$ be such that $f(x \ast y) \in \bigcap \{F_i \mid i \in \Lambda\}$. Then $f(x \ast y) \in F_i$ for all $i \in \Lambda$. It follows from Proposition 3.6 that $((y \ast x) \ast x) \ast y \in F_i$ for all $i \in \Lambda$. Hence $((y \ast x) \ast x) \ast y \in \bigcap\{F_i \mid i \in \Lambda\}$ and therefore $\bigcap\{F_i \mid i \in \Lambda\}$ is a prominent interior GE-filter in $(X, f)$ by Theorem 3.11. 

**Theorem 3.14.** If an interior GE-filter $F$ in an interior GE-algebra $(X, f)$ satisfies the condition (3.2), then $F$ is a prominent interior GE-filter in $(X, f)$. 

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Proof. Let $F$ be an interior GE-filter in $(X, f)$ that satisfies the condition (3.2). Let $x, y, z \in X$ be such that $x \ast (y \ast z) \in F$ and $x \in F$. Then $y \ast z \in F$ and thus $((z \ast y) \ast y) \ast z \in F$. Therefore $F$ is a prominent interior GE-filter in $(X, f)$. □

Given an interior GE-filter $F$ in an interior GE-algebra $(X, f)$, we consider an interior GE-filter $G$ which is greater than $F$ in $(X, f)$, that is, we take two interior GE-filters $F$ and $G$ such that $F \subseteq G$ in an interior GE-algebra $(X, f)$. We are now trying to find the condition that $G$ can be a prominent interior GE-filter in $(X, f)$.

**Theorem 3.15.** Let $(X, f)$ be an interior GE-algebra in which $X$ is transitive. Let $F$ and $G$ be interior GE-filters in $(X, f)$. If $F \subseteq G$ and $F$ is a prominent interior GE-filter in $(X, f)$, then $G$ is also a prominent interior GE-filter in $(X, f)$.

**Proof.** Assume that $F$ is a prominent interior GE-filter in $(X, f)$. Then it is an interior GE-filter in $(X, f)$ by Theorem 3.4. Let $x, y \in X$ be such that $f(x \ast y) \in G$. Then $x \ast y \in G$ by (2.22), and so $1 = (x \ast y) \ast (x \ast y) \leq x \ast ((x \ast y) \ast y)$ by (GE1) and (2.6). Since $1 \in F$, it follows from Lemma 2.6 that $x \ast ((x \ast y) \ast y) \in F$. Hence $(((x \ast y) \ast y) \ast x) \ast ((x \ast y) \ast y) \in F \subseteq G$ by Corollary 3.7. Since

$$(((x \ast y) \ast y) \ast x) \ast ((x \ast y) \ast y) \leq (x \ast y) \ast (((x \ast y) \ast y) \ast x) \ast y)$$

by (2.6), we have $(x \ast y) \ast (((x \ast y) \ast y) \ast x) \ast y) \in G$ by Lemma 2.6. Hence

$$(((x \ast y) \ast y) \ast x) \ast y \in G.$$

Since $y \leq (x \ast y) \ast y$, it follows from (2.11) that

$$(((x \ast y) \ast y) \ast x) \ast y \leq ((y \ast x) \ast x) \ast y.$$

Thus $((y \ast x) \ast x) \ast y \in G$ by Lemma 2.6. Therefore $G$ is a prominent interior GE-filter in $(X, f)$ by Theorem 3.11. □

The following example describes Theorem 3.15.

**Example 3.16.** Let $X = \{1, 2, 3, 4, 5\}$ be a set with the Cayley table which is given in Table 4.

**Table 4.** Cayley table for the binary operation “$\ast$”.

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and define a mapping $f$ on $X$ as follows:

$$f : X \to X, \ x \mapsto \begin{cases} 1 & \text{if } x = 1, \\ 3 & \text{if } x \in \{2, 3\}, \\ 5 & \text{if } x \in \{4, 5\}. \end{cases}$$
Then \((X, f)\) is an interior GE-algebra in which \(X\) is transitive, and \(F := \{1\}\) and \(G := \{1, 4, 5\}\) are interior GE-filters in \((X, f)\) with \(F \subseteq G\). Also we can observe that \(F\) and \(G\) are prominent interior GE-filters in \((X, f)\).

