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Research article

Polychromatic colorings of hypergraphs with high balance

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Abstract: Let m be a positive integer and $C = \{1, 2, \ldots, m\}$ be a set of m colors. A polychromatic m-coloring of a hypergraph is a coloring of its vertices in such a way that every hyperedge contains at least one vertex of each color in C. This problem is a generalization of 2-colorings of hypergraphs and has close relations with the longest lifetime problem for a wireless sensor network, cover decompositions problem of hypergraphs and vertex cover problem of hypergraphs. In this paper, a main work is to find the maximum m that a hypergraph H, with n hyperedges, admits a polychromatic m-coloring such that each color appears at least k times on each hyperedge. A $2\ln n$ approximation to the number is obtained when k is a fixed positive integer. For the case that $k = O(n\ln n)$, there exists an $O(\ln n)$ approximation algorithm; for the case that $k = \omega(n\ln n)$, there exists a $(2 + \sqrt{3})k$ approximation algorithm.

Keywords: hypergraph; polychromatic coloring; cover decomposition; balanced coloring; probabilistic method
Mathematics Subject Classification: 05C15, 05C65

1. Introduction

This work is inspired by recent developments concerning hypergraph vertex cover, disjoint edge cover of hypergraph, the longest lifetime problem for a wireless sensor network (WSN) with battery-limited sensors. A *hypergraph* $H = (V, E)$ consists of a ground set V of *vertices* and a collection E of *hyperedges*, where each hyperedge $f \in E$ is a subset of V. Let m be a positive integer. A *polychromatic m-coloring* of a hypergraph H is a coloring of vertices of H with m colors such that every hyperedge contains at least one vertex of each color. It is a generalization of 2-colorings of a hypergraph. Obviously, in a polychromatic coloring of hypergraph H, each color class is exactly a vertex cover of H. The *polychromatic number* of a hypergraph H is the maximum number m that H admits a polychromatic m-coloring and is denoted by $p(H)$.

The *rank* of a hypergraph H is $R(H) = \max_{f\in E}|f|$, the *anti-rank* of H is $S(H) = \min_{f\in E}|f|$. If $R(H) = S(H) = d$, that is, the size of every hyperedge in H is d, we say that the hypergraph H is a *d-uniform hypergraph*. The *degree* of a vertex $v \in V(H)$ is the number of hyperedges containing v in H, and is denoted by $d_H(v)$ or simply by $d(v)$. The *maximum degree*, *minimum degree*, of H is $\Delta(H) = \max_{v\in V(H)} d_H(v)$, $\delta(H) = \min_{v\in V(H)} d_H(v)$, respectively. A hypergraph in which each vertex has degree d is called a *d-regular hypergraph*. Throughout this paper, we denote the class of *d-regular d-uniform hypergraphs* by $\mathcal{H}_d$. Let f be a hyperedge in a hypergraph with anti-rank S. The operation *shrinking* f means to replace it with some $f' \subset f$. This operation is useful when considering polychromatic colorings of hypergraphs with the probabilistic method because it bounds the dependence degree. We could shrink each hyperedge f_j with $|f_j| > S$ to f'_j such that $|f'_j| = S$. Clearly, undoing shrinking preserves the property of being a hyperedge containing m colors. (For each hyperedge f'_j, its coloring is dependent on the colorings of its incident hyperedges. So its dependence degree is at most $S(\Delta - 1)$.) Since each hypergraph H with anti-rank $S = 1$ has $p(H) = 1$, we focus on the hypergraphs with anti-rank $S \geq 2$ throughout this paper.

