Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 6(11): 12471–12490.
DOI:10.3934/math.2021720
Received: 30 May 2021
Accepted: 08 July 2021
Published: 31 August 2021

Research article

Bioperators on soft topological spaces

Baravan A. Asaad[1,2], **Tareq M. Al-shami**[3,*] **and Abdelwaheb Mhemdi**[4]

[1] Department of Computer Science, College of Science, Cihan University-Duhok, Iraq
[2] Department of Mathematics, Faculty of Science, University of Zakho, Zakho, Iraq
[3] Department of Mathematics, Sana'a University, Sana'a, Yemen
[4] Department of Mathematics, College of Sciences and Humanities in Aflaj, Prince Sattam bin Abdulaziz University, Riyadh, Saudi Arabia

* **Correspondence:** Email: tareqalshami83@gmail.com.

Abstract: To contribute to soft topology, we originate the notion of soft bioperators $\tilde{\gamma}$ and $\tilde{\gamma}^{'}$. Then, we apply them to analyze soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open sets and study main properties. We also prove that every soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open set is soft open; however, the converse is true only when the soft topological space is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-regular. After that, we define and study two classes of soft closures namely $Cl_{(\tilde{\gamma},\tilde{\gamma}^{'})}$ and $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}^{'})}$-$Cl$ operators, and two classes of soft interior namely $Int_{(\tilde{\gamma},\tilde{\gamma}^{'})}$ and $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}^{'})}$-$Int$ operators. Moreover, we introduce the notions of soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed sets and soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$T_{\frac{1}{2}}$ spaces, and explore their fundamental properties. In general, we explain the relationships between these notions, and give some counterexamples.

Keywords: bioperators $\tilde{\gamma}$ and $\tilde{\gamma}^{'}$ on $\tilde{\tau}$; soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open sets; soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed sets; soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$T_{\frac{1}{2}}$ spaces
Mathematics Subject Classification: 54A05, 54D10

---

1. Introduction

Vagueness and uncertainty occupied the human mind for centuries. In modern society, we face uncertainty and vagueness in different areas such as economics, engineering, medical science, sociality, and environmental sciences. Over the years, mathematicians, engineers, and scientists, particularly those who focus on artificial intelligence are seeking for approaches to solve the problems that contain uncertainty or vagueness. They established many tools for this purpose such as soft sets which are the most popular of all these.

The concept of soft sets was first constructed by Molodtsov [29] in 1999 as a general mathematical tool for dealing with uncertain objects. He successfully applied the soft set theory in several directions of mathematics, such as smoothness of functions, game theory, operators research, Riemann integration, Perron integration, probability, theory of measurement, etc.

Maji et al. [27,28] presented an application of soft sets in decision making problems that is based on the reduction of parameters to keep the optimal choice objects. Chen [12] presented a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory. Pei and Miao [31] showed that soft sets are a class of special information systems. Kong et al. [26] introduced the notion of normal parameter reduction of soft sets and investigated the problem of sub-optimal choice. El-Shafei et al. [15] defined new relations between ordinary points and soft sets which leads to redefine many soft topological concepts. To keep more set-theoretic properties on soft set theory, Al-shami and El-Shafei [8] introduced the concepts of T-soft subset and T-equality relations. Also, they initiated soft linear system with respect to some soft equality relations. Al-shami [5] studied soft sets on ordered setting and applied to explore new types of compactness and expect missing values in the information systems.

The soft set theory has been applied to many different fields (for examples, [6–9, 14, 18, 23, 31, 36]). In 2011, Shabir and Naz [32] constituted the study of soft topological spaces. They defined a soft topology on the collection of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as soft open and soft closed sets, soft subspace, soft closure, soft neighbourhood of a point, soft separation axioms, soft regular spaces and soft normal spaces and established several properties for them. Kharal and Ahmad [25] defined mappings on soft classes utilizing two crisp maps, one of them between the universal sets and the second one the sets of parameters. Then, Zorlutuna and Cakir [36] studied continuity between soft topological spaces. Recently, Al-shami [2–4] has revised some foregoing results in connection with soft separation axioms and soft equality relations.

Kasahara [24] defined an operator associated with a topology, namely an α operator and initiated some definitions which are equivalent to their counterparts on topological spaces when the operator involved is the identity operator. He also studied α-closed graphs of α-continuous functions and α-compact spaces. Later, Jankovic [20] employed α operator to introduce α-closure of a set and give some characterizations on α-closed graph of functions. Then, Ogata [30] defined the notion of γ-open sets to study operator-functions and operator-separation. Umehara et al. [33] defined the concept of bioperatiors on topological spaces, and studied some bioperators-separation axioms. Recently, some researchers defined $\tilde{\gamma}$ operator on the soft topology $\tilde{\tau}$. By using this $\tilde{\gamma}$ operator, Benchalli et al. [11] and Kalaivani et al. [21] defined soft $\tilde{\gamma}$-open set via soft point e_F and studied some of its properties. Also, Kalavathia [22] introduced soft $\tilde{\gamma}$-open set via an ordinary point and established some of its properties.

We aim through this paper to achieve three goals: (1) introduce and investigate the concept of soft bioperators $\tilde{\gamma}$ and $\tilde{\gamma}'$; (2) present and discuss two classes of soft closures namely $Cl_{(\tilde{\gamma},\tilde{\gamma}')}$ and $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl$ operators, and two classes of soft interior namely $Int_{(\tilde{\gamma},\tilde{\gamma}')}$ and $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Int$ operators; (3) formulate and explore the concepts of soft $(\tilde{\gamma},\tilde{\gamma}')$-$g$.closed sets and soft $(\tilde{\gamma},\tilde{\gamma}')$-$T_{\frac{1}{2}}$ spaces.

This research paper consists of five sections. Section 2 contains the concepts and findings from both soft set theory and soft $\tilde{\gamma}$ operator. Section 3 puts forward two novel soft topological concepts, namely, bioperators $\tilde{\gamma}$ and $\tilde{\gamma}'$ and soft $(\tilde{\gamma},\tilde{\gamma}')$-regular spaces. Section 4 introduces and studies the concepts of soft $Cl_{(\tilde{\gamma},\tilde{\gamma}')}$ and $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl$ operators, and soft $Int_{(\tilde{\gamma},\tilde{\gamma}')}$ and $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Int$ operators. Section 5 presents soft $(\tilde{\gamma},\tilde{\gamma}')$-$g$.closed sets and soft $(\tilde{\gamma},\tilde{\gamma}')$-$T_{\frac{1}{2}}$ spaces, and discusses some of their characterizations. The goal of Section 6 is to outline our main findings and plan for future work.

2. Soft set and $\tilde{\gamma}$ operator

Definition 2.1. *[29] Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) is called a soft set over X, where if F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e) = \emptyset$. The family of all these soft sets over the universal set X is denoted by $SS(X)_A$.*

We call (F, A) a null soft set, denoted by $\tilde{\phi}$ if for all $e \in A$, $F(e) = \phi$, and we call it an absolute soft set, denoted by $\tilde{X}$ if for all $e \in A$, $F(e) = X$.

Definition 2.2. *[16] For two soft sets (F, A) and (G, B) over a common universe X, we say that (F, A) is a soft subset of (G, B) (we write $(F, A)\tilde{\subseteq}(G, B)$) if $A \subseteq B$, and $F(e) \subseteq G(e)$ for all $e \in A$. We also say that these two soft sets are soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).*

Definition 2.3. *[1] The complement of a soft set (F, A), denoted by $(F, A)^c$ or $\tilde{X}\tilde{\setminus}(F, A)$, is defined by $(F, A)^c = (F^c, A)$ where $F^c : A \to P(X)$ is a mapping given by $F^c(e) = X \setminus F(e)$ for all $e \in A$.*

Definition 2.4. *[28] The soft union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$, denoted by $(H, C) = (F, A) \tilde{\cup} (G, B)$, and is defined as for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & \text{if } e \in A \setminus B \\ G(e) & \text{if } e \in B \setminus A \\ F(e) \cup G(e) & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.5. *[1] The soft intersection of two soft sets (F, A) and (G, B) over a common universe X is the soft set (H, C) where $C = A \cap B \neq \emptyset$, denoted by $(H, C) = (F, A) \tilde{\cap} (G, B)$, and is defined as $H(e) = F(e) \cap G(e)$ for all $e \in C$.*

Definition 2.6. *[32] The soft difference of two soft sets (F, A) and (G, A) over X is the soft set (H, A), denoted by $(H, A) = (F, A)\tilde{\setminus}(G, A)$, and is defined as $H(e) = F(e) \setminus G(e)$ for all $e \in A$.*

Definition 2.7. *[13] A soft set $(P, A) \tilde{\in} SS(X)_A$ is called a soft point in $\tilde{X}$, denoted by P_e^x, if there exist $e \in A$ and $x \in X$ such that $P(e) = \{x\}$ and $P(e') = \phi$ for every $e' \in A \setminus \{e\}$. We write $P_e^x \tilde{\in} (F, A)$, if $x \in F(e)$.*

Definition 2.8. *[15, 32] Let $(F, A) \tilde{\in} SS(X)_A$, and let $x \in X$. We say that*

1. *$x \tilde{\in} (F, A)$ whenever $x \in F(e)$ for all $e \in A$.*
2. *$x \tilde{\Subset} (F, A)$ whenever $x \in F(e)$ for some $e \in A$.*

Note that $x \tilde{\notin} (F, A)$ if $x \notin F(e)$ for some $e \in A$, and $x \tilde{\not\Subset} (F, A)$ if $x \notin F(e)$ for all $e \in A$.

Definition 2.9. *[32] Let $x \in X$. Then (x, A) is the soft set over X for which $x(e) = \{x\}$ for all $e \in A$.*

Definition 2.10. *[32] Let $\tilde{\tau}$ be the collection of soft sets over X. Then $\tilde{\tau}$ is said to be a soft topology on X if it satisfies the following axioms:*

1. $\tilde{\phi}$, $\tilde{X}$ *belong to* $\tilde{\tau}$.
2. *The soft union of an arbitrary number of soft sets in* $\tilde{\tau}$ *belongs to* $\tilde{\tau}$.
3. *The soft intersection of a finite number of soft sets in* $\tilde{\tau}$ *belongs to* $\tilde{\tau}$.

The triple $(X, \tilde{\tau}, A)$ is said to be a soft topological space (or soft space, in short) over X. Every member of $\tilde{\tau}$ is called a soft open set. The complement of soft open set is called a soft closed set.

Definition 2.11. *[19, 32] Let* $(X, \tilde{\tau}, A)$ *be a soft topological space and let* $(F, A) \tilde{\in} SS(X)_A$.

1. *The soft closure of* (F, A) *is the soft set defined by* $Cl(F, A) = \tilde{\cap}\{(G, A) : (G, A)$ *is a soft closed set and* $(G, A) \tilde{\supseteq} (F, A)\}$. *Clearly* $Cl(F, A)$ *is the smallest soft closed set over X which contains* (F, A).
2. *The soft interior of* (F, A) *is the soft set defined by* $Int(F, A) = \tilde{\cup}\{(G, A) : (G, A)$ *is a soft open set and* $(F, A) \tilde{\supseteq} (G, A)\}$. *Thus* $Int(F, A)$ *is the largest soft open set contained in* (F, A).

Lemma 2.1. *[19] Let* $(F, A) \tilde{\in} SS(X)_A$. *Then* $Cl(F, A) = \tilde{X}\tilde{\setminus}Int(\tilde{X}\tilde{\setminus}(F, A))$ *and* $Int(F, A) = \tilde{X}\tilde{\setminus}Cl(\tilde{X}\tilde{\setminus}(F, A))$.

Lemma 2.2. *[34]* $Cl(F, A) \tilde{\cap} (U, A) \tilde{\subseteq} Cl((F, A) \tilde{\cap} (U, A))$ *for every soft open set* (U, A) *and every soft set* (F, A) *in* $(X, \tilde{\tau}, A)$.