In Theorem 3.15, if \(F\) is an interior GE-filter which is not prominent, then \(G\) is also not a prominent interior GE-filter in \((X, f)\) as shown in the next example.

**Example 3.17.** Let \(X = \{1, 2, 3, 4, 5\}\) be a set with the Cayley table which is given in Table 5,

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and define a mapping \(f\) on \(X\) as follows:

\[
f : X \to X, \ x \mapsto \begin{cases} 
1 \text{ if } x = 1, \\
3 \text{ if } x = 3, \\
4 \text{ if } x = 4, \\
2 \text{ if } x \in \{2, 5\}.
\end{cases}
\]

Then \((X, f)\) is an interior GE-algebra in which \(X\) is transitive, and \(F := \{1\}\) and \(G := \{1, 3\}\) are interior GE-filters in \((X, f)\) with \(F \subseteq G\). We can observe that \(F\) and \(G\) are not prominent interior GE-filters in \((X, f)\) since \(2 \ast 3 = 1 \in F\) but \((3 \ast 2) \ast 2 = (5 \ast 2) \ast 3 = 1 \ast 3 = 3 \notin F\), and \(4 \ast 2 = 1 \in G\) but \((2 \ast 4) \ast 4 \ast 2 = (4 \ast 4) \ast 2 = 1 \ast 2 = 2 \notin G\).

In Theorem 3.15, if \(X\) is not transitive, then Theorem 3.15 is false as seen in the following example.

**Example 3.18.** Let \(X = \{1, 2, 3, 4, 5, 6\}\) be a set with the Cayley table which is given in Table 6.

<table>
<thead>
<tr>
<th>*</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
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<tr>
<td>2</td>
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<td>6</td>
<td>6</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
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<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
If we define a mapping $f$ on $X$ as follows:

$$f : X \rightarrow X, \quad x \mapsto \begin{cases} 
1 & \text{if } x = 1, \\
4 & \text{if } x = 4, \\
5 & \text{if } x = 5, \\
6 & \text{if } x = 6, \\
2 & \text{if } x \in \{2, 3\}, 
\end{cases}$$

then $(X, f)$ is an interior GE-algebra in which $X$ is not transitive. Let $F := \{1\}$ and $G := \{1, 5, 6\}$. Then $F$ is a prominent interior GE-filter in $(X, f)$ and $G$ is an interior GE-filter in $(X, f)$ with $F \subseteq G$. But $G$ is not prominent interior GE-filter since $5 \ast (3 \ast 4) = 5 \ast 5 = 1 \in G$ and $5 \in G$ but $((4 \ast 3) \ast 3) \ast 4 = (3 \ast 3) \ast 4 = 1 \ast 4 = 4 \notin G$.

**Definition 3.19.** Let $(X, f)$ be an interior GE-algebra and let $F$ be a subset of $X$ which satisfies (2.15). Then $F$ is called:

- A prominent interior GE-filter of type 1 in $(X, f)$ if it satisfies:
  
  $$(\forall x, y, z \in X) (x \ast (y \ast f(z))) \in F, \ f(x) \in F \Rightarrow ((f(z) \ast y) \ast y) \ast f(z) \in F). \quad (3.3)$$

- A prominent interior GE-filter of type 2 in $(X, f)$ if it satisfies:
  
  $$(\forall x, y, z \in X) (x \ast (y \ast f(z))) \in F, \ f(x) \in F \Rightarrow ((z \ast f(y)) \ast f(y)) \ast z \in F). \quad (3.4)$$

**Example 3.20.** (1). Let $X = \{1, 2, 3, 4, 5\}$ be a set with the Cayley table which is given in Table 7,

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
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<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
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<tr>
<td>4</td>
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<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

and define a mapping $f$ on $X$ as follows:

$$f : X \rightarrow X, \quad x \mapsto \begin{cases} 
1 & \text{if } x \in \{1, 3\} \\
2 & \text{if } x = 2, \\
4 & \text{if } x = 4, \\
5 & \text{if } x = 5. 
\end{cases}$$

Then $(X, f)$ is an interior GE-algebra and $F := \{1, 3\}$ is a prominent interior GE-filter of type 1 in $(X, f)$.

