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Research article

Option pricing of geometric Asian options in a subdiffusive Brownian motion regime

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Abstract: In this paper, pricing problem of the geometric Asian option in a subdiffusive Brownian motion regime is discussed. The subdiffusive property is manifested by the random periods of time, during which the asset price does not change. Subdiffusive partial differential equations for geometric Asian option are derived by using delta-hedging strategy. Explicit formula for geometric Asian option is obtained by using partial differential equation method. Furthermore, numerical studies are performed to illustrate the performance of our proposed pricing model.

Keywords: hedging; option pricing; subdiffusive process; asian option
Mathematics Subject Classification: 91B26, 60H10, 58J35

1. Introduction

For modeling of fluctuations in movement of underlying asset price, Brownian motion has been used traditionally as the driving force [1]. However, based on some empirical studies it has been shown that financial data exhibiting periods of constant values which this property can not represent by Brownian motion [2]. In recent years, many researchers attempt to fix this gap by using subdiffusive Brownian motion. Magdziarz [3] introduced subdiffusive geometric Brownian motion as the model of underlying asset price, and obtained the corresponding subdiffusive Black-Scholes (BS) formula for the fair price of European options. Liang et al. [4] extended the model of [3] into a fractional regime. Based on the fractional Fokker-Planck equation, they obtained the corresponding BS formula for European options. Wang et al. [5] considered the European option pricing in subdiffusive fractional Brownian motion regime with transaction costs. They obtained the pricing formula for European options in continuous time. One can refer to [6–9] to see more about the option pricing model in subdiffusive regime.

Asian options are financial derivatives whose payoff depends on the average of the prices of the underlying asset over a pre-fixed time interval. We will denote by $\{S_t\}_{t\in[0,T]}$ the risky asset price process.

According to the payoffs on the expiration date, Asian options can be differentiated in to two classes: fixed strike price Asian options and floating strike price Asian options. The payoff for a fixed strike price Asian option is $(J_T - K)^+$ and $(K - J_T)^+$ for a call and put option respectively. The payoff for a floating strike price Asian option is $(S_T - J_T)^+$ and $(J_T - S_T)^+$ for a call and put option respectively. Where T is the expiration date, K is the strike price, J_t is the average price of the underlying asset over the predetermined interval. According to the definition of J_t, Asian options can again be divided into two types: the arithmetic average Asian option, where

$$J_t = \frac{1}{t}\int_0^t S_\tau d\tau,$$

and the geometric average Asian option, where

$$J_t = \exp\left\{\frac{1}{t}\int_0^t \ln S_\tau d\tau\right\}. \tag{1.1}$$

In recent years, scholars considered Asian option pricing under different models. Prakasa Rao [10] studied pricing model for geometric Asian power options under mixed fractional Brownian motion regime. They derived the pricing formula for European option when Hurst index $H > \frac{3}{4}$. Mao and Liang [11] discussed geometric Asian option under fractional Brownian motion framework. They derived a closed form solution for geometric Asian option. Zhang et al.[12] evaluated geometric Asian power option under fractional Brownian motion frame work.

To the best of our knowledge, pricing of geometric Asian option under subdiffusive regime has not been considered before. The main purpose of this paper is to evaluate the price of Asian power option under a subdiffusive regime.

The rest of the paper proceeds as follows: In section 2, the concept of subdiffusive Brownian motion and basic characteristics of inverse $\alpha-$stable subordinator are introduced. In section 3, the subdiffusive partial differential equations for geometric Asian option and the explicit formula for geometric Asian option are derived. In section 4, some numerical results are given.

