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Research article

Adaptive neural network control for nonlinear state constrained systems with unknown dead-zones input

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Abstract: In this paper, an adaptive neural network tracking control problem for a class of strict feedback systems is disposed. The neural network adaptive control method is introduced in this paper to simplify the controller design. The difficulty in this article is the constraint problem and how to resolve dead-zones in the system. In order to overcome these difficulties, the Barrier Lyapunov functions (BLF) and backstepping process are introduced to ensure that the full state constraint is implemented, meanwhile, keep the system output as close as possible to trace the desired trajectory. Dead-zone compensation method is also plays an important role in controller design. Delay constraint is introduced to solve the problem of uncertain initial state. In the end, the stability of the closed-loop system is proved. Simulation results show that the developed method is effective.

Keywords: adaptive control; dead-zone; barrier Lyapunov functions; neural network
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1. Introduction

In the last few years, the uncertainties general consist in the nonlinear systems, the most important is that it is also widely applied in constraint. By taking adaptive control [1], the parametric uncertainty of nonlinear system is presented in [2]. In the actual systems, out of the realistic need as well as the difficulty of operations, the unknown continuous functions are approximated, which is based on fuzzy logic systems (FLSs) or neural networks (NNs) in [3–7]. On the basis of NNs or FLSs, adaptive control algorithm comes up for nonlinear single-input single-output (SISO) systems [8]. As same as for MIMO nonlinear systems [9] with unknown functions and discrete-time systems, the above method also applies. Besides, a multilayer NNs estimator was first developed in [10] to improve the compensation accuracy of model-based feedforward control terms. However, the aforementioned results do not refer to constraint.

As one of the most important factors restricting the system performance, constraints extensive exist in actual systems, such as robotic manipulator system, nonuniform gantry crane and so on. Constraint cannot be omitted, otherwise it may have an impact on the equipment and something unexpected, thus, constraint control has become a significant portion of nonlinear control. As we all know, the Barrier Lyapunov function (BLF) is the significant tool to dispose constrained problem. So far it widespread application in nonlinear systems with output constraint in [11] and full state constraints in [12–13], and its effectiveness is also verified. The NNs or FLSs is used to design adaptive controllers for nonlinear systems using neural networks with constant constraint in [5]. Nevertheless, none of the above methods mention time-varying constraints in [14]. In this article, the time-varying with full state constraints are further studied. Adaptive controllers are designed for time-varying output constraints and time-varying full-state constraints [15], respectively. However, the above backstepping recursion method ignore the feasibility conditions of virtual controllers, namely, the virtual controller is within the given constraint bounded. In [16], a new coordinate transformation is introduced to completely remove this limitation.

The constraints of the above research are all direct constraints on the states, whether full state constraints or other types of constraints. Subsequent some studies are not limited to state constraints. At present, the state transfer function is introduced for coordinate transformation in [17] indirect processing constraint such as constrain the error. Compared with the above-mentioned research methods, the advantage of this approach is not only independent of initial tracking condition, but also suitable for asymmetric time-varying constraints, which is studied in [15]. However, in any of these cases, the effect of dead-zone on constraint is omitted.

Mention nonlinear input, the most common are dead-zones, saturation, time-delay, and so on. An innovative approach is to propose a dead-zone compensation in motion control systems using adaptive fuzzy logic control. In this paper, the dead-zone nonlinear input is our focus. The existing dead-zones prevent us from getting the desired control results, and the problem caused by it is serious. For instance, if the robot servo system has nonlinear links such as friction and unknown dead zone, it not only reduces the efficiency of the control system, but also lead to the instability of the system. In recent years, the study of dead-zone has become the focus of control research. In [18], for discrete-time plants with unknown dead-zone, a new control structure with adaptive dead region inverse is put forward. To this purpose, some adaptive control method is proposed, such as neural network control and adaptive fuzzy sliding mode control. Adaptive tracking of asymmetric dead-zone input nonlinear systems with uncertain parameters is proposed. As is well-known, for dead-zone in multi-input multi-output nonlinear system, which has lower triangular structure and asymmetric structure, a new control method is proposed. To eliminate dead-zone effects, the dead-zones compensation control is implemented for precision instrument control.

In this paper, the adaptive neural network control method is proposed, it is the realization of the control target. The design of the controller is beneficial. After comprehension of above achievements, the control scheme has the following advantages.

1) Compared with previous adaptive neural network control methods, this article takes into account more complex case. The effects of delay constraints and dead-zones on system performance are considered, which is more practical in line with the needs of the actual system.

2) State transition function is introduced; the appropriate time node is selected to ensure that the states are in the constraint bounds. Namely, the initial state is out of bounds, and then in the bounds. There is no mention of delay constraints in [16], the problem of delay constraints is considered in this paper. It provides convenience for error tracking that not widely involved in the previous research for nonlinear adaptive control.

3) In this paper, based on the introduction of BLF, potential dead-zones in the system is resolved, which affects the stability of the system and increases steady state error. Manipulative know m and unknown d , the difficult problem of controller design is solved.

2. Problem formulation and preliminaries

Consider a class of nonlinear strict-feedback systems with dead-zones as following:

$$\begin{cases} \dot{x}_i = g_i\left(\bar{x}_i\right)x_{i+1} + f_i\left(\bar{x}_i\right), i = 1, \ldots, n-1 \\ \dot{x}_n = g_n\left(\bar{x}_n\right)D\big(u(t)\big) + f_n\left(\bar{x}_n\right) \\ y = x_1 \end{cases} \tag{2.1}$$

where $x = [x_1, \ldots, x_n]^T \in R$ with $\bar{x}_i = [x_1, \ldots, x_j]^T$ $j = 1, \ldots, n$, $x_i \in R$, $D\big(u(t)\big) \in R$ and $y \in R$ are the state variables, the input and the output of the systems, respectively, $g_i\left(\bar{x}_i\right)$ are unknown control coefficients and $f_i\left(\bar{x}_i\right)$ are unknown smooth functions. $x_i \in \left(-\underline{k}_{ci}(t), \bar{k}_{ci}(t)\right)$, x_i is unconstrained when $t \in \left[0, T\right]$. Nevertheless, when $t \in [T, \infty)$, x_i are within the given bound. So as to ensure the validity of the constraint, the value of T is crucial. Let $-\underline{k}_{ci}(t) < x_i(t) < \bar{k}_{ci}(t)$, where $\underline{k}_{ci}(t)$ and $\bar{k}_{ci}(t)$ are given.

Remark 1: There are many factors that affect system performance, but one of the most significant is constraint. In order to ensure that constraint is not violated in the control process, the work that needs attention in the adaptive control strategy is proposed in [19–30]. In this paper, to make sure constraints are not violated, the transfer function is introduced for coordinate transformation. Constraints appear some time later, the system stability is improved.

