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Research article

Complexity of signed total k-Roman domination problem in graphs

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Abstract: Let G be a simple graph with finite vertex set $V(G)$ and $S = \{-1, 1, 2\}$. A signed total Roman k-dominating function (STRkDF) on a graph G is a function $f : V(G) \to S$ such that (i) any vertex y with $f(y) = -1$ is adjacent to at least one vertex t with $f(t) = 2$, (ii) $\sum_{t \in N(y)} f(t) \geq k$ holds for any vertex y. The *weight* of an STRkDF f, denoted by $\omega(f)$, is $\sum_{y \in V(G)} f(y)$, and the minimum weight of an STRkDF is the *signed total Roman k-domination number*, $\gamma^k_{stR}(G)$, of G. In this article, we prove that the decision problem for the signed total Roman k-domination is NP-complete on bipartite and chordal graphs for $k \in \{1, 2\}$.

Keywords: signed total Roman k-dominating function; signed total Roman k-domination number; complexity
Mathematics Subject Classification: 05C69

1. Introduction

In this paper, G is a simple graph with finite vertex set $V = V(G)$ and edge set $E = E(G)$. Assume $s \in V(G)$ and $S = \{-1, 1, 2\}$, we will use the following notations.

name	*symbol*	*definition*
order	n or $n(G)$	the vertex number of G
the *open neighborhood* of s	$N(s)$	$N(s) = \{u \in V(G) \mid us \in E(G)\}$
the *closed neighborhood* of s	$N[s]$	$N[s] = \{s\} \cup N(s)$
the *degree* of s	$\deg_G(s)$	$\deg_G(s) = \lvert N(s)\rvert$
a *leaf* of G		a vertex of degree 1
a *support vertex* of G		a vertex adjacent to a leaf
a *strong support vertex* of G		a vertex adjacent to at least two leaves
leaf neighbors	L_s	the set of leaves adjacent to s
the minimum degree of G	$\delta(G)$	$\delta(G) = \min\{\deg_G(s) \mid s \in V(G)\}$

If there is a function f meeting the following conditions: (i) each vertex y with $f(y) = -1$ has at least one neighbor t with $f(t) = 2$, (ii) $\sum_{t\in N(y)} f(t) \geq k$ for any vertex $y \in V$, and then f is called a signed total Roman k-dominating function (STRkDF). Let $\mathfrak{F}(G)$ denote the set of all the STRkDFs of G. The *weight* of an STRkDF f, denoted by $\omega(f)$, is defined to be the value $\sum_{y\in V(G)} f(y)$. The *signed total Roman k-domination number* of G, denoted $\gamma^k_{stR}(G)$, is weight of an STRkDF f where $\omega(f) = \min\{\omega(g) \mid g \in \mathfrak{F}(G)\}$. An STRkDF of weight $\gamma^k_{stR}(G)$ is called a $\gamma^k_{stR}(G)$-function. The signed total Roman k-domination number must exist if $\delta(G) \geq \frac{k}{2}$. For an STRkDF f, let $V_r = \{t \in V(G) \colon f(t) = r\}$ for $r \in S$. Because this partition determines f, we then write $f = (V_{-1}, V_1, V_2)$ equivalently. The signed total Roman domination number and signed total k-domination number was introduced and investigated in [12, 14]. This parameter is introduced and investigated in a more general setting [9, 11]. There are several works that considered the decision problems for the signed Roman domination parameters (see [1–3, 13]). For more details on Roman domination and its variants, we refer the reader to the recent book chapters and surveys [4–8].

In this article, we will show that the decision problems for the signed total Roman k-domination numbers for $k \in \{1, 2\}$ are NP-hard. In other words, there are no polynomial algorithms to compute this parameter unless P=NP.

2. Complexity result

In this section we will give the NP-complete result for the signed total Roman k-domination problem on bipartite and chordal graphs for $k \in \{1, 2\}$.

Signed total Roman k-domination problem(STRkDP) for k $\in \{1, 2\}$:
Instance: A *graph G and a positive integer* $\ell \leq |V(G)|$.
Question: Does G have an STRkDF with weight at most ℓ ?

