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Research article

Fixed point results on ordered Prešić type mappings

Seher Sultan Yeşilkaya[1,*], Cafer Aydın[2] and Adem Eroğlu[3]

[1] Institute of Science and Technology, Kahramanmaraş Sütçü İmam University, Kahramanmaraş, 46040, Turkey

[2] Department of Mathematics, Kahramanmaraş Sütçü İmam University, Kahramanmaraş, 46040, Turkey

[3] Department of Mathematics and Science Education, Tokat Gaziosmanpaşa University, Tokat, 60150, Turkey

* **Correspondence:** Email: sultanseher20@gmail.com.

Abstract: In the present study, we introduce a new concept of contractions called ordered Prešić type θ-contractivity and ordered Prešić type F-contractive on partial metric spaces. Then we give fixed point theorems for such mappings. Finally, some examples are presented to support the new results proved.

Keywords: fixed point; partial metric spaces; ordered Prešić type θ-contractivity; ordered Prešić type F-contraction; regular mapping
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1. Introduction

Banach [1] introduced a famous fundamental fixed point theorem, also known as the Banach contraction principle. There are various extensions and generalizations of the Banach contraction principle in the literature. Matthews [2] introduced the partial metric spaces and presented a fixed point theorem on partial metric space. After that, the fixed point results in partial metric spaces were studied by many other authors [3–7]. Ran and Reurings [8] proved a fixed point theorem on an ordered metric space. Thereafter, several authors obtained many fixed point theorems in ordered metric spaces. For more details see [9–11].

Considering the convergence of certain sequences S. B. Prešić [12] generalized Banach contraction principle as follows:

Theorem 1. *Let (X, d) be a complete metric space, k a positive integer and $T : X^k \to X$ a mapping* *satisfying the following contractive type condition*

$$d(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1})) \leqslant q_1 d(x_1, x_2) + q_2 d(x_2, x_3) + ... + q_k d(x_k, x_{k+1}), \tag{1.1}$$

for every $x_1, x_2, ..., x_{k+1}$ in X, where $q_1, q_2, ..., q_k$ are non negative constants such that $q_1 + q_2 + ... + q_k < 1$. Then there exist a unique point x in X such that $T(x, x, ..., x) = x$. Moreover, if $x_1, x_2, ..., x_k$, are arbitrary points in X and for $n \in N$,

$$x_{n+k} = T(x_n, x_{n+1}, ..., x_{n+k-1}), \quad (n = 1, 2, ...)$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$$\lim x_n = T(\lim x_n, \lim x_n, ..., \lim x_n).$$

Remark that condition (1.1) in the case $k = 1$ reduces to the well-known Banach contraction mapping principle. So, Theorem 1 is a generalization of the Banach fixed point theorem.

Ćirić and Prešić [13] generalized the above result as follows:

Theorem 2. *Let (X, d) be a complete metric space, k a positive integer and $T : X^k \to X$ a mapping satisfying the following contractive type condition*

$$d(T(x_1, x_2, ..., x_k), T(x_2, x_3, ..., x_{k+1})) \leqslant \lambda \max_{1 \leqslant i \leqslant k} \{d(x_i, x_{i+1})\}, \tag{1.2}$$

where $\lambda \in (0, 1)$ is constant and $x_1, x_2, ..., x_{k+1}$ are arbitrary elements in X. Then there exist a point x in X such that $T(x, x, ..., x) = x$. Moreover, if $x_1, x_2, ..., x_k$, are arbitrary points in X and for $n \in N$,

$$x_{n+k} = T(x_n, x_{n+1}, ..., x_{n+k-1}), \quad (n = 1, 2, ...)$$

then the sequence $\{x_n\}_{n=1}^{\infty}$ is convergent and

$$\lim x_n = T(\lim x_n, \lim x_n, ..., \lim x_n).$$

If in addition we suppose that on a diagonal $\triangle \subset X^k$

$$d(T(u, u, ..., u), T(v, v, ..., v)) < d(u, v) \tag{1.3}$$

holds for all $u, v \in X$, with $u \neq v$, then x is the unique point in X with $T(x, x, ..., x) = x$.

Later, Nazır and Abbas [14], proved common fixed point theorems of the Prešić type in partial metric space.

Recently, Jleli and Samet [15] introduced a new type of contraction which is called the θ-contractivity and proved a fixed point theorem for mappings of this type, for which the Banach contraction principle and some other known contractions conditions. Jleli and Samet denote the family of all functions, $\theta : (0, \infty) \to (1, \infty)$ satisfying the following properties by Θ:

(Θ_1) θ is non-decreasing;
(Θ_2) For each sequence $\{s_n\} \subset (0, \infty)$, $\lim_{n\to\infty} \theta(s_n) = 1$ if and only if $\lim_{n\to\infty} s_n = 0^+$;
(Θ_3) There exists $m \in (0, 1)$ and $z \in (0, \infty]$ such that $\lim_{s\to 0^+} \frac{\theta(s)-1}{s^m} = z$.

Wardowski [16] introduced concept of F-contractive mapping on metric space and proved a fixed point theorem for such a map on complete metric space. Let $\mathcal{F}$ be the set of all functions $F : \mathbb{R}_+ \to \mathbb{R}$ satisfying the following conditions:

(F1) F is strictly increasing. That is, $\beta < \gamma \Rightarrow F(\beta) < F(\gamma)$ for all $\beta, \gamma \in \mathbb{R}_+$
(F2) For every sequence $\{\beta_n\}_{n\in\mathbb{N}}$ in $\mathbb{R}_+$ we have $\lim_{n\to\infty} \beta_n = 0$ if and only if $\lim_{n\to\infty} F(\beta_n) = -\infty$
(F3) There exists a number $z \in (0, 1)$ such that $\lim_{\beta\to 0^+} \beta^z F(\beta) = 0$.

Durmaz et al. [17] introduced a new the concept of the ordered F-contractive on ordered metric spaces. For more study on F-contractions one may refer to [18, 19].

2. Ordered Prešić type θ-contractivity mappings

We give a fixed point theorem for ordered the Prešić type θ-contractivity mapping in partial metric space. Firstly, let us start with the definition of ordered the Prešić type θ-contractivity mapping.

