Research article

Existence solution of a system of differential equations using generalized Darbo's fixed point theorem

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Abstract: In this paper, we proposed a generalized of Darbo's fixed point theorem via the concept of operators $S(\bullet; .)$ associated with the measure of noncompactness. Using this generalized Darbo fixed point theorem, we have given the existence of solution of a system of differential equations. At the end, we have given an example which supports our findings.

1. Introduction

The measure of noncompactness (MNC) performs an important character in real world problems. First of all, the fundamental paper of Kuratowski [1] in 1930 open up a new direction of MNC to solve diffent type of Functional equations, which comes from the diffent real life problems. Using the notion of MNC, Darbo [2] in 1955 ensure that the endurance of fixed points, which is obtained by the generalization of Schauder fixed point theorem (SFPT) and banach contraction principle. Many authors using the notion of MNC generalize Darbo fixed point theorem (DFPT) which ensure that the endurance of fixed point to solve various kind of integal or differentail equations. Up to now, many authors have been published several papers using the notion of generalization of DFPT and MNC [3–14].

Our purpose of present paper is to extend the DFPT and we aaply our obtained results to find the existence of solutions of the functional differential equations.

At the beginning we provide concepts, notations, definitions and the preliminaries, which will be used all over the present paper.

2. Definition and preliminaries

The set of real numbers is symbolize by $\mathbb{R}$, $\mathbb{R}_+ = [0, \infty)$ and the set of natural numbers by $\mathbb{N}$. Let $(\Xi, \| \ . \ \|)$ be real Banach spaces. If Ω is a nonempty subset of Ξ then $\bar{\Omega}$ and $Conv\Omega$, symbolize the closure and convex closure of Ω respectively. Also, let $\mathcal{M}_\Xi$ symbolize the set of all nonempty and bounded subsets of Ξ and $\mathcal{N}_\Xi$ is the subset of all relatively compact sets.

Banas and Lecko [15] have given the definition of MNC which is given below.

Definition 2.1. A MNC is a mapping $\chi : \mathcal{M}_\Xi \to \mathbb{R}_+$ if it fulfills the following constraints for all $\Omega, \ \Omega_1, \ \Omega_2 \in \mathcal{M}_\Xi$.

(M_1) The family $\ker \chi = \{\Omega \in \mathcal{M}_\Xi : \chi(\Omega) = 0\}$ is nonempty and $\ker \chi \subset \mathcal{N}_\Xi$.

(M_2) $\Omega_1 \subset \Omega_2 \implies \chi(\Omega_1) \leq \chi(\Omega_2)$.

(M_3) $\chi(\bar{\Omega}) = \chi(\Omega)$.

(M_4) $\chi(Conv\Omega) = \chi(\Omega)$.

(M_5) $\chi(\kappa\Omega_1 + (1 - \kappa)\Omega_2) \leq \kappa\chi(\Omega_1) + (1 - \kappa)\chi(\Omega_2)$ for $\kappa \in [0, 1]$.

(M_6) if $\Omega_n \in \mathcal{M}_\Xi$, $\Omega_n = \bar{\Omega}_n$, $\Omega_{n+1} \subset \Omega_n$ for $n = 1, 2, 3, ...$ and $\lim_{n \to \infty} \chi(\Omega_n) = 0$, then $\bigcap_{n=1}^{\infty} \Omega_n \neq \emptyset$.

We are going to define the Concept of operator $S(\bullet; .)$ which was introduced by Altan and Turkoglu [16].

Definition 2.2. Let $A(\mathbb{R}_+)$ be the set of fuctions $f : \mathbb{R}_+ \to \mathbb{R}_+$ and let Z be the set of functions $S(\bullet; .) : A(\mathbb{R}_+) \to A(\mathbb{R}_+)$, which fulfills the following constraints:

(O_1) $S(f; \sigma) \geq 0$ for $\sigma > 0$ and $S(h; 0) = 0$,

(O_2) $S(f; \sigma_1) \leq S(f; \sigma_2)$ for $\sigma_1 \leq \sigma_2$,

(O_3) $\lim_{n \to \infty} S(f; \sigma_n) = S(f; \lim_{n \to \infty} \sigma_n)$,

(O_4) $S(f; \max\{\sigma_1, \sigma_2\}) = \max\{S(f; \sigma_1), S(f; \sigma_2)\}$ for some $f \in A(\mathbb{R}_+)$.

Theorem 2.1. *(Schauder) [17] A mapping $\Delta : \Omega \to \Omega$ which is compact and continuous has at least one fixed point for a nonempty, bounded, closed and convex (NBCC) subset Ω of a Banach space Ξ.*

DFPT is generalize by resting the compactness of Schauder's mapping and theorem is known as SFPT.

Theorem 2.2. *(Darbo) [18] Let $\Delta : \Omega \to \Omega$ be a continuous mapping and χ is an MNC. Then for any nonempty subset $\wp$ of Ω, there exists a $k \in [0, 1)$ having the inequality*

$$\chi(\Delta\wp) < k\chi(\wp).$$

Then the mapping Δ have a fixed point in Ω.

Isik et al. [10] introduce a function f to generalize the Banach contraction, we find various type of contraction mapping.

Theorem 2.3. *Let $\Delta : \Omega \to \Omega$ be a continuous self-mapping, where (Ω, ρ) is a complete metric space. Then for all $\gamma, \delta \in \Xi$ there exists a mapping $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\lim_{\tau \to 0_+} f(\tau) = 0$, $f(0) = 0$,*

$$\rho(\Delta\gamma, \Delta\delta) \leq f(\rho(\gamma, \delta)) - f(\rho(\Delta\gamma, \Delta\delta)).$$

Then Δ contains a unique fixed point.