A subfamily E_i of E in a hypergraph $H = (V, E)$ is called a *cover* in H if $\cup_{f\in E_i} f = V$. A *cover m-decomposition* of a hypergraph H is a partition of E into m covers in H, i.e. $E = \uplus_{i=1}^m E_i$ and $\cup_{f\in E_i} f = V$. The maximum integer m such that the hypergraph H admits a cover m-decomposition is called the *cover-decomposition number* of H and denoted by $\mathrm{cd}(H)$. The problem to determine the cover decomposition numbers of hypergraphs is called the maximum disjoint set cover problem (DSCP), which is NP-complete [8]. A hypergraph H can model a collection of sensors, with each hyperedge $f \in E$ corresponding to a sensor which can monitor the vertices (targets) in $f \subseteq V$. Since monitoring all vertices (targets) of V takes a cover in H, $\mathrm{cd}(H)$ is exactly the longest lifetime for a WSN corresponding to the hypergraph H if each sensor can only be turned on for a single time unit ([3, 7]).

Let $H = (V, E)$ be a hypergraph with $V = \{x_1, x_2, \ldots, x_n\}$ and $E = (f_1, f_2, ..., f_m)$. The *dual* of H is a hypergraph H^* whose vertices $\hat{f}_1, \hat{f}_2, ..., \hat{f}_m$ correspond to the hyperedges of H, and whose hyperedges $\hat{x}_i = \{\hat{f}_j | \ x_i \in f_j \text{ in } H\}$, $i = 1, 2, \ldots, n$. Clearly, $(H^*)^* = H$ and $\Delta(H^*) = R(H)$, $\delta(H^*) = S(H)$, $R(H^*) = \Delta(H)$, $S(H^*) = \delta(H)$.

Let $p_k(H^*)$ denote the maximum m such that H^* admits a polychromatic m-coloring satisfying that each color appears at least k times on each hyperedge and $\mathrm{cd}_k(H)$ denote the maximum m such that H has a cover m-decomposition satisfying that each cover contains at least k incident edges of each vertex. Clearly $\mathrm{cd}_k(H) = p_k(H^*)$.

Early in the 1970s, Erdős and Lovász [10] considered the existence of polychromatic colorings of hypergraphs and showed that, for each integer $m \geq 2$, every hypergraph with anti-rank $S \geq m$ and each of whose hyperedges intersecting at most $m^{S-1}/(4(m-1)^S)$ other hyperedges is polychromatic m-colorable. (The original version is formed on S-uniform hypergraphs. Via the operation "shrinking", it is easy to see that it could be stated with a slight generalization as above.) Moreover, for lattice point hypergraphs, Erdős and Lovász [10] gave a stronger version in which the existence of polychromatic colorings with high balance is guaranteed: For $\epsilon > 0, m > 2, n > 1$, there is an $r_0 = r_0(m, \epsilon)$ such that if T is any set of lattice points in the n-dimensional space with $|T| = S > r_0$ then the lattice points can be m-colored so that each set $T + \mathbf{a}$ obtained by translating T with an integer vector $\mathbf{a}$ contains at least $(1 - \epsilon)\frac{S}{m}$ points of any given color.

Henning and Yeo [13] considered polychromatic colorings of hypergraphs in $\mathcal{H}_d$ and showed every hypergraph $H \in \mathcal{H}_d$ ($d \geq 2$) has a polychromatic m-coloring for each $m \leq \frac{d}{\ln(d^3)}$. Using a randomized algorithm, Bagaria, Pananjady and Vaze [3] gave a $\ln n$ approximation result in polynomial time that each hypergraph H with n hyperedges and anti-rank S has $p(H) \geq S(1 - o(1))/\ln n$. For hypergraphs H with maximum degree at most Δ and anti-rank at least S, Li and Zhang [17] gave a lower bound $\lfloor S/\ln(c\Delta S^2)\rfloor$ for the polychromatic number of hypergraphs, where $0 < c = c(\Delta, S) < 1.5582 < e$, and, for polychromatic colorings with high balance, they showed that H has a polychromatic m-coloring such that every hyperedge in H contains at least $\lfloor \ln(e\Delta S^2)\rfloor$ vertices of each color for each $m \leq \frac{S}{\ln(e\Delta S^2)}$.