The delay constraint means that the constraint occurs over a period of time, which does not have to constrain the signal all the time. By designing the appropriate controller, the signal satisfies the constraint condition after a certain time.

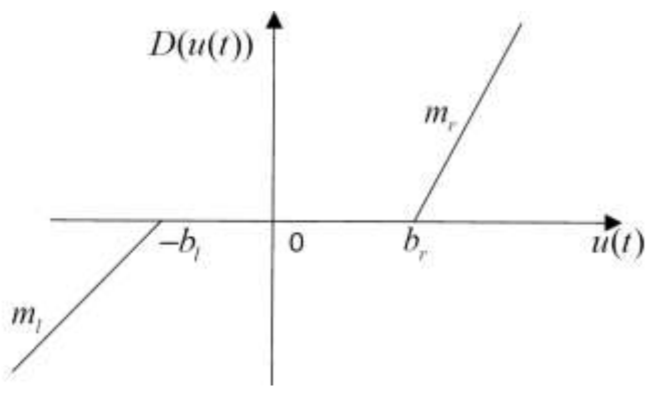

Figure 1. The structure diagram of dead zone.

All of state variables in the system (2.1) are constrained in the compact set. $D\left(u(t)\right)$ is a dead-zone defined as:

$$D\left(u(t)\right)=\begin{cases} m_r\left(u(t)-b_r\right), & u(t)\ge b_r \\ 0, & -b_l<u(t)<b \\ m_l\left(u(t)+b_l\right), & u(t)\le -b_l \end{cases} \tag{2.2}$$

where $u(t)$ is the dead-zone input, m_r and m_l respectively stand for dead-zone right slope and left slope, b_r and b_l represent the right and left cut point of the input nonlinearity. The detailed structure diagram of dead zone is given in Figure 1.

The aforementioned model can be transformed into the following form:

$$D\left(u(t)\right)=m(t)u(t)+b(t) \tag{2.3}$$

where

$$m(t)=\begin{cases} m_r, & u(t)>0 \\ m_l, & u(t)\le 0 \end{cases}$$

and

$$b(t)=\begin{cases} -m_r b_r, & u(t)\ge b_r \\ -m(t)u(t), & -b_l<u(t)<b_r \\ m_l b_l, & u(t)\le -b_l \end{cases}$$

Assumption 1. The function $m(t)$ is known, $b(t)$ is unknown and its up bound is $\bar{b}(t)$, namely, $\left|b(t)\right|\le \bar{b}(t)$.

3. Control objective

The task is to design an adaptive controller u, such that the system output y tracks a desired trajectory $y_d(t)$. All the signals in the closed-loop system are bounded, meanwhile the full state constraints are not violated. It holds that $-\underline{y}_d(t)\le y_d(t)\le \bar{y}_d(t)$, where $\underline{y}_d(t)$ and $\bar{y}_d(t)$ are continuous positive functions, with $\underline{k}_{c1}(t)\ge \underline{y}_d(t)$ and $\bar{k}_{c1}(t)>\bar{y}_d(t)$.

In this paper, on account of radial basis function neural networks (RBFNNs) approximate ability, it is chosen to approximate unknown and continuous function.

Consider a continuous function $h(z):R^q\to R$, the following form can be obtained:

$$h_{nn}(z)=\theta^{*T}S(z) \tag{3.1}$$

where the input variable $z\in\Omega_z\subset R^q$, desired weight matrix $\theta^*=[\theta_1,\theta_2,\dots\theta_l]^T\in R^l$, $l>1$ is the NN node number. In addition, $S(z)=\left[s_1(z),\dots,s_l(z)\right]^T$, it is often expressed by Gaussian function, which has the following form:

$$s_i(z)=\exp\left[\frac{-(z-\mu_i)^T(z-\mu_i)}{\eta_i^2}\right], i=1,2,\ldots,l$$

where $\mu_i=[\mu_{i1},\mu_{i2},\ldots,\mu_{iq}]$ is the center of NNs and η_i is the width of the Gaussian function.

According to the character of NNs, with regard to any continuous unknown function, it can be represented as

$$h(z)=\theta^{*T}S(z)+\varepsilon(z) \tag{3.2}$$

where $\varepsilon(z)$ is alluded to as the least approximate error and $|\varepsilon(z)|\le\varepsilon$ with $\varepsilon>0$ for any $z\in\Omega_z$. The weight matrix θ^* has the following representation

$$\theta^*\triangleq\arg\min_{\theta\in R^l}\left\{\sup_{Z\in\Omega_z}\left|h(z)-\theta^T S(z)\right|\right\}$$

Assumption 2. The function $g_i(\bar{x}_i)$ is unknown, time-varying, and bounded away from zero, respectively. Its upper bound and lower bound are $\bar{g}_i$ and $\underline{g}_i$, They are also unknown continuous function, such that $0<\underline{g}_i\le|g_i(\bar{x}_i)|\le\bar{g}_i$, $\forall\bar{x}_i\in\Omega\subset R^n$.

A new asymmetric Barrier Lyapunov function is introduced

$$V=\frac{\zeta^2(t)}{(F_1(t)+\zeta(t))(F_2(t)-\zeta(t))} \tag{3.3}$$

where $F_1(t)$ and $F_2(t)$ are positive functions, $\zeta(t)$ will be defined later. Note that V is valid in the interval $-F_1(t)<\zeta(t)<F_2(t)$.

Then, the transfer function is introduced to better constrain the states. The function is defined as

$$\zeta_i(t)=\begin{cases}0, & t=0\\ \tau(t)z_i(t), & 0<t<T\ (i=1,\ldots,n)\\ z_i(t), & t\ge T\end{cases} \tag{3.4}$$

where

$$\tau(t)=\begin{cases}1-\left(\dfrac{T-t}{T}\right)^{n+2}, & 0\le t<T\\ 1, & t\ge T\end{cases} \tag{3.5}$$

The effect of (3.4) is to solve the problem of uncertain initial conditions. In the meantime, $\tau(t)$ is a continuous and differentiable function.

Remark 2: The T is a time node. There exist two crucial features for $\tau(t)$, $\tau(0)=0$ and $\tau(t)=1$ for $t\ge T$. These two properties have important applications in the conversion of initial values, that is to transform a non-zero initial value into a zero initial value. In the end, their values will converge to a finite bounded.

4. Adaptive neural controller design and stability analysis

In the following study, adaptive control method and backstepping recursive are used to design virtual controllers α_i, actual controller u and adaptive laws $\hat{w}_i$. Adaptive control process has a total of n steps, the coordinate transformation is introduced as

$$z_i = x_i - \alpha_{i-1}\left(i = 2,...,n\right) \tag{4.1}$$

In the step 1, virtual controller α_1 and adaptive law $\hat{w}_1$ are defined as follows:

$$\alpha_1 = -k_1 z_1 - \psi_1 - \hat{w}_1 P_1 \tag{4.2}$$

$$\dot{\hat{w}}_1 = \lambda_1 G_1 \zeta_1 \tau P_1 - \delta_1 \hat{w}_1 \tag{4.3}$$

where k_1 is a positive constant, $\delta_1 > 0$ is a constant, $\lambda_1 > 0$ is a constant, $\hat{w}_1$ stands for adaptive parameter. In addition, ψ_1, P_1, and G_1 will be defined later.