Parvenah et al. [10] generalized DFPT as follows:

Theorem 2.4. *Let $\Delta : \Omega \to \Omega$ be a continuous operator defined on a NBCC subet Ω of Ξ having the inequality*

$$\chi(\Delta\wp) \leq f(\chi(\wp)) - f(\chi(\Delta\wp)),$$

for all $\wp \subset \Omega$, where $f : \mathbb{R}_+ \to \mathbb{R}_+$ with $\lim_{\tau \to 0_+} f(\tau) = 0$, $f(0) = 0$, and χ is an MNC. Then Δ contains a fixed point in Ω.

Remark 2.1. Remember that Theorem 2.4 generalize DFPT. Since $\Delta : \wp \to \wp$ is a Darbo mapping.
Then for all $\wp \subset \Xi$ there exists $k \in [0, 1)$ having the property $\chi(\Delta\wp) < k\chi(\wp)$.
So with the help of inequality, we have

$$\chi(\Delta\wp) \leq k\chi(\wp) \leq \frac{k}{1 + k - \sqrt{k}}\chi(\wp),$$

for all $\wp \subset \Xi$.
Consequently

$$k\chi(\Delta\wp) + (1 - \sqrt{k})\chi(\Delta\wp) \leq k\chi(\wp),$$
$$(1 - \sqrt{k})\chi(\Delta\wp) \leq k\chi(\wp) - k\chi(\Delta\wp).$$

So

$$\chi(\Delta\wp) \leq \frac{k}{1 - \sqrt{k}}\chi(\wp) - \frac{k}{1 - \sqrt{k}}\chi(\Delta\wp).$$

Taking $f(\tau) = \frac{k}{1 - \sqrt{k}}\tau$, we have $\chi(\Delta\wp) \leq f(\chi(\wp)) - f(\chi(\Delta\wp))$ for all $\wp \subset \Xi$. Therefore the Darbo Theorem is a specific case of contraction mapping of Theorem (2.4).

3. Main results

Let us recall an important theorem in this work which extends DFPT by taking the concept of $S(h; .)$.

Theorem 3.1. *Let $(\Xi, \| . \|)$ be a Banach space. Suppose $\Delta : \Xi \to \Xi$ is a continuous, nondecreasing and bounded mapping fulfills the following inequality*

$$S\left(h; \int_0^{\chi(\Delta\wp)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\Delta\wp)} \pi(\tau)d\tau\right)\right) \leq f\left(S\left(h; \int_0^{\chi(\wp)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\wp)} \pi(\tau)d\tau\right)\right)\right) - f\left(S\left(h; \int_0^{\chi(\Delta\wp)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\Delta\wp)} \pi(\tau)d\tau\right)\right)\right),$$

$$(3.1)$$

for all bounded $\wp$ of Ξ, where χ is MNC, $h \in A(\mathbb{R}_+)$, $S(\bullet;.) \in Z$, $\phi, \pi : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous functions and $f : \mathbb{R}_+ \to \mathbb{R}_+$ is a function as $\lim_{\tau \to 0_+} f(\tau) = 0$, $f(0) = 0$. Then Δ contains at least one fixed point.

Proof. Assume that $\wp_n$ with $\wp_0 = \wp$ and $\wp_{n+1} = conv(\Delta \wp_n)$ for all $n \geq 0$.

Also, $\Delta \wp_0 = \Delta \wp \subseteq \wp = \wp_0$, $\wp_1 = conv(\Delta \wp_0) \subseteq \wp = \wp_0$. Since $\wp_n$ is a closed and bounded subset in Ξ and

$$\wp_0 \supset \wp_1 \supset, ..., \supset \wp_n \supset, ... \tag{3.2}$$

Following (3.1), we have

$$S\left(h; \int_0^{\chi(\wp_{n+1})} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\wp_{n+1})} \pi(\tau)d\tau\right)\right) = S\left(h; \int_0^{\chi(conv(\Delta \wp_n))} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(conv(\Delta \wp_n))} \pi(\tau)d\tau\right)\right),$$

$$S = \left(h; \int_0^{\chi(\Delta \wp_n)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\Delta \wp_n)} \pi(\tau)d\tau\right)\right)$$

$$\leq f\left(S\left(h; \int_0^{\chi(\wp_n)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\wp_n)} \pi(\tau)d\tau\right)\right)\right) - f\left(S\left(h; \int_0^{\chi(\Delta \wp_n)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\Delta \wp_n)} \pi(\tau)d\tau\right)\right)\right).$$

Taking the limit as $n \to \infty$ on both the sides of this inequality, we have

$$\lim_{n \to \infty} S\left(h; \int_0^{\chi(\Delta \wp_n)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\Delta \wp_n)} \pi(\tau)d\tau\right)\right)$$

$$\leq \lim_{n \to \infty} f\left(S\left(h; \int_0^{\chi(\wp_n)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\wp_n)} \pi(\tau)d\tau\right)\right)\right) - \lim_{n \to \infty} f\left(S\left(h; \int_0^{\chi(\Delta \wp_n)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\Delta \wp_n)} \pi(\tau)d\tau\right)\right)\right).$$

Therefore

$$\lim_{n \to \infty} S\left(h; \int_0^{\chi(\wp_n)} \pi(\tau)d\tau + \phi\left(\int_0^{\chi(\wp_n)} \pi(\tau)d\tau\right)\right) = 0.$$

By the virtue of (*iii*) of Definition $S(h;.)$, we get

$$S\left(h; \lim_{n \to \infty} \int_0^{\chi(\wp_n)} \pi(\tau)d\tau + \lim_{n \to \infty} \phi\left(\int_0^{\chi(\wp_n)} \pi(\tau)d\tau\right)\right) = 0,$$

and therefore $\lim_{n \to \infty} \int_0^{\chi(\wp_n)} \pi(\tau)d\tau = 0$.

