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Research article

Comprehensive subclasses of analytic functions and coefficient bounds

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Abstract: In this paper, we introduce two general subclasses of analytic functions by means of the principle of subordination and investigate the coefficient bounds for functions in these classes. The well-known results are obtained as a corollary of our main results. Especially, we improve the results of Altıntaş and Kılıç [1].

Keywords: analytic functions; coefficient bounds; subordination
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1. Introduction

Let $\mathcal{A}$ be the family of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{D} = \{z : z \in \mathbb{C} \quad \text{and} \quad |z| < 1\}$.

For analytic functions f and g with $f(0) = g(0)$, f is said to be subordinate to g in $\mathbb{D}$ if there exists an analytic function $\mathfrak{h}$ on $\mathbb{D}$ such that

$$\mathfrak{h}(0) = 0, \quad |\mathfrak{h}(z)| < 1 \quad \text{and} \quad f(z) = g(\mathfrak{h}(z)) \qquad (z \in \mathbb{D}).$$

We denote the subordination by

$$f(z) \prec g(z) \qquad (z \in \mathbb{D}).$$

Note that if the function g is univalent in $\mathbb{D}$, then we have

$$f(z) \prec g(z) \quad (z \in \mathbb{D}) \Leftrightarrow f(0) = g(0) \quad \text{and} \quad f(\mathbb{D}) \subset g(\mathbb{D}).$$

Let $\mathcal{N}$ be the class consisting of analytic and univalent functions $\varphi : \mathbb{D} \to \mathbb{C}$ such that $\varphi(\mathbb{D})$ is convex with

$$\varphi(0) = 1 \qquad \text{and} \qquad \Re(\varphi(z)) > 0 \quad (z \in \mathbb{D}).$$

By means of functions belong to the class $\mathcal{N}$ and the principle of subordination, we consider following subclasses of analytic function class $\mathcal{A}$:

$$\mathcal{S}^*(\varphi) = \left\{ f \in \mathcal{A} : \frac{zf'(z)}{f(z)} \prec \varphi(z) \quad (\varphi \in \mathcal{N};\ z \in \mathbb{D}) \right\}, \tag{1.2}$$

$$\mathcal{K}(\varphi) = \left\{ f \in \mathcal{A} : 1 + \frac{zf''(z)}{f'(z)} \prec \varphi(z) \quad (\varphi \in \mathcal{N};\ z \in \mathbb{D}) \right\}, \tag{1.3}$$

$$C(\varphi, \psi) = \left\{ f \in \mathcal{A} : g \in \mathcal{K}(\psi) \ \wedge \ \frac{f'(z)}{g'(z)} \prec \varphi(z) \quad (\varphi, \psi \in \mathcal{N};\ z \in \mathbb{D}) \right\}, \tag{1.4}$$

$$\mathcal{CS}(\varphi, \psi) = \left\{ f \in \mathcal{A} : g \in \mathcal{S}^*(\psi) \ \wedge \ \frac{f(z)}{g(z)} \prec \varphi(z) \quad (\varphi, \psi \in \mathcal{N};\ z \in \mathbb{D}) \right\}, \tag{1.5}$$

$$\mathcal{QK}(\varphi, \psi) = \left\{ f \in \mathcal{A} : g \in \mathcal{K}(\psi) \ \wedge \ \frac{(zf'(z))'}{g'(z)} \prec \varphi(z) \quad (\varphi, \psi \in \mathcal{N};\ z \in \mathbb{D}) \right\}. \tag{1.6}$$

The classes $\mathcal{S}^*(\varphi)$ and $\mathcal{K}(\varphi)$ are introduced by Ma and Minda [2], and the class $C(\varphi, \psi)$ is introduced by Kim et al. [3]. Since

$$f(z) \in \mathcal{K}(\varphi) \Leftrightarrow zf'(z) \in \mathcal{S}^*(\varphi),$$

we also have

$$f(z) \in C(\varphi, \psi) \Leftrightarrow \exists g \in \mathcal{S}^*(\psi) \quad \text{s.t.} \quad \frac{zf'(z)}{g(z)} \prec \varphi(z) \quad (z \in \mathbb{D}).$$