2. Subdiffusive Brownian motion

Let $B(t)$ is a standard Brownian motion, then $B(T_\alpha(t))$ is called a subdiffusive Brownian motion. Where $T_\alpha(t)$ is the inverse α-stable subordinator defined as below

$$T_\alpha(t) = \inf\{\tau > 0 \colon U_\alpha(\tau) > t\}, \tag{2.1}$$

here $U_\alpha(\tau)_{\tau\geq 0}$ is a strictly increasing α-stable Lévy process [13–14] with Laplace transform: $\mathbb{E}(e^{-uU_\alpha(\tau)}) = e^{-\tau u^\alpha}$, $\alpha \in (0,1)$. $U_\alpha(t)$ is $\frac{1}{\alpha}$-self-similar $T_\alpha(t)$ is α-self-similar, that is for every $c > 0, U_\alpha(ct) \overset{d}{=} c^{\frac{1}{\alpha}} U_\alpha(t), T_\alpha(ct) \overset{d}{=} c^\alpha T_\alpha(t)$,where$\overset{d}{=}$ denotes "is identical in law to". The moments of the considered process can be found in [15]

$$E[T_\alpha^n(t)] = \frac{t^{n\alpha} n!}{\Gamma(n\alpha + 1)},$$

where $\Gamma(\cdot)$ is the gamma function. Moreover, the Laplace transform of $T_\alpha(t)$ equals

$$E\left(e^{-uT_\alpha(t)}\right) = E_\alpha(-ut^\alpha),$$

where the function $E_\alpha(z) = \sum_{n=0}^{\infty} \frac{z^n}{\Gamma(n\alpha+1)}$ is the Mittag-Leffler function [16]. Specially, when $\alpha \uparrow 1$, $T_\alpha(t)$ reduces to the "objective time"t.

3. Pricing the geometric average asian options under subdiffusive Brownian motion model

In this section, under the framework of PDE method and delta-hedging strategy, we will discuss the pricing problem of the geometric Asian call options in a subdiffusive environment.

Consider a subdiffusive version of the Black-Scholes model, i.e., a simple financial market model consists of a risk-less bond and a stock,whose price dynamics are respectively given by

$$dQ_t = rQ_t dt, Q(0) = Q_0, \tag{3.1}$$

with constant interest rate $r > 0$. The stock price $S_t = X(T_\alpha(t))$, in which $X(\tau)$ follows

$$dX(\tau) = \mu X(\tau)d\tau + \sigma X(\tau)dB(\tau), X(0) = S_0, \tag{3.2}$$

where μ, σ are constants.

In addition, we assume that the following assumptions holds:

(i) There are no transaction costs, margin requirements, and taxes; all securities are perfectly divisible; there are no penalties to short selling; the stock pays no dividends or other distributions; and all investors can borrow or lend at the same short rate.

(ii) The option can be exercised only at the time of maturity.

The value of a geometric average Asian call option at time t is function of time and of S_t and J_t , that is, $V_t = V(t, S_t, J_t)$, where $V(t, S, J)$ is a measurable function of $(t, S, J) \in [0, T] \times (0, +\infty) \times (0, +\infty)$. Then we can obtain the following result.

Theorem 3.1. *Suppose the stock price S follows the model given by Eq (3.2), the price of the geometric average Asian call option $V(t, S, J)$ satisfies the following PDE.*

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)} S^2 \frac{\partial^2 V}{\partial S^2} + J\frac{\ln S - \ln J}{t}\frac{\partial V}{\partial J} + rS\frac{\partial V}{\partial S} - rV = 0, \tag{3.3}$$

with the terminal condition

$$V(T, S, J) = \begin{cases} (J-K)^+, fixed\ strike\ Asian\ call\ option, \\ (S-J)^+, floating\ strike\ Asian\ call\ option, \end{cases} \tag{3.4}$$

where $\{0 < t < T, 0 < S < \infty, 0 < J < \infty\}$.

Proof of Theorem 3.1

Consider a replicating portfolio Π consists with one unit option $V_t = V(t, S_t, J_t)$ and Δ units of stock. At time t the value of this portfolio is

$$\Pi = V - \Delta S,$$

where to simplify the notation we omit t.

Suppose that Δ does not change over the time interval $(t, t + dt)$, then we will select appropriate Δ and make Π is risk-free over the time interval $(t, t + dt)$.