Definition 1. *Let $(X, \preceq, p)$ be an ordered partial metric space. We say that $M : X^r \to X$ is an ordered Prešić type θ-contractivity mapping, if $\theta \in \Theta$ and there exists $t \in (0, 1)$ such that $\forall(\digamma_{r+1}, \digamma_{r+2}) \in Z^*$ implies that*

$$\theta(p(M(\digamma_1, \digamma_2, ..., \digamma_r), M(\digamma_2, \digamma_3, ..., \digamma_{r+1}),)) \leqslant \left[\theta(\max_{1\leqslant i\leqslant r}\{p(\digamma_i, \digamma_{i+1})\})\right]^t, \tag{2.1}$$

where

$$Z^* = \{(\digamma_{r+1}, \digamma_{r+2}) \in X \times X : \digamma_{r+1} \preceq \digamma_{r+2},\ \ p(M(\digamma_1, \digamma_2, ..., \digamma_r), M(\digamma_2, \digamma_3, ..., \digamma_{r+1})) > 0\}. \tag{2.2}$$

Theorem 3. *Let $(X, \preceq, p)$ be an ordered complete partial metric spaces, $M : X^r \to X$ an ordered Prešić type θ-contractivity mapping where r a positive integer and M is non-decreasing mapping. There exists the sequence $(\digamma_{n+r})$ defined by*

$$\digamma_{n+r} = M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}), \quad (n = 1, 2, ...) \tag{2.3}$$

such that $\digamma_{n+r} \preceq M(\digamma_{n+r}, \digamma_{n+r}, ..., \digamma_{n+r})$, for any arbitrary points $\digamma_1, \digamma_2, ..., \digamma_r \in X$. If M is continuous then M has one and only one fixed point.

Proof: Firstly, we show that M has a fixed point. Let $\digamma_1, \digamma_2, ..., \digamma_r$ be arbitrary r elements in X. Using these points define a sequence $(\digamma_n)$ as follows:

$$\digamma_{n+r} = M(\digamma_n, \digamma_{n+1}, \ldots, \digamma_{n+r-1}), \qquad (n = 1, 2, \ldots).$$

If there exists $n_0 \in \{1, 2, \ldots r\}$ for which $\digamma_{n_0} = \digamma_{n_0+1}$ then,

$$\digamma_{n_0+r} = M(\digamma_{n_0}, \digamma_{n_0+1}, \ldots, \digamma_{n_0+r-1}) = M(\digamma_{n_0+r}, \digamma_{n_0+r}, \ldots, \digamma_{n_0+r})$$

that is, $\digamma_{n_0+r}$ is a fixed point of M.

We assume that $\digamma_{n+r} \neq \digamma_{n+r+1}$ for all $n \in \mathbb{N}$. Since $\digamma_{n+r} \preceq M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r})$ and M is non-decreasing, we obtain

$$\digamma_{n+1} \preceq \digamma_{n+2} \preceq \digamma_{n+3} \preceq \cdots \preceq \digamma_{n+r} \preceq \ldots.$$

Denote $\chi_{n+r} = p(\digamma_{n+r}, \digamma_{n+r+1})$, for $n = 1, 2, ...$ and

$$T = \max\{p(\digamma_1, \digamma_2), p(\digamma_2, \digamma_3), \ldots, p(\digamma_r, \digamma_{r+1})\}$$

then we have $\chi_{n+r} > 0$ for all $n \in \mathbb{N}$ and $T > 0$. Since $\digamma_{n+r} \preceq \digamma_{n+r+1}$ and

$$p(M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}), M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r})) > 0$$

for every $n \in \mathbb{N}$, then $(\digamma_n, \digamma_{n+1}) \in Z^*$ and so for $n \leqslant r$, we have the following inequalities:

$$\begin{aligned} \theta(\chi_{r+1}) &= \theta(p(\digamma_{r+1}, \digamma_{r+2})) \\ &= \theta(p(M(\digamma_1, \digamma_2, ..., \digamma_r), M(\digamma_2, \digamma_3, ..., \digamma_{r+1}))) \\ &\leqslant \left[\theta(\max_{1\leqslant i\leqslant r}\{p(\digamma_i, \digamma_{i+1})\})\right]^t \\ &= [\theta(T)]^t. \end{aligned}$$

and so on. Hence we obtain

$$\theta(\chi_{n+r}) \leqslant [\theta(\chi_{n+r-1})]^t \leqslant [\theta(\chi_{n+r-2})]^{t^2} \leqslant \ldots \leqslant [\theta(\chi_n)]^{t^r}.$$

Thus, we have

$$1 < \theta(\chi_{n+r}) \leqslant [\theta(\chi_n)]^{t^r}, \tag{2.4}$$

for all $r \in \mathbb{N}$. Letting $r \to \infty$ in (2.4), we obtain

$$\theta(\chi_{n+r}) \to 1$$

which implies from (Θ_2) that

$$\lim_{r\to\infty} \chi_{n+r} = 0^+. \tag{2.5}$$

From (Θ_3), there exist $a \in (0, 1)$ and $\wp \in (0, \infty]$ such that

$$\lim_{r\to\infty} \frac{\theta(\chi_{n+r}) - 1}{(\chi_{n+r})^a} = \wp. \tag{2.6}$$

Assumed that $\wp < \infty$. In this case, let $E = \frac{\wp}{2} > 0$. From the definition of the limit, there exists $n_0 \in \mathbb{N}$ such that

$$\left|\frac{\theta(\chi_{n+r}) - 1}{(\chi_{n+r})^a} - \wp\right| \leqslant E, \text{ for all } n + r \geqslant n_0.$$

This implies that

$$\frac{\theta(\chi_{n+r}) - 1}{(\chi_{n+r})^a} \geqslant \wp - E = E, \text{ for all } n + r \geqslant n_0.$$

Then

$$n(\chi_{n+r})^{a} \leqslant Fn[\theta(\chi_{n+r}) - 1],$$

for all $n + r \geqslant n_0$ where $F = \frac{1}{E}$. Assume that $\wp = \infty$. Let $E > 0$ be an arbitrary positive number. From the definition of the limit, there exists $n_0 \in \mathbb{N}$ such that

$$\frac{\theta(\chi_{n+r}) - 1}{(\chi_{n+r})^{a}} \geqslant E,$$

for all $n + r \geqslant n_0$. This implies that

$$n(\chi_{n+r})^{a} \leqslant Fn[\theta(\chi_{n+r}) - 1],$$

for all $n + r \geqslant n_0$, where $F = \frac{1}{E}$.
Thus, in all cases, there exist $F > 0$ and $n_0 \in \mathbb{N}$ such that

$$n(\chi_{n+r})^{a} \leqslant Fn[\theta(\chi_{n+r}) - 1],$$

for all $n + r \geqslant n_0$. Using (2.4), we obtain

$$n(\chi_{n+r})^{a} \leqslant Fn([\theta(\chi_{n})]^{t^{r}} - 1),$$

for all $n \geqslant n_0$. Letting $r \to \infty$ in the above inequality, we obtain

$$\lim_{r\to\infty} n(\chi_{n+r})^{a} = 0.$$

Thus, there exists $n_0 \in \mathbb{N}$ such that

$$\chi_{n+r} \leqslant \frac{1}{n^{\frac{1}{a}}}, \quad \text{for all } \ n + r \geqslant n_0. \tag{2.7}$$

