Mathematics

http://www.aimspress.com/journal/Math

AIMS Mathematics, 5(4): 3480–3494.
DOI:10.3934/math.2020226
Received: 01 January 2020
Accepted: 26 March 2020
Published: 08 April 2020

Research article

Common fixed point results for couples (f, g) and (S, T) satisfy strong common limit range property

Muhammad Shoaib[1], **Muhammad Sarwar**[1,*] **and Thabet Abdeljawad**[2,3,4,*]

[1] Department of Mathematics, University of Malakand, Chakdara Dir(L), Pakistan
[2] Department mathematics and General Sciences, Prince Sultan University, P. O. Box 66833, Riyadh 11586, Saudi Arabia
[3] Department of Medical Research, China Medical University, Taichung, Taiwan
[4] Department of Computer Science and Information Engineering, Asia University, Taichung, Taiwan

* **Correspondence:** Email: sarwarswati@gmail.com, tabdeljawad@psu.edu.sa;
Tel: +923444043268, +966114948523; Fax: +966114548317.

Abstract: In this manuscript, we introduce strong common limit range property for couples (f, g) and (S, T) and by means of this new concept we establish common fixed point results for hybrid pair via (F, φ)- contraction and rational type contraction conditions. Further, we give some examples to support and illustrate our result. Using the established results existence of solution to the system of integral and differential equations are also discussed. We provide example where the main theorem is applicable but relevant classic result in literature fail to have a common fixed point.

Keywords: Hybrid set-valued mapping; strong common limit range property; common fixed point
Mathematics Subject Classification: 47H10, 54H25

1. Introduction

In the area of fixed point theory the study of common fixed point for couple of mapping is a new research area (see [1, 7] and the references therein). For hybrid pair the (E. A) property was introduced by Kamran [6] and established coincidence and fixed points results via hybrid strict contractive conditions. To obtain common fixed point results for hybrid type contractive condition, Liu et al. [8] extended this property to common (E. A) property for hybrid pairs of (set-valued and single) mappings. For mapping of single-valued Sintunavarat and Kumam [13] put together the notion of the common limit range (CLR) property and showed its dominance on the property (E. A). For the mapping of hybrid pair the common limit range property defined by Imdad et al. [5] for set-valued fixed point mapping in the semi-metric (symmetric) spaces.

Afrah [2] generalized this property for single hybrid pair of mapping and then the same author in [3] extended the CLR_g property for two hybrid pairs of of mapping. In fuzzy metric spaces, by means of $CLRg$ property, Rold and Sintunavarat [14] established common fixed point results. In 2016, Shoaib and Sarwar [11] studied applications to two functional equations by using set-valued fixed point theorems for a pair of maps and made use of (CLR) property via generalized contractions. For generalized hybrid (F, φ)-contractions Nashine et al. [10] established common fixed point results and used the common limit range property. For more detail see([4, 12, 15–18]).

Motivated by above, using strong common limit range property, we derived set-valued common fixed point results in metric space for couples of maps. Using this property, established hybrid common fixed point results. Also explain the property by giving examples.

In the whole paper $CB(\tilde{N})$ shows the class of all bounded and closed subsets of $\tilde{N}$, $\mathbf{R}^{+}$ is the set of positive real numbers and $\mathbf{R}$ the set of real numbers respectively.

Definition 1.1. [9] Maps $f : \tilde{N} \to \tilde{N}$, $S : \tilde{N} \to CB(\tilde{N})$ are said to be occasionally S-weakly commuting if there exists $\xi \in \tilde{N}$ such that $f\xi \in S\xi$ and $ff\xi \in fS\xi$.

Definition 1.2. [13] If for a sequence $\{\xi_n\}$ in $\tilde{N}$, $\lim_{n\to\infty} f\xi_n = \lim_{n\to\infty} g\xi_n = fu$ for some $u \in \tilde{N}$. Then, $f, g : \tilde{N} \to \tilde{N}$ are said to have the common limit range property of f with respect to g (shortly, the (CLR_f)-property w.r.t to g).

The following two definitions can be found in [3].

Definition 1.3. If for a sequence $\{\xi_n\}$ in $\tilde{N}$ and $\Omega_1 \in CB(\tilde{N})$, $\lim_{n\to\infty} f\xi_n = fu \in \Omega_1 = \lim_{n\to\infty} S\xi_n$ for some $u \in \tilde{N}$. Then, $S : \tilde{N} \to CB(\tilde{N})$, $f : \tilde{N} \to \tilde{N}$, over metric space $(\tilde{N}, d)$ are said to have the common limit range property of f with respect to S (shortly, (CLR_f)-property w.r.t to S).

Definition 1.4. If for sequences $\{\xi_n\}, \{\zeta_n\}$ in $\tilde{N}$ and $\Omega_1, \Omega_2 \in CB(\tilde{N})$, $\lim_{n\to\infty} S\xi_n = \Omega_1$, $\lim_{n\to\infty} T\zeta_n = \Omega_2$, $\lim_{n\to\infty} f\xi_n = \lim_{n\to\infty} g\zeta_n = fu \in \Omega_1 \cap \Omega_2$, for some $u \in \tilde{N}$. Then, $S, T : \tilde{N} \to CB(\tilde{N})$, $f, g : \tilde{N} \to \tilde{N}$ on metric space $(\tilde{N}, d)$ are said to have the common limit in the range of f with respect to S (shortly, (CLR_f)-property w.r.t S).

The below definition is new and it is a modification of Definition 1.3 for couples of functions.

Definition 1.5. Assume $f, g : \tilde{N} \to \tilde{N}$ and $S, T : \tilde{N} \to CB(\tilde{N})$ are functions defined on metric space $(\tilde{N}, d)$. Then the couple (f, g) and the couple (S, T) are said to fulfil the common limit in the range of f with respect to (S, T) via g (shortly, (CLR_f)-property with respect to (S, T) via g) if there exist sequences $\{\xi_n\}$ and $\{\zeta_n\}$ in $\tilde{N}$ and $\Omega_1, \Omega_2 \in CB(\tilde{N})$ such that, for some $u \in \tilde{N}$ we have $\lim_{n\to\infty} S\xi_n = \Omega_1$, $\lim_{n\to\infty} T\zeta_n = \Omega_2$ and $\lim_{n\to\infty} f\xi_n = \lim_{n\to\infty} g\zeta_n = fu \in \Omega_1 \cap \Omega_2$.

Remark 1.6. Clearly, if $f = g$ and $S = T$ in Definition 1.4 then we reobtain Definition 1.3.

The below definition announces the so-called the strong common limit range property.