But for any $\epsilon > 0$, $\int_0^{\epsilon} \pi(\tau)d\tau > 0$, then $\chi(\wp_n) \to 0$ *as* $n \to \infty$.

Now since $\wp_n$ is nested sequence, by the definition of (MNC) of (M_6), we conclude that $\wp_\infty = \cap_{n=1}^\infty \wp_n$ is NBCC of Ξ. Also we aware that $\wp_\infty \in ker\chi$. Therefore $\wp_\infty$ is compact and invariant under the mapping Δ. Therefore by the SFPT, Δ has a fixed point in $\wp_\infty$. $\qquad\qquad\square$

Remark 3.1. Putting $\pi(\tau) = 1$ for $\tau \in [0, \infty)$ in Theorem 3.1, we have
$S(h; \chi(\Delta\wp) + \phi(\chi(\Delta\wp))) \le f(S(h; \chi(\wp) + \phi(\chi(\wp)))) - f(S(h; \chi(\Delta\wp) + \phi(\chi(\Delta\wp))))$.

Remark 3.2. Take $\phi = 0$, $S(h; \tau) = \tau$, $h = I$ in Remark(3.1), then we have

$$\chi(\Delta\wp) \le f(\chi(\wp)) - f(\chi(\Delta\wp)).$$

It is a generalization of the result given by Parvenah et al. [10].

Definition 3.1. [19] A mapping $\Delta : \Xi \times \Xi \times \Xi \to \Xi$ is said to have a TFP $(\gamma, \delta, \theta) \in \Xi^3$ if

$$\Delta(\gamma, \delta, \theta) = \gamma, \qquad \Delta(\gamma, \delta, \theta) = \delta, \qquad \Delta(\gamma, \delta, \theta) = \theta.$$

Theorem 3.2. *[18] Let $\chi_1, \chi_2, ..., \chi_n$ be the measure of noncompactness of $\Xi_1, \Xi_2, ..., \Xi_n$ respectively. Also assume that the function $\mathcal{B} : \mathbb{R}_+{}^\tau \to \mathbb{R}_+$ is convex and $\mathcal{B}(\gamma_1, \gamma_2, ..., \gamma_\tau) = 0$ if and only if $\gamma_r = 0$ for $r = 1, 2, ..., \tau$. Then*
$$\hat{\chi}(\Theta) = \mathcal{B}(\chi_1(\Theta_1), \chi_2(\Theta_2), ..., \chi_n(\Theta_n)).$$

Example 3.1. [20] Let $\mathcal{B}(\gamma, \delta, \theta) = \max\{\gamma, \delta, \theta\}$ for $(\gamma, \delta, \theta) \in \mathbb{R}_+{}^3$. Now $\mathcal{B}(\gamma, \delta, \theta) = \max\{\gamma, \delta, \theta\} = 0$ iff $\gamma = \delta = \theta = 0$. Then $\mathcal{B}$ is convex and satisfied all conditions of Theorem 3.2. Therefore $\hat{\chi}(\Theta) = \mathcal{B}(\chi_1(\Theta_1), \chi_2(\Theta_2), \chi_3(\Theta_3))$ is an MNC on $\Xi_1 \times \Xi_2 \times \Xi_3$, where χ be an MNC in Ξ and Θ_j is the natural projections of Z into Ξ_j for $j = 1, 2, 3$.

Example 3.2. [20] Let $\mathcal{B}(\gamma, \delta, \theta) = \gamma + \delta + \theta$ for $(\gamma, \delta, \theta) \in \mathbb{R}_+{}^3$. Now $\mathcal{B}(\gamma, \delta, \theta) = \gamma + \delta + \theta = 0$ iff $\gamma = \delta = \theta = 0$. Then $\mathcal{B}$ is convex and satisfied all conditions of Theorem 3.2. Therefore $\hat{\chi}(\Theta) = \mathcal{B}(\chi_1(\Theta_1), \chi_2(\Theta_2), \chi_3(\Theta_3))$ is an MNC on $\Xi_1 \times \Xi_2 \times \Xi_3$, where χ be an MNC in Ξ and X_j is the natural projections of Z into Ξ_j for $j = 1, 2, 3$.

Theorem 3.3. *Let C be a NBCC subset of a Banach space Ξ and let $\Delta : C \times C \times C \to C$ be a continuous mapping such that*

$$S(f; \chi(\Delta(\Theta_1 \times \Theta_2 \times \Theta_3))) = \omega[S(f; \chi(\Theta_1) + \chi(\Theta_2) + \chi(\Theta_3))] - \omega[S(f; \chi(\Delta\Theta_1) + \chi(\Delta\Theta_2) + \chi(\Delta\Theta_3))],$$

for all $\Theta_1, \Theta_2, \Theta_3 \in C$, χ is MNC and $\omega : [0, \infty) \to [0, \infty)$ is such that $\lim_{\tau \to 0_+} \omega(\tau) = 0$, $\omega(0) = 0$. Also $S(f; .) \in Z$ and $S(f; \tau_1 + \tau_2 + \tau_3) = S(f; \tau_1) + S(f; \tau_2) + S(f; \tau_3)$ for all $\tau_1, \tau_2, \tau_3 \ge 0$. Δ has at least a triple fixed point.