It follows from [5] that the differential of portfolio Π can be expressed as

$$\begin{aligned} d\Pi &= dV - \Delta dS \\ &= \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)} S^2 \frac{\partial^2 V}{\partial S^2}\right) dt + \frac{\partial V}{\partial S} dS + \frac{\partial V}{\partial J} dJ - \Delta dS \\ &= \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)} S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial J}\frac{dJ}{dt}\right) dt + \left(\frac{\partial V}{\partial S} - \Delta\right) dS \end{aligned} \tag{3.5}$$

Letting $\Delta = \frac{\partial V}{\partial S}$, as

$$d\Pi = r\Pi dt = r(V - \Delta S)dt,$$

then we can obtain

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)} S^2 \frac{\partial^2 V}{\partial S^2} + \frac{\partial V}{\partial J}\frac{dJ}{dt} + rS\frac{\partial V}{\partial S} - rV = 0. \tag{3.6}$$

From Eq (1.1) we know

$$\frac{dJ}{dt} = J\left[\frac{\ln S - \ln J}{t}\right]. \tag{3.7}$$

Substituting Eq (3.7) into Eq (3.6) we have

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)} S^2 \frac{\partial^2 V}{\partial S^2} + J\frac{\ln S - \ln J}{t}\frac{\partial V}{\partial J} + rS\frac{\partial V}{\partial S} - rV = 0. \tag{3.8}$$

This equation is written on the pair (S_t, J_t), but since it has a continuous distribution with support on $(0, +\infty) \times (0, +\infty)$, it follows that the function $V(t, S, J)$ solves the PDE (3.3) and this completes the proof.

Proof is completed.

Solving the terminal value problem of partial differential Eqs (3.3) and (3.4), we can obtain

Theorem 3.2. *If the stock price S follows the model given by Eq (3.2),then the price of a fixed strike geometric average Asian call option $V(t, S, J)$ is given by*

$$V(t, S, J) = \left(J^t S^{(T-t)}\right)^{\frac{1}{T}} e^{\delta(t) + \frac{\sigma^2}{2}(a^* + b^*) - r(T-t)}\Phi(d_1) - Ke^{-r(T-t)}\Phi(d_2), \tag{3.9}$$

where the function $\Phi(x)$ is the cumulative probability function for a standard normal distribution and

$$d_1 = \frac{\ln\left(\frac{J^{t/T} S^{(T-t)/T}}{K}\right) + \delta(t) + \sigma^2(a^* + b^*)}{\sigma\sqrt{a^* + b^*}},$$

$$d_2 = d_1 - \sigma\sqrt{a^* + b^*},$$

$$a^* = \frac{(T^\alpha - t^\alpha)}{\alpha\Gamma(\alpha)} - \frac{2}{T(\alpha+1)\Gamma(\alpha)}(T^{\alpha+1} - t^{\alpha+1}),$$

$$b^* = \frac{1}{T^2(\alpha+2)\Gamma(\alpha)}(T^{\alpha+2} - t^{\alpha+2}).$$

$$\delta(t) = \frac{r}{2T}(T-t)^2 - \frac{\sigma^2}{2\alpha\Gamma(\alpha)}(T^\alpha - t^\alpha) + \frac{\sigma^2}{2T(\alpha+1)\Gamma(\alpha)}(T^{\alpha+1} - t^{\alpha+1}).$$

Proof of theorem 3.2

We make the same transformation of variables as [12]

$$\xi = \frac{t \ln J + (T-t)\ln S}{T}, \tag{3.10}$$

and

$$V(t,S,J) = U(t,\xi). \tag{3.11}$$

Though calculating we can get the following Cauchy problem

$$\begin{cases} \frac{\partial U}{\partial t} + \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)} (\frac{T-t}{T})^2 \frac{\partial^2 U}{\partial \xi^2} + [r - \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)}] \frac{T-t}{T} \frac{\partial U}{\partial \xi} - rU = 0, \\ U(T,\xi) = (e^{\xi} - K)^+. \end{cases} \tag{3.12}$$

Furthermore, we apply the following transformation of variables

$$W = U e^{\beta(t)}, \tag{3.13}$$

$$\eta = \xi + \delta(t), \tag{3.14}$$

$$\tau = \gamma(t). \tag{3.15}$$

Substituting Eqs (3.13) and (3.15) into Eq (3.12) we can obtain

$$\begin{aligned} \gamma'(t)\frac{\partial W}{\partial \tau} + \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)} \left(\frac{T-t}{T}\right)^2 \frac{\partial^2 W}{\partial \eta^2} \quad &+ \quad \left[\left(r - \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)}\right) \frac{T-t}{T} + \delta'(t)\right] \frac{\partial W}{\partial \eta} \\ &- \quad (r + \beta'(t))\, W = 0, \end{aligned}$$