In the step $i\left(2 \le i \le n-1\right)$, virtual controller α_i and adaptive law $\hat{w}_i$ are defined as follows:

$$\alpha_i = -k_i z_i - \psi_i - \hat{w}_i P_i \tag{4.4}$$

$$\dot{\hat{w}}_i = \lambda_i G_i \zeta_i \tau P_i - \delta_i \hat{w}_i \tag{4.5}$$

where k_i is a positive constant, $\delta_i > 0$ is a constant, $\lambda_i > 0$ is a constant, $\hat{w}_i$ stands for adaptive parameter. In addition, ψ_i, P_i, and G_i will be defined later.

In the last step, actual controller u and adaptive law for $\hat{w}_n$ are defined as follows:

$$u = \frac{1}{m}\left(-k_n z_n - \psi_n - \hat{w}_n P_n\right) \tag{4.6}$$

$$\dot{\hat{w}}_n = \lambda_n G_n \zeta_n \tau P_n - \delta_n \hat{w}_n \tag{4.7}$$

where k_n is a positive constant, $\delta_n > 0$ is a constant, $\lambda_n > 0$ is a constant, $\hat{w}_n$ stands for adaptive parameter. In addition, ψ_n, P_n, and G_n will be defined later.

Let us consider tracking error $z_1 = x_1 - y_d$. From the first Eq (2.1), we get the derivation of z_1

$$\dot{z}_1 = g_1 x_2 + f_1 - \dot{y}_d = g_1\left(z_2 + \alpha_1\right) + f_1 - \dot{y}_d \tag{4.8}$$

From (3.4), we obtain

$$\dot{\zeta}_1 = \dot{\tau} z_1 + \tau g_1\left(z_2 + \alpha_1\right) + \tau\left(f_1 - \dot{y}_d\right) = g_1 \tau \alpha_1 + g_1 \zeta_2 + \dot{\tau} z_1 + \tau\left(f_1 - \dot{y}_d\right)$$

Remark 3: We just think about $0 < t < T$, so let $\zeta_i\left(t\right) = \tau\left(t\right) z_i\left(t\right)$. States are constrained indirectly through constraints on transition functions, this will be discussed below.

From $z_i = x_i - \alpha_{i-1}\left(i = 1,...,n\right)$, then we get

$$\begin{aligned}\dot{z}_i &= g_i x_{i+1} + f_i - \dot{\alpha}_{i-1} \\ &= g_i x_{i+1} + f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) - \Delta\alpha_{i-1} \\ &= g_i\left(z_{i+1} + \alpha_i\right) + f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) - \Delta\alpha_{i-1}\end{aligned} \tag{4.9}$$

where

$$\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) + \Delta\alpha_{i-1} \tag{4.10}$$

with

$$\Delta\alpha_{i-1} = \sum\nolimits_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(j)}} y_d^{(j+1)} + \sum\nolimits_{j=1}^{i-1} \sum\nolimits_{k=0}^{i-j} \frac{\partial \alpha_{i-1}}{\partial F_{j2}^{(k)}} F_{j2}^{(k+1)} + \sum\nolimits_{j=0}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \tau^{(j)}} \tau^{(j+1)} + \sum\nolimits_{j=1}^{i-1} \sum\nolimits_{k=0}^{i-j} \frac{\partial \alpha_{i-1}}{\partial F_{j1}^{(k)}} F_{j1}^{(k+1)}$$

From (3.4), we obtain the following dynamics

$$\begin{aligned}\dot{\zeta}_i &= \dot{\tau} z_i + \tau g_i\left(z_{i+1} + \alpha_i\right) + \tau\left(f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) - \Delta\alpha_{i-1}\right) \\ &= \dot{\tau} z_i + g_i \zeta_{i+1} + g_i \tau \alpha_i + \tau\left(f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) - \Delta\alpha_{i-1}\right)\end{aligned}$$

Substituting (2.3) into $z_n = x_n - \alpha_{n-1}$, one deduces that

$$\dot{z}_n = g_n\left(mu + b\right) + f_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) - \Delta\alpha_{n-1} \tag{4.11}$$

where

$$\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) + \Delta\alpha_{n-1} \tag{4.12}$$

with

$$\Delta\alpha_{n-1} = \sum\nolimits_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_d^{(j)}} y_d^{(j+1)} + \sum\nolimits_{j=1}^{n-1} \sum\nolimits_{k=0}^{n-j} \frac{\partial \alpha_{n-1}}{\partial F_{j2}^{(k)}} F_{j2}^{(k+1)} + \sum\nolimits_{j=1}^{n-1} \sum\nolimits_{k=0}^{n-j} \frac{\partial \alpha_{n-1}}{\partial F_{j1}^{(k)}} F_{j1}^{(k+1)} + \sum\nolimits_{j=0}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \tau^{(j)}} \tau^{(j+1)}$$

then, we obtain the following dynamics

$$\dot{\zeta}_n = \dot{\tau} z_n + \tau\left(f_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) - \Delta\alpha_{n-1}\right) + \tau g_n\left(mu + b\right)$$

Please see the appendix for the specific derivation process

Theorem 1: Consider a class of nonlinear system (2.1) with a dead-zone (2.2) under Assumptions 1–2, virtual controllers are designed in (4.2) and (4.4), and the actual controller is designed in (4.6). Meanwhile the adaption laws are constructed in (4.3), (4.5) and (4.7). If the design parameters are chosen appropriately, it ensures that all signals in the closed-loop system are UUB and bounded. The system output y tracks a desired trajectory $y_d(t)$.