Definition 2.12. *[11, 21, 22] Let* $(X, \tilde{\tau}, A)$ *be a soft topological space. An operator* $\tilde{\gamma}$ *on the soft topology* $\tilde{\tau}$ *is a mapping from* $\tilde{\tau}$ *into* $SS(X)_A$ *such that* $(V, A) \tilde{\subseteq} \tilde{\gamma}(V, A)$ *for all* $(V, A) \tilde{\in} \tilde{\tau}$, *where* $\tilde{\gamma}(V, A)$ *denotes the value of* $\tilde{\gamma}$ *at* (V, A). *This operator will be denoted by* $\tilde{\gamma}\colon \tilde{\tau} \to SS(X)_A$.

The main definitions and results about $\tilde{\gamma}$ operator on the soft topology $\tilde{\tau}$ can be found in [11,21,22]. Now, we will define the soft $\tilde{\gamma}$-open set with respect to a soft point P_e^x.

Definition 2.13. *Let* $(X, \tilde{\tau}, A)$ *be a soft topological space and* $\tilde{\gamma}\colon \tilde{\tau} \to SS(X)_A$ *be an operator on* $\tilde{\tau}$. *A soft set* (F, A) *of* $(X, \tilde{\tau}, A)$ *is said to be soft* $\tilde{\gamma}$*-open if for each* $P_e^x \tilde{\in} (F, A)$, *there exists* $(V, A) \tilde{\in} \tilde{\tau}$ *with* $P_e^x \tilde{\in} (V, A)$ *and* $\tilde{\gamma}(V, A) \tilde{\subseteq} (F, A)$.

$\tilde{\tau}_{\tilde{\gamma}}$ will be denoted by the class of all soft $\tilde{\gamma}$-open sets of a soft topological space $(X, \tilde{\tau}, A)$. It is clear that $\tilde{\tau}_{\tilde{\gamma}} \tilde{\subseteq} \tilde{\tau}$. The union of any soft $\tilde{\gamma}$-open sets is soft $\tilde{\gamma}$-open, but the intersection of any two soft $\tilde{\gamma}$-open sets need not be soft $\tilde{\gamma}$-open. Therefore, $\tilde{\tau}_{\tilde{\gamma}}$ is not a soft topology on $\tilde{X}$.

The definition of soft regular operator $\tilde{\gamma}$ on $\tilde{\tau}$ with respect to a soft point P_e^x is as follows.

Definition 2.14. *Let* $(X, \tilde{\tau}, A)$ *be any soft topological space. An operator* $\tilde{\gamma}$ *on* $\tilde{\tau}$ *is said to be soft regular if for every soft open neighborhoods* (U, A) *and* (V, A) *of each* $P_e^x \tilde{\in} \tilde{X}$, *there exists a soft open neighborhood* (W, A) *of* P_e^x *such that*

$$\tilde{\gamma}(W, A) \tilde{\subseteq} \tilde{\gamma}(U, A) \tilde{\cap} \tilde{\gamma}(V, A).$$

Proposition 2.1. *Let* $\tilde{\gamma}$ *be a soft regular operator on* $\tilde{\tau}$. *If* $(F, A) \tilde{\in} \tilde{\tau}_{\tilde{\gamma}}$ *and* $(G, A) \tilde{\in} \tilde{\tau}_{\tilde{\gamma}}$, *then* $(F, A) \tilde{\cap} (G, A) \tilde{\in} \tilde{\tau}_{\tilde{\gamma}}$. *Thus,* $\tilde{\tau}_{\tilde{\gamma}}$ *is a soft topology on* $\tilde{X}$.

Next, the definition of soft $\tilde{\gamma}$-regular space $(X, \tilde{\tau}, A)$ with respect to a soft point P_e^x is as follows.

Definition 2.15. *A soft topological space* $(X, \tilde{\tau}, A)$ *with an operator* $\tilde{\gamma}$ *on* $\tilde{\tau}$ *is said to be soft* $\tilde{\gamma}$*-regular if for every* $P_e^x \tilde{\in} \tilde{X}$ *and for every* $(U, A) \tilde{\in} \tilde{\tau}$ *with* $P_e^x \tilde{\in} (U, A)$, *there exists* $(W, A) \tilde{\in} \tilde{\tau}$ *with* $P_e^x \tilde{\in} (W, A)$ *and* $\tilde{\gamma}'(W, A) \tilde{\subseteq} (U, A)$.

Proposition 2.2. *A soft topological space $(X, \tilde{\tau}, A)$ is soft $\tilde{\gamma}$-regular if and only if $\tilde{\tau} = \tilde{\tau}_{\tilde{\gamma}}$.*

Definition 2.16. *Let $(X, \tilde{\tau}, A)$ be any soft topological space. An operator $\tilde{\gamma}$ on $\tilde{\tau}$ is said to be soft open if for each $P_e^x \tilde{\in} \tilde{X}$ and for each $(U, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U, A)$, there exists $(W, A) \tilde{\in} \tilde{\tau}_{\tilde{\gamma}}$ with $P_e^x \tilde{\in} (W, A)$ and $(W, A) \tilde{\subseteq} \tilde{\gamma}(U, A)$.*

Definition 2.17. *[22] Let $(F, A) \tilde{\in} SS(X)_A$ and $P_e^x \tilde{\in} \tilde{X}$. A soft point $P_e^x \tilde{\in} \tilde{X}$ is in the soft $\tilde{\gamma}$-closure of (F, A) if $\tilde{\gamma}(U, A) \tilde{\cap} (F, A) \tilde{\neq} \tilde{\phi}$ for every $(U, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U, A)$. The set of all soft $\tilde{\gamma}$-closure points of (F, A) is called the soft $\tilde{\gamma}$-closure of (F, A) and it is denoted by $Cl_{\tilde{\gamma}}(F, A)$.*

Definition 2.18. *[22] Let $(F, A) \tilde{\in} SS(X)_A$. The soft set $\tilde{\tau}_{\tilde{\gamma}}$-$Cl(F, A)$ denotes the soft intersection of all soft $\tilde{\gamma}$-closed sets of $(X, \tilde{\tau}, A)$ containing (F, A) and is defined as*
$\tilde{\tau}_{\tilde{\gamma}}\text{-}Cl(F, A) = \tilde{\bigcap}\{(K, A) : (F, A) \tilde{\subseteq} (K, A) \text{ and } \tilde{X}\tilde{\setminus}(K, A) \tilde{\in} \tilde{\tau}_{\tilde{\gamma}}\}$.

Definition 2.19. *[22] Let $(F, A) \tilde{\in} SS(X)_A$ and $P_e^x \tilde{\in} \tilde{X}$. A soft point $P_e^x \tilde{\in} (F, A)$ is said to be soft $\tilde{\gamma}$-interior point of (F, A) if there exists a soft open neighborhood (U, A) of P_e^x such that $\tilde{\gamma}(U, A) \tilde{\subseteq} (F, A)$. We denote the set of all soft $\tilde{\gamma}$-interior points of (F, A) by $Int_{\tilde{\gamma}}(F, A)$. That is,*
$Int_{\tilde{\gamma}}(F, A) = \{P_e^x \tilde{\in} (F, A) : \tilde{\gamma}(U, A) \tilde{\subseteq} (F, A) \text{ for some } (U, A) \tilde{\in} \tilde{\tau} \text{ with } P_e^x \tilde{\in} (U, A)\}$.

Definition 2.20. *[22] Let $(F, A) \tilde{\in} SS(X)_A$. Denote $\tilde{\tau}_{\tilde{\gamma}}$-$Int(F, A)$ by the soft union of all soft $\tilde{\gamma}$-open sets of $(X, \tilde{\tau}, A)$ contained in (F, A) and is defined as*
$\tilde{\tau}_{\tilde{\gamma}}\text{-}Int(F, A) = \tilde{\bigcup}\{(U, A) : (U, A) \tilde{\subseteq} (F, A) \text{ and } (U, A) \tilde{\in} \tilde{\tau}_{\tilde{\gamma}}\}$.

Definition 2.21. *[22] A soft set (F, A) of a soft space $(X, \tilde{\tau}, A)$ is said to be soft $\tilde{\gamma}$-g.closed if $Cl_{\tilde{\gamma}}(F, A) \tilde{\subseteq} (U, A)$ whenever $(F, A) \tilde{\subseteq} (U, A)$ and (U, A) is soft $\tilde{\gamma}$-open.*

Definition 2.22. *[22] A soft space $(X, \tilde{\tau}, A)$ is said to be soft $\tilde{\gamma}$-$T_{\frac{1}{2}}$ if every soft $\tilde{\gamma}$-g.closed set of $(X, \tilde{\tau}, A)$ is soft $\tilde{\gamma}$-closed.*

3. Soft $(\tilde{\gamma}, \tilde{\gamma}')$-open sets

Throughout this paper, let $\tilde{\gamma}$ and $\tilde{\gamma}'$ be given two operators on a soft topology $\tilde{\tau}$. That is, $\tilde{\gamma}: \tilde{\tau} \to SS(X)_A$ and $\tilde{\gamma}': \tilde{\tau} \to SS(X)_A$ are mappings such that $(U, A) \tilde{\subseteq} \tilde{\gamma}(U, A)$ and $(V, A) \tilde{\subseteq} \tilde{\gamma}'(V, A)$ for all $(U, A) \tilde{\in} \tilde{\tau}$ and for all $(V, A) \tilde{\in} \tilde{\tau}$.

We begin this section by presenting the following definition:

Definition 3.1. *Let $(X, \tilde{\tau}, A)$ be a soft topological space, and $\tilde{\gamma}$ and $\tilde{\gamma}'$ be operators on $\tilde{\tau}$. A non-null soft set (F, A) of $(X, \tilde{\tau}, A)$ is said to be soft $(\tilde{\gamma}, \tilde{\gamma}')$-open if for each soft point $P_e^x \tilde{\in} (F, A)$, there exist soft open neighborhoods (U, A) and (V, A) of P_e^x such that*

$$\tilde{\gamma}(U, A) \tilde{\cup} \tilde{\gamma}'(V, A) \tilde{\subseteq} (F, A).$$

Suppose that the null soft set $\tilde{\phi}$ is also soft $(\tilde{\gamma}, \tilde{\gamma}')$-open for any operators $\tilde{\gamma}$ and $\tilde{\gamma}'$ on $\tilde{\tau}$.

$\tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')}$ will be denoted by the class of all soft $(\tilde{\gamma}, \tilde{\gamma}')$-open sets of a soft topological space $(X, \tilde{\tau}, A)$.

Example 3.1. *Let $X = \{a_1, a_2, a_3\}$, $A = \{e_1, e_2\}$ and $\tilde{\tau} = \{\tilde{\phi}, \tilde{X}, (F_1, A), (F_2, A), (F_3, A), (F_4, A)\}$ be a soft topology on X, where (F_1, A), (F_2, A), (F_3, A) and (F_4, A) defined as follows:*
$(F_1, A) = \{(e_1, \{a_1\}), (e_2, \{a_1\})\}$,
$(F_2, A) = \{(e_1, \{a_2\}), (e_2, \{a_2\})\}$,

$(F_3, A) = \{(e_1, \{a_1, a_2\}), (e_2, \{a_1, a_2\})\}$ *and*
$(F_4, A) = \{(e_1, \{a_2, a_3\}), (e_2, \{a_2, a_3\})\}$.
Define operators $\tilde{\gamma}\colon \tilde{\tau} \to SS(X)_A$ *and* $\tilde{\gamma}^{'}\colon \tilde{\tau} \to SS(X)_A$ *as follows: For all* $(F, A) \tilde{\in} \tilde{\tau}$

$$\tilde{\gamma}(F, A) = \begin{cases} Cl(F, A) & \text{if } P_{e_1}^{a_2} \tilde{\in} (F, A) \\ (F, A) & \text{if } P_{e_1}^{a_2} \tilde{\notin} (F, A) \end{cases}$$

and

$$\tilde{\gamma}^{'}(F, A) = \begin{cases} (F, A) & \text{if } (F, A) = (F_2, A) \text{ or } (F, A) = (F_4, A) \\ \tilde{X} & \text{otherwise.} \end{cases}$$

Thus, $\tilde{\tau}_{\tilde{\gamma}} = \{\tilde{\phi}, \tilde{X}, (F_1, A), (F_4, A)\}$,
$\tilde{\tau}_{\tilde{\gamma}'} = \{\tilde{\phi}, \tilde{X}, (F_2, A), (F_4, A)\}$ *and*
$\tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')} = \{\tilde{\phi}, \tilde{X}, (F_4, A)\}$.