Letting

$$r + \beta'(t) = 0,$$

$$\left(r - \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)}\right) \frac{T-t}{T} + \delta'(t) = 0,$$

$$\gamma'(t) + \frac{\sigma^2}{2} \frac{t^{\alpha-1}}{\Gamma(\alpha)} \left(\frac{T-t}{T}\right)^2 = 0,$$

with the terminal condition

$$\beta(T) = \gamma(T) = \delta(T) = 0.$$

By calculating we have

$$\beta(t) = r(T-t),$$

$$\delta(t) = \frac{r}{2T}(T-t)^2 - \frac{\sigma^2}{2\alpha\Gamma(\alpha)}(T^\alpha - t^\alpha) + \frac{\sigma^2}{2T(\alpha+1)\Gamma(\alpha)}(T^{\alpha+1} - t^{\alpha+1}),$$

$$\gamma(t) \quad = \quad \frac{\sigma^2}{2\alpha\Gamma(\alpha)}(T^\alpha - t^\alpha) - \frac{\sigma^2}{T(\alpha+1)\Gamma(\alpha)}(T^{\alpha+1} - t^{\alpha+1})$$

$$+\frac{\sigma^2}{2T^2(\alpha+2)\Gamma(\alpha)}(T^{\alpha+2}-t^{\alpha+2}).$$

Then from Eqs (3.13) and (3.15), the cauchy problem Eq (3.12) changes into

$$\begin{cases} \frac{\partial W}{\partial \tau} - \frac{\partial^2 W}{\partial \eta^2} = 0, \\ W(0,\eta) = (e^{\eta} - K)^+. \end{cases} \tag{3.16}$$

According to heat equation theory, the solution to Eq (3.16) is given by

$$W(\tau,\eta) = \frac{1}{2\sqrt{\pi\tau}}\int_{-\infty}^{+\infty}(e^y - K)^+ e^{-\frac{(y-\eta)^2}{4\tau}}dy, \tag{3.17}$$

By utilizing the inverse transformation of variables and algebraic operation to Eq (3.17), we can obtain the pricing formula of a fixed strike geometric average Asian call option Eq (3.9).

Remark 3.1. *It is not to obtain that $\gamma'(t) < 0$, thus the maximum value of $\gamma(t)$ is $\gamma(0) = \frac{\sigma^2 T^\alpha}{\alpha(\alpha+1)(\alpha+2)\Gamma(\alpha)}$.*

Remark 3.2. *We can obtain the pricing formula for a floating strike geometric average Asian call option by using Fourier transform. Please refer to the Appendix for further details.*

Furthermore, we can obtain the following Theorem.

Theorem 3.3. *The put-call parity relationship for the fixed strike geometric average Asian option can be given by*

$$V(t,S,J) - P(t,S,J) = a(t)J^{\frac{t}{T}}S^{\frac{T-t}{T}} - Ke^{-r(T-t)}. \tag{3.18}$$

where $P(t,S,J)$ is the price of the fixed strike geometric average Asian put option and

$$a(t) = e^{\frac{\sigma^2}{2(\alpha+1)\Gamma(\alpha)T}(t^{\alpha+1}-T^{\alpha+1})+\frac{\sigma^2}{2(\alpha+2)\Gamma(\alpha)T^2}(T^{\alpha+2}-t^{\alpha+2})+\frac{r}{2T}(t^2-T^2)}.$$