Proof. It is obvious that $\tilde{w}_j \hat{w}_j = \tilde{w}_j\left(w_j - \tilde{w}_j\right) \le -\frac{\tilde{w}_j^2}{2} + \frac{w_j^2}{2}$ we can derive that

$$\begin{aligned} 2F_{j1}F_{j2} - F_{j1}\xi_j + F_{j2}\xi_j &\ge F_{j1}F_{j2} - F_{j1}\xi_j + F_{j2}\xi_j - \xi_j^2 \\ &= \left(F_{j1}+\xi_j\right)\left(F_{j2}-\xi_j\right) > 0 \end{aligned} \tag{4.13}$$

Then, we get

$$G_j \xi_j^2 = \frac{2F_{j1}F_{j2} - F_{j1}\xi_j + F_{j2}\xi_j}{[\left(F_{j1}+\xi_j\right)\left(F_{j2}-\xi_j\right)]^2}\xi_j^2 \ge \frac{\xi_j^2}{\left(F_{j1}+\xi_j\right)\left(F_{j2}-\xi_j\right)} \tag{4.14}$$

$$2F_{j1}F_{j2} - F_{j1}\xi_j + F_{j2}\xi_j / \left(F_{j1}+\xi_j\right)\left(F_{j2}-\xi_j\right) \ge 1 \tag{4.15}$$

Hence, we have the following inequality:

$$\begin{aligned} \dot{V}_n &\le -\sum_{j=1}^{n} k_j \underline{g}_j \frac{\xi_j^2}{\left(F_{j1}+\xi_j\right)\left(F_{j2}-\xi_j\right)} - \sum_{j=1}^{n} \frac{\underline{g}_j \delta_j}{2\lambda_j}\tilde{w}_j^2 + \sum_{j=1}^{n} \frac{\underline{g}_j \delta_j}{2\lambda_j} w_j^2 + \sum_{j=1}^{n} \Xi_j \\ &\le -\rho V_n + c \end{aligned} \tag{4.16}$$

where $\rho = \min\left\{k_j \underline{g}_j, \delta_j\right\} > 0$, $c = \sum_{j=1}^{n} \frac{\underline{g}_j \delta_j}{2\lambda_j} w_j^2 + \sum_{j=1}^{n} \Xi_j$

Besides, from (4.16), we get

$$0 \le V_n(t) \le c/\rho + \left(V_n(0) - c/\rho\right) e^{-\rho t} \tag{4.17}$$

Hence, we obtain that $V_n(t) \in l_\infty$, it implies that $\hat{w}_1 \in l_\infty$ and $V_{i0}(t) \in l_\infty$. The upper of $\zeta_i(t)$ is $-F_{i1}(t)$ and lower is $F_{i2}(t)$, so $-F_{i1}(t) < \zeta_i(t) < F_{i2}(t)$. Based on the shifting function $\tau(t)$, when $t \in [0,T)$, $\tau(t)$ is strictly increasing. And $\tau(0) = 0$, when and only when $t = 0$, then, $\zeta_i(0) = \tau(0) z_i(0) = 0$. We get $-F_{i1}(0) < \zeta_i(0) < F_{i2}(0)$. When $0 < t < T$, $\tau(t)$ is not zero. From (4.8), $z_i(t) = \zeta_i(t) / \tau(t)$, $z_i(t)$ is bounded because $\zeta_i(t)$ is bounded. When $t \ge T$, $\tau(t) = 1$, then we can get $z_i(t) = \zeta_i(t)$, so $-F_{i1}(t) < z_i(t) < F_{i2}(t)$. In conclusion, $z_i(t)$ is bounded anyway. According to the definition of F_{11} and F_{12}, thus F_{11}, F_{12}, $\dot{F}_{11}$ and $\dot{F}_{12}$ are bounded. From $z_1 = x_1 - y_d$, z_1 is bounded and x_1 is also bounded. Because F_{11}, F_{12}, $\dot{F}_{11}$, $\dot{F}_{12}$, $\hat{w}_1$, y_d, $\dot{y}_d$, τ and $\dot{\tau}$ are bounded, it follows that virtual controller α_1 and adaptive law $\dot{\hat{w}}_1$ are bounded. Then follow the definition of F_{21} and F_{22}, thus F_{21}, F_{22}, $\dot{F}_{21}$ and $\dot{F}_{22}$ are bounded. From $z_2 = x_2 - \alpha_1$, z_2 is bounded and x_2 is also bounded. In the similar way, α_2 and $\hat{w}_2$ can also be bounded. Continue this derivation, we can obtain that $x_i\left(i = 3,\cdots,n\right)$, $\alpha_i\left(i = 3,\cdots,n-1\right)$, $\dot{\hat{w}}_i\left(i = 3,\cdots,n\right)$ and control input u are bounded. In a word, all signals in closed-loop systems are bounded. The proof is completed.

5. Simulation example

In this paper, to prove the effectiveness of this method, the simulation experiment is given. Consider the following nonlinear systems

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{-mgl\sin(x_1)}{2M} - \dfrac{Bx_2}{M} + \dfrac{D(u(t))}{M} \\ y = x_1 \end{cases} \tag{5.1}$$

where y and u are the outputs and inputs of the system. Meanwhile, $D(u(t))$ is the output of the dead zone, described as

$$D(u(t)) = \begin{cases} 0.2(u\text{-}0.3), & u > 0.3 \\ 0, & -0.3 \le u \le 0.3 \\ 0.3(u+0.3), & u < -0.3 \end{cases} \tag{5.2}$$

and the states x_1 and x_2 are constrained by $\overline{k}_{c1}(t) < x_1(t) < -\underline{k}_{c1}(t)$ and $\overline{k}_{c2}(t) < x_2(t) < -\underline{k}_{c2}(t)$ with $\overline{k}_{c1}(t) = 0.07 + 0.01\sin(2t)$, $\underline{k}_{c1}(t) = 0.11+0.05\sin(4t)$, $\overline{k}_{c2}(t) = 0.3\sin(3t)+0.6$, $\underline{k}_{c2}(t) = 0.4\sin(2t)+0.72$. The ideal trajectory y_d is chosen as y_d=0.05cos$(6t)$. The initial states are chosen as $x_1(0) = 0.16$, $x_2(0) = 0.001$, $\hat{w}_1(0) = 0.6$, $\hat{w}_2(0) = 0.279$. The controllers and adaptive laws of simulation system are chosen as follow:

$$\begin{aligned} \alpha_1 &= -k_1 z_1 - G_1\zeta_1\tau_1 - \hat{w}_1 G_1\zeta_1\tau S_1^T(z_1) S_1(z_1) \\ u &= \frac{1}{m}\left(-k_2 z_2 - G_2\zeta_2\tau - \hat{w}_2 G_2\zeta_2\tau S_2^T(z_2) S_2(z_2)\right) \\ \dot{\hat{w}}_1 &= \lambda_1 G_1\zeta_1\tau P_1 - \delta_1\hat{w}_1 \\ \dot{\hat{w}}_2 &= \lambda_2 G_2\zeta_2\tau P_2 - \delta_2\hat{w}_2 \end{aligned}$$

In the simulation, the first NN includes 10 nodes with the center μ_1 evenly spaced on $[-1,1]\times[-1,1]\times[-1,1]$ with width $\eta_1 = 1$, the second NN includes 16 nodes with the center μ_2 evenly spaced on $[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]\times[-3,3]$ with width $\eta_2 = 1$. The parameters in the simulation system are given as $m = 1kg$, $g = 9.8m/s^2$, $l = 1m$, $M = 0.5kg.m^2$, $B = 1N.m.s$. The control parameters are selected as $k_1 = 29$, $k_2 = 44$, $\lambda_1 = 0.01$, $\lambda_2 = 0.01$, $\delta_1 = 0.1$, $\delta_2 = 0.5$. Delay constraint time $T = 1$, x_1 and x_2 are out of bounds, when $t \in (0,1)$ and while $T \ge 1$, x_1 and x_2 are expected completely within the bounds. The selection of the delay constraint time is based on the minimum tracking error and the best tracking performance. Besides, according to the desired trajectory, we obtain that $F_{11} = 0.01\sin(2t) - 1.93$ and $F_{12} = 0.05\sin(4t) - 1.89$, F_{21}=0.222+0.1sin$(5t)$ and F_{22}=0.25+0.05sin$(10t)$.