(2). Let $X = \{1, 2, 3, 4, 5\}$ be a set with the Cayley table which is given in Table 8,
Table 8. Cayley table for the binary operation “∗”.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

and define a mapping $f$ on $X$ as follows:

$$f : X \to X, \ x \mapsto \begin{cases} 1 & \text{if } x = 1, \\ 2 & \text{if } x \in \{2, 3, 4, 5\}. \end{cases}$$

Then $(X, f)$ is an interior GE-algebra and $F := \{1, 3\}$ is a prominent interior GE-filter of type 2 in $(X, f)$.

**Theorem 3.21.** In an interior GE-algebra, every prominent interior GE-filter is of type 1.

**Proof.** Let $F$ be a prominent interior GE-filter in an interior GE-algebra $(X, f)$. Let $x, y, z \in X$ be such that $x \ast (y \ast f(z)) \in F$ and $f(x) \in F$. Then $x \in F$ by (2.22). It follows from (2.18) that $((f(z) \ast y) \ast y) \ast f(z) \in F$. Hence $F$ is a prominent interior GE-filter of type 1 in $(X, f)$. □

The following example shows that the converse of Theorem 3.21 may not be true.

**Example 3.22.** Let $X = \{1, 2, 3, 4, 5\}$ be a set with the Cayley table which is given in Table 9,

Table 9. Cayley table for the binary operation “∗”.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
</tr>
</tbody>
</table>

and define a mapping $f$ on $X$ as follows:

$$f : X \to X, \ x \mapsto \begin{cases} 1 & \text{if } x = 1, \\ 2 & \text{if } x \in \{2, 3\}, \\ 5 & \text{if } x \in \{4, 5\}. \end{cases}$$

Then $(X, f)$ is an interior GE-algebra and $F := \{1\}$ is a prominent interior GE-filter of type 1 in $(X, f)$. But it is not a prominent interior GE-filter in $(X, f)$ since $1 \ast (3 \ast 4) = 1 \in F$ but $(4 \ast 3) \ast 3 \ast 4 = 4 \notin F$.

The following example shows that prominent interior GE-filter and prominent interior GE-filter of type 2 are independent of each other, i.e., prominent interior GE-filter is not prominent interior GE-filter of type 2 and neither is the inverse.
Example 3.23. (1). Let \( X = \{1, 2, 3, 4, 5\} \) be a set with the Cayley table which is given in the following Table 10,

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<tbody>
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<tr>
<td>4</td>
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<tr>
<td>5</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

and define a mapping \( f \) on \( X \) as follows:

\[
 f : X \rightarrow X, \quad x \mapsto \begin{cases} 
 1 & \text{if } x = 1, \\
 4 & \text{if } x \in \{3, 4\} \\
 5 & \text{if } x \in \{2, 5\} 
\end{cases}
\]

Then \((X, f)\) is an interior GE-algebra and \( F := \{1\} \) is a prominent interior GE-filter in \((X, f)\). But it is not a prominent interior GE-filter of type 2 since \( 1 \ast (5 \ast f(2)) = 5 \ast 5 = 1 \in F \) and \( f(1) = 1 \in F \) but \((2 \ast f(5)) \ast f(5)) \ast 2 = (2 \ast 5) \ast 2 = 1 \ast 5 = 2 = 3 \notin F \).

(2). Let \( X = \{1, 2, 3, 4, 5\} \) be a set with the Cayley table which is given in the following Table 11,

<table>
<thead>
<tr>
<th></th>
<th>1</th>
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<th>4</th>
<th>5</th>
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</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
<td>3</td>
<td>4</td>
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<tr>
<td>4</td>
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<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

and define a mapping \( f \) on \( X \) as follows:

\[
 f : X \rightarrow X, \quad x \mapsto \begin{cases} 
 1 & \text{if } x = 1, \\
 5 & \text{if } x \in \{2, 3, 4, 5\} 
\end{cases}
\]

Then \((X, f)\) is an interior GE-algebra and \( F := \{1\} \) is a prominent interior GE-filter of type 2 in \((X, f)\). But it is not a prominent interior GE-filter in \((X, f)\) since \( 1 \ast (5 \ast f(2)) = 5 \ast 5 = 1 \in F \) and \( f(1) = 1 \in F \) but \((3 \ast 2) \ast 3 = (2 \ast 2) \ast 3 = 1 \ast 3 = 3 \notin F \).

