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Research article

μ-extended fuzzy b-metric spaces and related fixed point results

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Abstract: This paper introduces the notion of μ-extended fuzzy b-metric space for extending the concept of fuzzy b-metric space and obtains an analogue of Banach fixed point result. Using functions $\alpha(x, y)$ and $\mu(x, y)$, the corresponding triangle inequality in μ-extended fuzzy b-metric space is given as follows

$$M(\upsilon, \omega, \alpha(\upsilon, \omega)s + \mu(\upsilon, \omega)t) \geq M(\upsilon, \nu, s) * M(\nu, \omega, t) \ \ \forall \upsilon, \nu, \omega \in X.$$

An analogue of Banach fixed point result is established. Besides, an example is given to confirm validity of this theorem.

Keywords: fuzzy metric space; extended fuzzy b-metric space; μ-extended fuzzy b-metric space; fixed point
Mathematics Subject Classification: 47H10, 54H25

1. Introduction and preliminaries

Fixed point theory is an extensively used mathematical tool in various fields of science and engeneering [1–3] Many researchers have generalized Banach contraction principle in various directions. Some have generalized the underlying space while some others have modified the contractive conditions [4–7].

Zadeh [8] initiated the notion of fuzzy set which lead to the evolution of fuzzy mathematics. Kramosil and Michalek [9] generalized probabilistic metric space via concept of fuzzy metric. George and Veeramani [10] defined Hausdorff topology on fuzzy metric space after slight modification in the definition of fuzzy metric presented in [9]. Heilpern [11] defined fuzzy mapping and establish fixed point result for it. Subsequently many concepts and results from general topology were generalized to fuzzy topological space.

Nădăban [12] generalized b-metric space by introducing fuzzy b-metric space in the setting of fuzzy metric space initiated by Michalek and Kramosil . Faisar Mehmood *et al.* [13] generalized fuzzy b-metric by introducing the concept of extended fuzzy b-metric. In this article we present the idea of μ-extended fuzzy b-metric space which extends the concepts of fuzzy b-metric and extended fuzzy b-metric spaces. We also establish a Banach-type fixed point result in the context of μ-extended fuzzy b-metric space.

First we recollect basic definitions and results which will be used in the sequel.

Definition 1.1. *[14] A binary operation* $* : [0,1]^2 \to [0,1]$ *is said to be continuous t-norm if* $([0,1], \leq\ , *)$ *is an ordered abelian topological monoid with unit 1.*

Some frequently used examples of continuous t-norm are $x *_L y = \max\{x+y-1, 0\}$, $x *_P y = xy$ and $x *_M y = \min\{x, y\}$. These are respectively called Lukasievicz t-norm, product t-norm and minimum t-norm

Definition 1.2. *[9] A fuzzy metric space is 3-tuple* $(S, \varsigma, *)$, *where* S *is a nonempty set,* $*$ *is continuous t-norm and* ς *is a fuzzy set on* $S \times S \times [0, \infty)$ *which satisfies the following conditions, for all* $p, q, r \in S$,

$(KM1)$ $\varsigma(p, q, 0) = 0$;

$(KM2)$ $\varsigma(p, q, \ell) = 1,$ *for all* $\ell > 0$ *if and only if* $p = q$;

$(KM3)$ $\varsigma(p, q, \ell) = \varsigma(q, p, \ell)$;

$(KM4)$ $\varsigma(p, r, \ell + t) \geq \varsigma(p, q, \ell) * \varsigma(q, r, t)$, *for all* $\ell, t > 0$;

$(KM5)$ $\varsigma(p, q, .) : [0, \infty) \to [0, 1]$ *is non-decreasing continuous;*

$(KM6)$ $\lim_{\ell\to\infty} \varsigma(r, y, \ell) = 1$.

Note that $\varsigma(p, q, \ell)$ indicates the degree of closeness between p and q with respect to $\ell \geq 0$.

Remark 1.1. *For* $p \neq q$ *and* $\ell > 0$, *it is always true that* $0 < \varsigma(p, q, \ell) < 1$.