Definition 1.7. If the couples (f, g) and (S, T) satisfy the (CLR_f)-property with respect to (S, T) via g and the (CLR_g) property with respect to (S, T) via f then we say the couples (f, g) and (S, T) fulfil the strong common limit range property.

Remark 1.8. f and S fulfil strong common limit range property if there exists $\{\xi_n\}$ and $\{\zeta_n\}$ in $\tilde{N}$ and $\Omega_1, \Omega_2 \in CB(\tilde{N})$ such that, for some $u \in \tilde{N}$ we have $\lim_{n\to\infty} S\xi_n = \Omega_1$, $\lim_{n\to\infty} S\zeta_n = \Omega_2$ and $\lim_{n\to\infty} f\xi_n = \lim_{n\to\infty} f\zeta_n = fu \in \Omega_1 \cap \Omega_2$.

Remark 1.9. Couples (f, g) and (S, T) said to be not strong common limit range property. if $\lim_{n\to\infty} f\xi_n$ and $\lim_{n\to\infty} g\zeta_n$ exist but not equal to fu or their does not exist $\{\xi_n\}$, $\{\zeta_n\}$ such that $\lim_{n\to\infty} f\xi_n = \lim_{n\to\infty} g\zeta_n = fu$.

Example 1.10. Let $\tilde{N} = [0, \infty)$ with the usual metric. Define $f, g : \tilde{N} \to \tilde{N}$ and $S, T : \tilde{N}\times\tilde{N} \to CB(\tilde{N})$ by $f(x) = 1 + x$, $g(x) = x^2$, $S(x) = [1, 2 + 2x]$, $T(x) = [1, 2 + \frac{3x}{4}]$, $\forall x \in \tilde{N}$.

Consider the sequences $\{\varsigma_n\} = \{1 + \frac{1}{n}\}$, $\{\zeta_n\} = \{2 + \frac{1}{n}\}$,

Clearly $\lim_{n\to\infty} S(\varsigma_n) = [1, 4] = \Omega_1$,

$\lim_{n\to\infty} T(\zeta_n) = [1, \frac{7}{2}] = \Omega_2$, $\lim_{n\to\infty} f(\varsigma_n) = \lim_{n\to\infty} g(\zeta_n) = 2 = f(1) = g(2) \in \Omega_1 \cap \Omega_2$.

Therefore couples the (f, g) and (S, T) fulfil the strong common limit range property.

Example 1.11. Let $\tilde{N} = [0, 10)$ with the usual metric. Define $f, g : \tilde{N} \to \tilde{N}$ and $S, T : \tilde{N}\times\tilde{N} \to CB(\tilde{N})$ by $f(x) = 1$, $g(x) = 2$, $S(x) = [2x, 2 + 2x]$, $T(x) = [x, 2 + \frac{3x}{4}]$, $\forall x \in \tilde{N}$.

For any choice of $\{\varsigma_n\}$ and $\{\zeta_n\}$, the couples the (f, g) and (S, T) does not hold the strong common limit range property.

Now, we provide definitions defined for set-valued mappings in a metric space $(\tilde{N}, d)$. Defined the function $H : CB(\tilde{N}) \times CB(\tilde{N}) \to \mathbf{R}^+$ for $\tilde{N}_1, \tilde{N}_2 \in CB(\tilde{N})$ by

$$H(\tilde{N}_1, \tilde{N}_2) = \max\{\sup_{\varsigma_1\in\tilde{N}_1} d(\varsigma_1, \tilde{N}_2), \sup_{\zeta_1\in\tilde{N}_2} d(\zeta_1, \tilde{N}_1)\},$$

where

$$d(\xi_1, \tilde{N}_1) = \inf\{d(\xi_1, \zeta_1) : \zeta_1 \in \tilde{N}_1\},$$

$$D(\tilde{N}_1, \tilde{N}_2) = \inf\{d(\varsigma_1, \zeta_1) : \varsigma_1 \in \tilde{N}_1, \zeta_1 \in \tilde{N}_2\}.$$

and

$$\delta(\tilde{N}_1, \tilde{N}_2) = \sup\{d(\varsigma_1, \zeta_1) : \varsigma_1 \in \tilde{N}_1, \zeta_1 \in \tilde{N}_2\}$$

Lemma 1.12. *[19] Let $(\tilde{N}, d)$ be a metric space. For any $\tilde{N}_1, \tilde{N}_2 \in CB(\tilde{N})$. We have $d(\xi, \tilde{N}_2) \le H(\tilde{N}_1, \tilde{N}_2)$, for all $\xi \in \tilde{N}_1$.*

Definition 1.13. [20] Let $\eta : \tilde{N} \to \tilde{N}$ and $\lambda : \tilde{N} \to CB(\tilde{N})$ be a mapping. Then η are known occasionally weakly λ- commuting $\Leftrightarrow$ there exist x in $\tilde{N}$ such that $\eta\eta x \in \lambda\eta x$ for $\eta x \in \lambda x$.

Theorem 1.14. *[3] Let $f, g : \tilde{N} \to \tilde{N}$ and $S, T : \tilde{N} \to CB(\tilde{N})$ be a maps on metric space $(\tilde{N}, d)$ satisfying the following condition.*

(a)The pairs (S, f) and (T, g) have common limit range property (CLR_f),

(b)for all $x, y \in \tilde{N}$

$$H^p(Sx, Ty)) \le \varphi(\Delta(x, y))), \tag{1.1}$$

where,

$$\Delta(x, y) = \max\left\{ d^p(fx, gy), \frac{d^p(fx,Sx)d^p(gy,Ty)}{1+d^p(fx,gy)}, \frac{d^p(fx,Ty)d^p(gy,Sx)}{1+d^p(fx,gy)} \right\}.$$

Here, $p \ge 1$, and $\varphi : [0, \infty) \to [0, \infty)$ is continuous monotone increasing function such that $\varphi(0) = 0$ and $\varphi(t) < t$ for all $t > 0$. If $f(\tilde{N})$ and $g(\tilde{N})$ are closed subsets of $\tilde{N}$, then we have the following:

(A_1) *(f, S) have coincidence point.*
(A_2) *(g, T) have coincidence point.*
(A_3) *f and S has a common fixed point, if $ffv = fv$ and f and S are weakly compatible at v;*
(A_4) *g and T has a common fixed point, if $ffv = fv$ and g and T are weakly compatible at v;*
(A_5) *if (A_3) and (A_4) holds. Then g, f, T and S have a common fixed point.*

Definition 1.15. [17] Let F_s denotes the class of all mapping $F_1 : \mathbf{R}^+ \to \mathbf{R}$, with the below conditions

(1) F_1 is strictly increasing and continuous;
(2) $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F_1(\alpha_n) = -\infty$;
(3) For $\{\alpha_n\} \subset \mathbf{R}^+$, $\lim_{n\to\infty} \alpha_n = 0$, there exists $q \in (0, 1)$, such that $\lim_{\alpha\to 0^+} (\alpha_n)^q F_1(\alpha_n) = 0$.