Remark 1. If we choose

$$\varphi(z) = \frac{1 + Az}{1 + Bz} \quad (-1 \leq B < A \leq 1)$$

in (1.2) and (1.3), then we get the classes of Janowski starlike functions and Janowski convex functions

$$\mathcal{S}^*\left(\frac{1 + Az}{1 + Bz}\right) = \mathcal{S}^*(A, B) \qquad \text{and} \qquad \mathcal{K}\left(\frac{1 + Az}{1 + Bz}\right) = \mathcal{K}(A, B),$$

respectively, introduced by Janowski [4].

Remark 2. If we choose

$$\varphi(z) = \frac{1 + Az}{1 + Bz} \quad (-1 \leq B < A \leq 1) \qquad \text{and} \qquad \psi(z) = \frac{1 + z}{1 - z}$$

in (1.4) and (1.5), then we obtain the classes

$$C\left(\frac{1 + Az}{1 + Bz}, \frac{1 + z}{1 - z}\right) = \mathcal{CCV}(A, B), \qquad \mathcal{CS}\left(\frac{1 + Az}{1 + Bz}, \frac{1 + z}{1 - z}\right) = \mathcal{CST}(A, B)$$

introduced by Reade [5]; and from (1.6), we have the class

$$\mathcal{QK}\left(\frac{1 + Az}{1 + Bz}, \frac{1 + z}{1 - z}\right) = \mathcal{QCV}(A, B)$$

introduced by Altıntaş and Kılıç [1].

Remark 3. If we choose

$$\varphi(z) = \frac{1+(1-2\alpha)z}{1-z} \quad (0 \leq \alpha < 1) \quad \text{and} \quad \psi(z) = \frac{1+(1-2\beta)z}{1-z} \quad (0 \leq \beta < 1)$$

in (1.4), then we obtain the class of close-to-convex functions of order α and type β,

$$\mathcal{C}\left(\frac{1+(1-2\alpha)z}{1-z}, \frac{1+(1-2\beta)z}{1-z}\right) = \mathcal{C}(\alpha,\beta),$$

introduced by Libera [6].

Remark 4. If we choose

$$\varphi(z) = \frac{1+z}{1-z} = \psi(z)$$

in (1.2) − (1.4), then we get the familiar class $\mathcal{S}^*$ consists of starlike functions in $\mathbb{D}$, $\mathcal{K}$ consists of convex functions in $\mathbb{D}$ and $\mathcal{C}$ consists of close-to-convex function in $\mathbb{D}$, respectively. Also, from (1.5) and (1.6), we get the class $\mathcal{CS}$ of close-to-starlike functions in $\mathbb{D}$ introduced by Reade [5], and the class $\mathcal{Q}$ of quasi-convex functions in $\mathbb{D}$ introduced by Noor and Thomas [7], respectively.

Throughout this paper

$$0 \leq \delta \leq \lambda \leq 1 \qquad \text{and} \qquad \varphi, \psi \in \mathcal{N}.$$

Now we define new comprehensive subclasses of analytic function class $\mathcal{A}$, as follows:

Definition 1. A function $f \in \mathcal{A}$ is said to be in the class $\mathcal{K}_{\lambda,\delta}(\varphi,\psi)$ if

$$\frac{f'(z) + (\lambda - \delta + 2\lambda\delta)zf''(z) + \lambda\delta z^2 f'''(z)}{g'(z)} \prec \varphi(z) \qquad (z \in \mathbb{D}), \tag{1.7}$$

where $g \in \mathcal{K}(\psi)$.

Definition 2. a function $f \in \mathcal{A}$ is said to be in the class $\mathcal{S}_{\lambda,\delta}(\varphi,\psi)$ if

$$\frac{(1-\lambda+\delta)f(z) + (\lambda-\delta)zf'(z) + \lambda\delta z^2 f''(z)}{g(z)} \prec \varphi(z) \qquad (z \in \mathbb{D}), \tag{1.8}$$

where $g \in \mathcal{S}^*(\psi)$.