We will prove that STRkDP is NP-complete by reducing the especial case of Exact Cover by 3-sets (X3C) to which we refer as X3C3. The NP-completeness of X3C3 was proven in 2008 by Hickey et al. [10].

X3C3
Instance: A set of elements X and a collection $\mathscr{C}$ of m 3-element subsets of X where $| X |= m = 3q$, with the condition that every element appears in exactly 3 members of $\mathscr{C}$.
Question: Does there exist a subcollection $\mathscr{C}' \subset \mathscr{C}$ with the condition that each element of X appears in exactly one member of $\mathscr{C}'$?

Now we show that the problem above is NP-hard, even when restricted to the case $k = 1$ and to bipartite and chordal graphs.

Theorem 1. *Problem STR1DP is NP-Complete for bipartite and chordal graphs.*

Proof. Clearly STR1DP is a member of NP since we can verify that a function $f : V(G) \to S$ has weight at most ℓ and determine whether $f \in \mathfrak{F}(G)$ in polynomial time. Now let us transform any instance of X3C3 into an instance G of STR1DP satisfying that STR1DP has a solution if and only if X3C3 has a solution. Let $X = \{x_1, x_2, \ldots, x_{3q}\}$ and $\mathscr{C} = \{C_1, C_2, \ldots, C_{3q}\}$ be an arbitrary instance of X3C3.

First we construct the bipartite graph G_1. For each $x_i \in X$, we create a single vertex x_i to which we associate a copy of the graph H_i, obtained from a cycle $u_i p_i v_i y_i u_i$ by adding two pendant edges at each of vertices u_i, p_i, v_i, as shown in Figure 1 by adding the edge $y_i x_i$. For each C_j, we create a vertex c_j to which we associate a copy of the graph H'_j as shown in Figure 1 by adding the edges $c_j z_j$ and $c_j w_j$. Now to obtain the graph G_1, we add edges $c_j x_i$ if $x_i \in C_j$. Since G_1 has no cycle of odd length, G_1 is a bipartite graph (see Figure 2). Now let $A = \{c_1, c_2, \ldots, c_{3q}\}$ and set $\ell = q$.

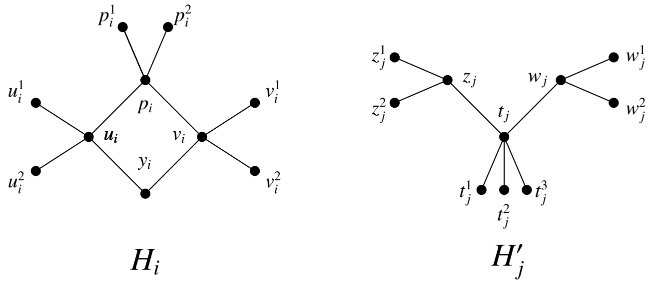

Figure 1. The graphs H_i and H'_j.

To prove that this is indeed a transformation, we only need to show that $\gamma^1_{stR}(G_1) \le \ell$ if, and only if, there is a truth assignment for X that satisfies all clauses in $\mathscr{C}$. This aim can be obtained the following:

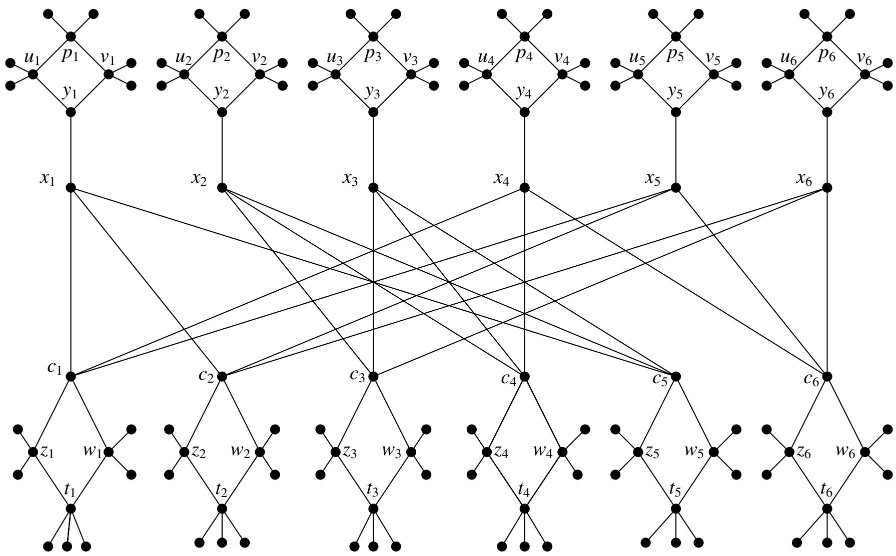

Figure 2. NP-completeness of STR1D for bipartite graphs, here $q = 2$ and $\gamma^1_{stR}(G_1) = 2$.