Given a plane graph G, a *face hypergraph* $\mathcal{F}(G)$ based on G is one whose vertex set is $V(G)$ and whose hyperedges are the vertex sets of G's faces. By virtue of the four-color theorem, Mohar and Škrekovski [19] proved that every simple plane graph is polychromatic 2-colorable. Later Bose et al. [6] proved this result without the use of the four-color theorem. For a family $\mathcal{H}$ of face hypergraphs with anti-rank S, Alon et al. [1] showed that $\lfloor \frac{3S-5}{4}\rfloor \leq \min_{H\in\mathcal{H}}\{p(H)\} \leq \lfloor \frac{3S+1}{4}\rfloor$. A *factor hypergraph* $\mathcal{H}_{\mathcal{F}}(G)$ based on G is one whose vertex set is $E(G)$ and whose hyperedges are the edge sets of G's F-factors. Axenovich et al. [2] determined the polychromatic number for the 1-factor hypergraph $\mathcal{H}_1(K_n)$ and bounded the polychromatic number for the 2-factor hypergraph $\mathcal{H}_2(K_n)$ and the Hamilton-cycle-factor hypergraph $\mathcal{H}_{C_n}(K_n)$.

On 2-colorings of hypergraphs, Vishwanathan [21] showed that, for each integer $d \geq 4$, every hypergraph in $\mathcal{H}_d$ is 2-colorable. The bound for d is sharp noting that Fano plane is in $\mathcal{H}_3$ but not 2-colorable. Henning and Yeo [13] discussed 2-coloring with high balance for the hypergraphs in $\mathcal{H}_d$ and observed that, for each integer $k \geq 2$, every hypergraph $H \in \mathcal{H}_d$ has a 2-coloring such that each hyperedge contains at least $k + 1$ vertices of each color if one of the following conditions holds: (i) $k \leq d/2 - \sqrt{d\ln(d\sqrt{2e})}$; ($ii$) $d \geq 2k + 3\sqrt{k\ln(k)} + 44.03$; ($iii$) $d \geq 2k + 4\sqrt{k\ln(k)} + 14.04$. Beck and Fiala [4] showed that every hypergraph with maximum degree $\Delta \geq 2$ has a 2-coloring such that each hyperedge $f \in E$ contains at least $|f|/2 - \Delta + 1$ vertices of each color. Chen, Du and Meng [9] gave a sufficient condition, each hyperedge meets at most $2^S/(e(S + 1)) - 2$ other hyperedge, to show a hypergraph with anti-rank $S \geq 4$ having a 2-coloring such that each color appears at least two times on each hyperedge.

There is much literature on cover decomposition number of (multi)graphs, using edge coloring method of (multi)graph. Gupta [11] showed every multigraph has a cover decomposition into at least $\min_{v\in V(G)}\{d(v) - \mu(v)\}$ covers, where $\mu(v) = \max_{u\in N(v)} |E(uv)|$. In [12], Gupta confirmed that each multigraph with minimum degree δ has a cover $\lfloor(3\delta + 1)/4\rfloor$-decomposition. Hilton [14] discussed cover decomposition of multigraphs such that each cover contains at least j incident edges of each vertex. Let $V_k = \{v \in V : k|d(v)\}$. Hilton and de Werra [15] showed every graph G with V_k independent has a cover m-decomposition such that each cover contains either $\lceil d(v)/m\rceil$ or $\lfloor d(v)/m\rfloor$ incident edges of each vertex $v \in V(G)$. Zhang and Liu [25] extended the conclusion to graphs G with $G[V_k]$ forests and, furthermore, peelable graphs G. Let g be a positive integer function defined on $V(G)$ such that $g(v) \leq d(v)$ for each $v \in V(G)$. Song and Liu [20] considered DSCP of multigraphs satisfying that each cover contains at least $g(v)$ incident edges for each vertex $v \in V(G)$, g-cover decomposition for short, and obtained a result with a form analogous to Gupta's one in [11]. Ma and Zhang [18] determined $\mathrm{cd}_g(G)$ for a class of graphs which extends the class of peelable graphs. Xu and Liu [23] discussed DSCP for multigraphs with $2 \leq \delta \leq 5$. Zhang and Zhang [26] considered DSCP for nearly bipartite graphs. A graph G is called g-critical on DSCP, if $\mathrm{cd}_g(G + uv) \geq \mathrm{cd}_g(G)$ for each pair of nonadjacent vertices u, v. Xu and Liu [22] gave some properties of 1-critical graphs on DSCP. Zhang [24] described completely disconnected g-critical graphs.