Thoroughly in this section Φ denote the below class

$$\Phi = \left\{ \begin{array}{l} \varphi : \mathbf{R}^+ \to \mathbf{R}^+, \textit{upper semi-contineous, increasing such that } \lim_{\kappa_1\to\tau_1^+} \varphi(\kappa_1) < \varphi(\tau_1), \varphi(\tau_1) < \tau_1, \\ for\ all\ \tau_1 > 0 \end{array} \right\}.$$

Theorem 1.16. *Let $f, g : \tilde{N} \to \tilde{N}$ and $S, T : \tilde{N} \to CB(\tilde{N})$ be a maps on metric space $(\tilde{N}, d)$. Suppose that (S, g) and (T, f) have strong common limit range property. Furthermore assume that*

$$\tau + F(H^p(Tx, Sy)) \leq F(\varphi(\Delta(x, y))), \tag{1.2}$$

where, $H^p(Tx, Sy) > 0$ and

$$\Delta(x, y) = \alpha[d^p(fx, gy)] + \beta\left[\frac{d^p(fx, Sy)d^p(gy, Tx)}{1 + d^p(fx, gy)}\right] + \gamma[d^p(fx, Tx) + d^p(gy, Sy)] + \sigma[d^p(fx, Sy)] + \eta[d^p(gy, Tx)].$$

Here, $\tau \in \mathbf{R}^+$, $\alpha + \beta + \gamma + \sigma + \eta < 1$, $p \geq 1$, $F \in F_s$ and $\varphi \in \Phi$. Then the below assumption holds.

(A_1) *(f, T) have coincidence point.*
(A_2) *(g, S) have coincidence point.*
(A_3) *T and f has a common fixed point, if $ffv = fv$ and f is occasionally T-weakly commuting at v;*
(A_4) *g and S has a common fixed point, if $ggu = gu$ and g is occasionally S-weakly commuting at w ;*
(A_5) *if (A_3) and (A_4) holds. Then g, f, T and S have a common fixed point.*

Proof. Since $f, g : \tilde{N} \to \tilde{N}$ and $T, S : \tilde{N} \to CB(\tilde{N})$ have strong common limit range property, therefore there exists a sequence $\{\xi_n\}$ and $\{\zeta_n\}$ in $\tilde{N}$ and $\Omega_1, \Omega_2 \in CB(\tilde{N})$ such that,

$$\lim_{n\to\infty} T\xi_n = \Omega_1, \lim_{n\to\infty} S\zeta_n = \Omega_2 \textit{ and } \lim_{n\to\infty} f\xi_n = \lim_{n\to\infty} g\zeta_n = fu = gv \in \Omega_1 \cap \Omega_2.$$

for some $u, v \in \tilde{N}$.

Now, we show $gv \in Sv$, suppose $gv \notin Sv$. then putting $x = \xi_n, y = v$ in inequality (1.2), we have

$$\tau + F(H^p(T\xi_n, Sv)) \leq F(\varphi(\Delta(\xi_n, v))), \tag{1.3}$$

where,

$$\Delta(\xi_n, v) = \alpha[d^p(f\xi_n, gv)] + \beta\Big[\frac{d^p(f\xi_n, S\zeta_n)d^p(gv, T\xi_n)}{1 + d^p(f\xi_n, gv)}\Big] + \gamma[d^p(f\xi_n, T\xi_n) + d^p(gv, Sv)] + \sigma[d^p(f\xi_n, Sv)] + \eta[d^p(gv, T\xi_n)].$$

By taking limit to Δ, we have

$$\lim_{n\to\infty}\Delta(\xi_n, v) = \alpha[d^p(fu, gv)] + \beta\Big[\frac{d^p(fu, \Omega_2)d^p(gv, \Omega_1)}{1 + d^p(fu, gv)}\Big] + \gamma[d^p(fu, \Omega_1) + d^p(gv, Sv)] + \sigma[d^p(fu, Sv)] + \eta[d^p(fu, \Omega_1)].$$

$$\lim_{n\to\infty}\Delta(\xi_n, v) = \gamma[d^p(gv, Sv)] + \sigma[d^p(fu, Sv)] + \eta[d^p(fu, \Omega_1].$$

$$\lim_{n\to\infty}\Delta(\xi_n, w) = (\gamma + \sigma)[d^p(gv, Sv)]. \tag{1.4}$$

Apply limit over (1.3) and by using (1.4), we have

$$\tau + F(H^p(\Omega_1, Sv)) \leq F(\varphi(\alpha((\gamma + \sigma)(d^p(gv, Sv))))).$$

Which implies that

$$F(H^p(\Omega_1, Sv)) \leq F(\varphi(\alpha((\gamma + \sigma)(d^p(gv, Sv))))).$$

Using definitions of F and φ, we have

$$H^p(\Omega_1, Sv) \leq \alpha((\gamma + \sigma)(d^p(gv, Sv))).$$

But $\alpha < 1$ and using Lemma 1.12

$$d^p(gv, Sv) \leq H^p(\Omega_1, Sv) \leq \alpha(d^p(gv, Sv)) < d^p(gv, Sv). \tag{1.5}$$

Which is contradiction. Hence, $gv \in Sv$.

Again from (1.2), we have

$$\tau + F(H^p(Tu, S\zeta_n)) \leq F(\varphi(\Delta(u, \zeta_n))), \tag{1.6}$$

where,

$$\Delta(u, \zeta_n) = \alpha[d^p(fu, g\zeta_n)] + \beta\Big[\frac{d^p(fu, S\zeta_n)d^p(g\zeta_n, Tu)}{1 + d^p(fu, g\zeta_n)}\Big] + \gamma[d^p(fu, Tu) + d^p(g\zeta_n, S\zeta_n)] + \sigma[d^p(fu, S\zeta_n)] + \eta[d^p(g\zeta_n, Tu)].$$

By taking limit to Δ, we have

$$\lim_{n\to\infty}\Delta(u, \zeta_n) = \alpha[d^p(fu, fu)] + \beta\Big[\frac{d^p(fu, \Omega_2)d^p(gu, Tu)}{1 + d^p(fu, gu)}\Big] + \gamma[d^p(fu, Tu) + d^p(fu, \Omega_2)] + \sigma[d^p(fu, \Omega_2)] + \eta[d^p(fu, Tu)].$$

$$\lim_{n\to\infty}\Delta(u,\zeta_n) = (\gamma+\eta)d^p(fu,Tu). \tag{1.7}$$

By taking limit to (1.6) and using (1.7), we have

$$\tau + F(H^p((Tu,\Omega_2)) \le F(\varphi(\alpha((\gamma+\eta)d^p(fu,Tu)))).$$

Which implies that

$$F(H^p((Tu,\Omega_2)) \le F(\varphi(\alpha((\gamma+\eta)d^p(fu,Tu)))).$$

Using definitions of F and φ, we have

$$H^p((Tu,\Omega_2)) \le \alpha((\gamma+\eta)d^p(fu,Tu)).$$

But $\alpha < 1$ and using Lemma 1.12

$$d^p(fu,Tu) \le H^p(Tu,\Omega_2) \le \alpha((\gamma+\eta)d^p(fu,Tu)) < d^p(fu,Tu).$$

Which is contradiction. Hence, $fu \in Tu$. Since $ffv = fv$ and $fv \in Tfv$, therefore $\gamma = f\gamma \in T\gamma$.