Proof of Theorem 3.3

Letting

$$H(t,S,J) = V(t,S,J) - P(t,S,J). \tag{3.19}$$

Theorem 3.1 leads that $H(t,S,J)$ satisfies the following PDE

$$\frac{\partial H}{\partial t} + \frac{1}{2}\sigma^2\frac{t^{\alpha-1}}{\Gamma(\alpha)}S^2\frac{\partial^2 H}{\partial S^2} + J\frac{\ln S - \ln J}{t}\frac{\partial H}{\partial J} + rS\frac{\partial H}{\partial S} - rH = 0, \tag{3.20}$$

with the terminal condition

$$H(T,S,J) = J - K. \tag{3.21}$$

Take the transformation Eq (3.10), we can obtain

$$\frac{\partial H}{\partial t} + \frac{1}{2}\sigma^2\frac{t^{\alpha-1}}{\Gamma(\alpha)}\left(\frac{T-t}{T}\right)^2\frac{\partial^2 H}{\partial \xi^2} + \left[r - \frac{1}{2}\sigma^2\frac{t^{\alpha-1}}{\Gamma(\alpha)}\right]\frac{T-t}{T}\frac{\partial H}{\partial \xi} - rH = 0, \tag{3.22}$$

and

$$H(T) = e^{\xi} - K. \tag{3.23}$$

Denote

$$H = a(t)e^{\xi} + b(t). \tag{3.24}$$

Substituting Eq (3.24) into Eq (3.22) we can obtain

$$\begin{cases} a'(t) + \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)}(\frac{T-t}{T})^2 a(t) + [r - \frac{1}{2}\sigma^2 \frac{t^{\alpha-1}}{\Gamma(\alpha)}]\frac{T-t}{T} a(t) - ra(t) = 0, \\ a(T) = 1. \end{cases} \tag{3.25}$$

and

$$\begin{cases} b'(t) - rb(t) = 0, \\ b(T) = -K. \end{cases} \tag{3.26}$$

Solving Eq (3.25) and Eq (3.26), we have

$$a(t) = e^{\frac{\sigma^2}{2(\alpha+1)\Gamma(\alpha)T}(t^{\alpha+1} - T^{\alpha+1}) + \frac{\sigma^2}{2(\alpha+2)\Gamma(\alpha)T^2}(T^{\alpha+2} - t^{\alpha+2}) + \frac{r}{2T}(t^2 - T^2)}, \tag{3.27}$$

$$b(t) = -Ke^{-r(T-t)}. \tag{3.28}$$

Substituting Eqs (3.27) and (3.28) into Eq (3.24), we can derive

$$V(t, S, J) - P(t, S, J) = a(t)J^{\frac{t}{T}} S^{\frac{T-t}{T}} + b(t). \tag{3.29}$$

Proof is completed.

4. Numerical analysis

In this section, we will give some numerical results. Theorem 3.2 leads the following results.

Corollary 4.1. *When $t = 0$, the price of a fixed strike geometric average Asian call option $\hat{V}(K, T)$ is given by*

$$\hat{V}(K, T) = S_0 e^{\delta(0) + \frac{\sigma^2}{2}(\hat{a}^* + \hat{b}^*) - rT}\Phi(\hat{d}_1) - Ke^{-rT}\Phi(\hat{d}_2), \tag{4.1}$$

where

$$\delta(0) = \frac{rT}{2} - \frac{\sigma^2}{2\alpha\Gamma(\alpha)}T^\alpha + \frac{\sigma^2 T^\alpha}{2(\alpha+1)\Gamma(\alpha)},$$

$$\hat{d}_1 = \frac{\ln(\frac{S_0}{K}) + \delta(0) + \sigma^2(\hat{a}^* + \hat{b}^*)}{\sigma\sqrt{\hat{a}^* + \hat{b}^*}},$$

$$\hat{d}_2 = \hat{d}_1 - \sigma\sqrt{\hat{a}^* + \hat{b}^*},$$

$$\hat{a}^* = \frac{T^\alpha}{\alpha\Gamma(\alpha)} - \frac{2T^\alpha}{(\alpha+1)\Gamma(\alpha)},$$

and

$$\hat{b}^* = \frac{T^\alpha}{(\alpha+2)\Gamma(\alpha)}.$$

Corollary 4.2. *Letting $\alpha \uparrow 1$, then from Theorem 3.2 we can obtain the price of a fixed strike geometric average Asian call option $V(t,S,J)$ is given by*