Suppose that $\mathscr{C}'$ is a solution of X3C3. Define the signed total 1- Roman dominating function f on G_1 of weight ℓ as follows: for every $i \in \{1, ..., 3q\}$, let $f(u_i) = f(v_i) = f(p_i) = f(z_i) = f(w_i) = f(t_i) = 2$ and $f(y_i) = 1$; for every $C_j \in \mathscr{C}'$, let $f(c_j) = 2$ and for every $C_j \notin \mathscr{C}'$ let $f(c_j) = 1$; and let $f(x_i) = -1$. Note that since $\mathscr{C}'$ exists, $|\mathscr{C}'| = q$, and so the number of c_j's with weight 2 is q, having disjoint neighborhoods in $\{x_1, x_2, \ldots, x_{3q}\}$. Now it is straightforward to see that f is a signed total Roman 1-dominating function with weight $\omega(f) = q = \ell$.

Conversely, assume there exists a function $h \in \mathfrak{F}(G_1)$ with $\omega(h) \le \ell$. Among all these functions, let $f = (V_{-1}, V_1, V_2)$ be one such function that assigns smallest possible values to the leaves of G_1. Clearly f assigns a positive value to each support vertex. We claim that $f(x) = -1$ for any leaf x of G_1. Suppose, to the contrary, that $f(x) \ge 1$ for some leaf of G_1. Without loss of generality that we may assume that $x \in V(H_1) \cup V(H_1')$. First let $x \in V(H_1)$. If $f(p_1^1) \geq 1$ and $f(p_1^2) \geq 1$, then the function g defined by $g(p_1) = 2, g(p_1^1) = -1$, $g(p_1^2) = 1$ and $g(u) = f(u)$ otherwise, is a STR1DF of G_1 of weight less than $\omega(f)$ which is a contradiction. Thus we may assume that $f(p_1^1) = -1$. It follows that $f(p_1) = 2$. Likewise, we may assume that $f(u_1^1) = f(v_1^1) = -1$ implying that $f(u_1) = f(v_1) = 2$. Since f has minimum weight, we deduce that $f(p_1^2) = -1$. Hence $x \in \{u_1^2, v_1^2\}$. If $f(y_1) \geq 1$, then similarly we have $f(u_1^2) = -1$ and $f(v_1^2) = -1$ which leads to a contradiction. Hence $f(y_1) = -1$. Now the function g defined by $g(x) = -1$, $g(y_1) = 1$ and $g(u) = f(u)$ otherwise, is a STR1DF of G_1 of weight $\omega(f)$ contradicting the choice of f. Now let $x \in V(H_1)$. If $f(t_1^i) \geq 1$ and $f(t_1^k) \geq 1$ for some $i \neq k$, then the function g defined by $g(t_1^i) = -1, g(t_1) = 2$ and $g(u) = f(u)$ otherwise, is a STR1DF of G_1 of weight less than $\omega(f)$ which is a contradiction. Thus we may assume without loss of generality that $f(t_1^1) = f(t_1^2) = -1$. It follows that $f(t_1) = 2$. Then we have $f(t_1) + f(c_1) \geq 1$. If $f(z_1^1) \geq 1$ and $f(z_1^2) \geq 1$, then the function g defined by $g(z_1^1) = -1, g(z_1) = 2$ and $g(u) = f(u)$ otherwise, is a STR1DF of G_1 of weight less than $\omega(f)$ which is a contradiction again. Hence we assume that $f(z_1^1) = -1$. Likewise, we may assume that $f(w_1^1) = -1$. Since f is a STR1DF of G_1, we must have $f(z_1) = f(w_1) = 2$. Since f has minimum weight, we deduce that $f(t_1^3) = -1$. Thus $x \in \{z_1^2, w_1^2\}$. If $f(c_1) \geq 1$, then the function g defined by $g(z_1^2) = g(w_1^2) = -1$ and $g(u) = f(u)$, otherwise is a STR1DF of G_1 of weight less than $\omega(f)$ which is a contradiction again. Hence $f(c_1) = -1$. Now the function g defined by $g(x) = -1$, $g(c_1) = 1$ and $g(u) = f(u)$ otherwise, is a STR1DF of G_1 of weight $\omega(f)$ contradicting the choice of f. This proves the claim. Thus $f(y) = 2$ for every support vertex y of G_1. Since $\sum_{u \in N(z_i)-\{c_j\}} f(u) = \sum_{u \in N(w_i)-\{c_j\}} f(u) = 0$, we must have $f(c_j) \geq 1$ for every j. Also, we observe that $\sum_{u \in N(u_i)-\{y_i\}} f(u) = 0$ and $\sum_{u \in N(v_i)-\{y_i\}} f(u) = 0$, and thus we must have $f(y_i) \geq 1$ for every i. It follows clearly that no x_i needs to be assigned a positive value under f, and thus $x_i \in V_{-1}$ for every i. If $y_i \in V_2$ for some i, then we can reassign y_i and any c_r adjacent to x_i the values 1 and 2, respectively. So we may assume that $y_i \in V_1$ for every i. Now, since f has weight at most $\ell = q$, and every x_i needs to be adjacent to a vertex assigned a 2, there must exist q vertices of A assigned a 2 under f and the remaining vertices of A belongs to V_1. On the other hand, since each c_j has exactly three neighbors in X, we conclude that $\mathscr{C}' = \{C_j : f(c_j) = 2\}$ is an exact cover for $\mathscr{C}$.