Proposition 3.1. *For any soft set* (F, A) *of* $(X, \tilde{\tau}, A)$*, the following hold.*

1. *If* (F, A) *is soft* $(\tilde{\gamma}, \tilde{\gamma}^{'})$*-open, then* (F, A) *is soft* $\tilde{\gamma}$*-open for any operator* $\tilde{\gamma}^{'}$.
2. *If* (F, A) *is soft* $(\tilde{\gamma}, \tilde{\gamma}^{'})$*-open, then* (F, A) *is soft* $\tilde{\gamma}^{'}$*-open for any operator* $\tilde{\gamma}$.
3. *(a)* (F, A) *is soft* $(\tilde{\gamma}, \tilde{\gamma}^{'})$*-open if and only if* (F, A) *is soft* $\tilde{\gamma}$*-open and soft* $\tilde{\gamma}^{'}$*-open.*
   (b) $\tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')} = \tilde{\tau}_{\tilde{\gamma}} \tilde{\cap} \tilde{\tau}_{\tilde{\gamma}'}$.
4. *If* (F, A) *is soft* $(\tilde{\gamma}, \tilde{\gamma}^{'})$*-open, then* (F, A) *is soft open.*

Proof. (1) Let $P_e^x \tilde{\in} (F, A)$. Then there exist soft open neighborhoods (U, A) and (V, A) of P_e^x such that $\tilde{\gamma}(U, A) \tilde{\cup} \tilde{\gamma}^{'}(V, A) \tilde{\subseteq} (F, A)$. Hence, $\tilde{\gamma}(U, A) \tilde{\subseteq} (F, A)$. This implies that (F, A) is soft $\tilde{\gamma}$-open.

(2) Let $P_e^x \tilde{\in} (F, A)$. Then there exist soft open neighborhoods (U, A) and (V, A) of P_e^x such that $\tilde{\gamma}(U, A) \tilde{\cup} \tilde{\gamma}^{'}(V, A) \tilde{\subseteq} (F, A)$. Hence, $\tilde{\gamma}^{'}(V, A) \tilde{\subseteq} (F, A)$. This implies that (F, A) is soft $\tilde{\gamma}^{'}$-open.

(3a) *Necessity:* Let $P_e^x \tilde{\in} (F, A)$. It follows from assumptions that there exist soft open neighborhoods (U, A) and (V, A) of P_e^x such that $\tilde{\gamma}(U, A) \tilde{\subseteq} (F, A)$ and $\tilde{\gamma}^{'}(V, A) \tilde{\subseteq} (F, A)$. Thus, $\tilde{\gamma}(U, A) \tilde{\cup} \tilde{\gamma}^{'}(V, A) \tilde{\subseteq} (F, A)$. Therefore, (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open.

Sufficiency: Let $P_e^x \tilde{\in} (F, A)$. Then from (1) and (2), we get $\tilde{\gamma}(U, A) \tilde{\subseteq} (F, A)$ and $\tilde{\gamma}^{'}(V, A) \tilde{\subseteq} (F, A)$. Therefore, (F, A) is soft $\tilde{\gamma}$-open and soft $\tilde{\gamma}^{'}$-open.

(3b) It is obvious.

(4) Since $\tilde{\tau}_{\tilde{\gamma}} \tilde{\subseteq} \tilde{\tau}$ and (F, A) is soft $\tilde{\gamma}$-open (by (1)), (F, A) is soft open. □

Remark 3.1. *The following relations are shown by Proposition 3.1 (3).*

$$\tilde{\tau}_{\tilde{\gamma}} \tilde{\cap} \tilde{\tau}_{\tilde{\gamma}'} = \tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')} \tilde{\subseteq} \tilde{\tau}_{\tilde{\gamma}} \tilde{\subseteq} \tilde{\tau}$$

and

$$\tilde{\tau}_{\tilde{\gamma}} \tilde{\cap} \tilde{\tau}_{\tilde{\gamma}'} = \tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')} \tilde{\subseteq} \tilde{\tau}_{\tilde{\gamma}'} \tilde{\subseteq} \tilde{\tau}.$$

The following example shows that the inverse inclusions of Remark 3.1 do not hold and the converses of Proposition 3.1 are not true in general.

Example 3.2. *Consider the soft topological space* $(X, \tilde{\tau}, A)$ *defined in Example 3.1. Then the soft set* (F_1, A) *is soft* $\tilde{\gamma}$*-open in* $\tilde{X}$*, but* (F_1, A) *is neither soft* $\tilde{\gamma}^{'}$*-open nor soft* $(\tilde{\gamma}, \tilde{\gamma}^{'})$*-open. Whereas, the soft set* (F_2, A) *is soft* $\tilde{\gamma}^{'}$*-open in* $\tilde{X}$*, but* (F_2, A) *is neither soft* $\tilde{\gamma}$*-open nor soft* $(\tilde{\gamma}, \tilde{\gamma}^{'})$*-open. Therefore,* $\tilde{\tau}_{\tilde{\gamma}} \tilde{\neq} \tilde{\tau}_{\tilde{\gamma}'}$, $\tilde{\tau}_{\tilde{\gamma}} \tilde{\nsubseteq} \tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')}$ *and* $\tilde{\tau}_{\tilde{\gamma}'} \tilde{\nsubseteq} \tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')}$*. Also,* (F_3, A) *is soft open, but it is not soft* $(\tilde{\gamma}, \tilde{\gamma}^{'})$*-open.*

Proposition 3.2. *For any soft set (F,A) of $(X,\tilde{\tau},A)$, the following statements are equivalent:*

1. (F,A) *is soft* $(\tilde{\gamma},\tilde{\gamma})$*-open.*
2. (F,A) *is soft* $\tilde{\gamma}$*-open.*
3. (F,A) *is soft* $(\tilde{\gamma},\tilde{id})$*-open, where* $\tilde{id}\colon \tilde{\tau} \to SS(X)_A$ *is the identity operator, i.e.* $\tilde{\gamma}(F,A) = (F,A)$ *for every* $(F,A) \tilde{\in} \tilde{\tau}$.

Proof. (1) $\Leftrightarrow$ (2) It is shown by setting $\tilde{\gamma}' = \tilde{\gamma}$ in Proposition 3.1 (3a).

(2) $\Leftrightarrow$ (3) It is shown by their definitions. □

Lemma 3.1. *If* (F_λ,A) *is soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open for every* $\lambda \in \Lambda$*, then* $\tilde{\bigcup}\{(F_\lambda,A) : \lambda \in \Lambda\}$ *is soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open.*

Proof. Let $P_e^x \tilde{\in} \tilde{\bigcup}_{\lambda\in\Lambda}(F_\lambda,A)$. Then $P_e^x \tilde{\in} (F_{\lambda_0},A)$ for some $\lambda_0 \in \Lambda$. Hence there exist soft open neighborhoods (U,A) and (V,A) of P_e^x such that

$$\tilde{\gamma}(U,A) \tilde{\cup} \tilde{\gamma}'(V,A) \tilde{\subseteq} (F_{\lambda_0},A) \tilde{\subseteq} \tilde{\bigcup}_{\lambda\tilde{\in}\Lambda} (F_\lambda,A).$$

Thus, $\tilde{\bigcup}_{\lambda\tilde{\in}\Lambda} (F_\lambda,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$. □

Remark 3.2. *The intersection of any two soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open sets need not be soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open as shown by the below example.*

Example 3.3. *Let* $(X,\tilde{\tau},A)$ *be same as given in Example 3.1. Define operators* $\tilde{\gamma}\colon \tilde{\tau} \to SS(X)_A$ *and* $\tilde{\gamma}' : \tilde{\tau} \to SS(X)_A$ *as follows: For all* $(F,A) \tilde{\in} \tilde{\tau}$

$$\tilde{\gamma}(F,A) = \begin{cases} (F,A) & \text{if } P_{e_1}^{a_1} \tilde{\in} (F,A) \\ Cl(F,A) & \text{if } P_{e_1}^{a_1} \tilde{\notin} (F,A) \end{cases}$$

and

$$\tilde{\gamma}'(F,A) = \begin{cases} (F,A) & \text{if } P_{e_1}^{a_2} \tilde{\in} (F,A) \\ \tilde{X} & \text{if } P_{e_1}^{a_2} \tilde{\notin} (F,A). \end{cases}$$

Thus, $\tilde{\tau}_{\tilde{\gamma}} = \{\tilde{\phi}, \tilde{X}, (F_1,A), (F_3,A), (F_4,A)\}$,
$\tilde{\tau}_{\tilde{\gamma}'} = \{\tilde{\phi}, \tilde{X}, (F_2,A), (F_3,A), (F_4,A)\}$ *and*
$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')} = \{\tilde{\phi}, \tilde{X}, (F_3,A), (F_4,A)\}$.
Then, (F_3,A) *and* (F_4,A) *are soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open sets. However, their intersection* $(F_3,A) \tilde{\cap} (F_4,A) = (F_2,A)$ *is not a soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open set in* $\tilde{X}$.

Remark 3.3. *It follows that* $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ *may not be a soft topology on* $\tilde{X}$*. According to Lemma 3.1* $(X,\tilde{\tau},A)$ *is a supra soft topological space.*

Proposition 3.3. *Let* $\tilde{\gamma}$ *and* $\tilde{\gamma}'$ *be soft regular operators on* $\tilde{\tau}$.

1. *If* $(F,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ *and* $(G,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$*, then* $(F,A) \tilde{\cap} (G,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$.
2. $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ *is a soft topology on* $\tilde{X}$.

Proof. (1) Assume that $(F,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ and $(G,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$. Let $P_e^x \tilde{\in} (F,A) \cap (G,A)$. Then $P_e^x \tilde{\in} (F,A)$ and $P_e^x \tilde{\in} (G,A)$. So, there exist soft open neighborhoods (U_1,A), (V_1,A), (U_2,A) and (V_2,A) of P_e^x such that

$$\tilde{\gamma}(U_1,A) \tilde{\cup} \tilde{\gamma}'(V_1,A) \tilde{\subseteq} (F,A)$$

and

$$\tilde{\gamma}(U_2, A) \tilde{\cup} \tilde{\gamma}'(V_2, A) \tilde{\subseteq} (G, A).$$

Since $\tilde{\gamma}$ and $\tilde{\gamma}'$ are soft regular operators on $\tilde{\tau}$, there exist soft open neighborhoods (W_1, A) and (W_2, A) of P_e^x such that

$$\tilde{\gamma}(W_1, A) \tilde{\subseteq} \tilde{\gamma}(U_1, A) \tilde{\cap} \tilde{\gamma}(V_1, A)$$

and

$$\tilde{\gamma}'(W_2, A) \tilde{\subseteq} \tilde{\gamma}'(U_2, A) \tilde{\cap} \tilde{\gamma}'(V_2, A).$$

So,

$$\begin{gathered} \tilde{\gamma}(W_1, A) \tilde{\cup} \tilde{\gamma}'(W_2, A) \\ \tilde{\subseteq} [\tilde{\gamma}(U_1, A) \tilde{\cap} \tilde{\gamma}(V_1, A)] \tilde{\cup} [\tilde{\gamma}'(U_2, A) \tilde{\cap} \tilde{\gamma}'(V_2, A)] \\ \tilde{\subseteq} [\tilde{\gamma}(U_1, A) \tilde{\cup} \tilde{\gamma}(U_2, A)] \tilde{\cap} [\tilde{\gamma}'(V_1, A) \tilde{\cup} \tilde{\gamma}'(V_2, A)] \\ \tilde{\subseteq} (F, A) \tilde{\cap} (G, A). \end{gathered}$$

Thus, $(F, A) \tilde{\cap} (G, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')}$.