Similarly $\gamma = g\gamma \in S\gamma$. (A_5) hold obviously. □

Example 1.17. Let $\tilde{N} = [0,\infty)$ is a metric(w.r.t) the usual metric. $S,T : \tilde{N} \to CB(\tilde{N})$, $f,g : \tilde{N} \to \tilde{N}$ and $\varphi : \mathbf{R}^+ \to \mathbf{R}^+$ define by $S(x) = [1, 5+2\beta x]$, $T(x) = [1, 5+\beta x]$, $f(x) = 4x$, $g(x) = 2x$ $\forall x \in \tilde{N}$ and $\varphi(t) = \beta t$, $0 < \beta < 1$.

Consider the sequences $\{\xi_n\} = \{1 + \frac{1}{n}\}$, $\{\zeta_n\} = \{2 + \frac{1}{n}\}$,

Now, $\lim_{n\to\infty} S(\xi_n) = [1, 5+2\beta]$ $\lim_{n\to\infty} T(\zeta_n) = [1, 5+2\beta]$, $\lim_{n\to\infty} g(\zeta_n) = \lim_{n\to\infty} f(\xi_n) = 4 = f(1) = g(2) \in [1, 5+\beta] \cap [1, 5+2\beta]$.

Therefore couples (f,g) and (S,T) satisfy the strong common limit range. Now

$$H(Sx,Ty) = H([1,5+2\beta x],[1,5+\beta y])$$

$$= \max\{d([1,5+2\beta x],[1,5+\beta y]), d([1,5+\beta x],[1,5+2\beta y])\},$$

$$= \max\{|\beta x - 2\beta y|, 0\},$$

$$= \frac{\beta}{2} d(gx,fu)$$

$$= \frac{1}{2}.\varphi(d(gx,fu))$$

$$\le \frac{1}{2}.\varphi(\Delta(x,y))$$

$$\le e^{\frac{-1}{6}}.\varphi(\Delta(x,y)).$$

By taking natural logarithm on both sides we conclude by Theorem 1.20 that $C(S,f) \ne \emptyset$ and $C(T,g) \ne \emptyset$. where $C(S,f)$ represent coincidence point of S and f.

Example 1.18. Let $\tilde{N} = (-11, 11)$ with the usual metric. Define $S, T : \tilde{N} \times \tilde{N} \to CB(\tilde{N})$, $f, g : \tilde{N} \to \tilde{N}$, $\varphi : \mathbf{R}^+ \to \mathbf{R}^+$ and $F : \mathbf{R}^+ \to \mathbf{R}$ by $S(x) = [-6, 2 + \frac{\beta x}{4}]$, $T(x) = [-6, 2 + \frac{\beta}{6}x]$, $f(x) = \frac{x}{2}$, $g(x) = \frac{x}{3}$ $\forall x \in \tilde{N}$, $\varphi(t) = \beta t$, $0 < \beta < 1$ and $F(t) = ln(t)$.

Consider the sequences $\{\xi_n\} = \{2 + \frac{1}{n}\}$, $\{\zeta_n\} = \{3 + \frac{1}{n}\}$,

Now, $\lim_{n\to\infty} S(\xi_n) = [-6, 2 + \frac{\beta}{2}]$ $\lim_{n\to\infty} T(\zeta_n) = [-6, 2 + \frac{\beta}{2}]$, $\lim_{n\to\infty} g(\zeta_n) = \lim_{n\to\infty} f(\xi_n) = 1 = f(2) = g(3) \in [-6, 2 + \frac{\beta}{2}] \cap [-6, 2 + \frac{\beta}{2}]$.

Therefore couples (f, g) and (S, T) satisfy the strong common limit range. Now

Now,

$$H(Sx, Ty) = H([-5, 1 + \frac{\alpha x}{4}], [-5, 1 + \frac{\alpha y}{6}])$$

$$= \max\{d([-5, 1 + \frac{\alpha x}{4}], [-5, 1 + \frac{\alpha y}{6}]), d([-5, 1 + \frac{\alpha x}{6}], [-5, 1 + \frac{\alpha y}{4}])\},$$

$$= \max\{|\frac{\alpha x}{6} - \frac{\alpha y}{4}|, 0\},$$

$$= \frac{\alpha}{2} d(gx, fy)$$

$$= \frac{1}{2}\varphi(d(gx, fy))$$

$$\leq \frac{1}{2}\varphi(\Delta(x, y))$$

$$\leq e^{\frac{-1}{6}}\varphi(\Delta(x, y)).$$

Taking logarithm on both sides and $p = 1$, we conclude that all the other condition of Theorem 1.16 are satisfied. Therefore (f, g) and (S, T) have coincidence point point.

Remark 1.19. From above examples it is clear that

- Theorem 1.14 is not applicable to Example 1.18 because $f(\tilde{N}) = (-\frac{11}{2}, \frac{11}{2})$ nor $g(\tilde{N}) = (\frac{-11}{3}, \frac{11}{3})$ are closed.

Theorem 1.20. *Let $f, g : \tilde{N} \to \tilde{N}$ and $S, T : \tilde{N} \to CB(\tilde{N})$ are mapping on metric space $(\tilde{N}, d)$. Furthermore assume that (S, g) and (T, f) have strong common limit range property and*

$$\tau + F(H^p(Tx, Sy) \leq F(\varphi(\Delta(x, y))), \tag{1.8}$$

where, $H^p(Tx, Sy) > 0$ and

$$\Delta(x, y) = \max\left\{\begin{array}{l} d^p(fx, Sy), d^p(gy, Tx), d^p(fx, gy), \\ \frac{d^p(fx,Sy)+d^p(gy,Tx)}{2}, \frac{d^p(fx,Sy)d^p(gy,Tx)}{1+d^p(fx,gy)}, \\ \frac{d^p(gy,Tx)d^p(fx,Sy)}{1+d^p(fx,gy)}, \frac{d^p(fx,Sy)d^p(gy,Tx)}{1+D^p(Sx,Tx)} \end{array}\right\}.$$

Here, $\tau \in \mathbf{R}^+$, $p \geq 1$, $F \in F_s$ and $\varphi \in \Phi$. Then the below condition holds.