Lemma 1.1. *[15] Let* S *be a nonempty set. Then* $\varsigma(p, q, .)$ *is nondecreasing for all* $p, q \in S$.

Example 1.1. *[16] Let* S *be a nonempty set and* $\varsigma : S \times S \times (0, \infty) \to [0, 1]$ *be fuzzy set defined on a metric space* (S, d) *such that*

$$\varsigma(x, y, \ell) = \frac{p\ell^q}{p\ell^q + rd(x, y)}, \quad \forall x, y \in S \, and \quad \ell > 0,$$

where p, q *and* r *are positive real numbers, and* $*$ *is product t-norm. This is a fuzzy metric induced by the metric d. The above fuzzy metric is also defined if minimum t-norm is used instead of product t-norm.*

If we take $p = q = r = 1$, then the above fuzzy metric becomes *standard fuzzy metric*.

Definition 1.3. *[12] Let S be a non-empty set and $b \geq 1$ be a given real number. A fuzzy set $\varsigma : S \times S \to [0, \infty)$ is said to be fuzzy b-metric if for all $p, q, r \in S$, the following conditions hold:*

(FbM_1) $\varsigma(p, q, 0) = 0$;

(FbM_2) $\varsigma(p, q, \ell) = 1$, *for all* $\ell > 0$ *if and only if* $q = p$;

(FbM_3) $\varsigma(p, q, \ell) = \varsigma(q, p, \ell)$;

(FbM_4) $\varsigma(p, r, b(\ell + t) \geq \varsigma(p, q, \ell) * \varsigma(q, r, t)$, *for all* $\ell, t > 0$;

(FbM_5) $\varsigma(p, q, .) : (0, \infty) \to [0, 1]$ *is continuous and* $\lim_{\ell\to\infty} \varsigma(p, q, \ell) = 1$.

Faisar Mehmood *et al.* [13] defined extended fuzzy *b*-metric as.

Definition 1.4. *[13] The ordered triple $(S, \varsigma, *)$ is called extended fuzzy b-metric space by function $\alpha : S \times S \to [1, \infty)$, where S is non-empty set, $*$ is continuous t-norm and $\varsigma : S \times S \to [0, \infty)$ is fuzzy set such that for all $x, y, z \in S$ the following conditions hold:*

(FbM_1) $\varsigma_\alpha(p, q, 0) = 0$;

(FbM_2) $\varsigma_\alpha(p, q, \ell) = 1$, *for all* $\ell > 0$ *if and only if* $q = p$;

(FbM_3) $\varsigma_\alpha(p, q, \ell) = \varsigma_\alpha(q, p, \ell)$;

(FbM_4) $\varsigma_\alpha(p, r, \alpha(p, r)(\ell + t) \geq \varsigma_\alpha(p, q, \ell) * \varsigma_\alpha(q, r, t)$, *for all* $\ell, t > 0$;

(FbM_5) $\varsigma_\alpha(p, q, .) : (0, \infty) \to [0, 1]$ *is continuous and* $\lim_{\ell\to\infty} \varsigma_\alpha(p, q, \ell) = 1$.

The authors in [13] established the following Banach type fixed point result in the setting of extended fuzzy *b*-metric space.

Theorem 1.1. *Let $(S, \varsigma_\alpha, *)$ be an extended fuzzy-b metric space by mapping $\alpha : X\times S \to [1, \infty)$ which is G-complete such that ς_α satisfies*

$$\lim_{t\to\infty} \varsigma_\alpha(p, q, t) = 1, \ \forall p, q \in S \text{ and } t > 0. \tag{1.1}$$

Let $f : S \to S$ be function such that

$$\varsigma_\alpha(fp, fq, kt) \geq \varsigma_\alpha(p, q, t), \ \forall p, q \in S \text{ and } t > 0, \tag{1.2}$$

where $k \in (0, 1)$. Moreover, if for $b_0 \in S$ and $n, p \in N$ with $\alpha(b_n, b_{n+p}) < \frac{1}{k}$, where $b_n = f^n b_o$. Then f will have a unique fixed point.