The simulation results are given in Figures 2–7. Figure 2 shows the trajectory y_d, output y and constrained intervals with a good tracking performance. Figure 3 shows the trajectory of error z_1. The trajectory of the dead-zone $D(u)$ is given in Figure 4. The trajectory of state x_2 is displayed in Figure 5. The trajectory of error z_2 and adaptive laws are shown in Figure 6 and Figure 7, respectively. Simulation results prove that all signals are bounded.

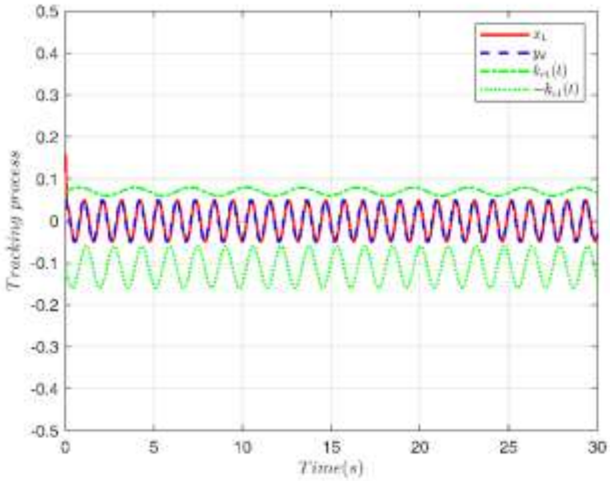

Figure 2. The trajectory y_d, output y and constrained intervals.

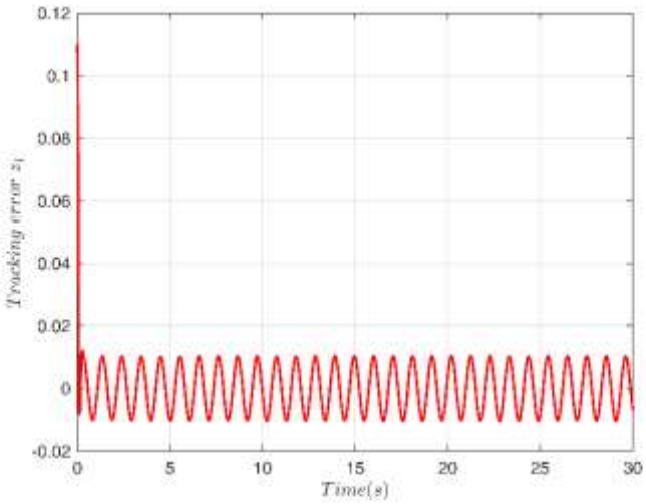

Figure 3. The trajectory of error z_1.

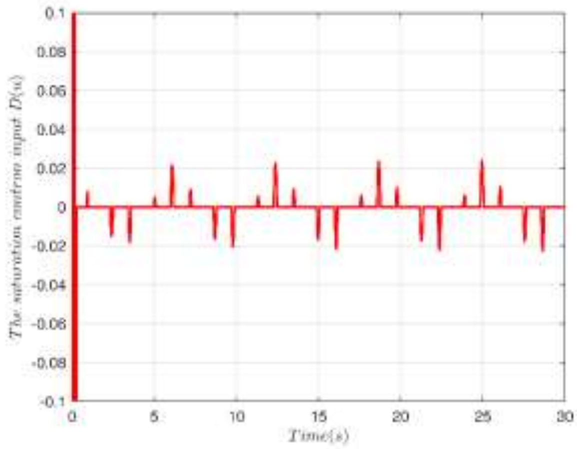

Figure 4. The trajectory of the dead-zone $D(u)$.

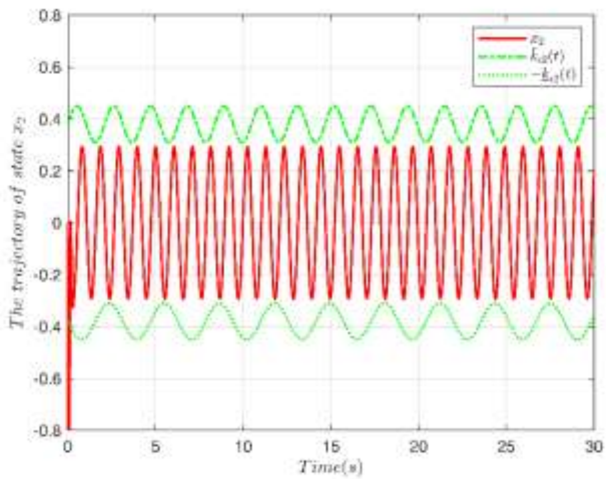

Figure 5. The trajectory of state x_2.

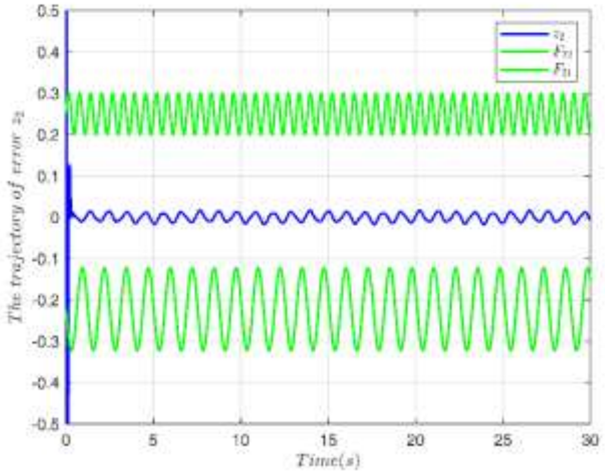

Figure 6. The trajectory of error z_2 and constrained intervals.