Bollobás et al. [5] researched cover decompositions of hypergraphs. We state their result in dual version: Let $\mathcal{H}$ be a family of hypergraphs with maximum Δ and anti-rank S. Then

(i) for all Δ, S and each $H \in \mathcal{H}$, $p(H) \geq S/(\ln \Delta + O(\ln \ln \Delta))$;
(ii) for all $\Delta \geq 2$, $S \geq 1$, $\min_{H \in \mathcal{H}}\{p(H)\} \leq \max\{1, O(S/\ln \Delta)\}$;
(iii) for each sequence $\Delta, S \to \infty$ with $S = \omega(\ln \Delta)$, $\min_{H \in \mathcal{H}}\{p(H)\} \leq (1 + o(1))S/\ln(\Delta)$.

In Section 2, we will prove the following result, which extends the result due to Bagaria, Pananjady and Vaze [3] to polychromatic colorings with high balance.

Theorem 1.1. *Let n, S, k be three positive integers and H be a hypergraph with n hyperedges and anti-rank S.*

(i) If k is a fixed positive integer, then $p_k(H) \geq S(1 - o(1))/(2 \ln n)$.
(ii) If $k = O(\ln(n \ln n))$, then $p_k(H) \geq S/O(\ln n)$.
(iii) If $k = \omega(\ln(n \ln n))$, then $p_k(H) \geq S(1 - o(1))/((2 + \sqrt{3})k)$.

2. The proof of the main result

Within the proof, we shall make use of the following classical tool of the probabilistic method–the Chernoff Bound. Let $X_1, X_2, \ldots, X_s$ be mutually independent Bernoulli variables such that $X_i = 1$ with probability p and $X_i = 0$ with probability $1 - p$. Define $X = \sum_{i=1}^{s} X_i$. Clearly, $E(X) = \sum_{i=1}^{s} E(X_i) = sp$.

Theorem 2.1. *[16](The Chernoff Bound) For any $0 \leq t \leq sp$, $Pr(X > sp + t) < e^{-\frac{t^2}{3sp}}$ and $Pr(X < sp - t) < e^{-\frac{t^2}{2sp}} \leq e^{-\frac{t^2}{3sp}}$.*

The proof of Theorem 1.1

Proof. By virtue of the operation *shrinking*, we can always assume that H is S-uniform.

Let n be large enough and $C = \{1, 2, \ldots, h\}$ be a color set. Color the vertices of H in such a way that each vertex is independently and uniformly assigned a color of C. For $f \in E$, $c \in C$, define $A_{f,c}$ to be the "bad" event that color c appears at most $k - 1$ times on hyperedge f. We want to avoid these "bad" events and achieve a polychromatic m_k-coloring with $m(\leq h)$ as large as possible. If we can show that with positive probability, each of m colors appears at least k times on every hyperedge, then we will be done. Let $X_{f,c}$ be the number of vertices colored with c on the hyperedge f. Then $E(X_{f,c}) = S/h$. Clearly, for each pair of $f \in E$ and $c \in C$,

$$Pr(A_{f,c}) = Pr(X_{f,c} < k) = Pr(X_{f,c} < \frac{S}{h} - (\frac{S}{h} - k))$$

and $\frac{S}{h} - k \leq \frac{S}{h}$. If $\frac{S}{h} - k \geq 0$, by the Chernoff Bound, the probability of event $A_{f,c}$ is the following.