Remark 5. If we set $\delta = 0$ and $\lambda = 1$ in Definition 1 and Definition 2, then we have the classes

$$\mathcal{K}_{1,0}(\varphi,\psi) = \mathcal{QK}(\varphi,\psi) \qquad \text{and} \qquad \mathcal{S}_{1,0}(\varphi,\psi) = \mathcal{C}(\varphi,\psi).$$

Also when $\delta = 0$ and $\lambda = 0$, we get the classes

$$\mathcal{K}_{0,0}(\varphi,\psi) = \mathcal{C}(\varphi,\psi) \qquad \text{and} \qquad \mathcal{S}_{0,0}(\varphi,\psi) = \mathcal{CS}(\varphi,\psi).$$

Remark 6. If we set $\delta = 0$ and

$$\varphi(z) = \frac{1+Az}{1+Bz} \quad (-1 \leq B < A \leq 1) \qquad \text{and} \qquad \psi(z) = \frac{1+z}{1-z}$$

in Definition 1 and Definition 2, then we obtain the classes $\mathcal{Q}_{\mathcal{CV}}(\lambda, A, B)$ and $\mathcal{Q}_{\mathcal{ST}}(\lambda, A, B)$, respectively, introduced very recently by Altıntaş and Kılıç [1]. These classes consist of functions $f \in \mathcal{A}$ satisfying

$$\frac{f'(z) + \lambda z f''(z)}{g'(z)} \prec \frac{1 + Az}{1 + Bz} \qquad (g \in \mathcal{K},\ z \in \mathbb{D})$$

and

$$\frac{(1-\lambda) f(z) + \lambda z f'(z)}{g(z)} \prec \frac{1 + Az}{1 + Bz} \qquad (g \in \mathcal{S}^*,\ z \in \mathbb{D}),$$

respectively.

Altıntaş and Kılıç [1] obtained following coefficient bounds for functions belong to the classes $\mathcal{Q}_{\mathcal{CV}}(\lambda, A, B)$ and $\mathcal{Q}_{\mathcal{ST}}(\lambda, A, B)$, as follows:

Theorem 1. *If $f \in \mathcal{Q}_{\mathcal{CV}}(\lambda, A, B)$, then*

$$|a_n| \leq \frac{1}{1 + (n-1)\lambda} \left(1 + \frac{(n-1)(A-B)}{1-B} \right) \qquad (n = 2, 3, \ldots).$$

Theorem 2. *If $f \in \mathcal{Q}_{\mathcal{ST}}(\lambda, A, B)$, then*

$$|a_n| \leq \frac{n}{1 + (n-1)\lambda} \left(1 + \frac{(n-1)(A-B)}{1-B} \right) \qquad (n = 2, 3, \ldots).$$

In this work, we obtain coefficient bounds for functions in the comprehensive subclasses $\mathcal{K}_{\lambda,\delta}(\varphi, \psi)$ and $\mathcal{S}_{\lambda,\delta}(\varphi, \psi)$ of analytic functions. Our results improve the results of Altıntaş and Kılıç [1] (Theorem 1 and Theorem 2).

2. Main results

Lemma 1. *[8] Let the function Φ given by*

$$\Phi(z) = \sum_{n=1}^{\infty} A_n z^n \qquad (z \in \mathbb{D})$$

be convex in $\mathbb{D}$. Also let the function Ψ given by

$$\Psi(z) = \sum_{n=1}^{\infty} B_n z^n \qquad (z \in \mathbb{D})$$

be holomorphic in $\mathbb{D}$. If

$$\Psi(z) \prec \Phi(z) \qquad (z \in \mathbb{D}),$$

then

$$|B_n| \leq |A_1| \qquad (n = 1, 2, \ldots).$$

Lemma 2. *[9] Let $f \in \mathcal{K}(\psi)$ and be of the form* (1.1)*, then*

$$|a_n| \leq \frac{\prod_{j=0}^{n-2} (j + |\psi'(0)|)}{n!} \qquad (n = 2, 3, \ldots).$$