Now we construct the chordal graph G_2. For each $x_i \in X$, we create a single vertex x_i to which we associate a copy of the graph F_i, obtained from a cycle $u_i p_i v_i y_i u_i$ by adding the edge $u_i v_i$, adding one pendant edge at p_i and three pendant edges at u_i and v_i, as shown in Figure 3 by adding the edge $y_i x_i$. For each C_j we create a vertex c_j to which we associate a copy of the graph F_j' as shown in Figure 3, by adding the edges $c_j z_j$ and $c_j w_j$. Now to obtain the graph G_2, we add edges $c_j x_i$ if $x_i \in C_j$ and all edges between vertices c_j's. Clearly, any cycle in G_2 with length at least four has a chord and hence G_2 is a chordal graph (see Figure 4). Let $A = \{c_1, c_2, \ldots, c_{3q}\}$ and set $\ell = q$.

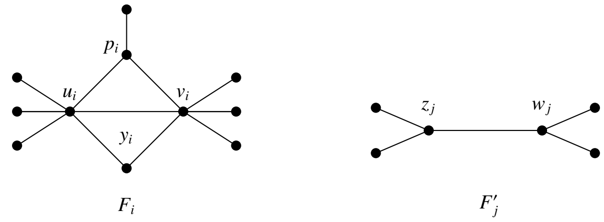

Figure 3. The graphs F_i and F_j'.