2. Main results

Motivated by the concept presented in [13] , we present μ-extended fuzzy *b*-metric space and generalize Banach contraction principle to it using the approach of Grabiec [17].

Definition 2.1. *Let $\alpha, \mu : X\times X \to [1, \infty)$ defined on a non-empty set X. A fuzzy set $\varsigma_\mu : X\times X\times[0, \infty) \to [0, 1]$ is said to be μ-extended fuzzy b-metric if for all $p, q, r \in X$, the following conditions hold:*

(μE_1) $\varsigma_\mu(p,q,0)=0$;

(μE_2) $\varsigma_\mu(p,q,\ell)=1$, *for all* $\ell>0$ *if and only if* $q=p$;

(μE_3) $\varsigma_\mu(p,q,\ell)=\varsigma_\mu(q,p,\ell)$;

(μE_4) $\varsigma_\mu(p,r,\alpha(p,r)\ell+\mu(p,r)\jmath)\geq\varsigma_\mu(p,q,\ell)*\varsigma_\mu(q,r,\jmath)$, *for all* $\ell,t>0$;

(μE_5) $\varsigma_\mu(p,q,.):(0,\infty)\to[0,1]$ *is continuous and* $\lim_{\ell\to\infty}\varsigma_\mu(p,q,\ell)=1$.

And $(X,\varsigma_\mu,*,\alpha,\mu)$ *is called* μ*-extended fuzzy b-metric space.*

Remark 2.1. *It is worth mentioning that fuzzy b-metric and extended fuzzy b-metric are special types of* μ*-extended fuzzy b-metric when* $\alpha(x,y)=\mu(x,y)=b$, *for some* $b\geq1$ *and* $\alpha(x,y)=\mu(x,y)$, *respectively.*

In the following we exemplify Definition 2.1.

Example 2.1. *Let* $S=\{1,2,3\}$ *and* $\alpha,\mu:S\times S\to[1,\infty)$ *be two functions defined by* $\alpha(m,n)=1+m+n$ *and* $\mu(m,n)=m+n-1$. *If* $\varsigma_\mu:S\times S\times[0,\infty)\to[0,1]$ *is a fuzzy set defined by*

$$\varsigma_\mu(m,n,\ell)=\frac{\min\{m,n\}+\ell}{\max\{m,n\}+\ell},$$

where contiuous t-norm $*$ *is defined as* $t_1*t_2=t_1\times t_2$, *for all* $t_1,t_2\in[0,1]$ *We show that* $(S,\varsigma_\mu,*,\alpha,\mu)$ *is* μ*-extended fuzzy b-metric space. Clearly* $\alpha(1,1)=3,\ \alpha(2,2)=5,\ \alpha(3,3)=7,\alpha(1,2)=\alpha(2,1)=4,\ \alpha(2,3)=\alpha(3,2)=6,\ \alpha(1,3)=\alpha(3,1)=5$, *and* $\mu(1,1)=1,\ \mu(2,2)=3,\ \mu(3,3)=5,\ \mu(1,2)=\mu(2,1)=2,\ \mu(2,3)=\mu(3,2)=4,\ \mu(1,3)=\mu(3,1)=3$. *One can easily verify that the conditions* $(\mu E_1),(\mu E_2),(\mu E_3)$ *and* (μE_5) *hold. In order to show that* $(S,\varsigma_\mu,\times,\alpha,\mu)$ *is* μ*-extended fuzzy b-metric space, it only remains to prove that* (μE_4) *is satisfied for all* $m,n,p\in S$. *For for all* $\ell,\jmath>0$, *it is clear that*

$$\varsigma_\mu(1,2,\alpha(1,2)\ell+\mu(1,2)\jmath)=\frac{1+4\ell+2\jmath}{2+4\ell+2\jmath}\geq\frac{2+\jmath+2\ell+\ell\jmath}{9+3\jmath+3\ell+\ell\jmath}=\varsigma_\mu(1,3,\ell)*\varsigma_\mu(3,2,\jmath),$$