(2) Since $\tilde{\phi}$ and $\tilde{X}$ are soft $(\tilde{\gamma}, \tilde{\gamma}')$-open sets, it is proved by (1) and Lemma 3.1 that $\tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')}$ is a soft topology on $\tilde{X}$. □

The following example shows that the soft regularity on $\tilde{\gamma}$ and $\tilde{\gamma}'$ of Proposition 3.3 cannot be removed in general.

Example 3.4. *Consider the soft topological space $(X, \tilde{\tau}, A)$ defined in Example 3.3. Then $\tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')}$ is not a soft topology on $\tilde{X}$ since $\tilde{\gamma}$ is not a soft regular operator on $\tilde{\tau}$. If we take soft open neighborhoods (F_2, A) and (F_3, A) of a soft point $P_{e_1}^{a_2}$, then $\tilde{\gamma}(F_3, A) \tilde{\cap} \tilde{\gamma}(F_4, A) = (F_3, A) \tilde{\cap} Cl(F_4, A) = (F_3, A) \tilde{\cap} (F_4, A)$ $= (F_2, A)$. But we cannot find a soft open neighborhood (W, A) of this soft point $P_{e_1}^{a_2}$ such that $\tilde{\gamma}(W, A)$ $\tilde{\subseteq} (F_2, A)$.*

Definition 3.2. *A soft topological space $(X, \tilde{\tau}, A)$ is said to be soft $(\tilde{\gamma}, \tilde{\gamma}')$-regular if for every $P_e^x \tilde{\in} \tilde{X}$ and for every $(U, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U, A)$, there exist $(V, A) \tilde{\in} \tilde{\tau}$ and $(W, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (V, A)$, $P_e^x \tilde{\in}$ (W, A) and $\tilde{\gamma}(V, A) \tilde{\cup} \tilde{\gamma}'(W, A) \tilde{\subseteq} (U, A)$.*

Theorem 3.1. *Let $(X, \tilde{\tau}, A)$ be a soft topological space. Then the following statements are equivalent:*

1. *$\tilde{\tau} = \tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')}$.*
2. *$(X, \tilde{\tau}, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}')$-regular.*
3. *For each $P_e^x \tilde{\in} \tilde{X}$ and for each $(U, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U, A)$, there exists $(W, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')}$ with $P_e^x \tilde{\in}$ (W, A) and $(W, A) \tilde{\subseteq} (U, A)$.*

Proof. (1) ⇒ (2) Let $P_e^x \tilde{\in} \tilde{X}$ and $(U, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U, A)$. It follows from assumption that $(U, A) \tilde{\in}$ $\tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')}$. This implies that there exist $(W, A) \tilde{\in} \tilde{\tau}$ and $(V, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (W, A)$, $P_e^x \tilde{\in} (V, A)$ and

$$\tilde{\gamma}(W, A) \tilde{\cup} \tilde{\gamma}'(V, A) \tilde{\subseteq} (U, A).$$

Thus, $(X, \tilde{\tau}, A)$ is a soft $(\tilde{\gamma}, \tilde{\gamma}')$-regular space.

(2) ⇒ (3) Let $P_e^x \tilde{\in} \tilde{X}$ and $(U, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U, A)$. Then by (2), there exist $(V_1, A) \tilde{\in} \tilde{\tau}$ and $(V_2, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (V_1, A)$, $P_e^x \tilde{\in} (V_2, A)$ and $\tilde{\gamma}(V_1, A) \tilde{\cup} \tilde{\gamma}'(V_2, A) \tilde{\subseteq} (U, A)$. Again, by using (2) for the soft sets (V_1, A) and (V_2, A), there exist soft open neighborhoods (S_1, A), (S_2, A), (T_1, A) and (T_2, A) of P_e^x such that

$$\tilde{\gamma}(S_1, A) \tilde{\cup} \tilde{\gamma}'(S_2, A) \tilde{\subseteq} (V_1, A)$$

and

$$\tilde{\gamma}(T_1, A) \tilde{\cup} \tilde{\gamma}'(T_2, A) \tilde{\subseteq} (V_2, A).$$

This means that $(V_1, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ and $(V_2, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$. Take $(W, A) = (V_1, A) \tilde{\cup} (V_2, A)$. Thus, by Lemma 3.1, $(W, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ with $P_e^x \tilde{\in} (W, A)$ and $(W, A) \tilde{\subseteq} (U, A)$.

(3) ⇒ (1) By using (3) and Lemma 3.1, it follows that $(U, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$. That is, $\tilde{\tau} \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$. Since $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')} \tilde{\subseteq} \tilde{\tau}$ (by Remark 3.1), $\tilde{\tau} = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$. □

Example 3.5. *Since $\tilde{\tau} \neq \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ as shown in Example 3.3, $(X, \tilde{\tau}, A)$ is not a soft $(\tilde{\gamma}, \tilde{\gamma}')$-regular space.*

Lemma 3.2. *$(X, \tilde{\tau}, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}')$-regular if and only if $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')} = \tilde{\tau}_{\tilde{\gamma}} = \tilde{\tau}_{\tilde{\gamma}'} = \tilde{\tau}$.*

Proof. The proof follows from Theorem 3.1 and Remark 3.1. □

Proposition 3.4. *$(X, \tilde{\tau}, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}')$-regular if and only if $(X, \tilde{\tau}, A)$ is both soft $\tilde{\gamma}$-regular and soft $\tilde{\gamma}'$-regular.*

Proof. The proof follows from Lemma 3.2 and Proposition 2.2. □

Proposition 3.5. *The following statements are equivalent:*

1. *$(X, \tilde{\tau}, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma})$-regular.*
2. *$(X, \tilde{\tau}, A)$ is soft $\tilde{\gamma}$-regular.*
3. *$(X, \tilde{\tau}, A)$ is soft $(\tilde{\gamma}, \tilde{id})$-regular.*

Proof. Since $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma})} = \tilde{\tau}_{\tilde{\gamma}} = \tilde{\tau}_{(\tilde{\gamma},\tilde{id})} \tilde{\subseteq} \tilde{\tau}$ holds in general, the equivalences are proved by using Theorem 3.1. □

4. Between $Cl_{(\tilde{\gamma},\tilde{\gamma}')}$ and $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl$ operators

In this section, we introduce two closure operators, namely, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}$ and $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl$, and two interior operators, namely, $Int_{(\tilde{\gamma},\tilde{\gamma}')}$ $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Int$. We illustrate the relationships between them and discuss main properties.

First, we define soft $(\tilde{\gamma}, \tilde{\gamma}')$-closed set.

Definition 4.1. *A soft subset (K, A) of a soft space $(X, \tilde{\tau}, A)$ is said to be soft $(\tilde{\gamma}, \tilde{\gamma}')$-closed if its complement $\tilde{X} \tilde{\setminus} (K, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}')$-open in $(X, \tilde{\tau}, A)$.*

Next, two classes of soft closures via bioperators $\tilde{\gamma}$ and $\tilde{\gamma}'$ are investigated.

Definition 4.2. *Let $(F, A) \tilde{\in} SS(X)_A$ and $P_e^x \tilde{\in} \tilde{X}$.*

1. *A soft point P_e^x is said to be a soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closure point of (F, A) if $(\tilde{\gamma}(U, A) \tilde{\cup} \tilde{\gamma}^{'}(V, A)) \tilde{\cap} (F, A) \tilde{\neq} \tilde{\phi}$ for every $(U, A) \tilde{\in} \tilde{\tau}$ and $(V, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U, A) \tilde{\cap} (V, A)$. The set of all soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closure points of (F, A) is called soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closure of (F, A) and it is denoted by $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)$.*
2. *$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$ denotes the soft intersection of all soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed sets of $(X, \tilde{\tau}, A)$ containing (F, A) and is defined as*
   $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A) = \tilde{\bigcap}\{(K, A) : (F, A) \tilde{\subseteq} (K, A)$ and $\tilde{X} \tilde{\setminus} (K, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\}$.

Theorem 4.1. *Let $(F, A) \tilde{\in} SS(X)_A$ and $P_e^x \tilde{\in} \tilde{X}$. Then $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$ if and only if $(F, A) \tilde{\cap} (U, A)$ $\tilde{\neq} \tilde{\phi}$ for each $(U, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ with $P_e^x \tilde{\in} (U, A)$.*

Proof. *Necessity:* Let $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$ and assume that $(F, A) \tilde{\cap} (U, A) = \tilde{\phi}$ for some $(U, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ with $P_e^x \tilde{\in} (U, A)$. Then $(F, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} (U, A)$ and $\tilde{X} \tilde{\setminus} (U, A)$ is a soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed set in $\tilde{X}$. Hence $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} (U, A)$. Thus, $P_e^x \tilde{\in} \tilde{X} \tilde{\setminus} (U, A)$. This is a contradiction. So, the proof is completed.

Sufficiency: Let $P_e^x \tilde{\notin} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$. So there exists a soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed set (K, A) containing (F, A) with $P_e^x \tilde{\notin} (K, A)$. Hence, $\tilde{X} \tilde{\setminus} (K, A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ with $P_e^x \tilde{\in} \tilde{X} \tilde{\setminus} (K, A)$ and $[\tilde{X} \tilde{\setminus} (K, A)] \tilde{\cap} (F, A) = \tilde{\phi}$, which is a contradiction by hypothesis. Thus, $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$. □

Lemma 4.1. *Let $(F, A), (G, A) \tilde{\in} SS(X)_A$. Then the following statements are true:*

1. *$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed in $\tilde{X}$ and $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)$ is soft closed in $\tilde{X}$.*
2. *(a) $(F, A) \tilde{\subseteq} Cl(F, A) \tilde{\subseteq} \tilde{\tau}_{\tilde{\gamma}}$-$Cl(F, A) \tilde{\subseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$.*
   (b) $(F, A) \tilde{\subseteq} Cl(F, A) \tilde{\subseteq} \tilde{\tau}_{\tilde{\gamma}'}$-$Cl(F, A) \tilde{\subseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$.
3. *The following are equivalent:*
   (a) (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed.
   (b) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A) = (F, A)$.
   (c) $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) = (F, A)$.
4. *If $(F, A) \tilde{\subseteq} (G, A)$, then $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(G, A)$ and $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(G, A)$.*
5. *(a) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl((F, A) \tilde{\cap} (G, A)) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A) \tilde{\cap} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(G, A)$.*
   (b) $Cl_{(\tilde{\gamma},\tilde{\gamma}')}((F, A) \tilde{\cap} (G, A)) \tilde{\subseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\cap} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(G, A)$.
6. *(a) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl((F, A) \tilde{\cup} (G, A)) \tilde{\supseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A) \tilde{\cup} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(G, A)$.*
   (b) $Cl_{(\tilde{\gamma},\tilde{\gamma}')}((F, A) \tilde{\cup} (G, A)) \tilde{\supseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\cup} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(G, A)$.
7. *$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$.*
8. *$Cl_{(\tilde{\gamma},\tilde{\gamma}')}(\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F, A)$.*

Proof. Straightforward. □

Theorem 4.2. *Let $(F, A) \tilde{\in} SS(X)_A$. Then,*

$$Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) = Cl_{\tilde{\gamma}}(F, A) \tilde{\cup} Cl_{\tilde{\gamma}'}(F, A).$$

Proof. We start by their definitions,
$P_e^x \tilde{\notin} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)$.
$\Leftrightarrow$ There exist $(U, A) \tilde{\in} \tilde{\tau}$ and $(V, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U, A)$ and $P_e^x \tilde{\in} (V, A)$ such that $(\tilde{\gamma}(U, A) \tilde{\cup} \tilde{\gamma}^{'}(V, A))$ $\tilde{\cap} (F, A) = \tilde{\phi}$.
$\Leftrightarrow$ There exist $(U, A) \tilde{\in} \tilde{\tau}$ and $(V, A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U, A)$ and $P_e^x \tilde{\in} (V, A)$ such that $\tilde{\gamma}(U, A) \tilde{\cap} (F, A) = \tilde{\phi}$ and $\tilde{\gamma}^{'}(V, A) \tilde{\cap} (F, A) = \tilde{\phi}$.
$\Leftrightarrow P_e^x \tilde{\notin} Cl_{\tilde{\gamma}}(F, A)$ and $P_e^x \tilde{\notin} Cl_{\tilde{\gamma}'}(F, A)$.