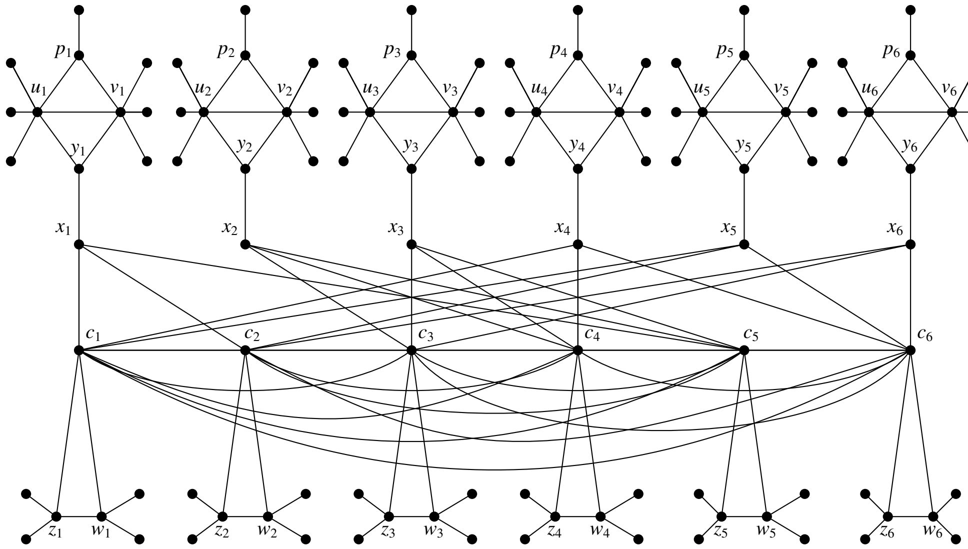

Figure 4. NP-completeness of STR1D for chordal graphs , here $q = 2$ and $\gamma^1_{stR}(G_2) = 2$.

To prove that this is indeed a transformation, we only need to show that $\gamma^1_{stR}(G_2) \leq \ell$ if, and only if, there is a truth assignment for X that satisfies all clauses in $\mathscr{C}$. This aim can be obtained the following: Suppose that $\mathscr{C}'$ is a solution of $X3C3$. Define a signed total 1-Roman dominating function g on G_2 of weight ℓ as follows: for every $i \in \{1, 2, \ldots, 3q\}, g(u_i) = g(v_i) = g(z_i) = g(w_i) = g(p_i) = 2, g(y_i) = 1$; if $C_j \in \mathscr{C}'$, then let $g(c_j) = 2$ and if $C_j \notin \mathscr{C}'$, then let $g(c_j) = 1$; and let $g(x_i) = -1$. Note that since $\mathscr{C}'$ exists, $|\mathscr{C}'| = q$, and so the number of c_j's with weight 2 is q, having disjoint neighborhoods in $\{x_1, x_2, \ldots, x_{3q}\}$. Now it is straightforward to see that g is a signed total Roman 1- dominating function with weight $\omega(g) = q = \ell$.

Conversely, assume there exists a function $h \in \mathfrak{F}(G_2)$ with $\omega(h) \leq \ell$. Among all these functions, let $g = (V_{-1}, V_1, V_2)$ be one such function that assigns smallest possible values to the leaves of G_2. As in the proof for bipartite graph, we can show that $g(x) = -1$ for any leaf x of G_2, and thus $g(y) = 2$ for every support vertex y of G_2. Since $\sum_{u\in N(z_i)-\{c_j\}} g(u) = \sum_{u\in N(w_i)-\{c_j\}} g(u) = 0$, we must have $g(c_j) \geq 1$ for every j. Also, for G_2 we observe that $\sum_{u\in N(u_i)-\{y_i\}} g(u) = 1$ and$\sum_{u\in N(v_i)-\{y_i\}} g(u) = 1$, and thus we must have $g(y_i) \geq 1$ for every i. It follows clearly that no x_i needs to be assigned a positive value under g and thus $x_i \in V_{-1}$ for every i. If $y_i \in V_2$ for some i, then we can reassign y_i and any c_r adjacent to x_i the values 1 and 2, respectively. So we may assume that $y_i \in V_1$ for every i. Now, since g has weight at most $\ell = q$, and every x_i needs to be adjacent to a vertex assigned a 2, there must exist q vertices of A assigned a 2 under g and the remaining vertices of A belongs to V_1. On the other hand, since each c_j has exactly three neighbors in X, we conclude that $\mathscr{C}' = \{C_j : g(c_j) = 2\}$ is an exact cover for $\mathscr{C}$.

The case $k = 2$

Theorem 2. *Problem STR2DP is NP-Complete for bipartite and chordal graphs.*

Proof. Similar as the proof of the Theorem 1, clearly STR2DP is a member of NP since we can verify that a function $f : V(G) \longrightarrow S$ has weight at most ℓ and determine whether $f \in \mathfrak{F}(G)$ in polynomial time. Now let us transform any instance of X3C3 into an instance G of STR2DP satisfying that STR2DP has a solution if and only if X3C3 has a solution. Let $X = \{x_1, x_2, \ldots, x_{3q}\}$ and $\mathscr{C} =$

$\{C_1, C_2, \ldots, C_{3q}\}$ be an arbitrary instance of X3C3. We now construct the bipartite graph G_3 and the chordal graph G_4, respectively.