Lemma 3. *[9] Let $f \in \mathcal{S}^*(\psi)$ and be of the form* (1.1)*, then*

$$|a_n| \leq \frac{\prod\limits_{j=0}^{n-2}(j+|\psi'(0)|)}{(n-1)!} \qquad (n=2,3,\ldots).$$

Theorem 3. *Let $f \in \mathcal{K}_{\lambda,\delta}(\varphi,\psi)$ and be of the form* (1.1)*, then*

$$\begin{aligned}&[1+(n-1)(\lambda-\delta+2\lambda\delta)+(n-1)(n-2)\lambda\delta]\,|a_n|\\ \leq\ & \frac{\prod\limits_{j=0}^{n-2}(j+|\psi'(0)|)}{n!}+\frac{|\varphi'(0)|}{n}\left(1+\sum_{k=1}^{n-2}\frac{\prod\limits_{j=0}^{n-k-2}(j+|\psi'(0)|)}{(n-k-1)!}\right) \quad (n=2,3,\ldots).\end{aligned} \tag{2.1}$$

Proof. Let the function $f \in \mathcal{K}_{\lambda,\delta}(\varphi,\psi)$ be defined by (1.1). Therefore, by Definition 1, there exists a function

$$g(z)=z+\sum_{n=2}^{\infty}b_n z^n \in \mathcal{K}(\psi),\ \psi \in \mathcal{M} \tag{2.2}$$

so that

$$\frac{f'(z)+(\lambda-\delta+2\lambda\delta)zf''(z)+\lambda\delta z^2 f'''(z)}{g'(z)} \prec \varphi(z) \qquad (z\in\mathbb{D}). \tag{2.3}$$

Note that by (2.2) and Lemma 2, we have

$$|b_n| \leq \frac{\prod\limits_{j=0}^{n-2}(j+|\psi'(0)|)}{n!} \qquad (n=2,3,\ldots). \tag{2.4}$$

Let us define the function $p(z)$ by

$$p(z)=\frac{f'(z)+(\lambda-\delta+2\lambda\delta)zf''(z)+\lambda\delta z^2 f'''(z)}{g'(z)} \qquad (z\in\mathbb{D}). \tag{2.5}$$

Then according to (2.3) and (2.5), we get

$$p(z)\prec\varphi(z) \qquad (z\in\mathbb{D}). \tag{2.6}$$

Hence, using Lemma 1, we obtain

$$\left|\frac{p^{(m)}(0)}{m!}\right| = |c_m| \leq |\varphi'(0)| \qquad (m=1,2,\ldots), \tag{2.7}$$

where

$$p(z)=1+c_1 z+c_2 z^2+\cdots \qquad (z\in\mathbb{D}). \tag{2.8}$$

Also from (2.5), we find

$$f'(z)+(\lambda-\delta+2\lambda\delta)zf''(z)+\lambda\delta z^2 f'''(z)=p(z)g'(z). \tag{2.9}$$

Since $a_1 = b_1 = 1$, in view of (2.9), we obtain

$$\begin{aligned} & n\left[1+(n-1)(\lambda-\delta+2\lambda\delta)+(n-1)(n-2)\lambda\delta\right]a_n - nb_n \\ = \; & c_{n-1} + 2c_{n-2}b_2 + \cdots + (n-1)\,c_1 b_{n-1} \\ = \; & \sum_{k=1}^{n-1}(n-k)\,c_k b_{n-k} \quad (n = 2,3,\ldots). \end{aligned} \tag{2.10}$$