$$V(t,S,J) = \left(J^t S^{(T-t)}\right)^{\frac{1}{T}} e^{\tilde{\delta}(t)+\frac{\sigma^2}{2}(\tilde{a}^*+\tilde{b}^*)-r(T-t)}\Phi(\tilde{d}_1) - Ke^{-r(T-t)}\Phi(\tilde{d}_2), \tag{4.2}$$

$$\tilde{\delta}(t) = \frac{r}{2T}(T-t)^2 - \frac{\sigma^2}{2}(T-t) + \frac{\sigma^2}{4T}(T^2-t^2),$$

$$\tilde{d}_1 = \frac{\ln\left(\frac{J^{t/T}S^{(T-t)/T}}{K}\right) + \tilde{\delta}(t) + \sigma^2(\tilde{a}^* + \tilde{b}^*)}{\sigma\sqrt{\tilde{a}^* + \tilde{b}^*}},$$

$$\tilde{d}_2 = \tilde{d}_1 - \sigma\sqrt{\tilde{a}^* + \tilde{b}^*},$$

$$\tilde{a}^* = \frac{t^2 - Tt}{T},$$

$$\tilde{b}^* = \frac{T^3 - t^3}{3T^2},$$

this is consistent with the result in [10,12].

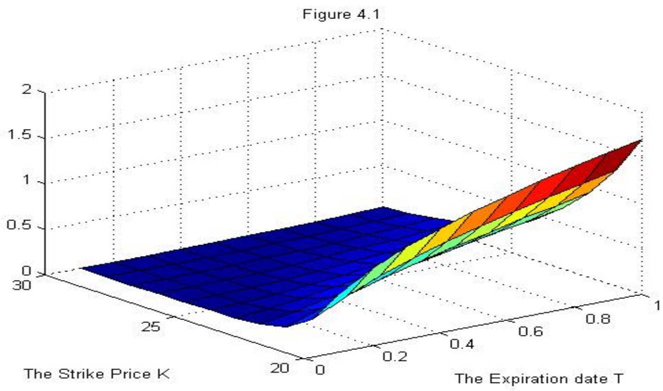

Figure 1. The price of a fixed strike geometric average Asian call option in subdiffusive regime $V_\alpha(K,T)$, according to the exercise date T and strike price K. Here, $S_0 = 20, r = 0.06, \sigma = 0.3, \alpha = 0.8$.

par From figure 1, we can see that the value of $V_\alpha(K,T)$ decreases with K increases and increases with T increases.

From Figure 2, we can see that the price of a fixed strike geometric average Asian call option in subdiffusive regime ($V_\alpha(K,T)$) is decrease with the increase of α. Furthermore, when $\alpha \uparrow 1$, reduces to the "objective time"t, then the price of a fixed strike geometric average Asian call option in subdiffusive regime is larger than the price of a fixed strike geometric average Asian call option in Brownian motion regime.

From Figure 3, it is obviously to see that the price of a fixed strike geometric average Asian call option in subdiffusive regime ($V_\alpha(K,T)$) is usually larger than that in Brownian motion regime $V_B(K,T)$. Specially, the value of $V_\alpha(K,T) - V_B(K,T)$ decrease as K increases.

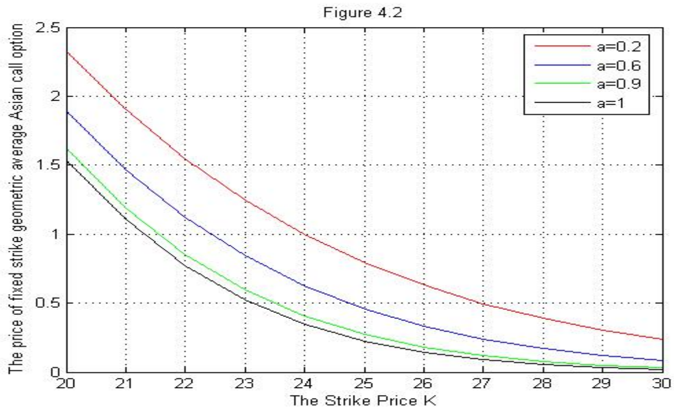

Figure 2. The price of a fixed strike geometric average Asian call option in subdiffusive regime $V_\alpha(K, T)$, according to the α parameter. Here, $S_0 = 20, r = 0.06, \sigma = 0.3, T = 1$ and $K \in [20, 30]$.