For any $n, m \in \mathbb{N}$ with $m > n \geqslant n_0$, we have

$$\begin{aligned}
p(\mp_{n+r}, \mp_{m+r}) &= p(M(\mp_{n}, ..., \mp_{n+r-1}), M(\mp_{m}, ..., \mp_{m+r-1})) \\
&\leqslant p(M(\mp_{n}, \mp_{n+1}, ..., \mp_{n+r-1}), M(\mp_{n+1}, \mp_{n+2}, ..., \mp_{n+r})) + \\
&\quad p(M(\mp_{n+1}, \mp_{n+2}, ..., \mp_{n+r}), M(\mp_{n+2}, \mp_{n+3}, ..., \mp_{n+r+1})) + \ldots + \\
&\quad p(M(\mp_{m-1}, \mp_{m}, ..., \mp_{m+r-2}), M(\mp_{m}, \mp_{m+1}, ..., \mp_{m+r-1})) - \\
&\quad \{p(M(\mp_{n+1}, \mp_{n+2}, ..., \mp_{n+r}), M(\mp_{n+1}, \mp_{n+2}, ..., \mp_{n+r})) + \\
&\quad p(M(\mp_{n+2}, \mp_{n+3}, ..., \mp_{n+r+1}), M(\mp_{n+2}, \mp_{n+3}, ..., \mp_{n+r+1})) + \ldots + \\
&\quad p(M(\mp_{m-1}, \mp_{m}, ..., \mp_{m+r-2}), M(\mp_{m-1}, \mp_{m}, ..., \mp_{m+r-2}))\} \\
&\leqslant p(M(\mp_{n}, \mp_{n+1}, ..., \mp_{n+r-1}), M(\mp_{n+1}, \mp_{n+2}, ..., \mp_{n+r})) + \\
&\quad p(M(\mp_{n+1}, \mp_{n+2}, ..., \mp_{n+r}), M(\mp_{n+2}, \mp_{n+3}, ..., \mp_{n+r+1})) + \ldots + \\
&\quad p(M(\mp_{m-2}, \mp_{m-1}, ..., \mp_{m+r-3}), M(\mp_{m-1}, \mp_{m}, ..., \mp_{m+r-2})) \\
&= p(\mp_{n+r}, \mp_{n+r+1}) + p(\mp_{n+r+1}, \mp_{n+r+2}) + \ldots + p(\mp_{m+r-2}, \mp_{m+r-1})
\end{aligned}$$

$$=\chi_{n+r}+\chi_{n+r+1}+\ldots+\chi_{m+r-2}<\sum_{i=n}^{\infty}\chi_{i+r}\leqslant\sum_{i=n}^{\infty}\frac{1}{i^{\frac{1}{a}}}\to 0.$$

This shows that $(\mathcal{T}_n)$ is a Cauchy sequence in (X,p). Since (X,p) is complete partial metric spaces the sequence $(\mathcal{T}_n)$ convergence to some point $e\in X$. That is

$$\lim_{n,m\to\infty}p(\mathcal{T}_{n+r},e)=0=\lim_{n,m\to\infty}p(\mathcal{T}_{n+r},\mathcal{T}_{m+r})=p(e,e).$$

Now if M is continuous, then we have

$$\begin{aligned}e=\lim_{n\to\infty}\mathcal{T}_{n+r}&=\lim_{n\to\infty}M(\mathcal{T}_n,\mathcal{T}_{n+1},...,\mathcal{T}_{n+r-1})\\&=M(\lim_{n\to\infty}\mathcal{T}_n,\lim_{n\to\infty}\mathcal{T}_{n+1},...,\lim_{n\to\infty}\mathcal{T}_{n+r-1})\\&=M(e,e,\ldots,e).\end{aligned}$$

Now let us show that the fixed point of M is uniqueness. Suppose that there exists another fixed point f of M distinct from e, such that $e=M(e,e,\ldots,e)$ and $f=M(f,f,\ldots,f)$ with $\forall(e,f)\in Z^*$, then

$$p(M(e,e,\ldots,e),M(f,f,\ldots,f))>0.$$

Then it follows from the assumption that

$$\theta(p(e,f))=\theta(p(M(e,e,\ldots,e),M(f,f,\ldots,f)))\leqslant[\theta(p(e,f))]^t<\theta(p(e,f)).$$

which is a contraction since $t\in(0,1)$. Thus M has a unique fixed point.

Example 1. *Let $X=\{u_n;n=1,2,\ldots\}$ and $p(d,h)=\max\{d,h\}$. Define an order relation $\preceq$ on X as*

$$u_s\preceq u_m\Leftrightarrow[u_s=u_m\ \ or\ \ u_s\leqslant u_m\ \ with\ \ u_s,u_m\in X],$$

where $\leqslant$ is usual order. Obviously, $(X,\preceq,p)$ be an ordered complete partial metric spaces. Let $k\in\mathbb{Z}^+$ and $M:X^k\to X$ be given by $M(u_1,u_1,...,u_1)=u_1$, for all $n\neq 1$, $M(u_n,u_n,...,u_n)=u_{n+1}$. Now we claim that an ordered Prešić type θ-contractivity mapping with $\theta(u):=e^{\sqrt{u}}$. Note that for $u_n=\frac{1}{n}$ and $u_s\preceq u_m$. Thus

$$p(M(u_s,u_s,...,u_s),M(u_m,u_m,...,u_m))>0,$$

we have

$$p(M(u_s,u_s,...,u_s),M(u_m,u_m,...,u_m))=\max\left\{\frac{1}{m+1},\frac{1}{s+1}\right\}=\frac{1}{s+1}$$

and

$$p(u_s,u_m)=\max\left\{\frac{1}{m},\frac{1}{s}\right\}=\frac{1}{s}.$$

Therefore,

$$\frac{s}{s+1}\leqslant t$$

for some $t\in(0,1)$. Therefore Theorem 3 implies that M has a unique fixed point. In this example u_1 is the unique fixed point of M.

Following is an example which illustrates that an ordered Prešić type θ-contractivity in partial metric space need not to be a Prešić type contraction in metric space.