Now we get the desired result given in (2.1) by using (2.4), (2.7) and (2.10). □

Theorem 4. *Let* $f \in \mathcal{S}_{\lambda,\delta}(\varphi,\psi)$ *and be of the form* (1.1)*, then*

$$\begin{aligned} & \left[1+(n-1)(\lambda-\delta+2\lambda\delta)+(n-1)(n-2)\lambda\delta\right]|a_n| \\ \leq \; & \frac{\prod_{j=0}^{n-2}(j+|\psi'(0)|)}{(n-1)!} + |\varphi'(0)|\left(1+\sum_{k=1}^{n-2}\frac{\prod_{j=0}^{n-k-2}(j+|\psi'(0)|)}{(n-k-1)!}\right) \quad (n = 2,3,\ldots). \end{aligned} \tag{2.11}$$

Proof. Let the function $f \in \mathcal{S}_{\lambda,\delta}(\varphi,\psi)$ be defined by (1.1). Therefore, by Definition 2, there exists a function

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \in \mathcal{S}^*(\psi),\ \psi \in \mathcal{M} \tag{2.12}$$

so that

$$\frac{(1-\lambda+\delta)\,f(z) + (\lambda-\delta)\,zf'(z) + \lambda\delta z^2 f''(z)}{g(z)} \prec \varphi(z) \qquad (z \in \mathbb{D}). \tag{2.13}$$

Note that by (2.12) and Lemma 3, we have

$$|b_n| \leq \frac{\prod_{j=0}^{n-2}(j+|\psi'(0)|)}{(n-1)!} \qquad (n = 2,3,\ldots). \tag{2.14}$$

Let us define the function $q(z)$ by

$$q(z) = \frac{(1-\lambda+\delta)\,f(z) + (\lambda-\delta)\,zf'(z) + \lambda\delta z^2 f''(z)}{g(z)} \qquad (z \in \mathbb{D}). \tag{2.15}$$

Then according to (2.13) and (2.15), we get

$$q(z) \prec \varphi(z) \qquad (z \in \mathbb{D}). \tag{2.16}$$

Hence, using Lemma 1, we obtain

$$\left|\frac{q^{(m)}(0)}{m!}\right| = |d_m| \leq |\varphi'(0)| \qquad (m = 1,2,\ldots), \tag{2.17}$$

where

$$q(z) = 1 + d_1 z + d_2 z^2 + \cdots \qquad (z \in \mathbb{D}). \tag{2.18}$$

Also from (2.15), we find

$$(1-\lambda+\delta) f(z)+(\lambda-\delta) z f'(z)+\lambda\delta z^2 f''(z)=q(z)g(z). \tag{2.19}$$

Since $a_1 = b_1 = 1$, in view of (2.19), we obtain

$$\begin{aligned} & [1-\lambda+\delta+n(\lambda-\delta)+n(n-1)\lambda\delta]\,a_n-b_n \\ = \;& c_{n-1}+c_{n-2}b_2+\cdots+c_1 b_{n-1} \\ = \;& \sum_{k=1}^{n-1} c_k b_{n-k} \qquad (n=2,3,\ldots). \end{aligned} \tag{2.20}$$

Now we get the desired result given in (2.11) by using (2.14), (2.17) and (2.20). □

3. Corollaries and consequences

Letting $\delta = 0$ and $\lambda = 1$ in Theorem 3 and Theorem 4, we obtain the following consequences, respectively.

Corollary 1. *Let $f \in \mathcal{QK}(\varphi,\psi)$ and be of the form* (1.1)*, then*

$$|a_n| \le \frac{\prod_{j=0}^{n-2}(j+|\psi'(0)|)}{n^2(n-1)!}+\frac{|\varphi'(0)|}{n^2}\left(1+\sum_{k=1}^{n-2}\frac{\prod_{j=0}^{n-k-2}(j+|\psi'(0)|)}{(n-k-1)!}\right) \qquad (n=2,3,\ldots).$$

Corollary 2. *Let $f \in C(\varphi,\psi)$ and be of the form* (1.1)*, then*

$$|a_n| \le \frac{\prod_{j=0}^{n-2}(j+|\psi'(0)|)}{n!}+\frac{|\varphi'(0)|}{n}\left(1+\sum_{k=1}^{n-2}\frac{\prod_{j=0}^{n-k-2}(j+|\psi'(0)|)}{(n-k-1)!}\right) \qquad (n=2,3,\ldots).$$

Letting $\delta = 0$ and $\lambda = 0$ in Theorem 4, we obtain the following consequence.