(A_1) *(f, T) have coincidence point.*
(A_2) *(g, S) have coincidence point.*
(A_3) *T and f has a common fixed point, if $ffv = fv$ and f is occasionally T-weakly commuting at v;*

(A_4) *S and g has a common fixed point, if $ggw = gw$ and g is occasionally S-weakly commuting at w ;*
(A_5) *if (A_3) and (A_4) holds. Then f, S, g and T have a common fixed point.*

Proof. Since $f, g : \tilde{N} \to \tilde{N}$ and $T, S : \tilde{N} \to CB(\tilde{N})$ have strong (CLR)-property, therefore there exists a sequence $\{\xi_n\}$ and $\{\zeta_n\}$ in $\tilde{N}$ and $\Omega_1, \Omega_2 \in CB(\tilde{N})$ such that,

$$\lim_{n\to\infty} T\xi_n = \Omega_1, \lim_{n\to\infty} S\zeta_n = \Omega_2 \text{ and } \lim_{n\to\infty} f\xi_n = \lim_{n\to\infty} g\zeta_n = fu = gv \in \Omega_1 \cap \Omega_2,$$

for some $u, v \in \tilde{N}$. Now, we show $gv \in Sv$, suppose $gv \notin Sv$, by putting $x = \xi_n, y = v$ in inequality (1.22), we have

$$\tau + F(H^p(T\xi_n, Sv) \le F(\varphi(\Delta(\xi_n, v))), \tag{1.9}$$

$$\Delta(\xi_n, v) = \max\left\{ \begin{array}{c} d^p(f\xi_n, Sv), d^p(gv, T\xi_n), d^p(f\xi_n, gv), \\ \frac{d^p(f\xi_n,Sv)+d^p(gv,T\xi_n)}{2}, \frac{d^p(f\xi_n,Sv)d^p(gv,T\xi_n)}{1+d^p(f\xi_n,gv)}, \\ \frac{d^p(gv,T\xi_n)d^p(f\xi_n,Sv)}{1+d^p(f\xi_n,gv)}, \frac{d^p(f\xi_n,Sv)d^p(gv,T\xi_n)}{1+D^p(S\xi_n,T\xi_n)} \end{array} \right\}.$$

Taking limit to Δ, we have

$$\lim_{n\to\infty} \Delta(\xi_n, v) = \max\left\{ \begin{array}{c} d^p(gv, Sv), d^p(gv, \Omega_1), d^p(gv, gv), \\ \frac{d^p(fv,Sv)+d^p(gv,\Omega_1)}{2}, \frac{d^p(gv,Sv)d^p(gv,\Omega_1)}{1+d^p(gv,gv)}, \\ \frac{d^p(gv,\Omega_1)d^p(gv,Sv)}{1+d^p(gv,gv)}, \frac{d^p(gv,Sv)d^p(gv,\Omega_1)}{1+D^p(\Omega_2,\Omega_1)} \end{array} \right\}.$$

$$\lim_{n\to\infty} \Delta(\xi_n, v) = \max\left\{ d^p(gv, Sv), \frac{d^p(gv,Sv)}{2} \right\}.$$

$$\lim_{n\to\infty} \Delta(\xi_n, v) = d^p(gv, Sv). \tag{1.10}$$

By taking limit over (1.9), and using (1.10), we have

$$\tau + F(H^p(\Omega_1, Sv)) \le F(\varphi(\alpha(d^p(gv, Sv)))).$$

Which implies that

$$F(H^p(\Omega_1, Sv)) \le F(\varphi(\alpha(d^p(gv, Sv)))).$$

Using definitions of F and φ, we have

$$H^p(\Omega_1, Sv) \le \alpha(d^p(gv, Sv)).$$

But $\alpha < 1$ and using Lemma 1.12

$$d^p(gv, Sv) \le H^p(\Omega_1, Sv) \le \alpha(d^p(gv, Sv)) < d^p(gv, Sv). \tag{1.11}$$

which is contradiction. Hence, $gv \in Sv$.

We show $fu \in Tu$, suppose $fu \notin Tu$. then putting $x = u, y = \zeta_n$ in inequality (1.22), we have

$$\tau + F(H^p(Tu, S\zeta_n) \le F(\varphi(\Delta(u, \zeta_n))), \tag{1.12}$$

$$\Delta(v, \zeta_n) = \max\left\{ \begin{array}{c} d^p(fu, S\zeta_n), d^p(g\zeta_n, Tu), d^p(fu, g\zeta_n), \\ \frac{d^p(fu,S\zeta_n)+d^p(g\zeta_n,Tu)}{2}, \frac{d^p(fu,S\zeta_n)d^p(g\zeta_n,Tu)}{1+d^p(fu,g\zeta_n)}, \\ \frac{d^p(g\zeta_n,Tu)d^p(fu,S\zeta_n)}{1+d^p(fu,g\zeta_n)}, \frac{d^p(fu,S\zeta_n)d^p(g\zeta_n,Tu)}{1+D^p(Su,Tu)} \end{array} \right\}.$$

By taking limit over Δ, we have

$$\lim_{n\to\infty}\Delta(u,\zeta_n) = \max\left\{\begin{array}{c} d^p(fu,\Omega_2), d^p(fu,Tu), d^p(fu,fu), \\ \frac{d^p(fu,\Omega_2)+d^p(fu,Tu)}{2}, \frac{d^p(fu,\Omega_2)d^p(fu,Tu)}{1+d^p(fu,fu)}, \\ \frac{d^p(fu,Tu)d^p(fu,\Omega_2)}{1+d^p(fu,fu)}, \frac{d^p(fu,\Omega_2)d^p(fu,Tu)}{1+D^p(Su,Tu)} \end{array}\right\}.$$

$$\lim_{n\to\infty}\Delta(u,\zeta_n) = \max\left\{ d^p(fu,Tu), \frac{d^p(fu,Tu)}{2} \right\}.$$

$$\lim_{n\to\infty}\Delta(u,\zeta_n) = d^p(fu,Tu) \tag{1.13}$$

By taking limit over (1.12) and using (1.13), we have

$$\tau + F(H^p(Tu,\Omega_2) \leq F(\varphi(\alpha d^p(fu,Tu))).$$

Which implies that

$$F(H^p(Tu,\Omega_2)) \leq F(\varphi(\alpha d^p(fu,Tu))).$$

Using definitions of F and φ, we have

$$H^p(Tu,\Omega_2) \leq \alpha d^p(fu,Tu).$$

But $\alpha < 1$ and using Lemma 1.12

$$d^p(fu,Tu) \leq H^p(Tu,\Omega_2) \leq \alpha d^p(fu,Tu) < d^p(fu,Tu).$$

Which is contradiction. Hence, $fu \in Tu$.