Proof. We define a mapping $\hat{\Delta} : C_3 \to C_3$ by
$\hat{\Delta}(\gamma, \delta, \theta) = (\Delta(\gamma, \delta, \theta), \Delta(\delta, \gamma, \theta), \Delta(\theta, \delta, \gamma))$ for all $(\gamma, \delta, \theta) \in C_3$.
$\hat{\Delta}$ is continuous, since Δ is continuous.
We know that $\hat{\chi}(\Theta) = \chi(\Theta_1) + \chi(\Theta_2) + \chi(\Theta_3)$,
where $\Theta_1, \Theta_2, \Theta_3$ denotes the natural projections of C. Suppose $\Theta \subset C^3$ be a nonempty subset.
Now using the Theorem 3.3, we get

$$S(f; \hat{\chi}(\hat{\Delta}\Theta)) \le S(f; \hat{\chi}(\Delta(\Theta_1 \times \Theta_2 \times \Theta_3) \times \Delta(\Theta_2 \times \Theta_1 \times \Theta_3) \times \Delta(\Theta_3 \times \Theta_2 \times \Theta_1)))$$

$$\leq S(f; \chi(\Delta(\Theta_1 \times \Theta_2 \times \Theta_3) + S(f; \chi(\Delta(\Theta_2 \times \Theta_1 \times \Theta_3))) + S(f; \chi(\Delta(\Theta_3 \times \Theta_2 \times \Theta_1))$$

$$\leq \omega(S(f; \chi(\Theta_1 \times \Theta_2 \times \Theta_3))) - \omega(S(f; \chi(\Delta(\Theta_1 \times \Theta_2 \times \Theta_3))))$$

$$+\omega(S(f; \chi(\Theta_2 \times \Theta_1 \times \Theta_3))) - \omega(S(f; \chi(\Delta(\Theta_2 \times \Theta_1 \times \Theta_3))))$$

$$+\omega(S(f; \chi(\Theta_3 \times \Theta_2 \times \Theta_1))) - \omega(S(f; \chi(\Delta(\Theta_3 \times \Theta_2 \times \Theta_1))))$$

$$\leq [\omega(S(f; \chi(\Theta_1) + \chi(\Theta_2) + \chi(\Theta_3))) - \omega(S(f; \chi(\Delta\Theta_1) + \chi(\Delta\Theta_2) + \chi(\Delta\Theta_3)))]$$

$$+[\omega(S(f; \chi(\Theta_2) + \chi(\Theta_1) + \chi(\Theta_3))) - \omega(S(f; \chi(\Delta\Theta_2) + \chi(\Delta\Theta_1) + \chi(\Delta\Theta_3)))]$$

$$+[\omega(S(f; \chi(\Theta_3) + \chi(\Theta_2) + \chi(\Theta_1))) - \omega(S(f; \chi(\Delta\Theta_3) + \chi(\Delta\Theta_2) + \chi(\Delta\Theta_1)))]$$

$$\leq 3\omega(S(f; \hat{\chi}(\hat{\Theta}))) - 3\omega(S(f; \hat{\chi}(\hat{\Delta}\Theta))).$$

Putting $\omega = \frac{1}{3}\nu$, we have

$$S(f; \hat{\chi}(\hat{\Delta}\Theta)) \leq \nu(S(f; \hat{\chi}(\Theta))) - \nu(S(f; \hat{\chi}(\hat{\Delta}\Theta))).$$

Now from the Theorem 3.1, we conclude that Δ has at least a triple fixed point. □

Remark 3.3. By taking $S(f; \tau) = \tau$, $\nu(\tau) = \tau$, $f = I$ in Theorem 3.3, we get the corollary which is given below.

Corollary 1. Let $\Delta : C \times C \times C \to C$ be a continuous function defined on a NBCC subset C of Ξ in such a way that

$$\chi(\Delta(\Theta_1 \times \Theta_2 \times \Theta_3)) \leq \frac{1}{2}[\chi(\Theta_1) + \chi(\Theta_2) + \chi(\Theta_3)].$$

Then Δ has a TFP.

4. Applications

This section contains the applicability of Theorem 3.1 and Corollary 1 by using the system of equations which is defined as

$$\begin{aligned}
\xi\prime(\gamma) &= h(\gamma, \xi(\zeta(\tau)), \nu(\zeta(\tau)), w(\zeta(\tau)), \xi\prime(\eta(\tau)), \nu\prime(\eta(\tau)), w\prime(\eta(\tau))), \\
\nu\prime(\gamma) &= h(\gamma, \nu(\zeta(\tau)), w(\zeta(\tau)), \xi(\zeta(\tau)), \nu\prime(\eta(\tau)), w\prime(\eta(\tau)), \xi\prime(\eta(\tau))), \\
w\prime(\gamma) &= h(\gamma, w(\zeta(\tau)), \xi(\zeta(\tau)), \nu(\zeta(\tau)), w\prime(\eta(\tau)), \xi\prime(\eta(\tau)), \nu\prime(\eta(\tau))),
\end{aligned} \tag{4.1}$$

where $\gamma \in [0, T]$ with the initial state $\xi(0) = \xi_0$, $\nu(0) = \nu_0$ and $w(0) = w_0$.