$$Pr(A_{f,c}) < e^{-\frac{(\frac{S}{h}-k)^2}{\frac{2S}{h}}} \tag{2.1}$$

$$= e^{-(\frac{S}{2h}-k+\frac{hk^2}{2S})}. \tag{2.2}$$

An invalid color is one which appears at most $k-1$ times in some hyperedge of H. Let L be the number of invalid colors in a random uniform h-coloring of H as described as above. Then the expectation of L

$$E(L) \leq \sum_{c\in C}\sum_{f\in E} Pr(A_{f,c}) \leq hn \max_{c\in C, f\in E} Pr(A_{f,c}).$$

Next, we discuss three cases according to the value of k, corresponding to which the function h will vary.

(i) *k is a fixed positive integer.*

Set $h = \frac{S}{2\ln(n\ln n)}$. Clearly, $\frac{S}{h} - k \geq 0$ as n is large enough. By Inequality (2), for each pair of $f \in E$ and $c \in C$,

$$Pr(A_{f,c}) < (n\ln n)^{-1} e^k e^{-\frac{k^2}{4\ln(n\ln n)}} < (n\ln n)^{-1} e^k.$$

Then

$$E(L) < hn(n\ln n)^{-1} e^k = he^k/\ln n.$$

Thus, with positive probability, we can get a coloring of H with at least $h - E(L)$ colors such that each of the colors appears at least k times on each hyperedge of H. That is to say,

$$\begin{aligned} p_k(H) &\geq h - E(L) \\ &> h(1 - \frac{e^k}{\ln n}) \\ &= \frac{S}{2\ln(n\ln n)}(\frac{\ln n - e^k}{\ln n}) \\ &= \frac{S}{2\ln n}\cdot\frac{\ln n - e^k}{\ln(n\ln n)} \\ &= \frac{S}{2\ln n}(1 - \frac{\ln\ln n + e^k}{\ln(n\ln n)}) \\ &= \frac{S}{2\ln n}(1 - o(1)) \end{aligned}$$

(ii) $k = O(\ln(n\ln n))$.

Then there exists a positive constant, say d, such that $k \leq d\ln(n\ln n)$ for large enough n. Set $h = \frac{S}{(d+\sqrt{2d+1}+1)\ln(n\ln n)}$. Clearly, $\frac{S}{h} - k > 0$. By Inequality (1), for each pair of $f \in E$ and $c \in C$,

$$\begin{aligned} Pr(A_{f,c}) &< e^{-\frac{(\frac{S}{h}-k)^2}{\frac{2S}{h}}} \\ &\leq e^{-\frac{h(\frac{S}{h}-d\ln(n\ln n))^2}{2S}} \\ &= e^{-(\frac{S}{2h} - d\ln(n\ln n) + \frac{hd^2\ln^2(n\ln n)}{2S})} \end{aligned}$$

$$= e^{-(\frac{(d+\sqrt{2d+1}+1)\ln(n\ln n)}{2} - d\ln(n\ln n) + \frac{d^2\ln^2(n\ln n)}{2(d+\sqrt{2d+1}+1)\ln(n\ln n)})}$$
$$= e^{-(\frac{(d+\sqrt{2d+1}+1)}{2} - d + \frac{d^2}{2(d+\sqrt{2d+1}+1)})\ln(n\ln n)}$$
$$= e^{-\ln(n\ln n)}$$
$$= (n\ln n)^{-1}.$$

So $E(L) < hn(n\ln n)^{-1} = h/\ln n$ and then there is

$$\begin{aligned}
p_k(H) &\geq h - E(L) \\
&> h(1 - \frac{1}{\ln n}) \\
&= \frac{S}{(d+\sqrt{2d+1}+1)\ln(n\ln n)}(\frac{\ln n - 1}{\ln n}) \\
&= \frac{S}{(d+\sqrt{2d+1}+1)\ln n}(1 - \frac{\ln\ln n + 1}{\ln(n\ln n)}) \\
&= \frac{S}{(d+\sqrt{2d+1}+1)\ln n}(1 - o(1)) \\
&= \frac{S}{O(\ln n)}
\end{aligned}$$