$$\varsigma_\mu(1,3,\alpha(1,3)\ell+\mu(1,3)\jmath)=\frac{1+5\ell+3\jmath}{3+5\ell+3\jmath}\geq\frac{2+\jmath+2\ell+\ell\jmath}{6+\jmath+2\ell+\ell\jmath}=\varsigma_\mu(1,2,\ell)*\varsigma_\mu(2,3,\jmath),$$

and

$$\varsigma_\mu(2,3,\alpha(2,3)\ell+\mu(2,3)\jmath)=\frac{2+6\ell+4\jmath}{3+6\ell+4\jmath}\geq\frac{1+\jmath+\ell+\ell\jmath}{6+2\jmath+3\ell+\ell\jmath}=\varsigma_\mu(2,1,\ell)*\varsigma_\mu(1,3,\jmath).$$

Hence ς_μ *is* μ*-extended fuzzy b-metric.*

Example 2.2. *Let* $S=\{1,2,3\}$ *and* $\alpha,\mu:S\times S\to[1,\infty)$ *be two functions defined by* $\alpha(m,n)=max\{m,n\}$ *and* $\mu(m,n)=min\{m,n\}$. *If* $\varsigma_\mu:S\times S\times[0,\infty)\to[0,1]$ *is a fuzzy set defined by*

$$\varsigma_\mu(m,n,\ell)=\begin{cases}1, & m=n,\\ 0, & \ell=0,\\ \frac{\ell}{2}, & ,0<\ell<2,\\ \frac{\max\{m,n\}}{\ell+1}, & 2<\ell<3,\\ \frac{\max\{m,n\}}{\ell}, & 3<\ell,\\ \frac{1}{\ell+1}, & \ell\in S,\end{cases}$$

where continuous t-norm $*$ *is defined to be the minimum , that is* $t_1 * t_2 = min(t_1, t_2)$*. Obviously conditions* $(\mu E_1), (\mu E_2), (\mu E_3)$ *and* (μE_5) *trivially hold. For* $p, q, r \in S$ *notice the following:*
Case 1: When $0 < \ell + \frac{\jmath}{2} < 1$*. Then*

$$\varsigma_\mu(1, 2, \alpha(1, 2)\ell + \mu(1, 2)\jmath) = \ell + \frac{\jmath}{2} \geq \min\{\frac{\ell}{2}, \frac{\jmath}{2}\} = \varsigma_\mu(1, 3, \ell) * \varsigma_\mu(3, 2, \jmath).$$

Case 2: When $1 < 2\ell + \jmath < \frac{3}{2}$*. Then*

$$\varsigma_\mu(1, 2, \alpha(1, 2)\ell + \mu(1, 2)\jmath) = \frac{2}{2\ell + \jmath + 1} \geq min\{\frac{\ell}{2}, \frac{\jmath}{2}\} = \varsigma_\mu(1, 3, \ell) * \varsigma_\mu(3, 2, \jmath).$$

Case 3: When $2\ell + \jmath > 3$ *such that* $\ell = 0$ *and* $\jmath > 3$*. Then*

$$\varsigma_\mu(1, 2, \alpha(1, 2)\ell + \mu(1, 2)\jmath) = \frac{2}{\jmath} > 0 = min\{0, \frac{3}{\jmath}\} = \varsigma_\mu(1, 3, \ell) * \varsigma_\mu(3, 2, \jmath).$$

Case 4: When $2\ell + \jmath > 3$ *such that* $\ell > 3$ *and* $\jmath = 0$*. Then*

$$\varsigma_\mu(1, 2, \alpha(1, 2)\ell + \mu(1, 2)\jmath) = \frac{1}{\ell} > 0 = min\{\frac{3}{\ell}, 0\} = \varsigma_\mu(1, 3, \ell) * \varsigma_\mu(3, 2, \jmath).$$

Similarly it can be easily verified that condition (μE_4) *is satisfied for all the remaining cases. Hence* $(S, \varsigma_\mu, *, \alpha, \mu)$ *is* μ*-extended fuzzy b-metric space.*

Before establishing an analog of Banach contraction principle in setting of μ-extended fuzzy b-metric space, we present the following concepts in the setting of μ-extended fuzzy b-metric space.