Suppose that the space of all bounded continuous function defined on $[0, T]$ is $C[0, T]$ equipped with the standard norm

$$\|\gamma\| = \sup\{|\gamma(\tau)| : \quad \tau \in [0, T]\}.$$

A function having Modulus of contiunity for $\gamma \in [0, T]$ is defined as

$$\omega(\gamma, \epsilon) = \sup\{|\gamma(\tau_1) - \gamma(\tau_2)| : \quad \tau_1, \tau_2 \in [0, T], |\tau_1 - \tau_2| \leq \epsilon\},$$

$\omega(\gamma, \epsilon) \to 0$ as $\epsilon \to 0$, because γ is continuously uniform on $[0, T]$. The Hausdorff MNC for every bounded subset $\wp$ of $C[0, T]$ is

$$\mu(\wp) = \lim_{\epsilon \to 0} \left\{ \sup_{\gamma \in \Theta} \omega(\gamma, \epsilon) \right\}.$$

Now, we construct the assumptions by which the system of integral Eq (4.1) will be studied.

(i) $\zeta, \eta : [0, T] \to [0, T]$ are the functions which are continuous.

(ii) For a continuous function $h : [0, T] \times \mathbb{R}^6 \to \mathbb{R}$ there exists a continuous function $\phi : \mathbb{R}_+ \to \mathbb{R}_+$ with $\phi(0) = 0$ *and* $\phi(\tau) < \tau$ for all $\tau > 0$ and also satisfy

$$S\left(f; |h(\tau, \gamma_1, ..., \gamma_6) - h(\tau, \delta_1, ..., \delta_6)|\right)$$
$$\leq \phi\left(S(f; \max_{1 \leq i \leq 3}\{|\gamma_i - \delta_i|\})\right) + \frac{1}{2}S\left(f; \max\{|\gamma_4 - \delta_4|, |\gamma_5 - \delta_5|, |\gamma_6 - \delta_6|\}\right).$$

(iii) $M = \sup\{S(f; |h(\tau, \xi_0, v_0, z_0, 0, 0)|)\} < \infty$, where $\tau \in [0, T]$ and $S(f; \epsilon) < \epsilon$.

(iv) There exists r_0 such that

$$\phi(S(f; \Delta r_0)) + \frac{1}{2}S(f; 3r_0) + M \leq r_0.$$

Theorem 4.1. *The system (4.1) with the assumptions* $(i) - (iv)$ *has at least one solution which belongs to the space* $\{C[0, T]\}^3$.

Proof. Assume that $\mathcal{U}(\tau) = \xi\prime(\tau), \; \mathcal{V}(\tau) = v\prime(\tau), \; \mathcal{W}(\tau) = w\prime(\tau)$. Then our system of Eq (4.1) can be written as the system of integral eqautions

$$\mathcal{U}(\tau) = h\left(\tau, \xi_0 + \int_0^{\zeta(\tau)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau)} w(\varrho)d\varrho, \xi(\eta(\tau)), v(\eta(\tau)), w(\eta(\tau))\right),$$

$$\mathcal{V}(\tau) = h\left(\tau, v_0 + \int_0^{\zeta(\tau)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau)} w(\varrho)d\varrho, \xi_0 + \int_0^{\zeta(\tau)} \xi(\varrho)d\varrho, v(\eta(\tau)), w(\eta(\tau)), \xi(\eta(\tau))\right), \qquad (4.2)$$

$$\mathcal{W}(\tau) = h\left(\tau, w_0 + \int_0^{\zeta(\tau)} w(\varrho)d\varrho, \xi_0 + \int_0^{\zeta(\tau)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau)} v(\varrho)d\varrho, w(\eta(\tau)), \xi(\eta(\tau)), v(\eta(\tau))\right),$$

where $\tau \in [0, T]$.

Assume $\Delta : C[0, T] \to C[0, T]$ be a operator with

$$\Delta(\xi, v, w)(\tau) = h\left(\tau, \xi_0 + \int_0^{\zeta(\tau)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau)} w(\varrho)d\varrho, \xi(\eta(\tau)), v(\eta(\tau)), w(\eta(\tau))\right).$$

We notice for every $\tau \in C[0, T]$, the mapping Δ is continuous i.e Δ maps the space $C[0, T]$ into itself.

For fixed arbitrary $\tau \in C[0, T]$ and $f \in \mathcal{F}([0, \infty))$, we have from the assumptions $(i) - (iv)$,

$$S\left(f; |\Delta(\xi, v, w)(\tau)|\right)$$

$$= S\left(f; \left|h\left(\tau, \xi_0 + \int_0^{\zeta(\tau)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau)} w(\varrho)d\varrho, \xi(\eta(\tau)), v(\eta(\tau)), w(\eta(\tau))\right)\right|\right)$$

$$\leq S\left(f; \left|h\left(\tau, \xi_0 + \int_0^{\zeta(\tau)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau)} w(\varrho)d\varrho, \xi(\eta(\tau)), v(\eta(\tau)), w(\eta(\tau))\right)\right.\right.$$

$$- h(\tau, \xi_0, v_0, w_0, 0, 0, 0) \Big| \Big) + S\left(f; |h(\tau, \xi_0, v_0, w_0, 0, 0, 0)|\right)$$

$$\leq \phi\left(S\left(f; \max\left\{\left|\int_0^{\zeta(\tau)} \xi(\varrho)d\varrho\right|, \left|\int_0^{\zeta(\tau)} v(\varrho)d\varrho\right|, \left|\int_0^{\zeta(\tau)} w(\varrho)d\varrho\right|\right\}\right)\right)$$

$$+ \frac{1}{2}S\left(f; |\xi(\eta(\tau))| + |v(\eta(\tau))| + |w(\eta(\tau))|\right) + S\left(f; |h(\tau, \xi_0, v_0, w_0, 0, 0, 0)|\right)$$

$$\leq \phi\left(S\left(f; \Delta \max\{\|\xi\|, \|v\|, \|w\|\}\right)\right) + \frac{1}{2}S\left(f; \|\xi\| + \|v\| + \|w\|\right) + M.$$