Example 2. *Let* $X = \{\digamma_r = \frac{2r^2+r}{2},\ r \in \mathbb{N}\} \cup \{0\}$ *and* $p(d,h) = \mid d-h \mid + \max\{d,h\}$. *Define an order relation* $\preceq$ *on* X *as*

$$\digamma_r \preceq \digamma_{r+1} \Leftrightarrow [\digamma_r = \digamma_{r+1} \ \ or \ \ \digamma_r \leqslant \digamma_{r+1} with \ \ \digamma_r, \digamma_{r+1} \in X],$$

here $\leqslant$ *is usual order. Clearly,* $(X, \preceq, p)$ *be an ordered complete partial metric spaces. Define the mapping* $M : X^2 \to X$ *by*

$$M(\digamma, \mathcal{A}) = \frac{\digamma_r + \mathcal{A}_r}{2} \quad for \ \ all \ \ \digamma_r, \mathcal{A}_r \in X.$$

We claim that M *is an ordered Prešić type* θ*-contractivity with respect to* $\theta(m) = e^{me^m}$ *and* $s = e^{-2} \in (0,1)$. *To see this, we shall prove that* M *satisfies the condition (2.1). Then we obtain*

$$e^{p(M(\digamma_{r-1},\digamma_r),M(\digamma_r,\digamma_{r+1}))e^{p(M(\digamma_{r-1},\digamma_r),M(\digamma_r,\digamma_{r+1}))}} \leqslant e^{s(\max\{p(\digamma_{r-1},\digamma_r),p(\digamma_r,\digamma_{r+1})\}e^{\max\{p(\digamma_{r-1},\digamma_r),p(\digamma_r,\digamma_{r+1})\}})},$$

for $s = e^{-2}$. *The above condition is equivalent to*

$$\begin{aligned} p(M(\digamma_{r-1},\digamma_r), M(\digamma_r,\digamma_{r+1}))&e^{p(M(\digamma_{r-1},\digamma_r),M(\digamma_r,\digamma_{r+1}))} \\ &\leqslant s\max\{p(\digamma_{r-1},\digamma_r), p(\digamma_r,\digamma_{r+1})\}e^{\max\{p(\digamma_{r-1},\digamma_r),p(\digamma_r,\digamma_{r+1})\}}. \end{aligned}$$

So, for $s = e^{-2}$, *we attain*

$$\frac{p(M(\digamma_{r-1},\digamma_r), M(\digamma_r,\digamma_{r+1}))}{\max\{p(\digamma_{r-1},\digamma_r), p(\digamma_r,\digamma_{r+1})\}} e^{p(M(\digamma_{r-1},\digamma_r),M(\digamma_r,\digamma_{r+1}))-\max\{p(\digamma_{r-1},\digamma_r),p(\digamma_r,\digamma_{r+1})\}} \leqslant s. \tag{2.8}$$

Then, we obtain

$$\begin{aligned} \frac{p(M(\digamma_{r-1},\digamma_r), M(\digamma_r,\digamma_{r+1}))}{\max\{p(\digamma_{r-1},\digamma_r), p(\digamma_r,\digamma_{r+1})\}} &e^{p(M(\digamma_{r-1},\digamma_r),M(\digamma_r,\digamma_{r+1}))-\max\{p(\digamma_{r-1},\digamma_r),p(\digamma_r,\digamma_{r+1})\}} \\ &= \frac{4r^2+14r+5}{4r^2+18r+12} e^{\frac{-4r-7}{4}} \leqslant e^{-2}. \end{aligned}$$

Thus the inequality (2.8) is satisfied with $s = e^{-2}$. *Therefore Theorem 3 implies that* M *has a unique fixed point, that is,* $M(0,0) = 0$.

On the other hand, it is not Prešić type contraction in metric spaces, where $d(d,h) = |d-h|$, *for all* $d, h \in X$. *To see this, we obtain*

$$\lim_{r\to\infty} \frac{d(M(\digamma_{r-1},\digamma_r), M(\digamma_r,\digamma_{r+1}))}{\max\{d(\digamma_{r-1},\digamma_r), d(\digamma_r,\digamma_{r+1})\}} = \lim_{r\to\infty} \frac{4r+1}{4r+3} = 1.$$

Then

$$d(M(\digamma_{r-1},\digamma_r), M(\digamma_r,\digamma_{r+1})) \leqslant q\max\{d(\digamma_{r-1},\digamma_r), d(\digamma_r,\digamma_{r+1})\}$$

does not hold for $q \in (0,1)$. *Hence the condition of Theorem 2 is not satisfied.*

Since

$$\lim_{r\to\infty}\frac{p(M(\mp_{r-1},\mp_r),M(\mp_r,\mp_{r+1}))}{\max\{p(\mp_{r-1},\mp_r),p(\mp_r,\mp_{r+1})\}}=\lim_{r\to\infty}\frac{4r^2+14r+5}{4r^2+18r+12}=1,$$

the condition of Theorem 2.1 *in [14] is not satisfied.*

This example shows the new class of ordered Prešić type θ-contractivity operators is not included in Prešić type classes of operators known in literature.

Corollary 1. *Let $(X,\preceq,p)$ be an ordered complete partial metric space, r positive integer and $M:X^r\to X$ a given mapping. Assume that there a exist $\theta\in\Theta$ and $t\in(0,1)$ such that*

$$\theta(p(M(\mp_1,\mp_2,...,\mp_r),M(\mp_2,\mp_3,...,\mp_{r+1}))\leqslant[\theta(\max_{1\leqslant i\leqslant r}\{p(\mp_i,\mp_{i+1})\})]^t,$$

for all $(\mp_{r+1},\mp_{r+2})\in Z^*$, where

$$p(M(\mp_1,\mp_2,...,\mp_r),M(\mp_2,\mp_3,...,\mp_{r+1}))>0.$$

Now let we show that the contractive mapping of Corollary 1. If M is a contractive there exists $\eta\in(0,1)$ such that

$$p(M(\mp_1,\mp_2,...,\mp_r),M(\mp_2,\mp_3,...,\mp_{r+1}))\leqslant\eta\max_{1\leqslant i\leqslant r}\{p(\mp_i,\mp_{i+1})\},\quad\forall\mp_{r+1},\mp_{r+2}\in X$$

then we have

$$e^{p(M(\mp_1,\mp_2,...,\mp_r),M(\mp_2,\mp_3,...,\mp_{r+1}))}\leqslant\left[e^{\max_{1\leqslant i\leqslant r}\{p(\mp_i,\mp_{i+1})\}}\right]^t.$$

Therefore the function $\theta:(0,\infty)\to(1,\infty)$ defined by $\theta(u):=e^{\sqrt{u}}$ belong to Θ. Also we obtain

$$\theta(p(M(e,e,\dots,e),M(f,f,\dots,f)))\leqslant[\theta(p(e,f))]^t,$$

for all $(e,f)\in Z^*$, where

$$p(M(e,e,\dots,e),M(f,f,\dots,f))>0.$$

Then M has one and only one fixed point. If M is a contractive there exists $\eta\in(0,1)$ such that

$$p(M(e,e,\dots,e),M(f,f,\dots,f))\leqslant\eta p(e,f),$$

then we have

$$e^{p(M(e,e,\dots,e),M(f,f,\dots,f))}\leqslant\left[e^{p(e,f)}\right]^t.$$

3. Ordered Prešić type F-contraction mappings

Recently, Abbas et al. [20] introduced a certain fixed point theorem for the Prešić type F-contractive mapping. Now we give a fixed point theorem for ordered the Prešić type F-contractive mapping in partial metric space. Firstly, let us start with the definition of the ordered Prešić type F-contraction mapping.