$\Leftrightarrow P_e^x \tilde{\notin} Cl_{\tilde{\gamma}}(F,A) \tilde{\cup} Cl_{\tilde{\gamma}'}(F,A)$.
So, we get the proof. □

Proposition 4.1. *Let $(F,A) \tilde{\in} SS(X)_A$. If $(X,\tilde{\tau},A)$ is a soft regular space, then $Cl(F,A) = \tilde{\tau}_{\tilde{\gamma}}\text{-}Cl(F,A) = \tilde{\tau}_{\tilde{\gamma}'}\text{-}Cl(F,A) = Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A)$.*

Proof. By using Theorem 3.1, $\tilde{\tau} = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$. Hence $Cl(F,A) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A)$. It follows from Lemma 4.1 (2) that $Cl(F,A) = \tilde{\tau}_{\tilde{\gamma}}\text{-}Cl(F,A) = \tilde{\tau}_{\tilde{\gamma}'}\text{-}Cl(F,A) = Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A)$. □

Lemma 4.2. *Let $\tilde{\gamma}$ and $\tilde{\gamma}'$ be soft regular operators on $\tilde{\tau}$. For any $(F,A),(G,A) \tilde{\in} SS(X)_A$, we have*

1. *$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\cup} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(G,A) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl((F,A) \tilde{\cup} (G,A))$.*
2. *$Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\cup} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(G,A) = Cl_{(\tilde{\gamma},\tilde{\gamma}')}((F,A) \tilde{\cup} (G,A))$.*

Proof. (1) It follows directly from Lemma 4.1 (6) that $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\cup} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(G,A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl((F,A) \tilde{\cup} (G,A))$. Then it is enough to prove that $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl((F,A) \tilde{\cup} (G,A)) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\cup} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(G,A)$. Let $P_e^x \tilde{\notin} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\cup} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(G,A)$. Then there exist $(U,A),(V,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ with $P_e^x \tilde{\in} (U,A)$, $P_e^x \tilde{\in} (V,A)$, $(F,A) \tilde{\cap} (U,A) = \tilde{\phi}$ and $(G,A) \tilde{\cap} (V,A) = \tilde{\phi}$. Since $\tilde{\gamma}$ and $\tilde{\gamma}'$ are soft regular operators on $\tilde{\tau}$, by Proposition 3.3 (1), $(U,A) \tilde{\cap} (V,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ such that

$$[(U,A) \tilde{\cap} (V,A)] \tilde{\cap} [(F,A) \tilde{\cup} (G,A)] = \tilde{\phi}.$$

This means that $P_e^x \tilde{\notin} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl((F,A) \tilde{\cup} (G,A))$. Hence,

$$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl((F,A) \tilde{\cup} (G,A)) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\cup} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(G,A).$$

(2) Let $P_e^x \tilde{\notin} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\cup} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(G,A)$. Then there exist soft open sets (U_1,A), (U_2,A), (V_1,A), (V_2,A) all containing P_e^x with $(\tilde{\gamma}(U_1,A) \tilde{\cup} \tilde{\gamma}'(U_2,A)) \tilde{\cap} (F,A) = \tilde{\phi}$ and $(\tilde{\gamma}(V_1,A) \tilde{\cup} \tilde{\gamma}'(V_2,A)) \tilde{\cap} (G,A) = \tilde{\phi}$. Since $\tilde{\gamma}$ and $\tilde{\gamma}'$ are soft regular operators on $\tilde{\tau}$, there exist $(W_1,A),(W_2,A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (W_1,A)$ and $P_e^x \tilde{\in} (W_2,A)$ such that

$$\tilde{\gamma}(W_1,A) \tilde{\subseteq} \tilde{\gamma}(U_1,A) \tilde{\cap} \tilde{\gamma}(V_1,A)$$

and

$$\tilde{\gamma}'(W_2,A) \tilde{\subseteq} \tilde{\gamma}'(U_2,A) \tilde{\cap} \tilde{\gamma}'(V_2,A).$$

Thus, we have

$$\begin{aligned}\tilde{\gamma}(W_1,A)\tilde{\cup}\tilde{\gamma}'(W_2,A) &\tilde{\subseteq}[\tilde{\gamma}(U_1,A)\tilde{\cap}\tilde{\gamma}(V_1,A)]\tilde{\cup}[\tilde{\gamma}'(U_2,A)\tilde{\cap}\tilde{\gamma}'(V_2,A)]\\ &\tilde{\subseteq}[\tilde{\gamma}(U_1,A)\tilde{\cup}\tilde{\gamma}'(U_2,A)]\tilde{\cap}[\tilde{\gamma}(V_1,A)\tilde{\cup}\tilde{\gamma}'(V_2,A)].\end{aligned}$$

Hence, $[(F,A) \tilde{\cup} (G,A)] \tilde{\cap} [\tilde{\gamma}(W_1,A) \tilde{\cup} \tilde{\gamma}'(W_2,A)] \tilde{\subseteq} [(F,A) \tilde{\cup} (G,A)] \tilde{\cap} [\tilde{\gamma}(U_1,A) \tilde{\cup} \tilde{\gamma}'(U_2,A)] \tilde{\cap} [\tilde{\gamma}(V_1,A) \tilde{\cup} \tilde{\gamma}'(V_2,A)]$.
The disjoint of $[(F,A) \tilde{\cup} (G,A)]$ and $[\tilde{\gamma}(U_1,A) \tilde{\cup} \tilde{\gamma}'(U_2,A)] \tilde{\cap} [\tilde{\gamma}(V_1,A) \tilde{\cup} \tilde{\gamma}'(V_2,A)]$ leads to

$$[(F,A) \tilde{\cup} (G,A)] \tilde{\cap} [\tilde{\gamma}(W_1,A) \tilde{\cup} \tilde{\gamma}'(W_2,A)] = \tilde{\phi}.$$

This means that $P_e^x \tilde{\notin} Cl_{(\tilde{\gamma},\tilde{\gamma}')}((F,A) \tilde{\cup} (G,A))$. Therefore, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}((F,A) \tilde{\cup} (G,A)) \tilde{\subseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(A) \tilde{\cup} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(G,A)$. From Lemma 4.1 (6), we obtain the equality. □

Lemma 4.3. *Let $\tilde{\gamma}$ and $\tilde{\gamma}'$ be soft regular operators on $\tilde{\tau}$, and let $(F,A) \tilde{\in} SS(X)_A$. Then*

$$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\cap} (U,A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl((F,A) \tilde{\cap} (U,A))$$

holds for each $(U,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$.

Proof. Suppose that $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\cap} (U,A)$ for each $(U,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$, then $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A)$ and $P_e^x \tilde{\in} (U,A)$. Let $(V,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ with $P_e^x \tilde{\in} (V,A)$. Since $\tilde{\gamma}$ and $\tilde{\gamma}'$ are soft regular operators on $\tilde{\tau}$, by Proposition 3.3 (1), $(U,A) \tilde{\cap} (V,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ with $P_e^x \tilde{\in} (U,A) \tilde{\cap} (V,A)$. Since $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A)$, by Theorem 4.1, we find that $(F,A) \tilde{\cap} ((U,A) \tilde{\cap} (V,A)) \tilde{\neq} \tilde{\phi}$. Therefore, $((F,A) \tilde{\cap} (U,A)) \tilde{\cap} (V,A) \tilde{\neq} \tilde{\phi}$. Thus, by Theorem 4.1, we have that $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl((F,A) \tilde{\cap} (U,A))$. Hence,

$$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\cap} (U,A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl((F,A) \tilde{\cap} (U,A)).$$

□

Theorem 4.3. *Let* $(F,A) \tilde{\in} SS(X)_A$*, then the following properties are equivalent:*

1. $(F,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$.
2. $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(\tilde{X} \tilde{\setminus} (F,A)) = \tilde{X} \tilde{\setminus} (F,A)$.
3. $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(\tilde{X} \tilde{\setminus} (F,A)) = \tilde{X} \tilde{\setminus} (F,A)$.
4. $\tilde{X} \tilde{\setminus} (F,A)$ *is soft* $(\tilde{\gamma},\tilde{\gamma}')$*-closed.*

Theorem 4.4. *Let* $\tilde{\gamma}$ *and* $\tilde{\gamma}'$ *be soft open operators on* $\tilde{\tau}$*, and let* $(F,A) \tilde{\in} SS(X)_A$*. If* $(X,\tilde{\tau},A)$ *is a soft* $(\tilde{\gamma},\tilde{\gamma}')$*-regular space, then the following hold:*

1. $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A)$.
2. $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)) = Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$.
3. $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$ *is soft* $(\tilde{\gamma},\tilde{\gamma}')$*-closed in* $\tilde{X}$.

Proof. (1) First we need to show that $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\subseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$. By Lemma 4.1 (2), we have $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A)$. Now let $P_e^x \tilde{\notin} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$, then there exist $(U,A) \tilde{\in} \tilde{\tau}$ and $(V,A) \tilde{\in} \tilde{\tau}$ with $P_e^x \tilde{\in} (U,A)$, $P_e^x \tilde{\in} (V,A)$ and $(\tilde{\gamma}(U,A) \tilde{\cup} \tilde{\gamma}'(V,A)) \tilde{\cap} (F,A) = \tilde{\phi}$. Since $\tilde{\gamma}$ and $\tilde{\gamma}'$ are soft open operators on $\tilde{\tau}$, there exist $(W_1,A) \tilde{\in} \tilde{\tau}_{\tilde{\gamma}}$ and $(W_2,A) \tilde{\in} \tilde{\tau}_{\tilde{\gamma}'}$ with $P_e^x \tilde{\in} (W_1,A)$, $P_e^x \tilde{\in} (W_2,A)$ such that $(W_1,A) \tilde{\subseteq} \tilde{\gamma}(U,A)$ and $(W_2,A) \tilde{\subseteq} \tilde{\gamma}'(V,A)$. So $(F,A) \tilde{\cap} ((W_1,A) \tilde{\cup} (W_2,A)) = \tilde{\phi}$. Since $(X,\tilde{\tau},A)$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-regular, by Lemma 3.2, $(W_1,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ and $(W_2,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$. Hence by Lemma 3.1, $((W_1,A) \tilde{\cup} (W_2,A)) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$ with $P_e^x \tilde{\in} ((W_1,A) \tilde{\cup} (W_2,A))$. Thus, by Theorem 4.1, $P_e^x \tilde{\notin} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A)$. Therefore, $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A) \tilde{\subseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$. Hence $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Cl(F,A)$.

(2) The proof follows from part (1) and Lemma 4.1 (7).

(3) By part (2) and Lemma 4.1 (3), we get $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-closed in $\tilde{X}$. □

In the end of this section, we introduce two classes of soft interior via bioperators $\tilde{\gamma}$ and $\tilde{\gamma}'$ called $Int_{(\tilde{\gamma},\tilde{\gamma}')}$ and $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}Int$ operators.

Definition 4.3. *Let* $(F,A) \tilde{\in} SS(X)_A$ *and* $P_e^x \tilde{\in} \tilde{X}$.