The following example shows that prominent interior GE-filter of type 1 and prominent interior GE-filter of type 2 are independent of each other.

Example 3.24. (1). Let \( X = \{1, 2, 3, 4, 5\} \) be a set with the Cayley table which is given in the following Table 12,
Table 12. Cayley table for the binary operation “∗”.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
<td>3</td>
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<td>2</td>
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<tr>
<td>3</td>
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</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
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<td>1</td>
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<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

and define a mapping $f$ on $X$ as follows:

$$f: X \to X, x \mapsto \begin{cases} 1 & \text{if } x = 1, \\ 3 & \text{if } x \in \{2, 3\}, \\ 5 & \text{if } x \in \{4, 5\}. \end{cases}$$

Then $(X, f)$ is an interior GE-algebra and $F := \{1, 2, 4\}$ is a prominent interior GE-filter of type 1 in $(X, f)$. But it is not a prominent interior GE-filter of type 2 since $1 \ast (5 \ast f(2)) = 1 \ast (5 \ast 3) = 1 \ast 1 = 1 \in F$ and $f(1) = 1 \in F$ but $((2 \ast f(5)) \ast f(5)) \ast 2 = ((2 \ast 5) \ast 5) \ast 2 = (5 \ast 5) \ast 2 = 1 \ast 2 = 2 \not\in F$.

(2). Let $X = \{1, 2, 3, 4, 5\}$ be a set with the Cayley table which is given in the following Table 13,

Table 13. Cayley table for the binary operation “∗”.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
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<td>1</td>
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<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>5</td>
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<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

and define a mapping $f$ on $X$ as follows:

$$f: X \to X, x \mapsto \begin{cases} 1 & \text{if } x = 1, \\ 2 & \text{if } x = 2, \\ 4 & \text{if } x = 4, \\ 3 & \text{if } x \in \{3, 5\}. \end{cases}$$

Then $(X, f)$ is an interior GE-algebra and $F := \{1\}$ is a prominent interior GE-filter of type 2 in $(X, f)$. But it is not a prominent interior GE-filter of type 1 in $(X, f)$ since $1 \ast (5 \ast f(2)) = 1 \ast (5 \ast 2) = 1 \ast 1 = 1 \in F$ and $f(1) \in F$ but $((f(2) \ast 5) \ast 5) \ast f(2) = ((2 \ast 5) \ast 5) \ast 2 = (5 \ast 5) \ast 2 = 1 \ast 2 = 2 \not\in F$.

The following example shows that interior GE-filter and prominent interior GE-filter of type 1 are independent of each other.

**Example 3.25.** (1). Let $X = \{1, 2, 3, 4, 5\}$ be a set with the Cayley table which is given in the following Table 14,
and define a mapping $f$ on $X$ as follows:

$$f : X \to X, \quad x \mapsto \begin{cases} 1 & \text{if } x = 1, \\ 2 & \text{if } x = 2, \\ 5 & \text{if } x \in \{3, 4, 5\}. \end{cases}$$

Then $(X, f)$ is an interior GE-algebra and $F := \{1\}$ is an interior GE-filter in $(X, f)$. But $F$ is not prominent interior GE-filter of type 1 since $1 \ast (5 \ast f(2)) = 1 \ast (5 \ast 2) = 1 \ast 1 = 1 \in F$ and $f(1) = 1 \in F$ but $(f(2) \ast 5) \ast 2 = (2 \ast 5) \ast 2 = (5 \ast 5) \ast 2 = 1 \ast 2 = 2 \notin F$.

(2). Let $X = \{1, 2, 3, 4, 5\}$ be a set with the Cayley table which is given in the following Table 15,

**Table 15.** Cayley table for the binary operation “$\ast$”.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<td>2</td>
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<td>3</td>
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<td>4</td>
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<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

and define a mapping $f$ on $X$ as follows:

$$f : X \to X, \quad x \mapsto \begin{cases} 1 & \text{if } x \in \{1, 2, 4\}, \\ 5 & \text{if } x \in \{3, 5\}. \end{cases}$$

Then $(X, f)$ is an interior GE-algebra and $F := \{1, 2\}$ is a prominent interior GE-filter of type 1 in $(X, f)$. But it is not an interior GE-filter in $(X, f)$ since $2 \ast 4 = 1$ and $2 \in F$ but $4 \notin F$.