Definition 2.2. *Let* $(S, \varsigma_\mu, *, \alpha, \mu)$ *be a* μ*-extended fuzzy b-metric space and* $\{a_n\}$ *be a sequence in* S*.*
(1) $\{a_n\}$ *is a G-convergent sequence if there exists* $a_0 \in S$ *such that*

$$\lim_{n\to\infty} \varsigma_\mu(a_n, a_0, \ell) = 1, \ \forall \ell > 0.$$

(2) $\{a_n\}$ *in X is called G-Cauchy if*

$$\lim_{n\to\infty} \varsigma_\mu(a_n, a_{n+p}, \ell) = 1, \text{ for each } p \in \mathbb{N} \text{ and } \ell > 0.$$

(3) S *is G-complete, if every Cauchy sequence in* S *converges.*

Next, we prove Banach fixed point Theorem in μ-extended fuzzy b-metric space.

Theorem 2.1. *Let* $(S, \varsigma_\mu, *, \alpha, \mu)$ *be a G-complete* μ*-extended fuzzy b-metric space with mappings* $\alpha, \mu : S \times S \to [1, \infty)$ *such that*

$$\lim_{t\to\infty} \varsigma_\mu(u, v, t) = 1, \ \forall u, v \in S \text{ and } t > 0. \tag{2.1}$$

Let $f : S \to S$ *be a mapping satisfying that there exists* $k \in (0, 1)$ *such that*

$$\varsigma_\mu(fu, fv, kt) \geq \varsigma_\mu(u, v, t), \ \forall u, v \in S \text{ and } t > 0. \tag{2.2}$$

If for any $b_0 \in S$ *and* $n, p \in N$,

$$max\{\sup_{p\geq 1} \lim_{i\to\infty} \alpha(b_i, b_{i+p}), \sup_{p\geq 1} \lim_{i\to\infty} \mu(b_i, b_{i+p})\} < \frac{1}{k},$$

where $b_n = f^n b_o$*, then* f *has a unique fixed point.*

Proof. Without loss of generality, assume that $b_{n+1} \neq b_n \ \forall n \geq 0$. From (2.2), it follows that, for any $n, q \in N$,

$$\begin{aligned}\varsigma_\mu(b_n, b_{n+1}, kt) =& \varsigma_\mu(fb_{n-1}, fb_n, kt)\\ \geq& \varsigma_\mu(b_{n-1}, b_n, t)\\ \geq& \varsigma_\mu\left(b_{n-2}, b_{n-1}, \frac{t}{k}\right)\\ \geq& \varsigma_\mu\left(b_{n-3}, b_{n-2}, \frac{t}{k^2}\right)\\ &\vdots\\ \geq& \varsigma_\mu\left(b_0, b_1, \frac{t}{k^{n-1}}\right).\end{aligned}$$

That is

$$\varsigma_\mu(b_n, b_{n+1}, kt) \geq \varsigma_\mu\left(b_0, b_1, \frac{t}{k^{n-1}}\right). \tag{2.3}$$

For any $p \in N$, applying (μE_4) yields that

$$\begin{aligned}&\varsigma_\mu\left(b_n, b_{n+p}, t\right)\\ =&\varsigma_\mu\left(b_n, b_{n+p}, \frac{pt}{p}\right) = \varsigma_\mu\left(b_n, b_{n+p}, \frac{t}{p} + \frac{pt-t}{p}\right)\\ \geq&\varsigma_\mu\left(b_n, b_{n+1}, \frac{t}{p\alpha\left(b_n, b_{n+p}\right)}\right) * \varsigma_\mu\left(b_{n+1}, b_{n+p}, \frac{pt-t}{p\mu\left(b_n, b_{n+p}\right)}\right)\\ \geq&\varsigma_\mu\left(b_n, b_{n+1}, \frac{t}{p\alpha\left(b_n, b_{n+p}\right)}\right) * \varsigma_\mu\left(b_{n+1}, b_{n+2}, \frac{t}{p\mu\left(b_n, b_{n+p}\right)\alpha\left(b_{n+1}, b_{n+p}\right)}\right)\\ &* \varsigma_\mu\left(b_{n+2}, b_{n+p}, \frac{pt-2t}{p\mu\left(b_n, b_{n+p}\right)\mu\left(b_{n+1}, b_{n+p}\right)}\right).\end{aligned}$$