Definition 2. *Let $(X, \preceq, p)$ be an ordered partial metric space. We say that $M : X^r \to X$ is an ordered Prešić type F-contraction mapping if $F \in \mathcal{F}$ and there exist $\tau > 0$ such that $\forall(\mp_{r+1}, \mp_{r+2}) \in S^*$ implies that*

$$\tau + F(p(M(\mp_1, \mp_2, ..., \mp_r), M(\mp_2, \mp_3, ..., \mp_{r+1}))) \leqslant F(\max_{1\leqslant t\leqslant r}\{p(\mp_t, \mp_{t+1})\}), \tag{3.1}$$

where

$$S^* = \{(\mp_{r+1}, \mp_{r+2}) \in X \times X : \mp_{r+1} \preceq \mp_{r+2},\ \ p(M(\mp_1, \mp_2, ..., \mp_r), M(\mp_2, \mp_3, ..., \mp_{r+1})) > 0\}. \tag{3.2}$$

Theorem 4. *Let $(X, \preceq, p)$ be an ordered complete partial metric spaces, $M : X^r \to X$ an ordered Prešić type F-contraction mapping, where r is a positive integer and M is non-decreasing mapping. There exists the sequence $(\mp_{n+r})$ defined by*

$$\mp_{n+r} = M(\mp_n, \mp_{n+1}, ..., \mp_{n+r-1}), \quad (n = 1, 2, ...) \tag{3.3}$$

such that $\mp_{n+r} \preceq M(\mp_{n+r}, \mp_{n+r}, ..., \mp_{n+r})$, for any arbitrary points $\mp_1, \mp_2, ..., \mp_r \in X$. If M is continuous or X is regular then M has a fixed point.

(A) If every pair of elements have a lower bound and upper bound, thus the fixed point of M is unique.

Moreover if $\forall(e, f) \in S^*$ implies that

$$\tau + F(p(M(e, e, ..., e), M(f, f, ..., f))) \leqslant F(p(e, f)),$$

then M has one and only one fixed point.

Proof: Firstly, we shows that M has a fixed point. Let $\mp_1, \mp_2, ..., \mp_r$, be arbitrary r elements in X. Using these points define a sequence $(\mp_n)$ as follows:

$$\mp_{n+r} = M(\mp_n, \mp_{n+1}, \ldots, \mp_{n+r-1}), \qquad (n = 1, 2, \ldots).$$

If there exists $n_0 \in \{1, 2, \ldots r\}$ for which $\mp_{n_0} = \mp_{n_0+1}$ then

$$\mp_{n_0+r} = M(\mp_{n_0}, \mp_{n_0+1}, \ldots, \mp_{n_0+r-1}) = M(\mp_{n_0+r}, \mp_{n_0+r}, \ldots, \mp_{n_0+r})$$

that is, $\mp_{n_0+r}$ is a fixed point of M.

We assume that $\mp_{n+r} \neq \mp_{n+r+1}$ for all $n \in \mathbb{N}$. Since $\mp_{n+r} \preceq M(\mp_{n+1}, \mp_{n+2}, ..., \mp_{n+r})$ and M is non-decreasing, we obtain

$$\mp_{n+1} \preceq \mp_{n+2} \preceq \mp_{n+3} \preceq \cdots \preceq \mp_{n+r} \preceq \ldots.$$

Denote $\kappa_{n+r} = p(\mp_{n+r}, \mp_{n+r+1})$, for $n = 1, 2, ...$ and

$$P = \max\{p(\mp_1, \mp_2), p(\mp_2, \mp_3), \ldots, p(\mp_r, \mp_{r+1})\}$$

then we have $\kappa_{n+r} > 0$ for all $n \in \mathbb{N}$ and $P > 0$. Since $\mp_{n+r} \preceq \mp_{n+r+1}$ and

$$p(M(\mp_n, \mp_{n+1}, ..., \mp_{n+r-1}), M(\mp_{n+1}, \mp_{n+2}, ..., \mp_{n+r})) > 0$$

for every $n \in \mathbb{N}$, then $(Ⱦ_n, Ⱦ_{n+1}) \in S^*$ and so for $n \leqslant r$, we have the following inequalities:

$$\begin{aligned} F(\kappa_{r+1}) &= F(p(Ⱦ_{r+1}, Ⱦ_{r+2})) \\ &= F(p(M(Ⱦ_1, Ⱦ_2, ..., Ⱦ_r), M(Ⱦ_2, Ⱦ_3, ..., Ⱦ_{r+1}))) \\ &\leqslant F(\max_{1\leqslant t\leqslant r}\{p(Ⱦ_t, Ⱦ_{t+1})\}) - \tau \\ &= F(P) - \tau \end{aligned}$$

$$\begin{aligned} F(\kappa_{r+2}) &= F(p(Ⱦ_{r+2}, Ⱦ_{r+3})) \\ &= F(p(M(Ⱦ_2, Ⱦ_3, ..., Ⱦ_{r+1}), M(Ⱦ_3, Ⱦ_4, ..., Ⱦ_{r+2}))) \\ &\leqslant F(\max_{2\leqslant t\leqslant r+1}\{p(Ⱦ_t, Ⱦ_{t+1})\}) - 2\tau \\ &\leqslant F(P) - 2\tau \end{aligned}$$

and so on. Thus we obtain

$$\begin{aligned} F(\kappa_{n+r}) &= F(p(Ⱦ_{n+r}, Ⱦ_{n+r+1})) \\ &= F(p(M(Ⱦ_n, Ⱦ_{n+1}, ..., Ⱦ_{n+r-1}), M(Ⱦ_{n+1}, Ⱦ_{n+2}, ..., Ⱦ_{n+r})) \\ &\leqslant F(\max_{n\leqslant t\leqslant n+r-1}\{p(Ⱦ_t, Ⱦ_{t+1})\}) - n\tau \\ &\leqslant F(P) - n\tau \end{aligned} \tag{3.4}$$

for $n \geqslant 1$. Letting $n \to \infty$ in (3.4) we obtain

$$\lim_{n\to\infty} F(\kappa_{n+r}) = -\infty \tag{3.5}$$

which implies from $(F2)$ that

$$\lim_{n\to\infty} \kappa_{n+r} = 0. \tag{3.6}$$

From $(F3)$ there exists $h \in (0,1)$ such that

$$\lim_{n\to\infty} \kappa_{n+r}^h F(\kappa_{n+r}) = 0. \tag{3.7}$$

By (3.4), we have

$$\kappa_{n+r}^h F(\kappa_{n+r}) - \kappa_{n+r}^h F(P) \leqslant -\kappa_{n+r}^h n\tau \leqslant 0. \tag{3.8}$$

On taking the limit as $n \to \infty$, we obtain

$$\lim_{n\to\infty} n\kappa_{n+r}^h = 0. \tag{3.9}$$

Thus from (3.9) there exists $n_0 \in \mathbb{N}$ such that $n\kappa_{n+r}^h \leqslant 1$ for all $n \geqslant n_0$. Consequently we have

$$\kappa_{n+r} \leqslant \frac{1}{n^{\frac{1}{h}}}$$

for all $n \geqslant n_0$.