Thus

$$S\left(f; \|\Delta(\xi, v, w)(\tau)\|\right) \leq \phi\left(S\left(f; \Delta \max\{\|\xi\|, \|v\|, \|w\|\}\right) + \frac{1}{2}S\left(f; \|\xi\| + \|v\| + \|w\|\right)\right) + M,$$

and

$$\Delta(\xi, v, w) \in C[0, T].$$

Due to the inequality $\phi\left(S\left(f; \Delta r_0\right)\right) + \frac{1}{2}S\left(f; 3r_0\right) + M \leq r_0$, the function Δ maps $(B_{r_0})^3$ into (B_{r_0}).
Now we prove that Δ is continuous on $(B_{r_0})^3$.
Let fixed arbitrary $\epsilon > 0$ and take $(\gamma, \delta, \theta), (\xi, v, w) \in (B_{r_0})^3$ such that

$$\max\{\|\gamma - \xi\|, \|\delta - v\|, \|\theta - w\|\} < \epsilon.$$

Therefore for every $t \in [0, T]$, we get

$$S\left(f; |\Delta(\gamma, \delta, \theta)(\tau) - \Delta(\xi, v, w)(\tau)|\right)$$

$$\leq S\left(f; \left|h\left(\tau, x_0 + \int_0^{\zeta(\tau)} x(\varrho)d\varrho, y_0 + \int_0^{\zeta(\tau)} y(\varrho)d\varrho, z_0 + \int_0^{\zeta(\tau)} z(\varrho)d\varrho, x(\eta(\tau)), y(\eta(\tau)), z(\eta(\tau))\right)\right|\right)$$

$$- S\left(f; \left|h\left(\tau, \xi_0 + \int_0^{\zeta(\tau)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau)} w(\varrho)d\varrho, \xi(\eta(\tau)), v(\eta(\tau)), w(\eta(\tau))\right)\right|\right)$$

$$\leq \phi\left(S\left(f; T \max\left\{|x_0 - \xi_0| + \int_0^{\zeta(\tau)} |x(s) - \xi(\varrho)d\varrho, |y_0 - v_0| + \int_0^{\zeta(\tau)} |y(s) - v(\varrho)d\varrho, |z_0 - w_0| + \int_0^{\zeta(\tau)} |z(s) - w(s)|ds\right\}\right)\right)$$

$$+ \frac{1}{2}S\left(f; \max\{|x(\eta(\tau)) - \xi(\eta(\tau))|, |y(\eta(\tau)) - v(\eta(\tau))|, |z(\eta(\tau)) - w(\eta(\tau))|\}\right)$$

$$\leq \phi\left(S\left(f; \epsilon + \Delta\epsilon\right)\right) + \frac{1}{2}S\left(f; \epsilon\right).$$

Thus, we have $\phi\left(S\left(f; \epsilon + \Delta\epsilon\right)\right) + \frac{1}{2}S\left(f; \epsilon\right) \to 0$ as $\epsilon \to 0$.
Therefore Δ is a continuous function on $(B_{r_0})^3$. Now, we shall show that Δ satisfy all the conditions of Corollary 1. To do this, let $\mathcal{U}$, $\mathcal{V}$ and W are nonempty and bounded subsets of (B_{r_0}) and $\epsilon > 0$ is

constant. Moreover we take $\tau_1, \tau_2 \in [0, T]$ with $|\tau_2 - \tau_1| \le \epsilon$ and $\xi \in \mathcal{U}, \quad v \in \mathcal{V} \quad and \quad w \in W$.

Then we have

$$
S\left(f; |\Delta(\gamma, \delta, \theta)(\tau) - \Delta(\xi, v, w)(\tau)| \right)
$$

$$
= S\Bigg(f; \Bigg| h\left(\tau_1, \xi_0 + \int_0^{\zeta(\tau_1)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau_1)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau_1)} w(\varrho)d\varrho, \xi(\eta(\tau_1)), v(\eta(\tau_1)), w(\eta(\tau_1)))\right.
$$

$$
\left. - h\left(\tau_1, \xi_0 + \int_0^{\zeta(\tau_1)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau_1)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau_1)} w(\varrho)d\varrho, \xi(\eta(\tau_2)), v(\eta(\tau_2)), w(\eta(\tau_2)))\right| \Bigg)
$$

$$
+ S\Bigg(f; \Bigg| h\left(\tau_1, \xi_0 + \int_0^{\zeta(\tau_1)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau_1)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau_1)} w(\varrho)d\varrho, \xi(\eta(\tau_2)), v(\eta(\tau_2)), w(\eta(t_2)))\right.
$$

$$
\left. - h\left(\tau_2, \xi_0 + \int_0^{\zeta(\tau_1)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau_1)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau_1)} w(\varrho)d\varrho, \xi(\eta(\tau_2)), v(\eta(\tau_2)), w(\eta(\tau_2)))\right| \Bigg)
$$

$$
+ S\Bigg(f; \Bigg| h\left(\tau_2, \xi_0 + \int_0^{\zeta(\tau_1)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau_1)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau_1)} w(\varrho)d\varrho, \xi(\eta(\tau_2)), v(\eta(\tau_2)), w(\eta(\tau_2)))\right.
$$