1. *A soft point* $P_e^x \tilde{\in} (F,A)$ *is said to be soft* $(\tilde{\gamma},\tilde{\gamma}')$*-interior point of* (F,A) *if there exist soft open neighborhoods* (U,A) *and* (V,A) *of* P_e^x *such that* $\tilde{\gamma}(U,A) \tilde{\cup} \tilde{\gamma}'(V,A) \tilde{\subseteq} (F,A)$*. We denote the set of all soft* $(\tilde{\gamma},\tilde{\gamma}')$*-interior points of* (F,A) *by* $Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$*. That is,*
   $Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) = \{P_e^x \tilde{\in} (F,A) : \tilde{\gamma}(U,A) \tilde{\cup} \tilde{\gamma}'(V,A) \tilde{\subseteq} (F,A)$ *for some* $(U,A),(V,A) \tilde{\in} \tilde{\tau}$ *with* $P_e^x \tilde{\in} (U,A)$ *and* $P_e^x \tilde{\in} (V,A)\}$.

2. $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A)$ *denotes the soft union of all soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open sets of* $(X,\tilde{\tau},A)$ *contained in* (F,A) *and is defined as*
$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A) = \tilde{\bigcup}\{(U,A) : (U,A) \tilde{\subseteq} (F,A)$ *and* $(U,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\}$.

Lemma 4.4. *Let* $(F,A),(G,A) \tilde{\in} SS(X)_A$*. Then the following statements are true:*

1. $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A)$ *is soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open in* $\tilde{X}$ *and* $Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$ *is soft open in* $\tilde{X}$*.*
2. (a) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A) \tilde{\subseteq} Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} \tilde{\tau}_{\tilde{\gamma}}$-*Int*$(F,A) \tilde{\subseteq} Int(F,A) \tilde{\subseteq} (F,A)$.
   (b) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A) \tilde{\subseteq} Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} \tilde{\tau}_{\tilde{\gamma}'}$-*Int*$(F,A) \tilde{\subseteq} Int(F,A) \tilde{\subseteq} (F,A)$.
3. *The following are equivalent:*
   (a) (F,A) *is soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open.*
   (b) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A) = (F,A)$.
   (c) $Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) = (F,A)$.
4. *If* $(F,A) \tilde{\subseteq} (G,A)$*, then* $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(G,A)$ *and* $Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} Int_{(\tilde{\gamma},\tilde{\gamma}')}(G,A)$.
5. (a) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$((F,A) \tilde{\cap} (G,A)) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A) \tilde{\cap} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(G,A)$.
   (b) $Int_{(\tilde{\gamma},\tilde{\gamma}')}((F,A) \tilde{\cap} (G,A)) \tilde{\subseteq} Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\cap} Int_{(\tilde{\gamma},\tilde{\gamma}')}(G,A)$.
6. (a) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$((F,A) \tilde{\cup} (G,A)) \tilde{\supseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A) \tilde{\cup} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(G,A)$.
   (b) $Int_{(\tilde{\gamma},\tilde{\gamma}')}((F,A) \tilde{\cup} (G,A)) \tilde{\supseteq} Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\cup} Int_{(\tilde{\gamma},\tilde{\gamma}')}(G,A)$.
7. (a) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A)) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A)$.
   (b) $Int_{(\tilde{\gamma},\tilde{\gamma}')}(Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)) = Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$.
8. (a) $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(\tilde{X}\tilde{\setminus}(F,A)) = \tilde{X}\tilde{\setminus}\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Cl*$(F,A)$.
   (b) $Int_{(\tilde{\gamma},\tilde{\gamma}')}(\tilde{X}\tilde{\setminus}(F,A)) = \tilde{X}\tilde{\setminus}Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$.

Proof. Straightforward. □

Lemma 4.5. *Let* $\tilde{\gamma}$ *and* $\tilde{\gamma}'$ *be soft regular operators on* $\tilde{\tau}$*. For any* $(F,A),(G,A) \tilde{\in} SS(X)_A$*, we have*

1. $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(F,A) \tilde{\cap} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$(G,A) = \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-*Int*$((F,A) \tilde{\cap} (G,A))$.
2. $Int_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\cap} Int_{(\tilde{\gamma},\tilde{\gamma}')}(G,A) = Int_{(\tilde{\gamma},\tilde{\gamma}')}((F,A) \tilde{\cap} (G,A))$.

Definition 4.4. *Let* $(X,\tilde{\tau},A)$ *be a soft space and a soft point* $P_e^x \tilde{\in} \tilde{X}$*. Then a soft* $(\tilde{\gamma},\tilde{\gamma}')$*-neighbourhood (soft* $(\tilde{\gamma},\tilde{\gamma}')$*-nbd, in short) of a soft point* P_e^x *is a soft set* (N,A) *which contains a soft* $(\tilde{\gamma},\tilde{\gamma}')$*-open set* (U,A) *in* $\tilde{X}$ *such that* $P_e^x \tilde{\in} (U,A)$*. Evidently, a soft set* (N,A) *is a soft* $(\tilde{\gamma},\tilde{\gamma}')$*-nbd of* P_e^x *if* $P_e^x \tilde{\in} Int_{(\tilde{\gamma},\tilde{\gamma}')}(N,A)$*.*

The class of all soft $(\tilde{\gamma},\tilde{\gamma}')$*-nbds of* P_e^x *is called the soft* $(\tilde{\gamma},\tilde{\gamma}')$*-nbd system at* P_e^x *and is denoted by* $(N_{P_e^x},A)$*.*

Theorem 4.5. *The soft* $(\tilde{\gamma},\tilde{\gamma}')$*-nbd system* $(N_{P_e^x},A)$ *at* P_e^x *in a soft space* $(X,\tilde{\tau},A)$ *has the following properties:*

1. *If* $(N,A) \tilde{\in} (N_{P_e^x},A)$*, then* $P_e^x \tilde{\in} (N,A)$*.*
2. *If* $(N,A),(M,A) \tilde{\in} (N_{P_e^x},A)$*, then* $(N,A) \tilde{\cap} (M,A) \tilde{\in} (N_{P_e^x},A)$*, where* $\tilde{\gamma}$ *and* $\tilde{\gamma}'$ *are soft regular operators on* $\tilde{\tau}$*.*
3. *If* $(N,A) \tilde{\in} (N_{P_e^x},A)$*, then there is* $(U,A) \tilde{\in} (N_{P_e^x},A)$ *such that* $(N,A) \tilde{\in} (N_{P_e^y},A)$ *for each* $P_e^y \tilde{\in} (U,A)$ *such that* $y \neq x$*.*
4. *If* $(N,A) \tilde{\in} (N_{P_e^x},A)$ *and* $(N,A) \tilde{\subseteq} (M,A)$*, then* $(M,A) \tilde{\in} (N_{P_e^x},A)$*.*

Proof. (1) It is clear.

(2) Let $(N, A), (M, A) \tilde{\in} (N_{P_e^x}, A)$. This means that $P_e^x \tilde{\in} Int_{(\tilde{\gamma},\tilde{\gamma}')}(N, A)$ and $P_e^x \tilde{\in} Int_{(\tilde{\gamma},\tilde{\gamma}')}(M, A)$ which imply that $P_e^x \tilde{\in} Int_{(\tilde{\gamma},\tilde{\gamma}')}(N, A) \tilde{\cap} Int_{(\tilde{\gamma},\tilde{\gamma}')}(M, A)$. Since $\tilde{\gamma}$ and $\tilde{\gamma}'$ are soft regular operators on $\tilde{\tau}$, by Lemma 4.5 (2), we have $P_e^x \tilde{\in} Int_{(\tilde{\gamma},\tilde{\gamma}')}((N, A) \tilde{\cap} (M, A))$. Thus, $(N, A) \tilde{\cap} (M, A) \tilde{\in} (N_{P_e^x}, A)$.

(3) Let $(N, A) \tilde{\in} (N_{P_e^x}, A)$. Take $(U, A) = Int_{(\tilde{\gamma},\tilde{\gamma}')}(N, A)$. Then for each $P_e^y \tilde{\in} (U, A)$ such that $y \neq x$, $P_e^y \tilde{\in} Int_{(\tilde{\gamma},\tilde{\gamma}')}(N, A)$ and hence $(N, A) \tilde{\in} (N_{P_e^y}, A)$.

(4) Let $(N, A) \tilde{\in} (N_{P_e^x}, A)$. This means that $P_e^x \tilde{\in} Int_{(\tilde{\gamma},\tilde{\gamma}')}(N, A)$. Since $(N, A) \tilde{\subseteq} (M, A)$, $Int_{(\tilde{\gamma},\tilde{\gamma}')}(N, A) \tilde{\subseteq} Int_{(\tilde{\gamma},\tilde{\gamma}')}(M, A)$ which obtains that $P_e^x \tilde{\in} Int_{(\tilde{\gamma},\tilde{\gamma}')}(M, A)$. Thus, $(M, A) \tilde{\in} (N_{P_e^x}, A)$. □

Theorem 4.6. *A soft set (N, A) is a soft $(\tilde{\gamma}, \tilde{\gamma}')$-open set in $\tilde{X}$ if and only if (N, A) is a soft $(\tilde{\gamma}, \tilde{\gamma}')$-nbd of each of its soft points.*

Proof. Necessity: If (N, A) is soft $(\tilde{\gamma}, \tilde{\gamma}')$-open, then $(N, A) = Int_{(\tilde{\gamma},\tilde{\gamma}')}(N, A)$ (by Lemma 4.4 (3)). Therefore, (N, A) is a soft $(\tilde{\gamma}, \tilde{\gamma}')$-nbd of each of its soft points.

Sufficiency: Let (N, A) be a soft $(\tilde{\gamma}, \tilde{\gamma}')$-nbd of each of its soft points. Then, (N, A) contains a soft $(\tilde{\gamma}, \tilde{\gamma}')$-open set (U, A) in $\tilde{X}$ such that $P_e^x \tilde{\in} (U, A)$ for each $P_e^x \tilde{\in} (N, A)$. Therefore, $(N, A) = \tilde{\bigcup}_{P_e^x \tilde{\in} (N,A)}(U_{P_e^x}, A)$ is a union of soft $(\tilde{\gamma}, \tilde{\gamma}')$-open sets and hence by Lemma 3.1, (N, A) is soft $(\tilde{\gamma}, \tilde{\gamma}')$-open. □

5. Soft $(\tilde{\gamma}, \tilde{\gamma}')$-$g$.closed sets and soft $(\tilde{\gamma}, \tilde{\gamma}')$-$T_{\frac{1}{2}}$ spaces

In this section, we introduce soft $(\tilde{\gamma}, \tilde{\gamma}')$-$g$.closed sets and soft $(\tilde{\gamma}, \tilde{\gamma}')$-$T_{\frac{1}{2}}$ spaces, and study some of their characterizations.

Definition 5.1. *A soft set (F, A) of a soft space $(X, \tilde{\tau}, A)$ is said to be soft $(\tilde{\gamma}, \tilde{\gamma}')$-g.closed if $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} (U, A)$ whenever $(F, A) \tilde{\subseteq} (U, A)$ and (U, A) is soft $(\tilde{\gamma}, \tilde{\gamma}')$-open.*

Remark 5.1. *Every soft $(\tilde{\gamma}, \tilde{\gamma}')$-closed set in $(X, \tilde{\tau}, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}')$-g.closed, but its converse is not true as may be shown from the following example.*

Example 5.1. *Consider the soft topological space $(X, \tilde{\tau}, A)$ defined in Example 3.3. Take $(F, A) \tilde{\in} SS(X)_A$ such that $(F, A) = \{(e_1, \{a_1, a_2\}), (e_2, \{a_2, a_3\})\}$. Then, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) = \tilde{X}$, and (F, A) is not soft $(\tilde{\gamma}, \tilde{\gamma}')$-closed. However, (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}')$-g.closed in $(X, \tilde{\tau}, A)$, because $\tilde{X}$ is the only soft $(\tilde{\gamma}, \tilde{\gamma}')$-open set containing (F, A).*

Proposition 5.1. *A soft set (F, A) of a soft space $(X, \tilde{\tau}, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma})$-g.closed if and only if (F, A) is soft $\tilde{\gamma}$-g.closed.*

Proof. The proof is immediate consequence of Proposition 3.1 (3). □

The following results characterize soft $(\tilde{\gamma}, \tilde{\gamma}')$-$g$.closed sets.