For any $n, m \in \mathbb{N}$ with $m > n \geqslant n_0$, we have

$$\begin{aligned}
p(\digamma_{n+r}, \digamma_{m+r}) &= p(M(\digamma_n, ..., \digamma_{n+r-1}), M(\digamma_m, ..., \digamma_{m+r-1}))) \\
&\leqslant p(M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}), M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}))+ \\
&\quad p(M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}), M(\digamma_{n+2}, \digamma_{n+3}, ..., \digamma_{n+r+1})) + \ldots + \\
&\quad p(M(\digamma_{m-1}, \digamma_m, ..., \digamma_{m+r-2}), M(\digamma_m, \digamma_{m+1}, ..., \digamma_{m+r-1}))- \\
&\quad \{p(M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}), M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}))+ \\
&\quad p(M(\digamma_{n+2}, \digamma_{n+3}, ..., \digamma_{n+r+1}), M(\digamma_{n+2}, \digamma_{n+3}, ..., \digamma_{n+r+1})) + \ldots + \\
&\quad p(M(\digamma_{m-1}, \digamma_m, ..., \digamma_{m+r-2}), M(\digamma_{m-1}, \digamma_m, ..., \digamma_{m+r-2}))\} \\
&\leqslant p(M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}), M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}))+ \\
&\quad p(M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}), M(\digamma_{n+2}, \digamma_{n+3}, ..., \digamma_{n+r+1})) + \ldots + \\
&\quad p(M(\digamma_{m-2}, \digamma_{m-1}, ..., \digamma_{m+r-3}), M(\digamma_{m-1}, \digamma_m, ..., \digamma_{m+r-2})) \\
&= p(\digamma_{n+r}, \digamma_{n+r+1}) + p(\digamma_{n+r+1}, \digamma_{n+r+2}) + \ldots + p(\digamma_{m+r-2}, \digamma_{m+r-1}) \\
&= \kappa_{n+r} + \kappa_{n+r+1} + \ldots + \kappa_{m+r-2} < \sum_{t=n}^{\infty} \kappa_{t+r} \leqslant \sum_{t=n}^{\infty} \frac{1}{t^{\frac{1}{h}}} \to 0.
\end{aligned}$$

This shows that $(\digamma_n)$ is a Cauchy sequence in (X, p). Since (X, p) is complete partial metric spaces, the sequence $(\digamma_{n+r})$ convergence to some point $e \in X$. That is

$$\lim_{n,m\to\infty} p(\digamma_{n+r}, e) = 0 = \lim_{n,m\to\infty} p(\digamma_{n+r}, \digamma_{m+r}) = p(e, e).$$

Now if M is continuous, then we have

$$\begin{aligned}
e = \lim_{n\to\infty} \digamma_{n+r} &= \lim_{n\to\infty} M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}) \\
&= M(\lim_{n\to\infty} \digamma_n, \lim_{n\to\infty} \digamma_{n+1}, ..., \lim_{n\to\infty} \digamma_{n+r-1}) \\
&= M(e, e, \ldots, e).
\end{aligned}$$

We stated that X is regular, if the ordered partial metric spaces $(X, \preceq, p)$ provides the following condition:
If $\{\digamma_n\} \subseteq X$ is a nondecreasing sequence with $\digamma_n \to e \in X$, then $\digamma_n \preceq e$ for all $n \in \mathbb{N}$. Assume $(X, \preceq, p)$ is regular, then $\digamma_n \preceq e$ for all $n \in \mathbb{N}$. Then two cases arised here.
Case 1. If there exists $n, r \in \mathbb{N}$ for which $\digamma_{n+r} = e$ then we obtain

$$M(e, e, ..., e) = M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}) = \digamma_{n+r+1} \preceq e.$$

Moreover, since $\digamma_{n+r} \preceq \digamma_{n+r+1}$, then $e \preceq M(e, e, ..., e)$ and thus, $e = M(e, e, ..., e)$.
Case 2. Assume that $\digamma_n \neq e$ for every $n \in \mathbb{N}$ and

$$p(e, M(e, e, ..., e)) > 0.$$

Since $\lim_{n\to\infty} \digamma_n = e$, then there exist $n_1 \in \mathbb{N}$ such that

$$p(\digamma_{n+r+1}, M(e, e, ..., e)) > 0$$

and

$$p(\digamma_n, e) < \frac{p(e, M(e, e, ..., e))}{2}$$

for all $n \geqslant n_1$, where $(\digamma_n, e) \in S^*$. Therefore by considering $(F1)$, we have, for $n \geqslant n_1$,

$$\begin{aligned}\tau + F(p(M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}), M(e, e, ..., e))) &\leqslant F(\max_{n+1 \leqslant t \leqslant n+r}\{p(\digamma_t, e)\}) \\ &\leqslant F\left(\frac{p(e, M(e, e, ..., e))}{2}\right),\end{aligned}$$

which yields

$$p(\digamma_{n+r+1}, M(e, e, ..., e)) \leqslant \frac{p(e, M(e, e, ..., e))}{2}.$$

Taking limit as $n \to \infty$, we deduce that

$$p(e, M(e, e, ..., e)) \leqslant \frac{p(e, M(e, e, ..., e))}{2}$$

a contraction. Therefore we conclude that $p(e, M(e, e, ..., e)) = 0$, that is, $e = M(e, e, ..., e)$. Now to see condition (A) it is sufficient to show that for every $\forall \digamma_{n+r} \in X$, $\lim_{n\to\infty} M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}) = e$ where e is the fixed point of M such that $e = \lim_{n\to\infty} M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r})$. For which two cases arise:

Let $\digamma_{n+r} \in X$ and $\digamma_{n+r+1}$ be as in Theorem 4.