$$
\left. - h\left(\tau_2, \xi_0 + \int_0^{\zeta(\tau_1)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau_1)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau_1)} w(\varrho)d\varrho, \xi(\eta(\tau_2)), v(\eta(\tau_2)), w(\eta(\tau_2)))\right| \Bigg)
$$

$$
+ S\Bigg(f; \Bigg| h\left(\tau_2, \xi_0 + \int_0^{\zeta(\tau_1)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau_1)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau_1)} w(\varrho)d\varrho, \xi(\eta(\tau_2)), v(\eta(\tau_2)), w(\eta(\tau_2)))\right.
$$

$$
\left. - h\left(\tau_2, \xi_0 + \int_0^{\zeta(\tau_1)} \xi(\varrho)d\varrho, v_0 + \int_0^{\zeta(\tau_1)} v(\varrho)d\varrho, w_0 + \int_0^{\zeta(\tau_1)} w(\varrho)d\varrho, \xi(\eta(\tau_2)), v(\eta(\tau_2)), w(\eta(\tau_2)))\right| \Bigg)
$$

$$
\le \frac{1}{2} S\left(f; \max \{|\xi(\eta(\tau_1)) - \xi(\eta(\tau_2))|, |v(\eta(\tau_1)) - v(\eta(\tau_2))|, |w(\eta(\tau_1)) - w(\eta(t_2))|\} \right)
$$

$$
+ S(f; \omega(h, \epsilon)) + \phi\Bigg(S\left(f; \max \left\{ \int_{\zeta(\tau_1)}^{\zeta(\tau_2)} |\xi(\varrho)|d\varrho, \int_{\zeta(\tau_1)}^{\zeta(\tau_2)} |v(\varrho)|d\varrho, \int_{\zeta(\tau_1)}^{\zeta(\tau_2)} |w(\varrho)|d\varrho \right\} \right) \Bigg)
$$

$$
\le \frac{1}{2} S\left(f; \max \{\omega(\xi, \omega(\eta, \epsilon)), \omega(v, \omega(\eta, \epsilon)), \omega(w, \omega(\eta, \epsilon))\} \right) + S(f; \omega(h, \epsilon)) + \phi(S(f; \max r_0\omega(\zeta, \epsilon))),
$$

where $\omega(\eta, \epsilon) = \sup \{|\eta(\tau_2) - \eta(\tau_1)| : \quad |\tau_1 - \tau_2| \le \epsilon \quad, \tau_1, \tau_2 \in [0, T]\}$,
$\omega(\zeta, \epsilon) = \sup \{|\zeta(\tau_2) - \zeta(\tau_1)| : \quad |\tau_1 - \tau_2| \le \epsilon \quad, \tau_1, \tau_2 \in [0, T]\}$,
$\omega(\xi, \omega(\eta, \epsilon)) = \sup \{|\xi(\tau_2) - \xi(\tau_1)| : \quad |\tau_1 - \tau_2| \le \omega(\eta, \epsilon) \quad, \tau_1, \tau_2 \in [0, \eta(T)]\}$,
$\omega(h, \epsilon) = \sup \{|h(\tau_1, \gamma_1, ..., \gamma_6) - h(\tau_2, \gamma_1, ..., \gamma_6)| : \quad |\tau_1 - \tau_2| \le \epsilon \quad, \tau_1, \tau_2 \in [0, T]\}$,

and $\gamma_1, ..., \gamma_6 \in [-r_0, r_0]$.

We infer that

$$S\left(f; |\Delta(\gamma, \delta, \theta)(\tau) - \Delta(\xi, v, w)(\tau)|\right)$$

$$\leq \frac{1}{2} S\left(f; \max\{\omega(\xi, \omega(\eta, \epsilon)), \omega(v, \omega(\eta, \epsilon)), \omega(w, \omega(\eta, \epsilon))\}\right) + S(f; \omega(h, \epsilon)) + \phi(S(f; \max r_0 \omega(\zeta, \epsilon))).$$

Therefore we get

$$S\left(f; \omega(\Delta(\mathcal{U} \times \mathcal{V} \times \mathcal{W}), \epsilon)\right)$$
$$\leq \frac{1}{2} S\left(f; \max\{\omega(\mathcal{U}, \omega(\eta, \epsilon)), \omega(\mathcal{V}, \omega(\eta, \epsilon)), \omega(\mathcal{W}, \omega(\eta, \epsilon))\}\right) + S(f; \omega(h, \epsilon)) + \phi(S(f; \max r_0 \omega(\zeta, \epsilon))).$$

Since h, η, ζ are uniformly continuous on $[0, T] \times [-r_0, r_0]^5, [0, T]$ *and* $[0, T]$ respectively, we get $\omega(h, \epsilon) \to 0$, $\omega(\eta, \epsilon) \to 0$ *and* $\omega(\zeta, \epsilon) \to 0$ *as* $\epsilon \to 0$.

By taking $S(f; \tau) = \tau$, $\Theta_1 = \mathcal{U}$, $\Theta_2 = \mathcal{V}$, $\Theta_3 = \mathcal{W}$, $f = I$ and from the MNC definition, we have $\chi(\Theta_1 \times \Theta_2 \times \Theta_3) \leq \frac{1}{2}\left(\max\{\chi(\Theta_1), \chi(\Theta_2), \chi(\Theta_3)\}\right)$.