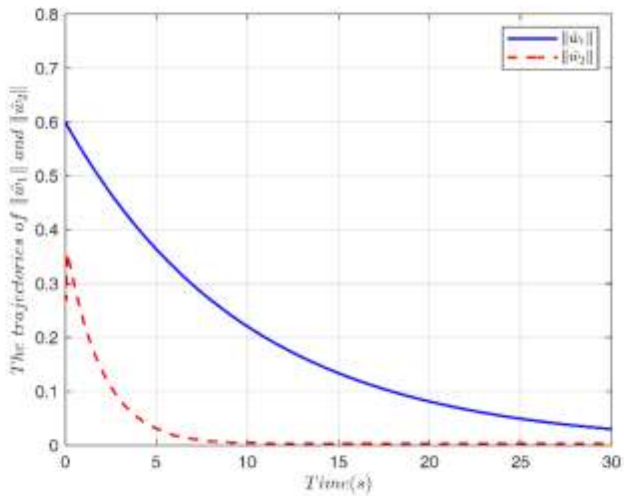

Figure 7. The trajectories of $\|\hat{w}_1\|$ and $\|\hat{w}_2\|$.

6. Conclusions

An adaptive neural network tracking control method is introduced for the strict feedback nonlinear systems with unknown dead-zones and full state constraints. The parameters of the dead-zones are unknown but bounded. Combining backsteping technique with neural network, it ensures that the state constraints are not violated. The feasibility of the control algorithm is proved. In the meantime, all signals in the closed-loop system are UUB. In this paper, delay constraint and BLF are combined. In other words, the constraint occurs after a period of time, it is not constrained from the beginning. Furthermore, the tracking error converges to a small area away from zero. In the end, simulation results verify the effectiveness of the design. We will further investigate the control performance by setting different delay times such as time-varying delay, which will enrich the applied range of the proposed control method. Besides, since the reliability is an interested topic and has been well discussed in [23–27], it will be taken into account in our future work.

7. Appendix

Step 1: In order to achieve the desired control objective, the following Barrier Lyapunov function is chosen

$$V_{10}=\frac{\zeta_1^2(t)}{\left(F_{11}(t)+\zeta_1(t)\right)\left(F_{12}(t)-\zeta_1(t)\right)}$$

where $F_{11}(t)$ and $F_{12}(t)$ are time varying continuous functions. Its definition will be given later. For $\zeta_1(t)$, its scope is $F_{11}(t)<\zeta_1(t)<F_{12}(t)$. The time-varying functions $F_{11}(t)$ and $F_{12}(t)$ are chosen such that

$$F_{11}(t)=\underline{k}_{c1}(t)-n\underline{y}_d,\ F_{12}(t)=\bar{k}_{c1}(t)-n\bar{y}_d$$

where n is a positive constant.

Consider the Barrier Lyapunov function as

$$V=V_{10}+\frac{g_1}{2\lambda_1}\tilde{w}_1^2 \tag{7.1}$$

where $\lambda_1>0$ is a constant, $w_1=\max_{k\in K}\left\{\left\|\theta_{K1}^*\right\|^2\right\}$, $\tilde{w}_1=w_1-\hat{w}_1$ stands for the estimation error, with $\hat{w}_1$ being the estimation of w_1. Based on (3.4), we get the time derivative of V_1

$$\begin{aligned}\dot{V}_1&=G_1\zeta_1\left(\dot{\zeta}_1+N_1\zeta_1\right)-\frac{g_1}{\lambda_1}\tilde{w}_1\dot{\hat{w}}_1\\&=G_1\zeta_1[g_1\tau\alpha_1+g_1\zeta_2+\dot{\tau}z_1+\tau\left(f_1-\dot{y}_d\right)+\zeta_1N_1]-\frac{g_1}{\lambda_1}\tilde{w}_1\dot{\hat{w}}_1\end{aligned} \tag{7.2}$$

where

$$G_1=\frac{2F_{11}F_{12}-F_{11}\zeta_1+F_{12}\zeta_1}{\left[\left(F_{11}+\zeta_1\right)\left(F_{12}-\zeta_1\right)\right]^2}$$

and

$$N_1=\frac{-\dot{F}_{11}F_{12}-F_{11}\dot{F}_{12}+\left(\dot{F}_{11}-\dot{F}_{12}\right)\zeta_1}{2F_{11}F_{12}-F_{11}\zeta_1+F_{12}\zeta_1}.$$

In this way, the design of controller and adaptive law are simplified.

Combining the Assumption 2, on the basis of Young's inequality, we get

$$G_1 g_1 \zeta_1 \zeta_2 \le \underline{g}_2 G_1^2 \zeta_1^2 \zeta_2^2 + \frac{g_1^2}{4\underline{g}_2} \tag{7.3}$$

$$G_1 \zeta_1 \dot{\tau} z_1 \le \underline{g}_1 G_1^2 \zeta_1^2 \dot{\tau}^2 z_1^2 + \frac{1}{4\underline{g}_1} \tag{7.4}$$

$$G_1 \zeta_1^2 N_1 \le \underline{g}_1 G_1^2 \zeta_1^4 N_1^2 + \frac{1}{4\underline{g}_1} \tag{7.5}$$

Substituting (7.3), (7.4) and (7.5) into (7.2) leads to

$$\dot{V}_1 \le G_1 \zeta_1 \tau (g_1 \alpha_1 + \underline{g}_1 G_1 \dot{\tau}^2 z_1^3 + \underline{g}_1 G_1 \zeta_1^2 N_1^2 z_1 + f_1 - \dot{y}_d) + \underline{g}_2 G_1^2 \zeta_1^2 \zeta_2^2 - \frac{g_1}{\lambda_1} \tilde{w}_1 \dot{\hat{w}}_1 + \frac{g_1^2}{4\underline{g}_2} + \frac{1}{2\underline{g}_1} \tag{7.6}$$

Let

$$h_1(z_1) = \underline{g}_1 G_1 \dot{\tau}^2 z_1^3 + \underline{g}_1 G_1 \zeta_1^2 N_1^2 z_1 + f_1 - \dot{y}_d \tag{7.7}$$

The unknown continuous function $h_1(z_1)$ can be approximated by an RBF NN as

$$h_1(z_1) = \theta_1^{*T} S_1(z_1) + \varepsilon_1(z_1) \tag{7.8}$$

Substituting (7.7) and (7.8) into (7.6) and using Young's inequality, one gets

$$G_1 \zeta_1 \tau \varepsilon_1(z_1) \le \underline{g}_1 G_1^2 \zeta_1^2 \tau^2 + \frac{\varepsilon_1^2}{4\underline{g}_1} \tag{7.9}$$

$$G_1 \zeta_1 \tau \theta_1^{*T} S_1(z_1) \le \underline{g}_1 G_1^2 \zeta_1^2 \tau^2 w_1 S_1^T(z_1) S_1(z_1) + \frac{1}{4\underline{g}_1} \tag{7.10}$$