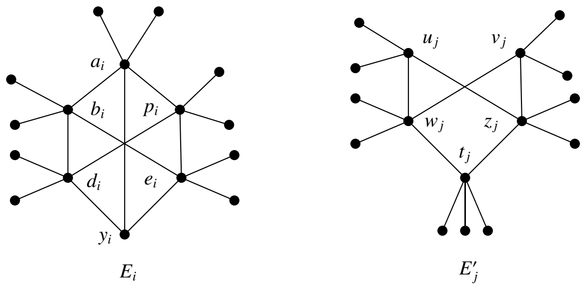

Figure 5. The graphs E_i and E'_j.

For each $x_i \in X$, we create a single vertex x_i to which we associate a copy of the graph E_i as shown in Figure 5 by adding the edge $y_i x_i$. For each C_j, we create a vertex c_j to which we associate a copy of the graph E'_j as shown in Figure 5 by adding the edges $c_j u_j$ and $c_j v_j$. Now to obtain the graph G_3, we add edges $c_j x_i$ if $x_i \in C_j$. It is clear that G_3 has no cycle of odd length and so G_3 is a bipartite graph (see Figure 6). Let $A = \{c_1, c_2, \ldots, c_{3q}\}$, and set $\ell = q$. To prove that this is indeed a transformation, we only need to show that $\gamma^2_{stR}(G_3) \leq \ell$ if, and only if, there is a truth assignment for X that satisfies all clauses in $\mathscr{C}$. This aim can be obtained the following:

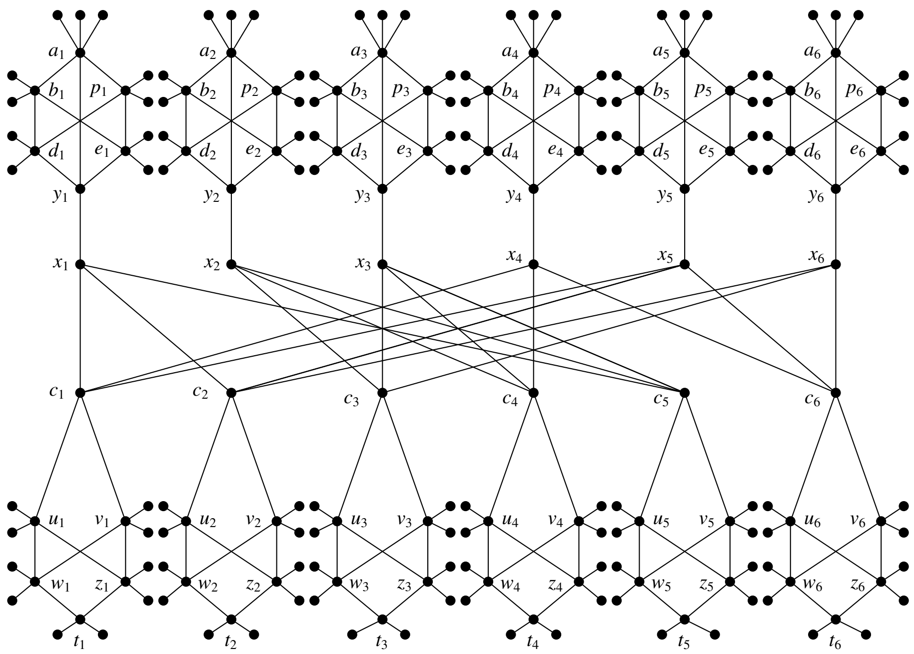

Figure 6. NP-completeness of STR2D for bipartite graphs , here $q = 2$ and $\gamma^2_{stR}(G_3) = 2$.