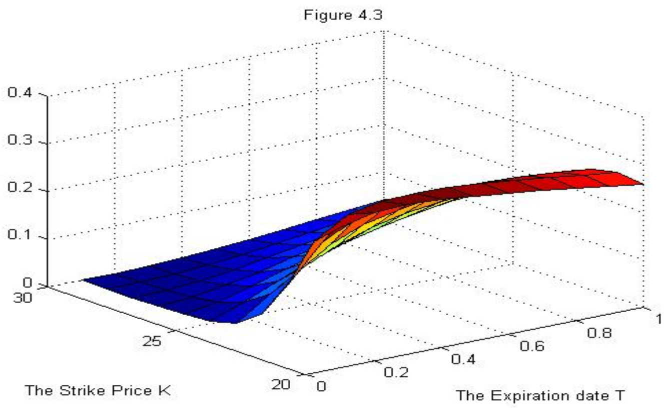

Figure 3. The difference between the price of a fixed strike geometric average Asian call option in subdiffusive regime $V_\alpha(K, T)$ and the price of a fixed strike geometric average Asian call option in Brownian motion regime $V_B(K, T)$, according to the exercise date T and strike price K. Here, $S_0 = 20, r = 0.06, \sigma = 0.3, \alpha = 0.7$.

5. Conclusion

The Asian options have been traded in major capital markets, and offer great flexibility to the market participants. Therefore, it is important to price them accurately and efficiently both in theory and practice. In order to capture the periods of constant values property in the dynamics of underlying asset price, this paper discuss the pricing problem of the fixed strike geometric Asian option under a subdiffusive regime. We derive both subdiffusive partial differential equations and explicit formula for geometric Asian option by using delta-hedging strategy and partial differential equation method. Furthermore, numerical studies are performed to illustrate the performance of our proposed pricing model.

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Conflict of interest

The authors declared that they have no conflicts of interest to this work.

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Appendix

Denoting

$$x = \ln S, \tag{5.1}$$

$$y = \frac{t \ln J + (T-t) \ln S}{T}, \tag{5.2}$$

$$V = U(x, y, t). \tag{5.3}$$

Substituting Eqs (5.1) and (5.3) into Eq (3.3) and Eq (3.4), we can obtain the following Cauchy problem

$$\frac{\partial U}{\partial t} + \frac{\sigma^2 t^{\alpha-1}}{2\Gamma(\alpha)} \left[\frac{\partial}{\partial x} + (\frac{T-t}{T})\frac{\partial}{\partial y}\right]^2 U + \left(r - \frac{\sigma^2 t^{\alpha-1}}{2\Gamma(\alpha)}\right)\left[\frac{\partial}{\partial x} + \left(\frac{T-t}{T}\right)\frac{\partial}{\partial y}\right] U - rU = 0, \tag{5.4}$$

and

$$U(T) = (e^x - e^y)^+ \stackrel{\text{def}}{=} U_0(x, y). \tag{5.5}$$

Letting

$$\hat{U}(\zeta, \vartheta, t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(x, y, t) e^{-i(\zeta x + \vartheta y)} dx dy. \tag{5.6}$$

Applying Fourier transform to Eqs (5.4) and (5.5), we can derive

$$\frac{d\hat{U}}{dt} - \frac{\sigma^2 t^{\alpha-1}}{2\Gamma(\alpha)} \left(\zeta + \frac{T-t}{T}\vartheta\right)^2 \hat{U} + i\left(r - \frac{\sigma^2 t^{\alpha-1}}{2\Gamma(\alpha)}\right)\left(\zeta + \frac{T-t}{T}\vartheta\right)\hat{U} - r\hat{U} = 0, \tag{5.7}$$

and

$$\hat{U}(T) = \hat{U}_0(\zeta, \vartheta), \tag{5.8}$$

$\hat{U}_0(\zeta, \vartheta)$ is the Fourier transform of $U_0(x, y)$.