Lemma 5.1. *A soft set (F, A) of a soft space $(X, \tilde{\tau}, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}')$-g.closed if and only if $(F, A) \tilde{\cap} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x) \tilde{\neq} \tilde{\phi}$ for every $P_e^x \tilde{\in} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)$.*

Proof. Necessity: Suppose that there exists a soft point $P_e^x \tilde{\in} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)$ such that $(F, A) \tilde{\cap} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x) = \tilde{\phi}$ implies $(F, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x)$. Since $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x)$ is soft $(\tilde{\gamma}, \tilde{\gamma}')$-closed, $\tilde{X} \tilde{\setminus} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x)$

is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open. Now, soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closedness of (F, A) in $\tilde{X}$ implies that $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x)$. Therefore, $P_e^x \tilde{\notin} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)$. This is a contradiction. Thus, $(F, A) \tilde{\cap} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x) \tilde{\neq} \tilde{\phi}$.

Sufficiency: Let (U, A) be a soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open set in $\tilde{X}$ such that $(F, A) \tilde{\subseteq} (U, A)$. To show that $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} (U, A)$, let $P_e^x \tilde{\in} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)$. Then by hypothesis, $(F, A) \tilde{\cap} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x) \tilde{\neq} \tilde{\phi}$. So, let $P_e^y \tilde{\in} (F, A) \tilde{\cap} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x)$ for a soft point $P_e^y \tilde{\in} \tilde{X}$ such that $y \neq x$. Thus, $P_e^y \tilde{\in} (F, A) \tilde{\subseteq} (U, A)$ and $P_e^y \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x)$. By Theorem 4.1, $P_e^x \tilde{\cap} (U, A) \tilde{\neq} \tilde{\phi}$ and so, $P_e^x \tilde{\in} (U, A)$. This implies that $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} (U, A)$. Thus, (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed. □

Theorem 5.1. *Let $(F, A) \tilde{\in} SS(X)_A$. Then the following hold:*

1. *If (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed, then $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A)$ does not contain any non-null soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed set in $(X, \tilde{\tau}, A)$.*
2. *If both $\tilde{\gamma}$ and $\tilde{\gamma}^{'}$ are soft open operators on $\tilde{\tau}$, and the soft space $(X, \tilde{\tau}, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-regular, then the converse of (1) is true.*

Proof. (1) Suppose that $(E, A) \tilde{\neq} \tilde{\phi}$ is a soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed set in $\tilde{X}$ such that $(E, A) \tilde{\subseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A)$. Then $(E, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} (F, A)$ and so, $(F, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} (E, A)$. Since $\tilde{X} \tilde{\setminus} (E, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open and (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} (E, A)$. That is, $(E, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)$. Therefore, $(E, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\cap} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\cap} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) = \tilde{\phi}$. Thus, $(E, A) = \tilde{\phi}$. But this is a contradiction. Hence, $(E, A) \tilde{\nsubseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A)$.

(2) Let (U, A) be soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open such that $(F, A) \tilde{\subseteq} (U, A)$. So, by hypothesis and Theorem 4.4 (3), $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed in $\tilde{X}$. Thus, we have $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\cap} \tilde{X} \tilde{\setminus} (U, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed in $\tilde{X}$. Since $\tilde{X} \tilde{\setminus} (U, A) \tilde{\subseteq} \tilde{X} \tilde{\setminus} (F, A)$, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\cap} \tilde{X} \tilde{\setminus} (U, A) \tilde{\subseteq} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A)$. Therefore, by using the assumption of the converse of (1), we obtain that $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) = \tilde{\phi}$. This implies that $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} (U, A)$. Thus, (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed in $(X, \tilde{\tau}, A)$. □

Corollary 5.1. *Let (F, A) be soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed of $(X, \tilde{\tau}, A)$. Then (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed if and only if $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed in $(X, \tilde{\tau}, A)$.*

Proof. Necessity: Let (F, A) be $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed in $(X, \tilde{\tau}, A)$. It follows from Lemma 4.1 (3) that $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) = (F, A)$ and hence $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A) = \tilde{\phi}$ which is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed.

Sufficiency: Suppose $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A)$ is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed and (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed. It follows from Theorem 5.1 (1) that $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A)$ does not contain any non-null soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed in $(X, \tilde{\tau}, A)$. Since $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A)$ is a soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed subset of itself, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\setminus} (F, A) = \tilde{\phi}$ implies $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\cap} \tilde{X} \tilde{\setminus} (F, A) = \tilde{\phi}$. Hence, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) = (F, A)$. Therefore, by Lemma 4.1 (3), we obtain (F, A) is a soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed set in $(X, \tilde{\tau}, A)$. □

Proposition 5.2. *If (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed and soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open set of $\tilde{X}$, then (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed.*

Proof. Since (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed and soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open set in $\tilde{X}$, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F, A) \tilde{\subseteq} (F, A)$ and hence by Lemma 4.1 (3), (F, A) is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed. □

Proposition 5.3. *For each $P_e^x \tilde{\in} \tilde{X}$, P_e^x is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed or $\tilde{X} \tilde{\setminus} P_e^x$ is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed.*

Proof. Suppose that P_e^x is not soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-closed. Then $\tilde{X} \tilde{\setminus} P_e^x$ is not soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open. So, $\tilde{X}$ is the only soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open set containing $\tilde{X} \tilde{\setminus} P_e^x$. Thus, $\tilde{X} \tilde{\setminus} P_e^x$ is soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed. □

Definition 5.2. *Let $(F,A) \tilde{\in} SS(X)_A$. Then the $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-kernel of (F,A), denoted by $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-ker$(F,A)$, is defined as follows:*

$$\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\text{-}ker(F,A) = \tilde{\bigcap}\{(U,A)\text{: } (F,A) \tilde{\subseteq} (U,A) \text{ and } (U,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}\}$$

That is, $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-ker$(F,A)$ is the intersection of all soft $(\tilde{\gamma},\tilde{\gamma}')$-open sets of $(X,\tilde{\tau},A)$ containing (F,A).

Theorem 5.2. *Let $(F,A) \tilde{\in} SS(X)_A$. Then (F,A) is soft $(\tilde{\gamma},\tilde{\gamma}')$-g.closed if and only if $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq}$ $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-ker$(F,A)$.*

Proof. Necessity: Suppose that (F,A) is soft $(\tilde{\gamma},\tilde{\gamma}')$-$g$.closed. Then $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} (U,A)$, whenever $(U,A) \tilde{\supseteq} (F,A)$ and (U,A) is soft $(\tilde{\gamma},\tilde{\gamma}')$-open. Let $P_e^x \tilde{\in} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$. Hence, by Lemma 5.1, $(F,A) \tilde{\cap}$ $\tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x) \tilde{\neq} \tilde{\phi}$. So, there exists a soft point $P_e^z \tilde{\in} \tilde{X}$ such that $z \neq x$ and $P_e^z \tilde{\in} (F,A) \tilde{\cap} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x)$ implies that $P_e^z \tilde{\in} (F,A) \tilde{\subseteq} (U,A)$ and $P_e^z \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(P_e^x)$. It follows from Theorem 4.1 that $P_e^x \tilde{\cap} (U,A)$ $\tilde{\neq} \tilde{\phi}$. Hence we show that $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$ker(F,A)$. Thus, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$ker(F,A)$.

Sufficiency: Let $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$ker(F,A)$. Let $(U,A) \tilde{\supseteq} (F,A)$ where (U,A) is soft $(\tilde{\gamma},\tilde{\gamma}')$-open in $\tilde{X}$. Let P_e^x be a soft point in $\tilde{X}$ such that $P_e^x \tilde{\in} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$. Then $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$ker(F,A)$. So, we have P_e^x $\tilde{\in} (U,A)$, because $(U,A) \tilde{\supseteq} (F,A)$ and $(U,A) \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$. That is, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$ker(F,A) \tilde{\subseteq} (U,A)$. Thus, (F,A) is soft $(\tilde{\gamma},\tilde{\gamma}')$-$g$.closed in $\tilde{X}$. □

Definition 5.3. *A soft set (K,A) of a soft space $(X,\tilde{\tau},A)$ is said to be soft $(\tilde{\gamma},\tilde{\gamma}')$-g.open if its complement $\tilde{X}\tilde{\setminus}(K,A)$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-g.closed in $(X,\tilde{\tau},A)$.*

Proposition 5.4. *A soft set (K,A) of a soft space $(X,\tilde{\tau},A)$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-g.open if and only if $(E,A) \tilde{\subseteq}$ $Int_{(\tilde{\gamma},\tilde{\gamma}')}(K,A)$ whenever $(E,A) \tilde{\subseteq} (K,A)$ and (E,A) is soft $(\tilde{\gamma},\tilde{\gamma}')$-closed.*

In the end of this section, we introduce the notion of soft $(\tilde{\gamma},\tilde{\gamma}')$-$T_{\frac{1}{2}}$ space and investigate some of its properties.

Definition 5.4. *A soft space $(X,\tilde{\tau},A)$ is said to be soft $(\tilde{\gamma},\tilde{\gamma}')$-$T_{\frac{1}{2}}$ if every soft $(\tilde{\gamma},\tilde{\gamma}')$-g.closed set of $(X,\tilde{\tau},A)$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-closed.*

Theorem 5.3. *A soft space $(X,\tilde{\tau},A)$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-$T_{\frac{1}{2}}$ if and only if for each $P_e^x \tilde{\in} \tilde{X}$, the soft set P_e^x is soft $(\tilde{\gamma},\tilde{\gamma}')$-closed or soft $(\tilde{\gamma},\tilde{\gamma}')$-open.*

Proof. Necessity: Suppose that P_e^x is not soft $(\tilde{\gamma},\tilde{\gamma}')$-closed in $(X,\tilde{\tau},A)$. By Proposition 5.3, we have $\tilde{X}\tilde{\setminus}P_e^x$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-$g$.closed. Since $(X,\tilde{\tau},A)$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-$T_{\frac{1}{2}}$, $\tilde{X}\tilde{\setminus}P_e^x$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-closed and hence P_e^x is soft $(\tilde{\gamma},\tilde{\gamma}')$-open.

Sufficiency: Let (F,A) be any soft $(\tilde{\gamma},\tilde{\gamma}')$-$g$.closed. Then, we claim that $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) = (F,A)$ holds. It is sufficient to show that $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} (F,A)$. Let $P_e^x \tilde{\in} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)$. By the assumption, P_e^x is soft $(\tilde{\gamma},\tilde{\gamma}')$-closed or soft $(\tilde{\gamma},\tilde{\gamma}')$-open. So there are two cases:

1[st] Case: If P_e^x is soft $(\tilde{\gamma},\tilde{\gamma}')$-closed and $P_e^x \tilde{\notin} (F,A)$, then $P_e^x \tilde{\in} Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A)\tilde{\setminus}(F,A)$ contains a non-null soft $(\tilde{\gamma},\tilde{\gamma}')$-closed set P_e^x. Since (F,A) is soft $(\tilde{\gamma},\tilde{\gamma}')$-$g$.closed and according to Theorem 5.1 (1), we obtain a contradiction. Hence, $P_e^x \tilde{\in} (F,A)$. Thus, $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} (F,A)$ and so $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) = (F,A)$. Hence by Lemma 4.1 (3), (F,A) is soft $(\tilde{\gamma},\tilde{\gamma}')$-closed in $(X,\tilde{\tau},A)$. Therefore, $(X,\tilde{\tau},A)$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-$T_{\frac{1}{2}}$.