From (2.3) and (μE_4),it follows that

$$\begin{aligned}&\varsigma_\mu\left(b_n, b_{n+p}, t\right)\\ \geq& \varsigma_\mu\left(b_0, b_1, \frac{t}{p\alpha\left(b_n, b_{n+p}\right)k^n}\right) * \varsigma_\mu\left(b_0, b_1, \frac{t}{p\mu\left(b_n, b_{n+p}\right)\alpha\left(b_{n+1}, b_{n+p}\right)k^{n+1}}\right)\\ &* \varsigma_\mu\left(b_0, b_1, \frac{t}{p\mu\left(b_n, b_{n+p}\right)\mu\left(b_{n+1}, b_{n+p}\right)\alpha\left(b_{n+2}, b_{n+p}\right)k^{n+2}}\right) * \cdots\\ &* \varsigma_\mu\left(b_0, b_1, \frac{t}{p\mu\left(b_n, b_{n+p}\right)\mu\left(b_{n+1}, b_{n+p}\right)\cdots\mu\left(b_{n+(p-3)}, b_{n+p}\right)\alpha\left(b_{n+(p-2)}, b_{n+p}\right)k^{n+(p-1)}}\right).\end{aligned}$$

Noting that for $k \in (0, 1)$, $\alpha(b_n, b_{n+p})k < 1$ and $\mu(b_n, b_{n+p})k < 1$ hold for all $n, p \in N$ and letting $n \to \infty$, applying Eq 3, it follows that

$$\lim_{n\to\infty} \varsigma_\mu(b_n, b_{n+p}, t) = 1 * 1 * \cdots * 1 = 1,$$

that is $\{b_n\}$ is Cauchy sequence. Due to the completeness of $(S, \varsigma_\mu, *, \alpha, \mu)$ there exists some $b \in S$ such that $b_n \to b$ as $n \to \infty$. We claim that b is unique fixed point of f. Applying Eq (2.1) and condition (μE_4), we have

$$\begin{aligned}\varsigma_\mu(fb, b, t) &\geq \varsigma_\mu\left(fb, b_{n+1}, \frac{t}{2\alpha(fb, b)}\right) * \varsigma_\mu\left(b_{n+1}, b, \frac{t}{2\mu(fb, b)}\right) \\ &\geq \varsigma_\mu\left(b, b_n, \frac{t}{2\alpha(fb, b)k}\right) * \varsigma_\mu\left(b_{n+1}, b, \frac{t}{2\mu(fb, b)}\right).\end{aligned}$$

Thus $\varsigma_\mu(fb, b, t) = 1$ and hence b is a fixed point of f. To show the uniqueness, let c be another fixed point of f. Applying inequality (2.3) yields that

$$\begin{aligned}\varsigma_\mu(b, c, t) &= \varsigma_\mu(fb, fc, t) \\ &\geq \varsigma_\mu\left(b, c, \frac{t}{k}\right) \\ &= \varsigma_\mu\left(fb, fc, \frac{t}{k}\right) \\ &\geq \varsigma_\mu\left(b, c, \frac{t}{k^2}\right) \\ &\vdots \\ &\geq \varsigma_\mu\left(b, c, \frac{t}{k^n}\right),\end{aligned}$$

which implies that $\varsigma_\mu(b, c, t) \to 1$, as $n \to \infty$, and hence $b = c$. □

Remark 2.2. *If $\alpha(u, v) = \mu(u, v)$ for all $u, v \in S$, then Theorem 2.1 reduces to Theorem 1.1.*

The following example illustrates Theorem 2.1.