Suppose that $\mathscr{C}'$ is a solution of $X3C3$. Define a signed 2-Roman dominating function f on G_3 of weight ℓ as follows: for every $i \in \{1, 2, \ldots, 3q\}$ let

$$f(x)=\begin{cases}2 & \text{if } x\in\{a_i,b_i,d_i,e_i,p_i,u_i,w_i,z_i,v_i,t_i\},\\ 1 & \text{if } x=y_i,\\ 2 & \text{if } x=c_j \text{ and } c_j\in\mathscr{C}',\\ 1 & \text{if } x=c_j \text{ and } c_j\notin\mathscr{C}',\\ -1 & \text{otherwise.}\end{cases}$$

Note that since $\mathscr{C}'$ exists and $|\mathscr{C}'|=q$, the number of c_j's with weight 2 is q, having disjoin neighborhoods in $\{x_1,x_2,\ldots,x_{3q}\}$. It is easy to see that f is signed total Roman 2-dominating function with weight $\omega(f)=q$.

Conversely, first assume there exists a function $h\in\mathfrak{F}(G_3)$ with $\omega(h)\le\ell$. Among all these functions, let $f=(V_{-1},V_1,V_2)$ be one such function that assigns smallest possible values to the leaves of G_3. As in the proof of Theorem 2 for bipartite graph, we can show that $f(x)=-1$ for any leaf x of G_3, and thus $f(y)=2$ for every support vertex y of G_3. Since $\sum_{u\in N(u_i)-\{c_j\}}f(u)=\sum_{u\in N(v_i)-\{c_j\}}f(u)=2$, we must have $f(c_j)\ge 1$ for every j. Also, we observe that $\sum_{u\in N(a_i)-\{y_i\}}f(u)=1$ and thus we must have $f(y_i)\ge 1$ for every i. It follows clearly that no x_i needs to be assigned a positive value under f, and thus $x_i\in V_{-1}$ for every i. If $y_i\in V_2$ for some i, then we can reassign y_i and any c_r adjacent to x_i the values 1 and 2, respectively. So we may assume that $y_i\in V_1$ for every i. Now, since f has weight at most $\ell=q$, and every x_i needs to be adjacent to a vertex assigned a 2, there must exist q vertices of A assigned a 2 under f and the remaining vertices of A belongs to V_1. Now since each c_j has exactly three neighbors in X, we conclude that $\mathscr{C}'=\{C_j : f(c_j)=2\}$ is an exact cover for $\mathscr{C}$.

Now we construct the chordal graph G_4.

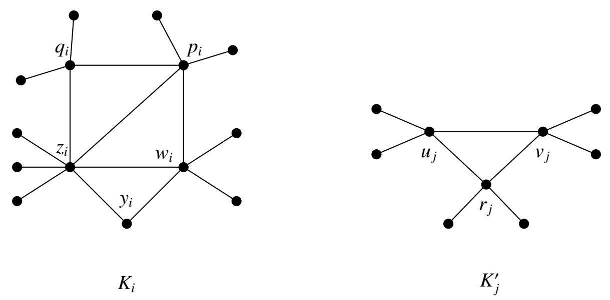

Figure 7. The graphs K_i and K'_j.

Similar as above, for each $x_i\in X$, we create a single vertex x_i to which we associate a copy of the graph K_i as shown in Figure 7 by adding the edge y_ix_i. For each C_j, we create a vertex c_j to which we associate a copy of the graph K'_j as shown in Figure 7 by adding the edges c_ju_j and c_jv_j. Now to obtain the graph G_4 we add edges c_jx_i if $x_i\in C_j$ and all edges between vertices c_j's. Clearly, any cycle of G_4 with length at least four has a chord and so G_4 is a chordal graph (see Figure 8). Let $A=\{c_1,c_2,\ldots,c_{3q}\}$, and set $\ell=q$. To prove that this is indeed a transformation, we only need to show that $\gamma^2_{stR}(G_4)\le\ell$ if, and only if, there is a truth assignment for X that satisfies all clauses in $\mathscr{C}$. This aim can be obtained the following:

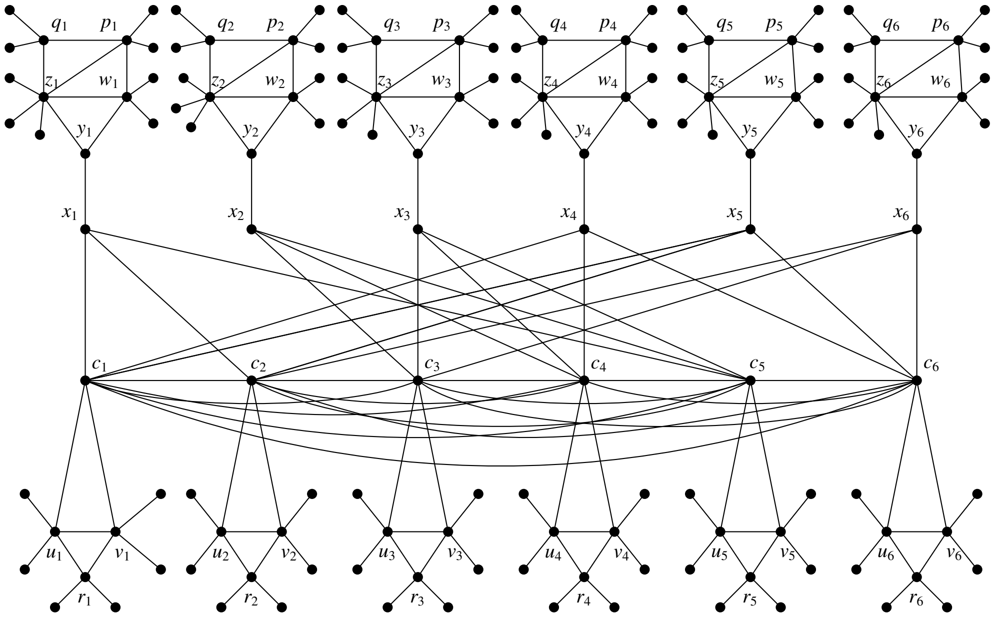

Figure 8. NP-completeness of STR2D for chordal graphs , here $q = 2$ and $\gamma^2_{stR}(G_4) = 2$.

Suppose that $\mathscr{C}'$ is a solution of $X3C3$. Define a signed 2-Roman dominating function g on G_4 of weight ℓ as follows: for every $i \in \{1, 2, \ldots, 3q\}$ let

$$g(x) = \begin{cases} 2 & \text{if } x \in \{q_i, p_i, z_i, w_i, u_i, v_i, r_i\}, \\ 1 & \text{if } x = y_i, \\ 2 & \text{if } x = c_j \text{ and } c_j \in \mathscr{C}', \\ 1 & \text{if } x = c_j \text{ and } c_j \notin \mathscr{C}', \\ -1 & \text{otherwise.} \end{cases}$$

Note that since $\mathscr{C}'$ exists and $|\mathscr{C}'| = q$, the number of c_j's with weight 2 is q, having disjoin neighborhoods in $\{x_1, x_2, \ldots, x_{3q}\}$. It is easy to see that g is signed total Roman 2-dominating function with weight $\omega(g) = q$.

Conversely, first assume there exists a function $h \in \mathfrak{F}(G_4)$ with $\omega(h) \leq \ell$. Among all these functions, let $g = (V_{-1}, V_1, V_2)$ be one such function that assigns smallest possible values to the leaves of G_4. As in the proof of Theorem 2 for bipartite graph , we can show that $g(x) = -1$ for any leaf x of G_4, and thus $g(y) = 2$ for every support vertex y of G_4. Since $\sum_{u \in N(u_i)-\{c_j\}} g(u) = \sum_{u \in N(v_i)-\{c_j\}} g(u) = 2$, we must have $g(c_j) \geq 1$ for every j. Also, we observe that $\sum_{u \in N(w_i)-\{y_i\}} g(u) =$ and thus we must have $g(y_i) \geq 1$ for every i. It follows clearly that no x_i needs to be assigned a positive value under g, and thus $x_i \in V_{-1}$ for every i. If $y_i \in V_2$ for some i, then we can reassign y_i and any c_r adjacent to x_i the values 1 and 2, respectively. So we may assume that $y_i \in V_1$ for every i. Now, since g has weight at most $\ell = q$, and every x_i needs to be adjacent to a vertex assigned a 2, there must exist q vertices of A assigned a 2 under g and the remaining vertices of A belongs to V_1. Now since each c_j has exactly three neighbors in X, we conclude that $\mathscr{C}' = \{C_j : g(c_j) = 2\}$ is an exact cover for $\mathscr{C}$. □

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Conflicts of interest

The authors declare that there are no known conflicts of interest associated with this paper and there has been no significant financial support for this work that could have influenced its outcome.

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