2[nd] Case: If P_e^x is soft $(\tilde{\gamma},\tilde{\gamma}')$-open, then by Theorem 4.1, $(F,A) \tilde{\cap} P_e^x \tilde{\neq} \tilde{\phi}$ because $P_e^x \tilde{\in} \tilde{\tau}_{(\tilde{\gamma},\tilde{\gamma}')}$-$Cl(F,A)$. This implies that $P_e^x \tilde{\in} (F,A)$. So $Cl_{(\tilde{\gamma},\tilde{\gamma}')}(F,A) \tilde{\subseteq} (F,A)$. Thus by Lemma 4.1 (3), (F,A) is soft $(\tilde{\gamma},\tilde{\gamma}')$-closed. Thus, $(X,\tilde{\tau},A)$ is soft $(\tilde{\gamma},\tilde{\gamma}')$-$T_{\frac{1}{2}}$. □

The following corollary follows directly from Theorem 5.3, Proposition 5.1 and Proposition 3.1 (3).

Corollary 5.2. *For any soft space $(X, \tilde{\tau}, A)$, the following are equivalent:*

1. $(X, \tilde{\tau}, A)$ *is soft* $(\tilde{\gamma}, \tilde{\gamma})$-$T_{\frac{1}{2}}$.
2. $(X, \tilde{\tau}, A)$ *is soft* $\tilde{\gamma}$-$T_{\frac{1}{2}}$.
3. *For each* $P_e^x \tilde{\in} \tilde{X}$, *the soft set* P_e^x *is soft* $\tilde{\gamma}$*-closed or soft* $\tilde{\gamma}$*-open.*

Proposition 5.5. *If* $(X, \tilde{\tau}, A)$ *is soft* $(\tilde{\gamma}, \tilde{\gamma}')$-$T_{\frac{1}{2}}$*, then it is soft* $\tilde{\gamma}$-$T_{\frac{1}{2}}$ *and soft* $\tilde{\gamma}'$-$T_{\frac{1}{2}}$.

Proof. It follows from Theorem 5.3, Corollary 5.2 and Proposition 3.1 (3). □

The following example shows that the converse of Proposition 5.5 is not true in general.

Example 5.2. *Let* $X = \{a_1, a_2\}$, $A = \{e_1, e_2\}$ *and* $\tilde{\tau} = \{\tilde{\phi}, \tilde{X}, (F_1, A), (F_2, A), (F_3, A), (F_4, A), (F_5, A), (F_6, A), (F_7, A), (F_8, A)\}$ *where*
$(F_1, A) = \{(e_1, \{a_1\}), (e_2, \phi)\}$,
$(F_2, A) = \{(e_1, \{a_2\}), (e_2, \phi)\}$,
$(F_3, A) = \{(e_1, \phi), (e_2, \{a_2\})\}$,
$(F_4, A) = \{(e_1, \{a_1\}), (e_2, \{a_2\})\}$,
$(F_5, A) = \{(e_1, \{a_2\}), (e_2, \{a_2\})\}$,
$(F_6, A) = \{(e_1, \{a_2\}), (e_2, X)\}$,
$(F_7, A) = \{(e_1, X), (e_2, \{a_2\})\}$ *and*
$(F_8, A) = \{(e_1, X), (e_2, \phi)\}$.
Then $(X, \tilde{\tau}, A)$ *is a soft topological space over* X. *Let* $\tilde{\gamma} : \tilde{\tau} \to SS(X)_A$ *and* $\tilde{\gamma}' : \tilde{\tau} \to SS(X)_A$ *be operators defined as follows: For all* $(F, A) \tilde{\in} \tilde{\tau}$,

$$\tilde{\gamma}(F,A) = \begin{cases} (F,A) & \text{if } P_{e_1}^{a_1} \tilde{\in} (F,A) \\ Int(Cl(F,A)) & \text{if } P_{e_1}^{a_1} \tilde{\notin} (F,A) \tilde{\neq} (F_6,A) \\ \tilde{X} & \text{if } (F,A) = (F_6,A) \end{cases}$$

and

$$\tilde{\gamma}'(F,A) = \begin{cases} (F,A) & \text{if } (F,A) = (F_2,A) \text{ or } (F,A) = (F_3,A) \\ & \text{or } (F,A) = (F_5,A) \text{ or } (F,A) = (F_7,A) \\ \tilde{X} & \text{otherwise.} \end{cases}$$

It is clear that $\tilde{\tau}_{\tilde{\gamma}} = \tilde{\tau} \tilde{\setminus} \{(F_5, A), (F_6, A)\}$ *and*
$\tilde{\tau}_{\tilde{\gamma}'} = \{\tilde{\phi}, \tilde{X}, (F_2, A), (F_3, A), (F_6, A), (F_7, A)\}$.
So, $\tilde{\tau}_{(\tilde{\gamma}, \tilde{\gamma}')} = \{\tilde{\phi}, \tilde{X}, (F_2, A), (F_3, A), (F_7, A)\}$. *Therefore,* $(X, \tilde{\tau}, A)$ *is both soft* $\tilde{\gamma}$-$T_{\frac{1}{2}}$ *and soft* $\tilde{\gamma}'$-$T_{\frac{1}{2}}$. *However,* $(X, \tilde{\tau}, A)$ *is not soft* $(\tilde{\gamma}, \tilde{\gamma}')$-$T_{\frac{1}{2}}$*, because the soft set* $P_{e_1}^{a_1} = (F_1, A)$ *is neither soft* $(\tilde{\gamma}, \tilde{\gamma}')$*-closed nor soft* $(\tilde{\gamma}, \tilde{\gamma}')$*-open.*

6. Conclusions

Researchers and scientists proposed different approaches to handle problems of uncertainty. Among them, soft set theory has received the attention of the topologists who always seek to generalize and apply the topological notions on different structures.

As a contribution to this area, we have presented and studied the concepts of bioperators $\tilde{\gamma}$ and $\tilde{\gamma}^{'}$ on soft topology $\tilde{\tau}$, and the notion of soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-open sets. Then, we have defined two soft closure and two soft interior operators, and elucidated the relationships between them. Finally, we have initiated the concepts of soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$g$.closed sets and soft $(\tilde{\gamma}, \tilde{\gamma}^{'})$-$T_{\frac{1}{2}}$ spaces and investigated main properties.

It was investigated in [10] the interchangeable property of soft interior and closure operators between soft sets and and their components. In the upcoming work, we will study, by making use of this property, the transmission of the concepts given herein from soft topology to its parametric topology and vise versa. Also, we will investigate this work in the contents of supra soft topology and fuzzy soft topology.

Conflict of interest

The authors declare that they have no competing interest.

Acknowledgements

The authors would like to thank the referees for their valuable comments which help us to improve the manuscript.

References

1. M. I. Ali, F. Feng, X. Liu, W. K. Min, M. Shabir, On some new operations in soft set theory, *Comput. Math. Appl.*, **57** (2009), 1547–1553.
2. T. M. Al-shami, Comments on "Soft mappings spaces", *Sci. World J.*, **2019** (2019), 6903809.
3. T. M. Al-shami, Investigation and corrigendum to some results related to g-soft equality and gf-soft equality relations, *Filomat*, **33** (2019), 3375–3383.
4. T. M. Al-shami, Comments on some results related to soft separation axioms, *Afrika Mat.*, **31** (2020), 1105–1119.
5. T. M. Al-shami, Compactness on soft topological ordered spaces and its application on the information system, *J. Math.*, **2021** (2021), 6699092.
6. T. M. Al-shami, On soft separation axioms and their applications on decision-making problem, *Math. Probl. Eng.*, **2021** (2021), 8876978.
7. T. M. Al-shami, E. A. Abo-Tabl, Soft α-separation axioms and α-fixed soft points, *AIMS Math.*, **6** (2021), 5675–5694.
8. T. M. Al-shami, M. E. El-Shafei, T-soft equality relation, *Turk. J. Math.*, **44** (2020), 1427–1441.
9. T. M. Al-shami, M. E. El-Shafei, M. Abo-Elhamayel, On soft topological ordered spaces, *J. King Saud Univ. Sci.*, **31** (2019), 556–566.
10. T. M. Al-shami, L. D. R. Kočinac, The equivalence between the enriched and extended soft topologies, *Appl. Comput. Math.*, **18** (2019), 149–162.
11. S. S. Benchalli, P. G. Patil, N. S. Kabbur, On soft γ-operations in soft topological spaces, *J. New Theory*, **6** (2015), 20–32.

12. B. Chen, The parametrization reduction of soft sets and its applications, *Comput. Math. Appl.*, **49** (2005), 757–763.
13. S. Das, S. K. Samanta, Soft metric, *Ann. Fuzzy Math. Inform.*, **6** (2013), 77–94.
14. M. E. El-Shafei, T. M. Al-shami, Applications of partial belong and total non-belong relations on soft separation axioms and decision-making problem, *Comput. Appl. Math.*, **39** (2020), 138.
15. M. E. El-Shafei, M. Abo-Elhamayel, T. M. Al-shami, Partial soft separation axioms and soft compact spaces, *Filomat*, **32** (2018), 4755–4771.
16. F. Feng, Y. B. Jun, X. Zhao, Soft semirings, *Comput. Math. Appl.*, **56** (2008), 2621–2628.
17. F. Feng, Y. M. Li, B. Davvaz, M. I. Ali, Soft sets combined with fuzzy sets and rough sets: a tentative approach, *Soft Comput.*, **14** (2010), 899–911.
18. D. N. Georgiou, A. C. Megaritis, V. I. Petropoulos, On soft topological spaces, *Appl. Math. Inform. Sci.*, **7** (2013), 1889–1901.
19. S. Hussain, B. Ahmad, Some properties of soft topological spaces, *Comput. Math. Appl.*, **62** (2011), 4058–4067.
20. D. S. Jankovic, On functions with α-closed graphs, *Glasnik Mat.*, **18** (1983), 141–148.
21. N. Kalaivani, D. Saravanakumar, G. S. S. Krishnan, On γ-operations in soft topological spaces, *Far East J. Math. Sci.*, **101** (2017), 2067–2077.
22. A. Kalavathia, *Studies on generalizations of soft closed sets and their operation approaches in soft topological spaces*, PhD Thesis, Anna University, 2017.
23. A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh, A. M. Abd El-latif, Soft semi compactness via soft ideals, *Appl. Math. Inform. Sci.*, **8** (2014), 2297–2306.
24. S. Kasahara, Operation compact spaces, *Math. Jpn.*, **24** (1979), 97–105.
25. A. Kharal, B. Ahmad, Mappings on soft classes, *New Math. Nat. Comput.*, **7** (2011), 471–481.
26. Z. Kong, L. Gao, L. Wong, S. Li, The normal parameter reduction of soft sets and its algorithm, *Comput. Math. Appl.*, **56** (2008), 3029–3037.
27. P. K. Maji, R. Biswas, R. Roy, An application of soft sets in a decision making problem, *Comput. Math. Appl.*, **44** (2002), 1077–1083.
28. P. K. Maji, R. Biswas, R. Roy, Soft set theory, *Comput. Math. Appl.*, **45** (2003), 555–562.
29. D. Molodtsov, Soft set theory-first results, *Comput. Math. Appl.*, **37** (1999), 19–31.
30. H. Ogata, Operation on topological spaces and associated topology, *Math. Jpn.*, **36** (1991), 175–184.
31. D. Pie, D. Miao, *From soft sets to information systems,* 2005 IEEE International Conference on Granular Computing, 617–621.
32. M. Shabir, M. Naz, On soft topological spaces, *Comput. Math. Appl.*, **61** (2011), 1786–1799.
33. J. Umehara, H. Maki, T. Noiri, Bioperations on topological spaces and some separation axioms, *Mem. Fac. Sci. Kochi Univ. Ser. A (Math.)*, **13** (1992), 45–59.
34. Y. Yumak, A. K. Kaymakci, Soft β-open sets and their applications, *J. New Theory*, **4** (2015), 80–89.

35. I. Zorlutuna, M. Akdag, W. K. Min, S. Atmaca, Remarks on soft topological spaces, *Ann. Fuzzy Math. Inform.*, **3** (2012), 171–185.

36. I. Zorlutuna, H. Cakir, On continuity of soft mappings, *Appl. Math. Inform. Sci.*, **9** (2015), 403–409.

AIMS Press