Solving the ODE problem Eqs (5.7) and (5.8), we have

$$\hat{U}(\zeta, \vartheta, t) = \hat{U}_0(\zeta, \vartheta) \exp\{-(h_1\zeta^2 + 2h_2\zeta\vartheta + h_3\vartheta^2) + i(h_4\zeta + h_5\vartheta) - h_6\}, \tag{5.9}$$

where

$$h_1 = \frac{\sigma^2}{2\alpha\Gamma(\alpha)}(T^\alpha - t^\alpha), \tag{5.10}$$

$$h_2 = \frac{\sigma^2}{2\alpha\Gamma(\alpha)}(T^\alpha - t^\alpha) - \frac{\sigma^2}{2T(\alpha+1)\Gamma(\alpha)}(T^{\alpha+1} - t^{\alpha+1}), \tag{5.11}$$

$$\begin{aligned} h_3 &= \frac{\sigma^2}{2\alpha\Gamma(\alpha)}(T^\alpha - t^\alpha) - \frac{\sigma^2}{T(\alpha+1)\Gamma(\alpha)}(T^{\alpha+1} - t^{\alpha+1}) \\ &+ \frac{\sigma^2}{2T^2(\alpha+2)\Gamma(\alpha)}(T^{\alpha+2} - t^{\alpha+2}), \end{aligned} \tag{5.12}$$

$$h_4 = r(T-t) - \frac{\sigma^2}{2\alpha\Gamma(\alpha)}(T^\alpha - t^\alpha), \tag{5.13}$$

$$h_5 = \frac{r}{2T}(T-t)^2 - \frac{\sigma^2}{2\alpha\Gamma(\alpha)}(T^\alpha - t^\alpha) + \frac{\sigma^2}{2T(\alpha+1)\Gamma(\alpha)}(T^{\alpha+1} - t^{\alpha+1}), \tag{5.14}$$

and

$$h_6 = r(T - t). \tag{5.15}$$

From the inverse Fourier transformation, we can obtain

$$U(x, y, t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} U_0(\mu, \kappa) G(x - \mu, y - \kappa) d\mu d\kappa, \tag{5.16}$$

where

$$U_0(\mu, \kappa) = (e^{\mu} - e^{\kappa})^{+}, \tag{5.17}$$

$$\begin{aligned} G(x, y) = \quad & \frac{1}{(2\pi)^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \exp\{-(h_1 \zeta^2 + 2h_2 \zeta \vartheta + h_3 \vartheta^2) + i(h_4 \zeta + h_5 \vartheta) - h_6\} \\ & \cdot \exp\{i(x\zeta + y\vartheta)\} d\zeta d\vartheta. \end{aligned} \tag{5.18}$$

Then from Eqs (5.1–5.3) and Eq (5.18) we can obtain the pricing formula for a floating strike geometric average Asian call option $V_f(t, S, J)$.

$$V_f(t, S, J) = S\Phi(-d_1) - \left(J^t S^{T-t}\right)^{\frac{1}{T}} \exp(\rho_1 - \rho_2) \Phi(-d_2), \tag{5.19}$$

where

$$d_1 = \frac{t \ln \frac{J}{S} - \frac{r}{2}(T^2 - t^2) - \frac{\sigma^2}{2(\alpha+1)\Gamma(\alpha)}(T^{\alpha+1} - t^{\alpha+1})}{\sigma \sqrt{\frac{1}{(\alpha+2)(\Gamma(\alpha))}(T^{\alpha+2} - t^{\alpha+2})}}, \tag{5.20}$$

$$d_2 = d_1 + \frac{\sigma}{T} \sqrt{\frac{1}{(\alpha + 2)(\Gamma(\alpha))}(T^{\alpha+2} - t^{\alpha+2})}, \tag{5.21}$$

$$\rho_1 = \frac{\sigma^2}{2T^2(\alpha + 2)(\Gamma(\alpha))}(T^{\alpha+2} - t^{\alpha+2}), \tag{5.22}$$

$$\rho_2 = \frac{r}{2T}(T^2 - t^2) + \frac{\sigma^2}{2T(\alpha + 1)\Gamma(\alpha)}(T^{\alpha+1} - t^{\alpha+1}). \tag{5.23}$$