Succeeding the parallel line of Theorem 1.16, we can achieved that S, T, f and g have common coupled fixed point. □

If $S = T$ and $f = g$ in Theorem 1.16, by using Remark 1.6, we have

Corollary 1.21. *Let $f : \tilde{N} \to \tilde{N}$ and $S : \tilde{N} \to CB(\tilde{N})$ be a maps on metric space $(\tilde{N}, d)$. Suppose (S, f) have strong common limit range property. Furthermore*

$$\tau + F(H^p(Sx,Sy)) \leq F(\varphi(\Delta(x,y))),$$

where, $H(Sx,Sy) > 0$ and

$$\Delta(x,y) = \alpha[d^p(fx,fy)] + \beta\left[\frac{d^p(fx,Sy)d^p(fy,Sx)}{1+d^p(fx,fy)}\right] + \gamma[d^p(fx,Sx) + d^p(fy,Sy)] + \sigma[d^p(fx,Sy)] + \eta[d^p(fy,Sx)].$$

Here, $\tau \in \boldsymbol{R}^+$, $\alpha+\beta+\gamma+\sigma+\eta < 1$, $p \geq 1$, $F \in F_s$ and $\varphi \in \Phi$. Then the below condition holds.

(A_1) *(f, S) have coincidence point.*

(A_2) *f and S has a common fixed point, if $ffv = fv$ and f is occasionally S-weakly commuting at v; Then f, S have a common fixed point.*

If $f = g$ and $S = T$ in Theorem 1.20 by using Remark 1.6, we have

Corollary 1.22. *Let $f : \tilde{N} \to \tilde{N}$ and $S : \tilde{N} \to CB(\tilde{N})$ be a maps on metric space $(\tilde{N}, d)$. Suppose (S, f) have strong common limit range property. Furthermore*

$$\tau + F(H^p(Sx, Sy) \leq F(\varphi(\Delta(x, y))),$$

where, $H(Sx, Sy) > 0$ and

$$\Delta(x,y) = \max\left\{\begin{array}{c} d^p(fx,Sy), d^p(fy,Sx), d^p(fx,fy), \\ \frac{d^p(fx,Sy)+d^p(fy,Sx)}{2}, \frac{d^p(fx,Sy)d^p(fy,Sx)}{1+d^p(fx,fy)}, \\ \frac{d^p(fy,Sx)d^p(fx,Sy)}{1+d^p(fx,fy)}, d^p(fx,Sy)d^p(fy,Sx) \end{array}\right\}.$$

Here, $\tau \in \boldsymbol{R}^+$, $p \geq 1$, $F \in F_s$ and $\varphi \in \Phi$. Then the below assumption holds.

(A_1) *(f, S) have coincidence point.*
(A_2) *f and S has a common fixed point, if $ffv = fv$ and f is occasionally S-weakly commuting at v; Then f, S have a common fixed point.*

Theorem 1.23. *Let $f, g : \tilde{N} \to \tilde{N}$ and $S, T : \tilde{N} \to CB(\tilde{N})$ be a maps on metric space $(\tilde{N}, d)$. Suppose that (S, g) and (T, f) have strong common limit range property. Moreover assume that*

$$H^p(Tx, Sy) \leq \alpha \frac{d^p(fx, Tx)d^p(fx, Sy) + d^p(gy, Sy)d^p(gy, Tx)}{1 + d^p(fx, Sy) + d^p(gy, Tx)}, \tag{1.14}$$

where, $0 < \alpha < 1$. Then

(A_1) *(f, T) have coincidence point.*
(A_2) *(g, S) have coincidence point.*
(A_3) *f and T has a common fixed point, if $ffv = fv$ and f is occasionally T-weakly commuting at v;*
(A_4) *g and S has a common fixed point, if $ggu = gu$ and g is occasionally S-weakly commuting at w ;*
(A_5) *if (A_3) and (A_4) holds. Then f, g, S and T have a common fixed point.*

Proof. Since $f, g : \tilde{N} \to \tilde{N}$ and $T, S : \tilde{N} \to CB(\tilde{N})$ have strong (CLR)-property, therefore there exists a sequence $\{\xi_n\}$ and $\{\zeta_n\}$ in $\tilde{N}$ and $\Omega_1, \Omega_2 \in CB(\tilde{N})$ such that,

$$\lim_{n\to\infty} T\xi_n = \Omega_1, \lim_{n\to\infty} S\zeta_n = \Omega_2 \text{ and } \lim_{n\to\infty} f\xi_n = \lim_{n\to\infty} g\zeta_n = fu = gv \in \Omega_1 \cap \Omega_2,$$

for some $u, v \in \tilde{N}$. Now, we show $gv \in Sv$, suppose $gv \notin Sv$, by putting $x = \xi_n, y = u$ in inequality (1.19), we have

$$H^p(T\xi_n, Sv) \leq \alpha \frac{d^p(f\xi_n, T\xi_n)d^p(f\xi_n, Sv) + d^p(gv, Sv)d^p(gv, T\xi_n)}{1 + d^p(f\xi_n, Sv) + d^p(gv, T\xi_n)}, \tag{1.15}$$

Applying limit, we have

$$H^p(\Omega_1, Sv) = 0.$$

By using Lemma 1.12

$$d^p(gv, Sv) \leq H^p(\Omega_1, Sv) = 0. \tag{1.16}$$

which is possible if $gv \in Sv$.

We show $fu \in Tu$, suppose $fu \notin Tu$. then putting $x = u, y = \zeta_n$ in inequality (1.19), we have

$$H^p(Tu, S\zeta_n) \leq \alpha \frac{d^p(fu, Tu)d^p(fu, S\zeta_n) + d^p(g\zeta_n, S\zeta_n)d^p(g\zeta_n, Tu)}{1 + d^p(fu, S\zeta_n) + d^p(g\zeta_n, Tu)}, \tag{1.17}$$

Taking limit, we have

$$H^p(Tu, \Omega_2) = 0.$$

Using Lemma 1.12, we have

$$H^p(fu, Tu) \leq H^p(Tu, \Omega_2). \tag{1.18}$$

Which is possible only if $fu \in Tu$.