Then, we obtain

$$\dot{V}_1 \le G_1 \zeta_1 \tau (g_1 \alpha_1 + \underline{g}_1 \psi_1 + \underline{g}_1 w_1 P_1) + \underline{g}_2 G_1^2 \zeta_1^2 \zeta_2^2 - \frac{g_1}{\lambda_1} \tilde{w}_1 \dot{\hat{w}}_1 + \Xi_1 \tag{7.11}$$

with

$$P_1 = G_1 \zeta_1 \tau S_1^T(z_1) S_1(z_1)$$

$$\psi_1 = G_1 \zeta_1 \tau$$

$$\Xi_1 = \frac{g_1^2}{4\underline{g}_2} + \frac{3}{4\underline{g}_1} + \frac{\varepsilon_1^2}{4\underline{g}_1}$$

From Assumption 2, we get

$$\begin{aligned} g_1 G_1 \zeta_1 \tau \alpha_1 &= -k_1 g_1 G_1 \zeta_1^2 - g_1 G_1 \zeta_1 \tau \psi_1 - g_1 G_1 \zeta_1 \tau \hat{w}_1 P_1 \\ &\le -k_1 \underline{g}_1 G_1 \zeta_1^2 - \underline{g}_1 G_1 \zeta_1 \tau \psi_1 - \underline{g}_1 G_1 \zeta_1 \tau \hat{w}_1 P_1 \end{aligned}$$

then

$$\dot{V}_1 \le -k_1 \underline{g}_1 G_1 \zeta_1^2 + \underline{g}_2 G_1^2 \zeta_1^2 \zeta_2^2 + \frac{\underline{g}_1}{\lambda_1} \delta_1 \tilde{w}_1 \hat{w}_1 + \Xi_1 \tag{7.12}$$

Step i $(i = 2, \dots, n-1)$: The following Barrier Lyapunov function is chosen as

$$V_{i0} = \frac{\zeta_i^2(t)}{\left(F_{i1}(t) + \zeta_i(t)\right)\left(F_{i2}(t) - \zeta_i(t)\right)}$$

Consider the BLF as

$$V_i = V_{i-1} + V_{i0} + \frac{\underline{g}_i}{2\lambda_i} \tilde{w}_i^2 \tag{7.13}$$

where $\lambda_i > 0$ is a constant, $w_i = \max_{k \in K} \left\{ \left\| \theta_{Ki}^* \right\|^2 \right\}$, $\tilde{w}_i = w_i - \hat{w}_i$ stands for the estimation error, with $\hat{w}_i$ being the estimating of w_i, Based on (3.4), we get the time derivative of V_i

$$\begin{aligned} \dot{V}_i &\le -\sum_{j=1}^{i-1} k_j \underline{g}_j G_j \zeta_j^2 + \underline{g}_i G_{i-1}^2 \zeta_{i-1}^2 \zeta_i^2 + \sum_{j=1}^{i-1} \frac{\underline{g}_j}{\lambda_j} \delta_j \tilde{w}_j \hat{w}_j + \sum_{j=1}^{i-1} \Xi_j + G_i \zeta_i \left(\dot{\zeta}_i + N_i \zeta_i \right) - \frac{\underline{g}_i}{\lambda_i} \tilde{w}_i \dot{\hat{w}}_i \\ &\le -\sum_{j=1}^{i-1} k_j \underline{g}_j G_j \zeta_j^2 + \sum_{j=1}^{i-1} \frac{\underline{g}_j}{\lambda_j} \delta_j \tilde{w}_j \hat{w}_j + \sum_{j=1}^{i-1} \Xi_j - \frac{\underline{g}_i}{\lambda_i} \tilde{w}_i \dot{\hat{w}}_i \\ &\quad + G_i \zeta_i \left(N_i \zeta_i + g_i \tau \alpha_i + g_i \zeta_{i+1} + \dot{\tau} z_i + \frac{\underline{g}_i G_{i-1}^2 \zeta_{i-1}^2 \zeta_i}{G_i} \right) \\ &\quad + G_i \zeta_i \tau \left(f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \left(g_j x_{j+1} + f_j \right) - \Delta \alpha_{i-1} \right) \end{aligned} \tag{7.14}$$

where

$$G_i = \frac{2F_{i1}F_{i2} - F_{i1}\zeta_i + F_{i2}\zeta_i}{\left[\left(F_{i1} + \zeta_i\right)\left(F_{i2} - \zeta_i\right)\right]^2}$$

and

$$N_i = \frac{-\dot{F}_{i1}F_{i2} - F_{i1}\dot{F}_{i2} + \left(\dot{F}_{i1} - \dot{F}_{i2}\right)\zeta_i}{2F_{i1}F_{i2} - F_{i1}\zeta_i + F_{i2}\zeta_i}$$

According to the Young's inequality, it yields

$$G_i g_i \zeta_i \zeta_{i+1} \le \underline{g}_{i+1} G_i^2 \zeta_i^2 \zeta_{i+1}^2 + \frac{g_i^2}{4\underline{g}_{i+1}} \tag{7.15}$$

$$G_i \zeta_i \dot{\tau} z_i \le \underline{g}_i G_i^2 \zeta_i^2 \dot{\tau}^2 z_i^2 + \frac{1}{4\underline{g}_i} \tag{7.16}$$

$$G_i \zeta_i^2 N_i \le \underline{g}_i G_i^2 \zeta_i^4 N_i^2 + \frac{1}{4\underline{g}_i} \tag{7.17}$$

Let $h_i(z_i) = f_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}\left(g_j x_{j+1} + f_j\right) - \Delta\alpha_{i-1} + \underline{g}_i G_i \dot{\tau}^2 z_i^3 + \underline{g}_i G_i \zeta_i^2 N_i^2 z_i + \frac{\underline{g}_i G_{i-1}^2 \zeta_{i-1}^2 z_i}{G_i}$ (7.18)

The unknown continuous function $h_i(z_i)$ can be approximated by an RBF NN as

$$h_i(z_i) = \theta_i^{*T} S_i\left(z_i\right) + \varepsilon_i\left(z_i\right) \tag{7.19}$$

Substituting (7.18) and (7.19) into (7.14) and using Young's inequality, one gets

$$G_i \zeta_i \tau \varepsilon_i \le \underline{g}_i G_i^2 \zeta_i^2 \tau^2 + \frac{\varepsilon_i^2}{4\underline{g}_i} \tag{7.20}$$

$$G_i \zeta_i \tau \theta_i^{*T} S_i\left(z_i\right) \le \underline{g}_i G_i^2 \zeta_i^2 \tau^2 w_i S_i^T\left(z_i\right) S_i\left(z_i\right) + \frac{1}{4\underline{g}_i} \tag{7.21}$$