(iii) $k = \omega(\ln(n\ln n))$.
In this case, set $h = \frac{S}{(2+\sqrt{3})k}$. Clearly, $\frac{S}{h} - k > 0$. By Inequality (1), for each pair of $f \in E$ and $c \in C$,

$$\begin{aligned}
Pr(A_{f,c}) &< e^{-\frac{(\frac{S}{h}-k)^2}{\frac{2S}{h}}} \\
&= e^{-\frac{((1+\sqrt{3})k)^2}{2(2+\sqrt{3})k}} \\
&= e^{-k} < e^{-\ln(n\ln n)} \\
&= (n\ln n)^{-1}.
\end{aligned}$$

Then $E(L) < h/\ln n$ and

$$\begin{aligned}
p_k(H) &\geq h - E(L) \\
&> h(1 - \frac{1}{\ln n}) \\
&= \frac{S(1 - o(1))}{(2+\sqrt{3})k}
\end{aligned}$$

□

3. Concluding remarks

Bagaria, Pananjady and Vaze [3] gave the following result for hypergraphs with n hyperedges.

Theorem 3.1. *[3] Let H be a hypergraph with n hyperedges and anti-rank S. Then $p(H) \geq S(1 - o(1))/\ln n$.*

From the proof of Theorem 1.1 (ii), we can deduce the following result.

Corollary 3.2. *Let n, S, k be three positive integers and d be a positive real. Let H be a hypergraph with n hyperedges and anti-rank S. If $k \leq d\ln(n\ln n))$, then $p_k(H) \geq \frac{S}{(d+\sqrt{2d+1}+1)\ln n}(1 - o(1))$. In particular, if $k \leq \ln(n\ln n))$, then $p_k(H) \geq \frac{S}{(2+\sqrt{3})\ln n}(1 - o(1))$.*

Let A be a nonempty set. An *equitable q-partition* of A is a collection $A_1, A_2, \ldots, A_q$ such that, for each $1 \leq i < j \leq q$, $A_i \cap A_j = \emptyset$, $||A_i| - |A_j|| \leq 1$ and $\cup_{1\leq i\leq q} A_i = A$. The operation *equitable q-splitting* a hyperedge f in a hypergraph means to replace f with an equitable q-partition of f. Let H be a hypergraph with n hyperedges and anti-rank S. Do an equitable k-splitting for each hyperedge of H and denote the resulting hypergraph by H_k. Clearly, H_k has kn hyperedges and $S(H_k) = \lfloor S/k \rfloor$. By Theorem 3.1, there is $p(H_k) \geq \lfloor \frac{S}{k} \rfloor (1 - o(1))/\ln(kn)$. It is easy to see that a polychromatic m-coloring of H_k is corresponding to a polychromatic m_k-coloring of H. So undoing equitable k-splitting could get a lower bound for $p_k(H)$, which is at most $S(1 - o(1))/(k\ln(kn))$. Obviously, for each $k \geq 2$, the lower bound shown in Theorem 1.1 is better.

By the dual relationship of H and H^*, we have the following result on cover decomposition of a hypergraph with high balance.

Theorem 3.3. *Let n, δ, k be three positive integers and H be a hypergraph with n vertices and minimum degree δ.*

(i) If k is a fixed positive integer, then $\mathrm{cd}_k(H) \geq \delta(1 - o(1))/(2\ln n)$.
(ii) If $k = O(\ln(n\ln n))$, then $\mathrm{cd}_k(H) \geq \delta/O(\ln n)$.
(iii) If $k = \omega(\ln(n\ln n))$, then $\mathrm{cd}_k(H) \geq \delta(1 - o(1))/((2 + \sqrt{3})k)$.

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Conflict of interest

All authors declare that there is no conflict of interest.

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