Case 1: If $\digamma_{n+r} \preceq \digamma_{n+r+1}$ or $\digamma_{n+r+1} \preceq \digamma_{n+r}$, then

$$M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}) \preceq M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r})$$

or

$$M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}) \preceq M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1})$$

for all $n \in \mathbb{N}$. If

$$M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}) = M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r})$$

for some $n \in \mathbb{N}$, then $M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}) \to e$. Now let

$$M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}) \neq M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r})$$

for all $n \in \mathbb{N}$, then

$$p(M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}), M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r})) > 0$$

and so $(M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}), M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r})) \in S^*$ for all $n \in \mathbb{N}$. Therefore from (3.1), we obtain

$$\begin{aligned}F(p(M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}), M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}))) &\leqslant F(\max_{n \leqslant t \leqslant n+r-1}\{p(\digamma_t, \digamma_{t+1})\}) - n\tau \\ &\leqslant F(P) - n\tau. \qquad (3.10)\end{aligned}$$

Taking into account $(F2)$, from (3.10) we obtain

$$\lim_{n\to\infty} p(M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}), M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r})) = 0$$

and then,

$$\lim_{n\to\infty} M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}) = \lim_{n\to\infty} M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}) = e.$$

Case 2: If $\digamma_{n+r} \npreceq \digamma_{n+r+1}$ or $\digamma_{n+r+1} \npreceq \digamma_{n+r}$ then from (A), there exist $\digamma_{m+r}, \digamma_{m+r+1} \in X$ such that $\digamma_{m+r+1} \preceq \digamma_{n+r} \preceq \digamma_{m+r}$ and $\digamma_{m+r+1} \preceq \digamma_{n+r+1} \preceq \digamma_{m+r}$. Therefore, as in the case 1, we can show that

$$\begin{aligned}\lim_{n\to\infty} M(\digamma_m, \digamma_{m+1}, ..., \digamma_{m+r-1}) &= \lim_{n\to\infty} M(\digamma_{m+1}, \digamma_{m+2}, ..., \digamma_{m+r}) \\ &= \lim_{n\to\infty} M(\digamma_n, \digamma_{n+1}, ..., \digamma_{n+r-1}) \\ &= \lim_{n\to\infty} M(\digamma_{n+1}, \digamma_{n+2}, ..., \digamma_{n+r}) = e.\end{aligned}$$

Also, we can show that the fixed point of M is unique the in this method. Suppose that $e = M(e, e, \ldots, e)$ and $f = M(f, f, \ldots, f)$ with $\forall (e, f) \in S^*$. Thus

$$p(M(e, e, \ldots, e), M(f, f, \ldots, f)) > 0.$$

Thus by given suppose we have

$$\tau + F(p(e, f)) = \tau + F(p(M(e, e, ..., e), M(f, f, ..., f))) \leqslant F(p(e, f)).$$

a contraction as $\tau > 0$, so $e = f$.

Example 3. *Let $X = [0, 4]$ and $p(d, h) = \max(d, h)$. Define an order relation $\preceq$ on X as*

$$\digamma_r \preceq \digamma_{r+1} \Leftrightarrow [\digamma_r = \digamma_{r+1} \;\; or \;\; \digamma_r \leqslant \digamma_{r+1} with \;\; \digamma_r, \digamma_{r+1} \in X],$$

here $\leqslant$ is usual order. Clearly, $(X, \preceq, p)$ be an ordered complete partial metric spaces. Let r positive integer and $M : X^r \to X$ be the mapping defined by

$$M(\digamma_1, \digamma_2, ..., \digamma_r) = \frac{\digamma_1 + \digamma_r}{8r} \quad for \;\; all \;\; \digamma_1, \digamma_2, ..., \digamma_r \in X.$$

Define $F : \mathbb{R}_+ \to \mathbb{R}$ by $F(\nu) = \nu + \ln(\nu)$. Note that for $\tau = \ln(4r)$ and $\digamma_r \preceq \digamma_{r+1}$. Thus

$$p(M(\digamma_1, \digamma_2, ..., \digamma_r), M(\digamma_2, \digamma_3, ..., \digamma_{r+1})) > 0,$$

we have

$$\begin{aligned}&\tau + F(p(M(\digamma_1, \digamma_2, ..., \digamma_r), M(\digamma_2, \digamma_3, ..., \digamma_{r+1}))) \\ &= \ln(4r) + F\left(\max\left\{\frac{\digamma_1 + \digamma_r}{8r}, \frac{\digamma_2 + \digamma_{r+1}}{8r}\right\}\right) \\ &= \ln(4r) + F\left(\frac{1}{8r}(\digamma_2 + \digamma_{r+1})\right) = \ln(4r) + F\left(\frac{1}{8r}(p(\digamma_1, \digamma_2) + p(\digamma_r, \digamma_{r+1}))\right)\end{aligned}$$

$$
\begin{aligned}
&\leqslant \ln(4r) + F(\frac{1}{4r}\,(p(\digamma_r, \digamma_{r+1})) = \ln(4r) + \frac{1}{4r}p(\digamma_r, \digamma_{r+1}) + \ln\frac{1}{4r}\,(p(\digamma_r, \digamma_{r+1}))\\
&= \frac{1}{4r}p(\digamma_r, \digamma_{r+1}) + \ln p(\digamma_r, \digamma_{r+1}) \leqslant \max_{1\leqslant t\leqslant r}\{p(\digamma_t, \digamma_{t+1})\} + \ln \max_{1\leqslant t\leqslant r}\{p(\digamma_t, \digamma_{t+1})\}\\
&= F(\max_{1\leqslant t\leqslant r}\{p(\digamma_t, \digamma_{t+1})\})
\end{aligned}
$$

In addition for all $e, f \in X$ with $e \preceq f$

$$p(M(e, e, ..., e), M(f, f, ..., f)) = \max\left\{\frac{e}{4r}, \frac{f}{4r}\right\} > 0$$

and

$$
\begin{aligned}
F(p(M(e, e, ..., e), M(f, f, ..., f))) &= F\left(\max\left\{\frac{e}{4r}, \frac{f}{4r}\right\}\right) = F\left(\frac{1}{4r}p(d, h)\right)\\
&= \frac{1}{4r}p(d, h) + \ln\left(\frac{1}{4r}p(d, h)\right)\\
&= \frac{1}{4r}p(d, h) + \ln(p(d, h)) - \ln(4r)\\
&\leqslant p(d, h) + \ln(p(d, h)) - \tau = F(p(d, h)) - \tau
\end{aligned}
$$

Thus all the required assumptions of Theorem 4 are satisfied. In addition, for any arbitrary points $\digamma_1, \digamma_2, ..., \digamma_r \in X$, the sequence $(\digamma_n)$ defined by (3.3) converges to $e = 0$, the unique fixed point of M.

Following is an example which illustrates that an ordered Prešić type F-contraction in partial metric space need not to be a Prešić type contraction in metric space.