After succeeding the similar lines of Theorem 1.16 we can obtained that S, T, f and g have common coupled fixed point. □

Theorem 1.24. *Let $f, g : \tilde{N} \to \tilde{N}$ and $S, T : \tilde{N} \to CB(\tilde{N})$ be a maps on metric space $(\tilde{N}, d)$. Suppose that (S, g) and (T, f) have strong common limit range property. Moreover assume that*

$$H^p(Tx, Sy) \leq \begin{cases} \alpha \dfrac{d^p(fx, Tx)d^p(fx, Sy) + d^p(gy, Sy)d^p(gy, Tx)}{d^p(fx, Sy) + d^p(gy, Tx)}, \\ if\ \Delta^* \neq 0, \\ 0, \\ if\ \Delta^* = 0. \end{cases}$$

Where $\Delta^* = d^p(fx, Sy) + d^p(gy, Tx)$, $0 < \alpha < 1$. Then

(A_1) *(f, T) have coincidence point.*
(A_2) *(g, S) have coincidence point.*
(A_3) *T and f has a common fixed point, if $ffv = fv$ and f is occasionally T-weakly commuting at v;*
(A_4) *S and g has a common fixed point, if $ggu = gu$ and g is occasionally S-weakly commuting at w ;*
(A_5) *if (A_3) and (A_4) holds. Then g, f, S and T have a common fixed point.*

Succeeding the steps of Theorem 1.23 we can obtained that S, T, f and g have common coupled fixed point.

If $f = g$, $T = S$ in Theorem 1.24 by using Remark 1.6, we get

Corollary 1.25. *Let $f : \tilde{N} \to \tilde{N}$ and $S : \tilde{N} \to CB(\tilde{N})$ be a maps on metric space $(\tilde{N}, d)$. Suppose that (S, f) have strong common limit range property. Furthermore*

$$H^p(Sx, Sy) \leq \begin{cases} \alpha \dfrac{d^p(fx, Sx)d^p(fx, Sy) + d^p(gy, Sy)d^p(fy, Sx)}{d^p(fx, Sy) + d^p(fy, Sx)}, \\ if\ \Delta^* \neq 0, \\ 0, \\ if\ \Delta^* = 0. \end{cases}$$

Where $\Delta^* = d^p(fx, Sy) + d^p(gy, Tx)$, $0 < \alpha < 1$. Then

(A_1) *(f, S) have coincidence point.*
(A_2) *S and f has a common fixed point, if $ffv = fv$ and f is occasionally S-weakly commuting at v. Then f, S have a common fixed point.*

If $T = S$, $f = g$ in Theorem 1.23. From Remark 1.6, we have

Corollary 1.26. *Let $f : \tilde{N} \to \tilde{N}$ and $S : \tilde{N} \to CB(\tilde{N})$ be a maps on metric space $(\tilde{N}, d)$. Suppose (S, f) have strong common limit range property. Furthermore*

$$H^p(Sx, Sy) \leq \alpha \frac{d^p(fx, Sx)d^p(fx, Sy) + d^p(fy, Sy)d^p(fy, Sx)}{1 + d^p(fx, Sy) + d^p(fy, Sx)}, \tag{1.19}$$

where, $0 < \alpha < 1$. Then

(A_1) *(f, S) have coincidence point.*
(A_2) *f and S has a common fixed point, if $ffv = fv$ and f is occasionally S-weakly commuting at v. Then S, f have a common fixed point.*

2. Application to system of integral and differential equation

Now, we study solutions of 2nd kind general nonlinear system of Fredholm integral equations given by

$$\begin{cases} x(t) = \phi(t) + \displaystyle\int_p^q Q_1(t, s, x(s))ds, t \in [p, q], \\ y(t) = \phi(t) + \displaystyle\int_p^q Q_2(t, s, y(s))ds, t \in [p, q], . \end{cases} \tag{2.1}$$

Let $\tilde{N} = C[p, q]$ be the set of all continuous function defined on $[p, q]$. Define $d : \tilde{N} \times \tilde{N} \to \mathbf{R}^+$, by

$$d(x, y) = \|x - y\|.$$

Where $\|x\| = \sup\{|x(t)| : t \in [p, q]\}$. Then $(\tilde{N}, d)$ is a complete d metric space on $\tilde{N}$. We give the following theorem.

Theorem 2.1. *Assume that the following assumptions hold*

(A_1) *$Q_j : [p, q] \times [p, q] \times \boldsymbol{R}^+ \to \boldsymbol{R}^+$, for $j = 1, 2$ and $\phi : \boldsymbol{R}^+ \to \boldsymbol{R}^+$ is continuous;*
(A_2) *there exist a continuous function $G : [p, q] \times [p, q] \to [0, \infty)$ such that,*

$$|Q_1(t, s, u) - Q_2(t, s, v)| \leq G(t, s)\gamma(|u - v|),$$

for each $t, s \in [p, q]$, $0 < \gamma < 1$,
(A_3) $\sup_{t,s \in [p,q]} \int_p^q |G(t, s)|ds \leq e^{-\tau}$ *for $\tau > 0$.*

Then the system of integral equations 2.1 has a common solution in $C([p, q])$.

Proof. Define $S, T : C([p, q]) \to C([p, q])$ by,

$$Sx(t) = \phi(t) + \int_p^q Q_1(t, s, x(s))ds, t \in [p, q].$$

$$Ty(t) = \phi(t) + \int_p^q Q_2(t, s, y(s))ds, t \in [p, q].$$

Now we have,

$$\begin{aligned}
d(Sx(t), Ty(t)) &= \sup_{t\in[p,q]} |Sx(t) - Ty(t)| \\
&\leq \sup_{t\in[p,q]} \int_p^q |Q_1(t, s, x(s)) - Q_2(t, s, y(s))|ds \\
&\leq \sup_{t\in[p,q]} \int_p^q G(t, s)\gamma(|x(s) - y(s)|)ds \\
&\leq \sup_{t\in[p,q]} \gamma(|x(t) - y(t)|) \sup_{t\in[p,q]} \int_p^q G(t, s)ds \\
&\leq \sup_{t\in[p,q]} \gamma(|x(t) - y(t)|)e^{-\tau} \\
&= \gamma(\|x(t) - y(t)\|)e^{-\tau} = \gamma(d(x(t), y(t)))e^{-\tau}.
\end{aligned}$$

By taking natural log to both side, we have

$$\tau + F(H^p(Sx, Ty) \leq F(\varphi(\Delta(x, y))).$$

Define $f(x) = g(x) = x$, $F(t) = ln(t)$, $\varphi(t) = \gamma t$, and $p = 1$ then by Theorem 1.20 the system (2.1) has a common solution in $\tilde{N}$. □