Example 2.3. *Let $S = [0, 1]$ and $\varsigma_\mu(u, v, t) = e^{\frac{-|u-v|}{t}}$, $\forall u, v \in S$. It can be easily verified that $(S, \varsigma_\mu, *, \alpha, \mu)$ is a G-complete μ-extended fuzzy b-metric space with mappings $\alpha, \mu : S \times S \to [1, \infty)$ defined by $\alpha(u, v) = 1 + uv$ and $\mu(u, v) = 1 + u + v$, respectively and continuous t-norm $*$ as usual product.*

Let $f : S \to S$ be such that $f(x, y) = 1 - \frac{1}{3}x$. For all $t > 0$ we have

$$\varsigma_\mu(fu, fv, \frac{1}{2}t) = e^{\frac{-\frac{2}{3}|u-v|}{t}} > e^{\frac{-|u-v|}{t}} = \varsigma_\mu(u, v, t).$$

That is all the conditions of Theorem 2.1 are satisfied. Therefore, f has unique fixed point $\frac{3}{4} \in [0, 1] = S$.

3. Conclusions

We introduce the concept of μ-extended fuzzy b-metric space and established fixed point result which generalizes Banach contraction principle to this newly introduced space. The concept we presented may lead to further investigation and applications. As the class of of μ-extended fuzzy b-metric spaces is wider than those of the fuzzy b-metric spaces and extended fuzzy b-metric spaces, therefore results established in this framework will generalize many results in the existing literature.

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Conflict of interest

The authors declare that they have no competing interest.

References

1. T. Abdeljawad, F. Jarad, D. Baleanu, *On the existence and the uniqueness theorem for fractional differential equations with bounded delay within Caputo derivatives*, Sci. China, Math., **51** (2008), 1775–1786.

2. T. Abdeljawad, F. Jarad, D. Baleanu, *Existence and uniqueness theorem for a class of delay fractional differential equations with left and right Caputo fractional derivatives*, J. Math. Phys., **49** (2008), 083507.

3. A. A. Kilbas, M. H. Srivastava, J. J. Tujillo, *Theory and application of fractional differential equations*, North Holand Mathematics Studies; Elsvier: Amsterdam, The Netherland, 2006.

4. G. bhaskar, V. Lakshmikantham, *Fixed point theorems in partially ordered metric spaces and applications*, Nonlinear Anal., **65** (2006), 1379–1393.

5. B. S. Chaudury, N. Metiya, M. Postolache, *A generalized weak contraction principle with applications coupled coincidence point problems*, Fixed Point Theory Appl., **2013** (2013), 152.

6. L. B. Ciric, *A generalization Banach's contraction principle*, Proc. Amer. Math. Soc., **45** (1974), 267–273.

7. T. Suzuki, *A generalized Banach contraction principle that characterizes metric completeness*, Proc. Amer. Math. Soc., **5** (2008), 1861–1869.

8. L. A. Zadeh, *Fuzzy sets*, Inf. Control, **8** (1965), 338–353.

9. I. Kramosil, J. Michalek, *Fuzzy metric and statistical metric spaces*, Kybernetika, **5** (1975), 336–344.

10. R. Vasuki, P. Veeramani, *Fixed point theorems and Cauchy sequences in fuzzy metric spaces*, Fuzzy Sets Syst., **3** (2003), 415–417.

11. S. Heilpern, *Fuzzy mappings and fixed point theorems*, J. Math. Anal. Appl., **83** (1981), 566–569.

12. S. Nădăban, *Fuzzy b-Metric spaces*, Int. J. Comput. Commun. Control., **2** (2016), 273–281.

13. F. Mehmood, R. A. Ali, C. Ionescu, et al. *Extended fuzzy b-Metric spaces*, J. Math. Anal., **8** (2017), 124–131.

14. B. Schweizer, A. Sklar, *Statistical metric spaces*, Pac. J. Math., **10** (1960), 313–334.

15. V. Gregori, A. Sapena, *On fixed point theorems in fuzzy metric spaces*, Fuzzy Sets Syst., **125** (2002), 245–252.

16. R. George, S. Radenović, K. P. Reshma, et al. *Rectangular b-metric space and contraction principles*, Nonlinear Sci. Appl., **6** (2015), 1005–1013.

17. M. Grabiec, *Fixed points in fuzzy metric spaces*, Fuzzy Sets Syst., **27** (1988), 385–389.