Example 4. *Let $X = \{\digamma_r = \frac{2r(r+1)}{4},\ \ r \in \mathbb{N}\}$ and $p(\digamma, \mathcal{A}) = \max\{\digamma, \mathcal{A}\}$. Define an order relation $\preceq$ on X as*

$$\digamma_r \preceq \digamma_{r+1} \Leftrightarrow [\digamma_r = \digamma_{r+1} \ \ or\ \ \digamma_r \leqslant \digamma_{r+1} with\ \ \digamma_r, \digamma_{r+1} \in X],$$

here $\leqslant$ is usual order. Clearly, $(X, \preceq, p)$ be an ordered complete partial metric spaces. Define the mapping $M : X^2 \to X$ by

$$M(\digamma, \mathcal{A}) = \frac{\digamma_r + \mathcal{A}_r}{2} \quad for\ \ all\ \ \digamma_r, \mathcal{A}_r \in X.$$

We claim that M is an ordered Prešić type F-contraction mapping with respect to $F(\nu) = \nu + \ln(\nu)$ and $\tau = \frac{1}{2}$. To see this, we shall prove that M satisfies the condition (3.1). Then we obtain

$$
\begin{aligned}
&p(M(\digamma_{r-1}, \digamma_r), M(\digamma_r, \digamma_{r+1}))e^{p(M(\digamma_{r-1}, \digamma_r), M(\digamma_r, \digamma_{r+1})) - \max\{p(\digamma_{r-1}, \digamma_r), p(\digamma_r, \digamma_{r+1})\}}\\
&\qquad = \frac{r^2 + 2r + 1}{2}e^{\frac{-r-1}{2}}\\
&\qquad < \frac{r^2 + 3r + 2}{2}e^{-\frac{1}{2}} = e^{-\frac{1}{2}}\max\{p(\digamma_{r-1}, \digamma_r), p(\digamma_r, \digamma_{r+1})\}.
\end{aligned}
$$

Therefore Theorem 3 implies that M has a unique fixed point, that is, $M(1, 1) = 1$.

On the other hand, it is not Prešić type contraction in metric spaces, where $d(d,h) = |d - h|$, *for all* $d, h \in X$. *Hence the condition of Theorem 2 is not satisfied. Since*

$$\lim_{r\to\infty} \frac{p(M(\mp_{r-1}, \mp_r), M(\mp_r, \mp_{r+1}))}{\max\{p(\mp_{r-1}, \mp_r), p(\mp_r, \mp_{r+1})\}} = \lim_{r\to\infty} \frac{2r^2 + 4r + 2}{2r^2 + 6r + 4} = 1,$$

the condition of Theorem 2.1 *in [14] is not satisfied.*

This example shows the new class of ordered Prešić type F*-contraction operators is not included in Prešić type classes of operators known in literature.*

The following results are an relation consequence of Theorem 4 by taking $F(\nu) = \ln \nu$.

Corollary 2. *Let* $(X, \preceq, p)$ *be an ordered complete partial metric space, r positive integer and* $M : X^r \to X$ *a given mapping. Assume that there exists* $\tau > 0$ *such that*

$$p(M(\mp_1, \mp_2, ..., \mp_r), M(\mp_2, \mp_3, ..., \mp_{r+1})) \leqslant e^{-\tau} \max_{1\leqslant i\leqslant r}\{p(\mp_i, \mp_{i+1})\}, \tag{3.11}$$

for all $(\mp_1, \mp_2, ..., \mp_{r+1}) \in X^{r+1}$ *with* $\mp_r \preceq \mp_{r+1}$. *Then for any arbitrary points* $\mp_1, \mp_2, ..., \mp_r \in X$, *the sequence* $(\mp_n)$ *defined by (3.3) converges to e, and e is a fixed point of M. That is,* $e = M(e, e, \ldots, e)$. *Moreover if*

$$p(M(e, e, ..., e), M(f, f, ..., f)) \leqslant e^{-\tau} p(e, f)$$

holds for all $e, f \in X$ *with* $e \preceq f$, *then e is the unique fixed point of M.*

Corollary 3. *Let* $(X, \preceq, p)$ *be an ordered complete partial metric space, r positive integer and* $M : X^r \to Xa$ *given mapping. Assume that there exists* $\delta_1, \delta_2, \ldots, \delta_k$ *non-negative constants with* $\delta_1 + \delta_2 + \cdots + \delta_r < 1$ *such that*

$$p(M(\mp_1, \mp_2, ..., \mp_r), M(\mp_2, \mp_3, ..., \mp_{r+1})) \leqslant \delta_1 p(\mp_1, \mp_2) + \delta_2 p(\mp_2, \mp_3) + \ldots + \delta_r p(\mp_k, \mp_{r+1}) \tag{3.12}$$

for all $(\mp_1, \mp_2, ..., \mp_{r+1}) \in X^{r+1}$ *with* $\mp_r \preceq \mp_{r+1}$. *Then for any arbitrary points* $\mp_1, \mp_2, ..., \mp_r \in X$, *the sequence* $(\mp_n)$ *defined by (3.3) converges to e, where e is the unique fixed point of M.*

Proof: Clearly condition (3.12) implies condition (3.11) with $\delta = \delta_1 + \delta_2 + \cdots + \delta_r$. Now, let $e, f \in X$ with $e \preceq f$. From (3.12), we have

$$\begin{aligned} p(M(e, e, ..., e), M(f, f, ..., f)) \leqslant\; & p(M(e, e, ..., e), M(e, e, ..., e, f)) + \\ & p(M(e, e, ..., e, f), M(e, e, ..., e, f, f)) + ... + \\ & p(M(e, f, ..., f), M(f, f, ..., f)) - \\ & \{p(M(e, e, ..., e, f), M(e, e, ..., e, f)) + \\ & p(M(e, e, ..., e, f, f), M(e, e, ..., e, f, f)) + ... + \\ & p(M(e, f, ..., f), M(f, f, ..., f))\} \\ \leqslant\; & p(M(e, e, ..., e), M(e, e, ..., e, f)) + \\ & p(M(e, e, ..., e, f), M(e, e, ..., e, f, f)) + ... + \\ & p(M(e, f, ..., f), M(f, f, ..., f)) \\ \leqslant\; & (\delta_1 + \delta_2 + \cdots + \delta_r) p(e, f) = \delta p(e, f), \end{aligned}$$

where $\delta = \delta_1 + \delta_2 + \cdots + \delta_r \in (0, 1)$. Therefore all the assumption of corollary 2 are satisfied.

4. Conclusions

In the present article, we prove the fixed point theorems for ordered Prešić type θ-contractivity and ordered Prešić type F-contraction mappings. Also, we provide examples showing that our main theorems are applicable.

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Conflict of interest

The authors declare that no competing interests exist.

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