With the help of Theorem 1.20, one can also solve the following coupled system of nonlinear fractional ordered differential equations given by

$$\begin{cases} {}^cD^\beta u(t) + \hat{g_1}(v(t)) = 0, \ 1 < \beta \leq 2, \ t \in [0, 1], \\ {}^cD^\beta v(t) + \hat{g_2}(w(t)) = 0, \ 1 < \beta \leq 2, \\ u(0) = v(0) = a, \ u(1) = v(1) = b, \\ \text{where } a, b \text{ are constant.} \end{cases} \tag{2.2}$$

Where $\hat{g_1}, \hat{g_2} : [0, 1]\times[0, \infty) \to [0, \infty)$. Then the equivalent system of integral equations corresponding to (2.2) is given by

$$\begin{cases} u(t) = \phi(t) + \int_0^1 \mathcal{G}(t, s)\hat{g_1}(v(s)ds, \ t \in [0, 1], \\ v(t) = \phi(t) + \int_0^1 \mathcal{G}(t, s)\hat{g_2}(w(s)ds, \ t \in [0, 1], \end{cases} \tag{2.3}$$

Where $\mathcal{G}(t, s)$ is the Green's function

$$\mathcal{G}(t, s) = \begin{cases} \dfrac{(t - s)^{\beta-1} - t(1 - s)^{\beta-1}}{\Gamma(\beta)}, & 0 \leq s \leq t \leq 1, \\ \dfrac{-t(1 - s)^{\beta-1}}{\Gamma(\beta)}, & 0 \leq t \leq s \leq 1, \end{cases}$$

and continuous on $[0,1] \times [0,1]$. Moreover $\sup_{t\in[0,1]} \int_0^1 |\mathcal{G}(t,s)|ds \leq 1$. Further, using $Q(t,s,x(s)) = \mathcal{G}(t,s)\hat{g_1}(v(s)$ etc. Then the system (2.3)become

$$\begin{cases} x(t) = \phi(t) + \displaystyle\int_0^1 Q_1(t,s,x(s))ds, \ t \in [0,1], \\ y(t) = \phi(t) + \displaystyle\int_0^1 Q_2(t,s,y(s))ds, \ t \in [0,1]. \end{cases} \tag{2.4}$$

Clearly by Theorem 1.20 the System(2.4) has a solution, which is the corresponding solution of the system of nonlinear fractional differential equation(2.2).

3. Conclusion

In this work, we introduced strong common limit range property for couples (f,g) and (S,T) to relaxed the conditions of completeness (closedness), the containment of the range of the mappings, convexity of the underline space and continuity of the mappings and by means of this new concept we established common fixed point results for hybrid pair via (F,φ)-contraction and rational type contraction conditions. Further, using the established results existence of solution to the system of integral and differential equations are also discussed. We provided example where the main theorem is applicable but relevant classic result in literature fail to have a common fixed point.

Acknowledgments

The third author would like to thank Prince Sultan University for funding this work through research group Nonlinear Analysis Methods in Applied Mathematics (NAMAM) group number RG-DES-2017-01-17.

Conflict of interest

The authors declare that they have no competing interest.

References

1. T. Abdeljawad, *Meir-Keeler $\alpha-$contractive fixed and common fixed point theorems*, Fixed Point Theory A., **2013** (2013), 1–10.
2. Afrah Ahmad Noan Abdou, *Common fixed point theorems for hybrid contractive pairs with the (CLR)-property*, Fixed Point Theory A., **2015** (2015), 1–9.
3. Afrah Ahmad Noan Abdou, *Common fixed point results for multi-valued mapping with some examples*, J. Nonlinear Sci. Appl., **9** (2016), 787–798.
4. S. Chandok, D. Kumar, *Some common fixed point results for rational type contraction mappings in complex valued metric spaces*, Journal of operators, **2013** (2013), 1–6.
5. M. Imdad, S. Chauhan, A. H. Soliman, et al. *Hybrid fixed point theorems in symmetric spaces via common limit range property*, Demonstratio Mathematica, **47** (2014), 949–962.

6. T. Kamran, *Coincidence and fixed points for hybrid strict contractions*, J. Math. Anal. Appl., **299** (2004), 235–241.
7. D. K. Patel, T. Abdeljawad, D. Gopal, *Common fixed points of generalized Meir-Keeler $\alpha-$contractions*, Fixed Point Theory A., **2013** (2013), 1–16.
8. Y. Liu, J. Wu, Z. Li, *Common fixed points of single-valued and multivalued maps*, Int. J. Math. Math. Sci., **2005** (2005), 3045–3055.
9. M. Samreen, T. Kamran, E. Karapinar, *Fixed point theorems for hybrid mappings*, Sci. World J., **2015** (2015), 1–7.
10. H. K Nashine, M. Imdad, M. Ahmadullah, *Common fixed point theorems for hybrid generalized (F, φ)-contractions under common limit range property with applications*, Ukr. math. J., **69** (2018), 1784–1804.
11. M. Shoaib, M. Sarwar, *Multivalued fixed Point theorems for generalized contractions and their applications*, J. Math., **2016** (2016), 1–8.
12. M. Shoaib, M. Sarwar, T. Abdeljawad, *Hybrid Coupled Fixed Point Theorems in Metric Spaces with Applications*, J. Funct. Space., **2019** (2019), 1–15.
13. W. Sintunavarat, P. Kumam, *Common fixed point theorems for a pair of weakly compatible mappings in fuzzy metric spaces*, J. Appl. Math., **2011** (2011), 1–14.
14. A. F. Roldán-López-de-Hierro, W. Sintunavarat, *Common fixed point theorems in fuzzy metric spaces using the CLRg property*, Fuzzy Set. Syst., **282** (2016), 13–142.
15. M. Sgroi, C. Vetro, *Multivalued F-contractions and the solution of certain functional and integral equations*, Filomat, **27** (2013), 1259–1268.
16. S. Shukla, S. Radenović, *Some common fixed point theorems for F-contraction type mappings on 0-complete partial metric spaces*, J. Math., **2013** (2013), 1–7.
17. D. Wardowski, *Fixed point of a new type of contractive mapping in complete metric space*, Fixed point theory A., **2012** (2012), 1–6.
18. D. Wardowski, N. V. Dung, *Fixed points of f-weak contractions on complete metric spaces*, Demonstratio Mathematica, **47** (2014), 146–155.
19. L. S. Dube, *Theorem on common fixed points of multi-valued mappings*, Ann. Soc. Sci. Bruxelles, Ser. 1., **89** (1975), 463–468.
20. M. Samreen, T. Kamran, E. Karapinar, *Fixed point theorems for hybrid mapping*, The Scientific World J., **2015** (2015), 1–7.