Then, we obtain

$$\begin{aligned} \dot{V}_i \le & -\sum_{j=1}^{i-1} k_j \underline{g}_j G_j \zeta_j^2 + \sum_{j=1}^{i-1} \frac{\underline{g}_j}{\lambda_j} \delta_j \tilde{w}_j \hat{w}_j + \underline{g}_{i+1} G_i^2 \zeta_i^2 \zeta_{i+1}^2 \\ & + G_i \zeta_i \tau \left(g_i \alpha_i + \underline{g}_i \psi_i + \underline{g}_i w_i P_i\right) - \frac{\underline{g}_i}{\lambda_i} \tilde{w}_i \dot{\hat{w}}_i + \sum_{j=1}^{i-1} \Xi_j \end{aligned} \tag{7.22}$$

with

$$P_i = G_i \zeta_i \tau S_i^T\left(z_i\right) S_i\left(z_i\right)$$

$$\psi_i = G_i \zeta_i \tau$$

$$\Xi_i = \frac{g_i^2}{4\underline{g}_{i+1}} + \frac{3}{4\underline{g}_i} + \frac{\varepsilon_i^2}{4\underline{g}_i}$$

Let

$$g_i G_i \zeta_i \tau \alpha_i = -g_i k_i G_i \zeta_i^2 - g_i G_i \zeta_i \tau \psi_i - g_i G_i \zeta_i \tau \hat{w}_i P_i \le \underline{g}_i k_i G_i \zeta_i^2 - \underline{g}_i G_i \zeta_i \tau \psi_i - \underline{g}_i G_i \zeta_i \tau \hat{w}_i P_i$$

Then, we get

$$\dot{V}_i \le -\sum_{j=1}^{i} k_j \underline{g}_j G_j \zeta_j^2 + \sum_{j=1}^{i} \frac{\underline{g}_j}{\lambda_j} \delta_j \tilde{w}_j \hat{w}_j + \sum_{j=1}^{i} \Xi_j + \underline{g}_{i+1} G_i^2 \zeta_i^2 \zeta_{i+1}^2 \tag{7.23}$$

Step n: Because $\dot{V}_n$ is similar to $\dot{V}_i$, we can figure out the following formula

$$\dot{V}_n \le G_n \zeta_n \left(\frac{\underline{g}_n G_{n-1}^2 \zeta_{n-1}^2 \zeta_n}{G_n} + \tau g_n (mu + b) + \dot{\tau} z_n \right) + G_n N_n \zeta_n^2 - \frac{\underline{g}_n}{\lambda_n} \tilde{w}_n \dot{\hat{w}}_n + \sum_{j=1}^{n-1} \Xi_j + \sum_{j=1}^{n-1} \frac{\underline{g}_j}{\lambda_j} \delta_j \tilde{w}_j \hat{w}_j$$
$$+ G_n \zeta_n \tau \left(f_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (g_j x_{j+1} + f_j) - \Delta \alpha_{n-1} \right) - \sum_{j=1}^{n-1} k_j \underline{g}_j G_j \zeta_j^2 \tag{7.24}$$

Since b is bounded, one obtains $b/m \le \bar{b}/m \triangleq \eta$ where η is a constant. Using Young's inequality get the following formula:

$$G_n \xi_n \tau g_n (mu + b) = G_n \xi_n \tau g_n mu + G_n \xi_n \tau g_n m \frac{b}{m} \le G_n \xi_n \tau g_n mu + G_n^2 \xi_n^2 \tau^2 m + \frac{m \eta^2 g_n^2}{4} \tag{7.25}$$

According to the Young's inequality, it causes

$$G_n \zeta_n \dot{\tau} z_n \le \underline{g}_n G_n^2 \zeta_n^2 \dot{\tau}^2 z_n^2 + \frac{1}{4\underline{g}_n} \tag{7.26}$$

$$G_n \zeta_n^2 N_n \le \underline{g}_n G_n^2 \zeta_n^4 N_n^2 + \frac{1}{4\underline{g}_n} \tag{7.27}$$

Denote

$$h_n(z_n) = f_n - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (g_j x_{j+1} + f_j) - \Delta \alpha_{n-1} + G_n \zeta_n \tau m + \underline{g}_n G_n \dot{\tau}^2 z_n^3 + \underline{g}_n G_n \zeta_n^2 N_n^2 z_n + \frac{\underline{g}_n G_{n-1}^2 \zeta_{n-1}^2 z_n}{G_n} \tag{7.28}$$

The unknown continuous function $h_n(z_n)$ can be approximated by an RBF NN as

$$h_n(z_n) = \theta_n^{*T} S_n(z_n) + \varepsilon_n(z_n) \tag{7.29}$$

Substituting (7.28) and (7.29) into **Error! Reference source not found.**, and using the Young's inequality, it yields

$$G_n \zeta_n \tau \varepsilon_n \le \underline{g}_n G_n^2 \zeta_n^2 \tau^2 + \frac{\varepsilon_n^2}{4\underline{g}_n} \tag{7.30}$$

$$G_n \zeta_n \tau \theta_n^{*T} S_n(z_n) \le \underline{g}_n G_n^2 \zeta_n^2 \tau^2 w_n S_n^T(z_n) S_n(z_n) + \frac{1}{4\underline{g}_n} \tag{7.31}$$

Then, using the above inequalities we get

$$\dot{V}_n \le -\sum_{j=1}^{n-1} k_j \underline{g}_j G_j \zeta_j^2 + \sum_{j=1}^{n-1} \frac{\underline{g}_j}{\lambda_j} \delta_j \tilde{w}_j \hat{w}_j + \sum_{j=1}^{n-1} \Xi_j + G_n \zeta_n \tau \left(g_n mu + \underline{g}_n \psi_n + \underline{g}_n w_n P_n \right) - \frac{\underline{g}_n}{\lambda_n} \tilde{w}_n \dot{\hat{w}}_n \tag{7.32}$$

where

$$P_n = G_n \zeta_n \tau S_n^T(z_n) S_n(z_n)$$

$$\psi_n = G_n \zeta_n \tau$$

$$\Xi_n = \frac{3}{4\underline{g}_n} + \frac{\varepsilon_n^2}{4\underline{g}_n} + \frac{m\eta^2 g_n^2}{4}$$

with

$$G_n \zeta_n \tau g_n mu = -k_n g_n G_n \zeta_n{}^2 - g_n G_n \zeta_n \tau \psi_n - g_n G_n \zeta_n \tau \hat{w}_n \mathrm{P}_n \le -k_n \underline{g}_n G_n \zeta_n{}^2 - \underline{g}_n G_n \zeta_n \tau \psi_n - \underline{g}_n G_n \zeta_n \tau \hat{w}_n \mathrm{P}_n$$

Then, we get

$$\dot{V}_n \le -\sum_{j=1}^{n} k_j \underline{g}_j G_j \zeta_j^2 + \sum_{j=1}^{n} \frac{\underline{g}_j}{\lambda_j} \delta_j \tilde{w}_j \hat{w}_j + \sum_{j=1}^{n} \Xi_j \tag{7.33}$$

Conflict of interest

All authors declare no conflicts of interest.

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