By the Corollary 1, Δ has at least a TFP. $\qquad\square$

5. Example

Example 5.1. Let the system of differentail equations is as

$$\begin{aligned}
\xi\prime(\tau) &= \tau^2 + \frac{\sqrt[3]{\xi(\tau)} + \sqrt[5]{v(\tau)} + \sqrt[7]{w(\tau)}}{3} + \frac{1}{6}\log(1 + |\tau\prime(\tau) + v\prime(\tau) + w\prime(\tau)|), \\
v\prime(\tau) &= \tau^2 + \frac{\sqrt[3]{\xi(\tau)} + \sqrt[5]{v(\tau)} + \sqrt[7]{w(\tau)}}{3} + \frac{1}{6}\log(1 + |v\prime(\tau) + w\prime(\tau) + \xi\prime(\tau)|), \\
w\prime(\tau) &= \tau^2 + \frac{\sqrt[3]{\xi(\tau)} + \sqrt[5]{v(\tau)} + \sqrt[7]{w(\tau)}}{3} + \frac{1}{6}\log(1 + |w\prime(\tau) + \xi\prime(\tau) + v\prime(\tau)|).
\end{aligned} \tag{5.1}$$

with the state condition $\xi(0) = 1$, $v(0) = 3$, $w(0) = 2$ and $\tau \in [0, 5]$.

System of Eq (5.1) is the particular case of Eq (4.1) where $\zeta(\tau) = \tau = \eta(\tau)$,

By the definition of ζ and β assumption (*i*) is satisfied.

$$h(\tau, \gamma_1, ..., \gamma_6) = \tau^2 + \frac{\sqrt[3]{\xi(\tau)} + \sqrt[5]{v(\tau)} + \sqrt[7]{w(\tau)}}{3} + \frac{1}{6}\log(1 + |\xi\prime(\tau) + v\prime(\tau) + w\prime(\tau)|).$$

Now assume that $\tau \in [0, T]$, $S(f; \tau) = \tau$, *and* $\phi(\tau) = \max_{i=3,5,7}\{\sqrt[i]{\tau}\}$, we get

$$\begin{aligned}
&|f(\tau, \gamma_1, ..., \gamma_6) - f(\tau, \gamma_1, ..., \gamma_6)| \\
&\leq \frac{|\sqrt[3]{\gamma_1} - \sqrt[3]{\delta_1}| + |\sqrt[5]{\gamma_2} - \sqrt[5]{\delta_2}| + |\sqrt[7]{\gamma_3} - \sqrt[7]{\delta_3}|}{3} + \frac{1}{6}|\log(1 + |\gamma_4 + \gamma_5 + \gamma_6|) - \log(1 + |\delta_4 + \delta_5 + \delta_6|) \\
&\leq \frac{|\sqrt[3]{\gamma_1} - \sqrt[3]{\delta_1}| + |\sqrt[5]{\gamma_2} - \sqrt[5]{\delta_2}| + |\sqrt[7]{\gamma_3} - \sqrt[7]{\delta_3}|}{3} + \frac{1}{6}\log\left(1 + \frac{|\gamma_4 + \gamma_5 + \gamma_6| - |\delta_4 + \delta_5 + \delta_6|}{1 + +|\delta_4 + \delta_5 + \delta_6|}\right) \\
&\leq \frac{\sqrt[3]{|\gamma_1 - \delta_1|} + \sqrt[5]{|\gamma_2 - \delta_2|} + \sqrt[7]{|\gamma_3 - \delta_3|}}{3} + \frac{1}{6}\log(1 + |\gamma_4 + \gamma_5 + \gamma_6 - (\delta_4 + \delta_5 + \delta_6)|) \\
&\leq \frac{\sqrt[3]{|\gamma_1 - \delta_1|} + \sqrt[5]{|\gamma_2 - \delta_2|} + \sqrt[7]{|\gamma_3 - \delta_3|}}{3} + \frac{1}{6}\log(1 + |\gamma_4 - \delta_4| + |\gamma_5 - \delta_5| + |\gamma_6 - \delta_6|) \\
&= \frac{1}{3}(\max \gamma\{|\gamma_4 - \delta_4|, |\gamma_5 - \delta_5|, |\gamma_6 - \delta_6|\}) + \phi(\max \gamma_{i=1,2,3}\{|\gamma_i - \delta_i|\}).
\end{aligned}$$

Hence assumption (*ii*) is satisfied. Moreover

$$
\begin{aligned}
M &= \sup\{|h(\tau, \gamma_0, \delta_0, \theta_0, 0, 0, 0) : \quad \tau \in [0, T]\} \\
&= \sup\{\tau^2 + \sqrt[3]{1} + \sqrt[5]{3} + \sqrt[7]{2} : \quad \tau \in [0, 5]\} \\
&\leq 29.
\end{aligned}
$$

It is simple to notice every number $r \geq 75$ fulfills the inequality given in (*iii*).
Now the inequality in assumption (*iv*) is $\phi\left(S(f; \Delta r_0)\right) + \frac{1}{6} S(f; 3r_0) + M$ is equal to

$$
\phi(5r) + \frac{1}{6}(3r) + 29 \leq r.
$$

Hence, as the number r_0 we can take $r_0 = 75$. Therefore, all the assumptions of Theorem 4.1 are satisfied. Hence the system of Eq (5.1) have at least one solution which belongs to $\{C[0, T]\}^3$ space.

6. Conclusions

The present paper concentrated on multiple FPT which is based on the generalization of DFPT via MNC. In this work, by using the concept of operators we extend DFPT by using MNC. We demonstrate the endurance of TFP by our extended DFPT and MNC. At the last we yield an example which fulfills our findings.

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Conflict of interest

All the authors declare that there is no conflict of interest.

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