Corollary 3. *Let $f \in \mathcal{CS}(\varphi,\psi)$ and be of the form* (1.1)*, then*

$$|a_n| \le \frac{\prod_{j=0}^{n-2}(j+|\psi'(0)|)}{(n-1)!}+|\varphi'(0)|\left(1+\sum_{k=1}^{n-2}\frac{\prod_{j=0}^{n-k-2}(j+|\psi'(0)|)}{(n-k-1)!}\right) \qquad (n=2,3,\ldots).$$

If we choose

$$\varphi(z)=\frac{1+(1-2\alpha)z}{1-z} \quad (0\le\alpha<1) \quad \text{and} \quad \psi(z)=\frac{1+(1-2\beta)z}{1-z} \quad (0\le\beta<1)$$

in Corollary 2, then we get following consequence.

Corollary 4. *[6] Let $f \in C(\alpha,\beta)$ $(0 \leq \alpha,\beta < 1)$ and be of the form* (1.1)*, then*

$$|a_n| \leq \frac{2\,(3-2\beta)\,(4-2\beta)\cdots(n-2\beta)}{n!}\left[n\,(1-\alpha)+(\alpha-\beta)\right] \qquad (n=2,3,\ldots).$$

Letting

$$\delta = 0, \quad \varphi(z) = \frac{1+Az}{1+Bz} \quad (-1 \leq B < A \leq 1), \quad \psi(z) = \frac{1+z}{1-z}$$

in Theorem 3 and Theorem 4, we obtain the following consequences, respectively.

Corollary 5. *Let $f \in \mathcal{Q}_{\mathcal{CV}}(\lambda, A, B)$ and be of the form* (1.1)*, then*

$$|a_n| \leq \frac{1}{1+(n-1)\,\lambda}\left(1+\frac{(n-1)\,(A-B)}{2}\right) \qquad (n=2,3,\ldots).$$

Corollary 6. *Let $f \in \mathcal{Q}_{\mathcal{ST}}(\lambda, A, B)$ and be of the form* (1.1)*, then*

$$|a_n| \leq \frac{n}{1+(n-1)\,\lambda}\left(1+\frac{(n-1)\,(A-B)}{2}\right) \qquad (n=2,3,\ldots).$$

Remark 7. It is clear that

$$1+\frac{(n-1)\,(A-B)}{2} \leq 1+\frac{(n-1)\,(A-B)}{1-B} \quad (-1 \leq B < A \leq 1,\ n=2,3,\ldots),$$

which would obviously yield significant improvements of Theorem 1 and Theorem 2.

Letting

$$\lambda = 0, \quad A = 1, \quad B = -1$$

in Corollary 5 and Corollary 6, we have following consequences, respectively.

Corollary 7. *[5] Let $f \in \mathcal{C}$ and be of the form* (1.1)*, then*

$$|a_n| \leq n \qquad (n=2,3,\ldots).$$

Corollary 8. *[5] Let $f \in \mathcal{CS}$ and be of the form* (1.1)*, then*

$$|a_n| \leq n^2 \qquad (n=2,3,\ldots).$$

Letting

$$\lambda = 1, \quad A = 1, \quad B = -1$$

in Corollary 5, we have following consequence.

Corollary 9. *[10] Let $f \in \mathcal{Q}$ and be of the form* (1.1)*, then*

$$|a_n| \leq 1 \qquad (n=2,3,\ldots).$$

4. Conclusions

In this paper, we introduce two comprehensive subclasses $\mathcal{K}_{\lambda,\delta}(\varphi,\psi)$ and $\mathcal{S}_{\lambda,\delta}(\varphi,\psi)$ of analytic functions by means of the principle of subordination, and obtain the coefficient bounds for functions in these classes. The well-known results are obtained as a corollary of our main results.

Conflict of interest

Author declares no conflicts of interest.

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