The following example shows that interior GE-filter and prominent interior GE-filter of type 2 are independent of each other.

**Example 3.26.** (1). Let $X = \{1, 2, 3, 4, 5\}$ be a set with the Cayley table which is given in the following Table 16,
Table 16. Cayley table for the binary operation “∗”.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
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<td>1</td>
<td>1</td>
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<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

and define a mapping \( f \) on \( X \) as follows:

\[
f : X \to X, \quad x \mapsto \begin{cases} 
1 & \text{if } x \in \{1, 4\} \\
2 & \text{if } x = 2, \\
3 & \text{if } x = 3, \\
5 & \text{if } x = 5.
\end{cases}
\]

Then \((X, f)\) is an interior GE-algebra and \( F := \{1, 4\} \) is an interior GE-filter in \((X, f)\). But \( F \) is not prominent interior GE-filter of type 2 since \( 4 \ast (2 \ast f(3)) = 4 \ast (2 \ast 3) = 4 \ast 1 = 1 \in F \) and \( f(4) = 1 \in F \) but \((3 \ast f(2)) \ast f(2)) \ast 3 = ((3 \ast 2) \ast f(2)) \ast 3 = (2 \ast 2) \ast 3 = 1 \ast 3 = 3 \notin F \).

(2). Let \( X = \{1, 2, 3, 4, 5\} \) be a set with the Cayley table which is given in the following Table 17,

Table 17. Cayley table for the binary operation “∗”.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
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<tbody>
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<tr>
<td>4</td>
<td>1</td>
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</table>

and define a mapping \( f \) on \( X \) as follows:

\[
f : X \to X, \quad x \mapsto \begin{cases} 
1 & \text{if } x = 1, \\
3 & \text{if } x \in \{2, 3, 4, 5\}.
\end{cases}
\]

Then \((X, f)\) is an interior GE-algebra and \( F := \{1, 2, 5\} \) is a prominent interior GE-filter of type 2 in \((X, f)\). But it is not an interior GE-filter in \((X, f)\) since \( 5 \ast 4 = 1 \) and \( 5 \in F \) but \( 4 \notin F \).

Before we conclude this paper, we raise the following question.

**Question.** Let \((X, f)\) be an interior GE-algebra. Let \( F \) and \( G \) be interior GE-filters in \((X, f)\). If \( F \subseteq G \) and \( F \) is a prominent interior GE-filter of type 1 (resp., type 2) in \((X, f)\), then is \( G \) also a prominent interior GE-filter of type 1 (resp., type 2) in \((X, f)\)?

### 4. Conclusions

We have introduced the concept of a prominent interior GE-filter (of type 1 and type 2), and have investigated their properties. We have discussed the relationship between a prominent GE-filter and a
prominent interior GE-filter and the relationship between an interior GE-filter and a prominent interior GE-filter. We have found and provide examples where any interior GE-filter is not a prominent interior GE-filter and any prominent GE-filter is not a prominent interior GE-filter. We have provided conditions for an interior GE-filter to be a prominent interior GE-filter. We have given conditions under which an internal GE-filter larger than a given internal GE filter can become a prominent internal GE-filter, and have provided an example describing it. We have considered the relationship between a prominent interior GE-filter and a prominent interior GE-filter of type 1. We have found and provide examples to verify that a prominent interior GE-filter of type 1 and a prominent interior GE-filter of type 2, a prominent interior GE-filter and a prominent interior GE-filter of type 2, an interior GE-filter and a prominent interior GE-filter of type 1, and an interior GE-filter and a prominent interior GE-filter of type 2 are independent each other. In future, we will study the prime and maximal prominent interior GE-filters and their topological properties. Moreover, based on the ideas and results obtained in this paper, we will study the interior operator theory in related algebraic systems such as MV-algebra, BL-algebra, EQ-algebra, etc. It will also be used for pseudo algebra systems and further to conduct research related to the very true operator theory.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

References


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